Overview of TMDPDFs and quasi-TMDPDFs

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Outline

Introduction: Collinear vs TMD factorization

Collinear vs TMD factorization

- Example process: Drell-Yan production $pp \to Z/\gamma^* \to l^+ l^-$

$$x_{a,b} = \frac{Q e^{\pm Y}}{E_{\rm cm}}$$

- Collinear factorization:

$$\frac{d\sigma_{pp}}{dQ^2 dY} = \sum_{i,j} \hat{\sigma}_{ij}(Q^2, \mu) \times f_i(x_a, \mu) f_j(x_b, \mu) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{Q}\Big)$$

  - Nonperturbative physics absorbed in PDF $f_i(x, \mu)$
  - Short-distance physics absorbed in $\hat{\sigma}_{q\bar{q}}$

Collinear vs TMD factorization

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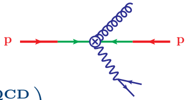
- Collinear factorization for $q_T \sim Q$



$$\frac{\mathrm{d}\sigma_{pp}}{\mathrm{d}Q^2 \mathrm{d}Y \mathrm{d}^2\vec{q}_T} = \sum_{i,j} \hat{\sigma}_{ij}(Q^2, \vec{q}_T, \mu) \times f_i(x_a, \mu) f_j(x_b, \mu) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{Q}, \frac{\Lambda_{\rm QCD}}{q_T}\Big)$$

  - $q_T \sim Q$ is perturbative scale $\to$ captured in $\hat{\sigma}_{q\bar{q}}$

Collinear vs TMD factorization

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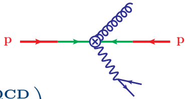

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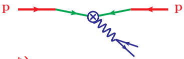
- TMD factorization for $q_T \ll Q$



$$\frac{\mathrm{d}\sigma_{pp}}{\mathrm{d}Q^2 \mathrm{d}Y \mathrm{d}^2\vec q_T} = \sum_{q} H_{q\bar q}(Q^2, \mu) \int \mathrm{d}^2\vec k_T \, f_q(x_a, \vec q_T - \vec k_T, \mu, \zeta) f_{\bar q}(x_b, \vec k_T, \mu, \zeta) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{Q}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)$$

$Y-$term
$\to$ see Ted's talk

  - q_T is nonperturbative scale $\to$ captured in TMDPDF $f_i(x, \vec q_T, \mu, \zeta)$
  - Vectorial nature of $\vec q_T$ $\to$ convolution structure

Collinear vs TMD factorization

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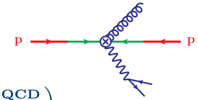

- Collinear factorization for $q_T \sim Q$

$$\frac{d\sigma_{pp}}{dQ^2 dY d^2\vec{q}_T} = \sum_{i,j} \hat{\sigma}_{ij}(Q^2,\vec{q}_T,\mu) \times f_i(x_a,\mu) f_j(x_b,\mu) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{Q}, \frac{\Lambda_{\rm QCD}}{q_T}\Big)$$

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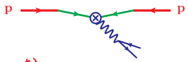
- TMD factorization for $q_T \ll Q$



$$\frac{d\sigma_{pp}}{dQ^2 dY d^2\vec{q}_T} = \sum_{q} H_{q\bar{q}}(Q^2,\mu) \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_q(x_a,\vec{b}_T,\mu,\zeta) f_{\bar{q}}(x_b,\vec{b}_T,\mu,\zeta)$$
$$+ \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{Q}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)$$

$Y-$term
$\to$ see Ted's talk

  - Convolution avoided in impact parameter space

Evolution of PDFs and TMDPDFs

Collinear PDFs:

- PDF obey DGLAP evolution: $$\frac{\mathrm{d} f_i(x,\mu)}{\mathrm{d}\ln\mu} = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\Big(\frac{x}{z},\mu\Big) f_j(z,\mu)$$
- Scale evolution is perturbative as long as $\mu \gg \Lambda_{\rm QCD}$

TMDPDFs:

- μ evolution: $$\frac{\mathrm{d} f_i(x,\vec b_T,\mu,\zeta)}{\mathrm{d}\ln\mu} = \gamma_\mu^i(\mu,\zeta) f_i(x,\vec b_T,\mu,\zeta)$$

Evolution of PDFs and TMDPDFs

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TMDPDFs:

- μ evolution: $$\frac{\mathrm{d} f_i(x,\vec b_T,\mu,\zeta)}{\mathrm{d}\ln\mu} = \gamma_\mu^i(\mu,\zeta) f_i(x,\vec b_T,\mu,\zeta)$$
- Collins-Soper evolution: $$\gamma_\mu^i(\mu,\zeta) = 2\Gamma^i_{\rm cusp}[\alpha_s(\mu)]\ln\frac{\mu^2}{\zeta} + \gamma_\mu^i[\alpha_s(\mu)]$$

$$\frac{\mathrm{d} f_i(x,\vec b_T,\mu,\zeta)}{\mathrm{d}\ln\zeta} = \frac{1}{2}\gamma_\zeta^i(b_T,\mu) f_i(x,\vec b_T,\mu,\zeta)$$

$$K(b_T,\mu) \equiv \frac{1}{2}\gamma_\zeta(b_T,\mu)$$

$$\frac{\mathrm{d}\gamma_\zeta^i(b_T,\mu)}{\mathrm{d}\ln\mu} = -2\Gamma^i_{\rm cusp}[\alpha_s(\mu)]$$

$$\gamma_K(\mu) \equiv 2\Gamma_{\rm cusp}[\alpha_s(\mu)]$$

- Closed solution for Collins-Soper kernel: $$\gamma_\zeta^i(b_T,\mu) = -2\int_{1/b_T}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(1/b_T)]$$
  - CS evolution is *nonperturbative* if $b_T \gtrsim \Lambda_{\rm QCD}^{-1}$!

Determining PDFs and TMDPDFs

Collinear PDFs:

- PDF obey DGLAP evolution: $$\frac{\mathrm{d} f_i(x,\mu)}{\mathrm{d}\ln\mu} = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\Big(\frac{x}{z},\mu\Big) f_j(z,\mu)$$
- Solve DGLAP evolution to evolve PDF $f_i(x,\mu_0)$, extracted at reference scale μ_0 (e.g. from lattice: $\mu_0 = \mathcal{O}(2\ \mathrm{GeV})$, to PDF $f_i(x,\mu)$ at desired scale $\mu \to$

TMDPDFs:

- Extract TMDPDFs at reference scales (μ_0, ζ_0)
- Evolve to desired scales (μ, ζ):

$$f_i(x,\vec{b}_T,\mu,\zeta) = f_i(x,\vec{b}_T,\mu_0,\zeta_0) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i(\mu',\zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu,b_T) \ln\frac{\zeta}{\zeta_0}\right]$$

  - Requires nonperturbative knowledge of $f_i(x,\vec{b}_T,\mu_0,\zeta_0)$
  - and nonperturbative knowledge of $\gamma_\zeta^i(\mu,b_T)$!

Determining PDFs and TMDPDFs

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TMDPDFs:

- Extract TMDPDFs at reference scales (μ_0,ζ_0)
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$$f_i(x,\vec{b}_T,\mu,\zeta) = f_i(x,\vec{b}_T,\mu_0,\zeta_0) \exp\bigg[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i(\mu',\zeta_0)\bigg] \exp\bigg[\frac{1}{2}\gamma_\zeta^i(\mu,b_T) \ln\frac{\zeta}{\zeta_0}\bigg]$$

  - Requires nonperturbative knowledge of $f_i(x,\vec{b}_T,\mu_0,\zeta_0)$
  - and nonperturbative knowledge of $\gamma_\zeta^i(\mu,b_T)$!
- *Remark:* For $\Lambda_{\mathrm{QCD}} \ll b_T^{-1} \ll Q$, TMDPDF are perturbatively calculable in terms of PDF $\rightarrow$ evolution then serves to resum large logarithms $\ln(b_T\mu) \sim \ln(Q/q_T)$
- In this regime, nonperturbative $\gamma_\zeta^i(\mu,b_T)$ is a major source of theory uncertainty

Definitions of TMDPDFs

TMD factorization revisited

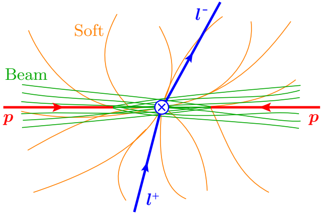
- TMD factorization in more detail for $pp \to Z/\gamma^* \to l^+ l^-$ (suppressing scales):

$$\frac{d\sigma_{pp}}{dQ^2 dY d^2\vec{q}_T} = H_{q\bar{q}}(Q^2) \int d^2\vec{b}_T \, e^{i\vec{b}_T\cdot\vec{q}_T} f_q(x_a, \vec{b}_T) f_{\bar{q}}(x_b, \vec{b}_T)$$

$$= H_{q\bar{q}}(Q^2) \int d^2\vec{b}_T \, e^{i\vec{b}_T\cdot\vec{q}_T} B_q(x_1, \vec{b}_T) B_{\bar{q}}(x_2, \vec{b}_T) S^q(b_T)$$

- Beam functions B_q: collinear radiation
  - Correspond to *unsubtracted TMDPDF* f_q^{unsub}
- Soft function S^q: soft radiation
  - Sensitive to both proton directions
  - Universal: same soft function for DIS
- TMDPDF defined by absorbing soft function:

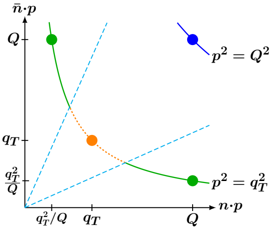
$$f_q(x, \vec{b}_T) = B_q(x, \vec{b}_T) \sqrt{S^q(b_T)}$$



- *Note:* CSS and SCET (Soft-Collinear Effective Theory) yield *equivalent* TMDPDFs, but there are many ways to define beam and soft functions individually [Collins '11; Becher, Neubert '10; Echevarria, Idilbi, Scimemi '11; Chiu et al '12; Li, Neill, Zhu '16]
- *Note:* [Collins, Soper, Sterman '85; Ji, Ma, Yuan '05] use a different scheme, but are perturbatively relatable [Collins, Rogers '17]

TMDs and rapidity regulators

- *Rapidity divergences* arise from expanding in regions:



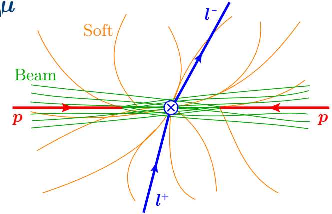
$$\int_{q_T}^{Q} \frac{\mathrm{d}k^+}{k^+} = \lim_{\tau\to 0} \int_0^Q \frac{\mathrm{d}k^+}{k^+} R(k^+,\tau) + \int_{q_T}^{\infty} \frac{\mathrm{d}k^+}{k^+} R(k^+,\tau)$$

$$= \ln \frac{Q}{q_T}$$

- TMDPDF defined s. t. rapidity divergences cancel:

$$f_q(x,\vec{b}_T,\mu,\zeta) = \lim_{\substack{\epsilon\to 0\\ \tau\to 0}} Z_{\mathrm{uv}}^q(\epsilon,\mu,\zeta) f_q^{\mathrm{unsub}}(x,\vec{b}_T,\epsilon,\tau)\sqrt{S^q(b_T,\epsilon,\tau)}$$

  - ϵ: UV regulator $\to$ renormalization scale μ
  - τ: rapidity regulator $\to$ Collins-Soper scale ζ



- Many definitions / schemes in the literature:
  - Wilson lines off light cone [Collins '11]
  - Δ regulator [Echevarria, Idilbi, Scimemi '11]
  - Analytic regulator [Becher, Bell '12]
  - η regulator [Chiu, Jain, Neill, Rothstein '12]
  - Exponential regulator [Li, Neill, Zhu '16]
- TMDPDF is scheme independent up to definition $\zeta = Q^2 e^{-2y_n}$

Definitions of TMDPDFs

$$b^\pm = b^0 \mp b^z$$
$$n = (1, 0, 0, 1)$$

- Definition of unsubtracted TMDPDF:

$$f_q^{\rm unsub}(x, \vec b_T) = \int \frac{{\rm d}b^+}{2\pi} e^{ib^+(xP_n^-)} \langle P(P_n) \big| \bar q(b^+, \vec b_T) W^{(0,\vec 0_T)}_{(b^+,\vec b_T)} \frac{\gamma^-}{2} q(0) \big| P(P_n) \rangle$$

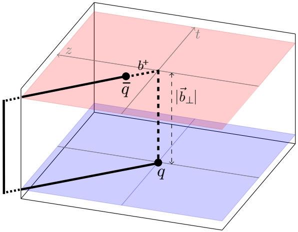
  - Proton matrix element (similar to collinear PDF)

- Definition of soft function:

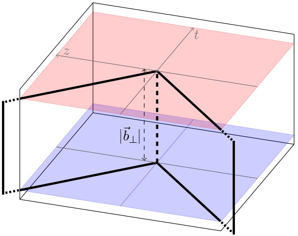
$$S_q(b_T) = \frac{1}{N_c} {\rm Tr}\, \langle 0 \big| [S_{\bar n}^\dagger S_n](\vec b_T) S^{(0,\vec b_T)}_{\perp,-\infty n} [S_n^\dagger S_{\bar n}](\vec 0_T) S^{(0,\vec b_T)}_{\perp,-\infty \bar n} \big| 0 \rangle$$

  - Vacuum matrix element (no analog in collinear factorization)

- Wilson line paths (prior to rapidity regularization):



Unsubtracted TMD



Soft function

Examples of TMD definitions

- TMDPDF defined such that rapidity divergences cancel:

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon \to 0 \\ \tau \to 0}} Z_{\rm uv}^q(\epsilon, \mu, \zeta) \frac{f_q^{\rm unsub}(x, \vec{b}_T, \epsilon, \tau)}{S^0(b_T, \epsilon, \tau)} \sqrt{S^q(b_T, \epsilon, \tau)}$$

  - S^0 removes overlap between collinear and soft matrix elements
- For more details / explicit one-loop examples, see [ME, Stewart, Zhao '19]

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- For more details / explicit one-loop examples, see [ME, Stewart, Zhao '19]
- Example 1: Wilson lines off the light cone [Collins '11]

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon \to 0 \\ y_B \to -\infty}} Z_{\rm uv}^q(\epsilon, \mu, \zeta) \frac{f_q^{\rm unsub}(x, \vec{b}_T, \epsilon, y_P - y_B)}{\sqrt{S^q(b_T, \epsilon, 2y_n - 2y_B)}}$$

(compact form due to [Buffing, Diehl, Kasemets '17])

  - Collins-Soper scale: $\zeta = (x m_p e^{y_P - y_n})^2 = (p^- e^{-y_n})^2$
  - Collins-Soper kernel:

$$\gamma_\zeta^q(\mu, b_T) = 2 \frac{\mathrm{d} \ln f_q}{\mathrm{d} \ln \zeta} = \frac{\mathrm{d} \ln f_q^{\rm unsub}}{\mathrm{d} y_P} = -\frac{\mathrm{d} \ln S^q}{\mathrm{d} y_n}$$

  - CS kernel can be obtained from *both* collinear and soft matrix element

Examples of TMD definitions

- TMDPDF defined such that rapidity divergences cancel:

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon \to 0 \\ \tau \to 0}} Z_{\rm uv}^q(\epsilon, \mu, \zeta) \frac{f_q^{\rm unsub}(x, \vec{b}_T, \epsilon, \tau)}{S^0(b_T, \epsilon, \tau)} \sqrt{S^q(b_T, \epsilon, \tau)}$$

  - S^0 removes overlap between collinear and soft matrix elements
- For more details / explicit one-loop examples, see [ME, Stewart, Zhao ’19]
- Example 2: Δ regulator [Echevarria, Idilbi, Scimemi ’11]

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon \to 0 \\ \delta \to 0}} Z_{\rm uv}^q(\epsilon, \mu, \zeta) \frac{f_q^{\rm unsub}(x, \vec{b}_T, \epsilon, \delta^-/p^-)}{\sqrt{S^q(b_T, \epsilon, \delta^- e^{-y_n})}}$$

  - Modify eikonal vertices from Wilson lines: $\dfrac{1}{k^- + \mathrm{i}0} \to \dfrac{1}{k^- + \mathrm{i}\delta^-}$
  - Collins-Soper scale: $\zeta = (p^- e^{-y_n})^2$
  - Collins-Soper kernel:

$$\gamma_\zeta^q(\mu, b_T) = 2\frac{\mathrm{d}\ln f_q}{\mathrm{d}\ln \zeta} = \frac{\mathrm{d}\ln f_q^{\rm unsub}}{\mathrm{d}p^-} = -\frac{\mathrm{d}\ln S^q}{\mathrm{d}y_n}$$

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Examples of TMD definitions

- TMDPDF defined such that rapidity divergences cancel:

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  - S^0 removes overlap between collinear and soft matrix elements
- For more details / explicit one-loop examples, see [ME, Stewart, Zhao '19]
- Example 3: η regulator [Chiu, Jain, Neill, Rothstein '12]

$$f_q(x, \vec{b}_T, \mu, \zeta) = B_q(x, \vec{b}_T, \mu, \nu^2/\zeta) \sqrt{S^q(b_T, \mu, \nu)}$$

  - Separately renormalize collinear and soft matrix elements
  - Modify eikonal vertices from Wilson lines: $\dfrac{1}{k^- + \mathrm{i}0} \to \dfrac{|k^z/\nu|^\eta}{k^- + \mathrm{i}\delta^-}$
  - Collins-Soper scale: $\zeta = (p^-)^2$
  - Collins-Soper kernel:

$$\gamma_\zeta^q(\mu, b_T) = 2\frac{\mathrm{d}\ln f_q}{\mathrm{d}\ln \zeta} = -\frac{\mathrm{d}\ln f_q^{\rm unsub}}{\mathrm{d}\ln \nu} = \frac{1}{2}\frac{\mathrm{d}\ln S^q}{\mathrm{d}\ln \nu}$$

  - CS kernel can be obtained from *both* collinear and soft matrix element

(Quasi-)TMDPDFs from lattice LCD

Based on [ME, Stewart, Zhao, PRD99 (2019); JHEP09 (2019) 037]

Reminder: Quasi PDFs

$b^\pm = b^0 \mp b^z$
$n = (1,0,0,1)$

- PDF:

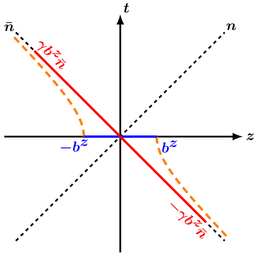
$$f_q(x,\mu) = \int \frac{\mathrm{d}b^+}{4\pi} e^{\mathrm{i}b^+(xP_n^-)} \langle P(P_n)|\, \bar q(b^+) W_n(b^+,0) \gamma^- q(0)\,|P(P_n)\rangle$$

- Quasi-PDF: Equal-time correlator [Ji '13, '14]

$$\tilde f_q(x,P_z,\mu) = \int \frac{\mathrm{d}b^z}{4\pi} e^{\mathrm{i}b^z(xP_z)} \langle P(P_n)\big|\bar q(b^z) W_z(z,0) \gamma_3 q(0)\big|P(P_n)\rangle$$

  - Related: pseudo PDF $\mathcal{P}_q(x,z^2,\mu)$ [Radyushkin '17]

- Operators can be related through a Lorentz boost



- Time-dependent PDF operator
  ⇔ Boosted equal-time operator
  ⇔ Equal-time operator in boosted proton state

- PDF and quasi-PDF describe same IR physics

- Difference in UV accounted for by perturbative matching

Reminder: Quasi PDFs

- Quasi-PDF can be perturbatively matched onto PDFs

  [Xiong, Ji, Zhang, Zhao '13; Ma, Qiu '14 '17; Izubuchu, Ji, Jin, Stewart, Zhao '18]
  - Rigorous proof using operator product expansion
- Matching relation:

$$\tilde{f}_i(x, P_z, \tilde{\mu}) = \int_0^1 \frac{\mathrm{d}y}{y} C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P_z}, \frac{\mu}{yP_z}\right) f_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$\tilde{f}_i(x, P_z, \tilde{\mu})$: Simulation/renormalization on lattice

C_{ij}: **Perturbative** matching coefficient

$f_j(y, \mu)$: PDF

$\mathcal{O}(\ldots)$: Higher-twist correction

- Significant progress in recent years:
  - Calculation at large P^z and physical pion mass [ETMC, LP3]
  - Nonperturbative renormalization of quasi-PDF on lattice / Calculation of matching coefficient

    [Ji, Zhang '15; Constantinou, Panagopoulos '17; Li, Ma, Qiu '18; ...]
  - Subtraction of higher-twist corrections

    [Chen et al '16; Radyushkin '17; Braun, Vladimirov, Zhang '18]
  - Inclusion in global PDF fits [Cichy, Debbio, Giani '19]

Towards Quasi TMDPDFs

Roadmap:

1. Construct quasi beam function $\tilde{B}_q(x, \vec{b}_T, a, L, P^z)$
   and quasi soft function $\tilde{S}^q(b_T, a, L)$
   - Must be computable on lattice
   - Regulate UV divergence by lattice spacing a
   - Regulate rapidity divergence by finite lattice size L
     (see also [Ji, Jin, Yuan, Zhang, Zhao '18])

2. Combine into quasi TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \lim_{\substack{a\to 0\\ L\to 0}} \tilde{Z}^q_{\mathrm{uv}}(\mu, P^z, a) \frac{\tilde{B}_q(x, \vec{b}_T, a, L, P^z)}{\sqrt{\tilde{S}^q(b_T, a, L)}}$$

   - Note: in practice needs nonperturbative lattice renormalization
     $\to$ see talk by [M. Wagman]

3. Derive *perturbative* matching

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = (C_q \otimes f_q)(x, \vec{b}_T, \mu, \zeta)$$

4. Determine Collins-Soper kernel to relate TMDs at different energies

Constructing the quasi beam function

$b^{\pm} = b^0 \mp b^z$
$n = (1, 0, 0, 1)$

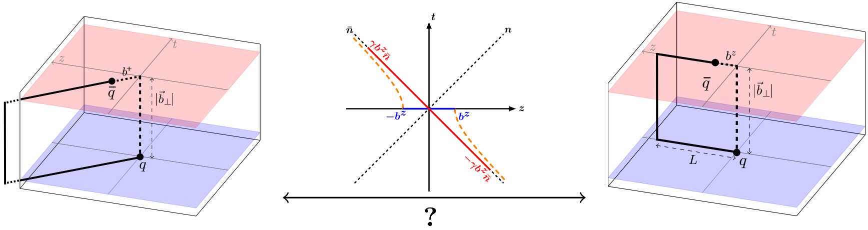
- Beam function: (light-cone correlator)

$$B_q(x, \vec{b}_T) = \int \frac{\mathrm{d}b^+}{4\pi} e^{-\frac{\mathrm{i}}{2} b^+ (xP^-)} \Big\langle p(P) \Big| \bar{q}(b^+, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^+,\vec{b}_T)} \gamma^- q(0) \Big| p(P) \Big\rangle$$

- Quasi beam function: (equal-time correlator)

$$\tilde{B}_q(x, \vec{b}_T) = \int \frac{\mathrm{d}b^z}{4\pi} e^{\mathrm{i} b^z (xP^z)} \Big\langle p(P) \Big| \bar{q}(b^z, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^z,\vec{b}_T)} \gamma^3 q(0) \Big| p(P) \Big\rangle$$

- Wilson line path:
  - Finite lattice size requires to truncate at length L
  - Bare operators related by Lorentz boost



Beam function | Quasi beam function

Constructing the quasi soft function

$n = (1,0,0,1)$
$\bar{n} = (1,0,0,-1)$

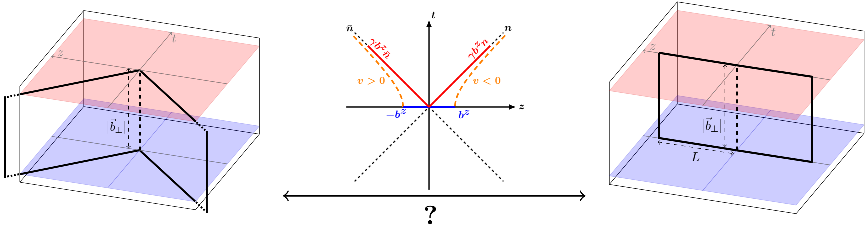
- Soft function: (light-cone correlator)

$$S^q(b_T) = \frac{1}{N_c} \mathrm{Tr}\, \langle 0 | [S_n^\dagger S_{\bar{n}}](\vec{b}_T)\, S_T(-\bar{n}\infty)\, [S_{\bar{n}}^\dagger S_n](\vec{0}_T)\, S_T^\dagger(-n\infty) | 0 \rangle$$

- Quasi soft function: (equal-time correlator)

$$\tilde{S}^q(b_T) = \frac{1}{N_c} \mathrm{Tr}\, \langle 0 | [S_{\hat{z}}^\dagger S_{-\hat{z}}](\vec{b}_T)\, S_T(-\hat{z}L)\, [S_{-\hat{z}}^\dagger S_{\hat{z}}](\vec{0}_T)\, S_T^\dagger(\hat{z}L) | 0 \rangle$$

- Wilson line path:
  - Finite lattice size requires to truncate at length L
  - Bare operators *not* related by Lorentz boost (more on this later)



Soft function

Quasi soft function

Constructing the quasi soft function

$n = (1, 0, 0, 1)$
$\bar{n} = (1, 0, 0, -1)$

- Soft function: (light-cone correlator)

$$S^q(b_T) = \frac{1}{N_c} \mathrm{Tr}\, \langle 0 \big| [S_n^\dagger S_{\bar{n}}](\vec{b}_T)\, S_T(-\bar{n}\infty)\, [S_{\bar{n}}^\dagger S_n](\vec{0}_T)\, S_T^\dagger(-n\infty) \big| 0 \rangle$$

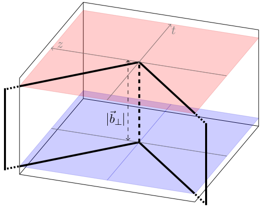
- Quasi soft function: (equal-time correlator)

$$\tilde{S}^q(b_T) = \frac{1}{N_c} \mathrm{Tr}\, \langle 0 \big| [S_{\hat{z}}^\dagger S_{-\hat{z}}](\vec{b}_T)\, S_T(-\hat{z}L)\, [S_{-\hat{z}}^\dagger S_{\hat{z}}](\vec{0}_T)\, S_T^\dagger(\hat{z}L) \big| 0 \rangle$$

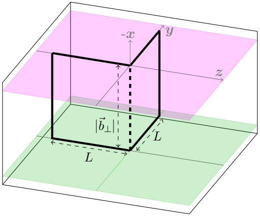
- Alternative: bent soft function (equal-time correlator)

$$\tilde{S}^q_{\rm bent}(b_T) = \frac{1}{N_c} \mathrm{Tr}\, \langle 0 \big| [S_{\hat{z}}^\dagger S_y](\vec{b}_T)\, S_T(-\hat{z}L)\, [S_y^\dagger S_{\hat{z}}](\vec{0}_T)\, S_T^\dagger(\hat{y}L) \big| 0 \rangle$$

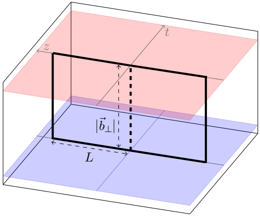
  - Agrees with the soft function in [Ji,Sun,Xiong,Yuan '14]



Soft function



Bent soft function



Quasi soft function

Relating quasi TMDPDF and TMDPDF

- Goal: perturbative matching between TMDPDF and quasi TMDPDF
- Due to soft mismatch: expect *nonperturbative* relation

Quasi-TMD from lattice → $\tilde{f}_q(x, \vec{b}_T, \mu, P^z)$; Perturbative kernel → $C_q(\mu, xP^z)$; TMDPDF → $f_q(x, \vec{b}_T, \mu, \zeta)$

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C_q(\mu, xP^z)\, f_q(x, \vec{b}_T, \mu, \zeta) \times g_q^S(b_T, \mu) \exp\left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma_\zeta^q(\mu, b_T)\right]$$

Soft mismatch: $g_q^S(b_T, \mu)$

Collins-Soper evolution (ensures ζ independence): $\exp\left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma_\zeta^q(\mu, b_T)\right]$

- *Not* proven, but validated explicitly at one loop, finding
  - Quasi soft function: $g_q^S = 1 + \frac{\alpha_s C_F}{2\pi} \ln(b_T^2 \mu^2 / b_0^2)$
  - Bent soft function: $g_q^S = 1$
- Perturbative matching requires $g_q^S = 1$ *and* $\zeta = (2xP^z)^2$
  - Not feasible on lattice, where $P^z \sim \mathcal{O}(1\ \mathrm{GeV})$, but $\zeta \sim m_Z^2$

Collins-Soper kernel from Lattice QCD

- Reminder: quark (quasi) TMDPDFs related by (at NLO)

$$\frac{\tilde{f}_q(x, \vec{b}_T, \mu, P^z)}{f_q(x, \vec{b}_T, \mu, \zeta)} = C_q(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right]$$

- Nonperturbative factor $g_q^S(b_T, \mu)$ cancels in ratios with same b_T and μ
  - Can similarly study other ratios of TMDPDFs, e.g. spin structures (ratios of moments studied in [Musch et al '10 '12; Engelhardt et al '15; Yoon et al '17])
  - See talk by [M. Engelhardt]

- Factor out Collins-Soper kernel by varying proton momenta $P_1^z \neq P_2^z$:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C_q(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C_q(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

- Independent of hadron state: Can use pion state for simplicity
  - Yield much better signal-to-noise ratios than protons
- Formally independent of x, P_1^z, P_2^z, L
  - Many handles to study systematic uncertainties
- First exploratory studies: see talk by [M. Wagman]

Summary

Summary

TMDPDFs:

- Definition of TMDPDFs more complicated than for collinear PDFs
  - Collinear proton matrix element $f_i^{\rm unsub}(x, \vec{b}_T, \epsilon, \tau)$
  - Soft vacuum matrix element $S_q(b_T, \epsilon, \tau)$
  - Requires a rapidity regulator τ
- Rapidity divergences give rise to Collins-Soper evolution
  - Collins-Soper kernel $\gamma_\zeta^i(b_T, \mu)$ is nonperturbative for $b_T^{-1} \lesssim \Lambda_{\rm QCD}$
  - Nonperturbative knowledge of $\gamma_\zeta^i(b_T, \mu)$ crucial to evolve TMDPDFs / resummation of large logs $\ln(Q/q_T)$
- Variety of *equivalent* definitions available in literature

(Quasi-)TMDPDFs from lattice:

- No straightforward definition of quasi-TMDPDF due to soft sector
- Perturbative matching using bent soft function only verified at NLO
  - Still requires to fix $\zeta = (xP^z)^2$
- Ratios of (quasi) TMDPDFs are feasible
  - Can study e.g. spin and flavor dependence [see talk by M. Engelhardt]
  - Calculation of $\gamma_\zeta^i(b_T, \mu)$ from lattice [see talk by M. Wagman]

Backup slides

Impact of nonperturbative Collins-Soper kernel

- Recall: TMD factorization for $pp \to Z/\gamma^* \to l^+ l^-$:

$$\frac{d\sigma_{pp}}{dQ^2 dY d^2\vec{q}_T} = \hat{\sigma}_{q\bar{q}}(Q^2, \mu) \int d^2\vec{b}_T \, e^{i\vec{b}_T \cdot \vec{q}_T} f_q(x_a, \vec{b}_T, \mu, \zeta) f_{\bar{q}}(x_b, \vec{b}_T, \mu, \zeta)$$

- Recall TMDPDF evolution equations:

$$\mu \frac{d}{d\mu} f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta) f_q(x, \vec{b}_T, \mu, \zeta)$$

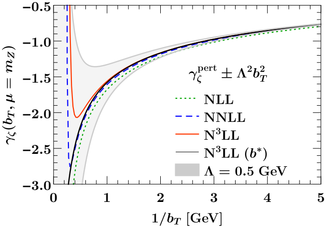
$$\zeta \frac{d}{d\zeta} f_q(x, \vec{b}_T, \mu, \zeta) = \frac{1}{2} \gamma_\zeta^q(\mu, b_T) f_q(x, \vec{b}_T, \mu, \zeta)$$

- All-order form of CS kernel:

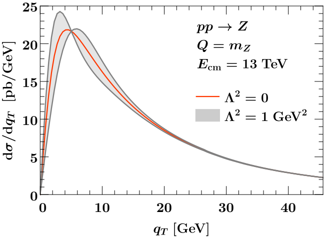
$$\gamma_\zeta^i(b_T, \mu) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^i[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(1/b_T)]$$

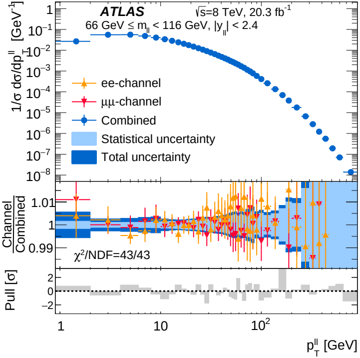
- Applications of CS evolution:
  - Evolve TMD measured from data / extracted from lattice
  - For $\Lambda_{\rm QCD} \ll q_T \ll Q$: resum large logarithms $\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$
- Key ingredient: nowledge of γ_ζ
  - Perturbatively known at N^3LO in QCD [Li, Zhu '16; Vladimirov '16]

Impact of nonperturbative Collins-Soper kernel



$\longleftarrow \gamma_\zeta^{\rm pert}(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma[\alpha_s(\mu')] + \gamma_\zeta[\alpha_s(1/b_T)]$





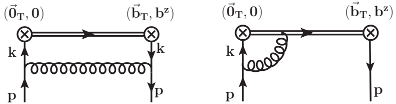
- Nonperturbative correction to $\gamma_\zeta^q(\mu, b_T)$ is a key uncertainty for predicting the q_T spectrum at the LHC
- So far: fitted from data or "neglected"

Verification of matching at NLO

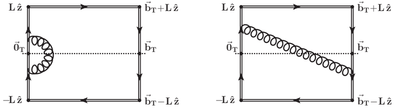
- Test matching relation perturbatively at NLO
  - Work in $\overline{\mathrm{MS}}$ scheme, not lattice renormalization
  - On-shell external quark state
  - Ignore mixing with gluons
- Quasi-TMDPDF becomes simple product:

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \lim_{\substack{\epsilon \to 0 \\ L \to 0}} \tilde{Z}^q_{\mathrm{uv}}(\mu, P^z, \epsilon) \frac{\tilde{B}_q(x, \vec{b}_T, \epsilon, L, P^z)}{\sqrt{\tilde{S}(b_T, \epsilon, L)}}$$

  - Precise form fixed by cancellation of divergences L/b_T
- Example diagrams:



Quasi beam function



Quasi soft function

Verification of matching at NLO

- Result at one loop:

$$\frac{\tilde{f}_q(x, \vec{b}_T, \mu, P^z)}{f_q(x, \vec{b}_T, \mu, \zeta)} = 1 + \frac{\alpha_s C_F}{2\pi}\Big[-\frac{1}{2}\ln^2\frac{(2xP^z)^2}{\mu^2} + \ln\frac{(2xP^z)^2}{\mu^2} + \ln(b_T^2\mu^2) - \ln(b_T^2\mu^2)\ln\frac{(2xP^z)^2}{\zeta} + \cdots\Big]$$

- Compare to matching formula:

$$\frac{\tilde{f}_q(x, \vec{b}_T, \mu, P^z)}{f_q(x, \vec{b}_T, \mu, \zeta)} = C_q(\mu, xP^z)\, g_q^S(b_T, \mu)\, \exp\Big[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma_\zeta^q(\mu, b_T)\Big]$$

- Comparison:
  - Perturbative kernel C_q
  - Nonperturbative Collins-Soper kernel γ_ζ^q
  - Leftover nonperturbative logarithm $\ln(b_T^2\mu^2)$
- Interpretation: remnant of failure of relating soft factors S and $\tilde{S}$
- Possible solution: can modify soft function to remove $\ln(b_T^2\mu^2)$ at NLO
  - Valid to all orders?

More details on rapidity regulators

Collinear/soft Wilson lines

$$W_n = P \exp\left[ig \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot A(\bar{n}s)\right] = \sum_{\text{perms}} \exp\left[-g \frac{\bar{n} \cdot A(k)}{\bar{n} \cdot k}\right]$$

$$S_n = P \exp\left[ig \int_{-\infty}^{0} \mathrm{d}s\, n \cdot A(ns)\right] = \sum_{\text{perms}} \exp\left[-g \frac{n \cdot A(k)}{n \cdot k}\right]$$

η regulator [Chiu, Jain, Neill, Rothstein '12]

$$W_n \to \sum_{\text{perms}} \exp\left[-g w^2 \frac{|\bar{n} \cdot \mathcal{P}|^{-\eta}}{\nu^{-\eta}} \frac{\bar{n} \cdot A(k)}{\bar{n} \cdot k}\right]$$

$$S_n \to \sum_{\text{perms}} \exp\left[-g w \frac{|2\mathcal{P}_z|^{-\eta/2}}{\nu^{-\eta/2}} \frac{n \cdot A(k)}{n \cdot k}\right]$$

More details on rapidity regulators

Collinear/soft Wilson lines

$$W_n = P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot A(\bar{n}s)\right] = \sum_{\text{perms}} \exp\left[-g \frac{\bar{n} \cdot A(k)}{\bar{n} \cdot k}\right]$$

$$S_n = P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, n \cdot A(ns)\right] = \sum_{\text{perms}} \exp\left[-g \frac{n \cdot A(k)}{n \cdot k}\right]$$

δ regulator [Echevarria, Idilbi, Scimemi ’11; Echevarria, Scimemi, Vladimirov ’16]

$$W_n \to P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot A(\bar{n}s) e^{-\delta^+ s}\right]$$

$$S_n \to P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, n \cdot A(ns) e^{+\delta^- s}\right]$$

More details on rapidity regulators

Collinear/soft Wilson lines

$$W_n = P\exp\left[ig\int_{-\infty}^{0} \mathrm{d}s\, \bar n\cdot A(\bar n s)\right] = \sum_{\text{perms}} \exp\left[-g\frac{\bar n\cdot A(k)}{\bar n\cdot k}\right]$$

$$S_n = P\exp\left[ig\int_{-\infty}^{0} \mathrm{d}s\, n\cdot A(n s)\right] = \sum_{\text{perms}} \exp\left[-g\frac{n\cdot A(k)}{n\cdot k}\right]$$

Exponential regulator [Li, Neill, Zhu '16]

- Does not modify Wilson lines, but phase space integrals ($b_0 = 2e^{-\gamma_E}$):

$$\int \mathrm{d}^d k \to \lim_{\tau\to 0}\int \mathrm{d}^d k\, e^{-k^0 \tau\, b_0}$$

More details on rapidity regulators

Collinear/soft Wilson lines

$$W_n = P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot A(\bar{n}s)\right] = \sum_{\text{perms}} \exp\left[-g \frac{\bar{n} \cdot A(k)}{\bar{n} \cdot k}\right]$$

$$S_n = P \exp\left[\mathrm{i}g \int_{-\infty}^{0} \mathrm{d}s\, n \cdot A(ns)\right] = \sum_{\text{perms}} \exp\left[-g \frac{n \cdot A(k)}{n \cdot k}\right]$$

Analytic regulator [Becher, Neubert '10]

- Replace eikonal propagator (p = momentum along Wilson line)

$$\frac{1}{n \cdot k - \mathrm{i}0} \to \frac{\nu_1^{2\alpha} \bar{n} \cdot p}{[(n \cdot k)(\bar{n} \cdot p) - \mathrm{i}0]^{1+\alpha}}$$

$$\frac{1}{\bar{n} \cdot k - \mathrm{i}0} \to \frac{\nu_2^{2\beta} n \cdot p}{[(\bar{n} \cdot k)(n \cdot p) - \mathrm{i}0]^{1+\beta}}$$

- $S^q = 1$ in this regulator
- Asymmetric: take first $\beta \to 0$, then $\alpha \to 0$ (or vice versa)
  $\to S^q = 1$ absorbed into *one* TMDPDF

Rapidity divergences at finite L

- Recall: Rapidity divergences arise from integrals of type

$$I_{\rm div} = \int \frac{dk^+ dk^-}{(k^+k^-)^\epsilon} \frac{f(k^+k^-)}{k^+k^-} = \int \frac{d(k^+/k^-)}{2\,k^+/k^-} \int \frac{d(k^+k^-)}{(k^+k^-)^\epsilon} \frac{f(k^+k^-)}{k^+k^-}$$

  - Integrand depends only on product k^+k^-

- Eikonal propagator for $L < \infty$:

$$\frac{1}{k^\pm + i0} \to \frac{1 - e^{ik^\pm L}}{k^\pm}$$

- With finite L:

$$I_{\rm div} \to \int dk^+ dk^- \frac{f(k^+k^-)}{(k^+k^-)^\epsilon} \frac{1 - e^{ik^+ L}}{k^+} \frac{1 - e^{-ik^- L}}{k^-}$$

  - Finite L fully regulates $k^\pm \to 0$
  - No rapidity divergences for finite L

Comparison to Ji et al

- Quasi TMDPDF was previously studied in [Ji, Jin, Yuan, Zhang, Zhao '18]
  - do not separately consider $\tilde{B}_q$ and $\tilde{S}^q$
  - but directly absorb $\tilde{f}_q = \frac{B_q}{\sqrt{S^q}}$
- Matching relation at NLO: $\zeta = (2xP^z)^2, \mu = \sqrt{\zeta}$

$$\tilde{f}_q(x, \vec{b}_T; \zeta) = e^{-S_w^q(\zeta, \vec{b}_T)} \left[1 - \frac{\alpha_s C_F}{\pi}\right] f_q(x, \vec{b}_T; \zeta)$$

- Matching kernel involves nonperturbative component for $b_T \sim \Lambda_{\mathrm{QCD}}^{-1}$

$$e^{-S_w^q(\zeta, \vec{b}_T)} = \exp\left[\int_{c_0/b_T}^{\zeta} \frac{\mathrm{d}\mu}{\mu} \frac{\alpha_s(\mu') C_F}{\pi}\right]$$

- Matching requires nonperturbative knowledge of S_w^q, in agreement with our interpretation