Ioffe time behavior of PDFs and GPDs

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PDFLattice 2019
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Outline

- Ioffe time behavior of PDFs
- Reconstructing PDFs from Mellin moments calculated in lattice QCD
- Generating x and ξ dependence of GPDs
- Pseudo PDFs in a diquark model

In collaboration with Simonetta Liuti, University of Virginia

Ioffe Time Distributions

- Due to invariance under Lorentz transformations, the matrix element depends on two scalars

$$\mathcal{M}\left((pz), z^2\right) = \langle p|\phi(0)\phi(z)|p\rangle$$

Ioffe time

Fourier Transform

Radyushkin, Phys Rev D 96 (2017)

Orginos et al, Phys Rev D 96 (2017)

Braun et al, Phys Rev D 51 (1995)

$$\int_{-1}^{1} dx e^{-ix(pz)} \mathcal{P}(x, z^2) = \mathcal{M}\left((pz), z^2\right)$$

On the light cone

$$\mathcal{P}(x, 0) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-ix\nu}\mathcal{M}(\nu, 0)$$

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Ioffe Time Distributions

$$T_c + iT_s = \int_0^1 dx\, f(x) e^{i(x\nu)}$$

$\nu = (Pz)$

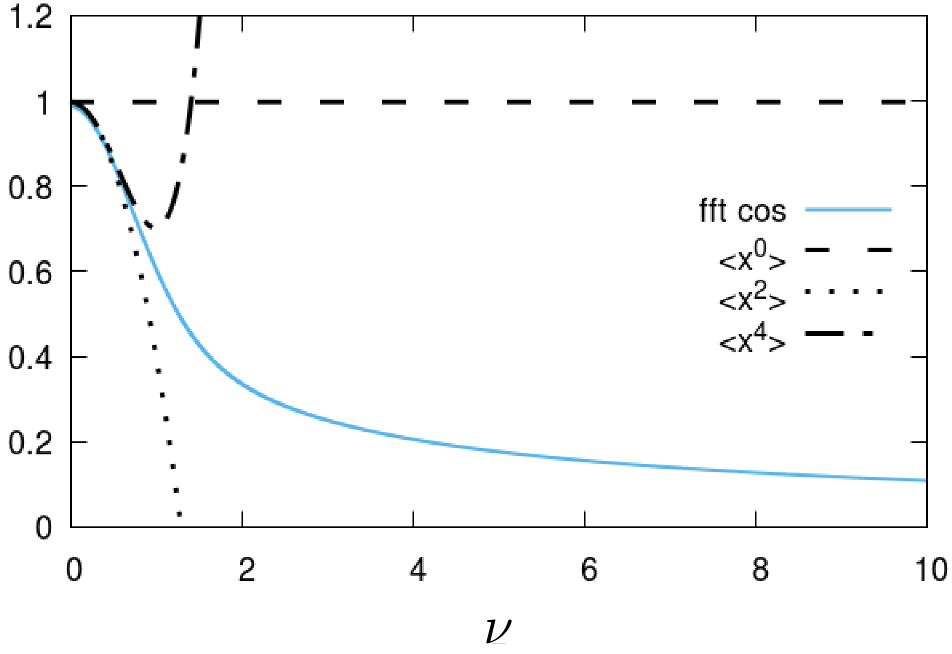
$$M_n = \int dx\, x^{n-1}\, f(x)$$

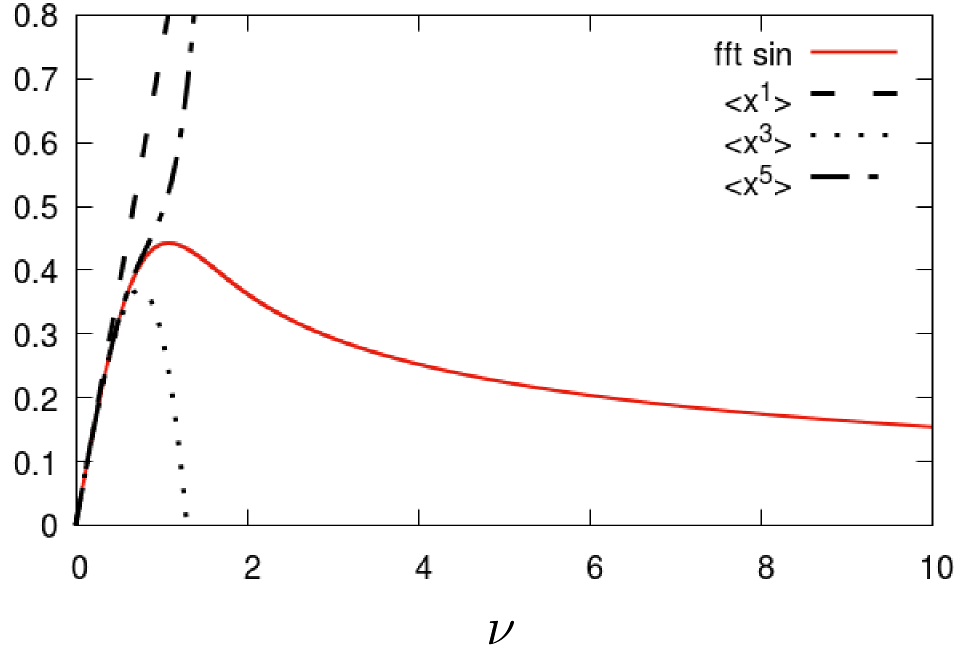
Taylor expansion for small ν

$$T_s(x) = \int_0^1 dx f(x) sin(x\nu) = M_2\nu - \frac{1}{3!} M_4 \nu^3 + \ldots$$

$$T_c(x) = \int_0^1 dx f(x) cos(x\nu) = M_1 - \frac{1}{2!} M_3 \nu^2 + \ldots$$

Describing Ioffe Time Distributions using Mellin Moments



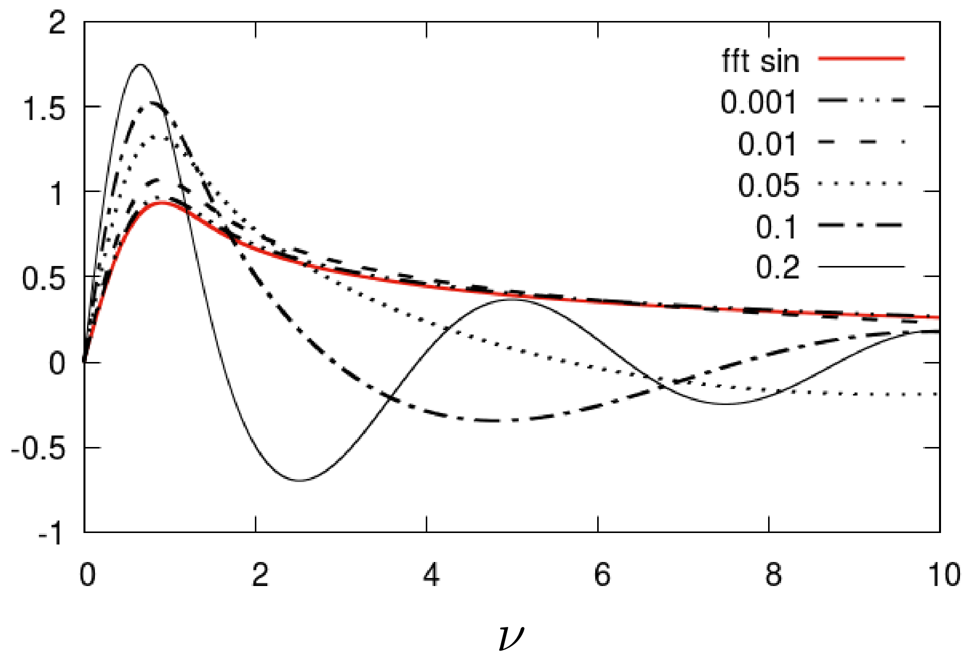


$$T_c(\nu) = M_1 - \frac{1}{2!}M_3\nu^2 + \dots$$

$$T_s(\nu) = M_2\nu - \frac{1}{3!}M_4\nu^3 + \dots$$

From x space to Ioffe Time ν

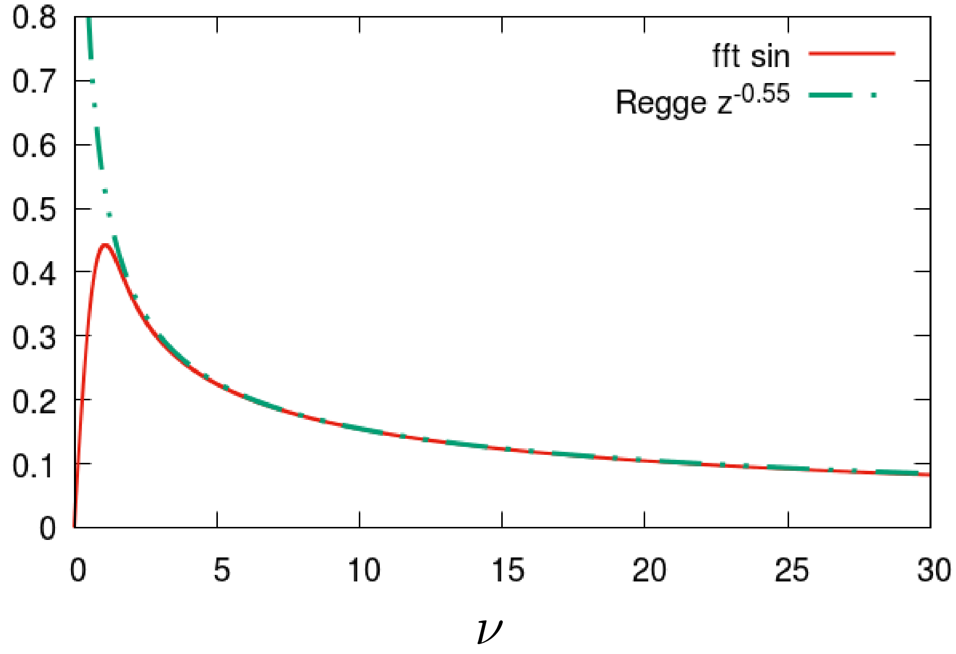
Truncating the PDF at an x_{min}



$$\nu = (Pz)$$

Large z / small x behavior dominated by Regge factor

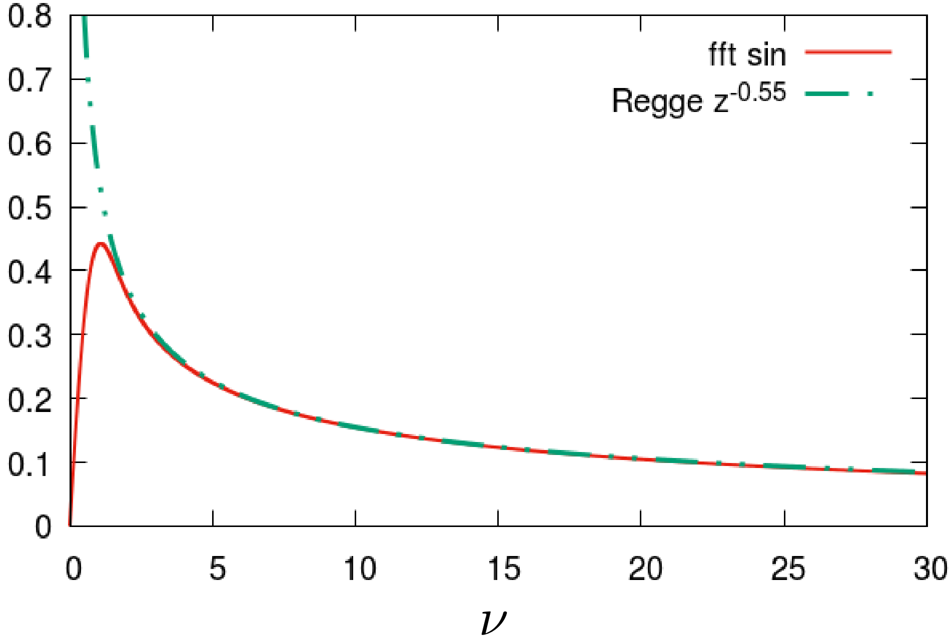
From x space to Ioffe Time ν



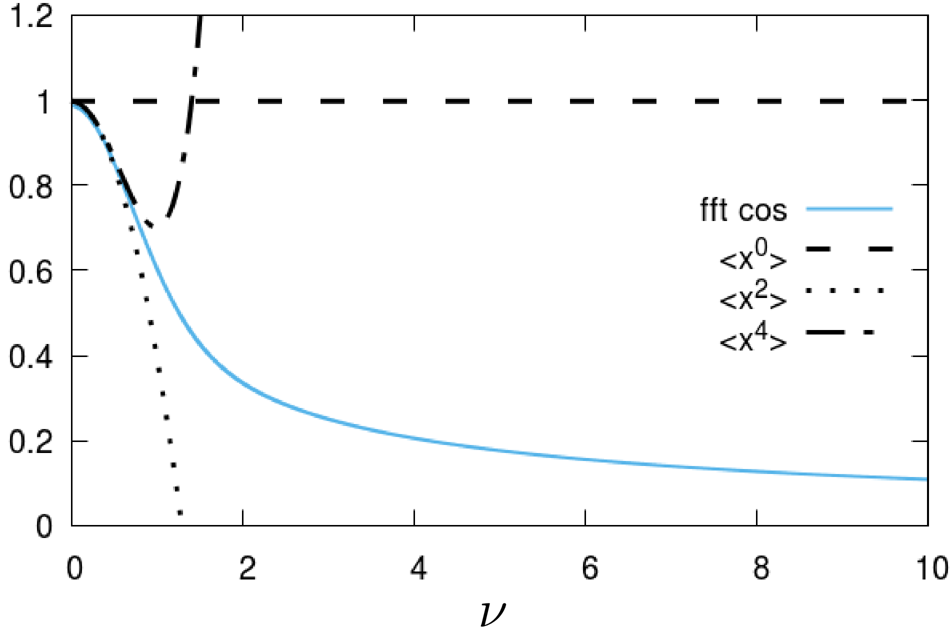
$\nu = (Pz)$

Large z / small x behavior dominated by Regge factor

Reconstructing PDFs from Mellin moments

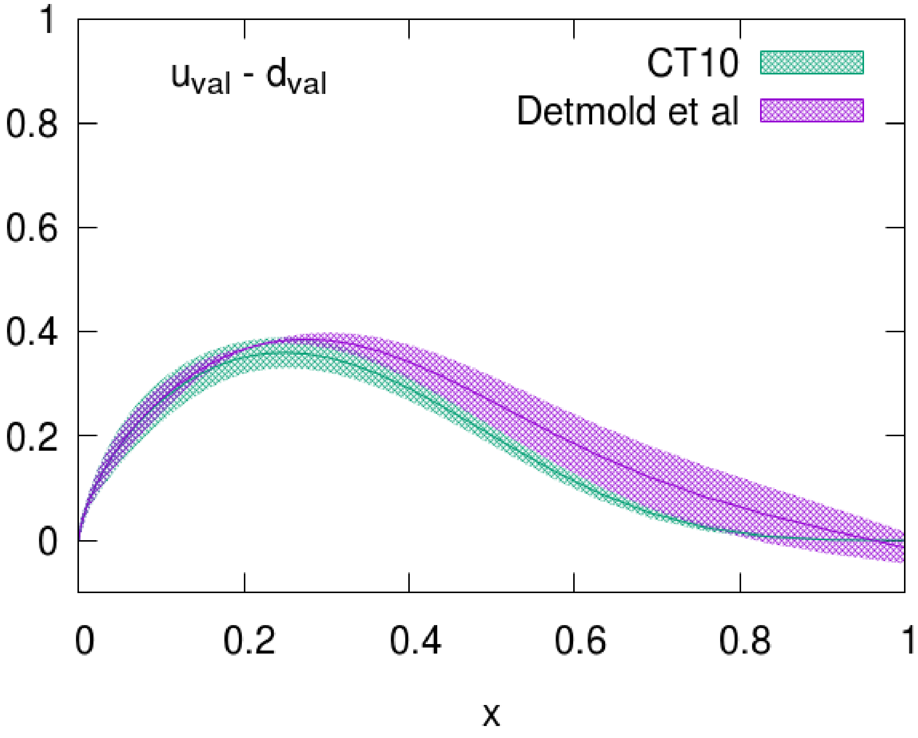


Large z / small x behavior dominated by Regge factor

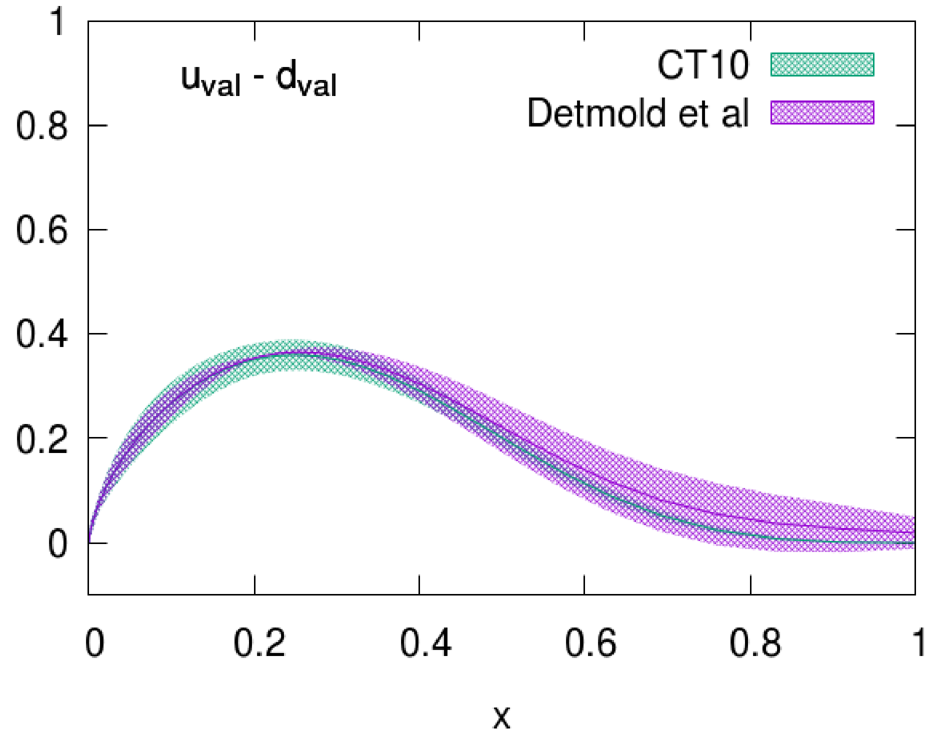


Small z behavior fixed by Mellin moments

Reconstructing PDFs from Mellin moments



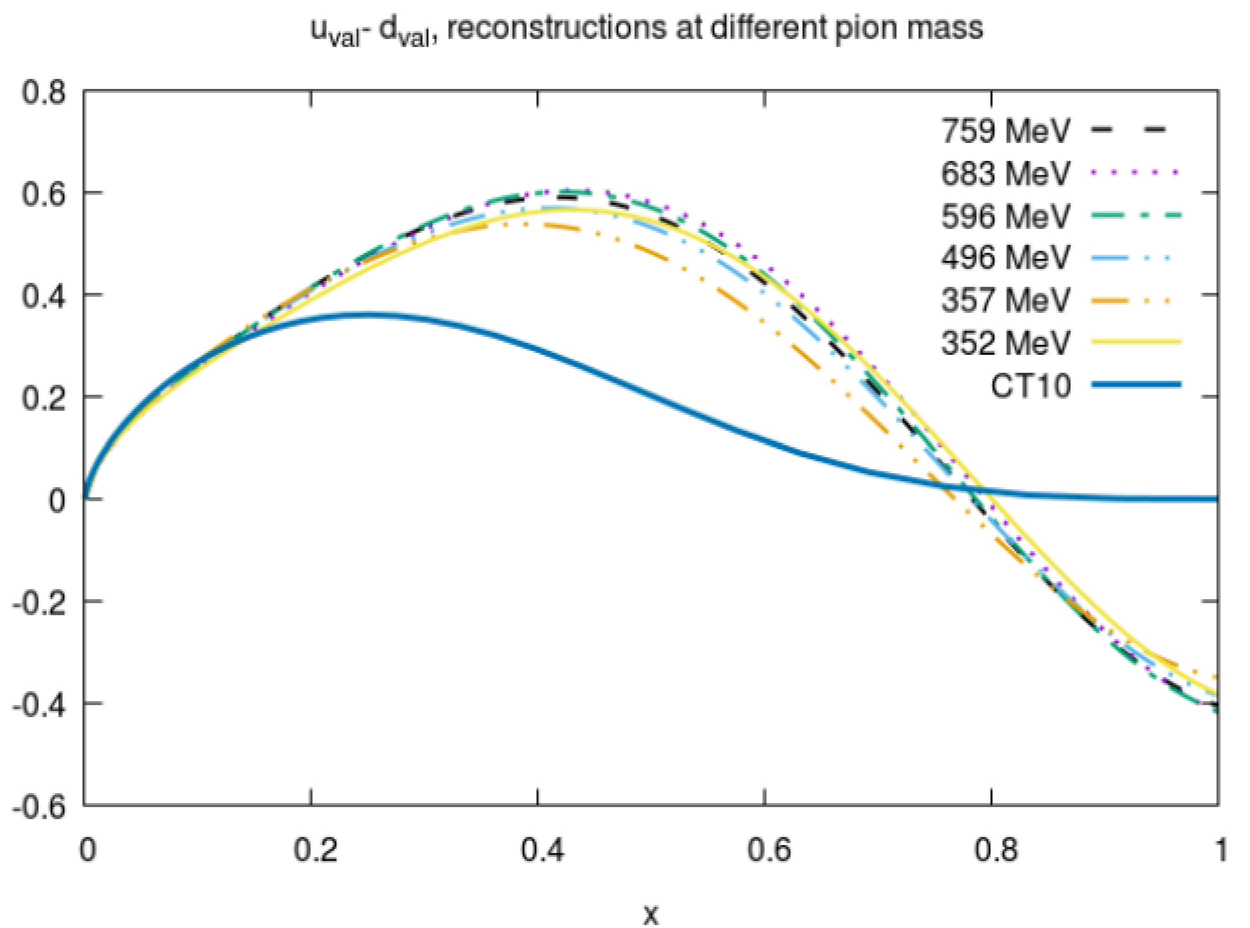
3 moments



4 moments

W Detmold et al., Eur. Phys. J.3 (2001), Mod. Phys. Lett. A18 (2003)

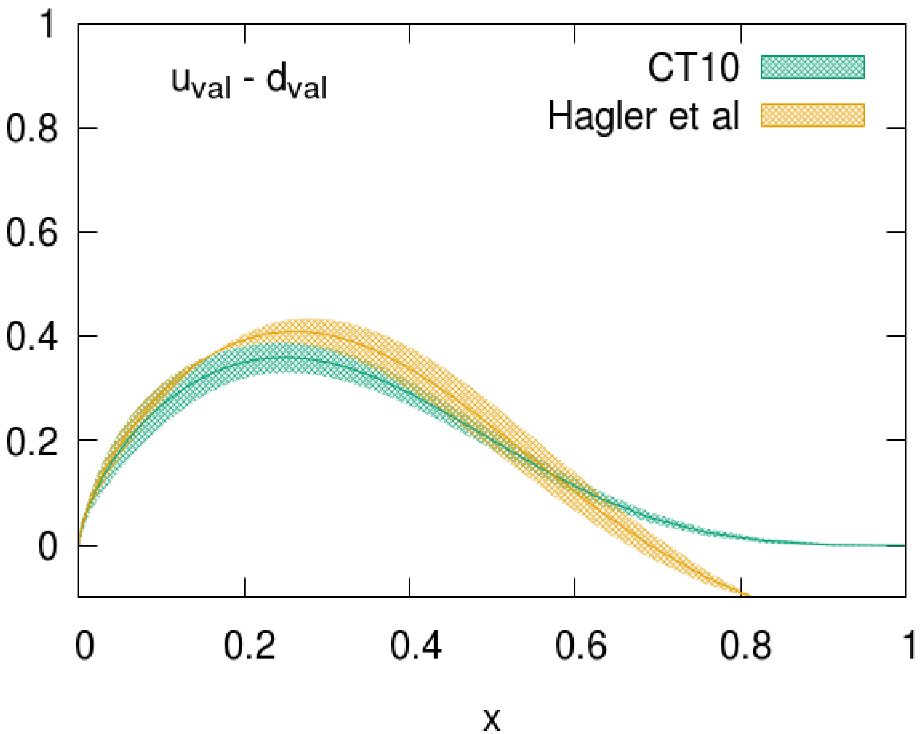
Reconstructing PDFs from Mellin moments



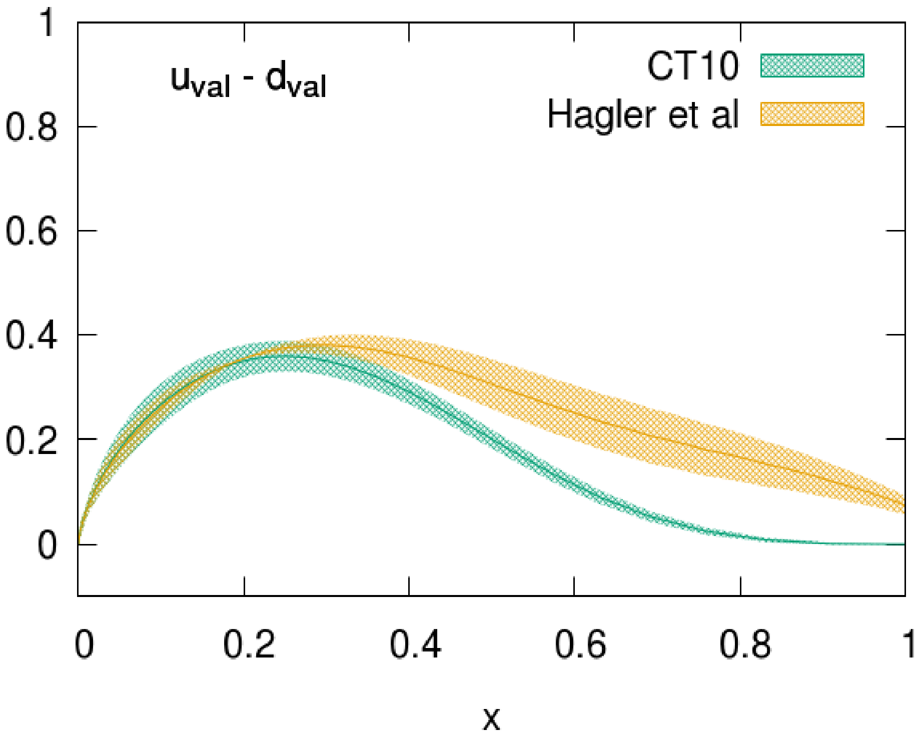
Using 2 moments at different pion mass

LHPC (Ph. Hagler et al.) Phys. Rev. D. 77 (2008)

Reconstructing PDFs from Mellin moments



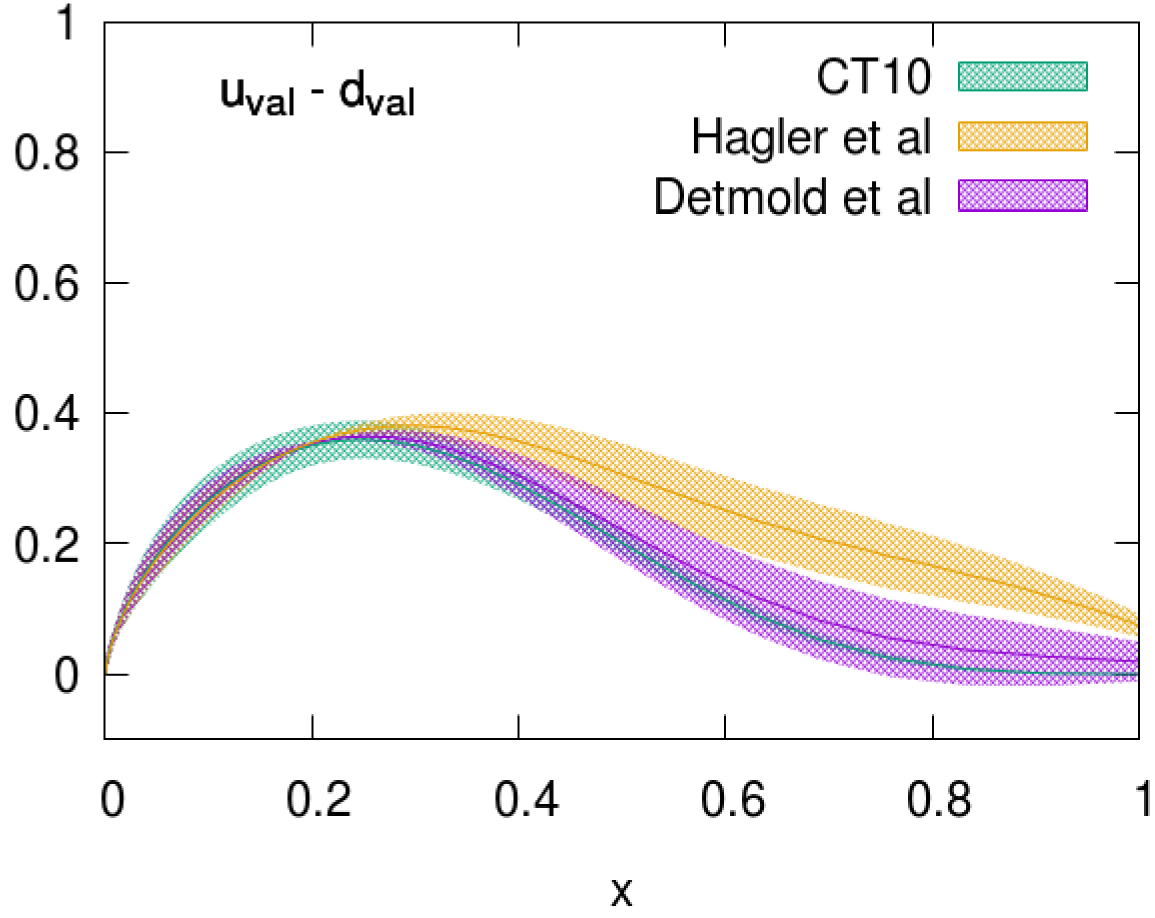
2 moments



3 moments (third moment not extrapolated to physical pion mass)

LHPC (Ph. Hagler et al.) Phys. Rev. D. 77 (2008)

Reconstructing PDFs from Mellin moments - comparison



Comparing the two calculations

Extending to GPDs

The GPD moments are a polynomial in ξ

$$\int dx\, H(x,\xi,t) = A_{10}$$

$$\int dx\, x\, H(x,\xi,t) = A_{20} + 4\xi^2 C_{20}$$

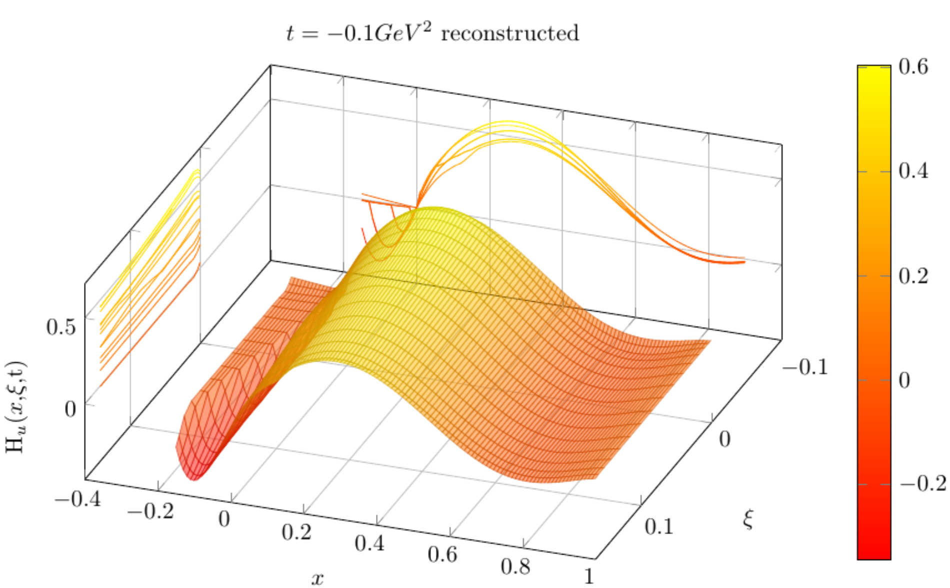
$$\vdots$$

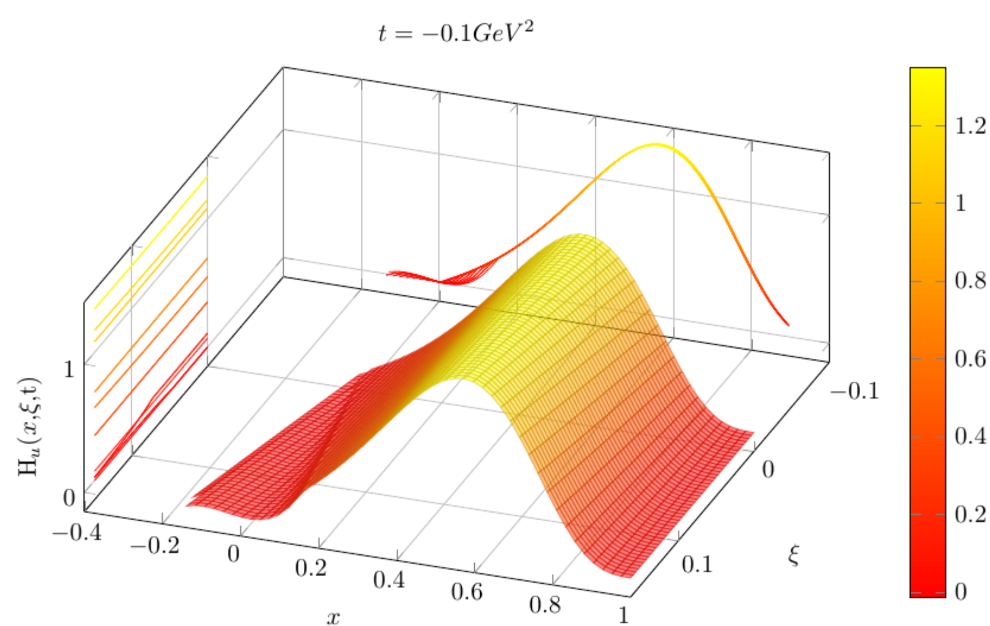
Generalized form factors are a function of t.

LHPC (Ph. Hagler et al.), 2008

Detmold and Shanahan, 2018

ETMC (C. Alexandrou et al.), 2019





Pseudo PDFs in a phenomenological model

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Ioffe time

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On the light cone

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Pseudo PDFs

- Pseudo PDFs generalize the lightcone PDFs onto space like intervals $z = (0, 0, 0, z_3)$

Radyushkin, Phys Rev D 96 (2017)

Orginos et al, Phys Rev D 96 (2017)

$$\mathcal{M}(\nu, z_3^2) \longrightarrow \mathcal{P}(x, z_3^2)$$

- Reduce z_3^2 dependence by taking ratios $\dfrac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$

- By rotational invariance, one can equivalently take an interval of the form $z = (0, z_1, z_2, 0)$

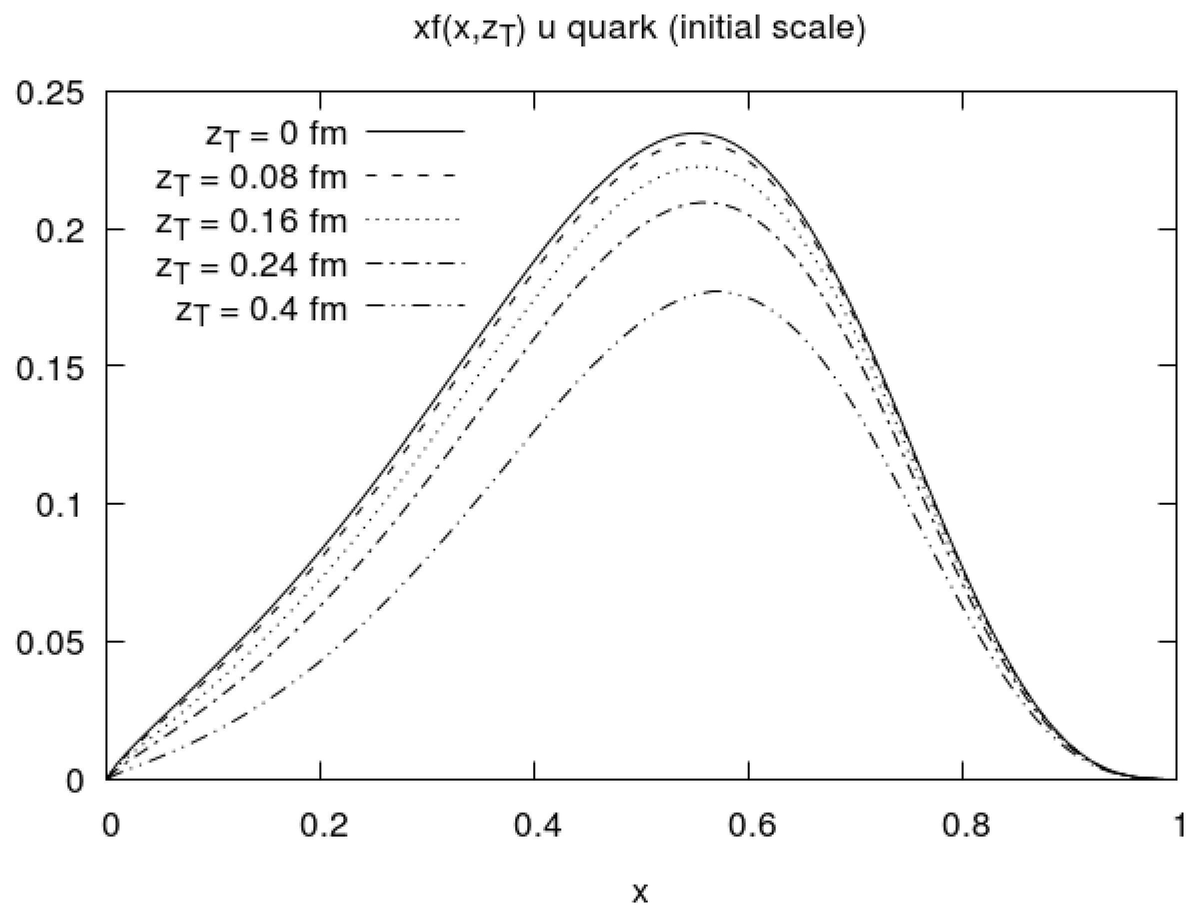
$$\mathcal{M}(\nu, z_T^2) \longrightarrow \mathcal{P}(x, z_T^2)$$

Pseudo PDFs in a diquark model

$$f(x, k_T^2) = \frac{(m + xM)^2 + k_T^2}{(k^2 - M_\Lambda^2)^4}$$

2D Fourier transform

$$\tilde{f}(x, z_T)$$



Extension to GPDs in progress!

Summary

- PDFs receive contributions from both large and short longitudinal spacing, the two regions correspond to very different physics.
- Ioffe time distributions provide a natural way to single out each of them separately. Mellin moments play a key role in this.
- A way to model GPDs using lattice calculations.
- Model studies used to probe the dependence of Pseudo PDFs on z^2

Thanks!