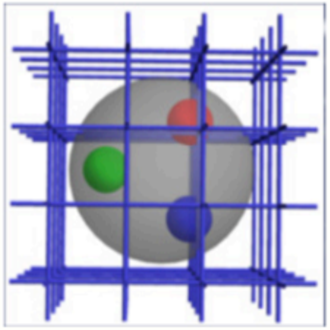

W. K . Kellogg Biological Station

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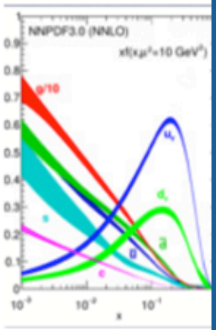


Parton Distributions and Lattice Calculations (PDFLattice 2019)

25-27 September 2019

GPDs – Overview

Jianwei Qiu

Theory Center, Jefferson Lab

September 26, 2019

Jefferson Lab

TMD Collaboration

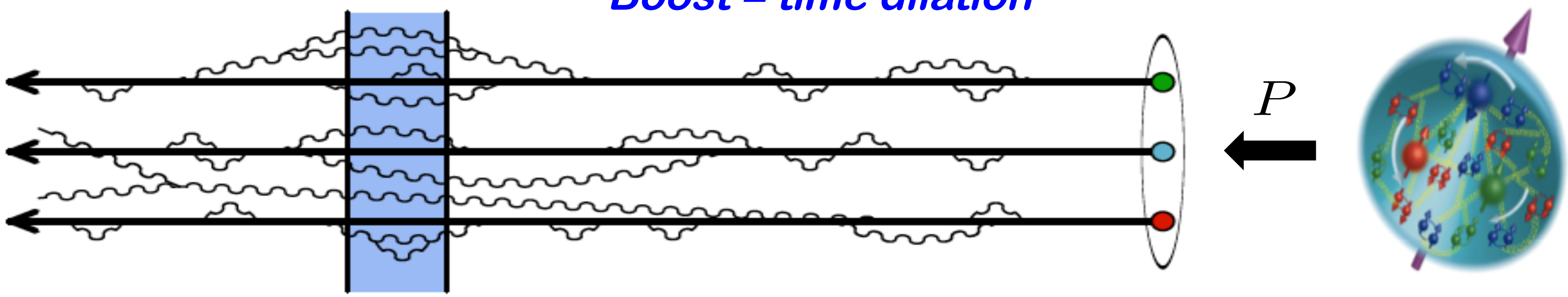
U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

How to “see” 3D partonic structure of hadrons?

- **Hard probes to “catch” the quantum fluctuation:**

Boost = time dilation

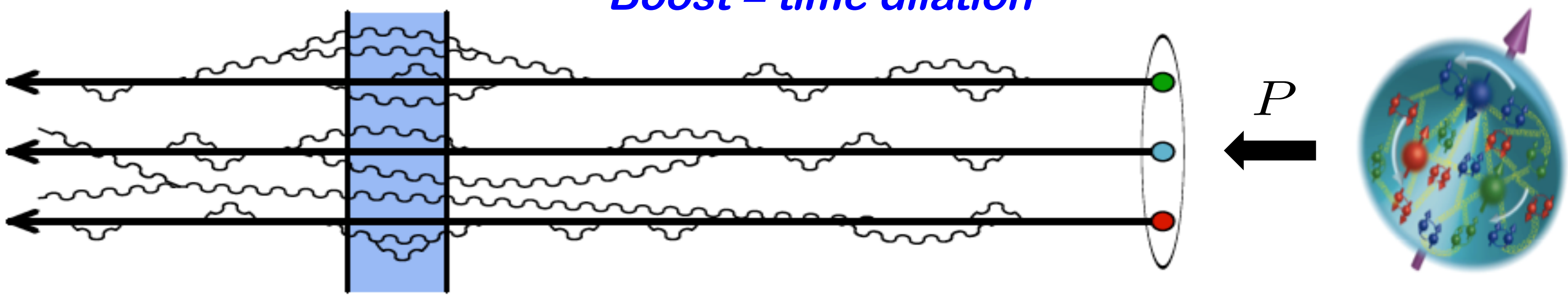


Hard probe ($t \sim 1/Q <$ fm) ➡ Probability to “catch” the parton!

How to "see" 3D partonic structure of hadrons?

- **Hard probes to "catch" the quantum fluctuation:**

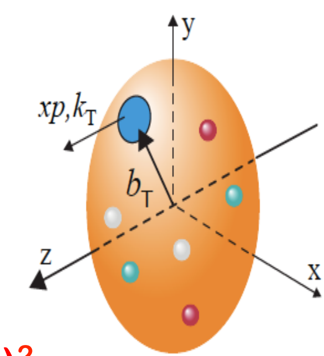
Boost = time dilation



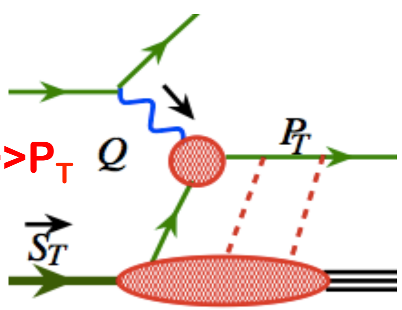
Hard probe ($t \sim 1/Q <$ fm) → Probability to "catch" the parton!

- **Observables with two momentum scales:** $Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$

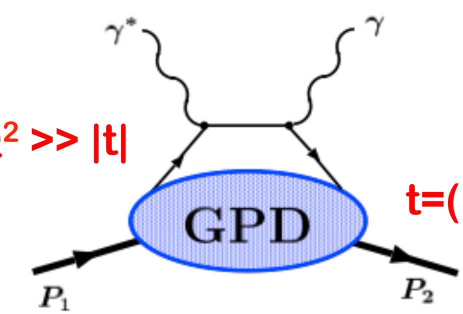
  - ✧ **Hard scale:** Q_1 "see" particle nature of "partons"
  - ✧ **Soft scale:** Q_2 "sensitive" to the fermi-scale structure



SIDIS: $Q \gg P_T$



DVCS: $Q^2 \gg |t|$



$t=(p_1-p_2)^2$

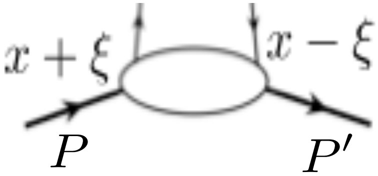
TMDs – Confined motion

GPDs – Spatial imaging

Definition of GPDs

❑ Quark "form factor":

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+ q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

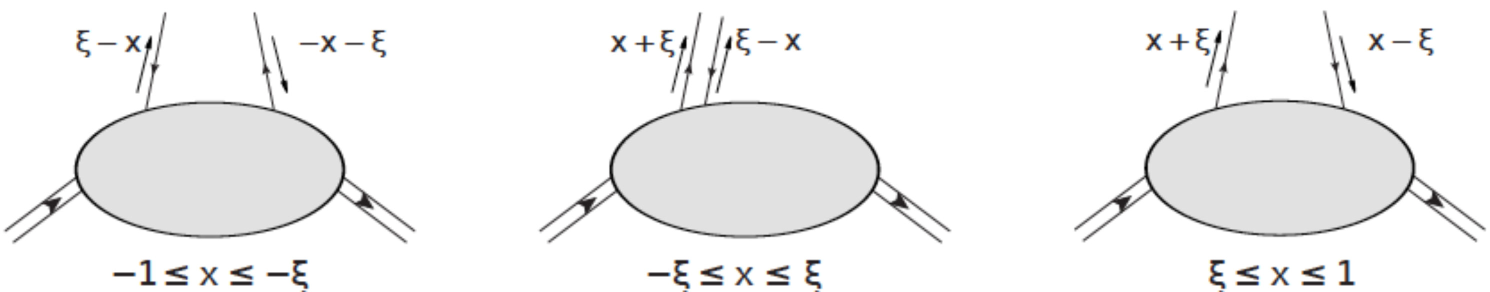


$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

with $\xi = (P'-P)\cdot n/2 \;\; \text{and} \;\; t = (P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2 \;\; \text{if} \;\; \xi \to 0$

Gauge link: $W[a,b] = P\exp\left(ig\int_b^a dx^- A^+(x^- n_-)\right)$

❑ Kinematics:



Two more for quarks: $\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$

with $\gamma\cdot n \longrightarrow \gamma\cdot n\,\gamma_5$

Definition of GPDs

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

❑ Gluon "form factor":

$$F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|G^{+\mu}(-\tfrac{1}{2}z)G_\mu{}^{+}(\tfrac{1}{2}z)|p\rangle \Big|_{z^+=0,\mathbf{z}=0}$$

$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Two more for gluons: $\tilde{H}^g(x,\xi,t)$ $\tilde{E}^g(x,\xi,t)$

with the two gluon field strength contracted anti-symmetrically

❑ Forward limit – connection to collinear PDFs:

$$H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x) \qquad \text{for } x > 0$$

$$H^q(x,0,0) = -\bar{q}(-x), \quad \tilde{H}^q(x,0,0) = \Delta\bar{q}(-x) \quad \text{for } x < 0$$

$$H^g(x,0,0) = xg(x), \quad \tilde{H}^g(x,0,0) = x\Delta g(x) \quad \text{for } x > 0$$

The factorization scale dependence is suppressed

Properties of GPDs

❑ Connection to Dirac and Pauli form factors:

$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \quad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t)$$

Where $$\langle p'|\bar{q}(0)\gamma^\mu q(0)|p\rangle = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{\mathrm{i}\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right]u(p)$$

And the axial and pseudoscalar version:

$$\int_{-1}^{1} \mathrm{d}x\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \quad \int_{-1}^{1} \mathrm{d}x\, \tilde{E}^q(x,\xi,t) = g_P^q(t)$$

Where $$\langle p'|\bar{q}(0)\gamma^\mu\gamma_5 q(0)|p\rangle = \bar{u}(p')\left[g_A^q(t)\gamma^\mu\gamma_5 + g_P^q(t)\frac{\gamma_5\Delta^\mu}{2m}\right]u(p)$$

❑ Some symmetry properties:

$$H^g(x,\xi,t) = H^g(-x,\xi,t) \qquad E^g(x,\xi,t) = E^g(-x,\xi,t)$$

$$\tilde{H}^g(x,\xi,t) = -\tilde{H}^g(-x,\xi,t) \qquad \tilde{E}^g(x,\xi,t) = -\tilde{E}^g(-x,\xi,t)$$

$$H^{q,g}(x,\xi,t) = H^{q,g}(x,-\xi,t), \ldots$$

$$H^{q,g}(x,\xi,t)^* = H^{q,g}(x,-\xi,t), \ldots$$

GPDs are real value functions

Properties of GPDs

- **QCD energy-momentum tensor:**

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{q}\gamma^{(\mu} \mathrm{i}\overleftrightarrow{D}^{\nu)} q$$

$$T_g^{\mu\nu} = G^{\mu\alpha} G_\alpha{}^\nu + \tfrac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta}$$

- **Form factors:**

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = A_{q,g}(t) \bar{u} P^{(\mu}\gamma^{\nu)} u + B_{q,g}(t) \bar{u} \frac{P^{(\mu} \mathrm{i}\sigma^{\nu)\alpha} \Delta_\alpha}{2m} u + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{m} \bar{u}u + \bar{C}_{q,g}(t) m g^{\mu\nu} \bar{u}u$$

- **Light-cone helicity operator:**

$$J^3 = \int \mathrm{d}x^- \, \mathrm{d}^2\mathbf{x} \, M^{+12}(x) \quad \textbf{with} \quad M^{\alpha\mu\nu} = T^{\alpha\nu} x^\mu - T^{\alpha\mu} x^\nu$$

- **Connection to the proton spin:**

Ji, PRL78, 1997

$$\langle J_q^3 \rangle = \tfrac{1}{2}[A_q(0) + B_q(0)] \,, \quad \langle J_g^3 \rangle = \tfrac{1}{2}[A_g(0) + B_g(0)]$$

$$A_q(t) + B_q(t) = \int_{-1}^{1} \mathrm{d}x \, x [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

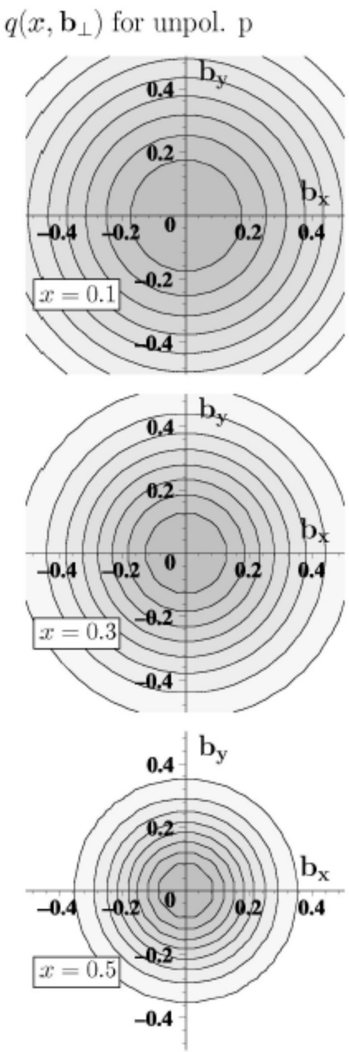
$$A_g(t) + B_g(t) = \int_{0}^{1} \mathrm{d}x [H_g(x,\xi,t) + E_g(x,\xi,t)]$$

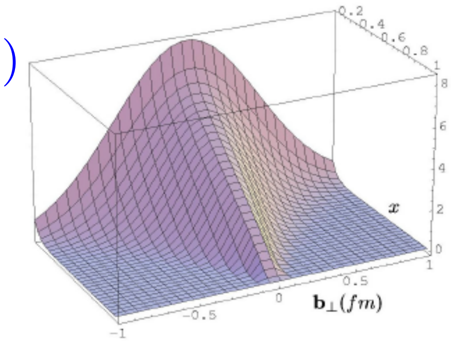
Spatial imaging from GPDs

M. Burkdart, PRD 2000

❑ Impact parameter dependent quark distribution:

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$





Unpolarized proton

- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

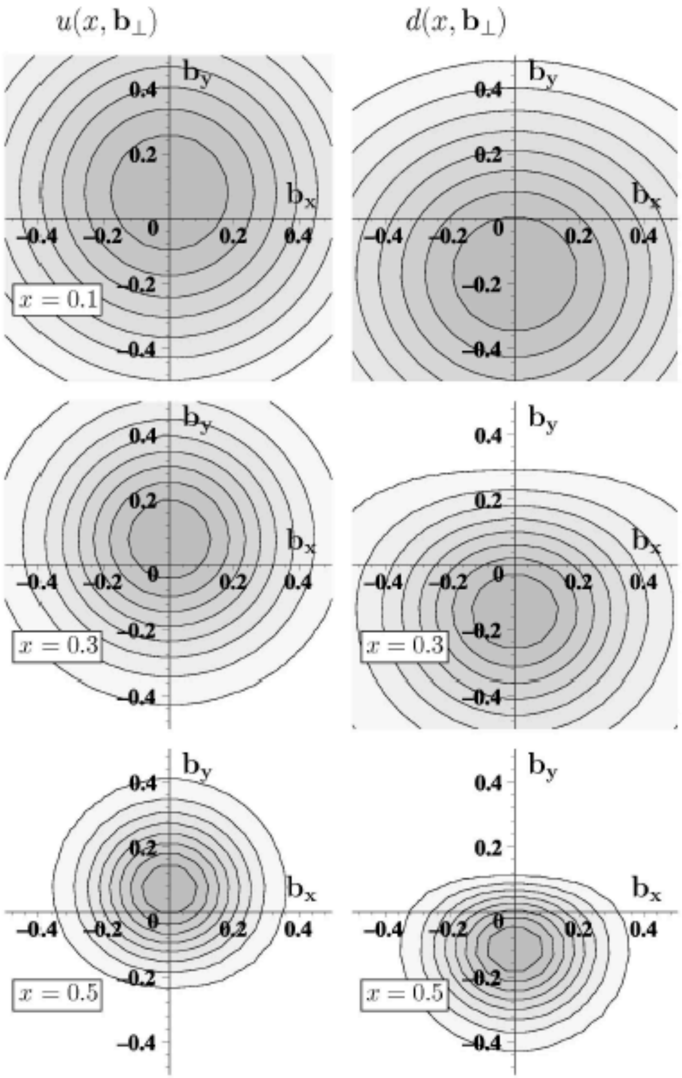
$\to$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$

Spatial imaging from GPDs

M. Burkdart, PRD 2000

❑ Impact parameter dependent quark distribution:

Proton polarized in +x direction



$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

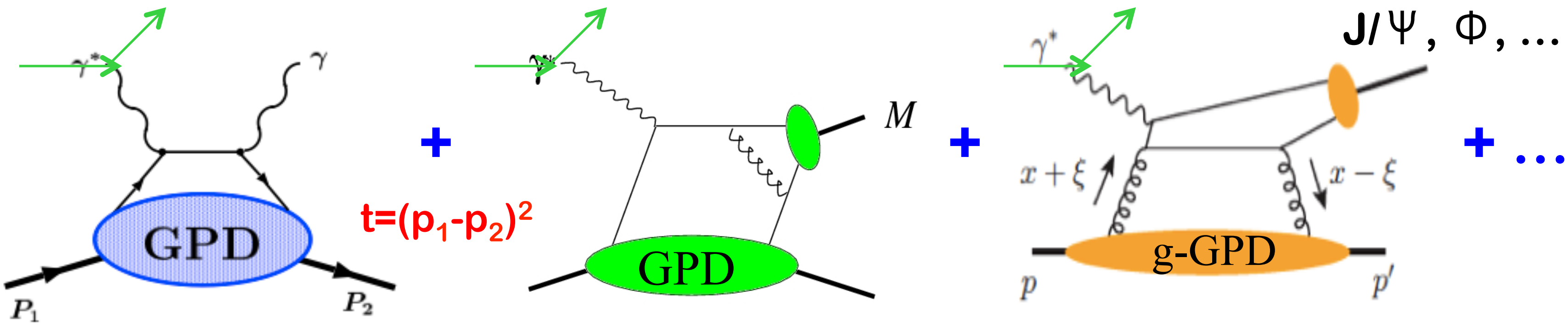
Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

Sign and magnitude of the averaged shift related to the hadron's magnetic moment:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$

Hunting for GPDs – Exclusive DIS

- **Experimental access to GPDs:**



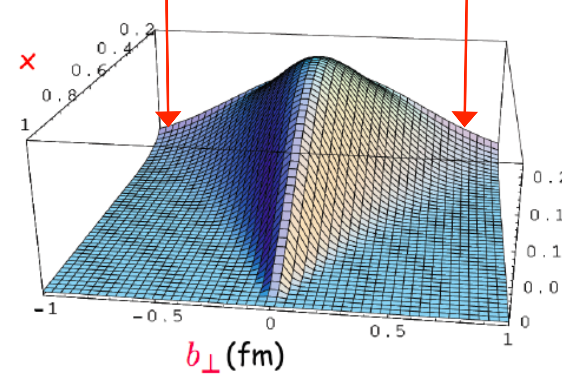
DVCS: $Q^2 >> |t|$ DVEM: $Q^2 >> |t|$ EHMP: $Q^2 >> |t|$

- **Much more complicated – (x, ξ, t) variables:**
  - Challenge to derive GPDs from data

- **GPDs could tell us:**
  - Orbital contribution to proton's spin
  - Proton radius of quark & gluon density
  - Hints for color confining radius/mechanism
  - Origin of nuclear force, …
  - …

How far does glue density spread?

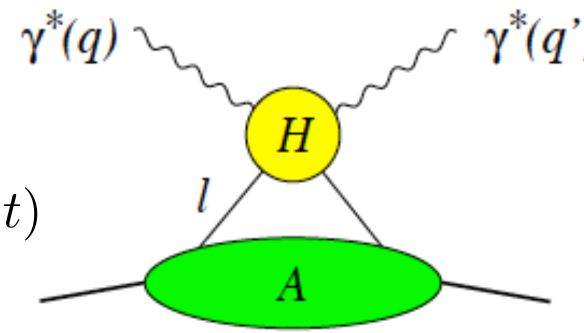
How fast does glue density fall?



QCD factorization

- **Deep Virtual Compton Scattering (DVCS):**

$$\gamma^*(q) + p(p) \to \gamma^*(q') + p(p')$$



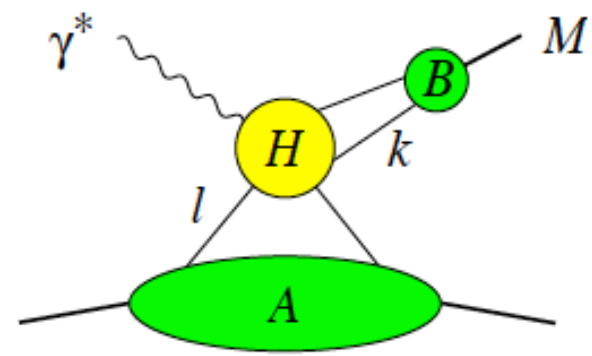
- **Factorization:**

$$\mathcal{A}(\gamma^* p \to \gamma p) = \sum_i \int_{-1}^{1} dx\, T^i(x,\xi,\rho,Q^2)\, F^i(x,\xi,t)$$

$$\rho = -(q+q')^2/2(p+p')\cdot(q+q')$$

- **Deep Virtual Meson Production (DVMP):**

$$\gamma^*(q) + p(p) \to M(q') + p(p')$$



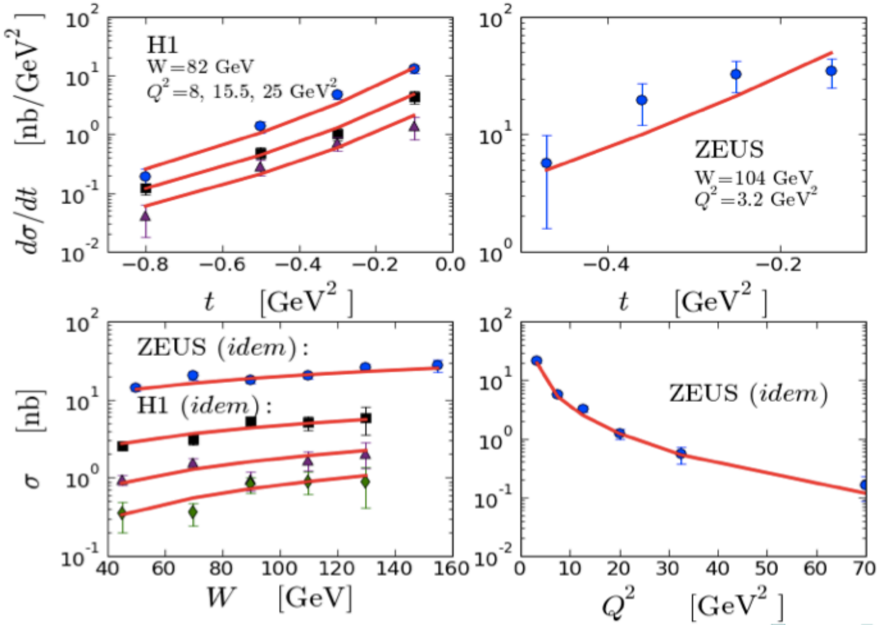
- **Factorization:**

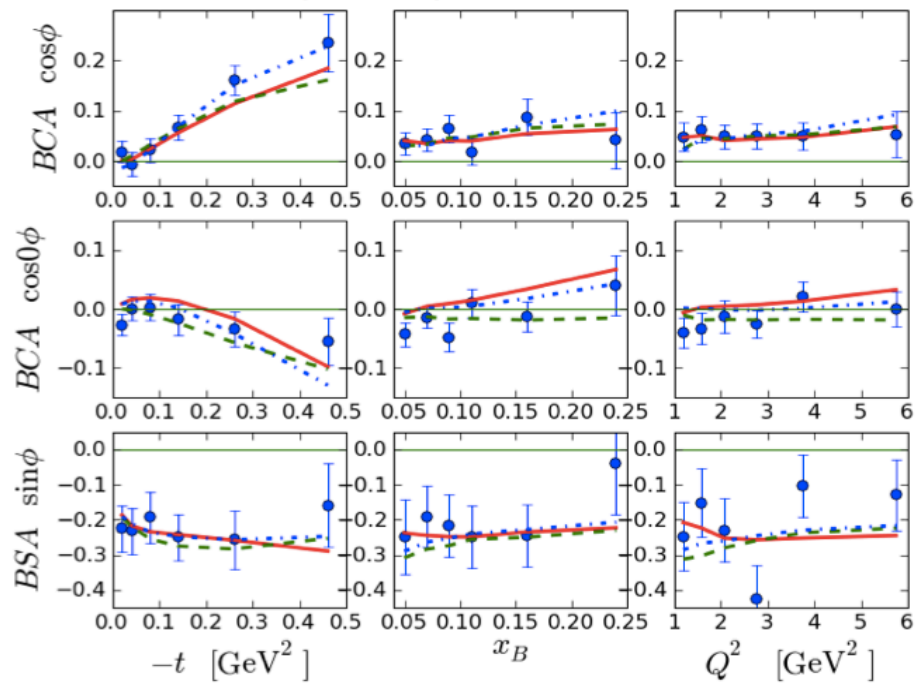
$$\mathscr{A}(\gamma_L^* p \to M_L p) = \frac{1}{Q}\sum_{ij}\int_{-1}^{1} \mathrm{d}x \times \int_0^1 \mathrm{d}z\, T^{ij}(x,\xi,z,Q^2) F^i(x,\xi,t) \Phi^j(z)$$

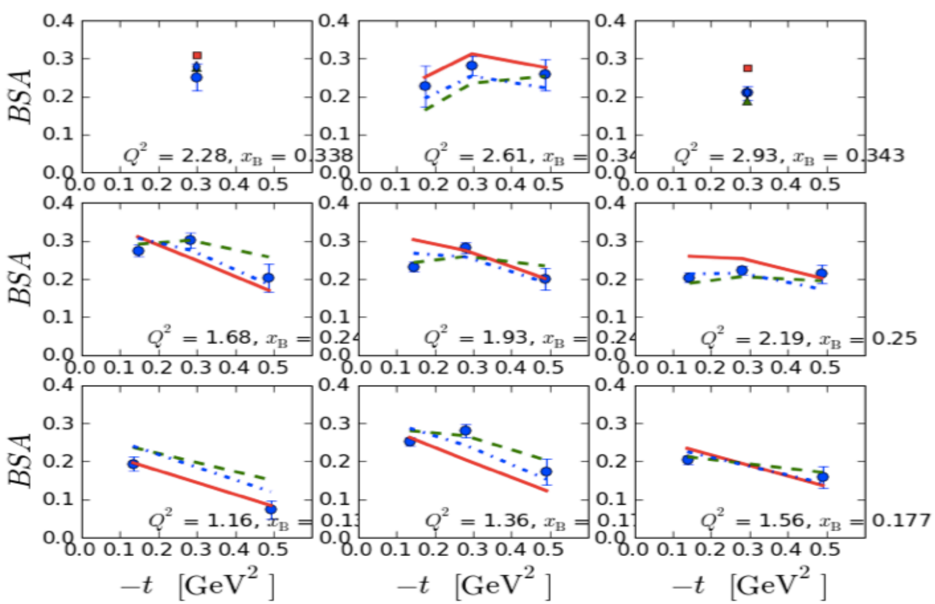
- **Evolution:**

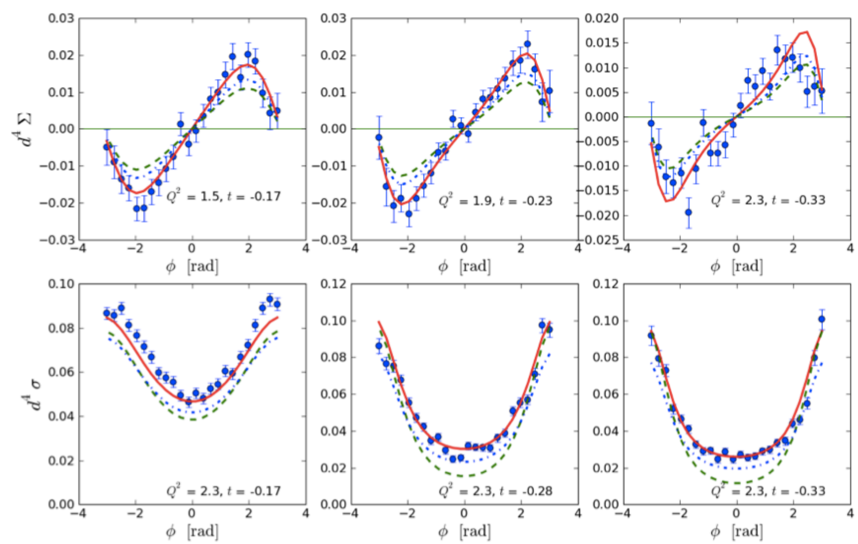
Factorization nationally lead to evolution equations for GPDs

Data: just the beginning



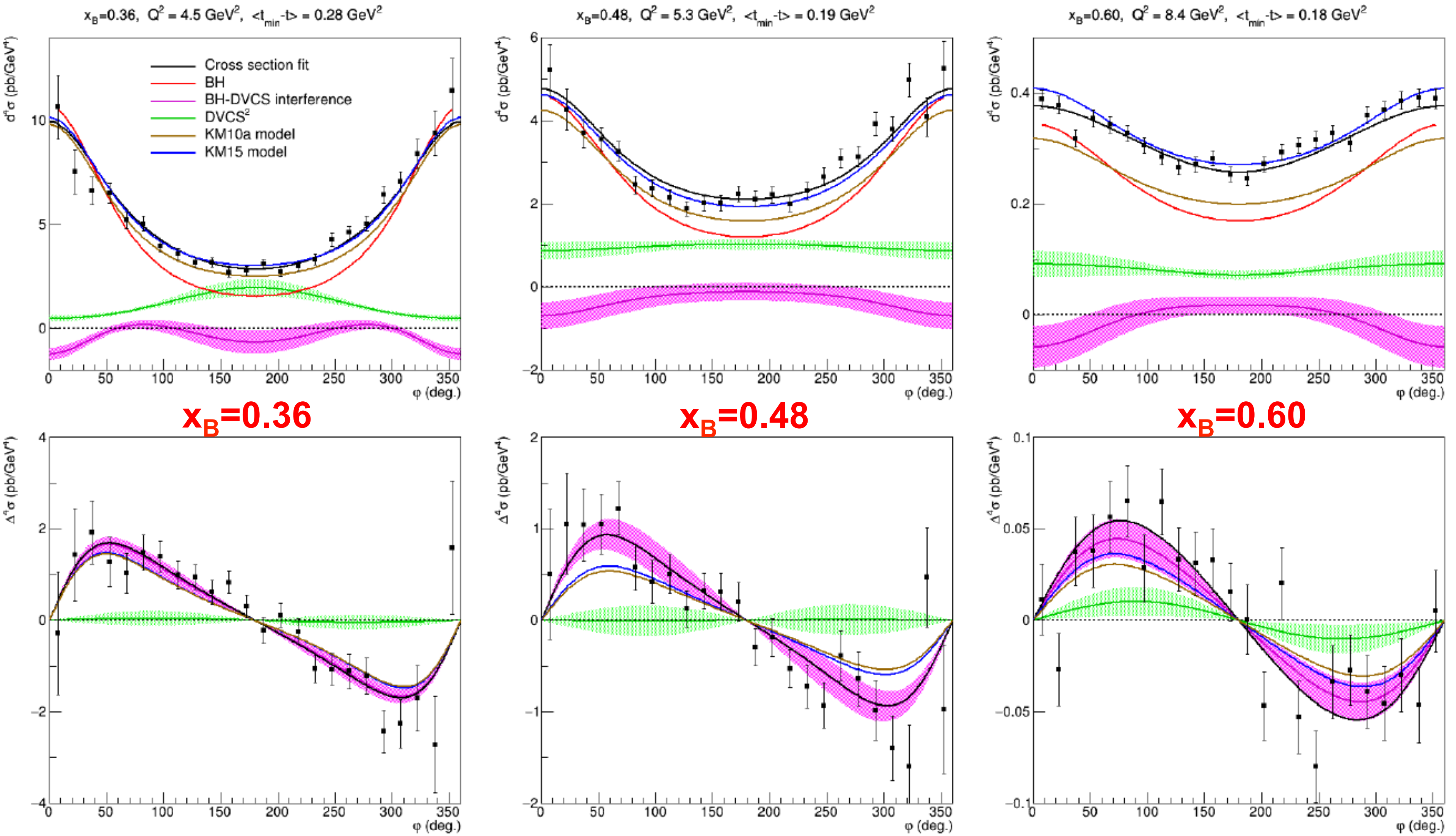






JLab E12-06-114 DVCS/Hall A Experiment at 11 GeV

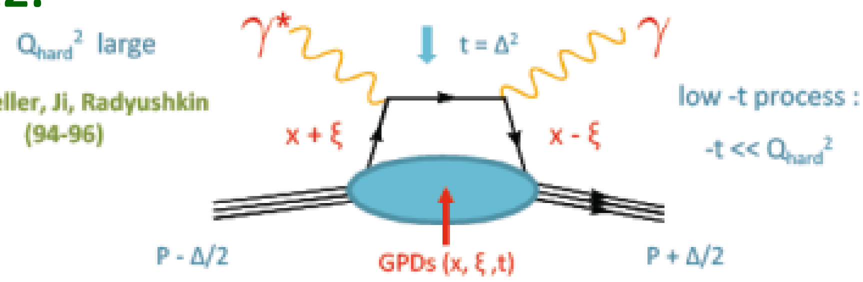
Sample of cross-section results:



Jefferson Lab

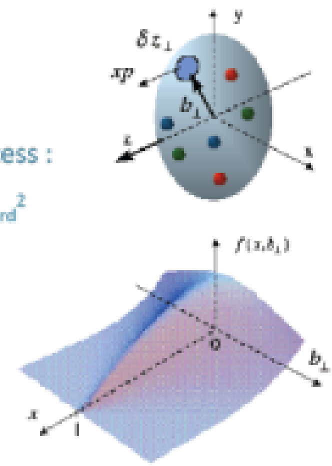
JLab DVCS/Hall B Experiment at 11 GeV

❑ **CLAS12:**



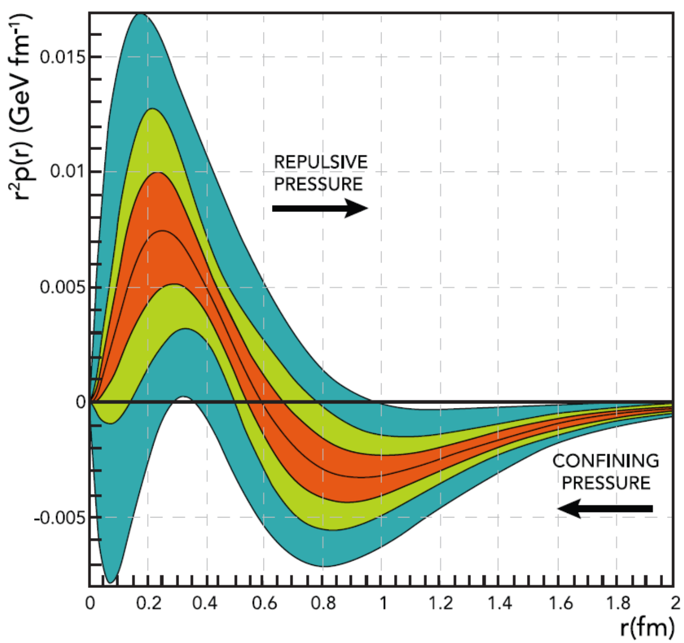
Mostly H for unpolarized proton target

Mostly $\tilde{H}$ for polarized proton target



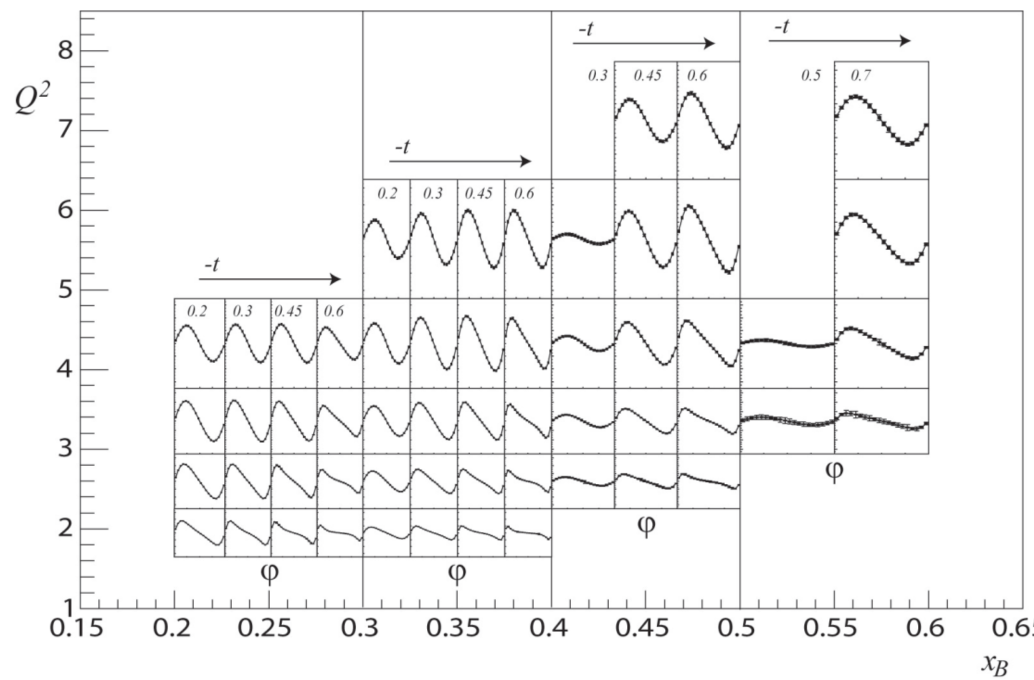
Hall A DVCS scaling check completed

Hall B DVCS on H 50% complete



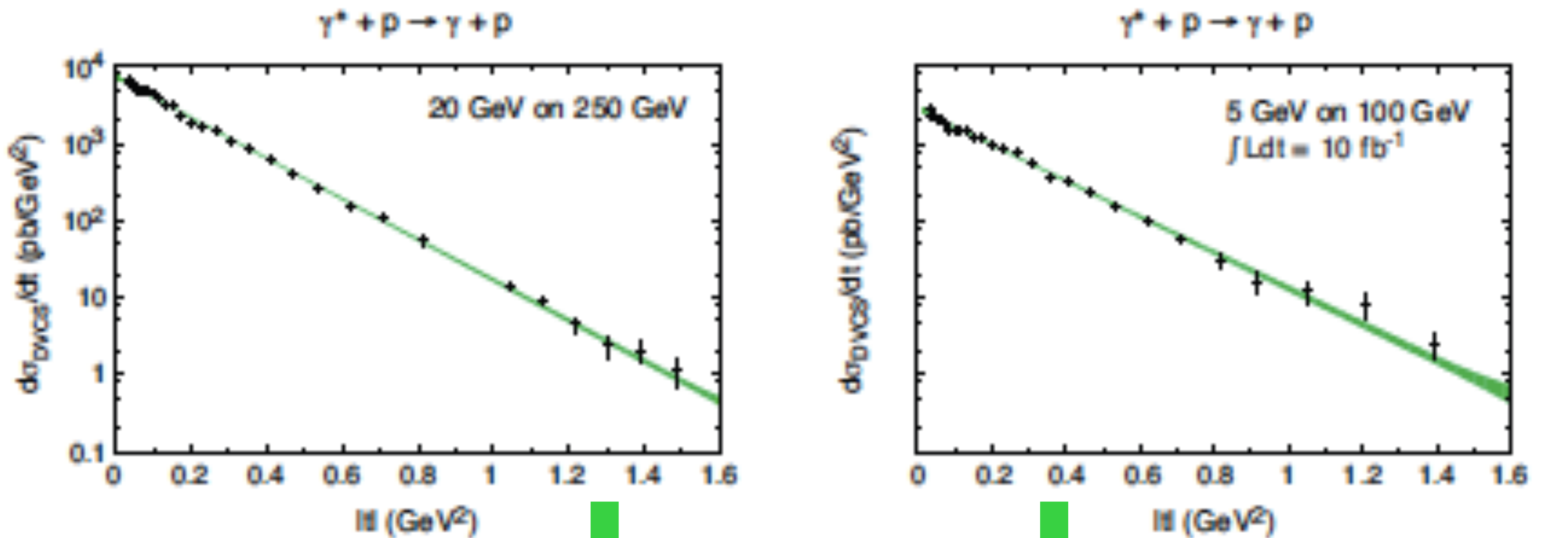
Nature **557**, 396-399 (2018)

CLAS12 (projected)

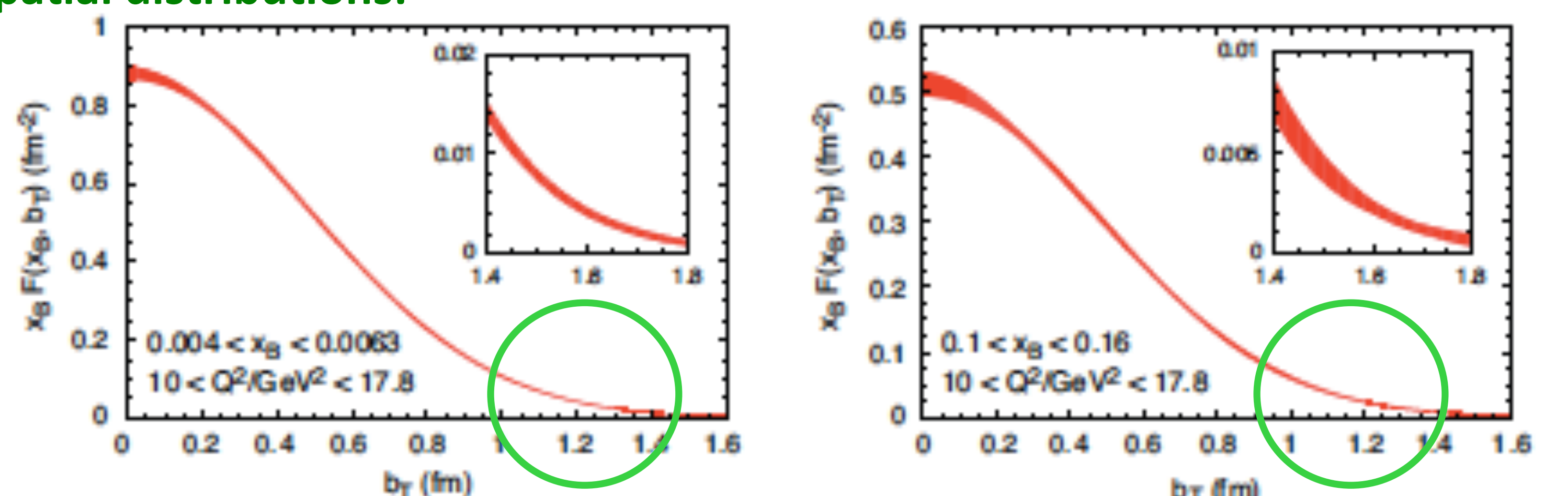


DVCS at future EIC (White Paper)

Cross Sections:



Spatial distributions:



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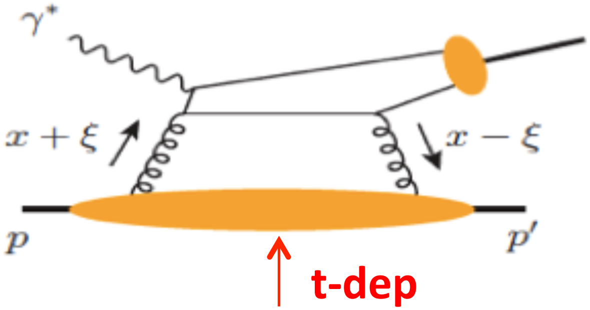
Imaging the gluon (White Paper)

- ❑ **Exclusive vector meson production:**



J/Ψ, Φ, …

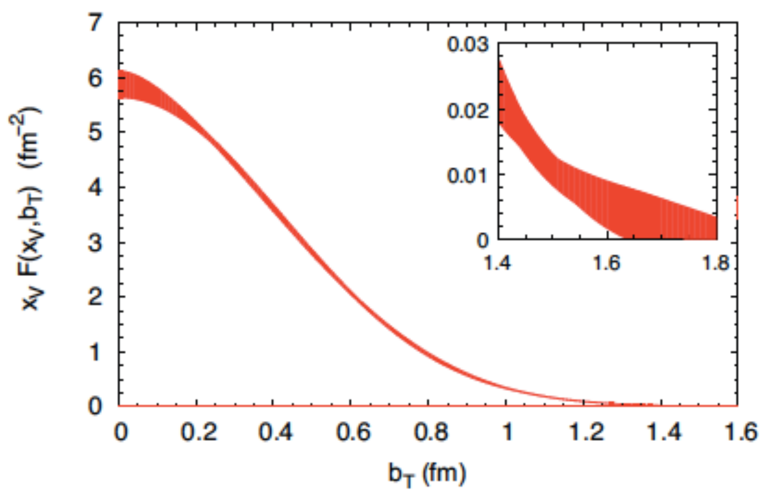
$$\frac{d\sigma}{dx_B dQ^2 dt}$$

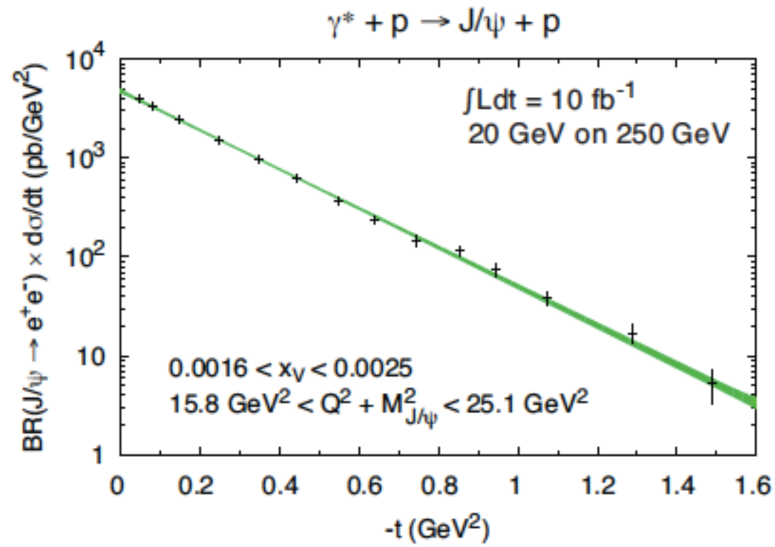
✧ **Fourier transform of the t-dep**

➡ **Spatial imaging of glue density**

✧ **Resolution ~ 1/Q or 1/M_Q**

- ❑ **Gluon imaging from simulation:**





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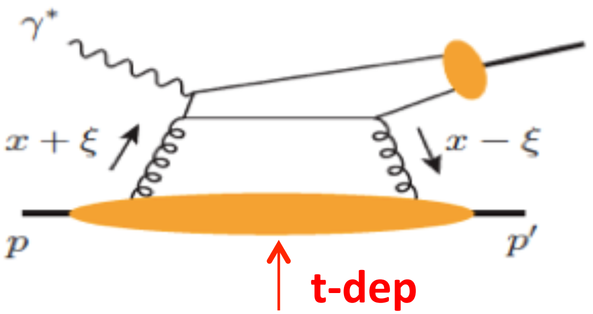

J/Ψ, Φ, …

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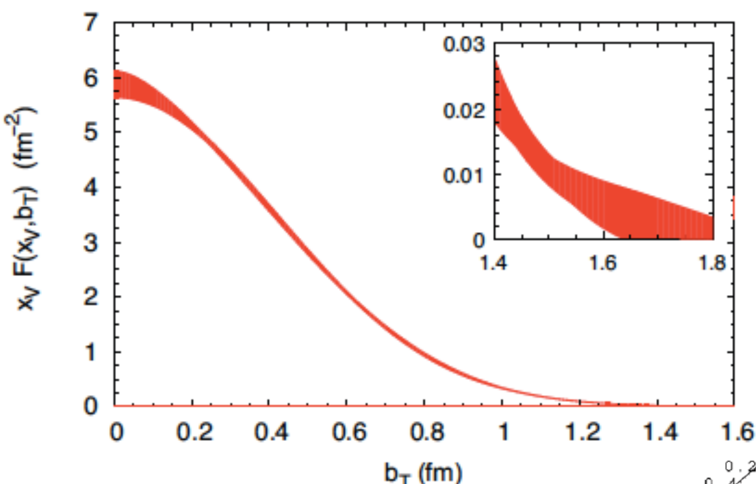
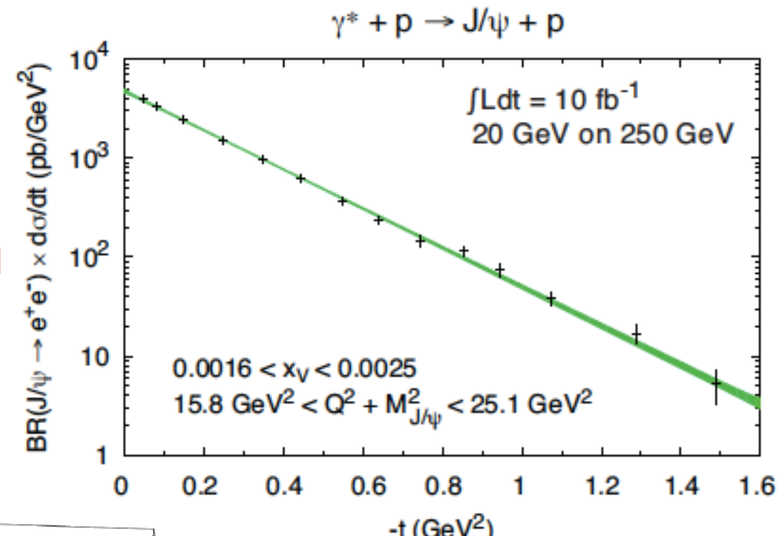
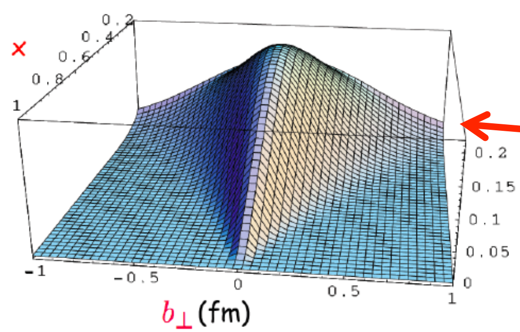




Only possible at the EIC

Gluon radius?

Gluon radius (x)!



How spread at small-x?
Color confinement

Jefferson Lab

GPDs from Lattice QCD

Also see talks by Martha and Anatoly

Y.-S. Liu et al.
arXiv:1902.00307

- **Definition:**

$$\widetilde{F}(\Gamma, x, \widetilde{\xi}, t, P^z, \widetilde{\mu}) = \frac{1}{N}\int \frac{dz}{4\pi} e^{ixzP^z} \langle P'', S''|\bar{\psi}\left(\frac{z}{2}\right)\Gamma\lambda^a W_z\left(\frac{z}{2}, -\frac{z}{2}\right)\psi\left(-\frac{z}{2}\right)|P', S'\rangle$$

GPDs could be extracted from LQCD calculation of j-j correlation

- **Decomposition:**

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \widetilde{\mu}) = \frac{1}{2P^t}\bar{u}(P'', S'')\Big\{\widetilde{H}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\Gamma + \widetilde{E}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\frac{[\Delta\!\!\!/, \Gamma]}{4M}$$
$$+ \widetilde{H}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{P^{[z}\Delta^{\perp]}}{M^2} + \widetilde{E}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{\gamma^{[z}P^{\perp]}}{M}\Big\}u(P', S')$$

- **Factorization:**

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \mu_R, p_R^z) = \int_{-1}^{1}\frac{dy}{|y|}C_\Gamma\left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z}\right)F(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}\right)$$

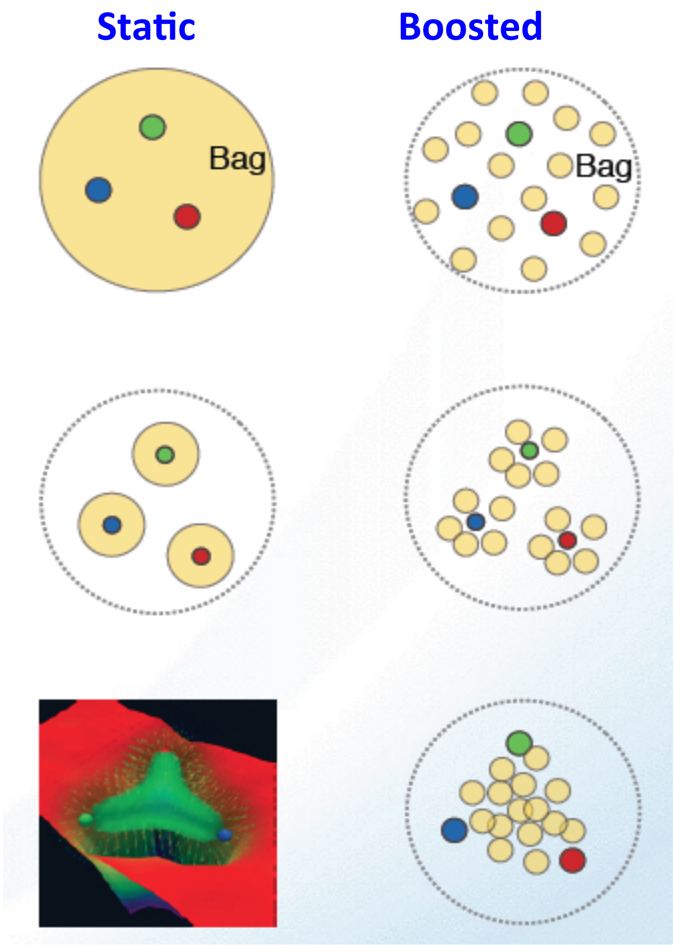
- **Matching coefficient – NLO – RI/MOM renormalization:**

$$C_\Gamma\left(x, \xi, r, \frac{p^z}{\mu}, \frac{p^z}{p_R^z}\right) = \delta(1-x) + \left[f_1\left(\Gamma, x, \xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_R^z}\right| f_2\left(\Gamma, \frac{p^z}{p_R^z}(x-1)+1, r\right)\right]_+$$
$$+ \delta_{\Gamma, i\sigma^{z\perp}}\delta(1-x)\frac{\alpha_s C_F}{4\pi}\ln\left(\frac{\mu^2}{\mu_R^2}\right) + \mathcal{O}(\alpha_s^2)$$

where the functions f1, f2 and δ_Γ are given in the paper (1902.00307)

Beyond the 3D picture

❑ Spatial distributions of quarks and gluons:



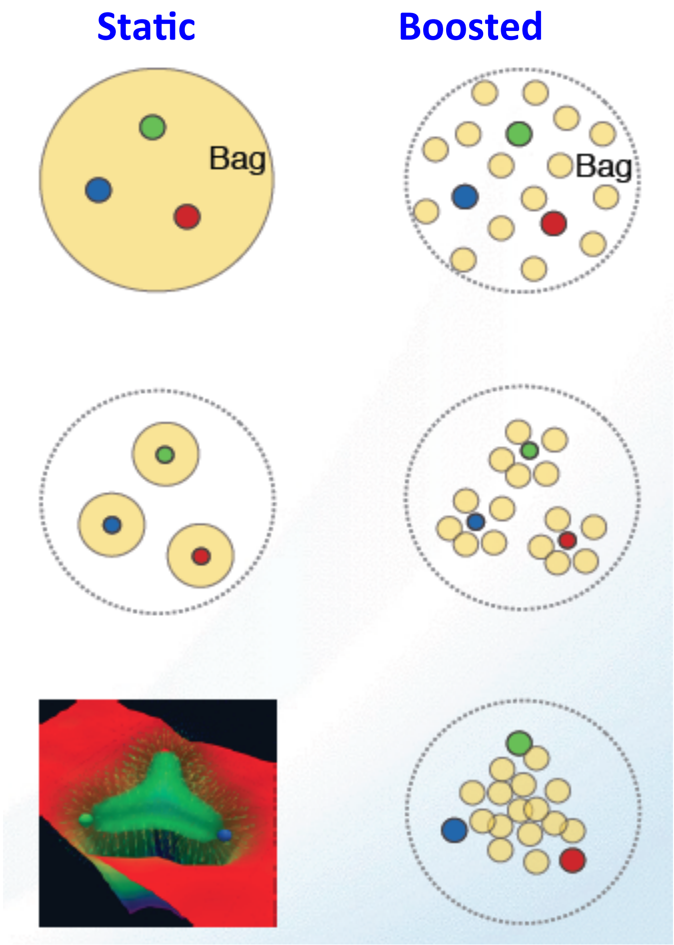
Bag Model:
Gluon field distribution is wider than the fast moving quarks.
Gluon radius > Charge Radius

Constituent Quark Model:
Gluons and sea quarks hide inside massive quarks.
Gluon radius ~ Charge Radius

Lattice Gauge theory (with slow moving quarks):
Gluons more concentrated inside the quarks
Gluon radius < Charge Radius

Beyond the 3D picture

❑ **Spatial distributions of quarks and gluons:**



Bag Model:

Gluon field distribution is wider than the fast moving quarks.

Gluon radius > Charge Radius

Constituent Quark Model:

Gluons and sea quarks hide inside massive quarks.

Gluon radius ~ Charge Radius

Lattice Gauge theory (with slow moving quarks):

Gluons more concentrated inside the quarks

Gluon radius < Charge Radius

3D confined motion (TMDs) + spatial distribution (GPDs)

Hints on the color confining mechanism

Relation between charge radius, quark radius (x), and gluon radius (x)?

Summary

- ❑ **GPDs are fundamental quantum probability distributions**

  Carry important information on spatial imaging of hadron's partonic structure

- ❑ **Need exclusive processes with a unbroken hadron under hard collisions**

  - ✧ **Need lepton-hadron facilities**
  - ✧ **Need well-controlled exclusive processes in lepton-hadron collisions**
  - ✧ **DVCS, DDVCS, DVMP, Diffractive heavy vector boson production, …**
  - ✧ **JLab12, COMPASS, and future EIC will produce a lot of data on GPDs**

- ❑ **GPDs could be extracted from LQCD calculations**

  Work just got started – more efforts are needed!

Thank you!