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Design of a multipass optical cavity for spin-flip spectroscopy of muonic hydrogen in FAMU experiment

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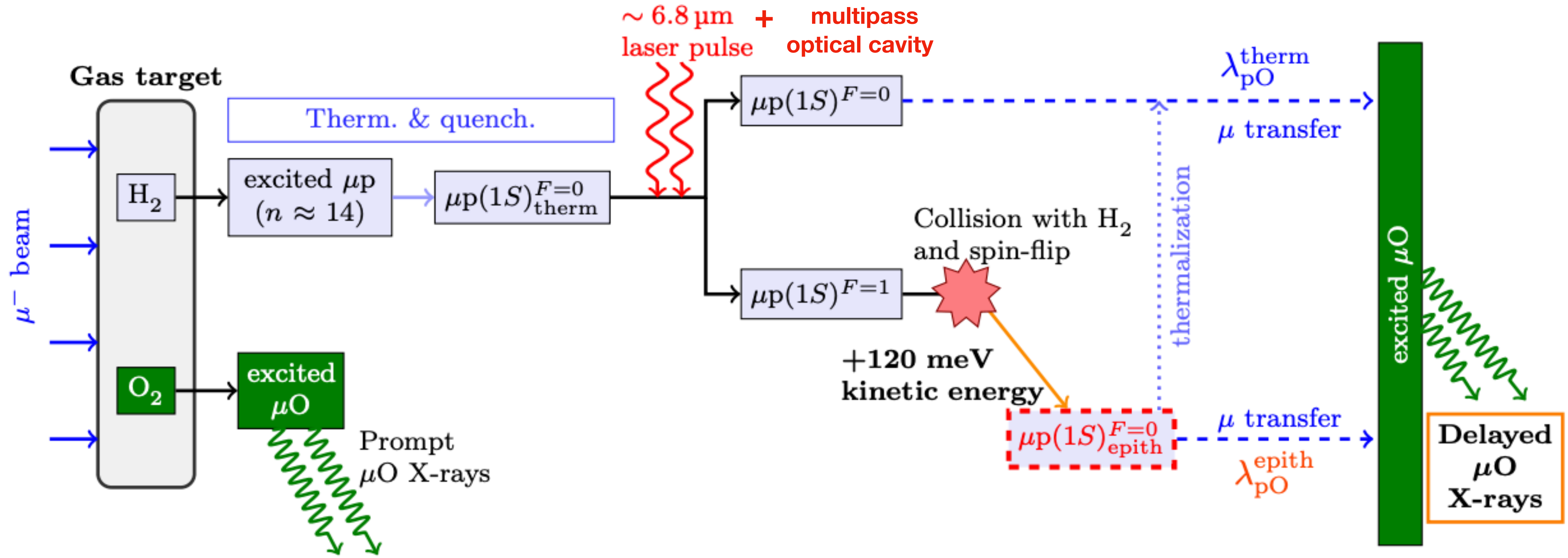
Outline

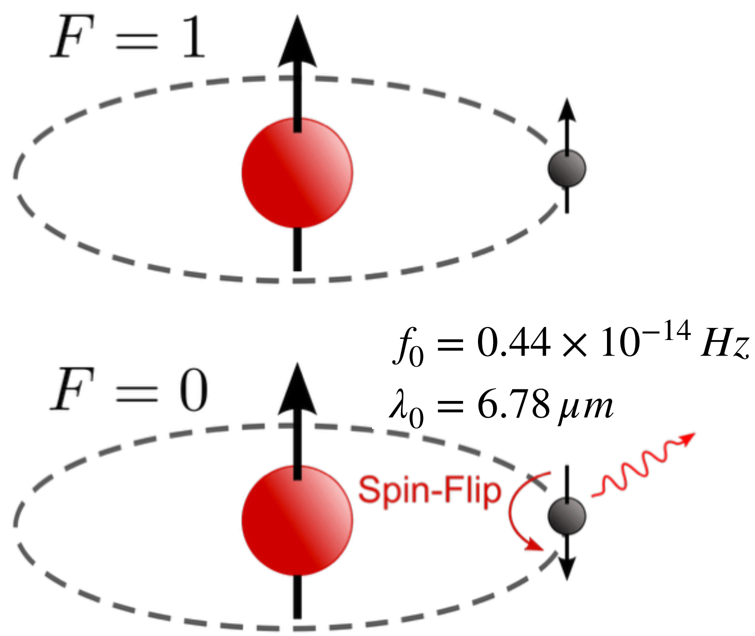
- FAMU Experiment Workflow;
- Spin-flip transition probability in μp;
- Requirements about the MOC for FAMU experiment;
- Scheme of experimental setup;
- Design of different prototype of MOC;
- HR Mirror fabrication;
- Ray-tracing simulation;
- Light injection system;

FAMU experiment workflow



The FAMU experiment is based on the capability to detect the difference of muon transfer rate (λ_{pO}) from µp to µO as function of µp kinetic energy.

spin-flip transition



$$\sigma_{sf} = 6.58 \times 10^{-22} \mathrm{cm}^2$$

cross-section of spin-flip transition at 80 K from Adamczak *et al.*, NIMPR B 281 72-76 (2012)

Transition probability of one muonic atom:

$$\bar{P} = \frac{\sigma_{sf}}{h\nu} D = 2.24 \times 10^{-5} \left[\frac{\mathrm{cm}^2}{\mathrm{mJ}} \right] \cdot D = \frac{D}{D_{sat}}$$

- D: laser fluence
- D_{sat}: saturation fluence

$$h\nu = 2.9 \times 10^{-20} \mathrm{J} \rightarrow D_{sat} = \frac{h\nu}{\sigma_{sf}} = 4.47 \times 10^4 \left[\frac{\mathrm{mJ}}{\mathrm{cm}^2} \right]$$

In PSI experiment, the saturation fluence of transition is 16.5 mJ/cm^2

Cavity enhancement effect at glance

E_l : laser pulse energy

S_{ill} : illuminated surface

N_R : number of reflection ($\propto 1/\alpha$)

$$\left.\begin{matrix} E_l \\ S_{ill} \\ N_R \end{matrix}\right\} \rightarrow D_{cav} = \frac{N_R E_l}{S_{ill}}$$

α is the losses per pass

Transition probability in cavity:

$$\overline{P}_{cav} = \frac{D_{cav}}{D_{sat}}$$

PSI experiment: $\overline{P} = \frac{0.15\mathrm{mJ} \times 670 / 12.3\mathrm{cm}^2}{16.5} \approx 0.50$

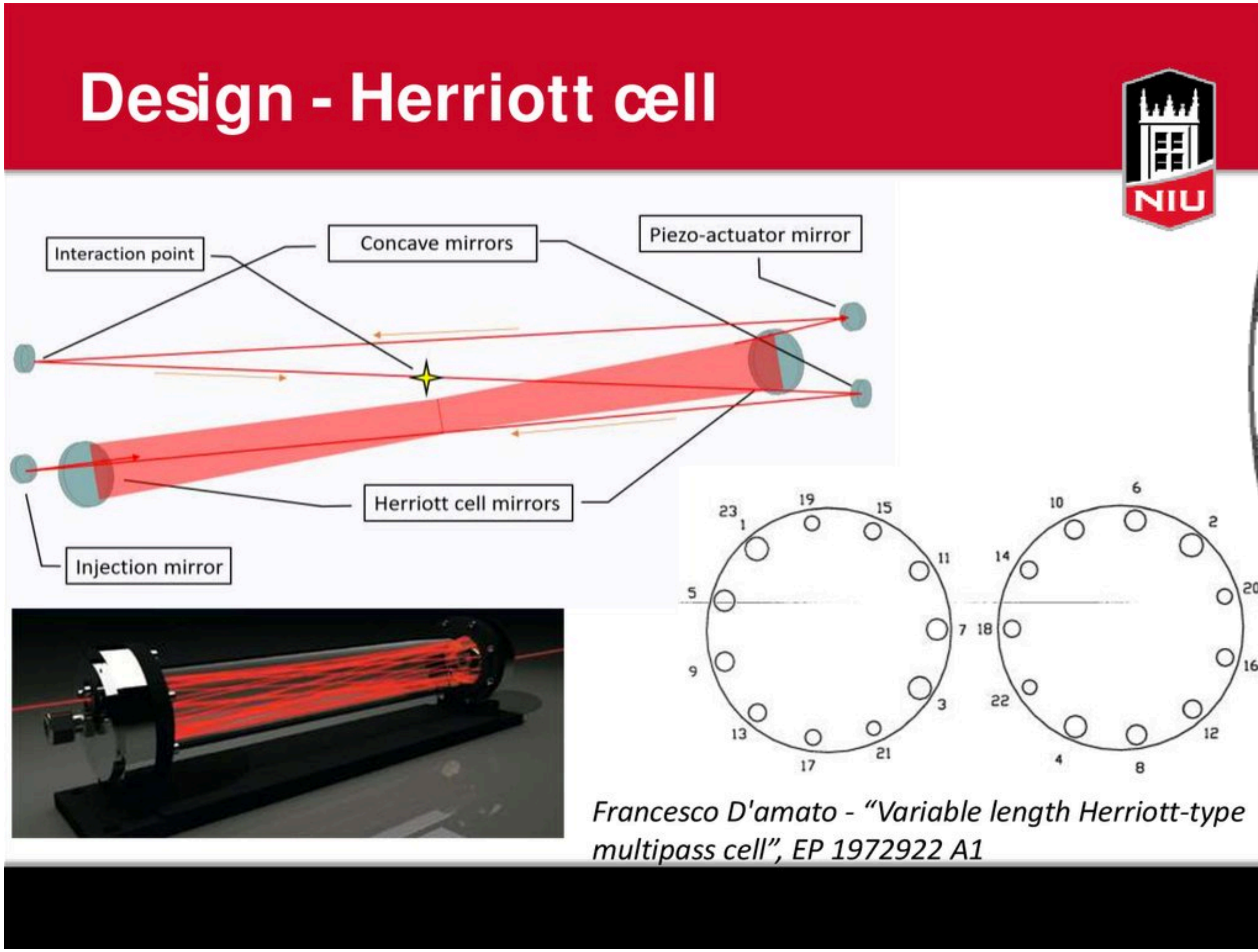
Vogelsang *et al.* *OpEx* **22** 13050 (2014)
R. Pohl *et al.* *Nature* **466** 213 (2010)

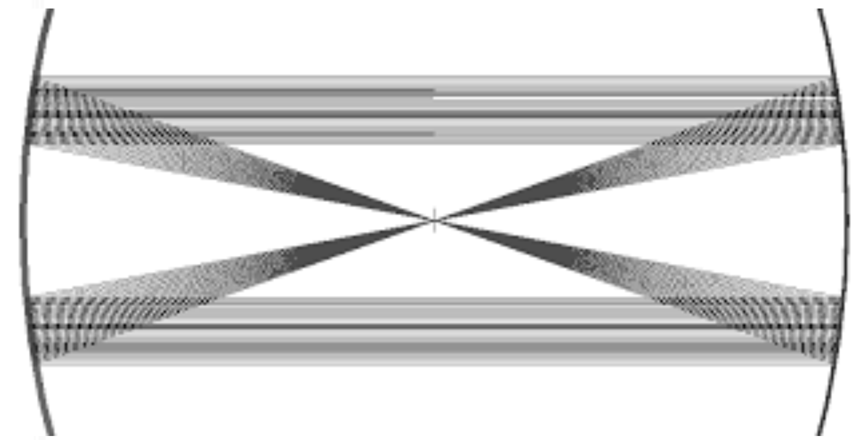
Requirements of MOC

The Multipass Optical Cavity for atomic muon spectroscopy must have several peculiar characteristics:

- High photon energy density—> to reduce the mirror surface;
- Low losses per pass —> high reflectivity mirrors; no injection hole;
- Low and non-uniform density of μp—>The light must filled uniformly the cavity;
- No active system for the mirror stabilisation—> No resonant cavity;
- The photon cavity lifetime smaller than muon lifetime—> Time window of measurement.

Conventional Multipass Optical Cavity

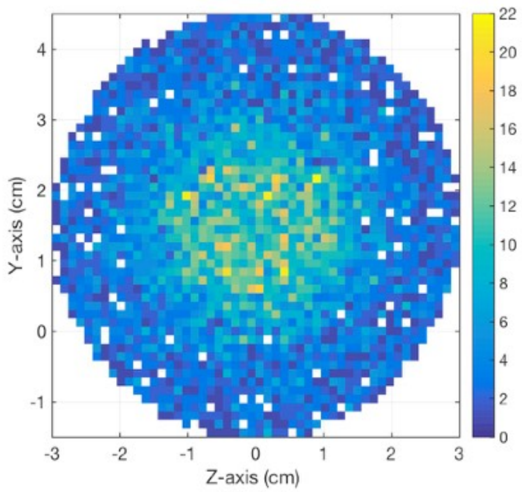


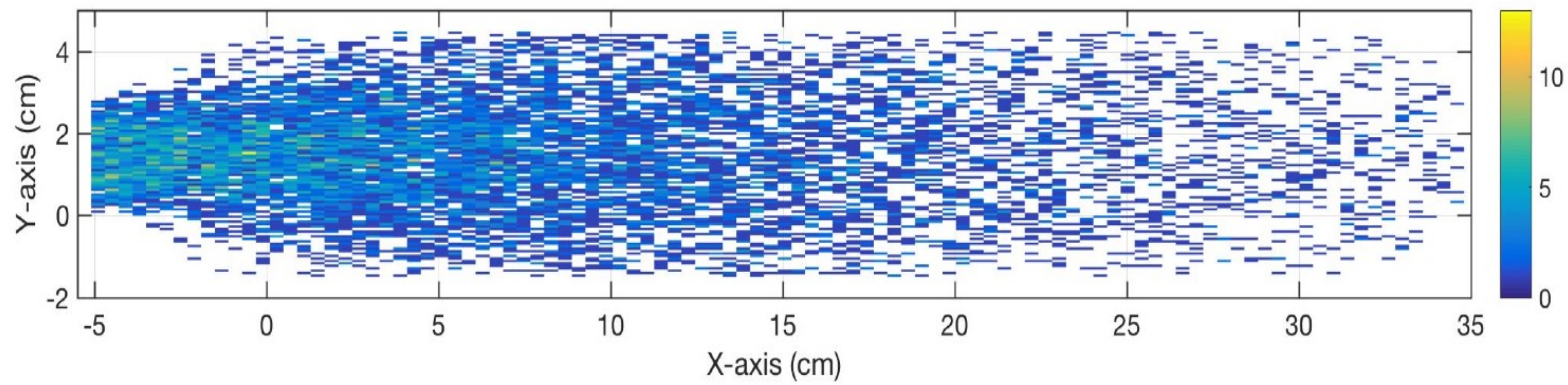


- **Injection hole in the mirror**
- **No uniform filling of light**

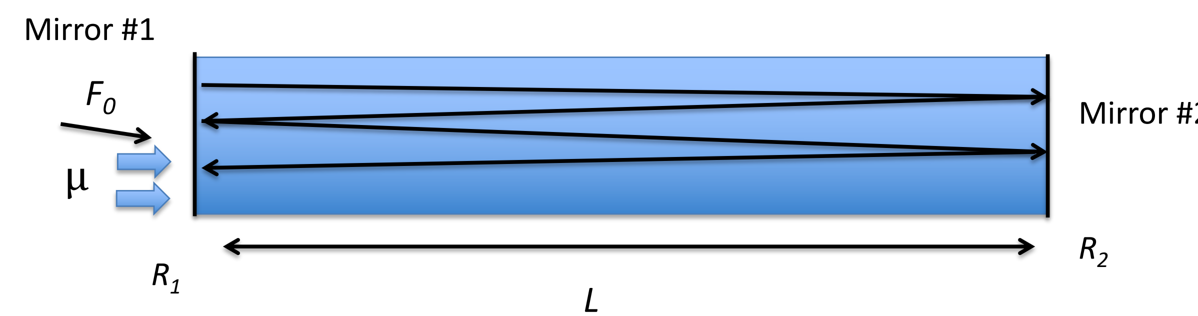
Geometry of MOC

Simulation of muon distribution in the target



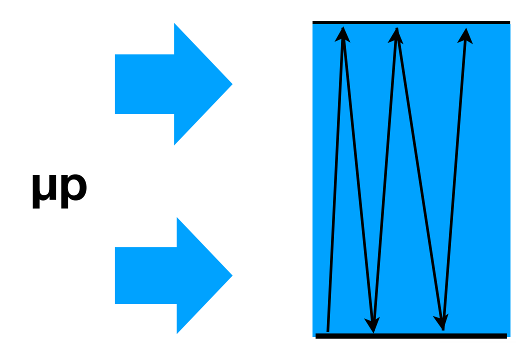


Longitudinal cavity$(L = 10\,cm)$:



- The muons are stopped into the substrate mirrors (FuSi) providing a X-ray background signal.

Transversal cavity$(L = 5\,cm)$:



- The length of cavity is not enough to design a open-cavity. Therefore is necessary to have lateral mirrors.

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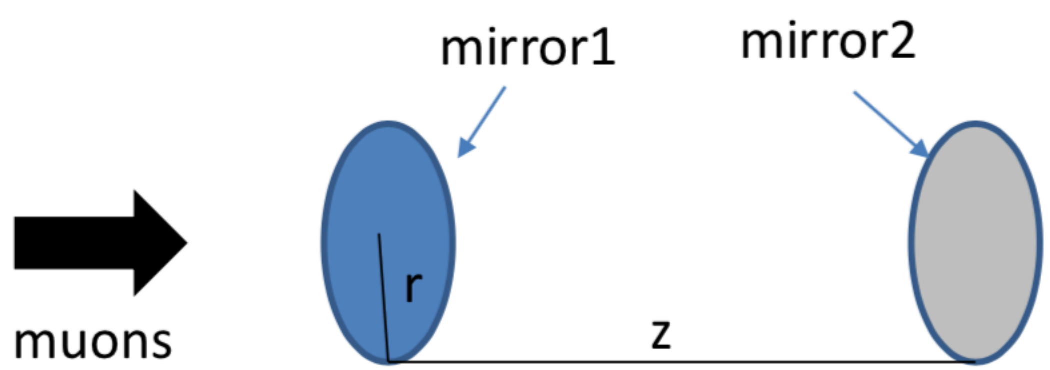
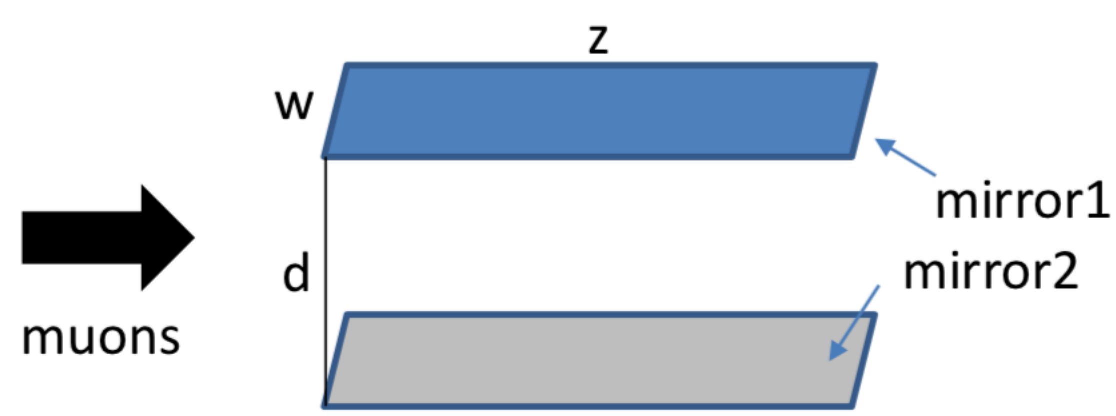
Muon beam simulation (1)

GEANT simulation about the best signal-to-noise ratio taking into account the muons beam shape, mirrors material and size, laser energy, etc.

Longitudinal cavity



Transversal cavity



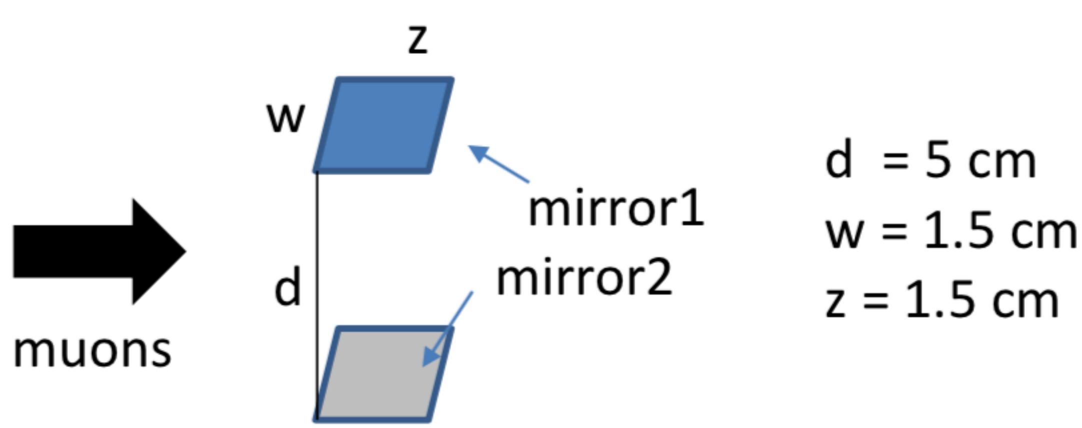
Muon beam simulation (2)

Working with very close mirrors is difficult because of:

1) Construction of the mirrors
2) Low statistics in one day of data taking

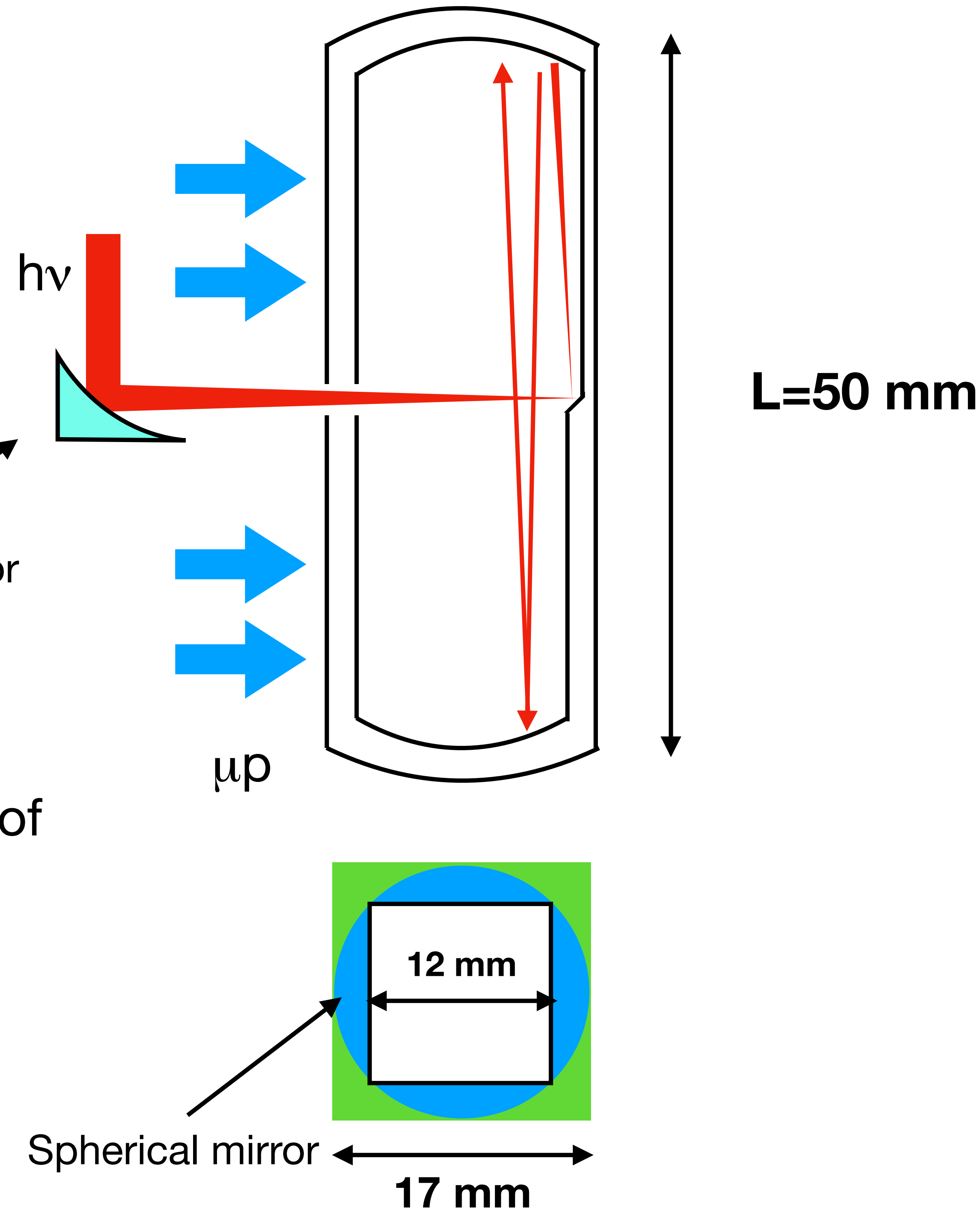
hence the SNR is not too relevant, we will not be able to work with small time windows!

Eventually the mirrors must be small, their distance should be (in my opinion) the largest to collect as many muons as possible (consider also the beam divergence once entering the gas).
Target and cavity MUST be the same thing (to minimize the non-illuminated gas).
Cavity must be completely filled by light.



Sketch of transversal closed cavity

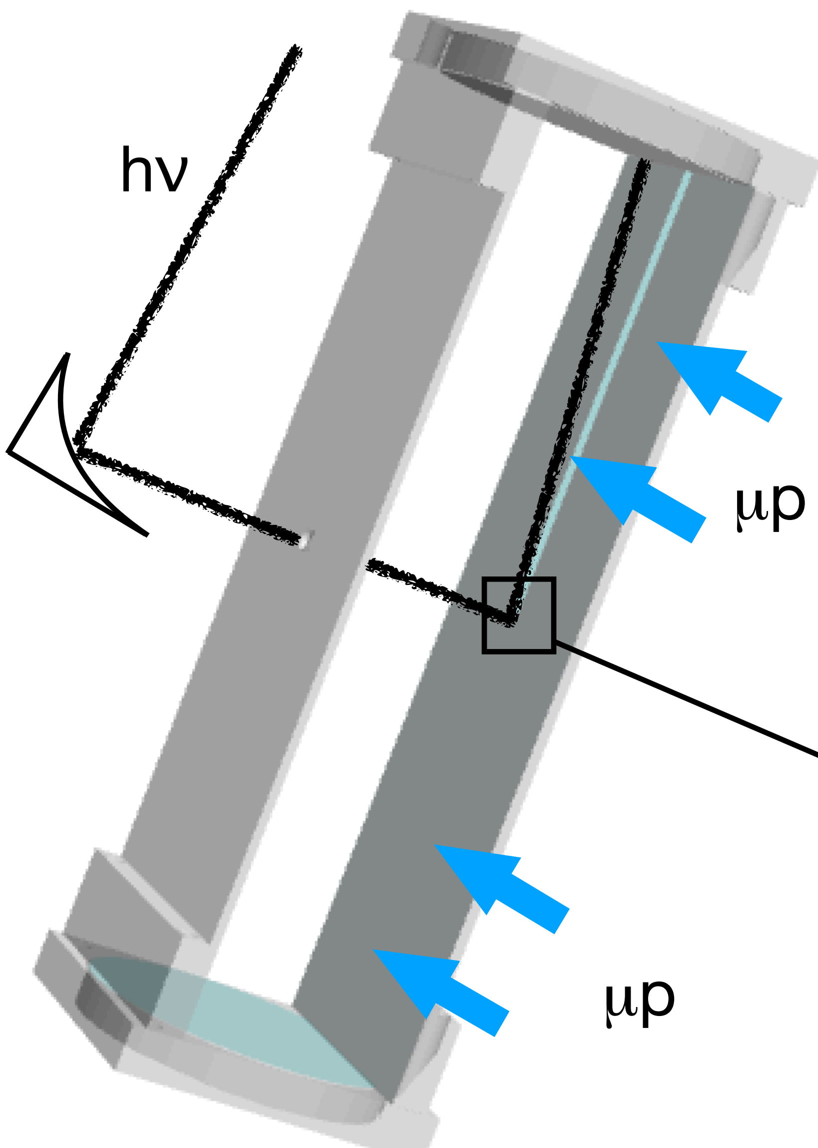
Transversal open cavity with the requested size it is very critical in terms of fabrication specifications.



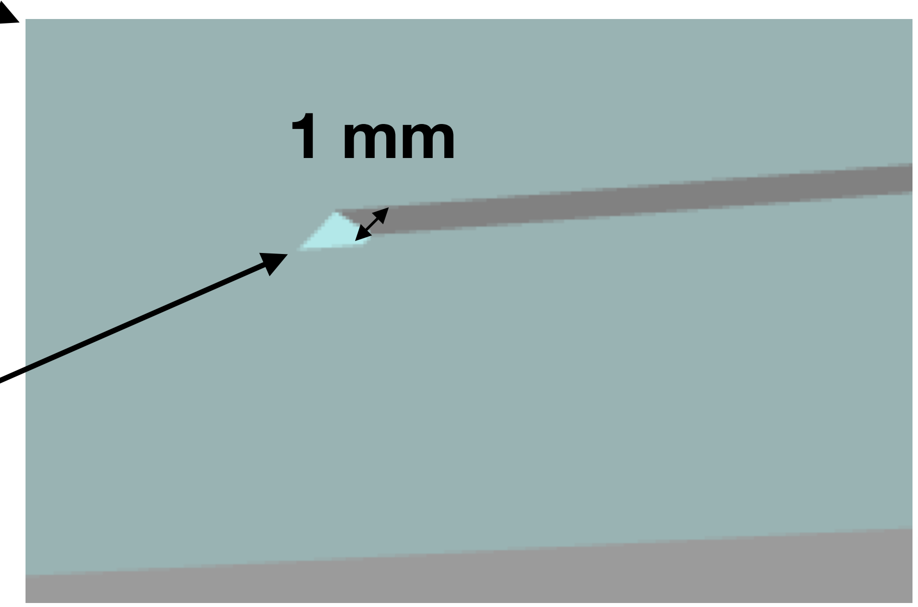
R_t, R_b curvature radius of mirrors 760 mm

Design of Transversal closed MOC

- **Thickness of mirror : 2 mm**
- **Material: FuSi or Stainless steel**
- **Mirror surface, (12x12) mm²**
- **lenght of cavity, 50 mm**



The quantity of FuSi should be to minimise in order to reduce the X-ray background signal due to the Bremsstrahlung of electrons in the FuSi substrate.

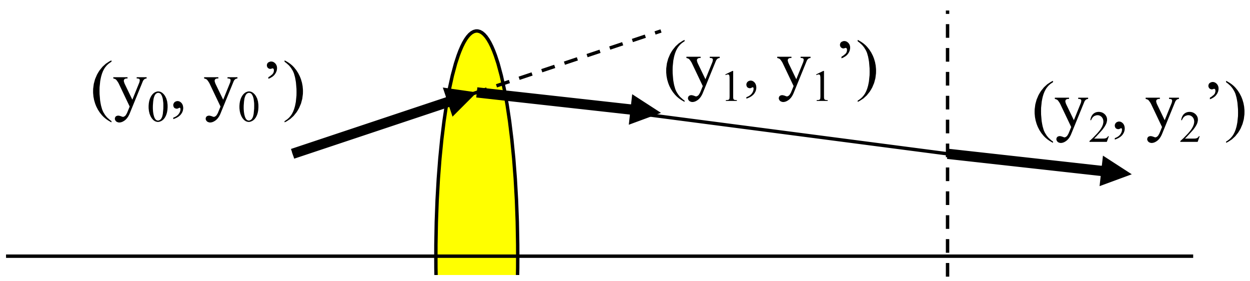


45°-mirror

Ray-tracing simulation

- Generalize $\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$

- Can cascade to make single matrix for system
- Example: go through lens and propagate distance L = *f*



$$\begin{pmatrix} y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix} = \begin{pmatrix} 1 & f \\ -1/f & 0 \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

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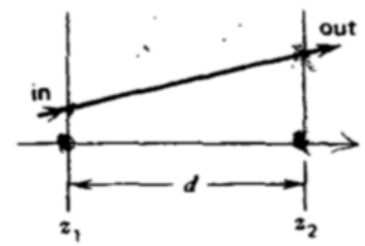
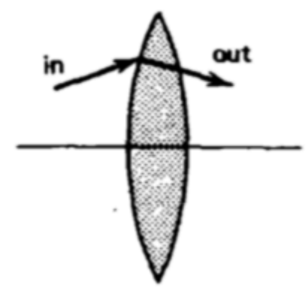
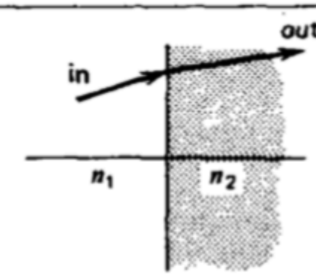
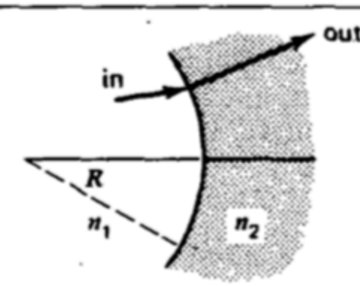
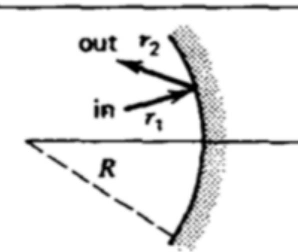
ABCD matrix

TABLE 6.1. Ray Matrices for Some Common Optical Elements and Media

(1) Homogeneous Medium: Length d		$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
(2) Thin Lens: Focal length f ($f>0$, converging; $f<0$, diverging)		$\begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix}$
(3) Dielectric Interface: Refractive indices n_1, n_2		$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$
(4) Spherical Dielectric Interface: Radius R		$\begin{bmatrix} 1 & 0 \\ \frac{n_2-n_1}{n_2}\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}$
(5) Spherical Mirror: Radius of curvature R		$\begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$

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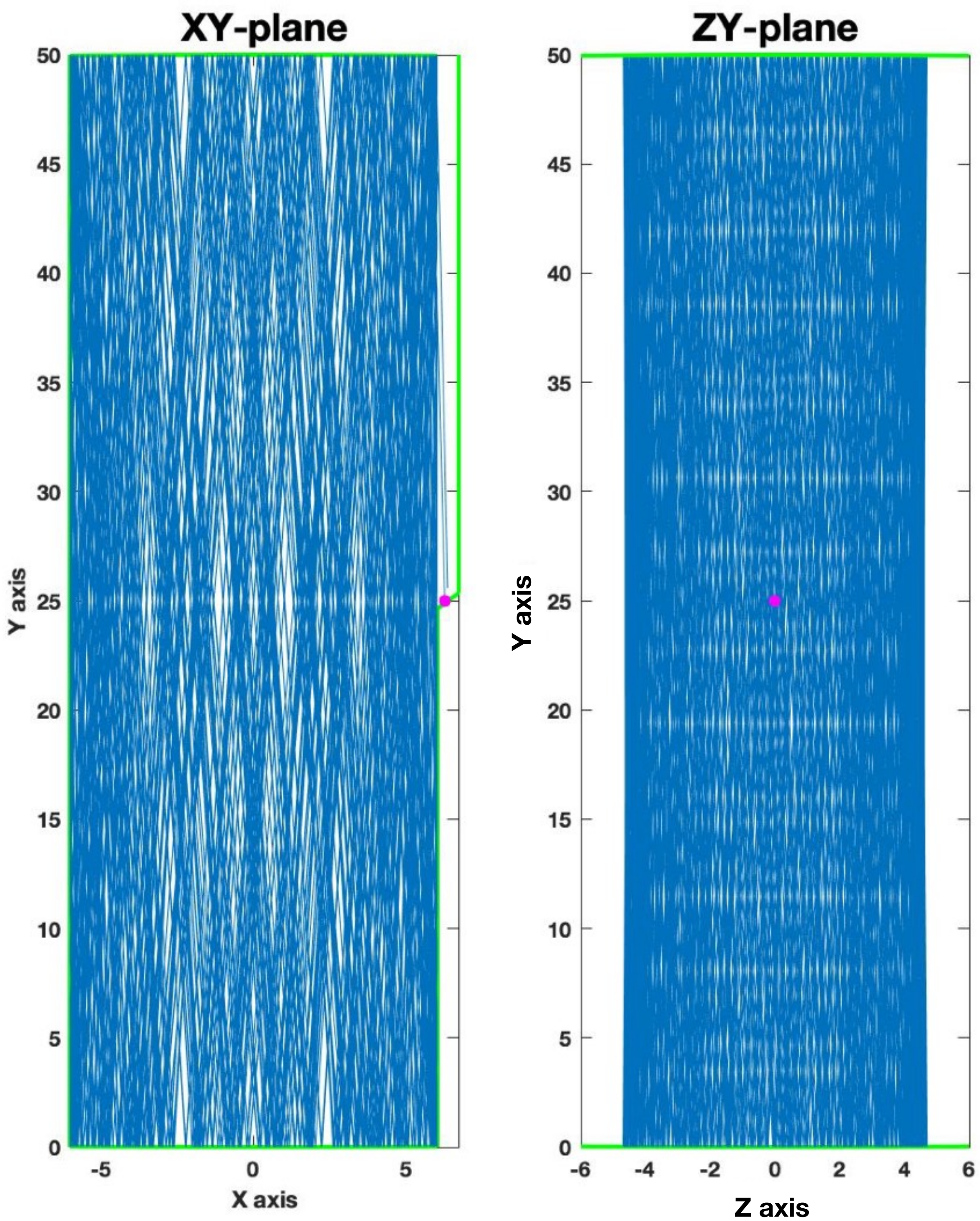
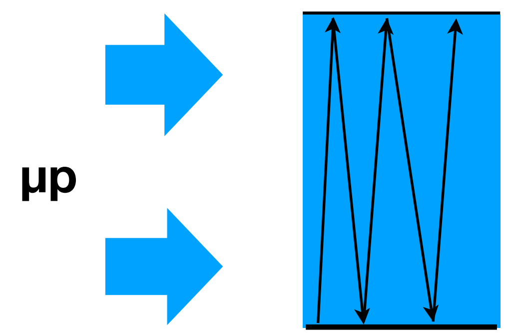
Ray-tracing (1)



Injection angle: $\alpha_{xy} = 20\ mrad; \alpha_{zy} = 32\ mrad$

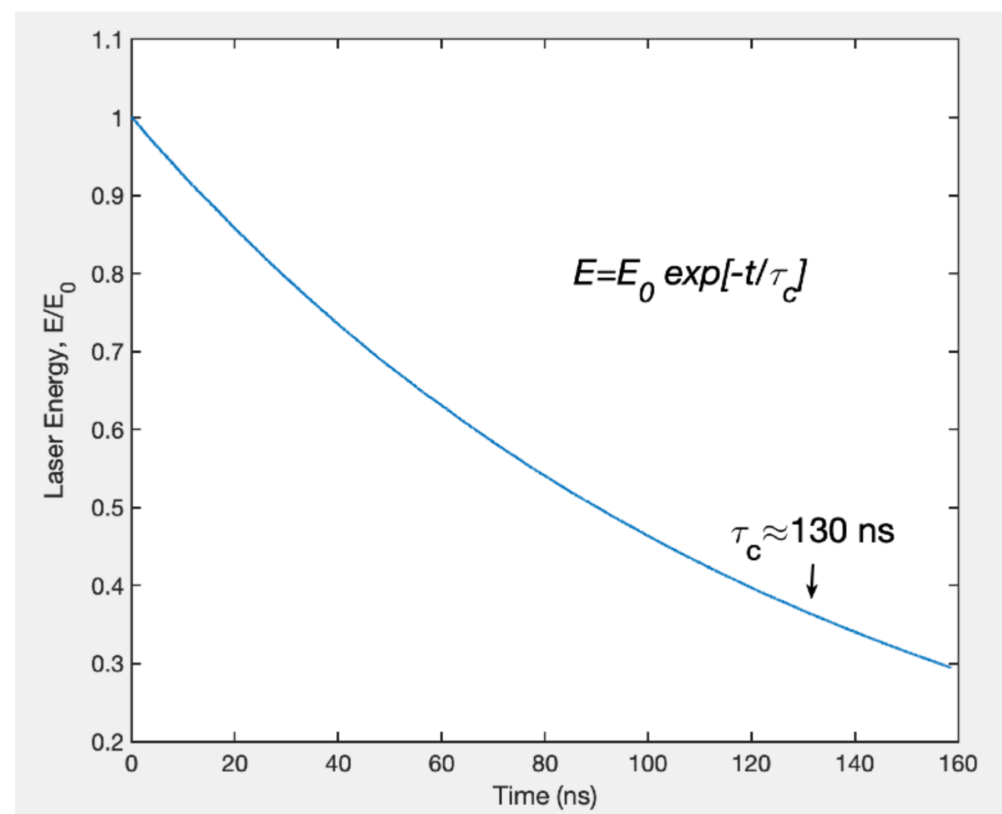
No critical alignment

transversal cavity$(L = 5\,cm)$:



$$\tau_c \approx 130\,ns$$

$$L_{eff} = c\tau_c \approx 39\,m$$



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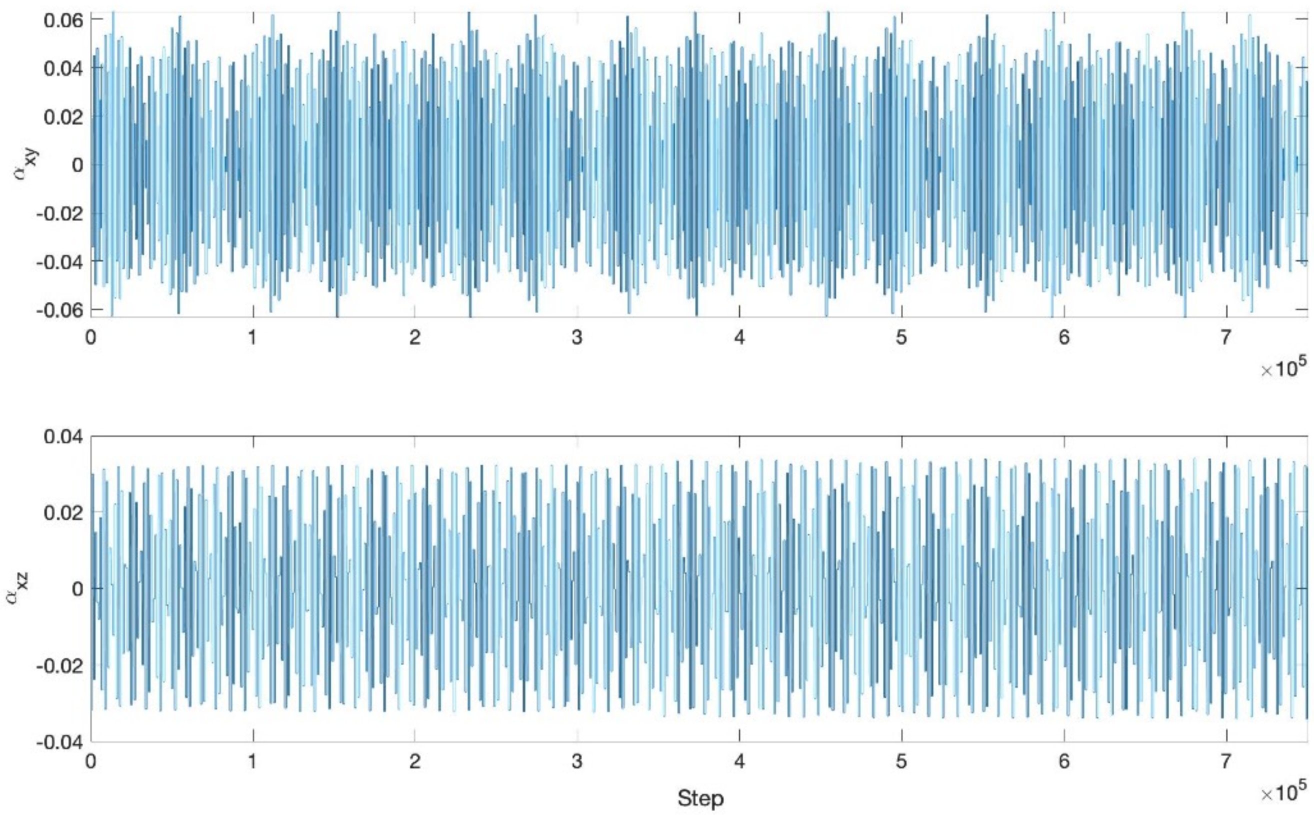
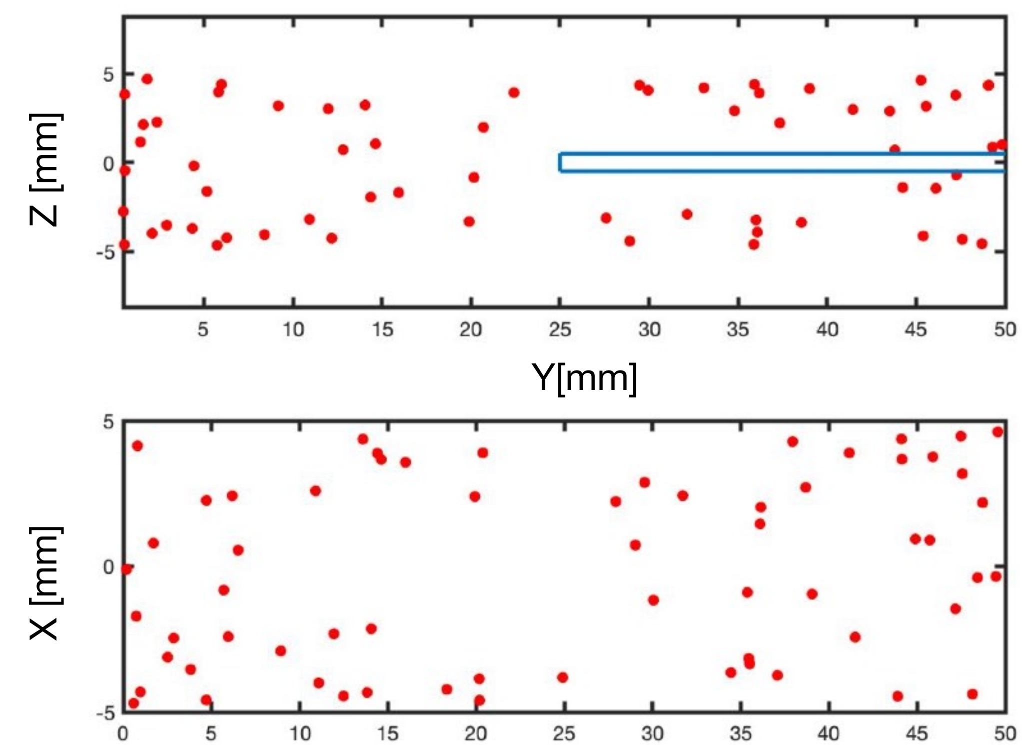
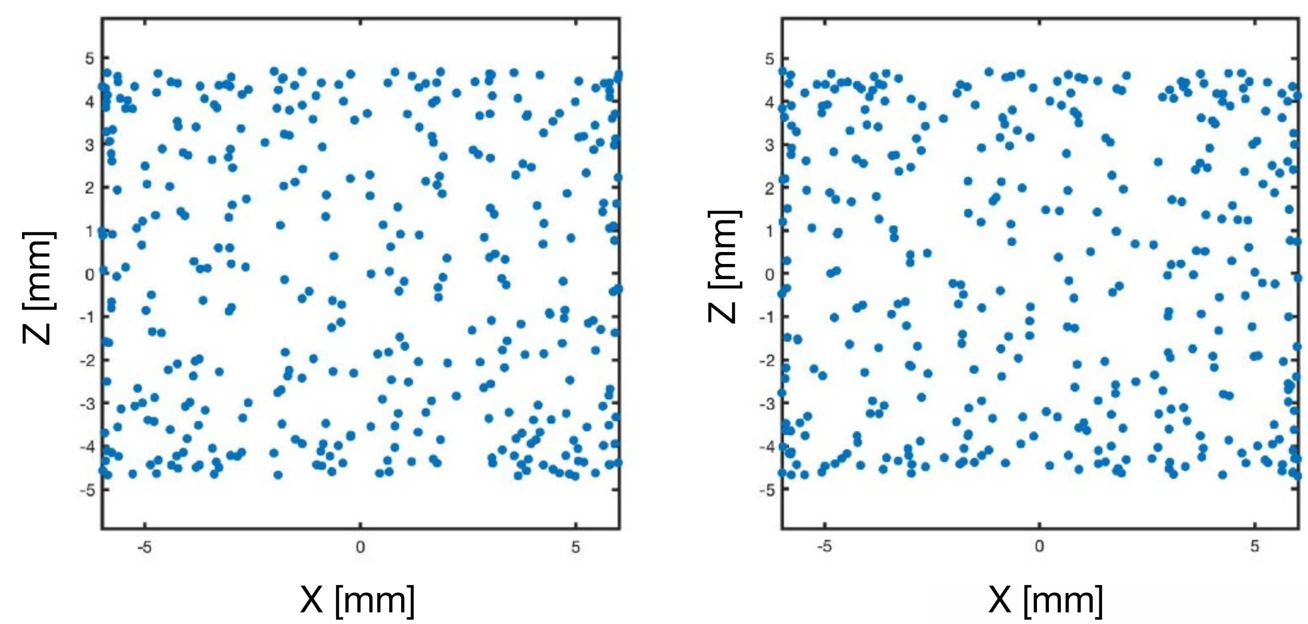
Ray-tracing (2)



Reflections spots on the lateral mirrors



Reflections spots on the spherical mirror



max value of xy-angle: 3.6°
max value of xz-angle: 2°
The paraxial approximation is preserved

$$S_{ill} = (1.1 \times 1.4)\, cm^2$$

About 1000 reflections;
14% of reflection on the lateral walls

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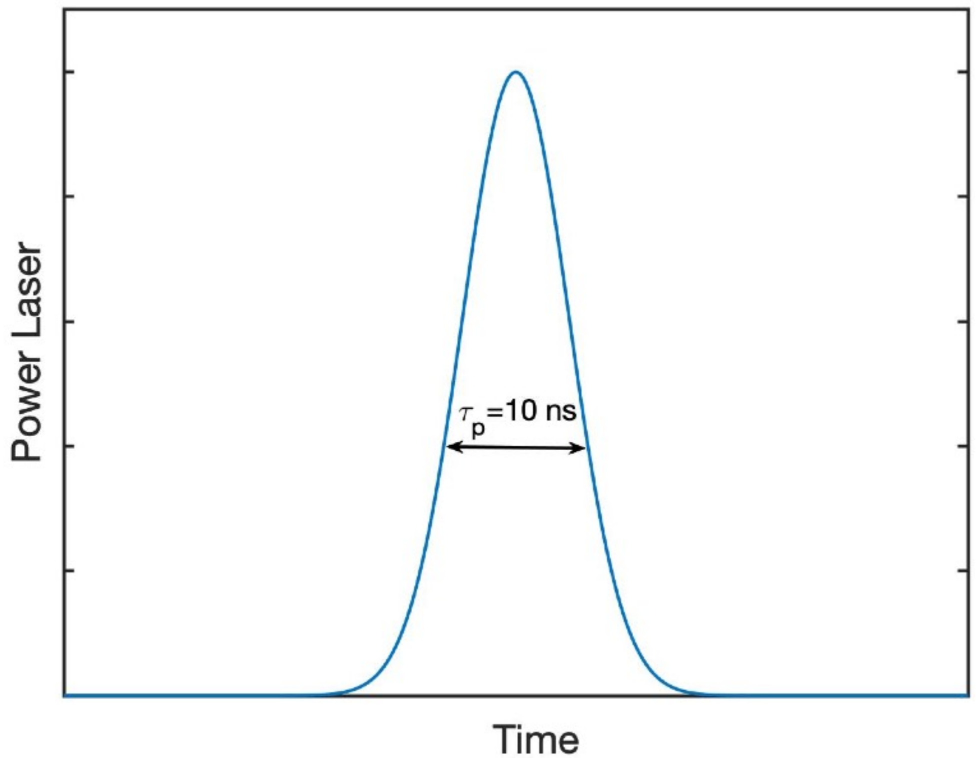
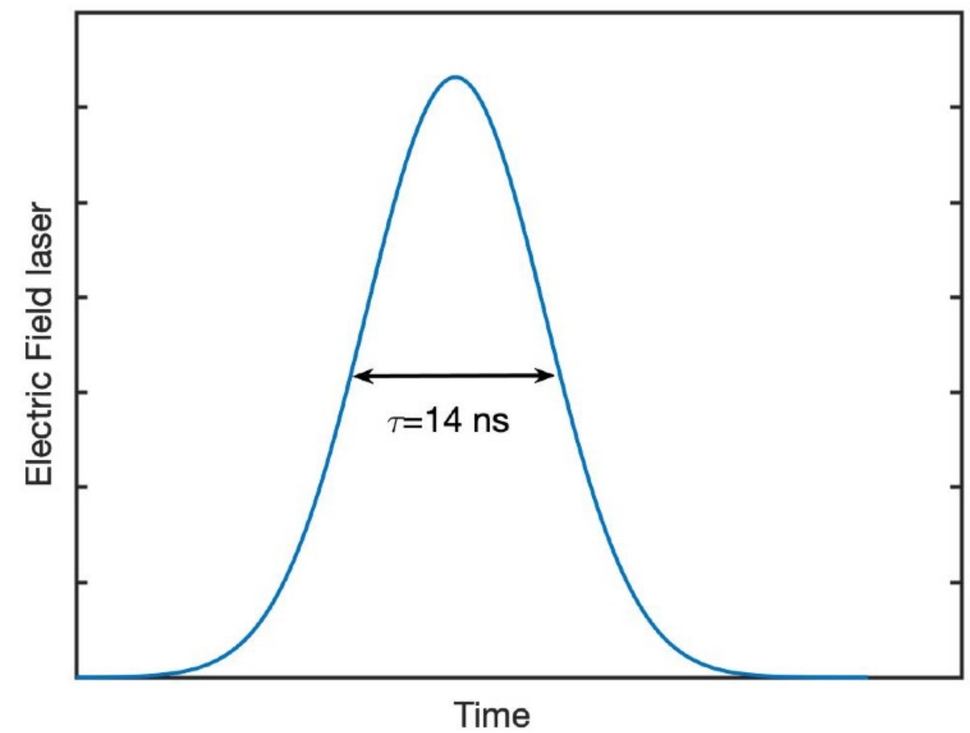
Evaluation of interference effect





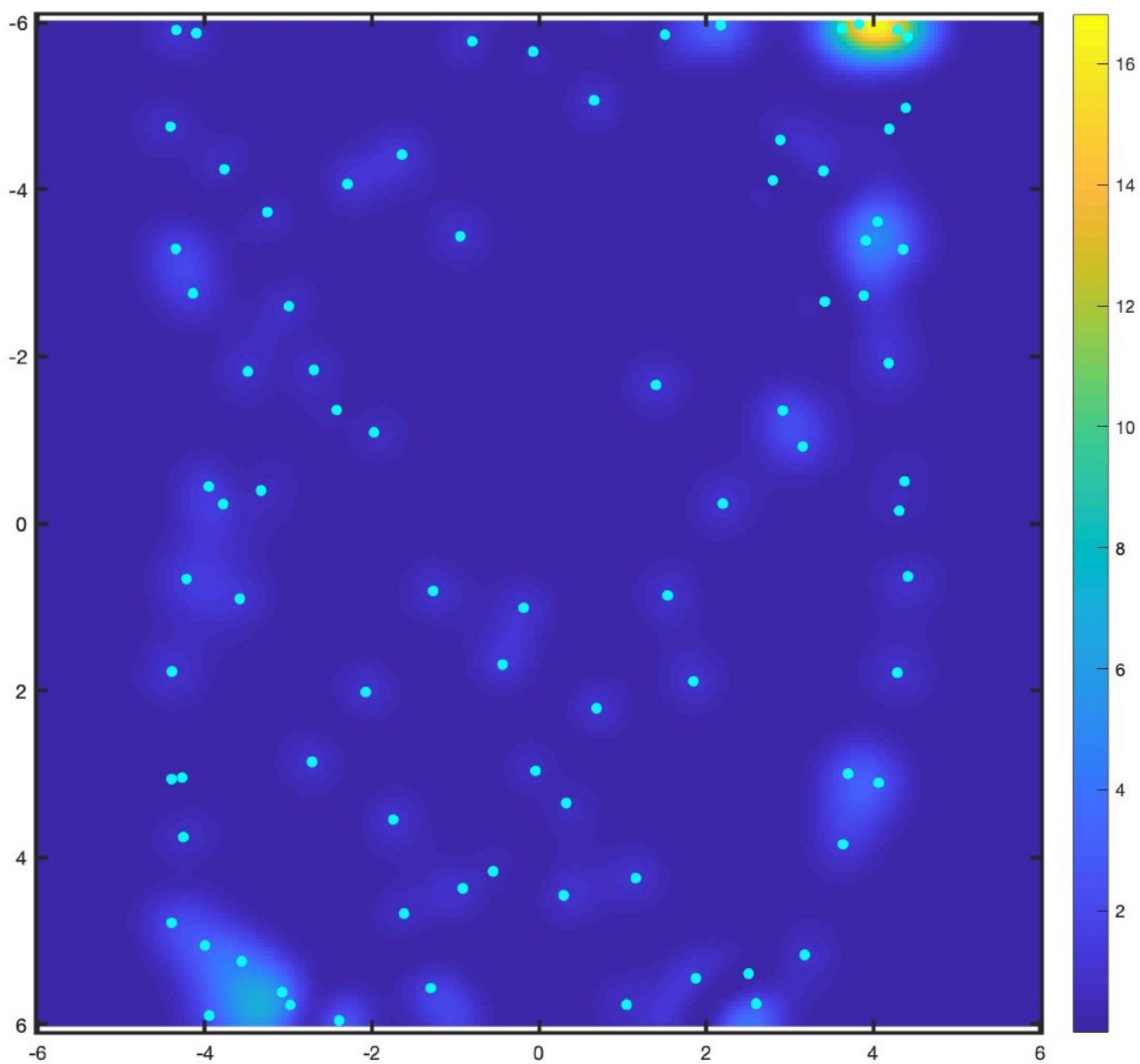
$$P_l(t) \propto e^{-\frac{t^2}{2\tau_p^2}}$$

$$E_l(t) \propto \sqrt{P_l(t)} \propto e^{-\frac{t^2}{\tau_p^2}}$$

The time between two reflection in cavity is about 0.17 ns.
The electric field interferes with himself if there is superimposition of the counter propagating wave during the travel of light in cavity.
After about 100 reflection (the duration of time pulse) the amplitude of electric field is quite low. Therefore, to take into account the interference effect we sum the electric field profile from the first 100 reflections.

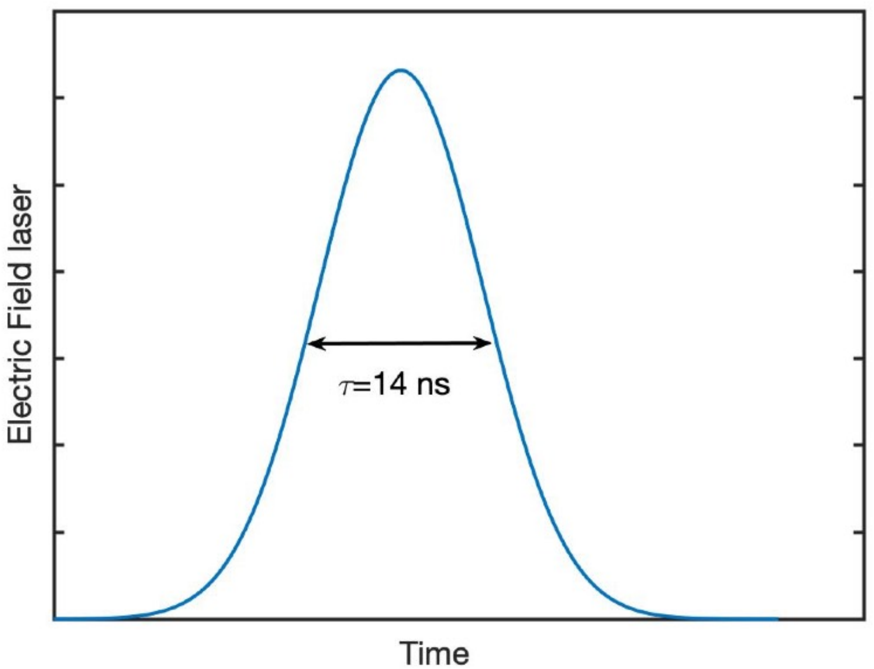
Interference effect

Beam waist with radius of 1 mm



Light intensity on XZ plane in the middle of cavity

Number of reflection involved in the interference effect: 98



$$E_t = \sum_{i=0}^{98} R^{\frac{n}{2}} E_n(\mathbf{r})$$

About the substrate of HR mirrors

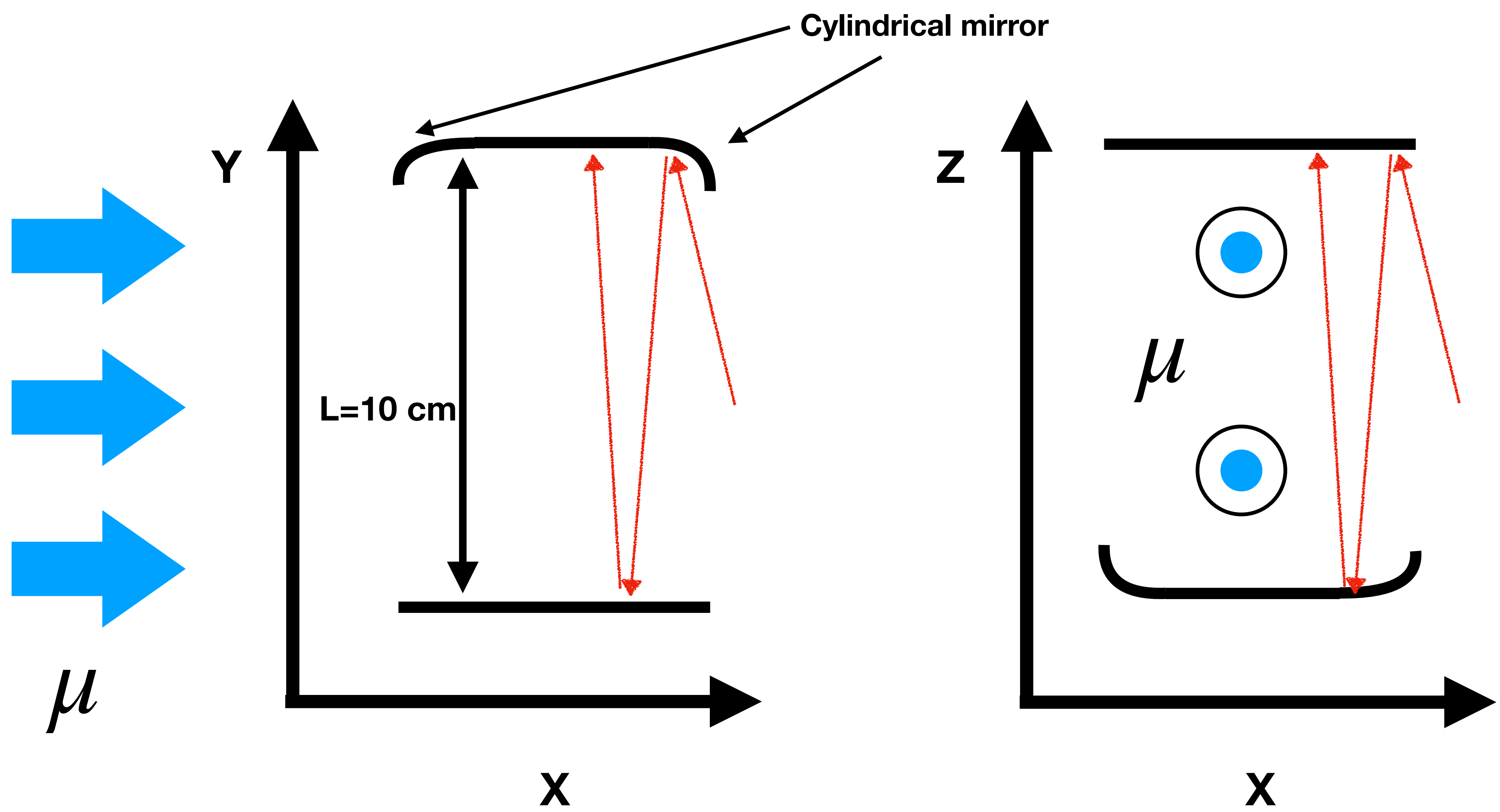
The lateral mirrors are a strong disturbance for the measurement

- Fused Silica: Increase the lifetime of muons and it provides an increasing of X-ray background noise due to Bremsstrahlung.

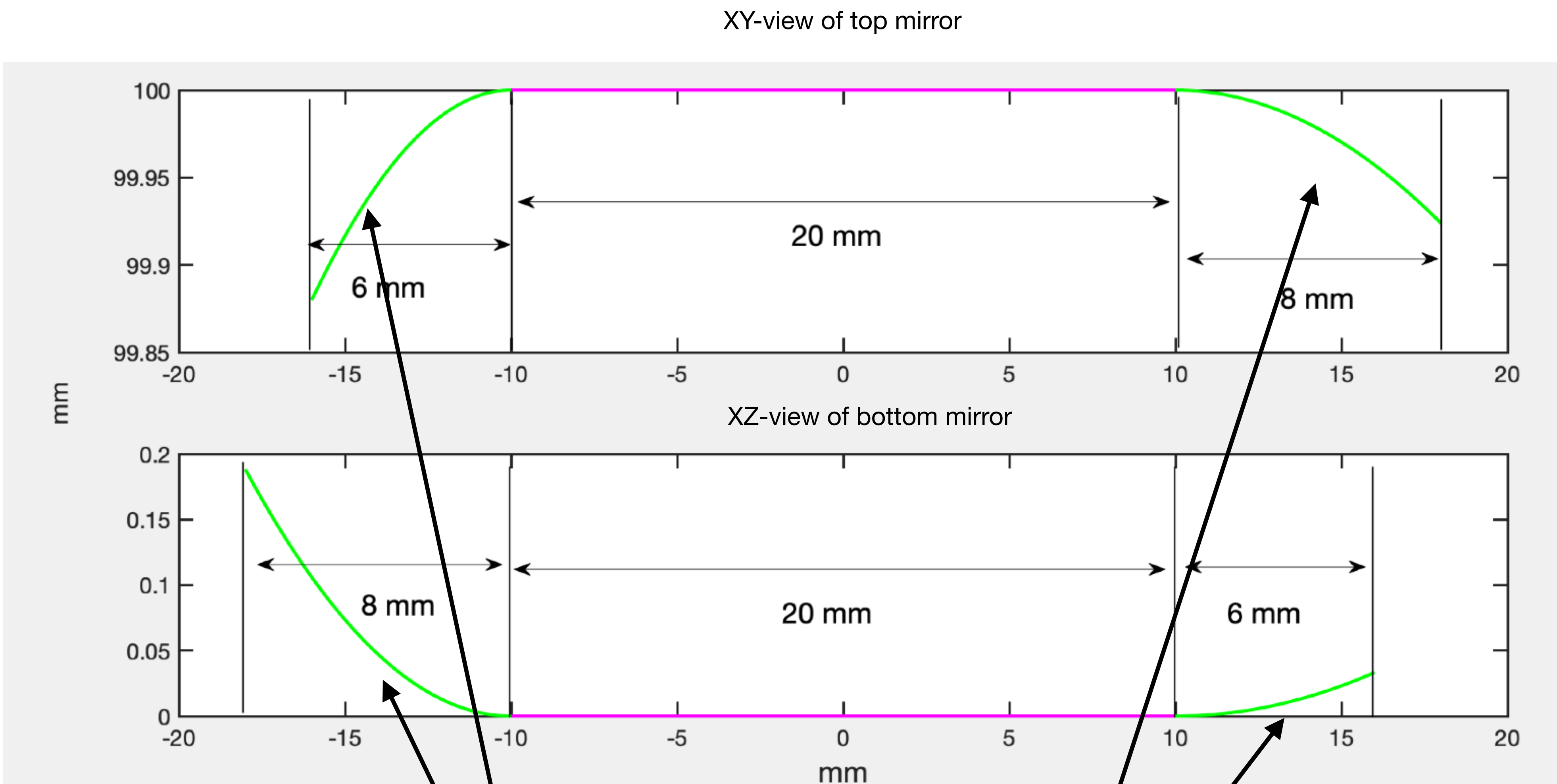
- Stainless steel: screening and therefore reduce the X-ray signal.

- We are investigated the feasibility of making the substrate mirrors with more heavier metals (Tungsten).

Sketch of open transversal cavity

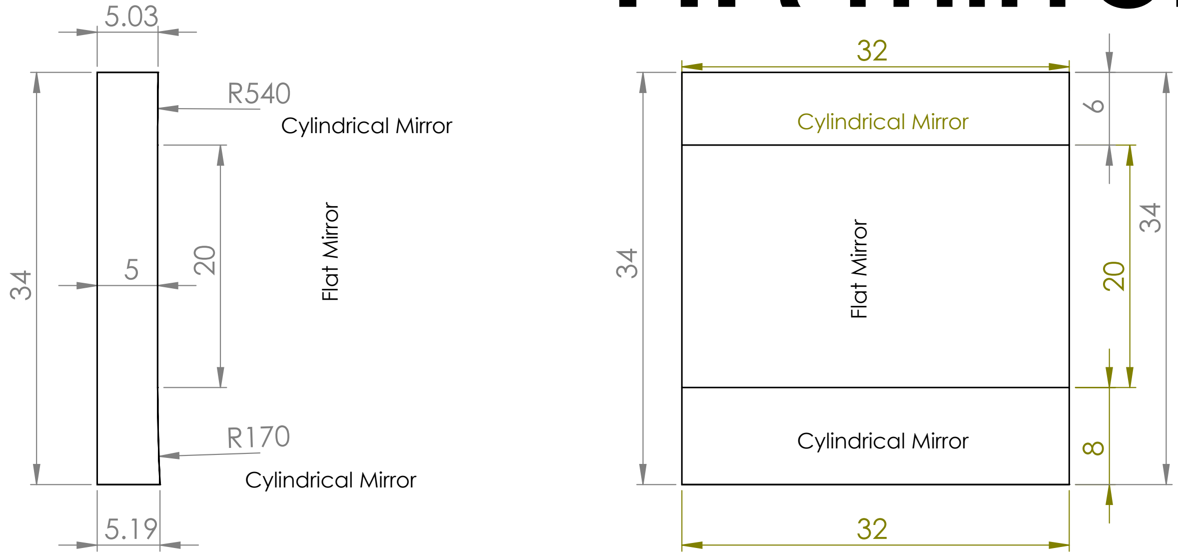


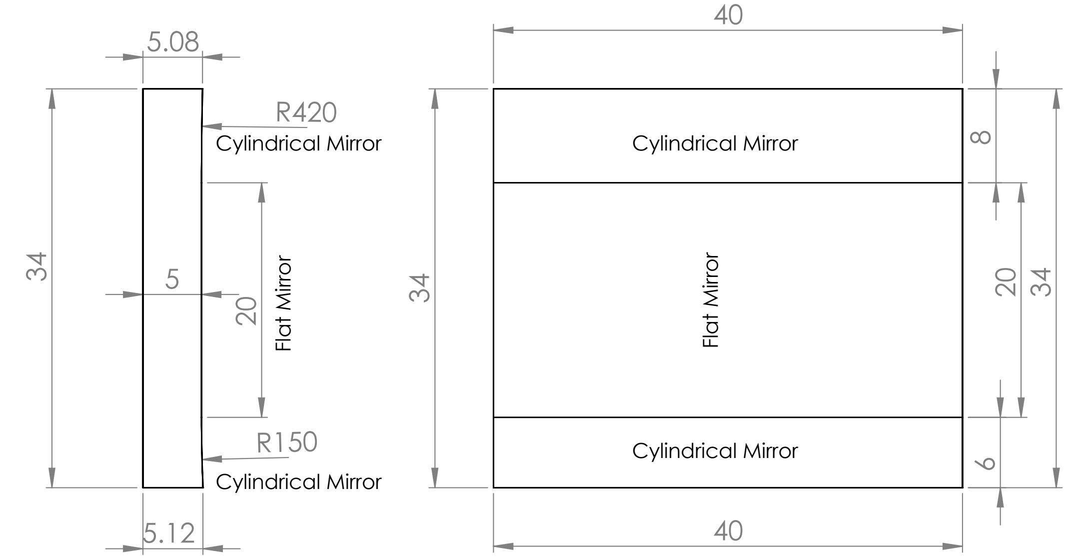
HR mirror design-1

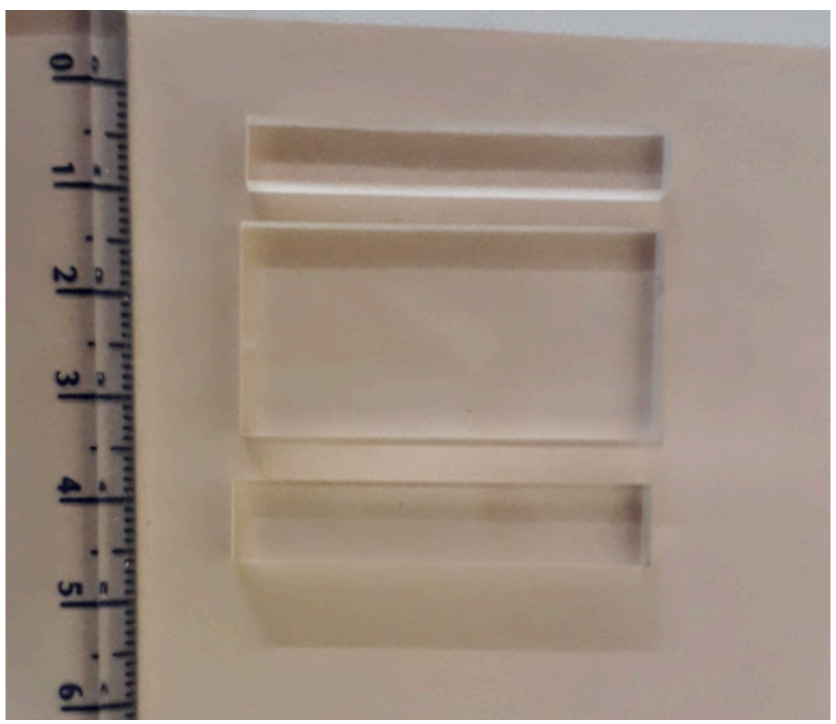


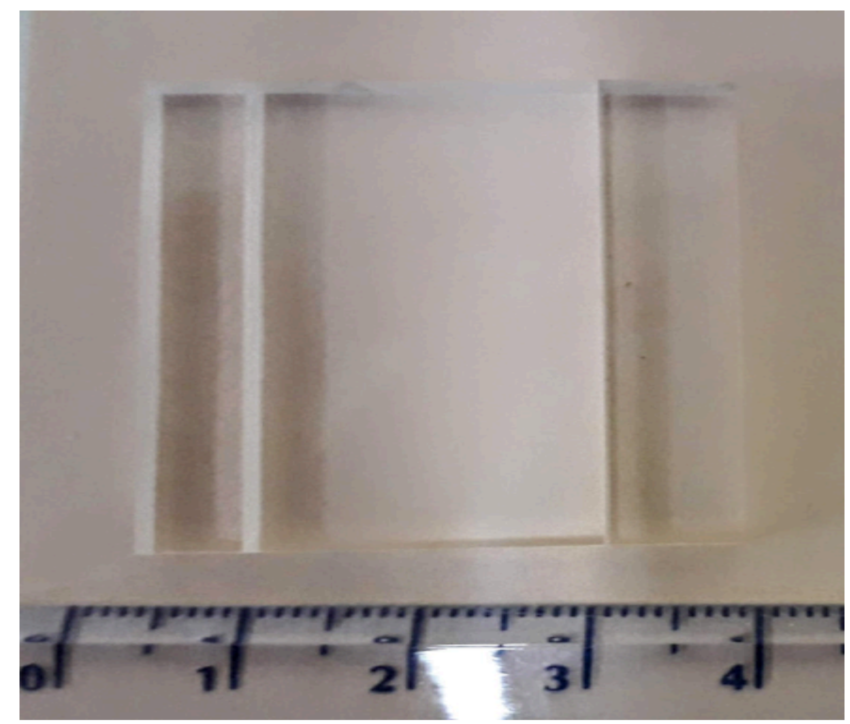
Cylindrical part of mirrors

HR mirror design-2









- Substrate material: FuSi
- HR coating: ZnS/Ge

LENS-Optics

LohnStar Optics
A Solution-Driven, Precision Coating Company INCORPORATED

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Details about of HR coating

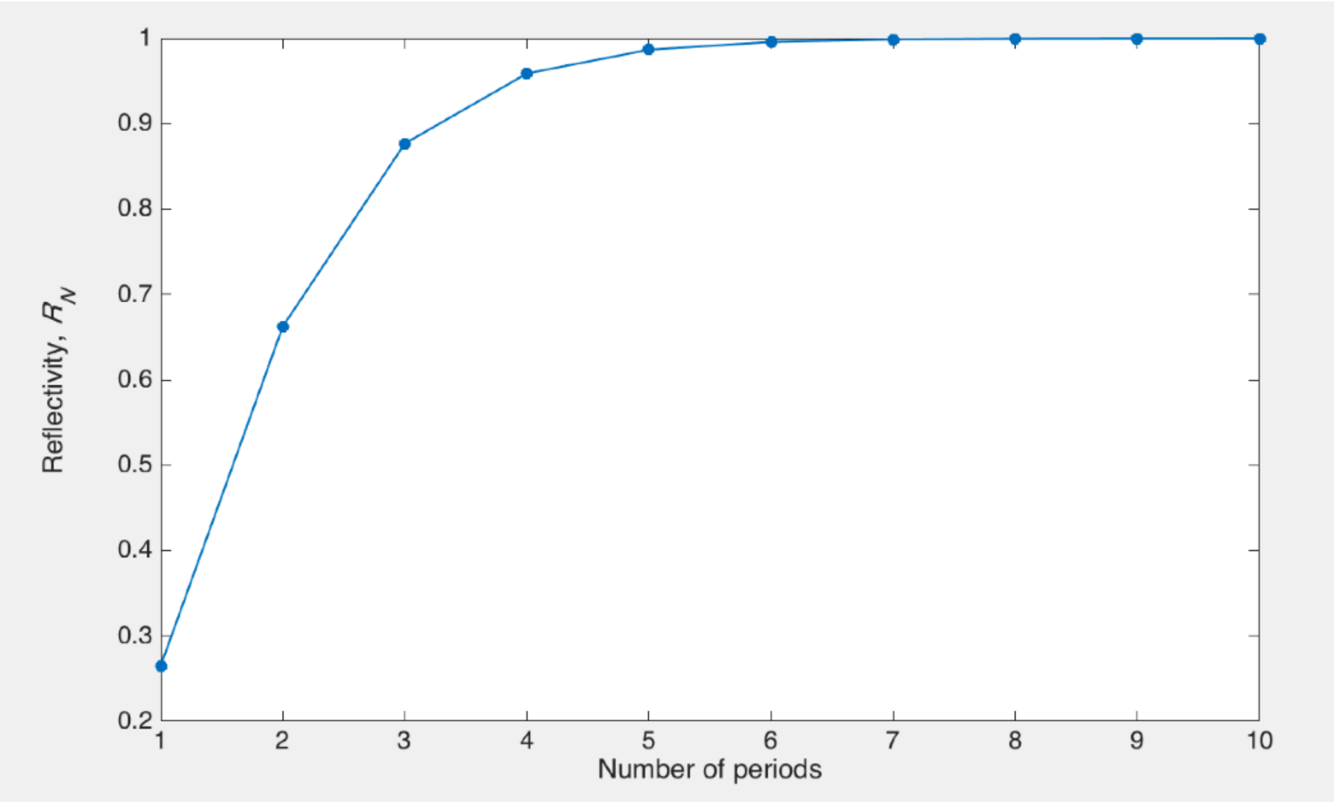
- **Substrate :** Fused Silica;
- **HR coating:** Semiconductor multilayer Ge/ZnS;

$$d_{ZnS} = \frac{\lambda}{4n_{ZnS}} = 0.75\mu m$$

$$\lambda = 6.73\mu m \qquad n_{ZnS} = 2.23 \qquad n_{Ge} = 3.94$$

$$d_{Ge} = \frac{\lambda}{4n_{Ge}} = 0.43\mu m$$

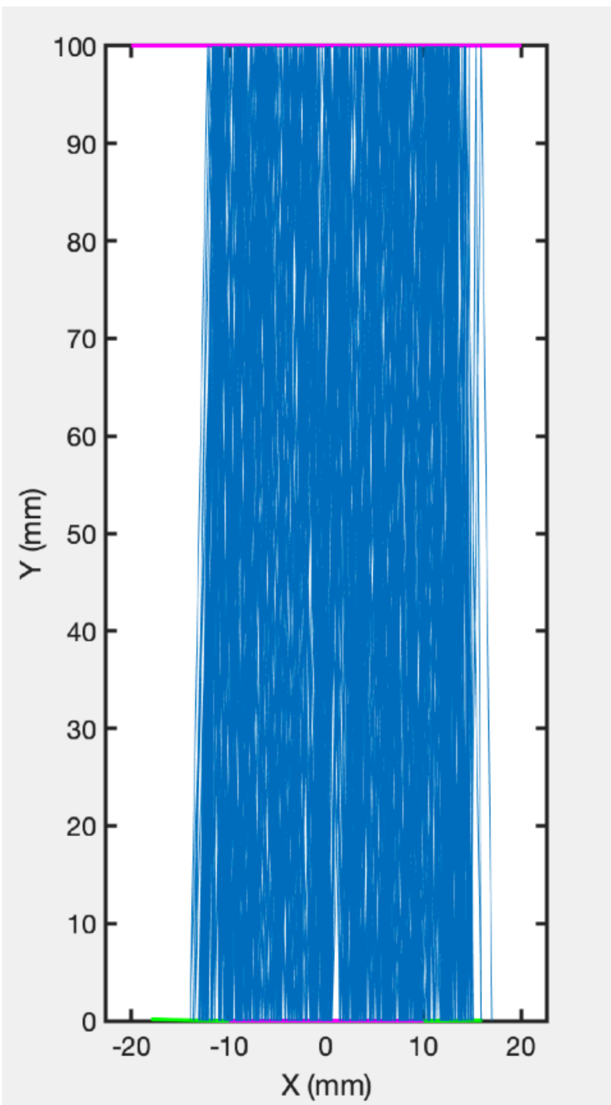
$$R_N \approx \left[\frac{1-\left(\frac{n_{Ge}}{n_{ZnS}}\right)^{2N}}{1+\left(\frac{n_{Ge}}{n_{ZnS}}\right)^{2N}} \right]^2$$



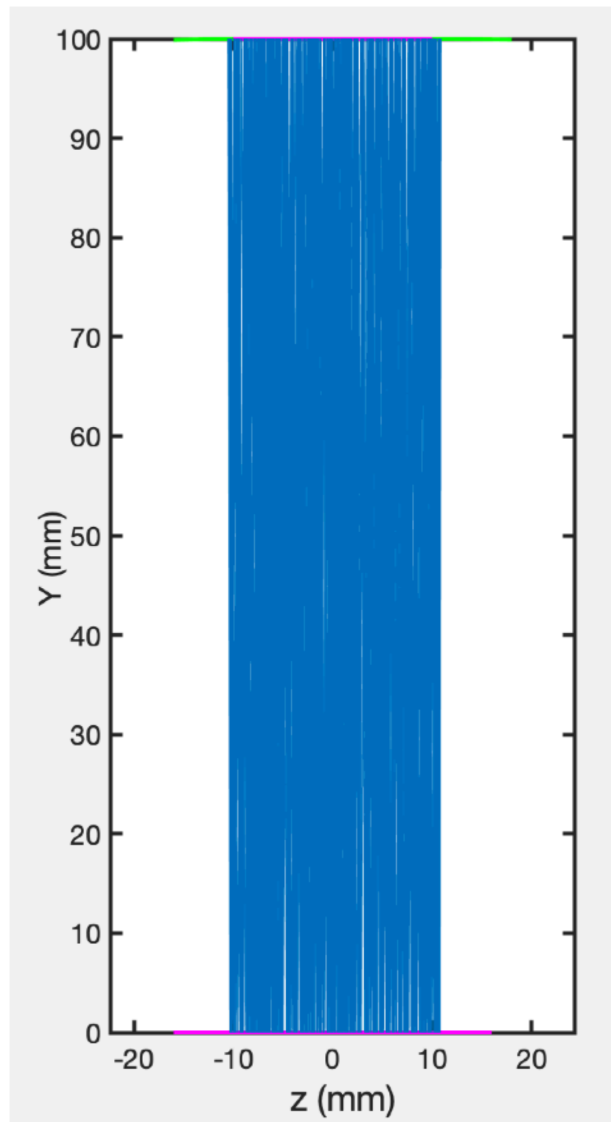
$$N = 4 \qquad D_{coat} = N\left(d_{ZnS} + d_{Ge}\right) \simeq 4\mu m$$

Ray-tracing simulation

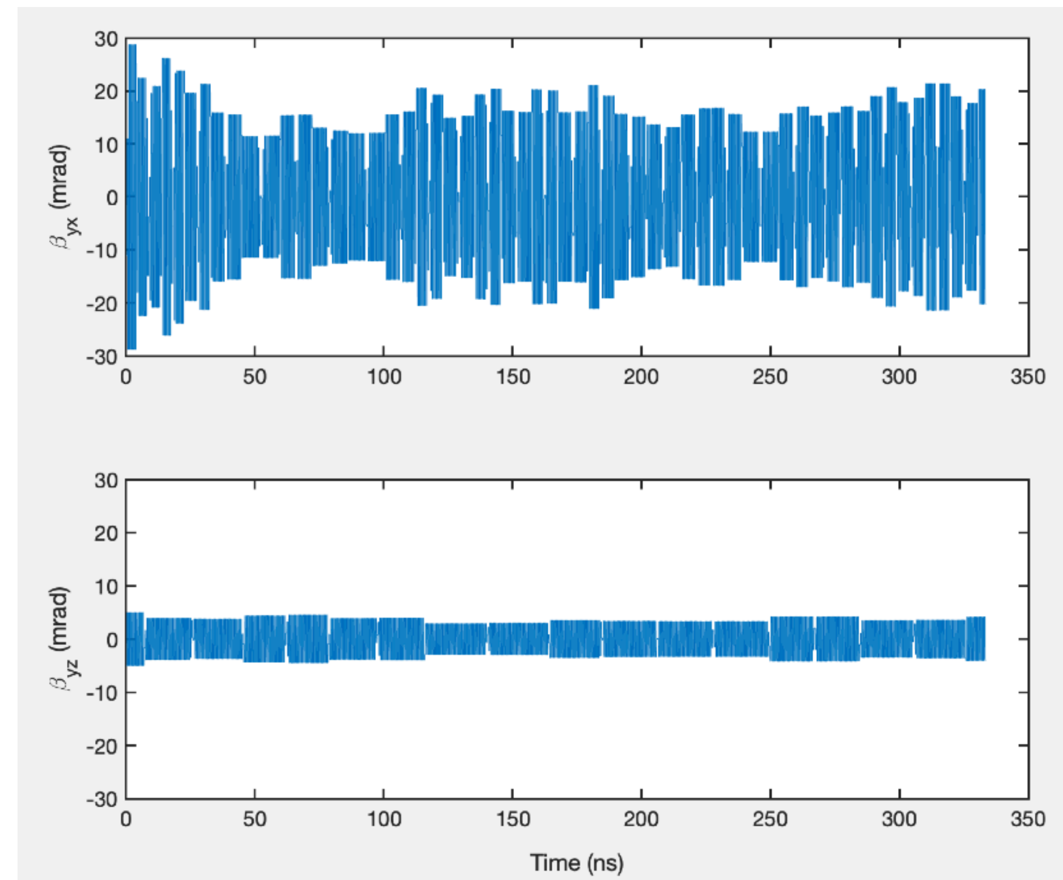
XY plane



ZY plane



Evolution of optical ray slope



injection angle

$$\beta_{yx} = 5\,mrad \quad \beta_{yz} = 11\,mrad$$

It is preserved the paraxial propagation.

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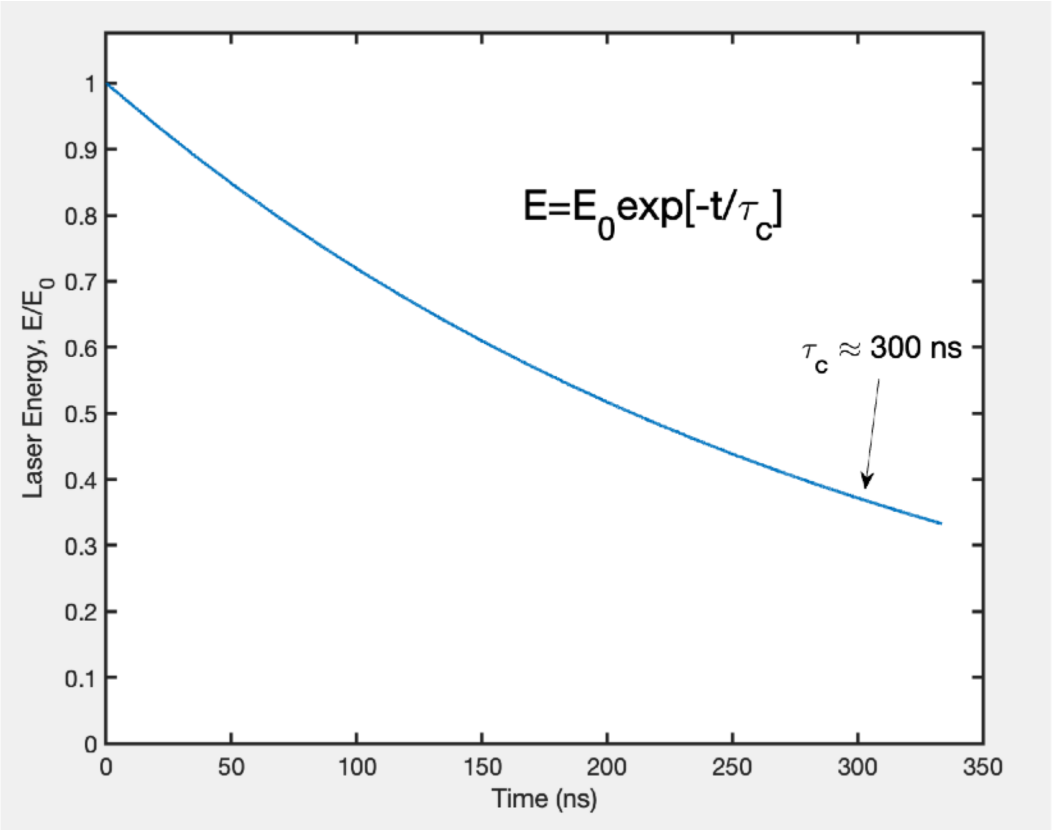
Laser energy in cavity and illuminated surface

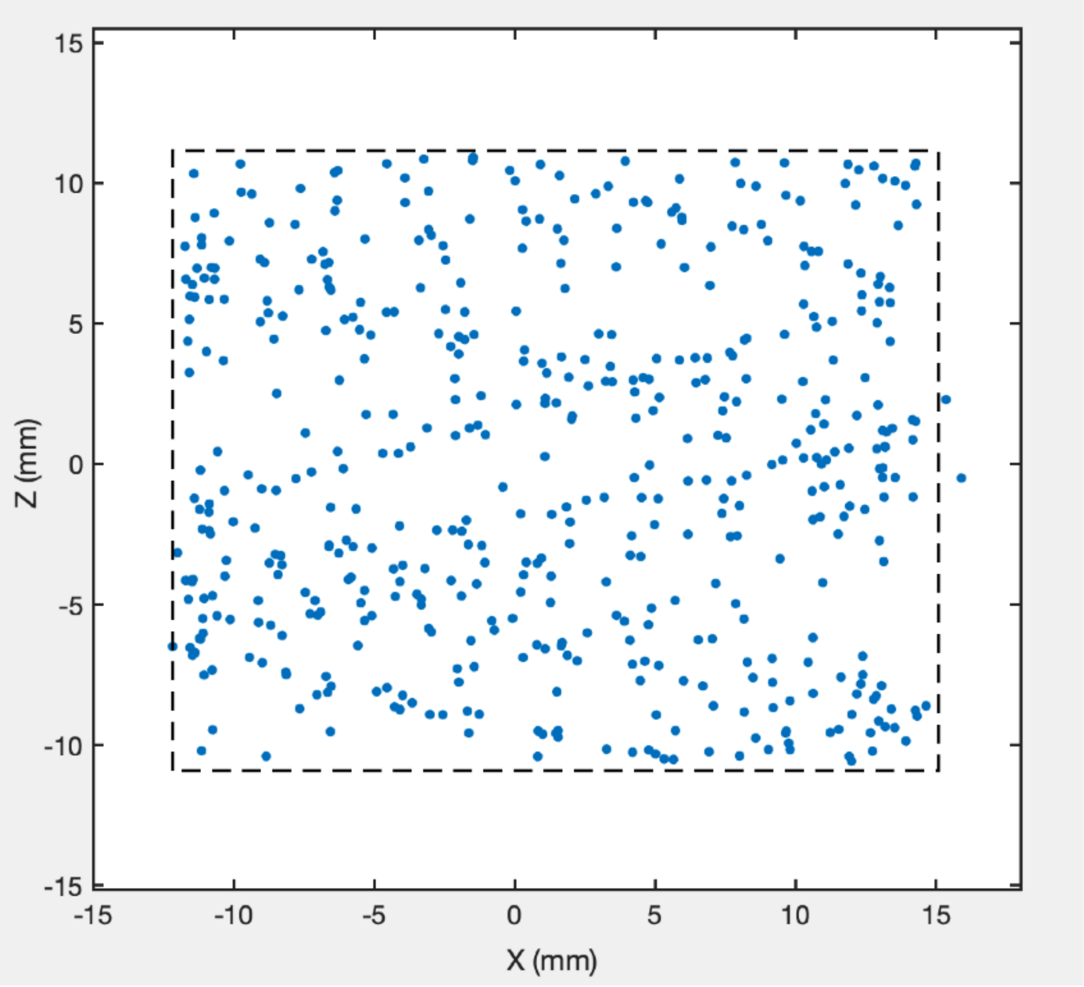
Photon lifetime is related with the cavity losses and the reflectivity of mirrors



$$L_{eff} = c\tau_c \approx 90\,m$$

$$\alpha = 1 - \sqrt{R_1 R_2} = 11 \times 10^{-4}$$

Reflection points on the bottom mirror



$$S_{ill} \approx (27 \times 21)\,cm^2$$

spin-flip transition

In order to take into account the lifetime of muonic atoms and the cavity photon lifetime:

$$\frac{dN_{sf}}{dt} = n_\mu(t)\frac{\sigma_{sf}}{h\nu}I(t)$$

$$N_{sf}(t) = \frac{\sigma_{sf}}{h\nu}\int_0^t dt' n_\mu(t')I(t')$$

$$n_\mu(t) = n_\mu(0)\exp(-t/\tau_\mu)$$

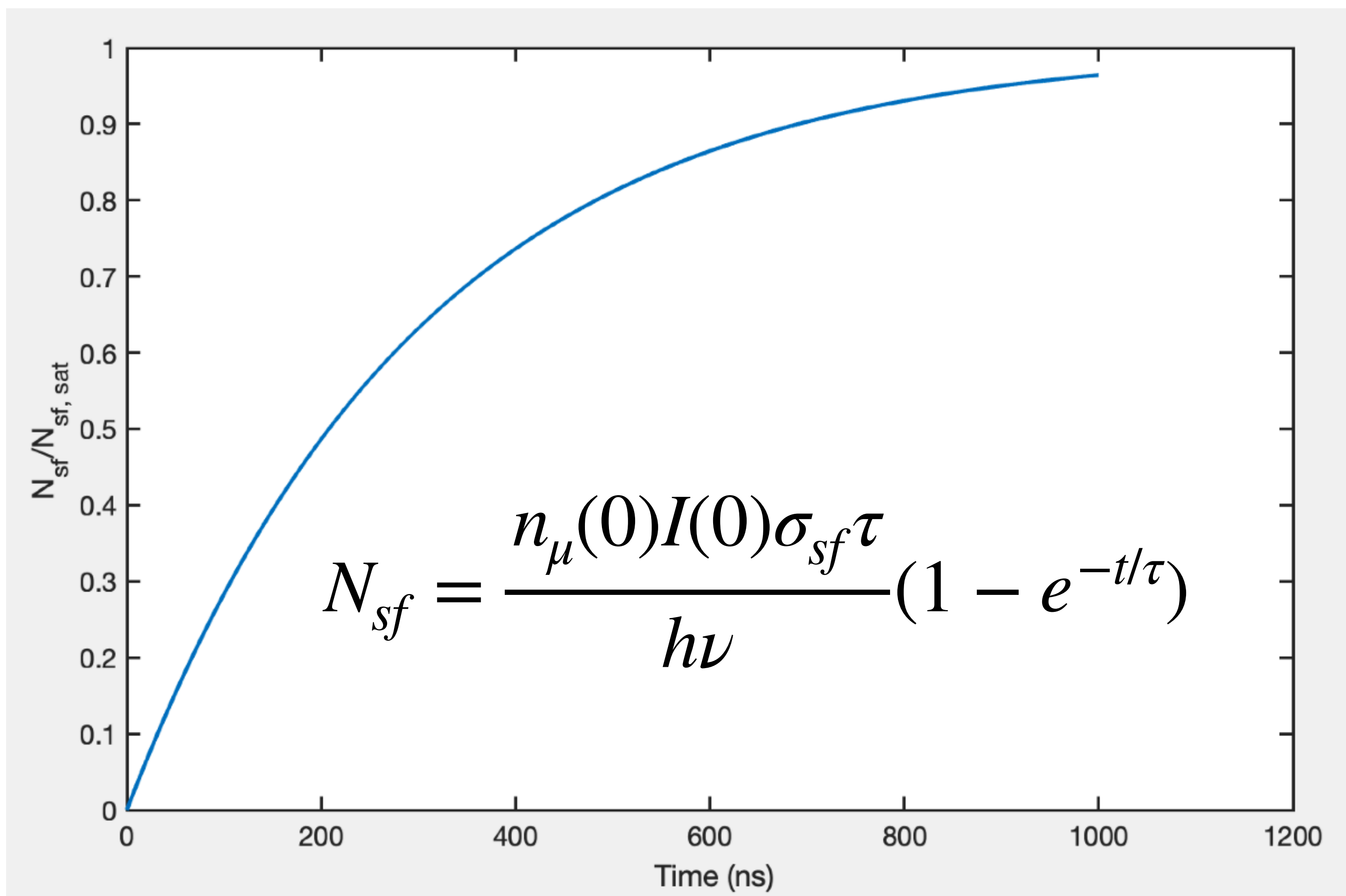
$$I(t) = I_0 \exp(-t/t_c)$$

$$\tau_\mu = 2.2\,\mu s$$

$$\tau_c = 300\,ns$$

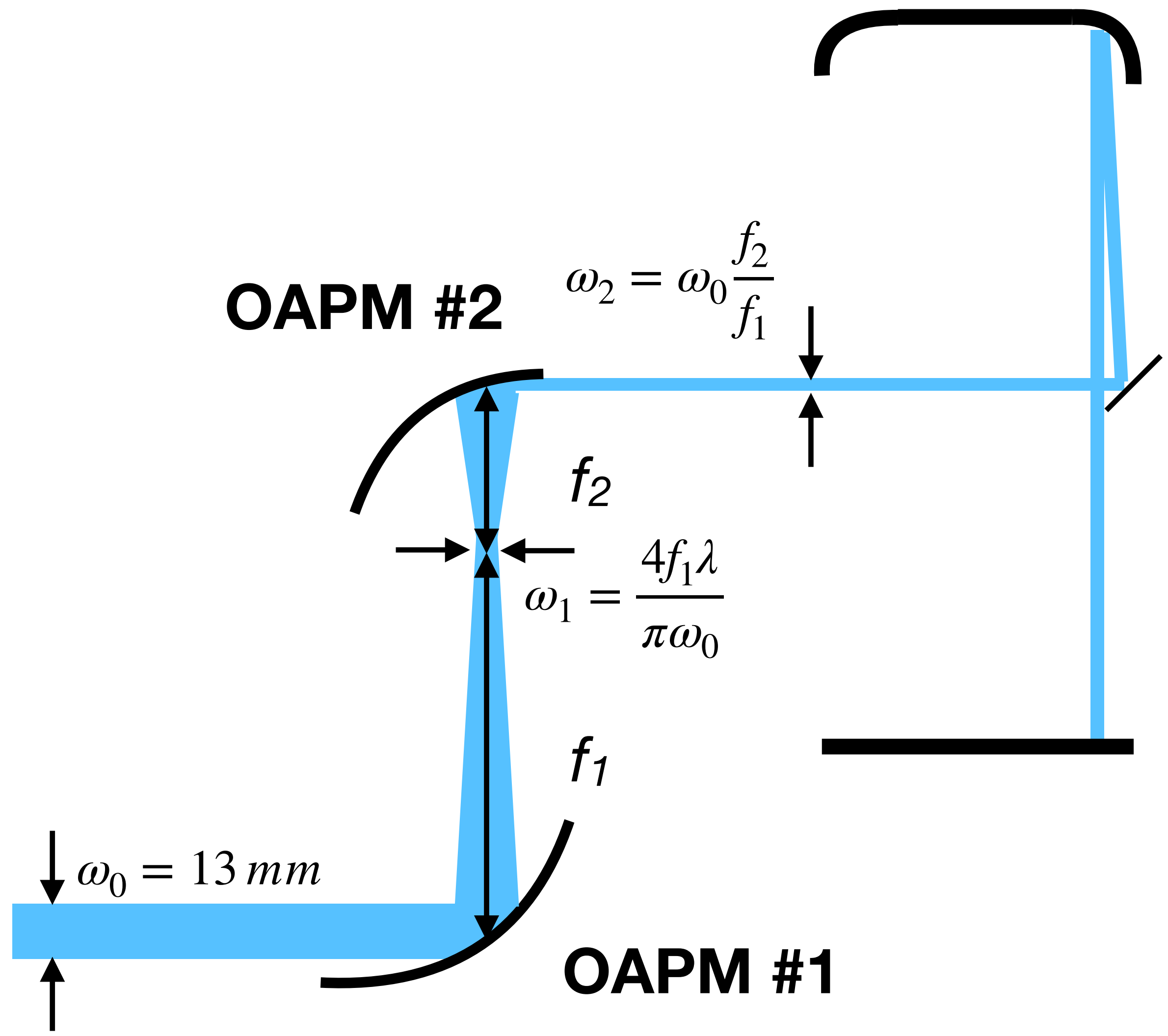
$$\frac{1}{\tau} = \frac{1}{\tau_\mu} + \frac{1}{\tau_c}$$

$$\tau \approx 270\,ns$$



$$N_{sf} = \frac{n_\mu(0)I(0)\sigma_{sf}\tau}{h\nu}(1 - e^{-t/\tau})$$

Injection system of light in cavity



The injection system of light will be realised with a couple of Off-Axis Parabolic Mirrors

- Simple system for the alignment
- Reduced losses respect the use of lens

The size of beam inside the cavity depends from the ratio between focal lengths of the OAP mirrors

Conclusions and Perspectives

- MOC is a crucial device to increase the spin-flip probability transition;
- the best solution could be a transversal MOC with a length of 10 cm with FuSi substrate;
- thermal characterisation and laser power damage will be carried out on HR mirrors.

Thank You

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