A discussion of precision top quark mass extraction

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What we need to measure the top mass

- A top mass sensitive observable $O(m_t)$
- The ability to measure it.
- The ability to compute it.

Generally

$$O(m_t) = c_0 + c_1\alpha_s + c_2\alpha_s^2 \ldots + \left(\frac{\Lambda}{m_t}\right)^m .$$

with

$$\alpha_s = \frac{1}{b_0 \log \frac{m_t^2}{\Lambda^2}}$$

Precision is limited by missing HO ($c_k\alpha_s^k$, Higher Orders) and NP ($\left(\frac{\Lambda}{m_t}\right)^m$, Non-Perturbative) terms.

For top mass measurements, only $m = 1$ is relevant (Linear Power Corrections).

Interplay between HO and NP corrections

The behaviour of the perturbative expansion at high orders is believed to be related to the power of non-perturbative corrections:

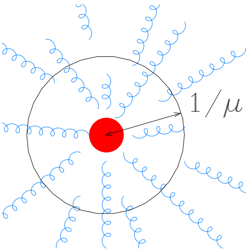
$$\begin{aligned} c_k \alpha_s^k &\approx (mb_0)^k k! \alpha_s^k \\ \text{minimal term at} \quad k &= \frac{1}{mb_0 \alpha(m_t)} \quad \text{(typically 6-8)} \\ \text{at the minimum:} \quad c_k \alpha_s^k &= \left(\frac{\Lambda}{m_t} \right)^m \end{aligned}$$

where m is a positive integer. The connection between the factorial growth and power corrections is in turn related to the growth of the effective coupling constant as large distances, and thus to soft emissions and exchanges.

For top, only $m = 1$ is relevant (linear power corrections).

The mass scheme

The mass of a heavy quark is also carried by the gluon field that is accompanying it.



We can decide to include all the field accompanying the quark down to infinite distance. This is the POLE MASS.

Or we can cut it off, keeping only contributions at distance below some scale $1/\mu$ (or equivalently, keeping only wavelength above μ. These are the SHORT DISTANCE MASSES.

The pole mass has a linear power correction related to the large coupling of the field at long distances.

There are short distance masses that are very close to the pole mass, since they include radiation effects down to very small scales. As precision increases, at some point one should switch to some of these schemes.

The mass scheme

The pictorial view of the definition of the short distance mass is more than just a suggestive picture. If we cut off long range radiation by giving a mass μ to the gluon, and compute the top self energy we get at order α_s

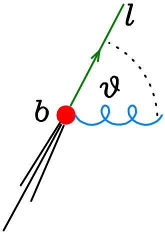
$$m_{\rm pole} = m(\mu)|_{\mu=0} = m(\mu) + \frac{2}{3}\mu\alpha_s. \qquad (1)$$

One can check explicitly that this correction can be computed classically as the difference between the vacuum energies of the Coulomb field in the massless and massive (i.e. screened) case.

Can we get rid of linear corrections?

Example: lepton spectrum in B decay.
(reminded by Melnikov in Zurich talk.)
How come? (There are theorems about this, but what is the underlying physics?) A tentative explanation:

- A b quark in a B meson undergoes Fermi motion, i.e. it has momentum of order Λ. But its kinetic energy is of order Λ^2/m_b, because it is non-relativistic. So, no linear power corrections there.
- The decay can take place in a time fraction when the b is in a virtual state associated with the emission of a soft gluon.



The decay product are boosted with velocity $v = k/m_b$, where k is the soft gluon momentum. The corresponding change in the lepton momentum is $\delta p_l \approx vp\cos\theta$. Thus effects linear in v vanish under azimuthal average.

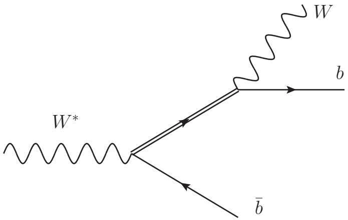
As a result, the semileptonic spectrum has no linear power corrections *if expressed in terms of a short distance mass*

Thus this have to do with the B being rotational invariant? The explanation also holds for heavy quarks produced on-shell, since their soft radiation pattern does not depend upon its spin.

Linear Power Corrections

Ferrario Ravasio, Oleari, P.N.2019

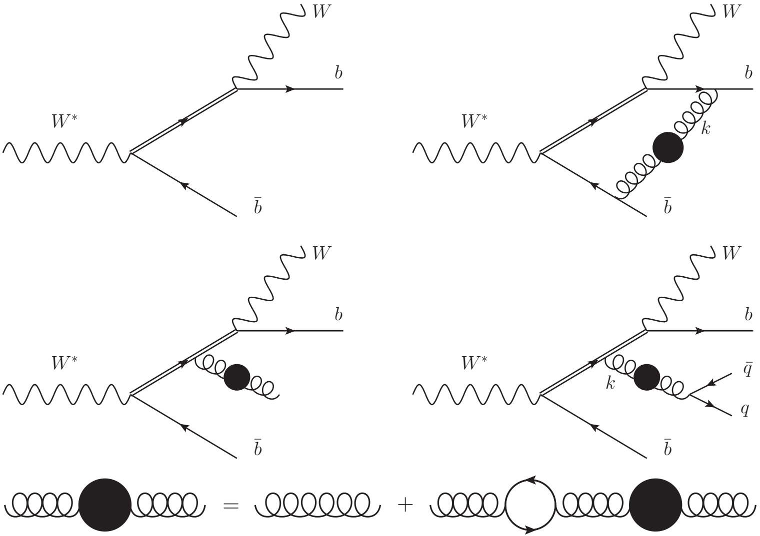
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can examine any infrared safe observable, no matter how complex.

Diagrams up to leading N_f one gluon correction:



Linear Power Corrections

We find:

- The total cross section (in the simplified model!) does not have them. However, if cuts are present (even if only on the lepton!) they are there.
- Jets have large linear power corrections with coefficients of order $1/R$. These have some sort of universality, and may be controlled by calibration. However, power corrections with no $1/R$ enhancement are also there, and are not universal.

Linear Power Corrections

- Leptonic observables have linear power corrections. These are seen to be absent for distributions defined in the top rest frame, consistently with the B decay example.
- In general, the top finite width screens the linear power corrections due to top emissions. Thus, for observables not involving jets, like the leptonic observables, we see that the linear corrections disappear for finite with.

Can this fact be exploited to perform top mass measurements free of linear power corrections?

At the moment the answer is not known.

Shower generators

Questions for shower generators:

- Are soft and non-perturbative effects in Shower Generator consistent with our field-theoretic knowledge?
- Would the cancellation of linear power corrections in top decays at rest be properly modelled by shower Monte Carlo's?
- Can we relate the modeling of soft radiation from top to more rigorous field theoretical ones, and thus relate the Monte Carlo mass parameters to a more rigorously defined one?

The last question has been investigated by Hoang and collaborators in a sequel of papers. These investigations are carried out in the framework of boosted jets, and in simplified production contexts.

At the moment very little is known for realistic cases.

Theory status: consensus in the theory community

- Top mass issues have been characterized by conflicting opinions among theorists.
- Some progress in understanding differences has been made with the 4 pages document in HE-LHC Working Group report, authored by G. Corcella, A. Hoang, H. Yokoya and myself.

Theory status: consensus in the theory community

Some points where we agree:

- As far as HO corrections are concerned (i.e. corrections of relative order $\alpha_s^k(m_t)$, to the top mass), direct measurements can be viewed as accessing the top pole mass.
- The scheme issue (advocated by Hoang and collaborators) has to do with short distance masses evaluated at low scales (of the order of the shower cut-off). These masses are very close to the pole mass.

Theory status: understanding linear power corrections

- Many questions with no answer (see also Kirill's talk).
- There are frameworks where we can make some theoretical progress in understanding how to deal with linear power corrections. However, much more work is needed.
- μὴ εἶναι βασιλικὴν ἀτραπὸν ἐπί γεωμετρίαν
("There is no Royal short-cut to geometry" Euclid, 300 BC).
The problem looks difficult because it is. It involves words like renormalons, mass schemes, etc. However, at variance with what Euclid said, we should try to find short cuts, (i.e. simpler interpretations of theoretical results) that may be transferred to simulation programs.

Direct Measurements

When we talk about linear power corrections, we do not really know if we are talking about 100 MeV or 1 GeV effects. Monte Carlo model can provide an estimate of the size of this effects.

In Ferrario Ravasio,Jezo,Oleari,P.N,2017, we find that the particle level peak position of the reconstructed mass of the top spans a range of about 250 MeV when switching between Pythia8, PYTHIA6, Herwig7 and HERWIG6. This suggests a contribution to the theory error of ± 120 MeV.

There are good reason to insist with direct measurements, and, lacking for the moment a better method to estimate linear power corrections, use Shower Monte Carlo's.

Top mass from differential distributions

These are very interesting, but be aware of the many ways in which they may induce us to wrongfully think that they bypasse the linear power corrections issue.

- In a fixed order calculation HO corrections have a lower bound that is reached near the 6th-8th order, and that is matched to a power correction. How can one possibly obtain uncertainties that are smaller than typical hadronic scales when stopping the expansion at the first or second order?
- "Unfolding to the parton level top": this has problems relating to the interpretation of the accompanying Coulomb field (i.e. ultimately about the mass scheme).

Parton level top: the boosted case.

Few simple observation inspired by Hoang,Plätzer,Samitz 2018:

- In the highly boosted limit the Coulomb field accompanying the quark can be view as a superposition of quanta (method of virtual quanta, or Fermi-Weiszäker-Williams). The shower algorithm is fully consistent with this view, and the shower cutoff plays a role similar to the short-distance mass scale μ.
- Away from the boosted regime this identification is absent. It is not clear whether and how the Shower represents the Coulomb energy. Thus there is a problem identifying the scheme in which the parton level top is defined.

If you wonder how can we have a “mass scheme” problem with an NLO or NNLO calculation performed in a definite renormalization scheme, keep in mind that we can introduce short distance mass schemes defined at low (but still perturbative) scales. Differences in results using different schemes will show up at orders $n = log(m_t/\mu)$. If μ is near a GeV, this is order 5.

Other observables

The example of B physics (and also the results of Ferrario Ravasio,Jezo,Oleari,P.N,2017), suggests that leptonic observables in the top rest frame may be free from power corrections.
It may be interesting to see if this property survives also with an approximate reconstruction of the top rest frame, or if there are suitably defined lepton observables that satisfy this property.

See for example if the observable proposed in Kawabata,Shimizu,Sumino,Yokoya 2015 has this property.

Conclusions

- Top mass measurements at hadron colliders, when the precision approaches feew hundred MeV's, pose difficult and profound theoretical problems, involving our understanding of non-perturbative corrections in QCD, and of how they are implemented in shower generators.
- The traditional method: aim at an observable, measure it, extract its value from a perturbative calculation, and estimate power corrections using a shower Monte Carlo, is still a valuable strategy to follow, as long as better ways of doing it are not in sight.
- Theoretical studies of the form of linear power corrections and to what extent they can/are implemented in shower Monte Carlo are at a primitive stage, but they are promising. They can help us to understand the limitations of current measurements, and they can help to identify better observables.