The Many Worlds of Quantum Mechanics

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Before there was quantum mechanics: classical (Newtonian) mechanics

A simple theory. Two main elements:

• A state (point in phase space).

• Equations of motion.

Laplace’s Demon:
Given the complete state of the universe (positions and velocity of every particle & field), we could predict the future and past perfectly.
Quantum mechanics messes everything up

As before we have two main elements:

• A state (wave function in Hilbert space).

• Equations of motion (Schrödinger).

But we need extra rules to deal with observations:

• Only certain quantities are observable.

• Observational outcomes are probabilities, not certainties. **Born Rule:** probability = |wave function|^2.

• After observation, wave function “collapses.”
This is terrible.

- What counts as an “observation”?
- When exactly does an observation occur?
- What divides “classical” from “quantum”?
- Why probabilistic?

Thus, “interpretations of quantum mechanics.”
Feynman:
“I think I can safely say that nobody understands quantum mechanics.”
Everything is Quantum: Everett’s Many-Worlds Interpretation

- There is no “classical realm.” Everything is quantum, including you, the observer.
- Wave functions never “collapse.” Only smooth, deterministic evolution.
- *Apparent* collapse due to entanglement/decoherence.
- Unobserved possibilities – other “worlds” – still exist.
<table>
<thead>
<tr>
<th><strong>Textbook QM</strong></th>
<th>vs.</th>
<th><strong>Everettian QM</strong></th>
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<tr>
<td>1. Hilbert space $\mathcal{H}$.</td>
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<td>2. Schrödinger equation: $H\ket{\Psi} = i\partial_t \ket{\Psi}$</td>
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<td>3. Measurements associated with an operator $A$ return eigenvalues: $A\ket{A_n} = a_n \ket{A_n}$</td>
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<td>That’s all.</td>
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<td>4. Born Rule: probability of $a_n$ is given by $</td>
<td>\langle A_n</td>
<td>\Psi \rangle</td>
</tr>
<tr>
<td>5. Collapse: after measurement, system is in state $\ket{A_n}$</td>
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A classical cat is in a definite awake/asleep state:

\[
[\text{cat}] = \begin{bmatrix} \text{awake} \\ \text{asleep} \end{bmatrix}
\] -or-

\[
[\text{cat}] = \begin{bmatrix} \text{asleep} \\ \text{awake} \end{bmatrix}
\]

A quantum cat can be in a superposition:

\[
(\text{cat}) = \left( \begin{array}{c} \text{awake} \\ + \\ \text{asleep} \end{array} \right)
\]

Ruth Schrödinger: “I think my father just didn’t like cats.”
Thinking Quantum-Mechanically

Classical intuition:

“Cats are either awake or asleep, but quantum mechanics allows for superpositions. That’s weird.”

\[
\text{(cat)} = (\text{awake}) + (\text{asleep})
\]

Quantum intuition:

“Cats are in arbitrary quantum superpositions, but when we look we only see them either awake or asleep. That’s weird.”
Schrödinger’s Cat: Textbook (Copenhagen) version

The cat is in a superposition of (awake) and (asleep), then observed. (Hilbert space = all such superpositions.)

\[ (\text{cat})[\text{observer}] = \left( \begin{array}{c}
\text{awake} \\
\text{asleep}
\end{array} \right) [\text{Hilbert space}] \]

\[ \text{observation/collapse} \]

- or -

\[ (\text{cat})[\text{observer}] = \left( \begin{array}{c}
\text{awake} \\
\text{asleep}
\end{array} \right) [\text{Hilbert space}] \]
Schrödinger’s Cat: Many-Worlds version

Now the cat and the observer are both quantum.

\[(\text{cat})(\text{obs}) = (\text{cat} + \text{obs})(\text{cat} + \text{obs})\]

Why don’t we ever “feel like” we’re in a superposition?
Decoherence (environmental entanglement) implies “branching” into distinct worlds.

Consider the cat, an observer, and an environment.

\[
(\text{cat})(\text{obs})(\text{env}) = (\text{cat}) + (\text{obs})(\text{env})
\]

\[
(\text{cat,obs})(\text{env}) = (\text{cat,obs}) + (\text{cat})(\text{obs,env})
\]

\[
(\text{cat,obs,env}) = (\text{cat,obs,env}) + (\text{cat,obs,env})
\]
Bottom line: Everettian QM is just wave functions smoothly evolving through time. No collapse, no mention of measurement, nothing unpredictable.

If a system becomes entangled with a messy, permanent, external environment, states corresponding to different outcomes will never interfere with each other, nor affect each other in any way.

It’s as if they have become part of separate worlds.
Misguided objections to Many-Worlds

1. That’s too many universes!

   The dimensionality (size) of Hilbert space remains fixed. Every version of QM includes states that describe many worlds. The only question is whether you let them happen.

2. This can’t be tested!

   EQM is just wave functions obeying Schrödinger. Easy to falsify by detecting collapse or extra variables. If you have an alternative, we can test that! (Dynamical collapse models, hidden variables.)
Reasonable questions for Many-Worlds

1. Why are *probabilities* given by the square of the wave function? For that matter, why are there probabilities *at all*? The theory is completely deterministic!

2. How does the *classical world* emerge? Why does the wave function branch into quasiclassical realms? Why is there spacetime?
Probability is epistemic, not objective.

**Self-locating uncertainty**: you know the wave function of the universe, but not where you are within it.

\[
\text{(cat)(env)(obs)} = (\text{\begin{array}{c} \text{cat} \\ + \\ \text{env} \end{array}}) (\text{\begin{array}{c} \text{obs} \\ \text{amplitude}^2 \end{array}})
\]

**decoherence**

\[
\text{(cat,env,obs)} = (\text{\begin{array}{c} \text{cat} \\ \text{env} \\ \text{obs} \end{array}}) + (\text{\begin{array}{c} \text{cat} \\ \text{env} \\ \text{obs} \end{array}})
\]

Two identical individuals; assign credences for being on each branch weighted by \(|\text{amplitude}|^2\).

[Sebens & Carroll]
Classical mechanics: every subsystem has its own state, a set of positions and momenta.

Quantum mechanics: just one state, the “wave function of the universe.”

It’s a vector in Hilbert space. Featureless. How do we know what it describes?
Usual strategy: take a classical system and “quantize” it.

Take configuration space of $N$ particles (positions or momenta, not both): $x_1, x_2, x_3...$

The wave function assigns a complex number to every configuration:

$$\Psi : \{x_1, x_2, x_3 \ldots\} \rightarrow \mathbb{C}$$

Probability of finding that configuration is the wave function squared (the Born Rule):

$$P(x_1, x_2, x_3 \ldots) = |\Psi(x_1, x_2, x_3 \ldots)|^2$$
Another problem: quantum gravity

Einstein: gravity is the curvature of spacetime.

How do you quantize spacetime itself?
Maybe the problems cancel each other out?

Don’t try to “quantize gravity.”

Instead, find gravity within quantum mechanics.

Key feature: entanglement.
Recall:

\[(\text{cat})(\text{obs}) = (\text{cat}) + (\text{obs})\]

That final state features entanglement between the cat and the observer. There is a correlation between the observable features of the two systems. A crucial concept in understanding quantum reality.
In quantum field theory, empty space is a busy place.
Almost all degrees of freedom in the universe are field modes in their vacuum states.

Number of particle degrees of freedom in a person ≈ $10^{28}$

Number of field degrees of freedom in a person-sized box ≈ $10^{105}$
Modes in a quantum field theory are highly entangled, and the closer they are to each other, the more entangled they get.

Idea: turn this around.

*Define* “closer together” as “more entangled.”
So the picture is this:

Hilbert space (the space of all quantum states) is a product of different “factors,” representing different degrees of freedom (modes).

Measure how entangled they are.

More entangled = more nearby.

Not really vibrating “things” – just abstract quantum degrees of freedom.

Space, particles, etc. – all are emergent phenomena. Cf. fluids emerging from atoms & molecules.

[Cao, Carroll, Mikhalakis]
Quantifying entanglement

John von Neumann: the amount of entanglement between two systems is related to the entropy of either one of them.

Entropy = ignorance of exact state.

If systems are unentangled, can specify their individual states exactly.

Impossible for entangled systems.
Clue: black hole entropy

Hawking & Bekenstein: entropy of a black hole is proportional to the area of the event horizon.

\[ S_{BH} = \frac{A}{4G} \sim \left( \frac{L}{L_P} \right)^2 \]
Conjecture: the emergent area of any surface $\Sigma$ is proportional to the entropy of the region inside.

Knowing the area of every surfaces fixes the geometry.
So we have:

Geometry $\Leftrightarrow$ Entanglement

Entanglement $\Leftrightarrow$ Entropy

Final ingredient:

Entropy $\Leftrightarrow$ Energy
Entropy is naturally related to energy (heat): $dS = dQ/T$

Continues to be true in quantum mechanics: change in entanglement is proportional to change in energy.

Vibrating modes are naturally entangled; it takes energy to break the entanglement.

Putting it all together we’re left with:

Geometry $\approx$ Energy
But that motto

Geometry = Energy

is just the plain-English version of Einstein’s equation for general relativity:

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ \uparrow \]

curvature of spacetime

\[ \uparrow \]

energy and momentum
The aspiration, in brief:

Starting with nothing but a quantum wave function,

Using entanglement between different parts of the wave function to define an emergent geometry,

We are naturally led to Einstein’s general relativity in the classical limit.

Many issues remain, and the program could crash at any time. But perhaps we’re not far from understanding the quantum origin of spacetime itself.
SOMETHING DEEPLY HIDDEN

Quantum Worlds and the Emergence of Spacetime

SEAN CARROLL

New York Times bestselling author of The Big Picture