

Extracting the proton charge and magnetic radius using dispersively improved chiral effective field theory

Jose M. Alarcón, Douglas W. Higinbotham, **Christian Weiss**, and **Zhihong Ye**

Phys. Rev. C **99** (2019) 044303; <https://doi.org/10.1103/PhysRevC.99.044303>

The following presentation has been approved for all audiences.



(i.e. I am an experimentalists, so this is not going to be a scary theory talk.)

Proton Radius Puzzle

R. Pohl *et al.*, Nature **466** (2010) 213–216.

The signal was nearly not found as they had been scanning frequencies assuming a large radius.

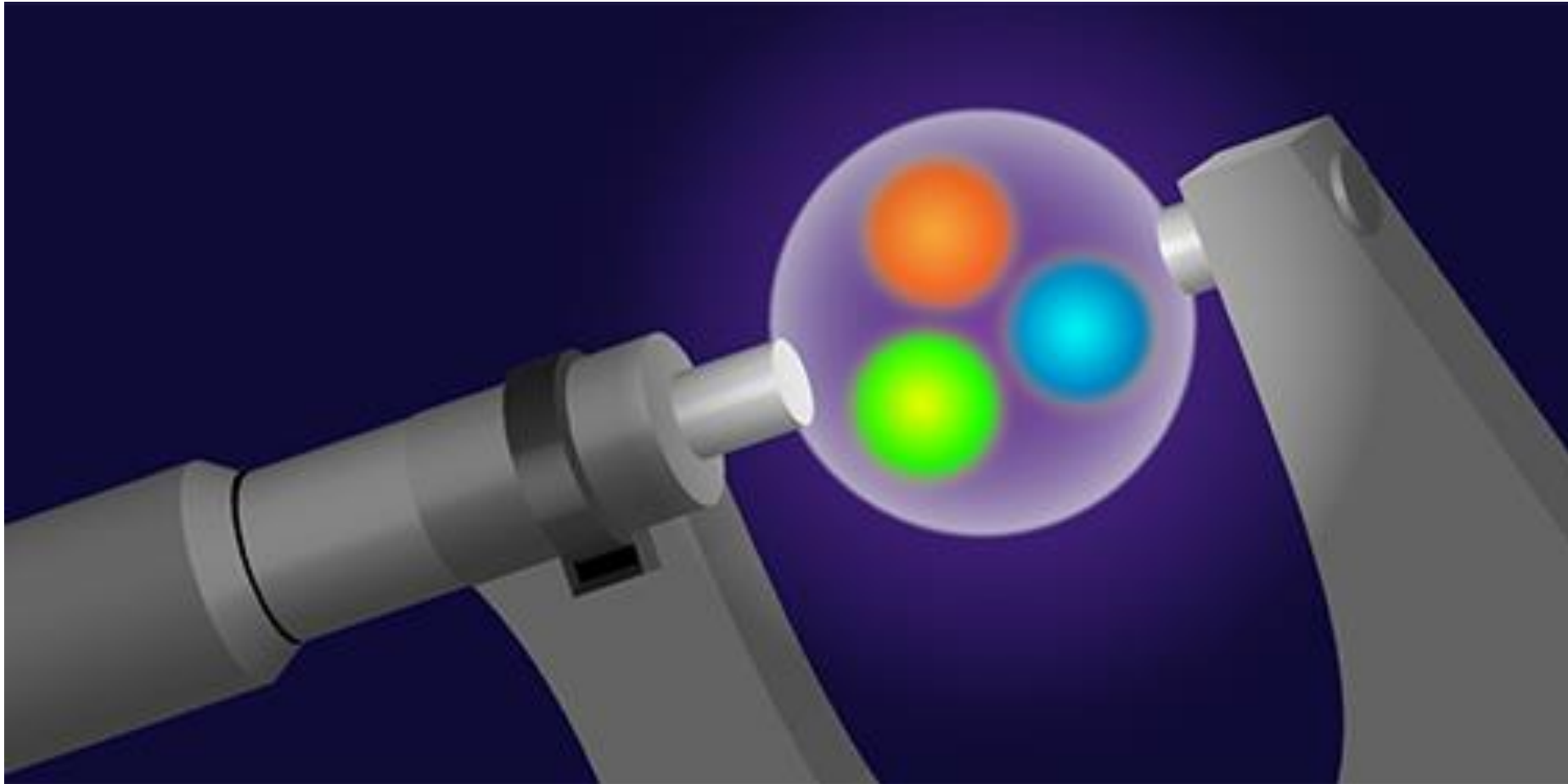
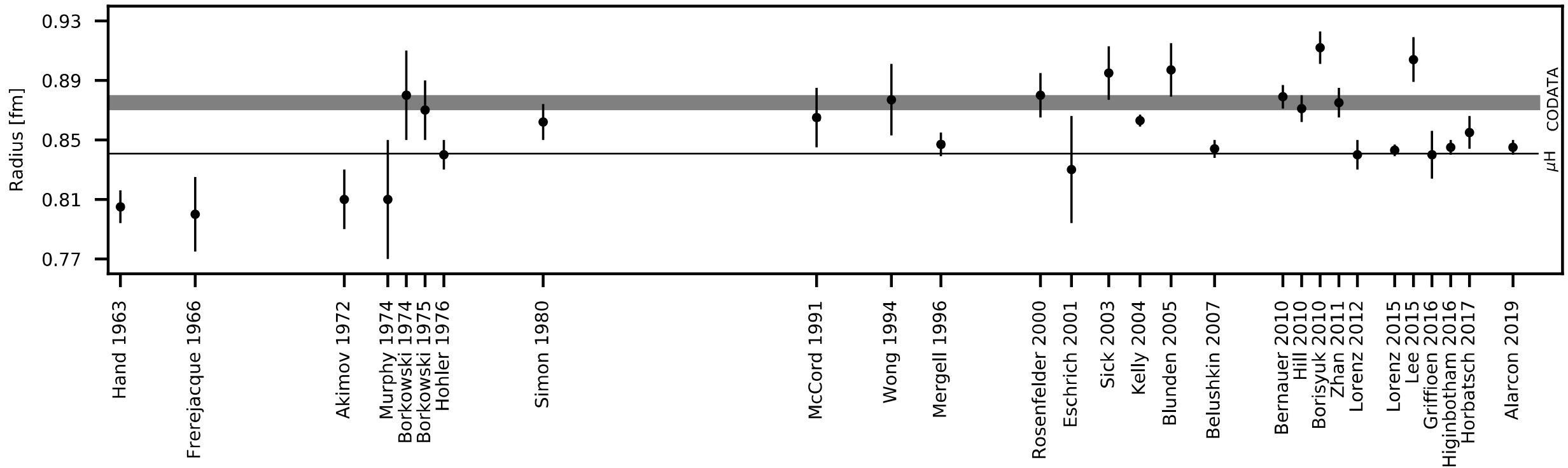


Figure by APS/Alan Stonebraker

Radii from Electron Scattering Is A Story Of Many Results With Almost As Many Methods

L. Hand *et al.*, Rev. Mod. Phys. **35** (1963) 335 is the radius of standard dipole.

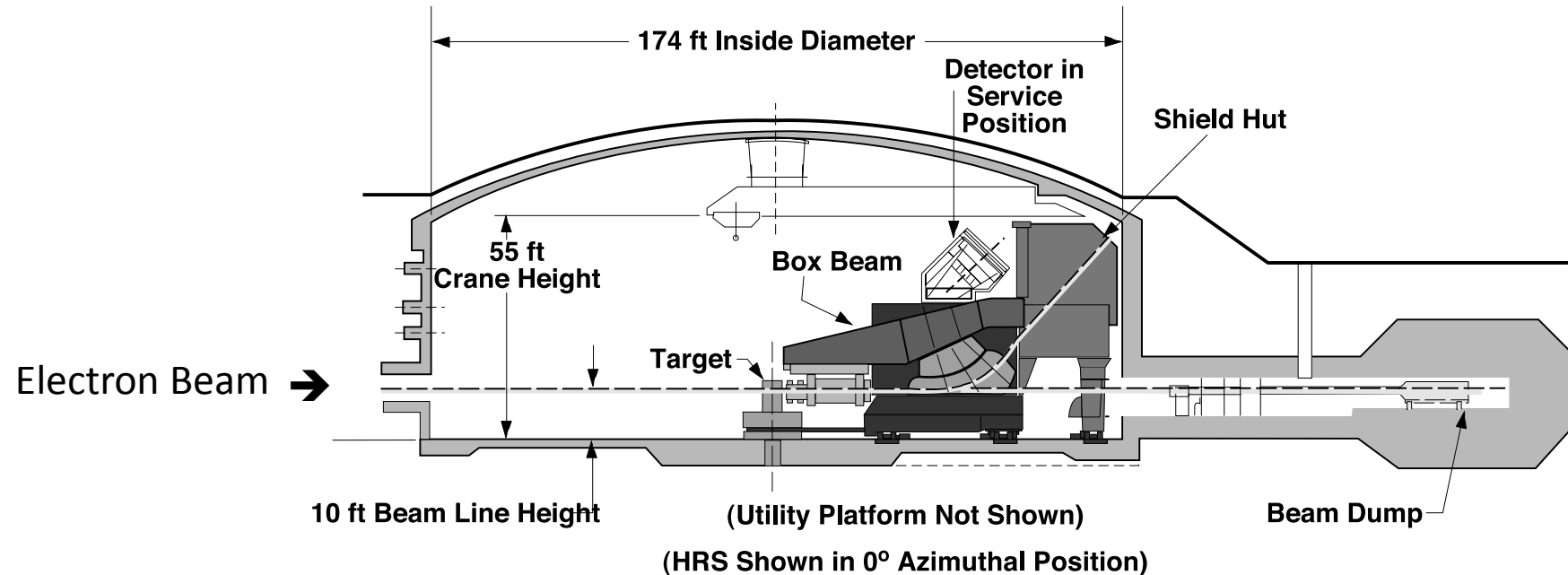
(which Franziska Hagelstein nicely ruled out yesterday during her talk on determining a lower limit for the radius)



<https://gist.github.com/dhiginbotham/264d3f897cf79e3ae08c80b2793a9b77#file-scatteringresults-py>

How do we make the electron scattering measurements?

- Beam of electrons from an accelerator (E)
- Place target material in the beam
 - Foils are easy, nearly point (typically thin) targets and thickness is easy to determine
 - Cryo-targets are challenging (e.g. boiling effects, energy loss)
 - **Since these thickness cannot be determined to less than ~1 percent, floating normalizations are often used.**
- For elastic hydrogen measure scattered electron (E') and/or proton.
 - Over determined reaction
- Spectrometers are typically used (PRad is an exception)
 - Magnetic fields, wire-chambers, reconstructed tracks, sieve data, etc.

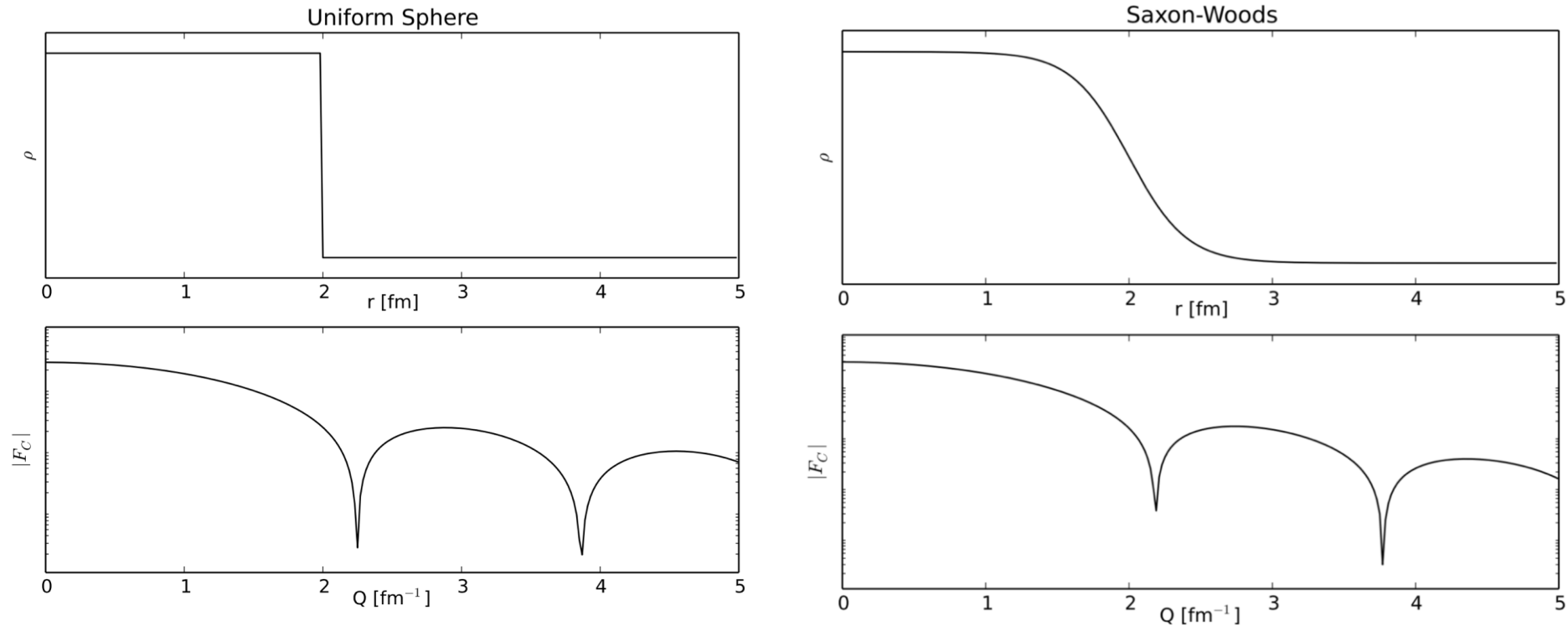


Jefferson Lab Hall A High Resolution Spectrometer



Electron Scattering Charge Radii from Nuclei

Fourier Transformation of Ideal Charge Distributions.



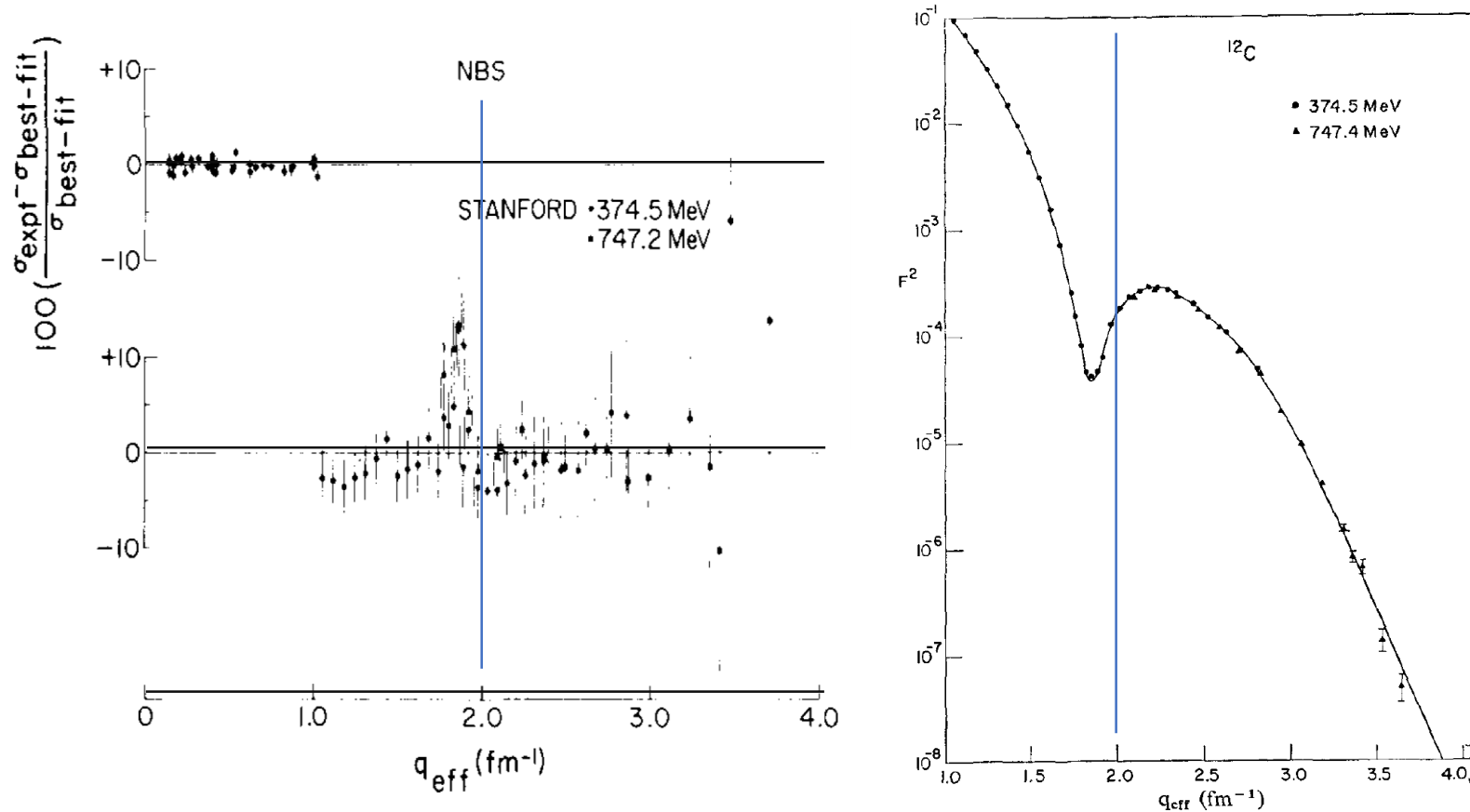
Example Plots Made By R. Evan McClellan (Jefferson Lab Postdoc)

e.g. for Carbon: Stanford high Q^2 data from I. Sick and J.S. McCarthy, Nucl. Phys. **A150** (1970) 631.
National Bureau of Standards low Q^2 data from L. Cardman *et al.*, Phys. Lett. **B91** (1980) 203.

Determining the Charge Radius of Carbon

Stanford high Q^2 data from I. Sick and J.S. McCarthy, Nucl. Phys. **A150** (1970) 631.

National Bureau of Standards (NBS) low Q^2 data from L. Cardman et. al., Phys. Lett. **B91** (1980) 203.



See the L. Cardman's paper for details of the carbon radius (2.46 fm) analysis.

Charge Radius of the Proton

- Proton G_E has no measured minima and it is too light for the Fourier transformation to work in a model independent way.
- Thus for the proton we make use of the fact that as Q^2 goes to zero the charge radius is proportional to the slope of G_E

$$r_p \equiv \left(-6 \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0} \right)^{1/2}$$

This definition of r_p has been shown to be consistent with the radius extracted from the muonic hydrogen data.

Regression Model Selection

Table from Richard J. Hill and Gil Paz, Phys. Rev. D 82 (2010) 113005

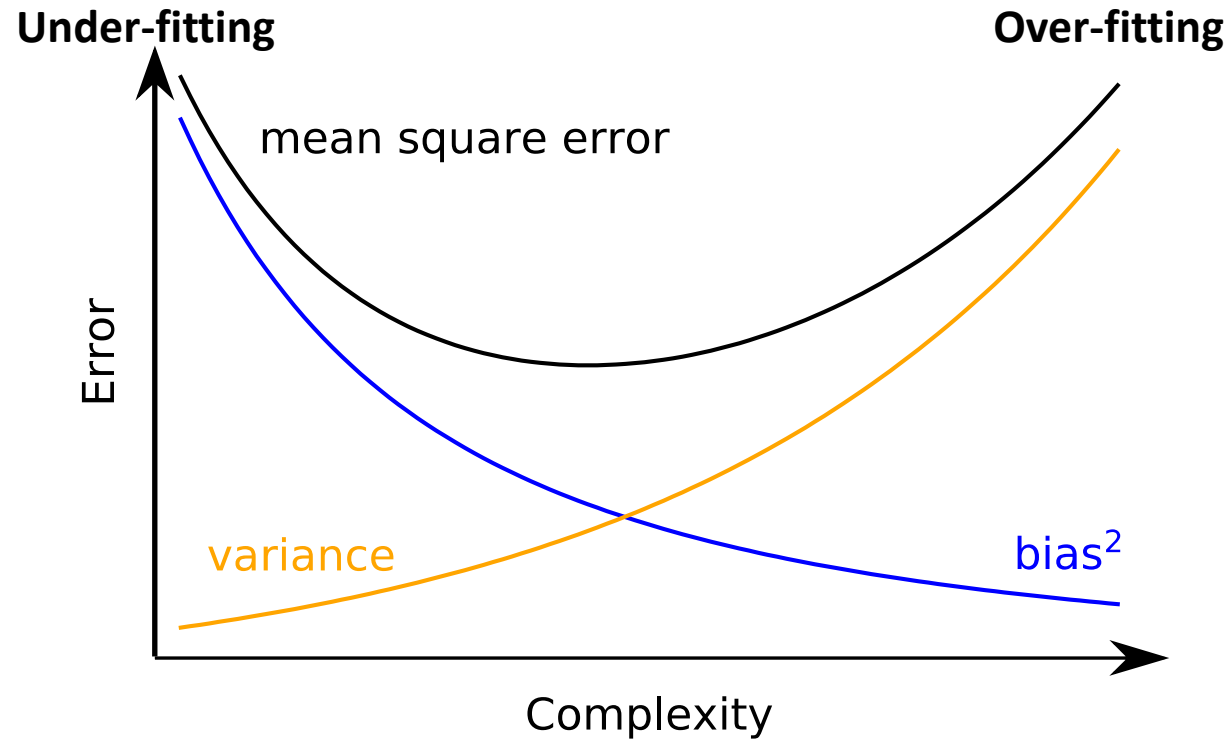
Fit of 70 points of classic nuclear physics data with $Q^2 < 1 \text{ fm}^{-2}$

	$k_{\max} = 1$	2	3	4	5
polynomial	836_{-9}^{+8}	867_{-24}^{+23}	866_{-56}^{+52}	959_{-93}^{+85}	1122_{-137}^{+122}
	$\chi^2 = 34.49$	32.51	32.51	31.10	28.99
continued fraction	882_{-10}^{+10}	869_{-25}^{+26}	—	—	—
	$\chi^2 = 32.81$	32.51			
z expansion (no bound)	918_{-9}^{+9}	868_{-29}^{+28}	879_{-69}^{+64}	1022_{-114}^{+102}	1193_{-174}^{+152}
	$\chi^2 = 36.14$	32.52	32.48	30.35	28.92
z expansion ($ a_k \leq 10$)	918_{-9}^{+9}	868_{-29}^{+28}	879_{-59}^{+38}	880_{-61}^{+39}	880_{-62}^{+39}
	$\chi^2 = 36.14$	32.52	32.48	32.46	32.45

By most model selection techniques (F-test, AIC, BIC) one is going to stop by $k_{\max} = 2$; though the fact reduced χ^2 is much less than one for even $k_{\max} = 1$ says more checks in order (**residuals, normal-qq, leverage, etc.**). On the other hand, without a clear model selection technique, it isn't clear why the table doesn't continue.

NOTE: The continued fraction regression was stuck in local minimum for $k > 2$.

Bias-Variance Trade-Off



Monte Carlo and Root Mean Square Error is a standard tool for picking appropriate model(s).

From the Monte Carlo studies like this one, it has been shown F-Tests, AIC, BIC are all reasonable ways of picking the appropriate function from a single set of data.

$$\chi^2 = \sum_{n=1}^N ((\text{data}_i - \text{model})/\sigma_i)^2,$$

$$\text{reduced } \chi^2 = \chi^2 / (N - N_{\text{var}}),$$

$$\text{AIC} = N \log(\chi^2/N) + 2N_{\text{var}},$$

$$\text{BIC} = N \log(\chi^2/N) + \log(N)N_{\text{var}},$$

Bias In Statistics Is Not A Pejorative Term

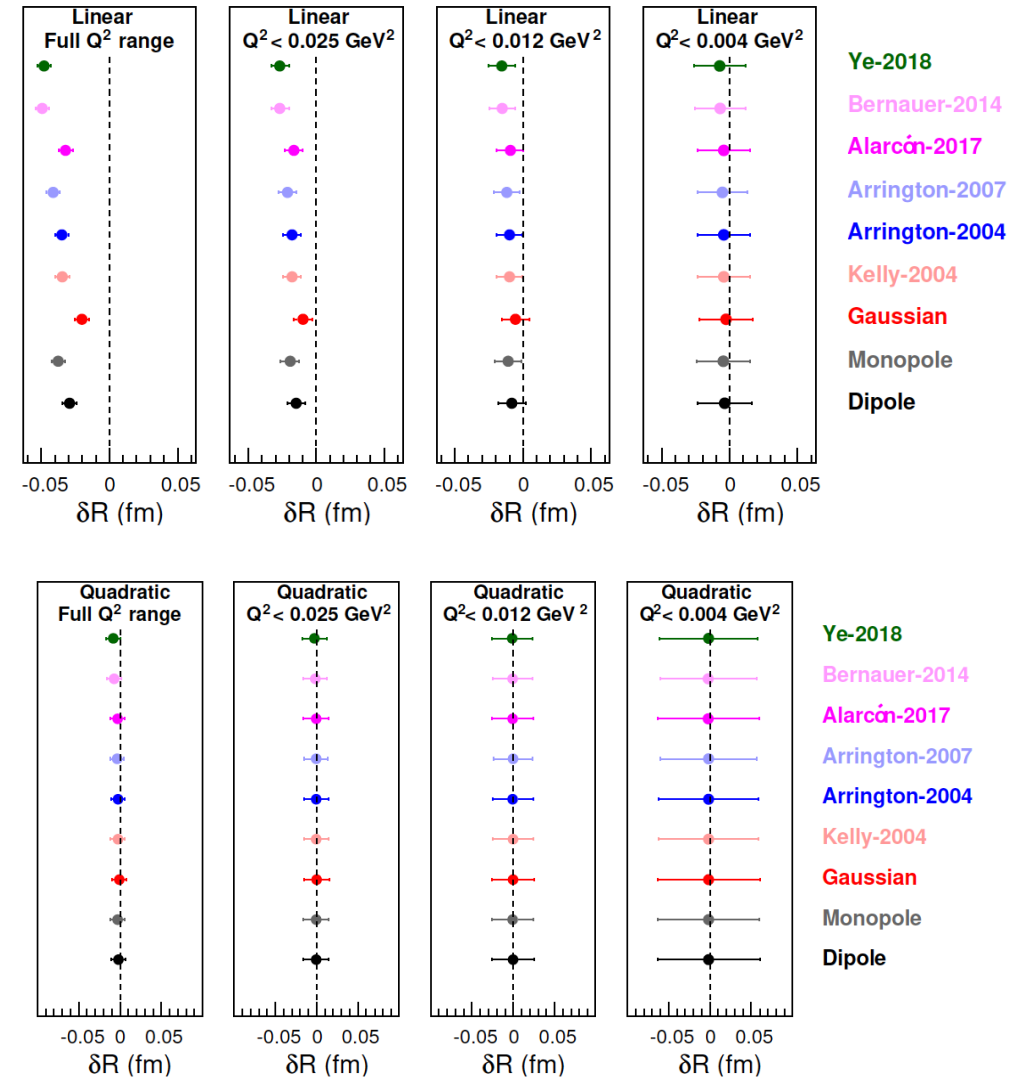
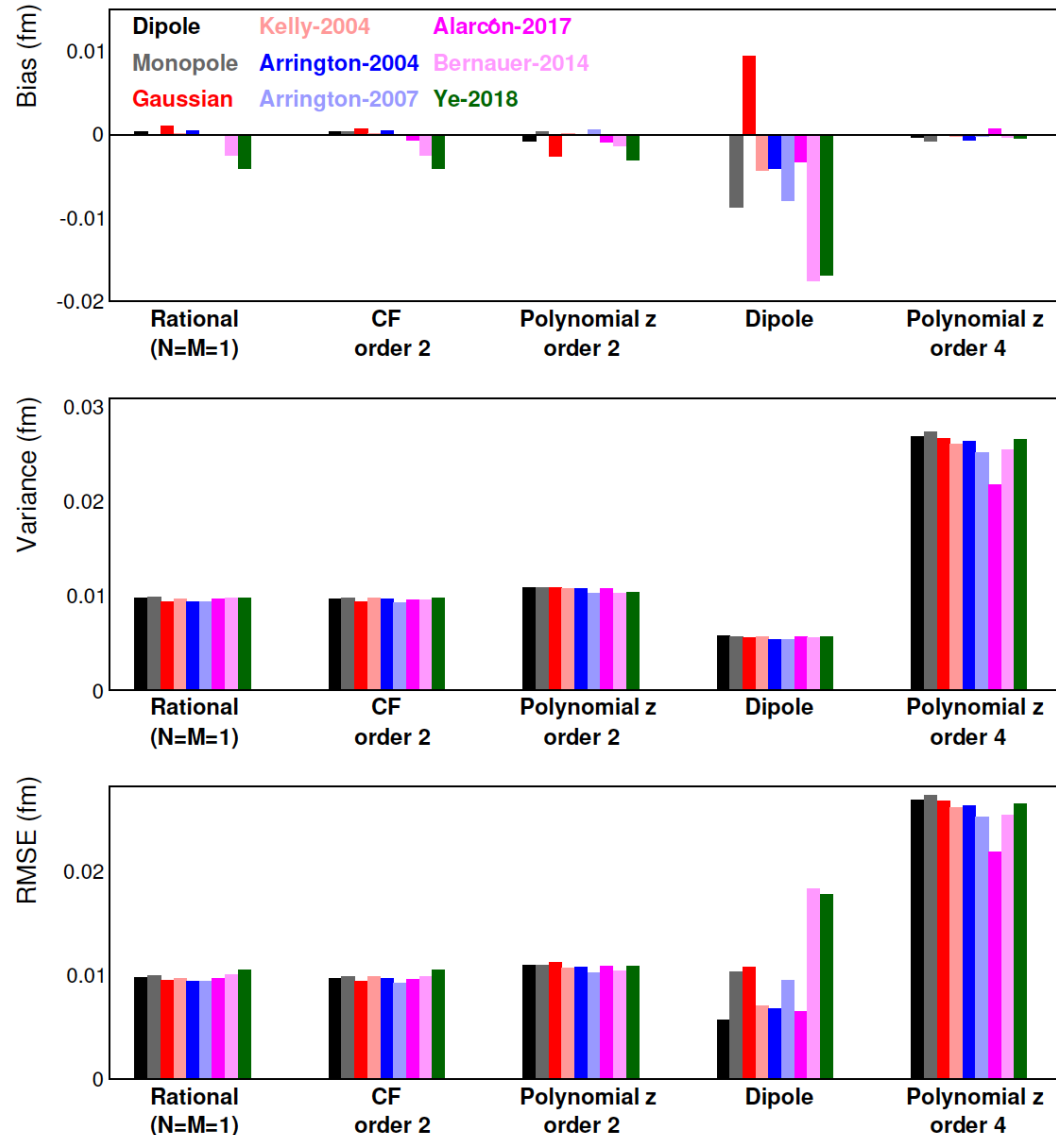
- **Common Usage:** unreasonably hostile feelings or opinions about a social group, prejudice
- **Statistics / Machine Learning:** the difference between the mean extracted value and the true value of the parameter being estimated

Model Selection For PRad BEFORE Seeing The Data

Z. Yan *et al.*, Phys. Rev. C 98 (2018) 025204; <https://doi.org/10.1103/PhysRevC.98.025204>

Shown are a subset of the result of fitting the full range.

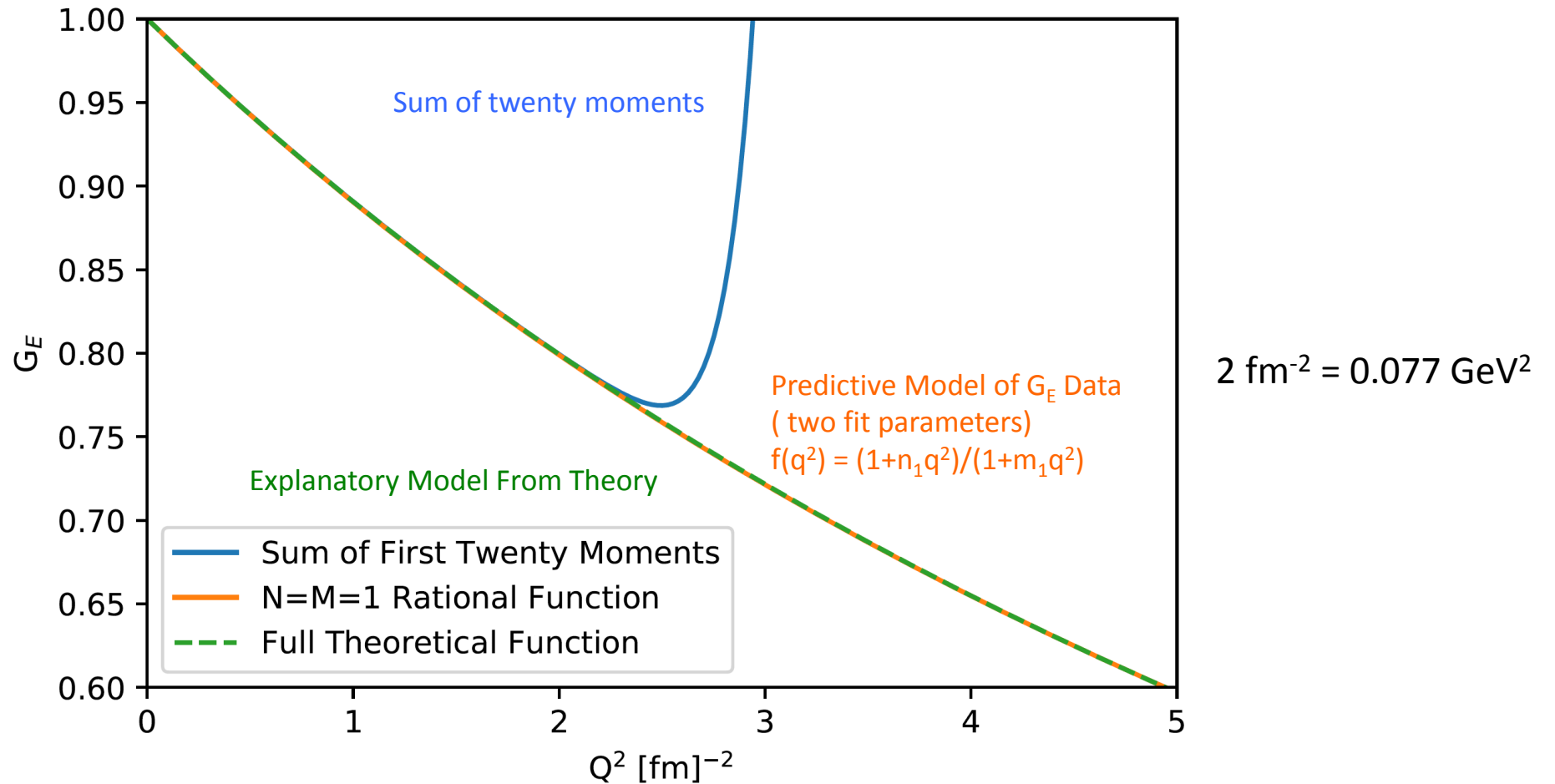
Linear and Quadratic fits for different ranges.



Predictive vs. Explanatory Models

Explanatory Model: J. M. Alarcón and C. Weiss, <https://doi.org/10.1103/PhysRevC.97.055203>.

Predictive Model: Higinbotham *et al.*, <https://doi.org/10.1103/PhysRevC.93.055207>.

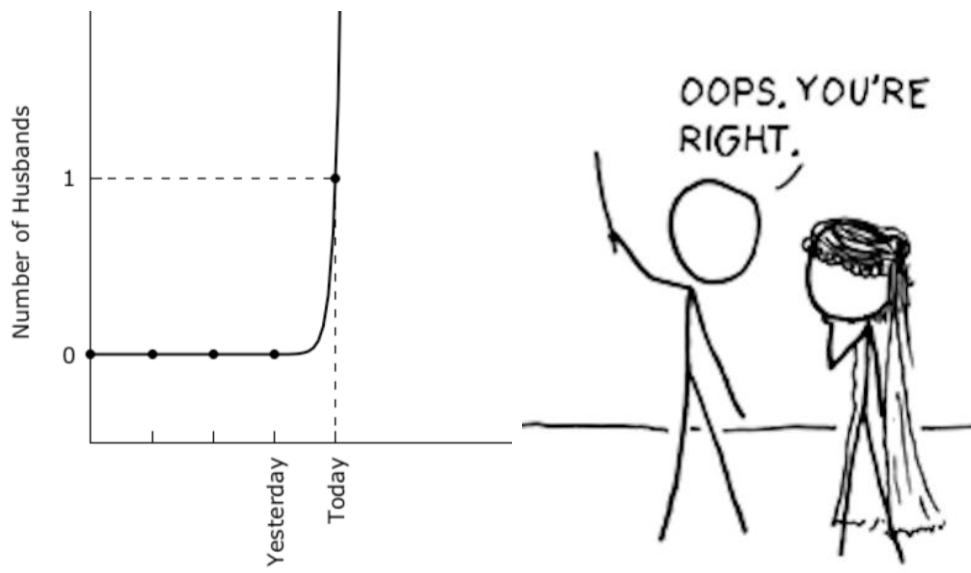
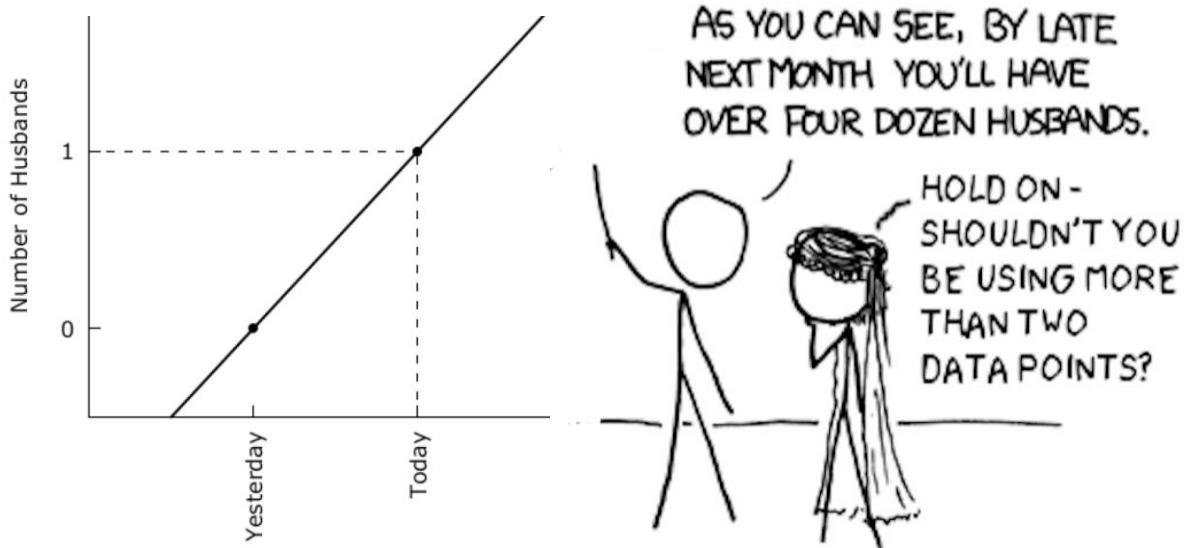


Sanjoy Manhajan (Olin College Applied Science & Engineering Professor) originally suggested the rational functions when I first showed him the data.

Blind Validation Test by Theorist Jerry Miller

- Two sets of Faux G_E Were Set
- Used the binning and uncertainties of the PRad Experiment
- Able To Correctly Tell His Input Radius Using The Robust Functions
- Caveat that the electromagnetic form factors are reasonably well behaved.

XFCD “My Hobby: Extrapolating”



GNUPLOT OVERFITTING CODE
Using 101,600 Iterations To Converge

```
#
# gnuplot overfitting of xkcd Husband Data
# modified from https://xkcd.com/605/
# by
# Douglas W. Higinbotham
#

set terminal wxt enhanced font "verdana,12" size 900,450
set nokey
set xtic rotate 90
set ytic 0,1,1
set border 3

set xtics nomirror
set ytics nomirror

set multiplot layout 1,2

set ylabel "Number of Husbands"

f(x)=f0+f1*x
g(x)=g0*exp(g1*x)

fit f(x) '1.dat' using 1:3 via f0,f1
fit g(x) '2.dat' using 1:3 via g0,g1

set arrow from 0,1 to 2,1 nohead dashtype 7 lc 'black'
set arrow from 2,-0.5 to 2,1 nohead dashtype 7 lc 'black'
set xrange [0:3]
set yrange [-0.5:2]
plot '1.dat' using 1:3:xtic(2) lt 7 lc 'black', f(x) lw 2 lc 'black'
unset arrow

set xrange [-2:6]
set arrow from -2,1 to 2,1 nohead dashtype 7 lc 'black'
set arrow from 2,-0.5 to 2,1 nohead dashtype 7 lc 'black'
plot '2.dat' using 1:3:xtic(2) lt 7 lc 'black', g(x) lw 2 lc 'black'

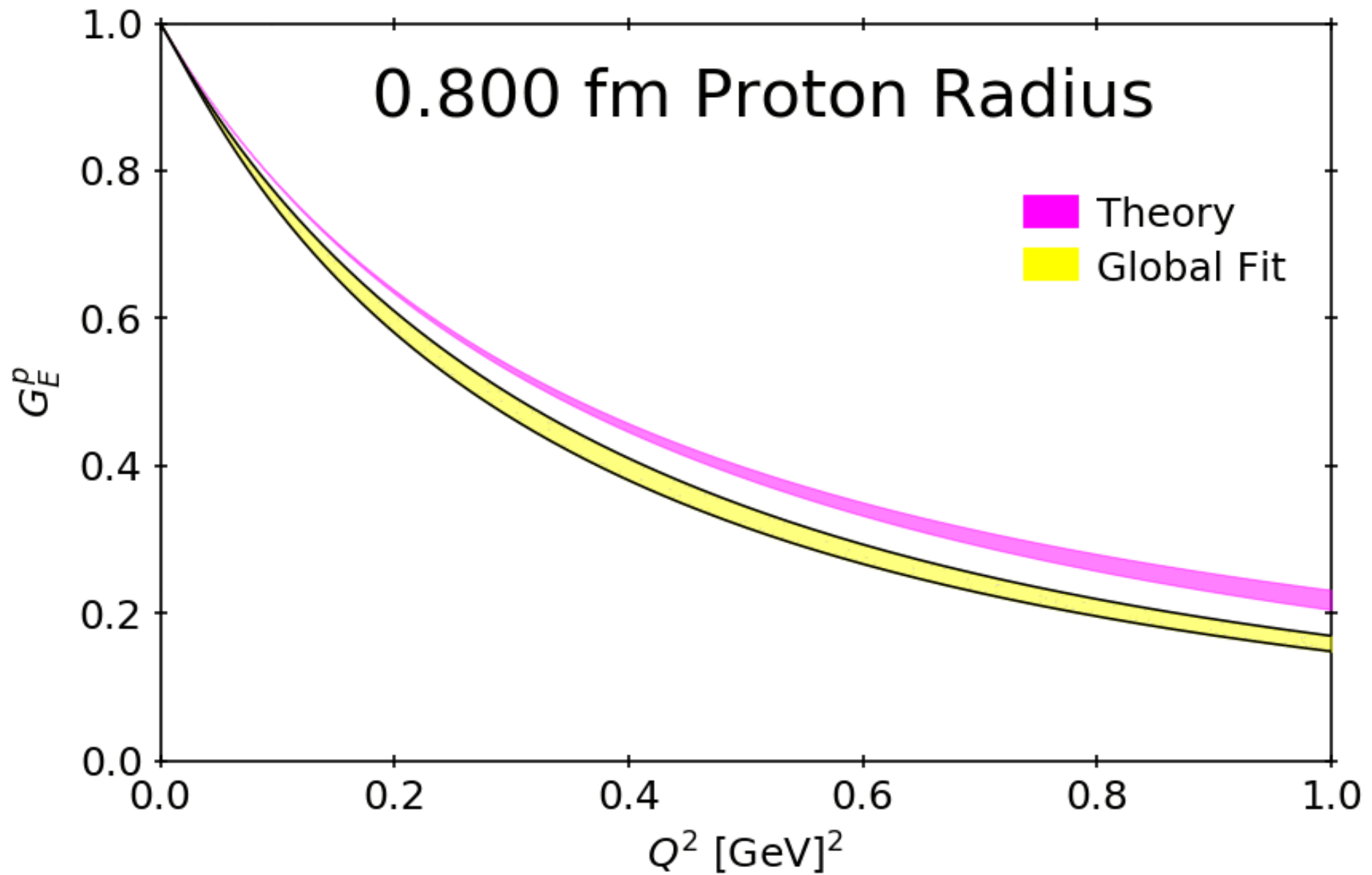
unset multiplot
pause -1
```

Is There Some Way To Avoid The Extrapolations?!

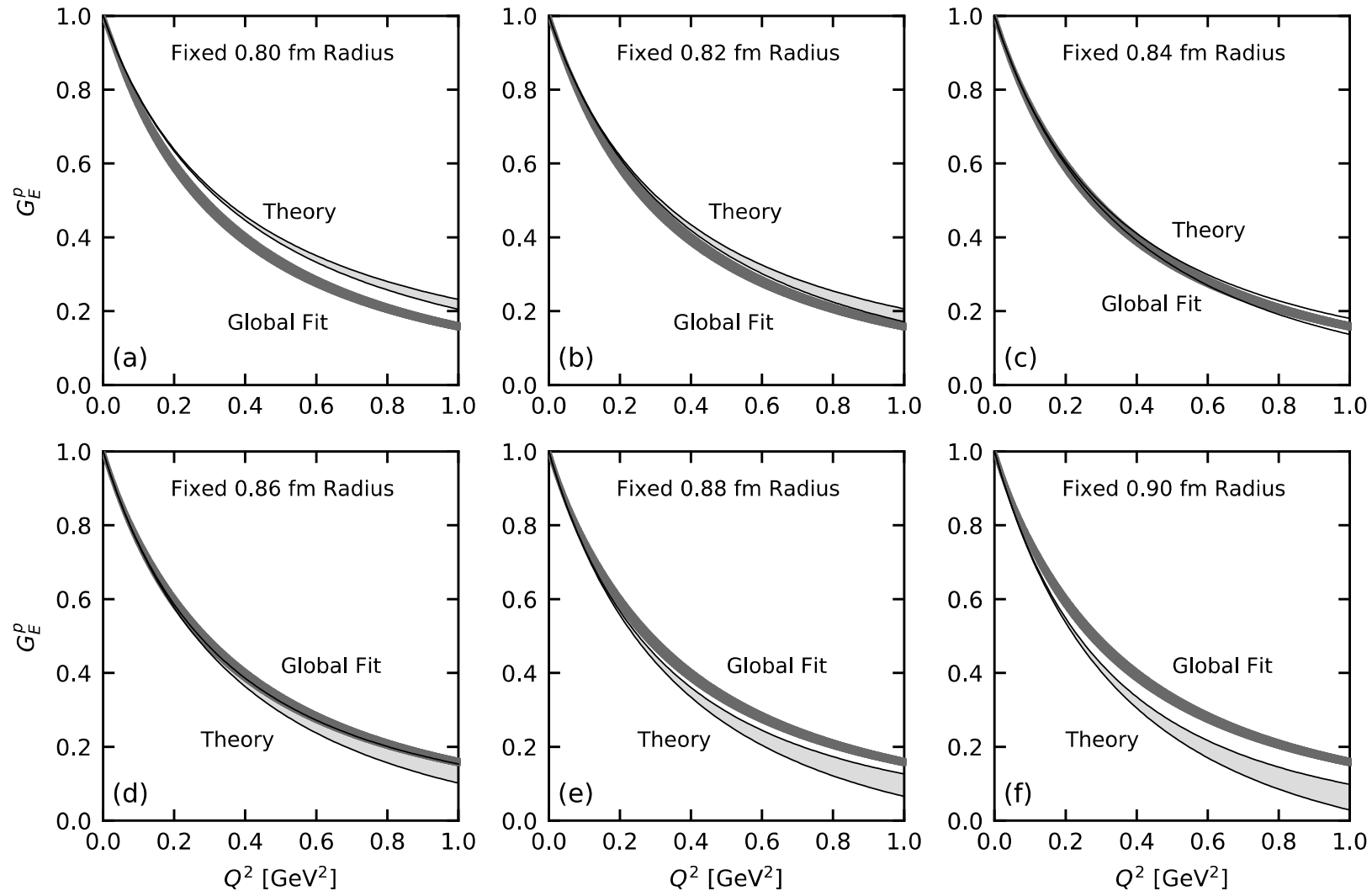
- Recent Global Fit
 - Z. Ye *et al.*, Phys. Lett. B **777** (2018) 8.
 - Used world data
 - **Fixed the Radius** to the current CODATA value
- Recent Theory Work
 - Jose Alarcon and Christian Weiss, Phys. Lett. B **784** (2018) 373-377
 - Dispersively Improved Chiral Effective Theory
 - Proper analytic behavior of the Form Factors
 - Work was **Expanded In Radius** (i.e. the radius is there unknown parameter.

The Goldilocks Method

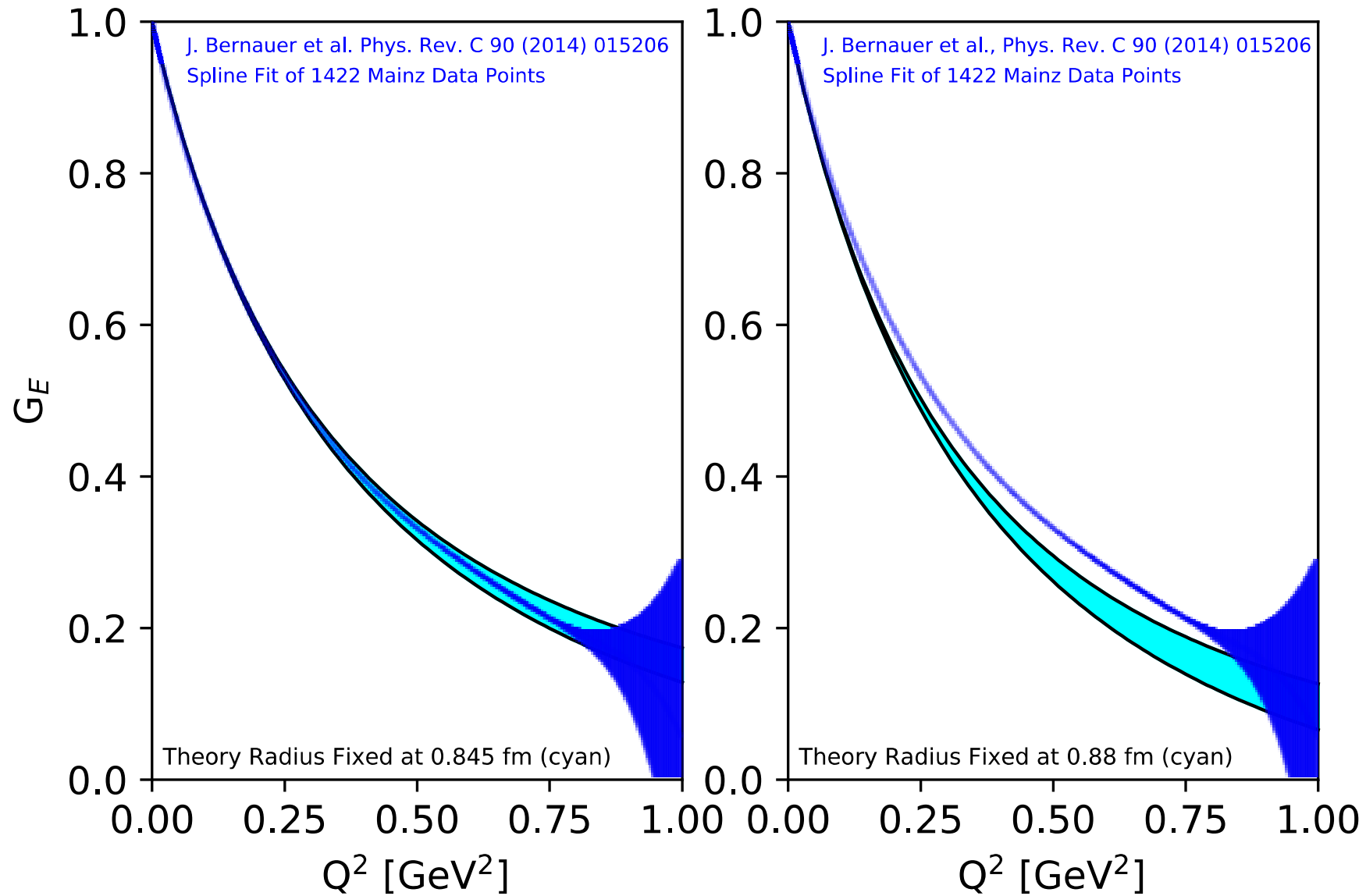
- Take the global fit of Zhihong Ye and fix the radius to different values
 - Values 0.80 to 0.90 fm (completely did the regression for each radius)
 - Different radii slight change the normalization (*i.e. floating normalizations*)
 - z-expansion
 - world data
- Ask Alarcon & Weiss to fix the radius to different values in their model.
 - Generate many different curves for different radii
 - Each curves has all the proper physical properties.
- Compare
 - Is there a curve where the global fit and the theory agree? (i.e. are just right)
 - If so, what is the radius of that curve?



J. M. Alarcón, DWH, C. Weiss, and Zhihong Ye, Phys. Rev. C **99** (2019) 044303
an open access paper thanks to SCOAP



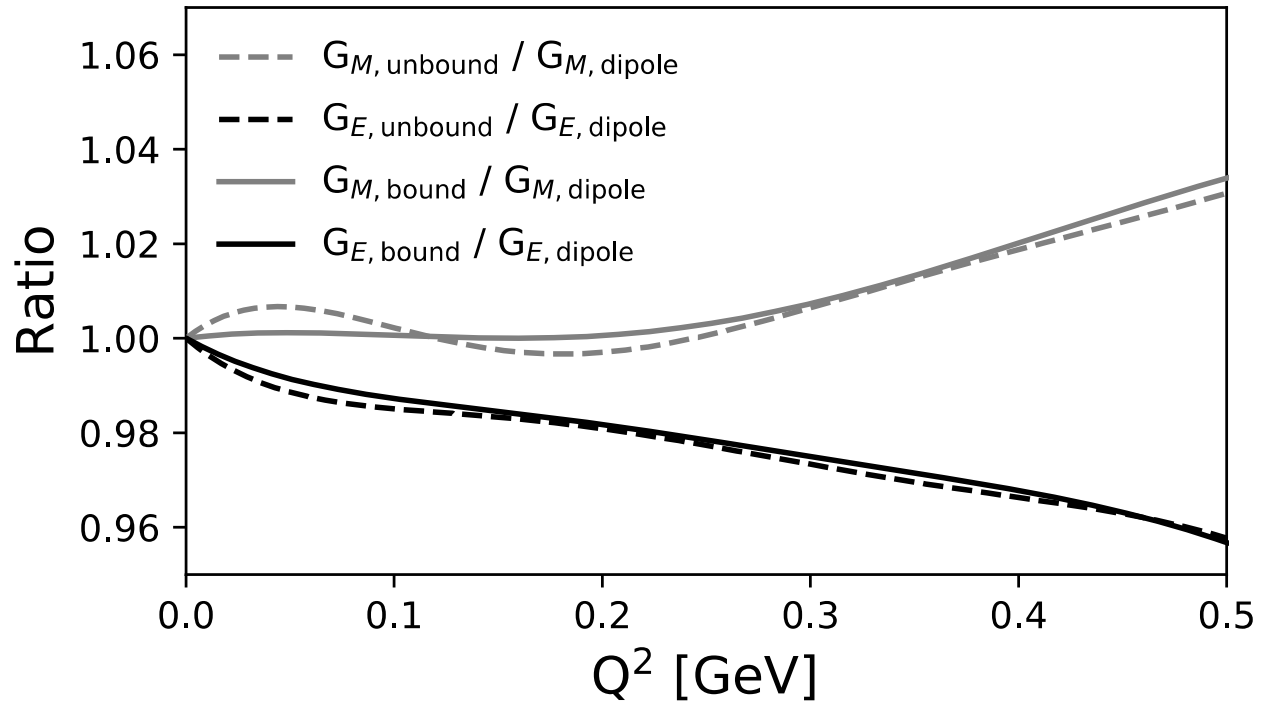
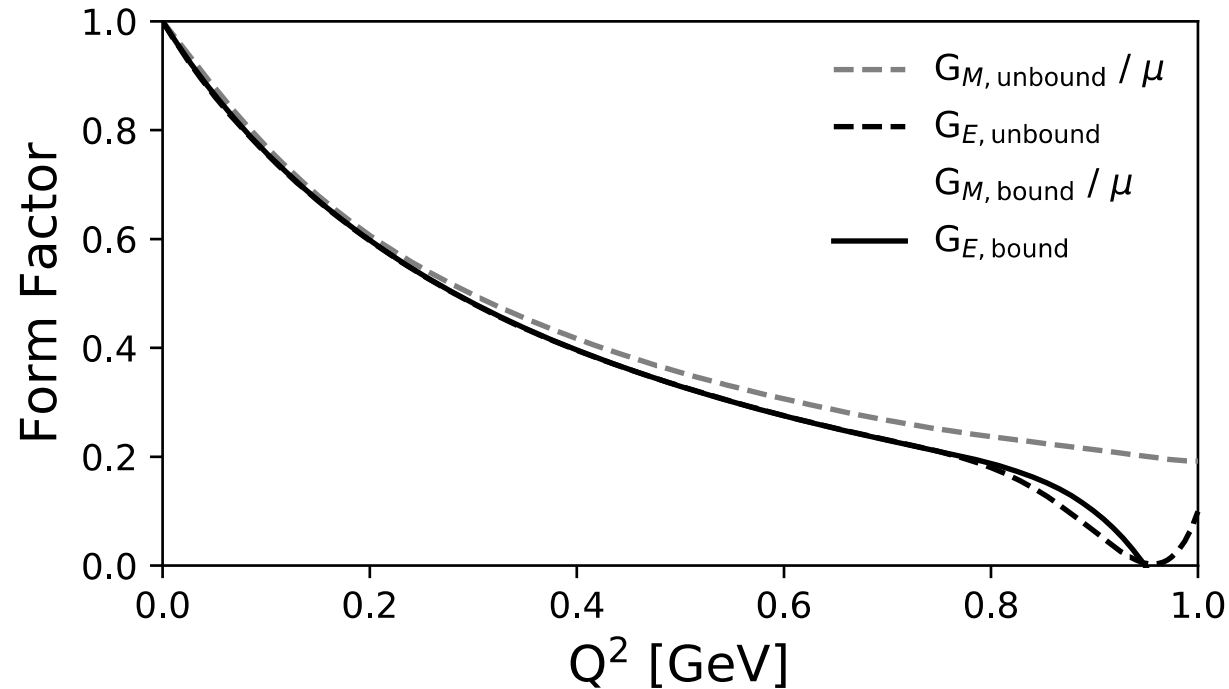
Comparison to Mainz Analysis



“There must be some way out of here
Said the joker to the thief
There's too much confusion
I can't get no relief” – Bob Dylan

Oddly Enough, The Way Out May Lie With G_M^p

Full fit to the Mainz data: 1422 points, 31 normalizations and 11th order polynomials with the mean values shown

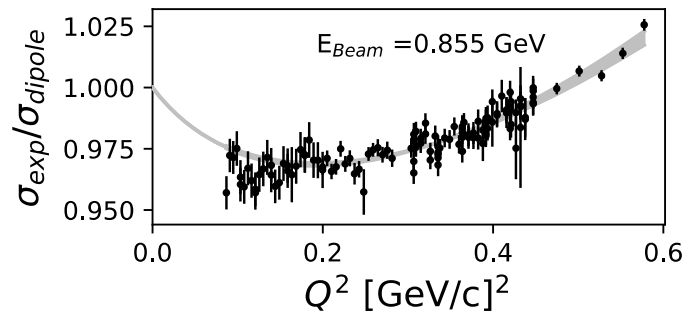
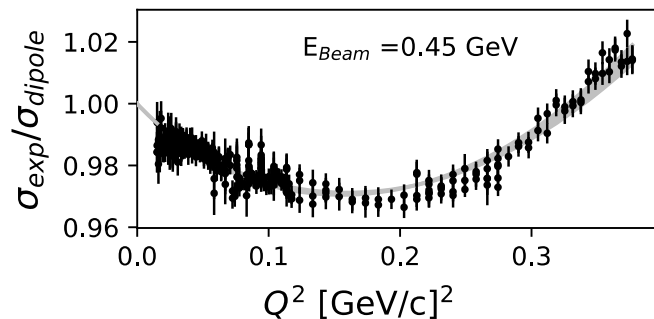
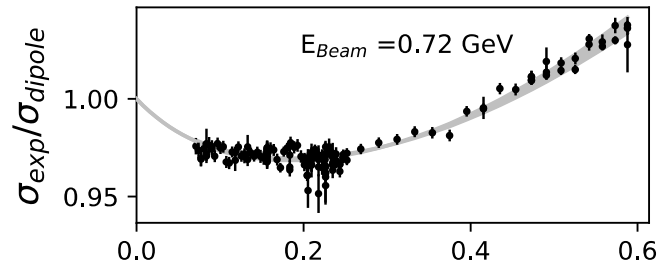
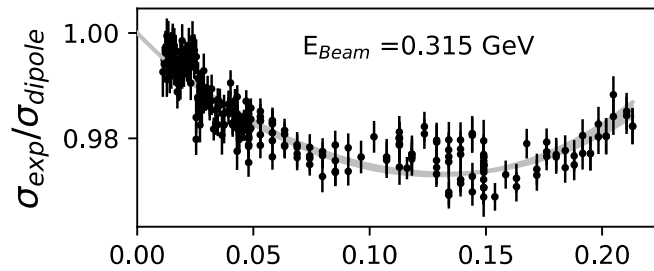
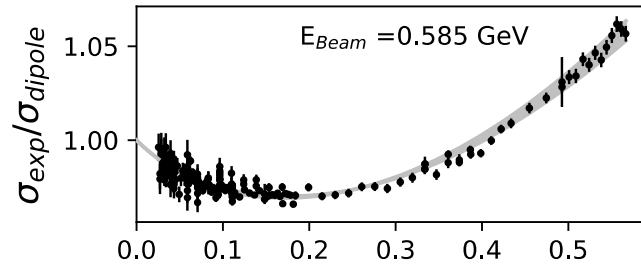
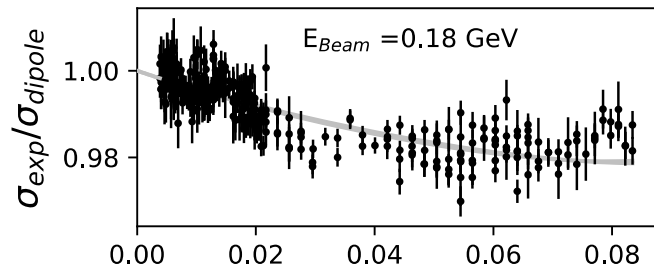


The bounded fit is completely monotone, $(-1)^n f^n(Q^2) \geq 0$, while the unbounded is not. The physical model of Alarcon and Weiss is completely monotone, so it is not an unreasonable requirement.

Scott Barcus, DH, Randall E. McClellan, <https://arxiv.org/abs/1902.08185>

Apply $D\chi$ XEFT To Mainz Data Set

Simply Replacing The Polynomial Fit Functions With The Model Removes 18 Parameters From The Regression
Fits done with both Bernauer's original binning and the rebinned version of Lee, Arrington, and Hill, Phys. Rev. D **92** (2015) 013013

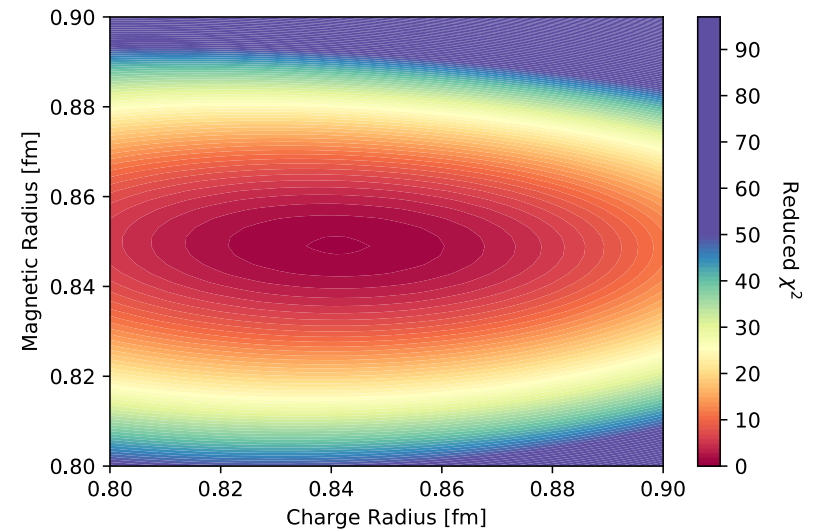


For original set:

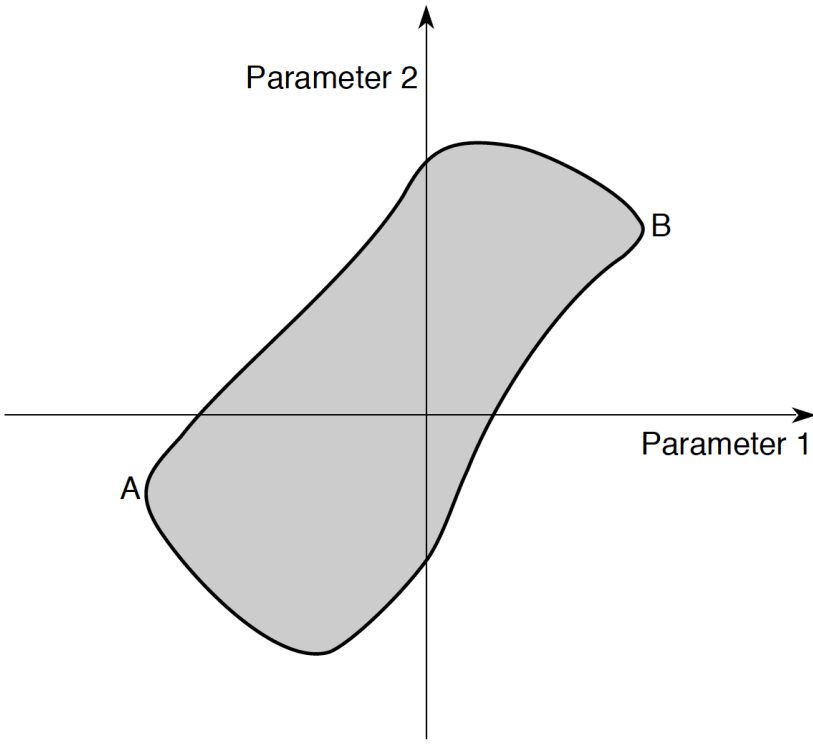
$R_E = 0.842(2)$ fm, $R_M = 0.850(1)$
reduced chi2 of 1.43

For rebinned set:

$R_E = 0.841(3)$ fm, $R_M = 0.849(1)$
reduced chi2 of 1.00



Multivariate Errors



The Interpretation of Errors in Minuit (2004 by James)

<http://seal.cern.ch/documents/minuit/mnerror.pdf>

In ROOT: **SetDefaultErrorDef(real #)**

Default is 1 and doesn't change unless you change it!

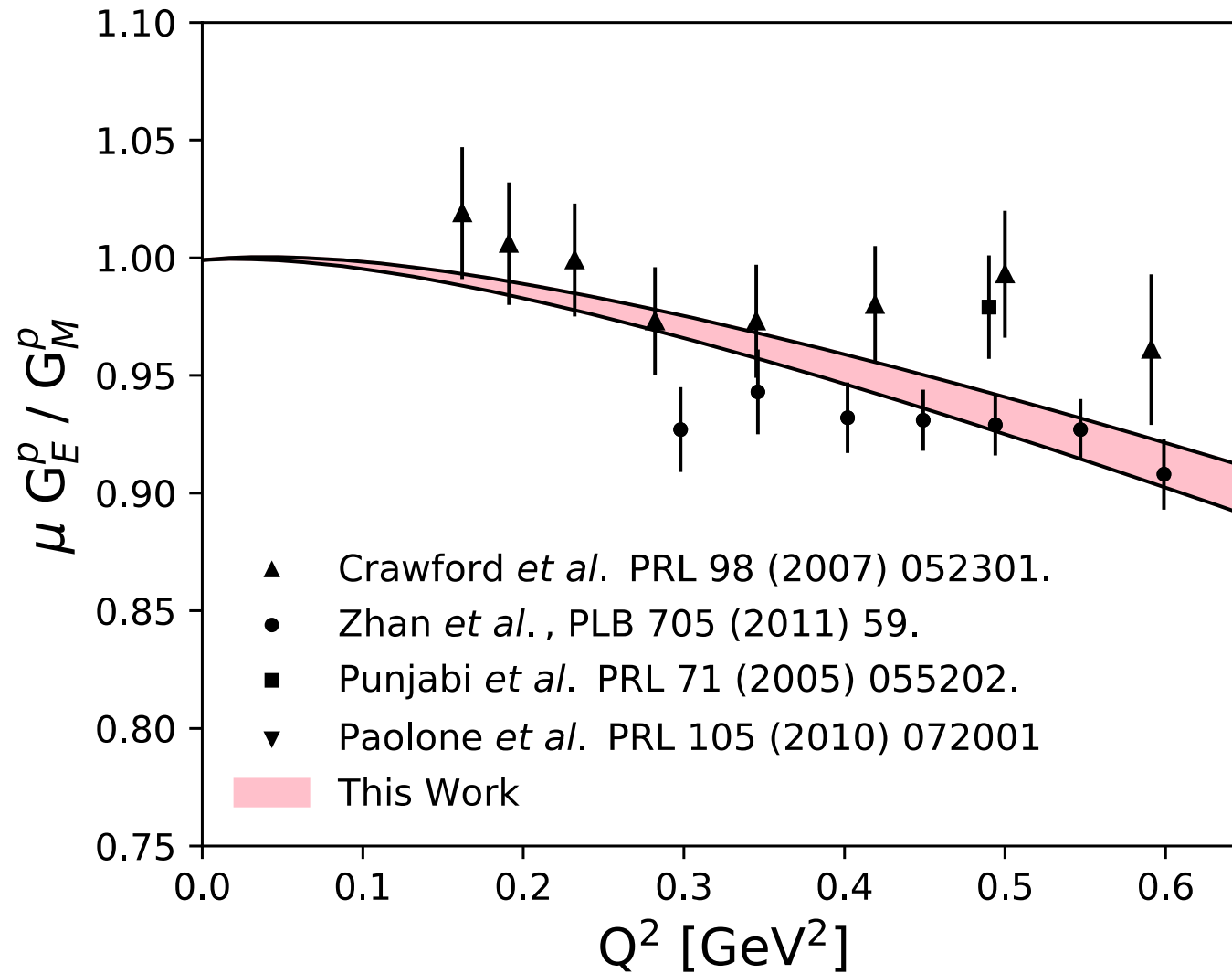
As per the particle data handbook, one should be using a co-variance matrix and calculating the probably content of the hyper-contour of the fit. Default setting of Minuit of “up”(often call $\Delta\chi^2$)is one.

Also note standard Errors often underestimate true uncertainties. (manual of gnuplot fitting has an explicate warning about this)

Number of Parameters	Confidence level (probability contents desired inside hypercontour of $\chi^2 = \chi_{\min}^2 + \text{up}$)				
	50%	70%	90%	95%	99%
1	0.46	1.07	2.70	3.84	6.63
2	1.39	2.41	4.61	5.99	9.21
3	2.37	3.67	6.25	7.82	11.36
4	3.36	4.88	7.78	9.49	13.28
5	4.35	6.06	9.24	11.07	15.09
6	5.35	7.23	10.65	12.59	16.81
7	6.35	8.38	12.02	14.07	18.49
8	7.34	9.52	13.36	15.51	20.09
9	8.34	10.66	14.68	16.92	21.67
10	9.34	11.78	15.99	18.31	23.21
11	10.34	12.88	17.29	19.68	24.71
If FCN is $-\log(\text{likelihood})$ instead of χ^2 , all values of up should be divided by 2.					

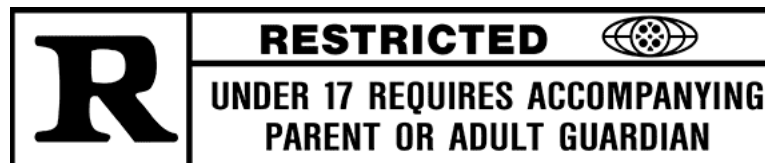
With modern computing power, an easy solution is doing a statistical bootstrap (sampling with replacement).

Validation Test: Checking Result With Data Not Included In The Fits



Summary and Outlook

- Dispersively Improved Chiral Effective Field Theory is consistent with a smaller radius and for a model with on
- Many more results coming!
 - PRad results are nearly finalized (Haiyan's talk on Thursday)
 - MUSE, New Mainz A1 Data, MESA, Compass, and more
 - And of course new atomic Lamb shift measurements.
- And I end with a scary R-rating warning:
 - Nuclear physicists tend judge regressions from a single number (i.e. reduced χ^2)
 - Other fields have had the same issue with p-values.
 - Statisticians warn against this constantly...



Anscombe's Quartet

F. J. Anscombe, The American Statistician 27 (1973) 17 with added dy terms

Data set	1-3	1	2	3	4	4	1-5
Variable	x	y	y	y	x	y	dy

Obs. no.	1:	10.0	8.04	9.14	7.46	8.0	6.58	1.235
	2:	8.0	6.95	8.14	6.77	8.0	5.76	1.235
	3:	13.0	7.58	8.74	12.74	8.0	7.71	1.235
	4:	9.0	8.81	8.77	7.11	8.0	8.84	1.235
	5:	11.0	8.33	9.26	7.81	8.0	8.47	1.235
	6:	14.0	9.96	8.10	8.84	8.0	7.04	1.235
	7:	6.0	7.24	6.13	6.08	8.0	5.25	1.235
	8:	4.0	4.26	3.10	5.39	19.0	12.50	1.235
	9:	12.0	10.84	9.13	8.15	8.0	5.56	1.235
	10:	7.0	4.82	7.26	6.42	8.0	7.91	1.235
	11:	5.0	5.68	4.74	5.73	8.0	6.89	1.235

Slope, Intercept, χ^2 are all the same

Standard error assumes that the data are normally disturbed about the fit function.

