Factorization with TMD PDFs

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- TMD basics
- Large Transverse Momentum

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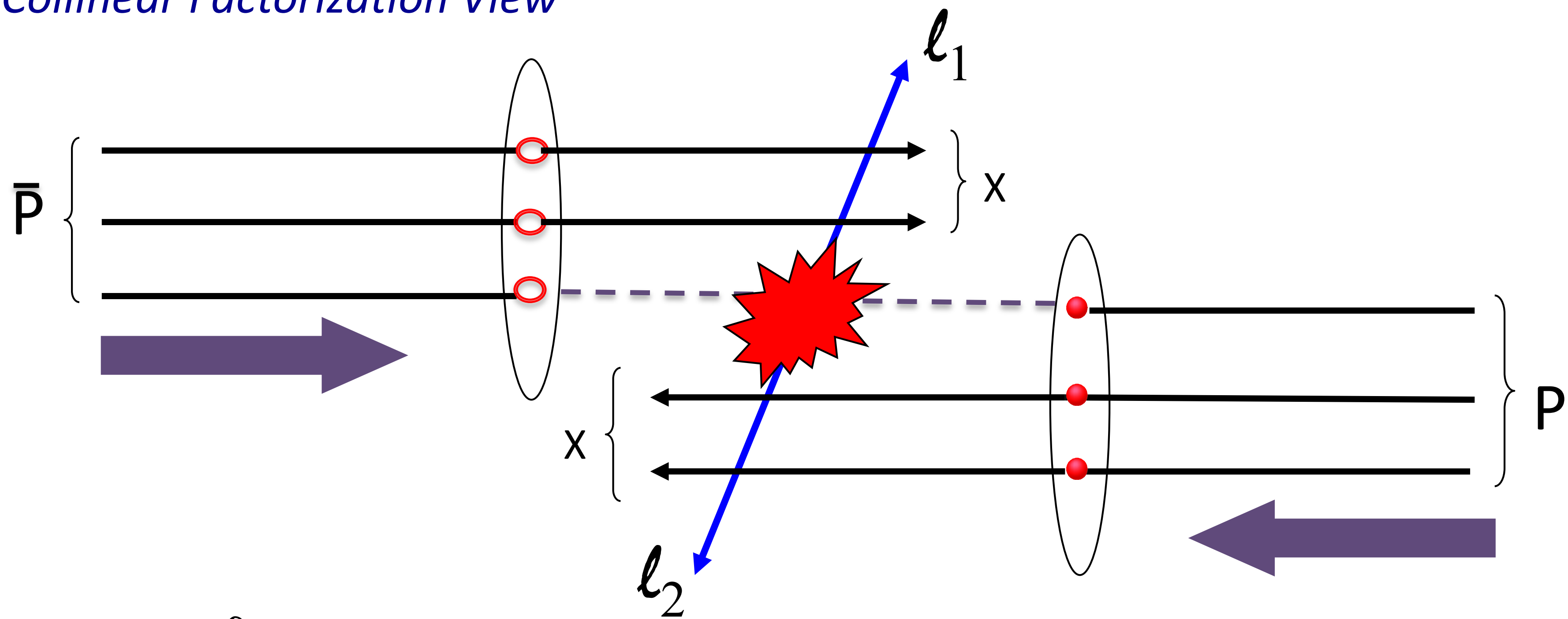
Transverse Momentum Dependent Parton Densities

1) Needed to calculate transversely differential cross sections in pQCD

2) More detail about hadron structure than standard parton densities

Example: Drell-Yan

Collinear Factorization View

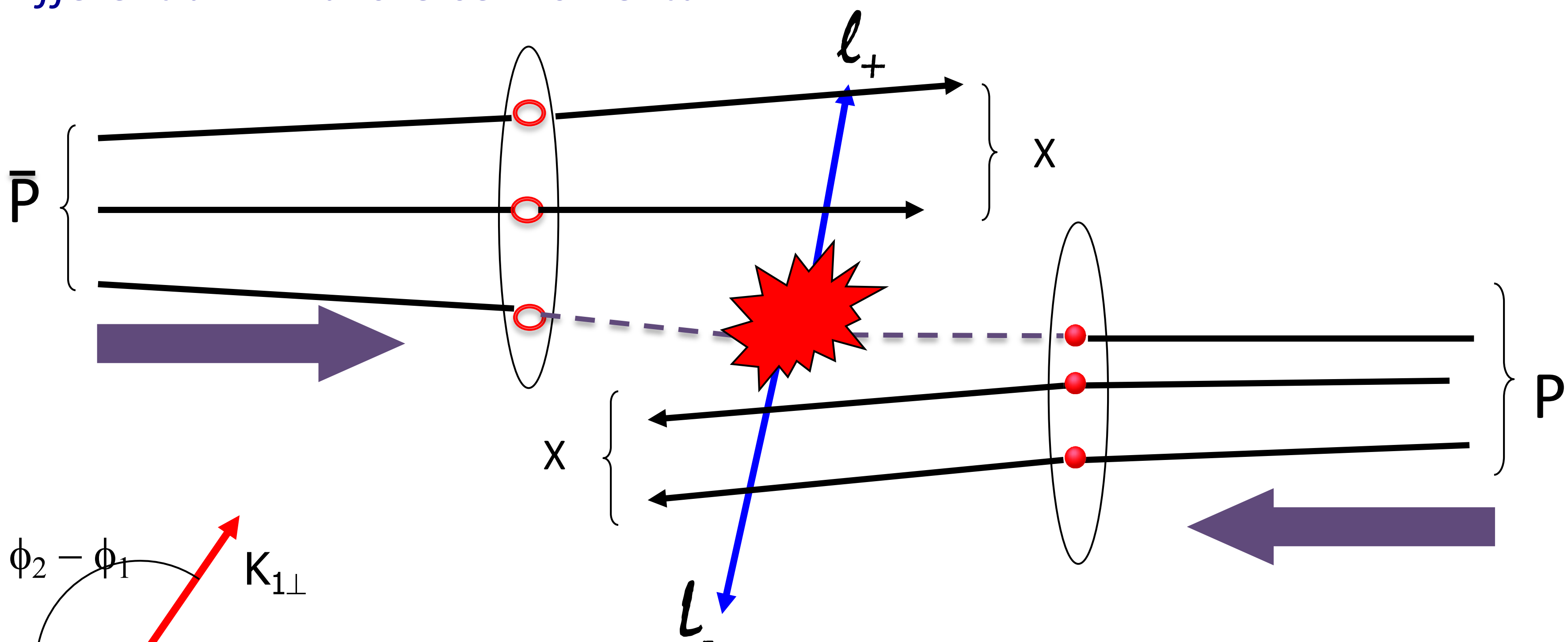


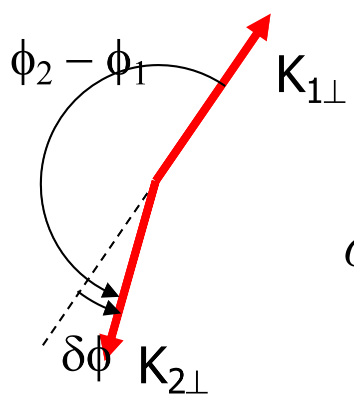
$$\sigma \sim \int \mathcal{H} \otimes f_{q/P}(x_1) \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}$$

Scale Dependence: DGLAP

TMD Example: Drell-Yan

Differential In Transverse Momentum

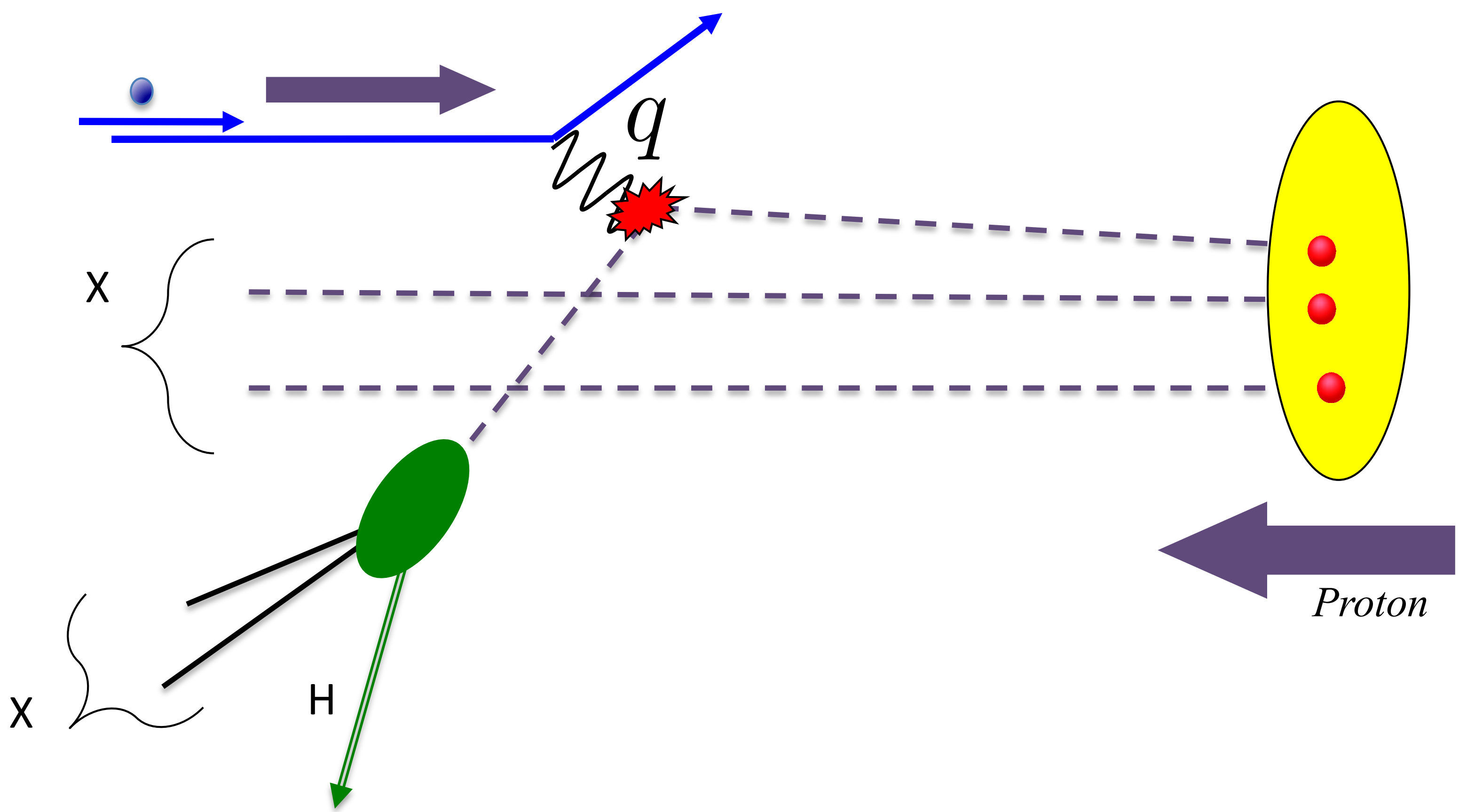




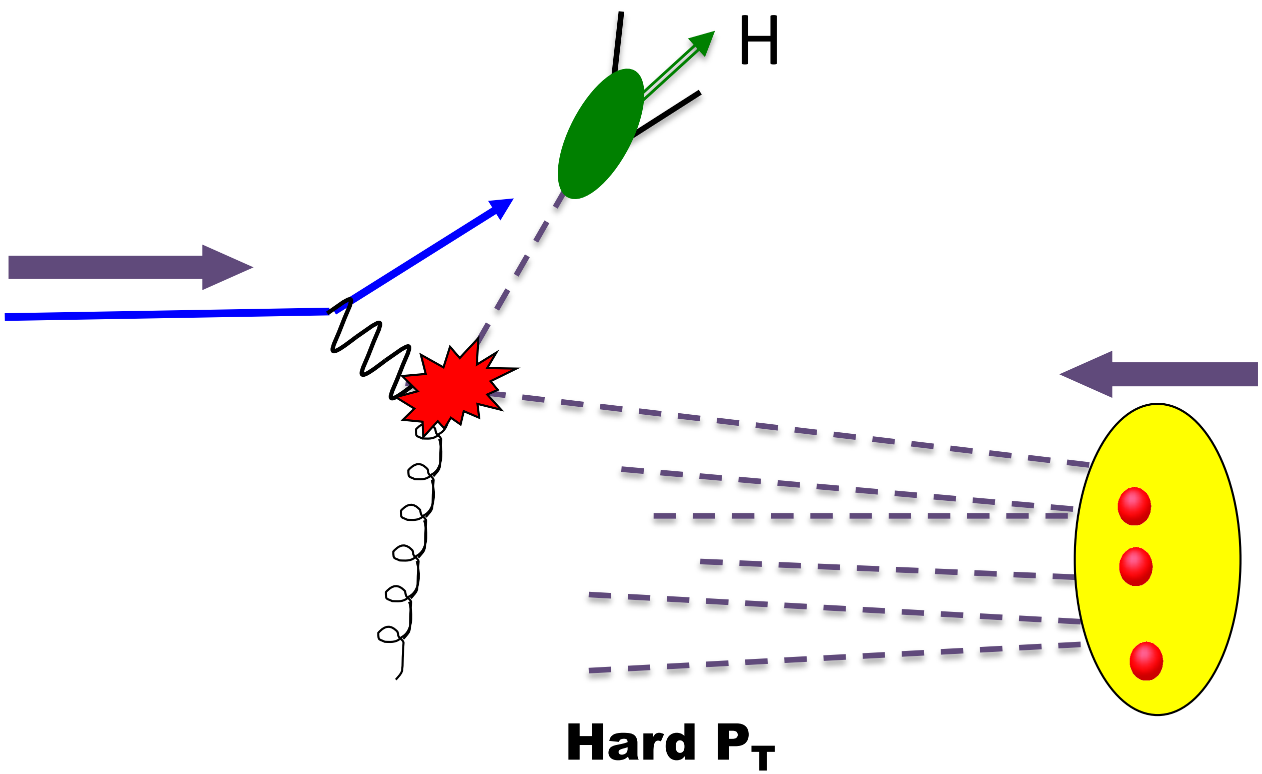
$$\sigma \sim \int \mathcal{H} \otimes F_{q/P}(x_1, \mathbf{k}_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})$$

Scale Dependence: TMD Evolution

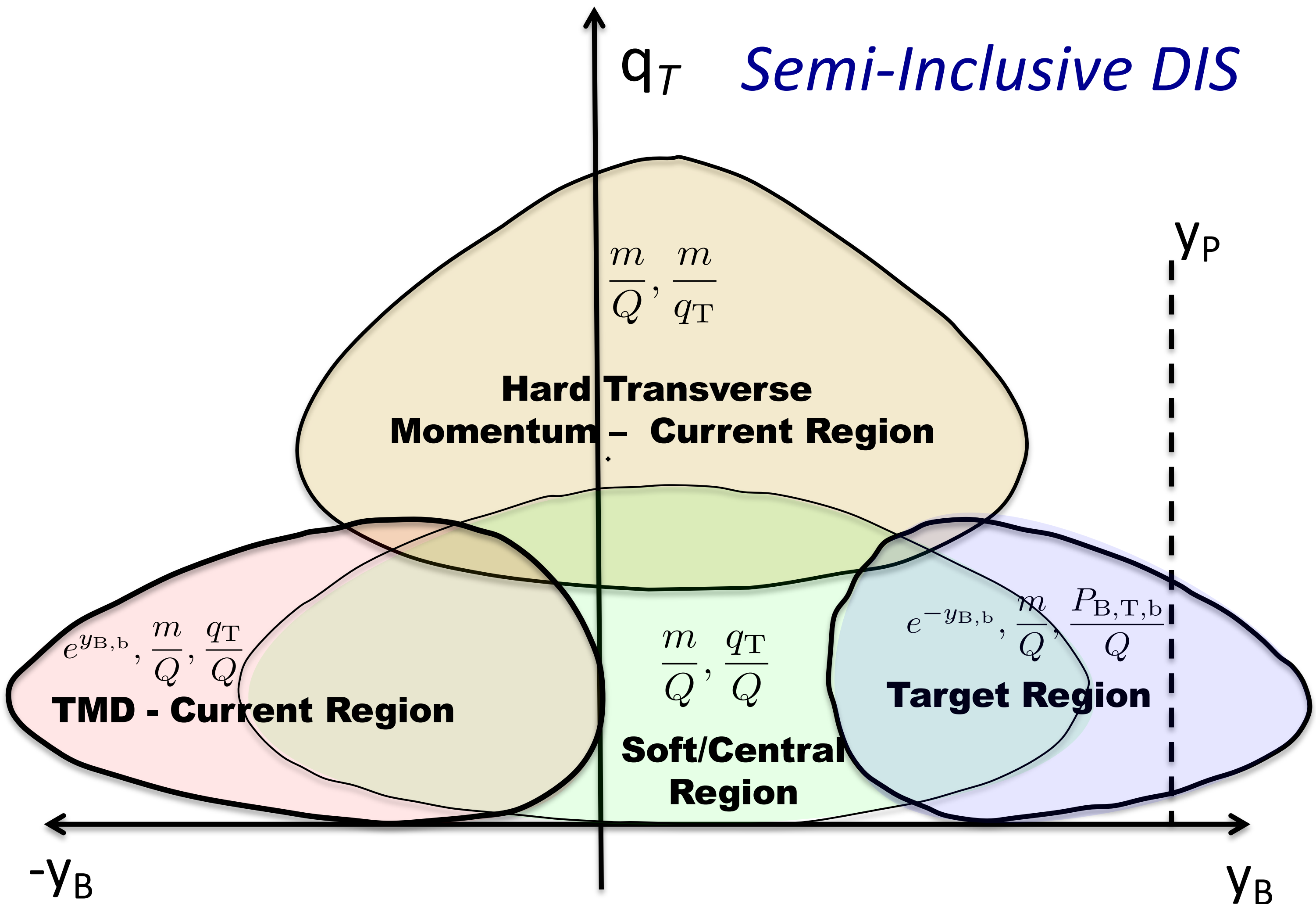
Example: SIDIS



Example: SIDIS



Catalogue of Regions



Catalogue of Regions: Boglione et al
(To appear in JHEP)

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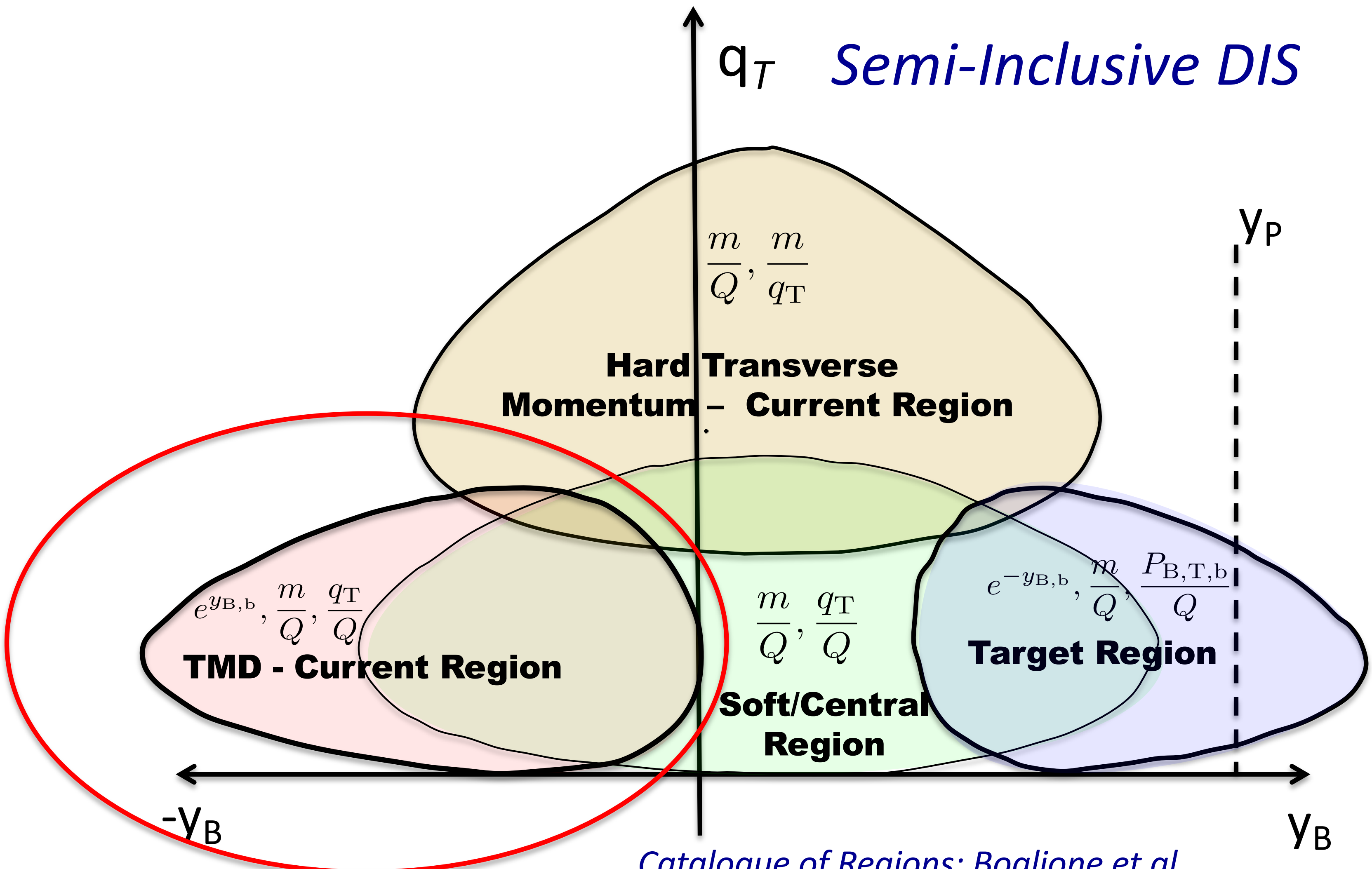

Catalogue of Regions: Boglione et al
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Transverse Momentum Dependent Evolution

- Collinear / DGLAP, Evolution with Scale:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

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- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\frac{\mathrm{d}\tilde{K}(b_T;\mu)}{\mathrm{d}\ln\mu} = -\gamma_K(g(\mu))$$

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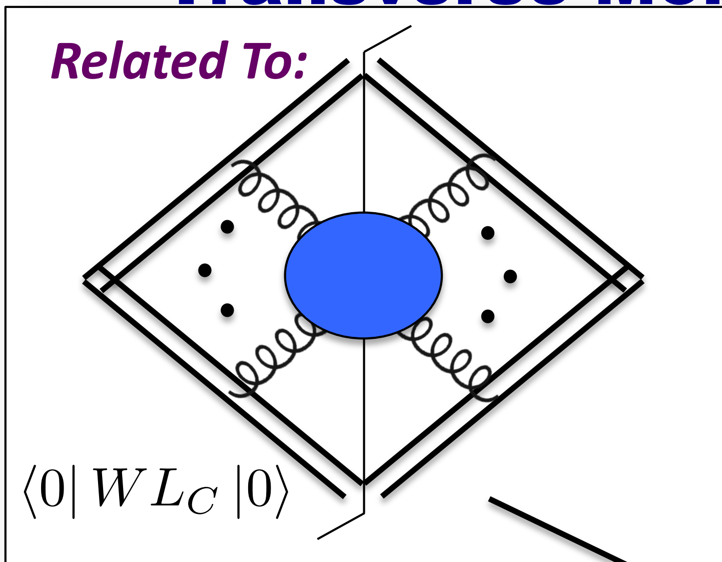
Transverse Momentum Dependent Evolution



volution with Scale:

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$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

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One TMD PDF: Solution to Evolution

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma \quad (g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -g_{f/P}(x, b_T; b_{\max}) - g_K(b_T; b_{\max}) \ln \frac{Q}{Q_0} \right\}$$

One TMD PDF: Solution to Evolution

Finite coefficient

Collinear PDFs

$$\tilde{F}_{f/P}(x, \mathbf{b}_{\mathrm{T}}; Q, Q^2) =$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma\ \ (g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -g_{f/P}(x, b_T; b_{\mathrm{max}}) - g_K(b_T; b_{\mathrm{max}}) \ln \frac{Q}{Q_0} \right\}$$

One TMD PDF: Solution to Evolution

Cutoff

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

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Logarithms

One TMD PDF: Solution to Evolution

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Nonperturbative parts, large b_T

Organization of Factors in a Cross Section

- Example: CSS1

$$\sigma \overset{??}{\sim} \int \mathcal{H} \otimes F_{q/P}(x_1, \mathbf{k}_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})$$

$$\mu_Q \equiv C_2 Q$$
$$\mu_b \equiv C_1/b_{\mathrm{T}}$$
$$\mu_{b_*} \equiv C_1/b_*$$

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = {} & \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
& \times \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{CSS1,\ DY}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{CSS1,\ DY}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{ -\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \left[A_{\mathrm{CSS1}}(a_s(\mu'); C_1) \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\mathrm{CSS1,\ DY}}(a_s(\mu'); C_1, C_2) \right] \right\} \\
& \times \exp\left[-g^{\mathrm{CSS1}}_{j/A}(x_A, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_{\bar{\jmath}/B}(x_B, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_K(b_{\mathrm{T}}; b_{\max}) \ln(Q^2/Q_0^2) \right] \\
& + \text{suppressed corrections.}
\end{aligned}$$

No explicit hard part

CSS1 = Collins-Soper-Sterman (≈1985)

Organization of Factors in a Cross Section

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&\times \exp\left\{ - \int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \left[\underline{A_{\mathrm{CSS1}}(a_s(\mu'); C_1)} \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + \underline{B_{\mathrm{CSS1,\ DY}}(a_s(\mu'); C_1, C_2)} \right] \right\} \\
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& \times \exp\left[-g^{\mathrm{CSS1}}_{j/A}(x_A, b_\mathrm{T}; b_\mathrm{max}) - g^{\mathrm{CSS1}}_{\bar{\jmath}/B}(x_B, b_\mathrm{T}; b_\mathrm{max}) - g_K^{\mathrm{CSS1}}(b_\mathrm{T}; b_\mathrm{max}) \ln(Q^2/Q_0^2) \right] \\
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Organization of Factors with Explicit Factorization

- Example: CSS2

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_j \underline{H_{j\bar{\jmath}}^{\mathrm{DY}}(Q, \mu_Q, a_s(\mu_Q))} \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}\, e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\, \underline{\tilde{f}_{j/A}(x_A, b_{\mathrm{T}}; Q^2, \mu_Q)}\, \underline{\tilde{f}_{\bar{\jmath}/B}(x_B, b_{\mathrm{T}}; Q^2, \mu_Q)}$$
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$$+ \text{ suppressed corrections,}$$

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$$\times\, e^{-g_{j/A}(x_A, b_{\mathrm{T}}; b_{\max})} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B, b_{\mathrm{T}}; b_{\max})} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B; \mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{ -g_K(b_{\mathrm{T}}; b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*; \mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu')) \right] \right\}$$

$$+ \text{ suppressed corrections.}$$

Collins, TCR (2017)

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$$\times\, e^{-g_{\bar{\jmath}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} \underline{f_{j_B/B}(\xi_B;\mu_{b_*})}\ \underline{\tilde{C}^{\mathrm{PDF}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)}$$

$$\times\, \exp\left\{ -g_K(b_{\mathrm{T}};b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu')) \right] \right\}$$

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Collins, TCR (2017)

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$$\times\, e^{-g_{j/A}(x_A, b_\mathrm{T}; b_\mathrm{max})} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
$$\times\, e^{-g_{\bar{j}/B}(x_B, b_\mathrm{T}; b_\mathrm{max})} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B; \mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
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$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H^\mathrm{DY}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q))\int\frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}$$
$$\times\, e^{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})}\int_{x_A}^1\frac{\mathrm{d}\xi_A}{\xi_A}f_{j_A/A}(\xi_A;\mu_{b_*})\,\tilde{C}^\mathrm{PDF}_{j/j_A}\left(\frac{x_A}{\xi_A},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, e^{-g_{\bar{j}/B}(x_B,b_\mathrm{T};b_\mathrm{max})}\int_{x_B}^1\frac{\mathrm{d}\xi_B}{\xi_B}f_{j_B/B}(\xi_B;\mu_{b_*})\,\tilde{C}^\mathrm{PDF}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\exp\left\{-g_K(b_\mathrm{T};b_\mathrm{max})\ln\frac{Q^2}{Q_0^2}+\tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2}+\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu'))-\ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$
$$+\text{ suppressed corrections.}$$

Collins, TCR (2017)

Translation

Example

$$\tilde{C}^{\rm PDF}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) = \frac{\tilde{C}^{\rm CSS1,\ DY}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)}{\sqrt{(1/e_j^2) H^{\rm DY}_{j\bar{j}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*}))}} \exp\left[\tilde{K}(b_*; \mu_{b_*}) \ln C_2\right]$$

CSS2 (Collins, 2011)

- Translation also available for
  - Nonperturbative (g) functions
  - A, B
  - Other resummation formulations

Collins, TCR (2017)

TMD Factorization: Translation of Results

Translate different versions of TMD results: Collins, TCR (2017)

Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

α_s^2 Wilson Coefficients from Collinear Factorization: (Catani et al, (2012)), and SCET (Echevarria, Scimemi, Vladimirov (2016))

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} &= \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
&\times e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
&\times e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
&\times \exp\left\{ -g_K(b_{\mathrm{T}};b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu')) \right] \right\} \\
&+ \text{suppressed corrections.}
\end{aligned}
$$

Ex: Konychev, Nadolsky (2006)
ResBos extractions (and others)

Li, Zhu (2017)
Vladimirov (2017)

From
Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

TMD Factorization: Translation of Results

Translate different versions of TMD results: Collins, TCR (2017)

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = {} & \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
& \times e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
& \times e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{ -g_K(b_{\mathrm{T}};b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu')) \right] \right\} \\
& + \text{suppressed corrections.}
\end{aligned}
$$

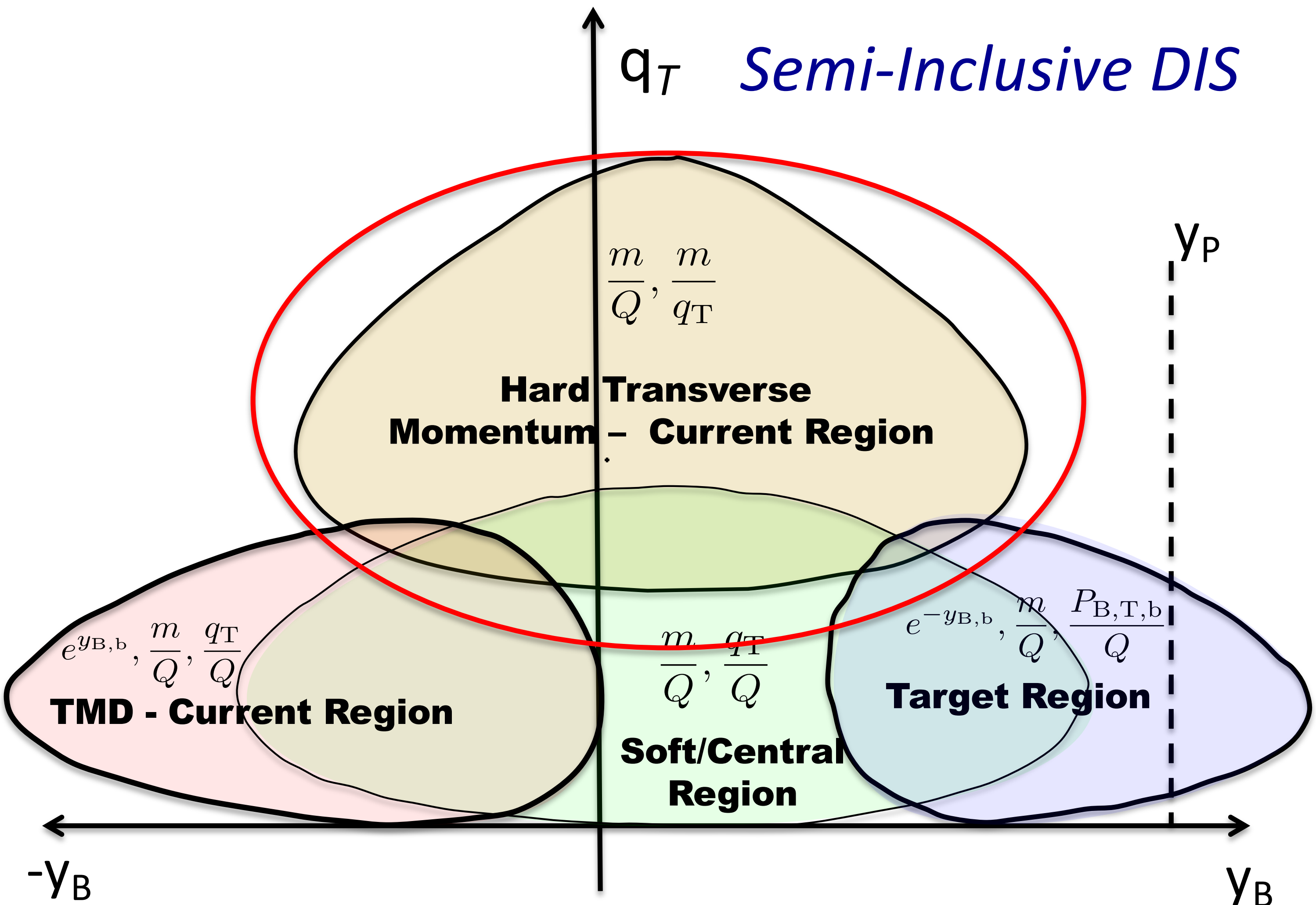
Lattice Calculations, e.g.

Ji, Sun, Xiong, Yuan, PR91 (2015)
Ebert, Stewart and Zhao., JHEP09(2019)037

Open Problems

- How low can scales be?
- Description of large transverse momentum

What is the relevant description?



Catalogue of Regions: Boglione et al (To appear in JHEP)

Large and Small Transverse Momentum

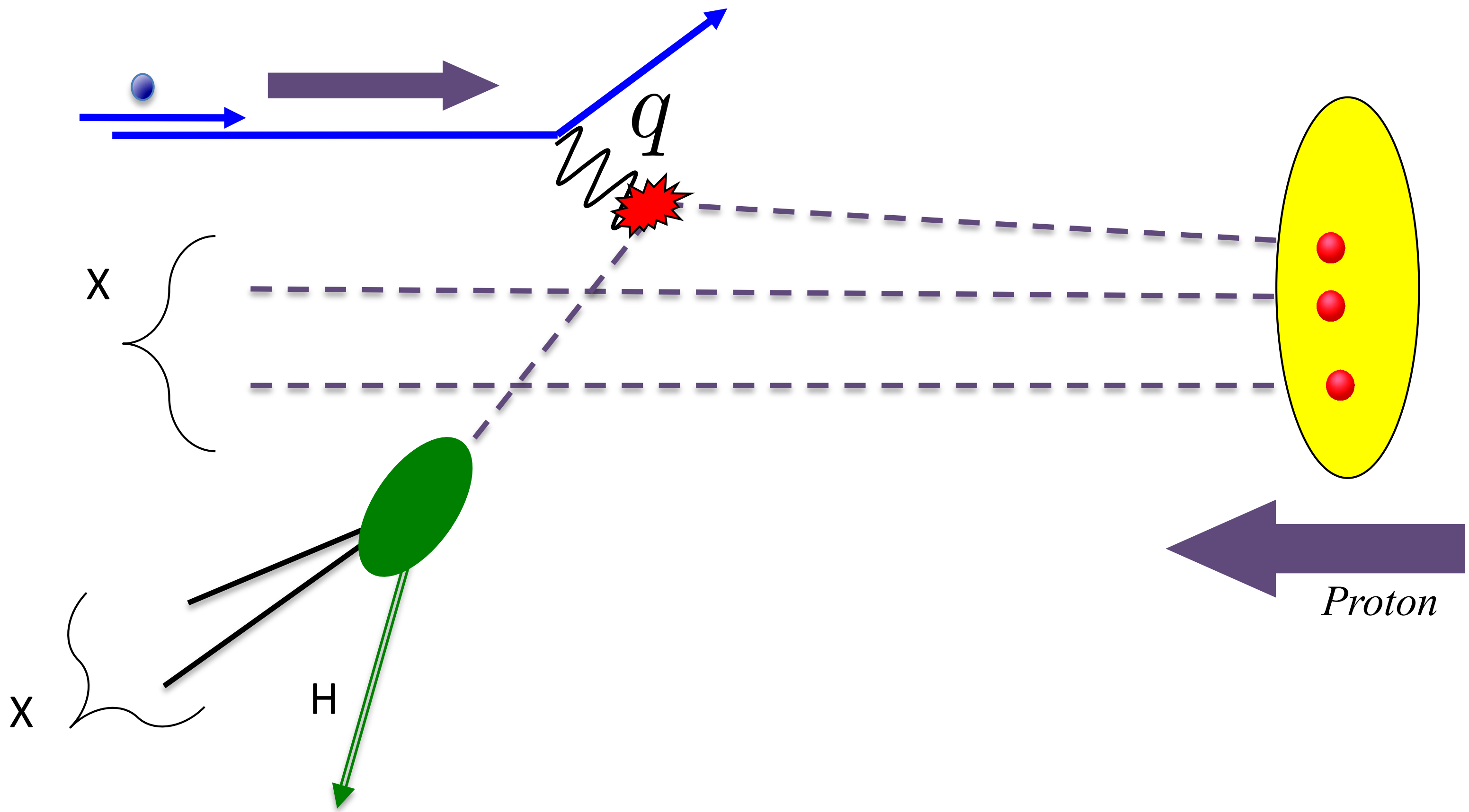
- TMD pdfs in physical processes
  - Small q_T/Q expansion
    - Need to separate small and large q_T/Q

$$\frac{d\sigma}{dx\, dz\, dQ\, dq_T} = \underbrace{H(Q) f(x, k_{1T}) \otimes d(z, z\, k_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{approximation} \\ \text{(TMD factorization)}}} + \underbrace{Y(x, z, q_T)}_{\substack{q_T \sim Q \\ \text{correction} \\ \text{(collinear factorization)}}} + \text{P.S.C.}$$

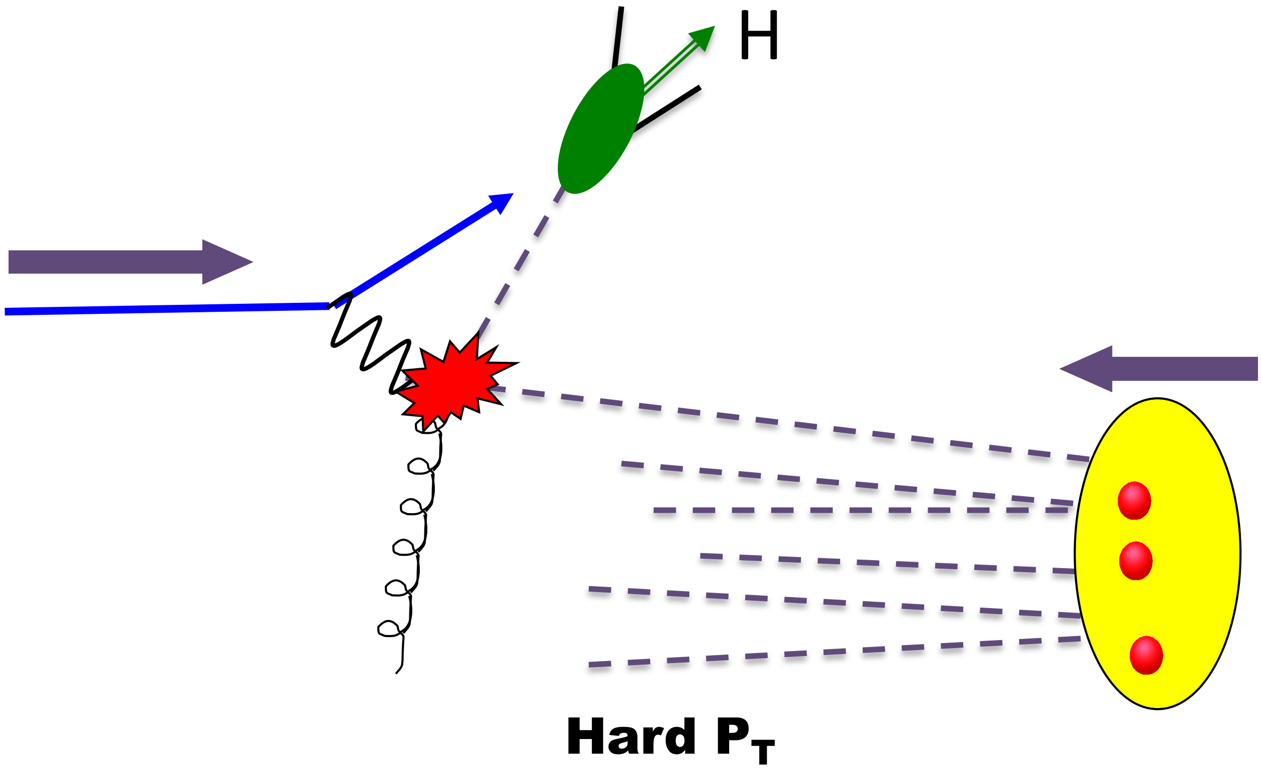
- Works best if there is a region of overlap between large and small q_T/Q methods:

$$Y(x, z, q_T) = \text{Large } \ q_T/Q \ \text{ Approx.} \ - \text{Small and Large } \ q_T/Q \ \text{ Approx.}$$

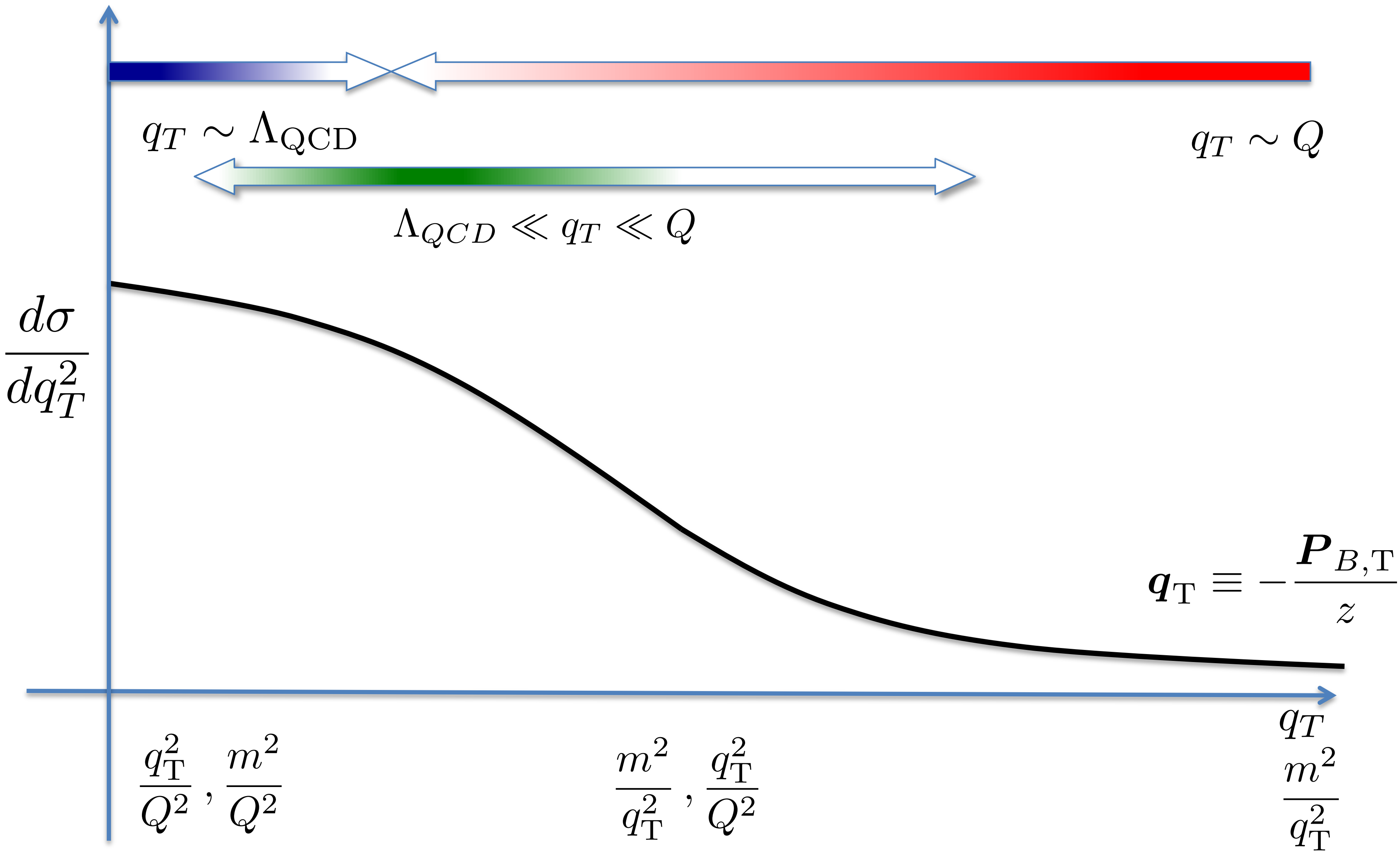
Example: SIDIS



Example: SIDIS

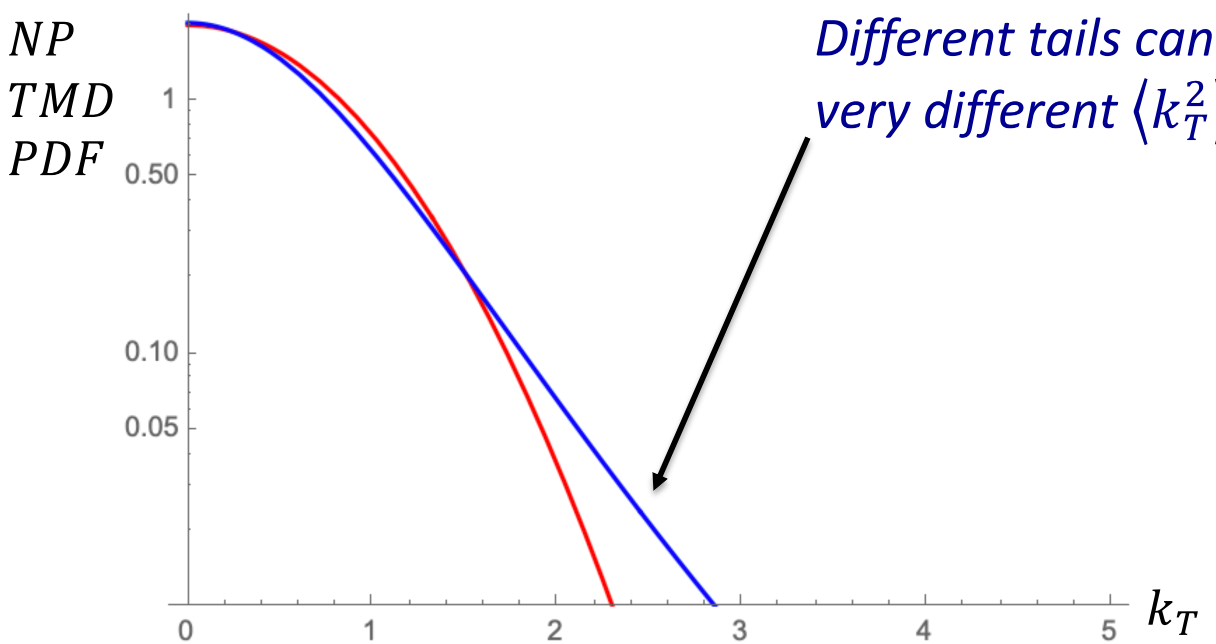


Large and Small Transverse Momentum



Why Large Transverse Momentum?

- Example: Shapes of TMD distributions:
  - Sea versus valence?
  - Different flavors?



Why Large Transverse Momentum?

- Interpretation of integrals

$$\int d^2\mathbf{k}_T f(x,k_T) = f(x), \quad \int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x), \quad \ldots$$

- In generalized parton model:

$$\frac{d\sigma}{dx\,dz\,dQ\,dq_T} = H(Q)f(x,k_{1T}) \otimes d(z,z\,k_{2T}) + Y(x,z,q_T) + \text{P.S.C.}$$

$$= H(Q)\int d^2\mathbf{k}_{1T}\int d^2\mathbf{k}_{2T}\, f(x,k_{1T})d(z,k_{2T})\delta^{(2)}(\mathbf{k}_{2T}-\mathbf{k}_{1T}-\mathbf{q}_T)$$

Assume all contributions to transverse momentum dependence are intrinsic

Why Large Transverse Momentum?

- Integrated cross section in generalized parton model

$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\,dz\,dQ\,dq_T}\right) = H(Q)\left(\int d^2\mathbf{k}_{1T}\, f(x,k_{1T})\right)\left(\int d^2\mathbf{k}_{2T}\, d(z,z\,k_{2T})\right)$$

$$= H(Q)f(x)d(z)$$

- Full integral

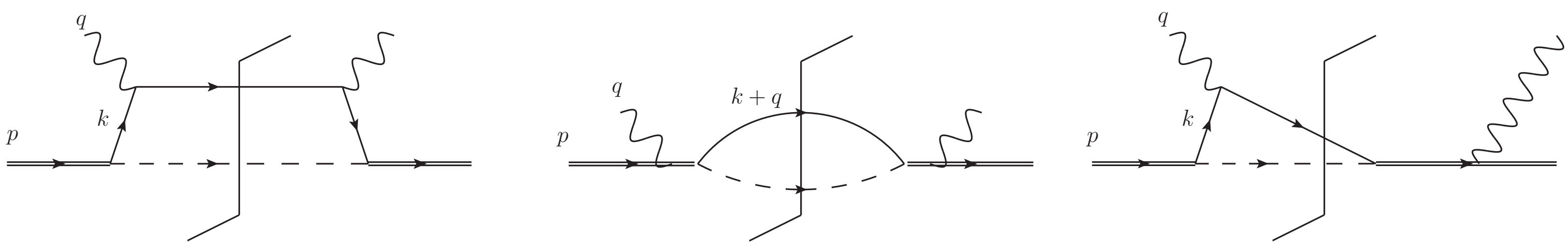
$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\,dz\,dQ\,dq_T}\right) = \int d^2\boldsymbol{q}_T \left(H(Q)f(x,k_{1T}) \otimes d(z,z\,k_{2T}) + Y(x,z,q_T)\right)$$

Cutoff dependence cancels between terms

Large Transverse Momentum

- $q_T \approx Q$ outside the region where TMD factorization is applicable

- Still needed for TMD pdf identification



(a) (b) (c)

$$\mathcal{L}_{\rm int} = -\lambda\,\overline{\Psi}_N\,\psi_q\,\phi \;+\; \text{H.C.}$$

- Exact $O(\lambda^2)$ cross section is easy to calculate

Example

- Collinear Factorization

$$F_1(x,Q) = \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi - x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

$$\times \underbrace{\left\{\delta(1-\xi)\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q + \xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

Partonic structure function

Parton Distribution

Effect from integrating $k_T \to \infty$ cancels

Example

- TMD Factorization

$$F_1^W(x,z,\boldsymbol{k}_{\mathrm{T}},Q) = \hat{F}_1^W \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}}\delta^{(2)}(\boldsymbol{k}_{1\mathrm{T}}+\boldsymbol{k}_{\mathrm{T}}-\boldsymbol{k}_{2\mathrm{T}})f(x,\boldsymbol{k}_{1\mathrm{T}};\mu)d(z,z\boldsymbol{k}_{2\mathrm{T}};\mu)$$

$$\hat{F}_1^W = \frac{1}{2}$$

$$f(x,\boldsymbol{k}_{1\mathrm{T}};\mu) = \frac{a_\lambda(\mu)}{\pi}\frac{(1-x)\left[k_{\mathrm{T}}^2+(m_q+xm_p)^2\right]}{\left[k_{\mathrm{T}}^2+xm_s^2+(1-x)\,m_q^2+x(x-1)m_p^2\right]^2}$$

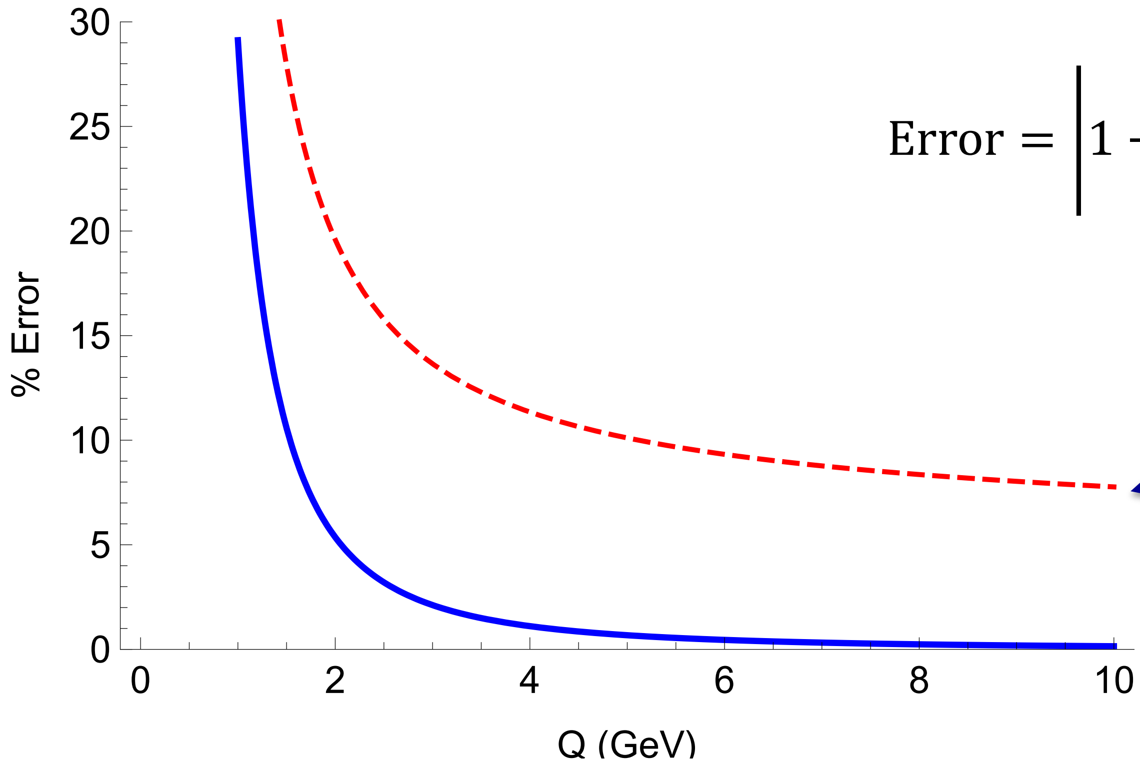
$$d(z,z\boldsymbol{k}_{2\mathrm{T}};\mu) = \delta(1-z)\delta^{(2)}(z\boldsymbol{k}_{2\mathrm{T}})$$

Example

- Accuracy

Collinear Factorization

Integrated TMD factorization



$$\text{Error} = \left| 1 - \frac{F_1^{\text{Factorized}}}{F_1^{\text{Exact}}} \right|$$

$$\int_0^Q d^2 k_T \, [F_1(x, k_T)]_{\text{TMD fact.}}$$

Simplest Processes with Transverse Momentum

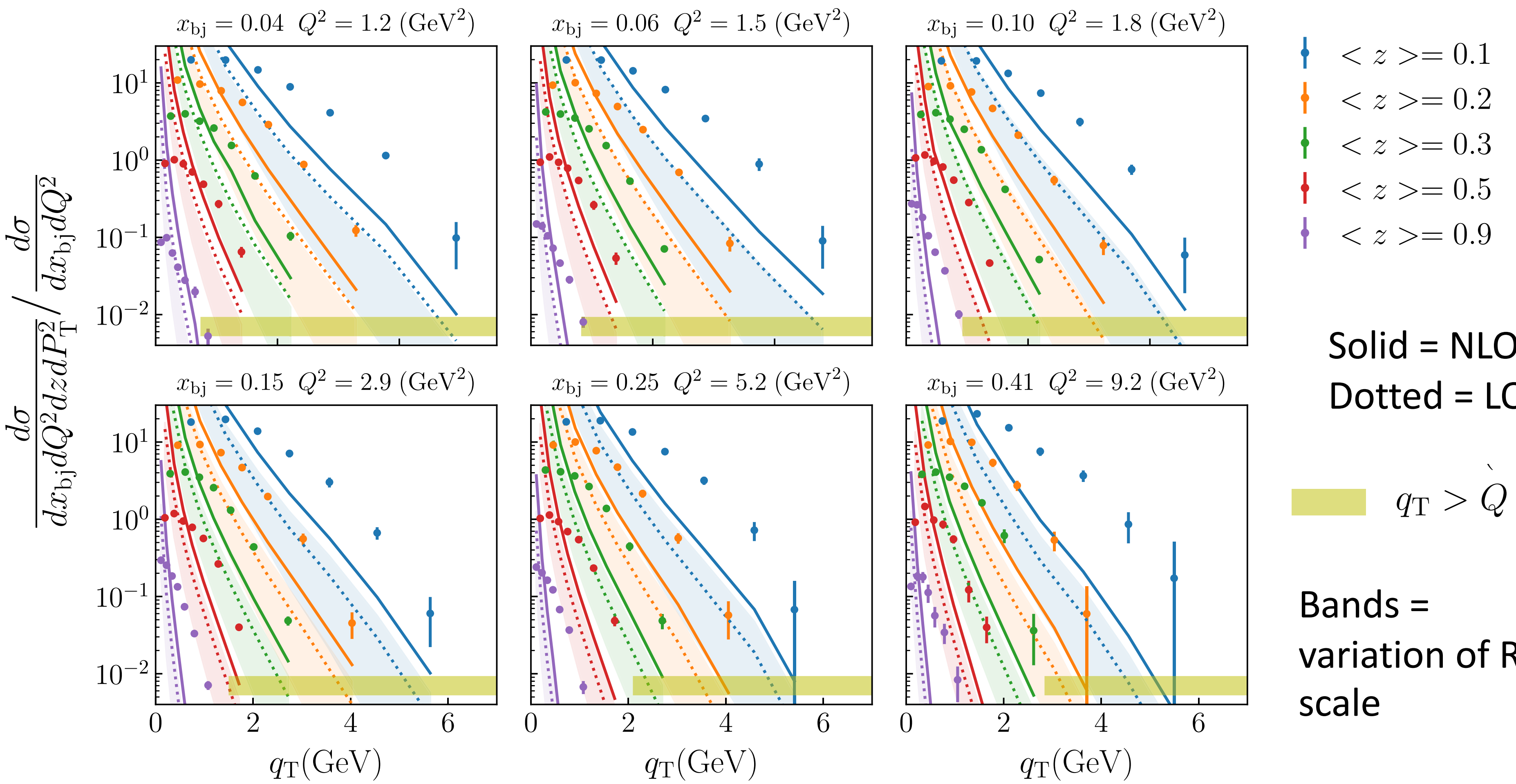
- Semi-inclusive DIS (JLab, EIC,…)
- Drell-Yan
- Dihadron production in e^+e^- annihilation (Belle)

SIDIS

- NLO study near the valence region.

B. Wang, J. O. Gonzalez-Hernandez, TR, N. Sato
Phys. Rev. D 99, 094029

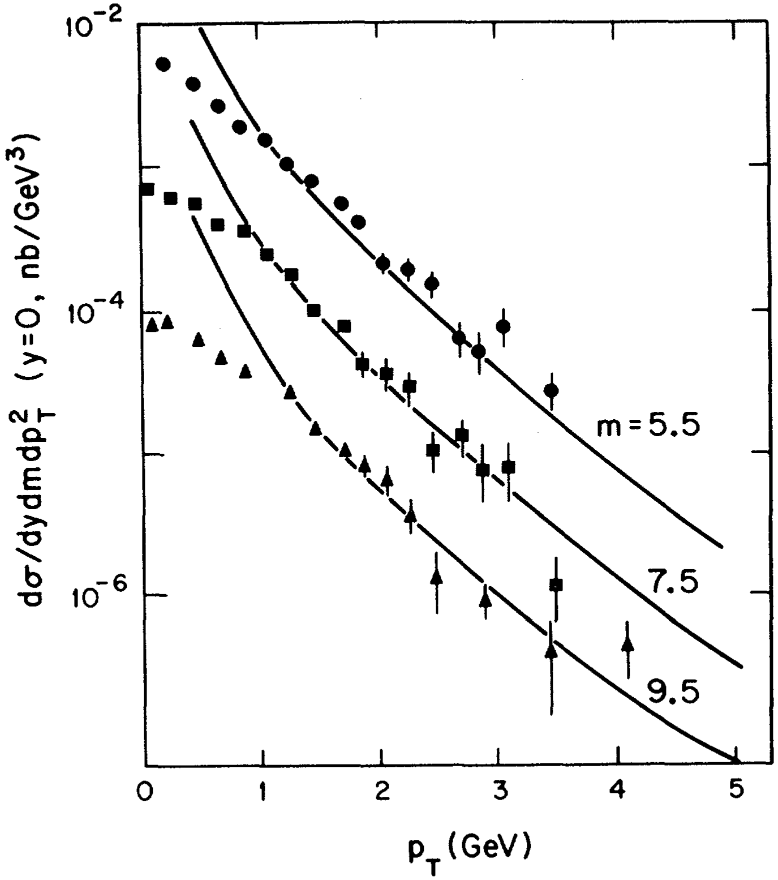
https://jeffersonlab.github.io/BigTMD/_build/html/index.html



Solid = NLO
Dotted = LO

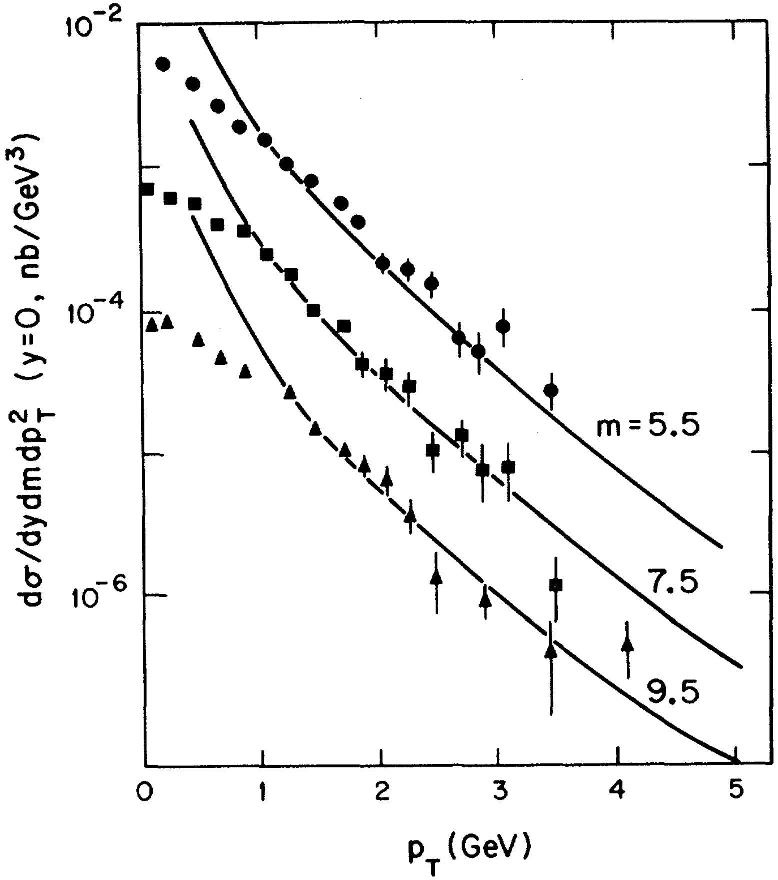
Bands = variation of RG scale

Drell-Yan: Old Results

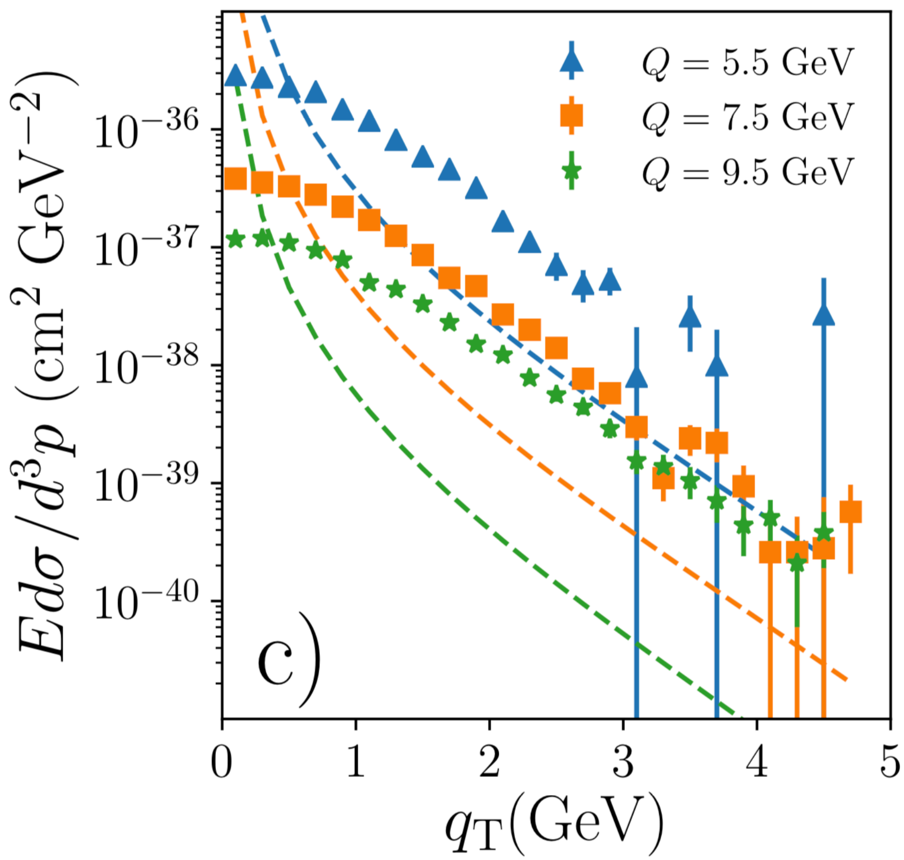


- Halzen & Scott, Phys. Rev. Lett. 40, 1117 (1978)

Drell-Yan: Old Results



- Halzen & Scott, Phys. Rev. Lett. 40, 1117 (1978)

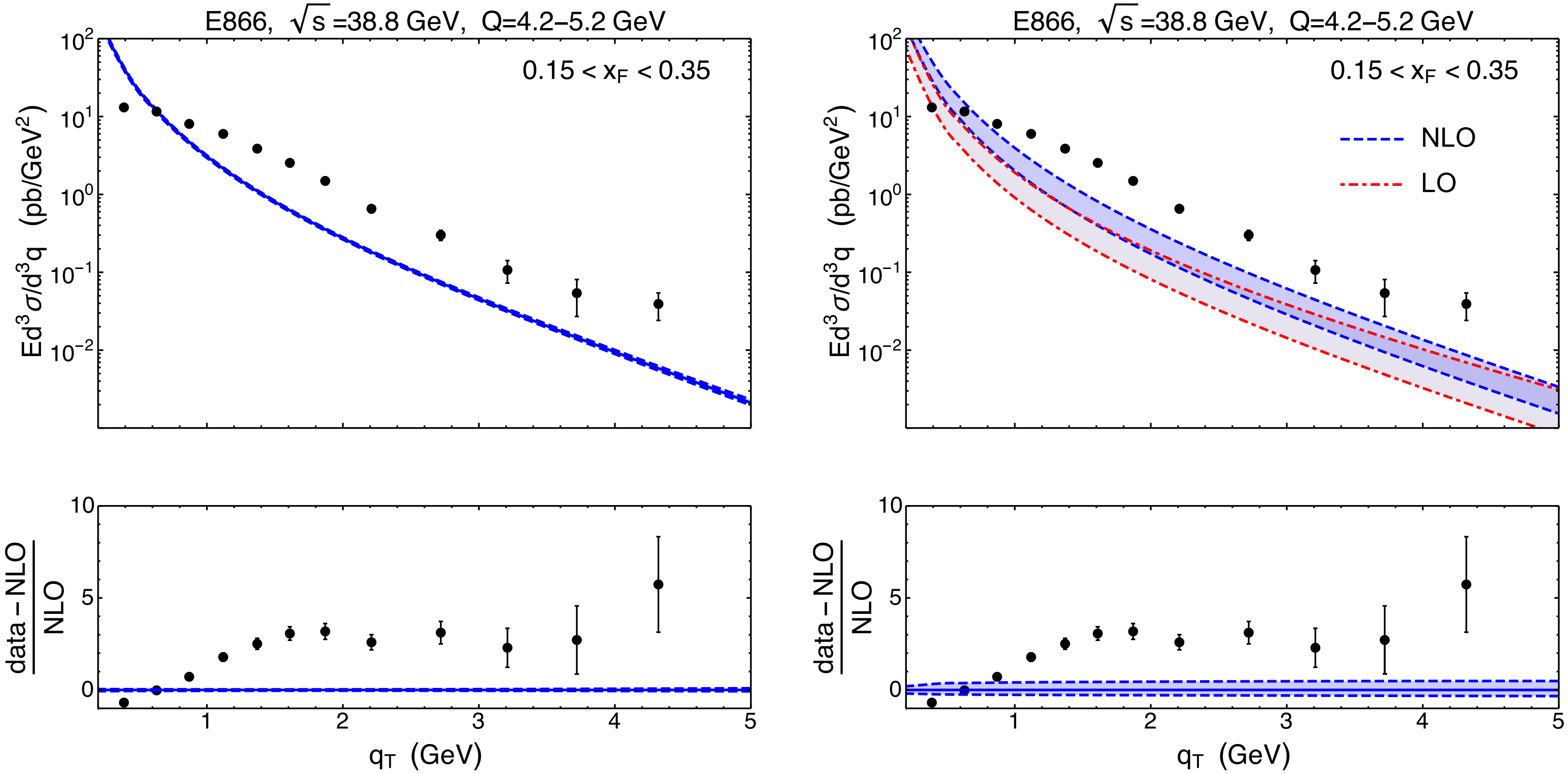


Corrected data
+ modern pdf set

Data from A. S. Ito et al., Phys. Rev. D23, 604 (1981)

Drell-Yan: New Results

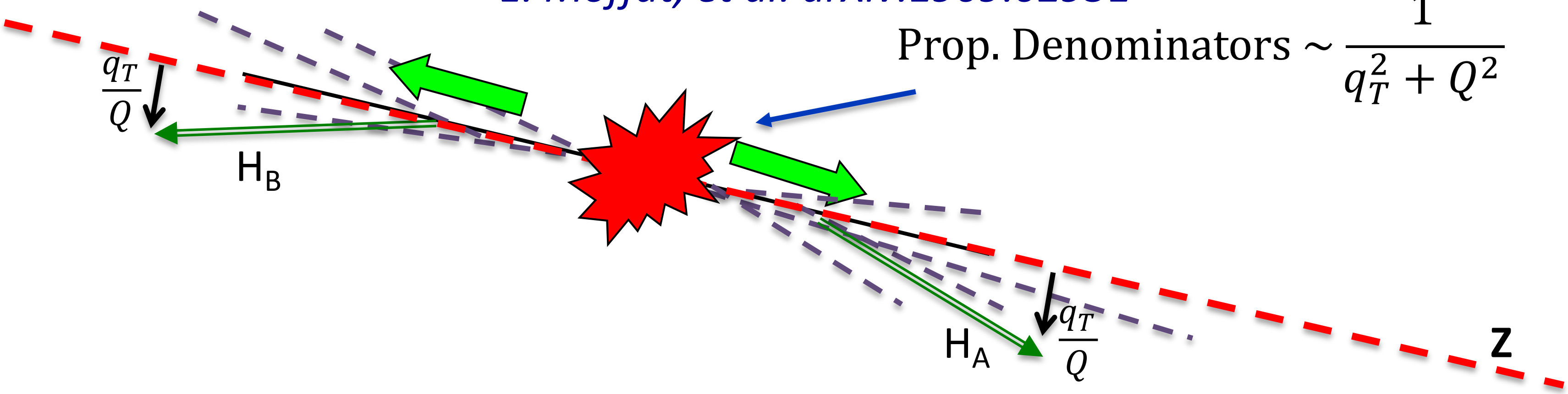
- Similar trend



Bacchetta et al., Phys.Rev. D100 (2019) no.1, 014018

Dihadron Production in e^+e^- Annihilation

E. Moffat, et al: arXiv:1909.02951



- If q_T/Q is small, TMD fragmentation functions are needed.

- If q_T/Q is large, there is recoil against a high momentum parton, and collinear factorization is needed:

$$z_A = \frac{p_A \cdot p_B}{q \cdot p_B}$$

$$\hat{z}_A = z_A/\zeta_A$$

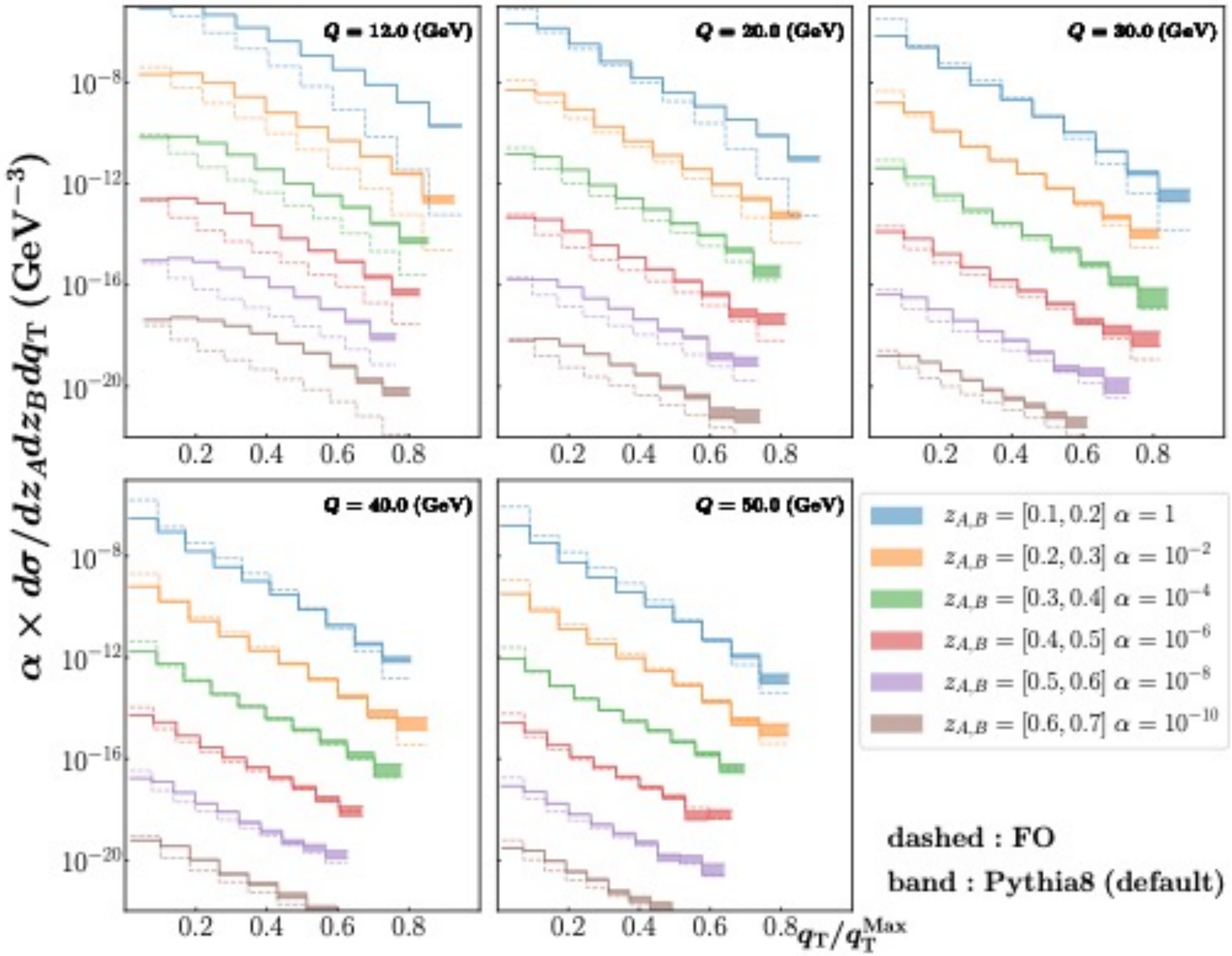
$$z_B = \frac{p_A \cdot p_B}{q \cdot p_A}$$

$$\hat{z}_B = z_B/\zeta_B$$

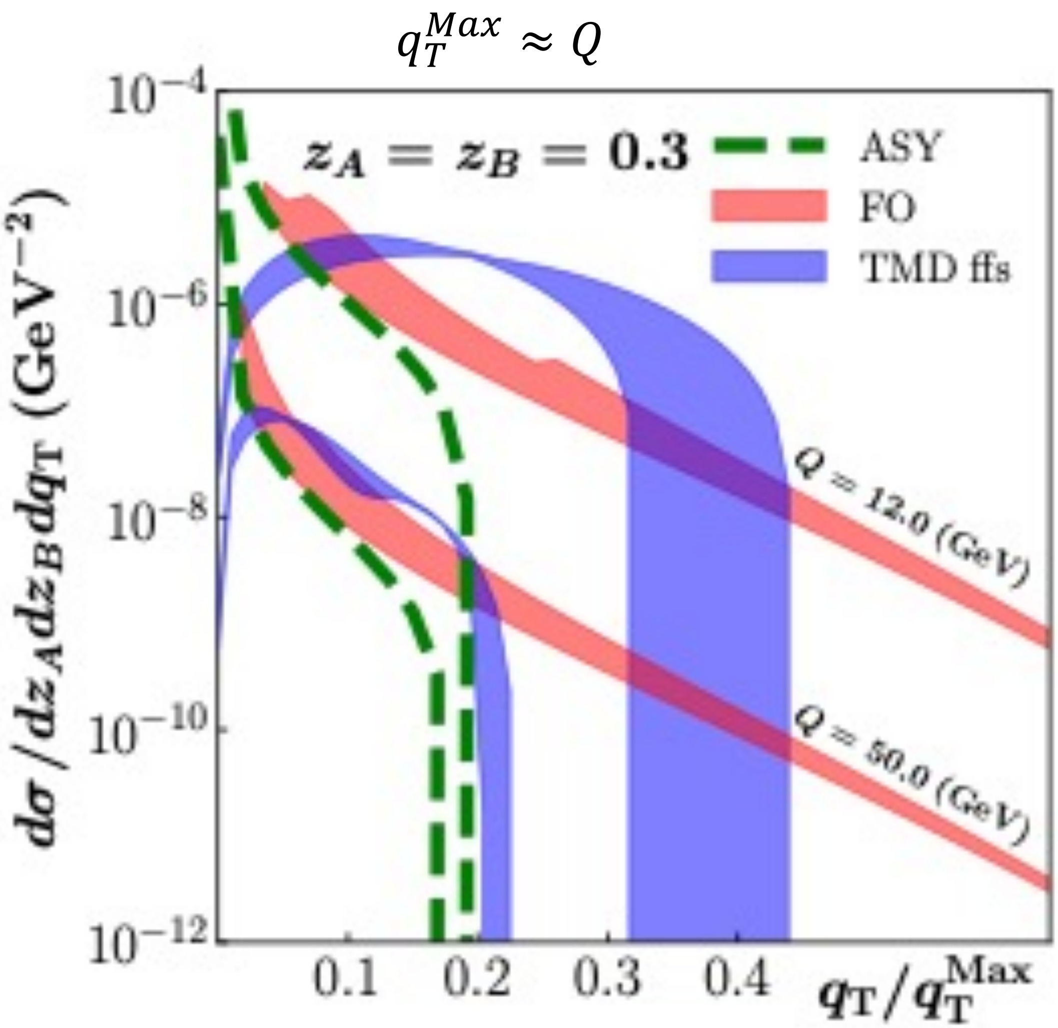
$$E_A E_B \frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}^3\boldsymbol{p}_A \mathrm{d}^3\boldsymbol{p}_B} = \sum_{i,j} \int_{z_A}^{1} \mathrm{d}\zeta_A \int_{z_B}^{1} \mathrm{d}\zeta_B \left(E_A E_B \frac{\mathrm{d}\hat{\sigma}_{ij}(\hat{z}_A, \hat{z}_B)}{\mathrm{d}^3\boldsymbol{p}_A \mathrm{d}^3\boldsymbol{p}_B} \right) d_{H_A/i}(\zeta_A) d_{H_B/j}(\zeta_B)$$

Dihadron Production in e^+e^- Annihilation

- Validate with event generator



Electron-Positron Annihilation



- Blue band:
  - from survey of non-perturbative fits
- Pink band:
  - Large TM calculation, width from varying RG scale
- Green:
  - Small $q_T/Q \to 0$ asymptote
- No overlap in the transition region for smaller Q

Summary

- Translation prescription between different versions of TMD factorization is established
- Can large transverse momentum be explained with standard collinear perturbation theory?
- Focus more on collinear factorization and understand the transition from small to large Q