Examining parton distribution functions

Davison E. Soper
University of Oregon

Workshop on Parton Distributions and Lattice Calculations
MSU Kellogg Biological Station, September 2019

Parton distribution functions

- They are defined as proton matrix elements of a certain operator.

- For quarks,

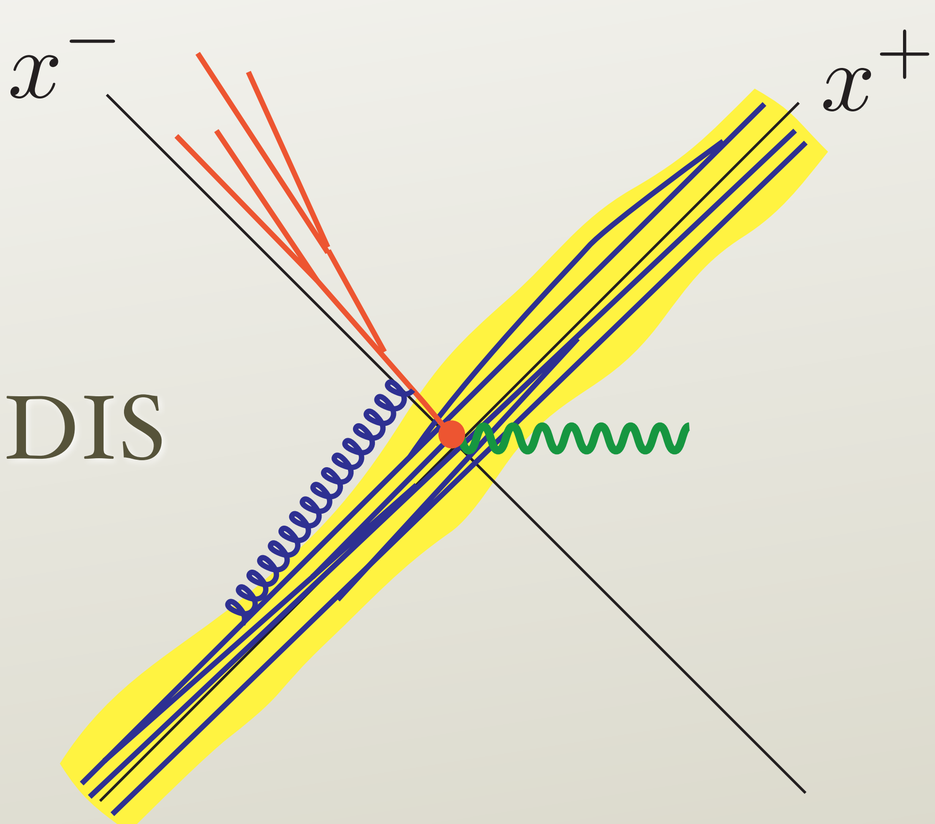
$$f_{i/h}(\xi, \mu_F) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{\psi}_i(0, y^-, \mathbf{0})\gamma^+ F(y^-, 0)\psi_i(0)|p\rangle$$

$$F(y^-, 0) = \mathcal{P}\exp\left(-ig\int_0^{y^-} dz^-\, A_a^+(0, z^-, \mathbf{0})\, t_a\right)$$

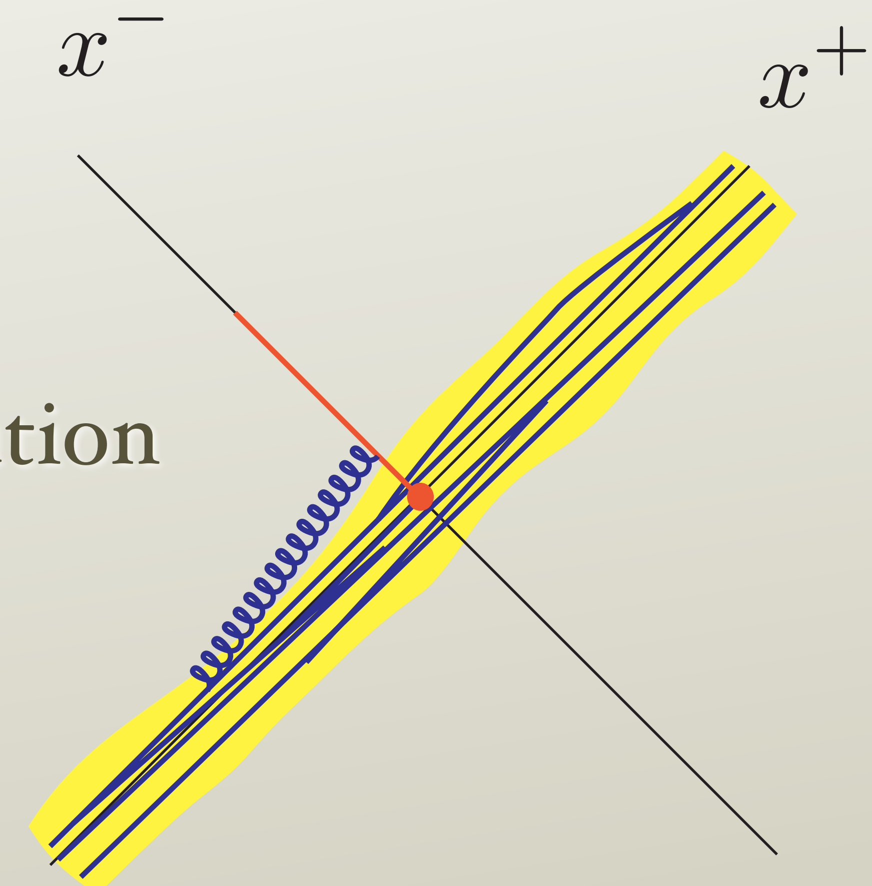
- For gluons, a similar definition.
- Renormalize with the $\overline{\mathrm{MS}}$ prescription with scale μ_F.

The definition in pictures.

- Here we depict the production of the final state which illustrates the connection to DIS.



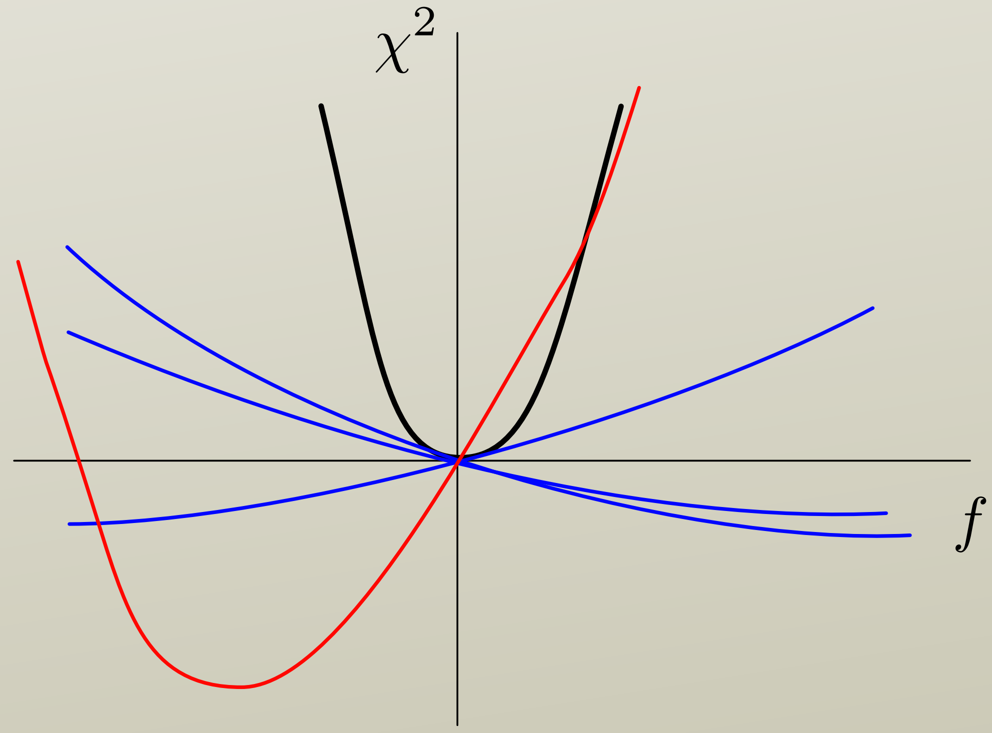
quark distribution function



$$f_{i/h}(\xi,\mu_F) = \frac{1}{2}\sum_N \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{\psi}_i(0,y^-,\mathbf{0})\gamma^+ F(y^-,\infty)\big|N\big\rangle$$
$$\times \big\langle N\big|F(\infty,0)\psi_i(0)|p\rangle$$

Fitting parton distributions

- See talk of C.-P. Yuan
- These are fit to data by CTEQ, MMHT, NNPDF, ABMP, ...
- We saw new results from the CT18 fit.
- Not everything is perfect: there can be tensions between different experiments that are outside of the quoted errors.



- One can also fit data on nuclei. For example EPPS, nCTEQ, DSSZ, HKN …
- See talk of Emanuele Nocera about NNPDF fit.
- A nucleus is not the same as a bag of non-interacting protons and neutrons.
- For instance, a small x gluon has a wave function that extends through much of the nucleus.
- Better knowledge of PDFs in nuclei can inform us about the nuclear corrections needed for data on ν interactions in iron.

- One can go beyond the inclusive proton PDFs to consider TMDs, GPDs etc. A major program would be useful to fit these from data. There was lots of discussion about this.

- Abha Rajan and Simonetta Liuti discussed steps in this direction.

- Alberto Accardi updated us on JAM fits to multiple functions describing hadron structure in one program.

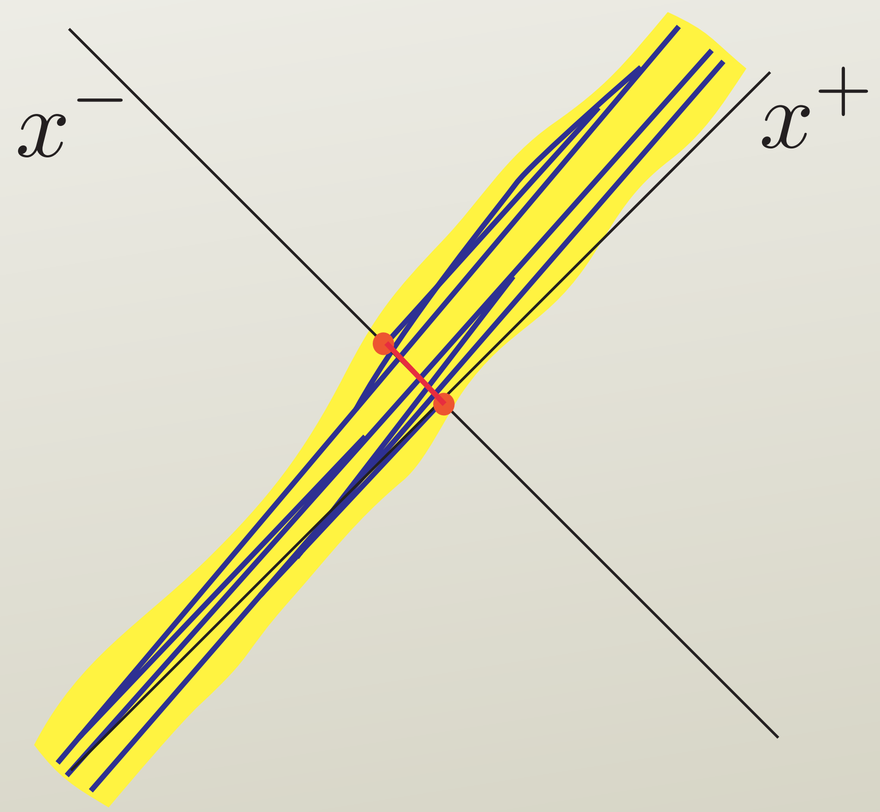
- Tim Hobbs discussed the prospects for combining results of lattice calculations and results of collider experiments.

- Tommaso Giani tested how to use the NNPDF fitting procedure to fit lattice data.

- One can also measure spin dependent PDFs.
- Marco Radici summarized measurement of the transversity distribution.
- This is the probability to find a transversely polarized quark in a transversely polarized proton.
- The transversity distribution is not so straightforward to measure.
- Aurore Courtoy described a fit to the transversity distribution from dihadron production in DIS.

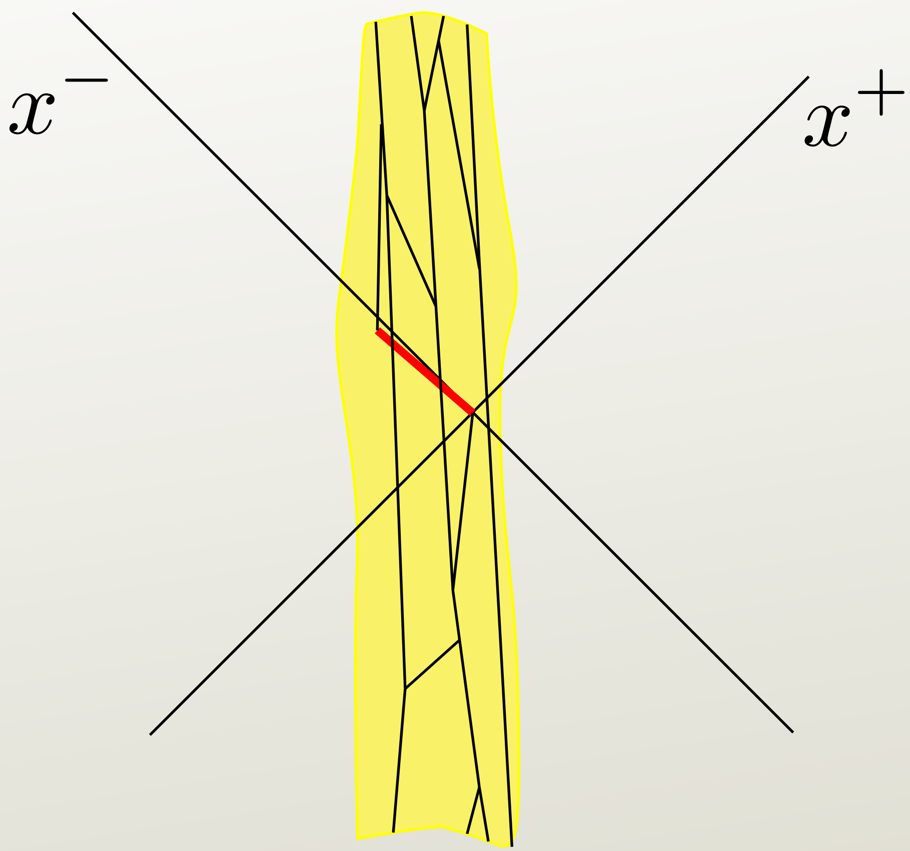
PDFs from lattice

- Here we depict whole operator for the PDF.

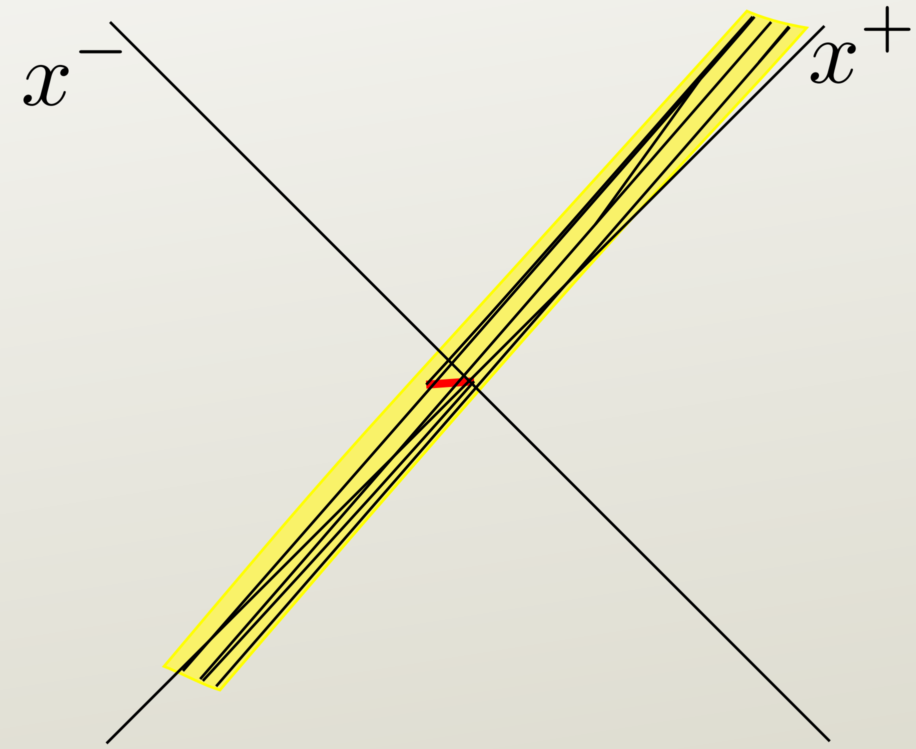


$$f_{i/h}(\xi, \mu_F) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p | \bar{\psi}_i(0, y^-, \mathbf{0}) \gamma^+ F(y^-, 0) \psi_i(0) | p \rangle$$

Approximate version of this for lattice calculations



Almost lightlike line
in proton rest frame



becomes line in z direction
in highly boosted frame.

$$f_{i/h}(\xi,\mu_F) \approx \frac{1}{2}\int \frac{dy^3}{2\pi} e^{i\xi p^3 y^3}\langle p|\bar{\psi}_i(0,\mathbf{0},y^3)\gamma^3 F(y^3,0)\psi_i(0)|p\rangle$$

$$F(y^3,0) = \mathcal{P}\exp\left(-ig\int_0^{y^3} dz^3\, A_a^+(0,\mathbf{0},z^3)\, t_a\right)$$

- Jian Liang described prospects for a lattice calculation of the hadronic tensor that describes DIS.

- Jianhui Zhang described calculation of the transversity distribution on the lattice.

Generalized Parton Distributions

- Jain-wei Qiu introduced these.
- Built from

$$W = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z} \langle p', \Lambda' | \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{U} \psi \left(\frac{z}{2}\right) | p, \Lambda \rangle$$

- These can be measured, for example, in deeply virtual Compton scattering.

- One feature is that one can look at transverse position distributions. The key idea is to look at

$$\rho(\boldsymbol{b}) = {}_R\big\langle p'^+, -\boldsymbol{b}_\perp \big| \bar{\psi}(0, -z^-/2, \mathbf{0}_\perp)\Gamma\psi(0, z^-/2, \mathbf{0}_\perp) \big| p^+, -\boldsymbol{b}_\perp \big\rangle_R$$

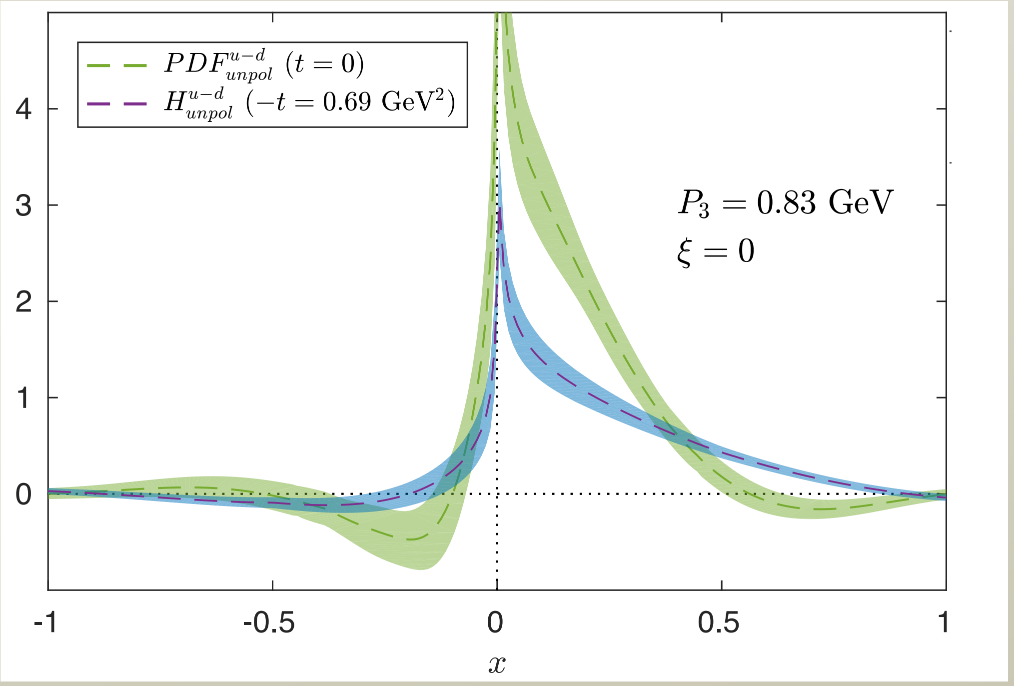
where $\big|p^+, -\boldsymbol{b}_\perp\big\rangle_R$ is a proton state at a definite transverse position $-\boldsymbol{b}_\perp$:

$$\big|p^+, -\boldsymbol{b}\big\rangle_R = \int \frac{d\boldsymbol{p}_\perp}{(2\pi)^2} e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{b}} \big|p^+, \boldsymbol{p}_\perp\big\rangle$$

- This gives

$$\rho(\boldsymbol{b}) = \int \frac{d\boldsymbol{p}'_\perp}{(2\pi)^2} \int \frac{d\boldsymbol{p}_\perp}{(2\pi)^2}\, e^{-i(\boldsymbol{p}'_\perp - \boldsymbol{p}_\perp)\cdot\boldsymbol{b}} \times \big\langle p'^+, \boldsymbol{p}'_\perp \big| \bar{\psi}(0, -z^-/2, \mathbf{0}_\perp)\Gamma\psi(0, z^-/2, \mathbf{0}_\perp) \big| p^+, \boldsymbol{p}_\perp \big\rangle$$

- Abha Rajan and Simonetta Liuti discussed how GPDs can be extracted from data.
- Anatoly Radyushkin described a method for calculating GPDs on the lattice.
- Martha Constantinou showed results from lattice calculations of GPDs.



Transverse Momentum Dependent Parton Distributions

- These give the density of partons as a function of momentum fraction and transverse momentum.
- Marcus Ebert gave us the background for TMDs, explaining how their multiple formulations match up.
- Ted Rogers explained more about the formalism, including factorization of cross sections and issues with how well the formalism works.

- Evolution of TMDs is more subtle than with collinear PDFs.

$$f_i(x, \boldsymbol{k}_\perp, \mu^2, \zeta) \to f_i(x, \boldsymbol{b}_\perp, \mu^2, \zeta)$$

$$\frac{\partial f_i(x, \boldsymbol{b}_\perp, \mu^2, \zeta)}{\partial \log(\mu)} = \gamma_\mu^i(\mu^2, \zeta) f_i(x, \boldsymbol{b}_\perp, \mu^2, \zeta)$$

$$\frac{\partial f_i(x, \boldsymbol{b}_\perp, \mu^2, \zeta)}{\partial \log(\zeta)} = \frac{1}{2}\gamma_\zeta^i(b_\perp, \mu^2) f_i(x, \boldsymbol{b}_\perp, \mu^2, \zeta)$$

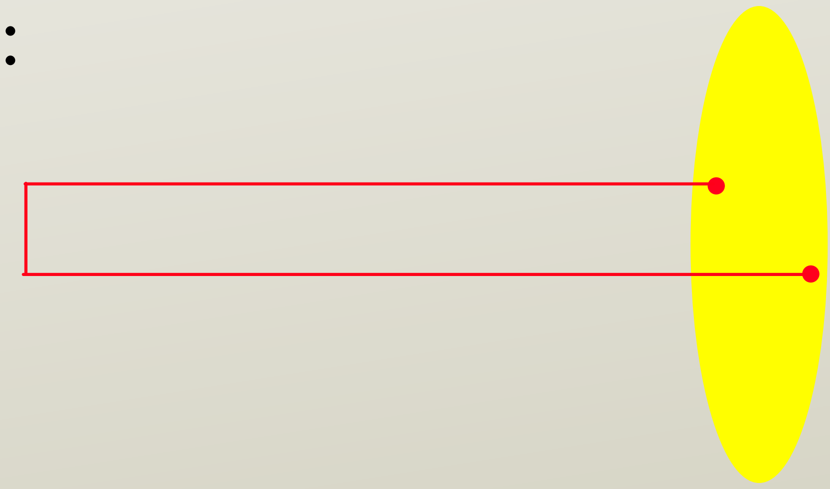
$$\frac{\partial \gamma_\zeta^i(b_\perp, \mu^2)}{\partial \log(\mu)} = -2\Gamma^i_{\mathrm{cusp}}(\alpha_s(\mu^2))$$

- Solve with boundary value $\gamma_\zeta^i(b_\perp, 1/b_\perp^2)$.
- This is non-perturbative for large $b_\perp^2$.

TMD parton distributions for lattice calculations

$$f_{i/h}(\xi, k_\perp) \approx \frac{1}{2}\int \frac{dy^3}{2\pi}\int d\boldsymbol{y}_\perp e^{i(\xi p^3 y^3 + \boldsymbol{k}_\perp \cdot \boldsymbol{y}_\perp)}$$
$$\times \langle p|\bar{\psi}_i(0, \boldsymbol{y}_\perp, y^3)\gamma^3 F(\boldsymbol{y}_\perp, y^3, 0)\psi_i(0)|p\rangle$$

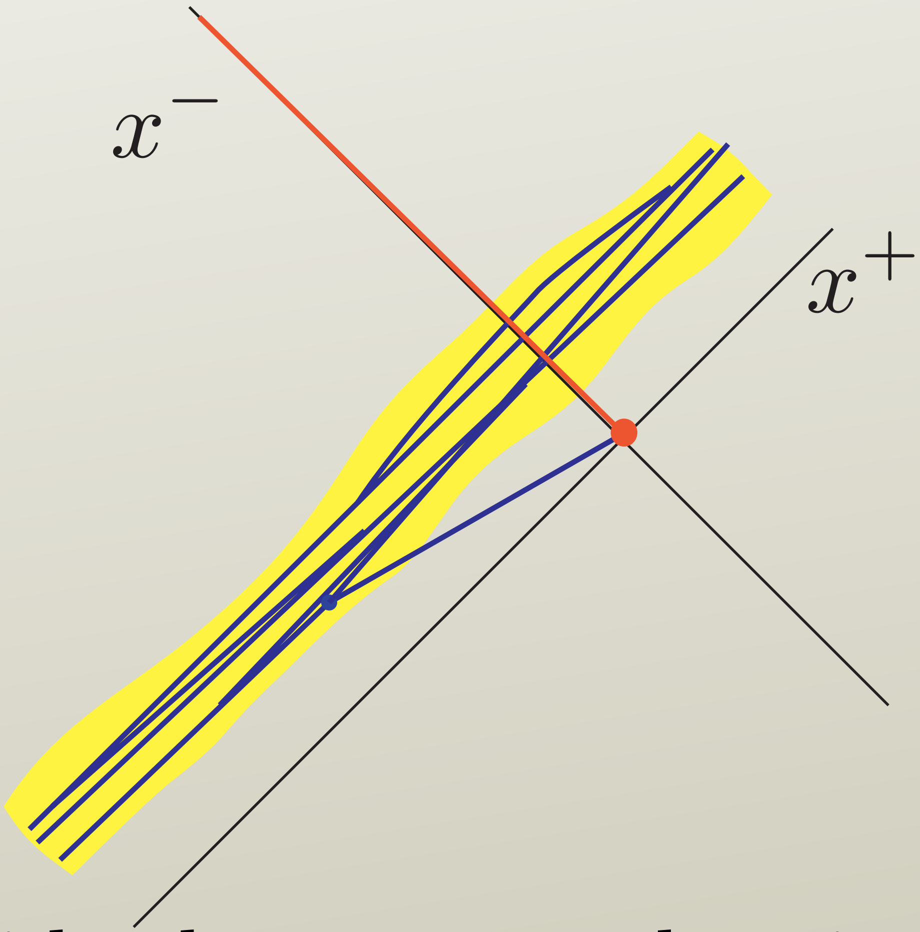
$F(\boldsymbol{y}_\perp, y^3, 0)$:



- Michael Wagman discussed lattice calculations of $\gamma^i_\zeta(b_\perp, \mu^2)$.

Parton distribution functions at small x.

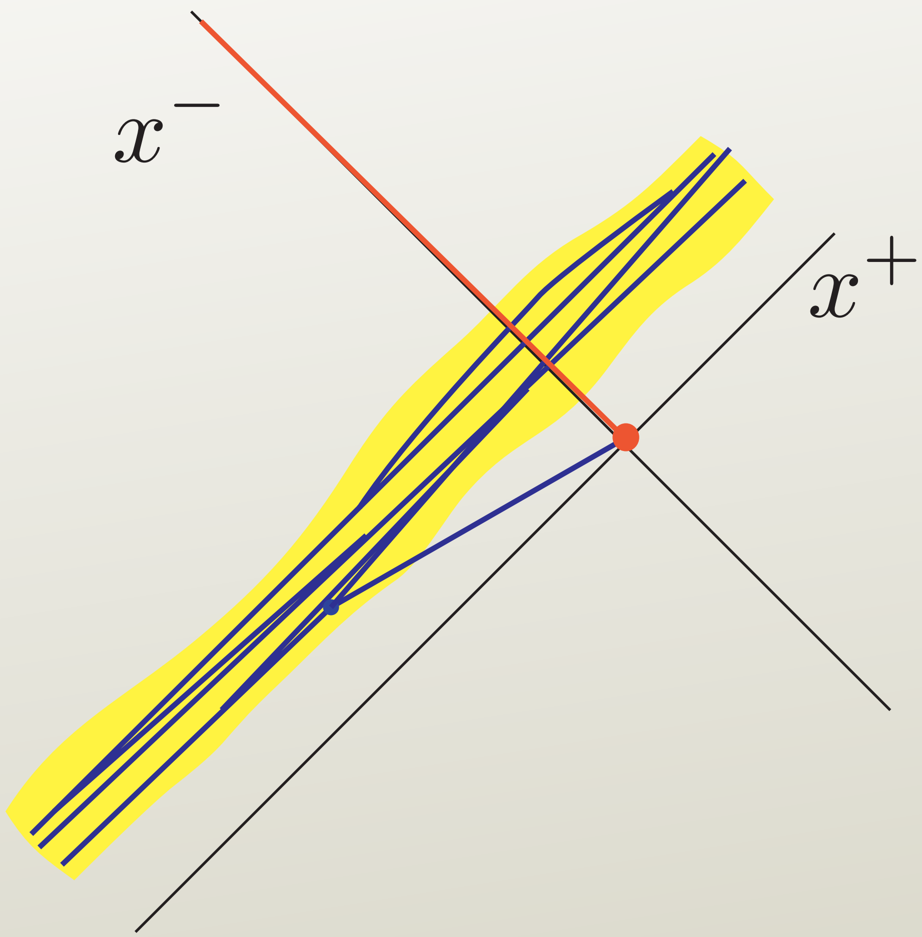
- Gerrit Schierholz discussed problems with lattice calculations at small x.



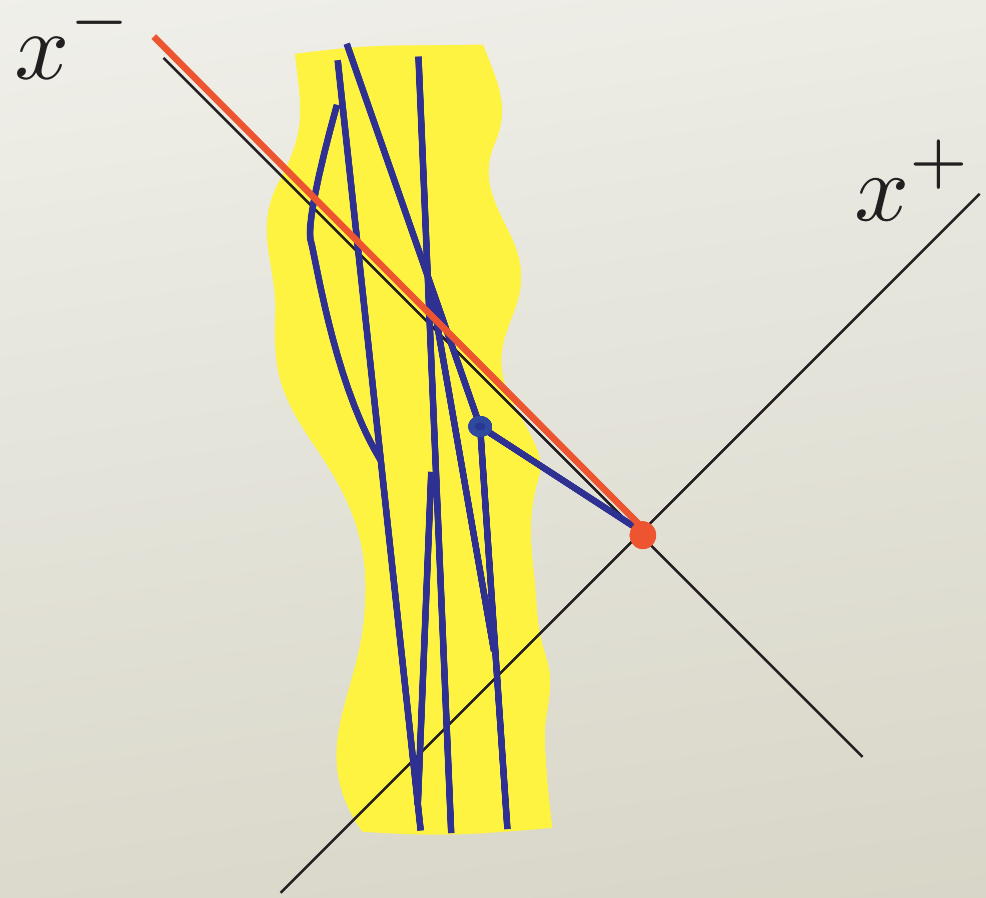
- The quark is outside the proton when it is annihilated.

$$\frac{1}{xP^+} > R_{\rm p}\,\frac{m_{\rm p}}{P^+}$$

The view from the proton rest frame



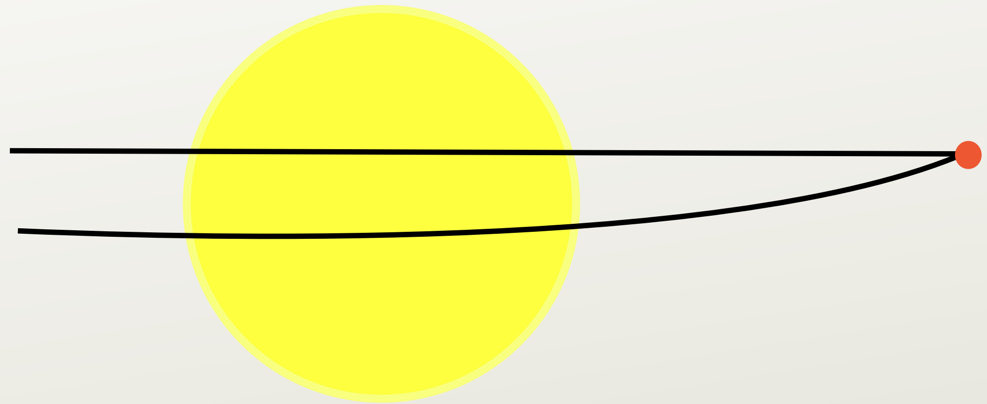
Fast proton



Proton at rest

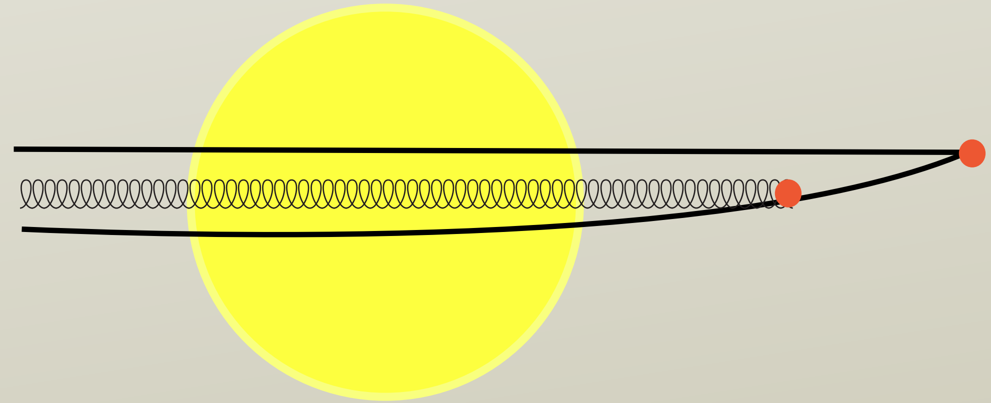
- Note the time ordering.
- We make a color 3 eikonal line and a fast antiquark.

For $x \ll 1$ we get a dipole picture.



- The PDF operator creates a color 3 eikonal line and a high energy antiquark that shoots through the proton.

- This color dipole can develop further according to vacuum QCD.



- The proton structure part is about how the dipole gets through the proton.

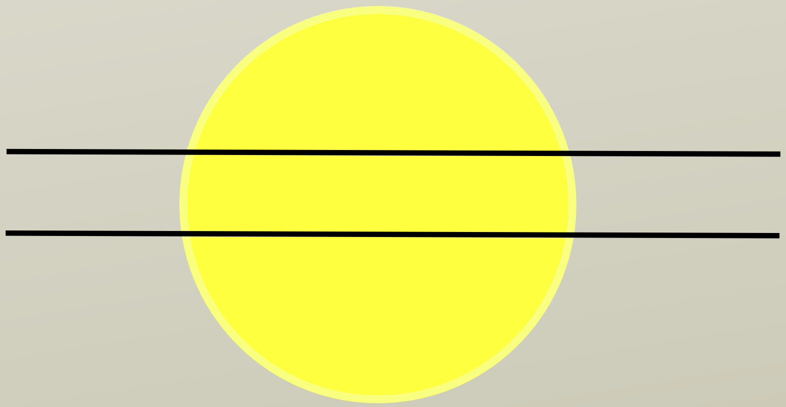
Dipole picture

- E. Iancu, A. H. Mueller and S. Munier,
  "Universal behavior of QCD amplitudes at high energy from general tools of statistical physics,"
  Phys. Lett. B **606**, 342 (2005) [hep-ph/0410018].
- A. H. Mueller, A. I. Shoshi and S. M. H. Wong,
  "Extension of the JIMWLK equation in the low gluon density region,"
  Nucl. Phys. B **715**, 440 (2005) [hep-ph/0501088].
- Y. Hatta, E. Iancu, L. McLerran, A. Stasto and D. N. Triantafyllopoulos,
  "Effective Hamiltonian for QCD evolution at high energy,"
  Nucl. Phys. A **764**, 423 (2006) [hep-ph/0504182].
- K. J. Golec-Biernat and M. Wusthoff,
  "Saturation in diffractive deep inelastic scattering,"
  Phys. Rev. D **60**, 114023 (1999) [hep-ph/9903358].
- A. H. Mueller,
  "Parton saturation at small x and in large nuclei,"
  Nucl. Phys. B **558**, 285 (1999) [hep-ph/9904404].
- F. Hautmann and D. E. Soper,
  "Parton distribution function for quarks in an s-channel approach,"
  Phys. Rev. D **75**, 074020 (2007) [hep-ph/0702077 [HEP-PH]].

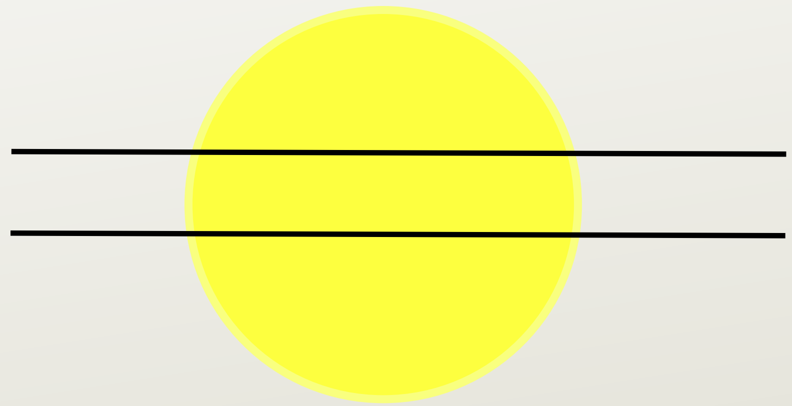
$f_{q/\mathrm{p}}(x,\mu^2)$ in the dipole picture

$$x f_{q/\mathrm{p}}(x,\mu^2) \approx \frac{N_\mathrm{c}}{3\pi^4}\int d\boldsymbol{\Delta}\;\Theta(\Delta^2\mu^2 > 2e^{1/6-\gamma})\frac{1}{\Delta^4} \times \int d\boldsymbol{b}\;\Xi(\boldsymbol{b},\boldsymbol{\Delta})$$

- $1/\boldsymbol{\Delta}^4$ is the dipole wave function.
- $\Xi(\boldsymbol{b},\boldsymbol{\Delta})$ gives the proton structure.



- $\Xi(\boldsymbol{b},\boldsymbol{\Delta})$ represents the probability that the dipole gets through the proton leaving the proton intact.



$$\Xi(\boldsymbol{b},\boldsymbol{\Delta}) = \frac{1}{N_{\rm c}}\left(\frac{1}{2}\sum_s\right)\int\frac{dP'^+}{(2\pi)2P'^+}\,\langle P'^+,\boldsymbol{x}=\boldsymbol{0},s|$$
$$\times\,{\rm Tr}\left[1-F(\boldsymbol{b}+\boldsymbol{\Delta}/2)^\dagger F(\boldsymbol{b}-\boldsymbol{\Delta}/2)\right]\left|P'^+,\boldsymbol{p}_{\rm T}=\boldsymbol{0},s\right\rangle$$

$$F(\boldsymbol{\Delta}) = \mathcal{P}\exp\left[-{\rm i}g\int_{-\infty}^{+\infty}dz^-A_a^+(0,z^-,\boldsymbol{\Delta})\,t_a\right]$$

- There are models for this.
- Perhaps one could calculate it on the lattice.