A primer on LIGO and Virgo gravitational wave science

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Lecture 1

- Gravity and General Relativity
- Linearized gravity and gravitational waves
- How to measure gravitational waves
- Scientific goals and future outlook

Full calculations can be found in


“Gravitational Waves: Theory and Experiments”, Michele Maggiore, Oxford University Press

Introductory books:

“A First Course in General Relativity”, Bernard Schutz, Cambridge University Press

“Gravity: An Introduction to Einstein’s General Relativity”, James B. Hartle, Pearson

General public introductory books:

“A Journey into Gravity and Space-time”, J. A. Wheeler, Scientific American Library

Gravity

Gravity is the least understood fundamental interaction with many open questions. Should we not now investigate general relativity experimentally, in ways it was never tested before?

Gravity
- Main organizing principle in the Universe
  - Structure formation
- Most important open problems in contemporary science
  - Acceleration of the Universe is attributed to Dark Energy
  - Standard Model of Cosmology features Dark Matter
  - Or does this signal a breakdown of general relativity?

Large world-wide intellectual activity
- Theoretical: combining GR + QFT, cosmology, …
- Experimental: astronomy (CMB, Euclid, LSST), particle physics (LHC), Dark Matter searches (Xenon1T), …

Gravitational waves
- Dynamical part of gravitation, all space is filled with GW
- Ideal information carrier, almost no scattering or attenuation
- The entire universe has been transparent for GWs, all the way back to the Big Bang

Gravitational wave science can impact
- Fundamental physics: black holes, spacetime, horizons
- Cosmology: Hubble parameter, Dark Matter, Dark Energy
Einstein’s theory of general relativity

Einstein discovers deep connections between space, time, light, and gravity

Einstein’s Gravity

- Space and time are physical objects
- Gravity as a geometry

Predictions

- Gravitation is curvature of spacetime
- Light bends around the Sun
- Expansion of the Universe
- Black holes, wormholes, structure formation, …
- Gravitational waves: curvature perturbations in the spacetime metric
Arthur Eddington observes bending of light in May 1919

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<td>30 Jun 1973</td>
<td>Mauritania</td>
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Einstein arrives in New York

Einstein is now world famous

“Relatively speaking... he's delighted!”

EINSTEIN - famous German Scientist faces barrage of camera-men on arrival.
Einstein predicts existence of gravitational waves

Einstein publishes his discovery in Sitzungberichte Preussische Akademie der Wissenschaften, 22 June 1916 and on 14 February 1918.
Gravitational radiation exists: PSR B1913+16

Russell A. Hulse
Joseph H. Taylor, Jr.
In 1974 discovery of the first pulsar in a binary system

Period $\sim 8$ h

GW emission shortens the period

Indirect detection of GWs
Nobel 1993

$\Delta t_p [s]$ Periastron advance

Deviation $<0.2\%$
Robert Oppenheimer predicts existence of black holes

Together with Snyder he predicts in 1939 the creation of black holes for neutron stars with masses above approximately $3M_\odot$ (Tolman-Oppenheimer-Volkoff limit)

Black holes are made of spacetime. According to GR all matter has disappeared in a singularity

Finkelstein (1958) identified Schwarzschild surface as an event horizon

Discovery of pulsars (1967) and identification as neutron stars (1969)
Evolution of stars

Compact objects are the product of stellar evolution: white dwarfs, neutron stars, and black holes.
We have observed over 2,000 pulsars (NS) in our Milky Way. Thus NS exist and there are probably billions of NS per galaxy. Before GW150914 many of us expected the first GW event would be a BNS.

Astronomers discovered 17 binary neutron stars (BNS), e.g. Hulse Taylor BNS. These systems undergo strong quadrupole-type acceleration. After a certain time, both NS will collide. In the process a black hole may be created.


Numerical simulation representing the binary neutron star coalescence and merger which resulted in the gravitational-wave event GW170817 and gamma ray burst GRB170817A.

Credits: Numerical Relativity Simulation: T. Dietrich (Max Planck Institute for Gravitational Physics) and the BAM collaboration

Scientific Visualization: T. Dietrich, S. Ossokine, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics)
Interferometric gravitational wave detectors
Tiny vibrations in space can now be observed by using the kilometer-scale laser-based interferometers of LIGO and Virgo. This enables an entire new field in science
Eleven detections…so far
First gravitational wave detection with GW150914 and first binary neutron star GW170817
Event GW150914
Chirp-signal from gravitational waves from two coalescing black holes were observed with the LIGO detectors on September 14, 2015
Intermezzo: what are gravitational waves?
Metric tensor and geometry
Geometry

Geometry is the study of shapes and spatial relations

Euclidean geometry
- sum of internal angles of a triangle is $180^\circ$
- circumference of a circle with radius $R$ has length $2\pi R$
- a sphere with radius $R$ has area $A = 4\pi R^2$
- Newton: space is Euclidean and time is universal
- Newton thus assumes a flat spacetime

Euclidean geometry is not the only kind of geometry
- Bolyai, Lobachevsky, Gauss discovered other geometries

Especially, Carl Friedrich Gauss (1777 – 1855) and Bernhard Riemann (1826 – 1866) have made important contributions to the development of geometry. The defined differential geometry

Determining the geometric properties of the space (or even spacetime) around us is an experimental enterprise. It is not the subject of pure mathematics
**Differential geometry and the line element**

Line element is the central element of any geometry

Example: Length of a curve in Euclidean flat geometry in 2D
- Cartesian coordinates
- divide the curve in segments $\Delta l_i$
- use Pythagoras for each segment $(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2$
- sum in order to find the entire length of the curve

$$L_C(P, Q) \approx \sum_{i=1}^{n} \Delta l_i$$

Take the limit to infinitesimal elements
- this defines the line element

$$L_C(P, Q) = \int_{P}^{Q} dl$$

\[ dl^2 = dx^2 + dy^2 \quad \Rightarrow \quad dl = (dx^2 + dy^2)^{1/2} \]

Now we only need to know how to evaluate such an integral. When we add up the line elements, we need to account for the direction of the line element, since it gives different contributions $dx$ and $dy$
Differential geometry and the line element

Line element is the central element of any geometry

Length of a curve in Euclidean 2D plane

The simplest solution is to make use of a parametrized curve. The coordinates become functions of a continuous varying parameter \( u \)

We then have \( x(u) \) and \( y(u) \)

**Example:** the function \( y = x^2 \) gives \( x(u) = u, \ y(u) = u^2 \)

**Example:** the function \( x^2 + y^2 = 1 \) gives \( x(u) = \cos(u), \ y(u) = \sin(u) \)

One has \( \Delta x = \frac{\Delta x}{\Delta u} \Delta u \)

\[ \begin{align*}
\Delta x &= \frac{\Delta x}{\Delta u} \Delta u \\
\Delta y &= \frac{\Delta y}{\Delta u} \Delta u
\end{align*} \]

\[ dx = \frac{dx}{du} du, \quad dy = \frac{dy}{du} du, \]

\[ dl = (dx^2 + dy^2)^{1/2} \]

\[ dl = \left( \left( \frac{dx}{du} \right)^2 du^2 + \left( \frac{dy}{du} \right)^2 du^2 \right)^{1/2} = \left( \left( \frac{dx}{du} \right)^2 + \left( \frac{dy}{du} \right)^2 \right)^{1/2} du \]

\[ L_C(P, Q) = \int_P^Q \, dl = \int_{u_P}^{u_Q} \left( \left( \frac{dx}{du} \right)^2 + \left( \frac{dy}{du} \right)^2 \right)^{1/2} \, du \]
Consider 3D Euclidean (flat) space

Cartesian coordinates  \[ dl^2 = dx^2 + dy^2 + dz^2 \]

Spherical coordinates
\[
\begin{align*}
x &= r \sin \theta \cos \phi, \\
y &= r \sin \theta \sin \phi, \\
z &= r \cos \theta.
\end{align*}
\]

Using this line element we can again determine the length of a curve in the 3D flat space. We then can build up the entire 3D Euclidean geometry. Gauss realized that the line element is the key concept.

We can go further and also consider the geometry of 2D curved spaces in the 3D flat space. To this end we fix \( r = R \) and find for the line element

\[ dl^2 = R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2 \]
Curved spaces

Line element for curved surface leads to different geometry

Consider 2D curved surface (space) of a sphere

Line element \( dl^2 = R^2 \, d\theta^2 + R^2 \sin^2 \theta \, d\phi^2 \)

Coordinates are \( \theta \) and \( \phi \)

Example: calculate the circumference of circle \( C \)

Answer: \( \theta \) is constant, thus

\[
C = \int_0^{2\pi} R \sin \theta \, d\phi = R \sin \theta [\phi]_0^{2\pi} = 2\pi R \sin \theta
\]

When we measure the radius and circumference in the same curved surface, the we discover that the circumference is different than the Euclidean result

The sum of the internal angles of a triangle is larger than \( 180^\circ \)

The shortest path between two points is the segment of a circle through both point, where the center of the circle coincides with the center of the sphere. This is a *great circle*

Note that curvature is an *intrinsic* property of the surface
Metric and Riemannian geometry

Line element is the starting point for any geometry

We have seen several line elements

We deduced expressions for these line elements from the known properties of these spaces

Line element is expressed as the sum of the squares of coordinate differences, in analogy to the expression by Pythagoras

With the line element we can determine the lengths of curves and shortest paths, and with this the properties of circles and triangles, and in fact the entire geometry of a particular (curved) space

Riemann realized the one also can take the line element as starting point for a geometry (and not only as a summary)

An $n$-dimensional Riemann space is a space for which

The functions $g_{ij}$ are called the metric coefficients and must be symmetric $g_{ij} = g_{ji}$ so there are $n(n+1)/2$ independent coefficients

The set of metric coefficients is called the metric, sometimes also the metric tensor, and it determines the complete geometry of the space
Metric perturbations and gravitational waves
Metric of Special and General Relativity

Special Relativity uses the metric of flat spacetime: Minkowski metric

Line element measures the distance between two infinitesimally close events in spacetime

For the global Cartesian coordinates \( ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 \)

With the Minkowski metric \( \eta_{\alpha\beta} \) we can write the line element as \( ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta \)

With \( \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

In general spacetime is not flat and we write the metric as \( g_{\mu\nu} \)

The metric is a rank 2 tensor. The curvature is defined by the rank 4 Riemann tensor tensor \( R_{\alpha\beta\gamma\delta} \), which depends on \( g_{\alpha\beta} \), and also by its contractions, the Ricci tensor \( R_{\alpha\beta} \) and curvature scalar \( R \)

What generates curvature of spacetime?

General Relativity to the rescue ...

Einstein equations \( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \) with \( T_{\mu\nu} \) the energy momentum tensor
Linearized gravity

Einstein field equations can be written as a wave equation for metric perturbations

We assume that the metric $g_{\mu\nu}$ can be described as flat $\eta_{\mu\nu}$ with a small perturbation $h_{\mu\nu} \ll 1$ encoding the effect of gravitation.

Start from the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Linearize by replacing $g_{\alpha\beta}$ with $\eta_{\alpha\beta} + h_{\alpha\beta}$ and removing higher order terms in $h_{\alpha\beta}$.

The equivalent equation is still complicated: change variables $h_{\alpha\beta} \rightarrow \tilde{h}_{\alpha\beta}$.

The trace-reversed variables are defined by $\tilde{h}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} h$.

In GR physics does not depend choice of coordinates (gauge).

Choose a specific set of coordinates systems that meet the condition $\frac{\partial}{\partial x^\alpha} \tilde{h}^{\alpha\beta} = \partial_\alpha \tilde{h}^{\alpha\beta} = 0$.

The field equations may now be written as

$$\square \tilde{h}_{\alpha\beta} = \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \tilde{h}_{\alpha\beta} = -2 \left( \frac{8\pi G}{c^4} \right) T_{\alpha\beta}$$

In vacuum the EFE reduce to a wave equation for the metric perturbation $h_{\alpha\beta}$.
Gravitational waves

GW polarizations can be derived in Transverse Traceless (TT) gauge (coordinate system).

In vacuum we have $T_{\alpha\beta} = 0$ and the EFE reduce to the wave equation $\Box \bar{h}_{\alpha\beta} = \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{h}_{\alpha\beta} = 0$

This wave equation obeys the gauge condition $\partial_\alpha \bar{h}^{\alpha\beta} = 0$

We consider solutions $\bar{h}_{\alpha\beta} = \Re \left( \epsilon_{\alpha\beta} e^{-i k_{\rho} x^\rho} \right)$ with wave vector $k^\rho = \begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix}$

Then satisfying the wave equation $\Box \bar{h}_{\alpha\beta} = 0$ implies $k_\rho k^\rho = 0$. This gives $\omega^2 = c^2 \left| \vec{k} \right|^2$ and thus a wave propagating at the speed of light $c$

Using the gauge condition $\partial_\alpha \bar{h}^{\alpha\beta} = 0$ leads to $k_\rho \epsilon^{\rho\sigma} = 0$ and we have 6 remaining independent elements in the polarization tensor.
Among the set of coordinate systems, it is possible to choose one for which $\varepsilon_{0\sigma} = 0$. This reduces the number of independent elements to 2 denoted “plus” and “cross” polarization.

General solution for a wave traveling along the $z$-axis is $\varepsilon_{\alpha\beta} e^{-ik_\rho x^\rho} = \left( h_+ \varepsilon_{\alpha\beta}^+ + h_\times \varepsilon_{\alpha\beta}^\times \right) e^{-ik_\rho x^\rho}$

Here is $\varepsilon_{\alpha\beta}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\varepsilon_{\alpha\beta}^\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ are a basis for the polarization tensor.

Gravitational waves

GW polarizations can be derived in Transverse Traceless (TT) gauge (coordinate system).

Interferometers are well suited to measure such a tidal effect: beamsplitter in the center and mirrors attached to mass points.
Gravitational waves: effect on two test masses

Metric perturbation $h$ is the relative variation in proper length between the test masses.

Consider the proper length between two test masses in free fall.

Line element $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$.

Consider a plus-polarized gravitational wave.

We only consider the $x$-direction and for proper length ($dt = 0$) we have $ds^2 = g_{xx} dx^2$.

We have $L = \int_0^{\Delta x} \sqrt{g_{xx}} \, dx = \int_0^{\Delta x} \sqrt{1 + h_{xx}^{TT}(t, z = 0)} \, dx$.

We find $L \approx \int_0^{\Delta x} \left(1 + \frac{1}{2} h_{xx}^{TT}(t, z = 0)\right) dx = \left(1 + \frac{1}{2} h_{xx}^{TT}(t, z = 0)\right) \Delta x$.

The TT-coordinate system has coordinates “fixed” at the test masses.

The proper length changes, because the metric changes. This is observable.

Note that for a given strain $h$, the displacement that must be measured scales with $L$. 
End of intermezzo
For decades relativists doubted whether GWs are real

**Gravitational waves**

In Eddington’s early textbook on relativity, he quipped that some people thought that “gravitational waves travel at the speed of thought”

Einstein predicted GWs in 1916 … but then doubted their existence for the rest of his life. He proposed many experiments, including really hard ones, but never suggested a search for gravitational waves

The controversy lasted four decades, until the Chapel Hill Conference in 1957

**Felix Pirani**

Solved the problem of the reality of gravitational waves

He showed relativists that gravitational waves must have physical reality, because you could invent a (thought) experiment that could detect them

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If now one introduces an orthonormal frame on $\zeta$, $v^\mu$ being the timelike vector of the frame, and assumes that the frame is parallely propagated along $\zeta$ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (1) becomes

$$\frac{d^2 \eta^a}{d\tau^2} + R^a_{\text{obo}} \eta^b = 0 \quad (a, b = 1, 2, 3)$$

Here $\eta^a$ are the physical components of the infinitesimal displacement and $R^a_{\text{obo}}$ some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.
Weber bars

Joe Weber, co-inventor of the maser, was working with John Wheeler at Princeton on gravitational waves. The two of them were at Chapel Hill, and listened well to Pirani’s talk.

Weber’s gravitational wave detector was a cylinder of aluminum. Each end is like a test mass, while the center is like a spring. PZT’s around the midline are Bondi’s dashpots, absorbing energy to send to an electrical amplifier.

A massive (aluminum) cylinder. Vibrating in its gravest longitudinal mode, its two ends are like two test masses connected by a spring.
Resonant bar detectors

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.
Rainer Weiss

In 1957, Rai Weiss was a grad student of Jerrold Zacharias at MIT, working on atomic beams.

In the early ‘60’s, he spent two years working with Bob Dicke at Princeton on gravity experiments.

In 1964, Rai was back at MIT as a professor. He was assigned to teach general relativity. He didn’t know it, so he had to learn it one day ahead of the students.

He asked, What’s really measurable in general relativity? He found the answer in Pirani’s papers presented at Chapel Hill in 1957.

In Pirani’s papers, he didn’t “put in” either a spring or a dashpot between the test masses. Instead, he said: “It is assumed that an observer, by the use of light signals or otherwise, determine the coordinates of a neighboring particle in his local Cartesian coordinate system.”

Zach’s lab at MIT was in the thick of the new field of lasers. Rai read Pirani, and knew that lasers could do the job.
Gravitational waves: a 50 year quest

GW research started with Joseph Weber in the late 1960s with the use of resonant bars

GW resonant bar antenna’s

- Weber published various detections

- Discredited
  - IBM, Levine and Garwin (1973)
  - Bell Labs, Tyson (1973)

- Room-temperature and cryogenic bars
  - RT12: Russia, US, UK, Japan, Germany, Italy, China, France
  - CT06: Stanford, LSU, UK, Rome, Regina, Legnaro
  - Triple (94%) and fourfold operations (57%): 2005 - 2007
  - Plans: TIGA, GRAVITON, (mini)GRAIL and SFERA

- Further claims
  - SN1987A: Rome, Maryland with neutrino’s
  - P. Astone et al., Class. Quantum Grav. 19 (2002) 5449

12. Lunar Surface Gravimeter Experiment

John J. Gigante, J. V. Larson, J. P. Richard, and J. Weber
Apollo 17 Prelim. Sci. Rept. SP-330

The primary objective of the lunar surface gravimeter (LSG) is to use the Moon as an instrumented antenna (refs. 12-1 to 12-8) to detect gravitational waves predicted by Einstein’s general relativity theory. A secondary objective is to measure tidal deformation of the Moon. Einstein’s theory describes gravitation as propagating with the speed of light. Gravitational waves carry energy, momentum, and information concerning changes in the configuration of their source. In these respects, such waves are similar to electromagnetic waves; however, electromagnetic waves only interact with electric charges and electric currents. Gravitational waves are predicted to interact with all forms of energy.
Interferometers as detectors for gravitational waves

Interferometric detectors were first suggested in the early 1960s and the 1970s

Interferometers

• First proposed by Gertsenshtein and Pustovoit (1962)
  - Sov. Phys. – JETP 878 16, 433 (1962) 879
• First built by Forward at Hughes Research Laboratories
• A study of the noise and performance of such detectors
• First interferometer studies
  - MPG Garching: early 1980s: a 3 m and later a 30 m ITF
  - UK: 1 m and later 10 m ITF with FP cavities
  - Caltech: 40 m ITF; later Japan
• LIGO
  - Caltech and MIT agreement 1984
  - Congress funding in 1991 for M$ 23
  - NSF funding in 1994 for M$ 395
• Virgo
  - Approved CNRS (1993) and INFN (1994)
  - Nikhef joined in 2007

SOVIEI PHYSICS JETP  VOLUME 16, NUMBER 2     FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEIN and V. I. PUSTOVOIT

Submitted to JETP editor: March 3, 1962

It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocystals is ten orders of magnitude worse than that estimated by Weber. In the low frequency range, it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.
LIGO: Laser Interferometer Gravitational Observatory

LIGO realized two interferometers with 4 km long arms in the USA (Livingston LA and Hanford WA)
Detecting gravitational waves with an interferometer

During the measurement of GW150914 our mirrors moved by about $10^{-18}$ m

Sensitivity of the LIGO instruments during O1
Detecting gravitational waves with an interferometer

During the measurement of GW150914 our mirrors moved by about $10^{-18}$ m

Sensitivity of the LIGO instruments during O1

$$\Delta f_S = 10^{-23}/\sqrt{\text{Hz}}$$

$$\Delta L = L\sqrt{\overline{h^2}} = 4 \times 10^{-19} \text{ m}$$
Detecting gravitational waves with an interferometer

During the measurement of GW150914 our mirrors moved by about $10^{-18}$ m.
Event GW150914
On September 14th 2015 the gravitational waves generated by a binary black hole merger, located about 1.4 Gly from Earth, crossed the two LIGO detectors displacing their test masses by a small fraction of the radius of a proton.

Measuring intervals must be smaller than 0.01 seconds.
Intermezzo: let’s have a very basic look at the actual data
Data analysis

The interferometers produce a raw $h(t)$ (so called “hoft”) time series. Note the scale of the amplitudes.
Data analysis
In the frequency domain we have an amplitude spectral density (ASD). We whiten the data (i.e. divide out the median noise curve)
Data analysis
We can get rid of a lot of noise just by restricting where to look: this is called bandpassing
Data analysis

Back in the time domain the bandpassed data look much cleaner. Note the scale of the amplitude. However we observe some spiky-ness. This is indicated by the arrows.
Data analysis
In the frequency domain, we observe some sharp lines. What do you think is the effect of these lines?
Data analysis

The sharp lines are due to instrumentation effects. Let’s get rid of them by using the following transfer function.

![Transfer Function Graph]
Data analysis

Data in the frequency domain after applying a collection of notch files and one bandpass filter

Alex L. Urban, Louisiana State University
LIGO, DCC Document Number G1701155
Data analysis
Back in the time domain, the data look much cleaner! But those spikes are still there. Let’s zoom in …
Data analysis

After these basic filtering steps we clearly see the first detected gravitational wave event in the raw data streams.
Data analysis
There is a coherent signal in both detector streams at this time
Data analysis

Data in comparison to results of numerical relativity calculations

Alex L. Urban, Louisiana State University
LIGO, DCC Document Number G1701155
Parameter estimation: GW150914

Once a gravitational wave signal has been identified in our data, the next step is to measure the properties of the system. This is done by comparing the signal to millions of different waveforms predicted by general relativity and seeing which match the data.
End of intermezzo
Numerical relativity

Numerical relativity example for GW150914: parameters can be determined by matching millions of trial waveforms in 15-dimensional parameter space.
Discovery of gravitational waves


“Testing General Relativity” analysis pipelines developed at Nikhef
Discovery of gravitational waves


Binary Black Hole Mergers in the first Advanced LIGO Observing Run

\[ \sqrt{S(f)} \text{ and } 2|h(f)|\sqrt{f} \text{ (strain/}\sqrt{\text{Hz}}) \]

- Hanford
- Livingston

\[ \text{Frequency (Hz)} \]

\[ \text{Time from 30 Hz (s)} \]

GW150914
LVT151012
GW151226
Observation of GW from a Binary Black Hole merger

First direct detection of GW and the first observation of a binary black hole merger. Two types of searches have been used: unmodeled and modeled. Detector noise is non-stationary and non-Gaussian. Empirical determination with time-shift technique. Trials factor 3

Modeled search” (which makes use of waveform predictions) uses 16 days of coincident Livingston-Hanford data
- False alarm rate < 1 in 203,000 years
- Significance > 5.1σ

![Generic transient search](image1)

![Binary coalescence search](image2)
Recovered gravitational waveforms

Wavelet estimate for the waveform without assuming a particular source in comparison to results if we assume the event is a binary black hole (BBH) as predicted by general relativity

See “Properties of the Binary Black Hole Merger GW150914” [arXiv][1]

The basic physics of binary black hole merger GW150914

The system will lose energy due to emission of gravitational waves. The black holes get closer and their velocity speeds up. Masses and spins can be determined from this inspiral phase

- Total mass $M = M_1 + M_2$
- Reduced mass $\mu = M_1 M_2 / M$
- Chirp mass $M_S^{5/3} = \mu M^{2/3}$

- Chirp $\dot{f} \approx f^{11/3} M_S^{5/3}$
- Maximum frequency $f_{ISCO} = \frac{1}{6^{3/2} \pi M}$

- Speed $\frac{v}{c} = \left( \frac{GM\pi f}{c^3} \right)^{1/3}$
- Separation $R_S = \frac{2GM}{c^2}$

- Orbital phase (post Newtonian expansion)
  \[
  \Phi(v) = \left( \frac{v}{c} \right)^{-5} \sum_{n=0}^{\infty} \left[ \varphi_n + \varphi_n^{(l)} \ln \left( \frac{v}{c} \right) \right] \left( \frac{v}{c} \right)^n
  \]

- Strain $h \approx \frac{M_S^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{rf^3}$
In summary: GW150914 merger of two black holes

Two black holes collided about 1.3 billion years ago and merged to a single black holes. Spacetime is strongly distorted in the process. We have observed this in the form of gravitational waves

1. Collision of two black holes
2. Formation of a black hole
3. Most powerful event ever detected
4. Gravitational waves
5. Ultimate test of Einstein’s theory

https://shenovafashion.com/products/gravitational-waves-dress
Thank you!