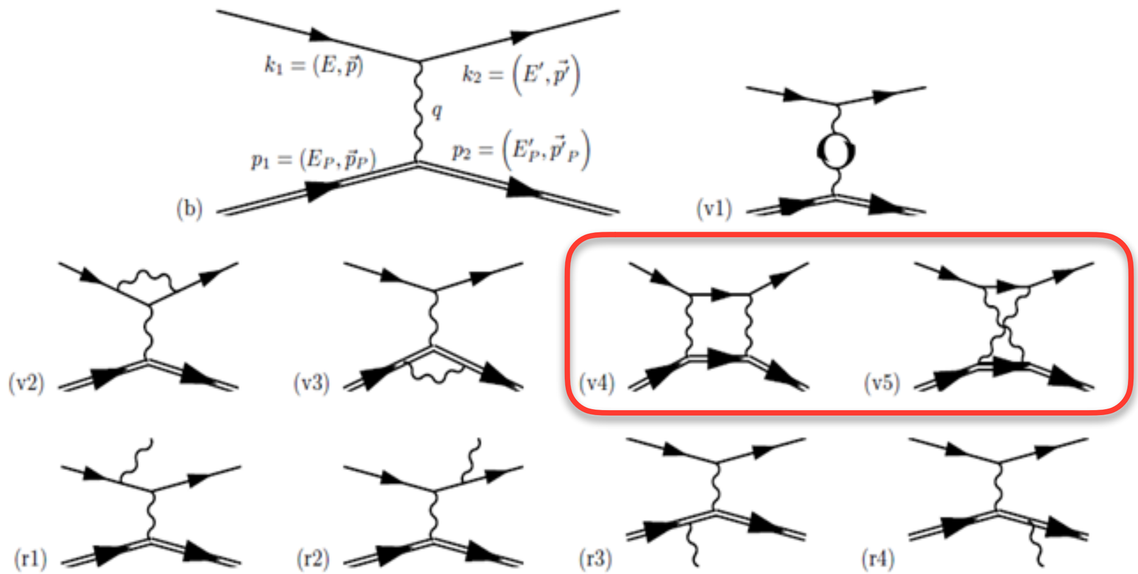
Two-photon exchange in muonic-hydrogen and lepton-proton scattering

Vladimir Pascalutsa

Institute for Nuclear Physics
University of Mainz, Germany

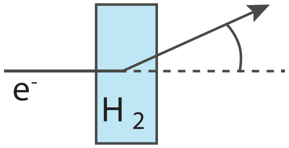
with
Franziska Hagelstein, Vadim Lensky, Marc Vanderhaeghen

Radiative corrections

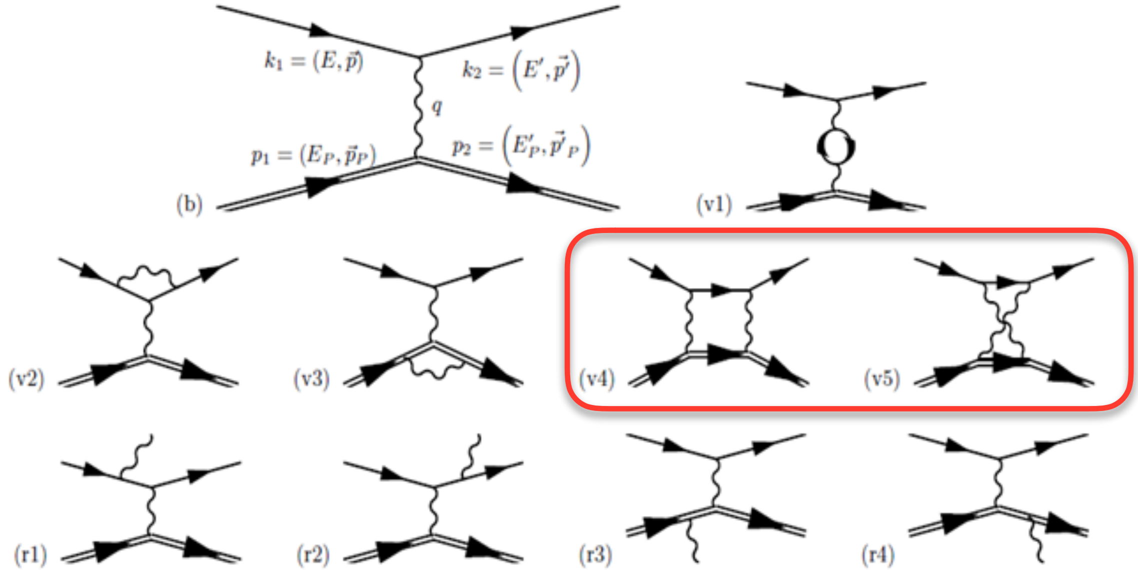


$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

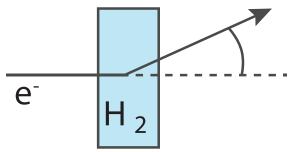
Radiative corrections



e⁻-p scattering

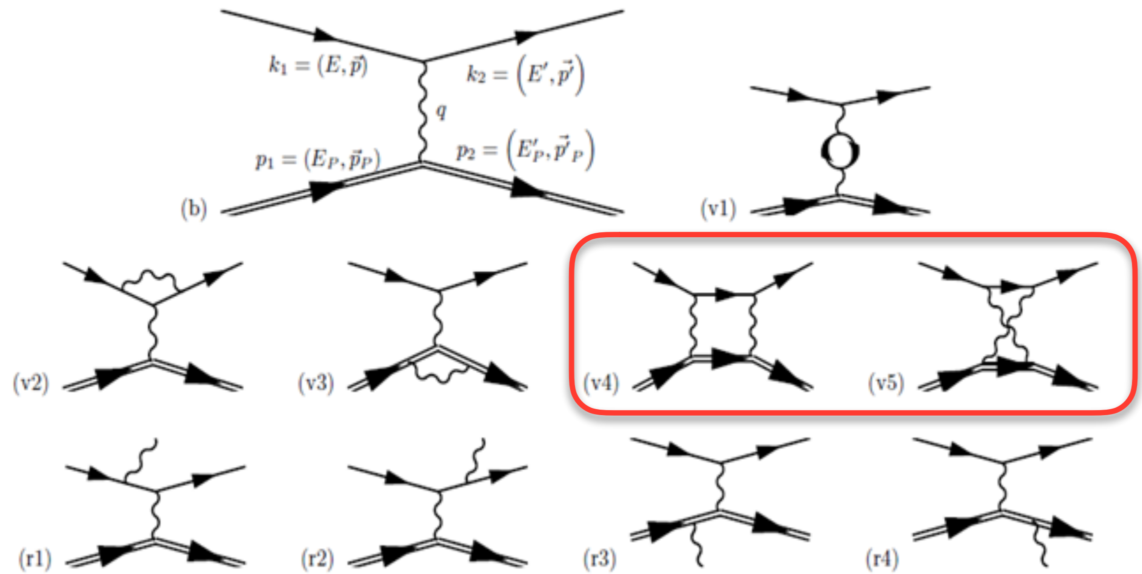


$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$



e^--p scattering

Radiative corrections

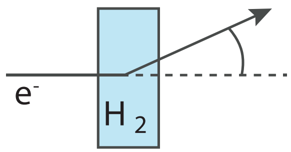


$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

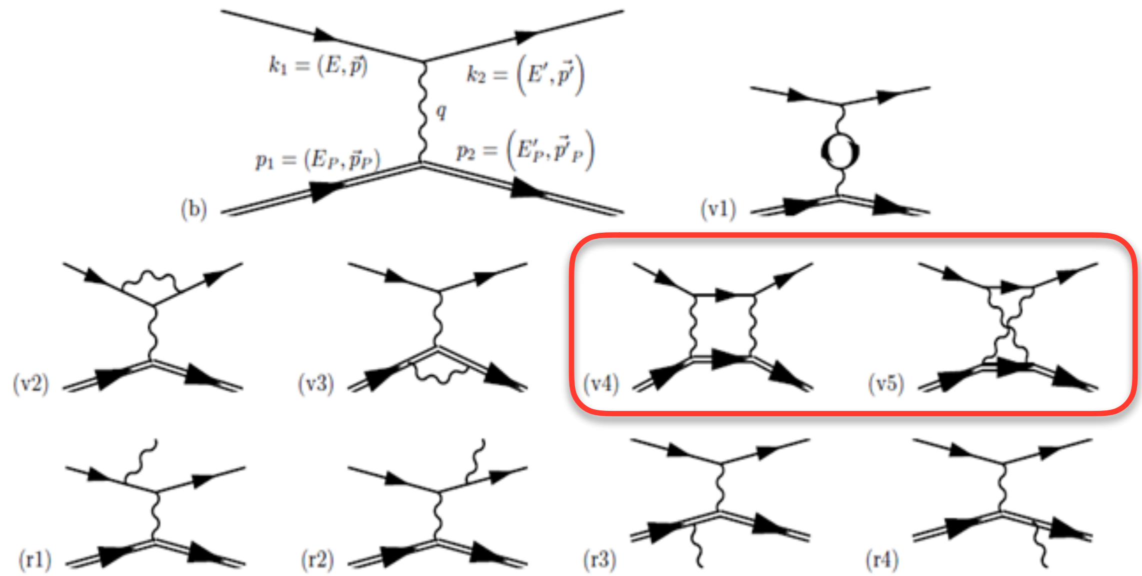
- Soft corrections are well understood Tsai(1961), Mo & Tsai (1968), Maximon & Tjon (2000)
- 2γ (or, TPE) involve the nucleon structure
- TPE not calculable (at present) from first principles, *viz*. QCD
- Nevertheless, there are many model calculations

Blunden, Melnitchouk, Tjon (2003); Guichon & Vanderhaeghen (2003); Kondratyuk & Blunden (2007); Borisyuk & Kobushkin (2006, 2007, 2008, 2012); Bystritsky, Kuraev, Tomasi-Gustaffson (2006) … many more recently

Radiative corrections

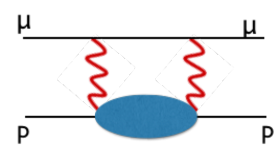


e⁻-p scattering



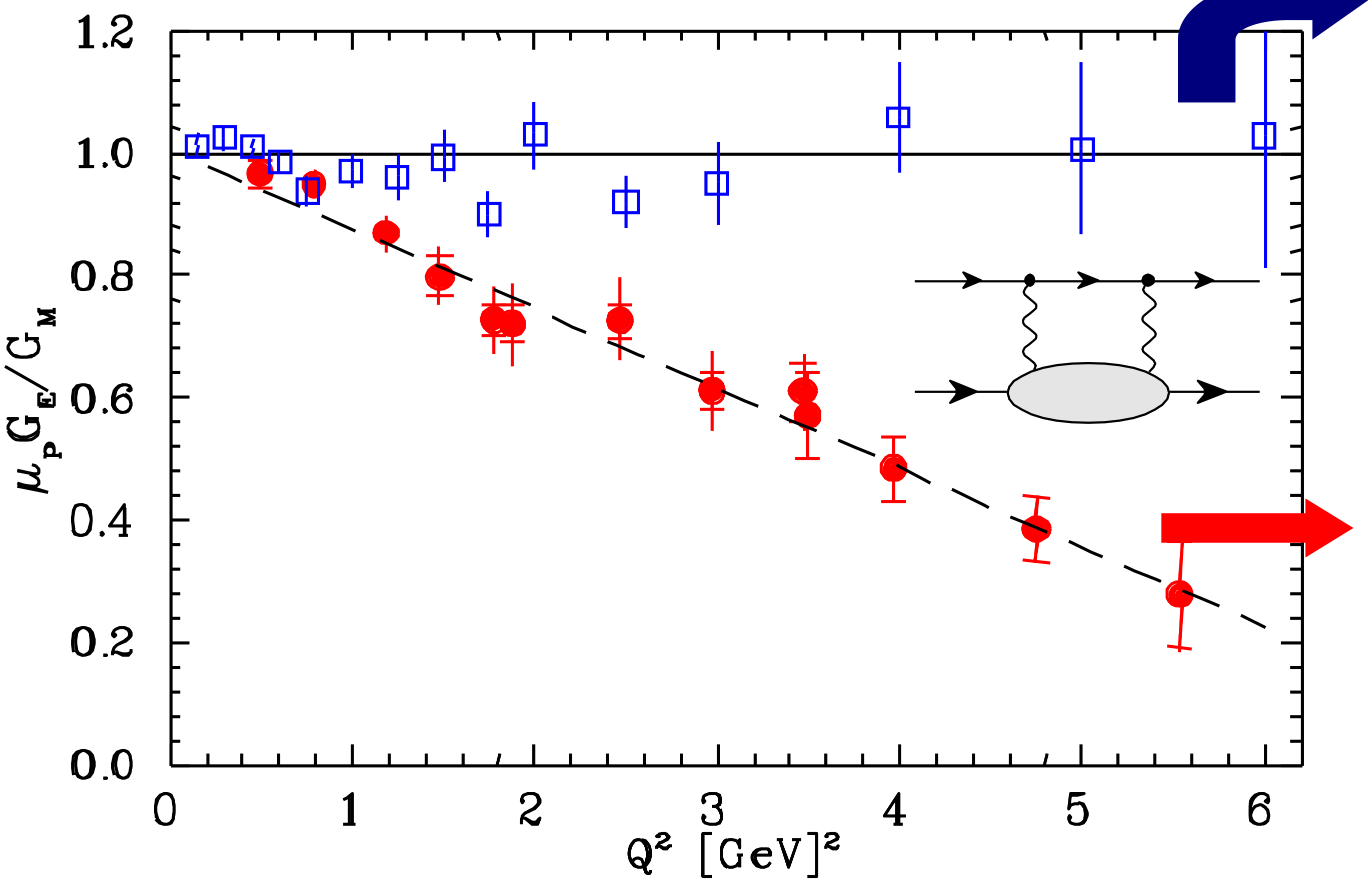
$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

- Soft corrections are well understood Tsai(1961), Mo & Tsai (1968), Maximon & Tjon (2000)
- 2γ (or, TPE) involve the nucleon structure
- TPE not calculable (at present) from first principles, *viz*. QCD
- Nevertheless, there are many model calculations



Blunden, Melnitchouk, Tjon (2003); Guichon & Vanderhaeghen (2003); Kondratyuk & Blunden (2007); Borisyuk & Kobushkin (2006, 2007, 2008, 2012); Bystritsky, Kuraev, Tomasi-Gustaffson (2006) … many more recently

TPE?



SLAC

Rosenbluth

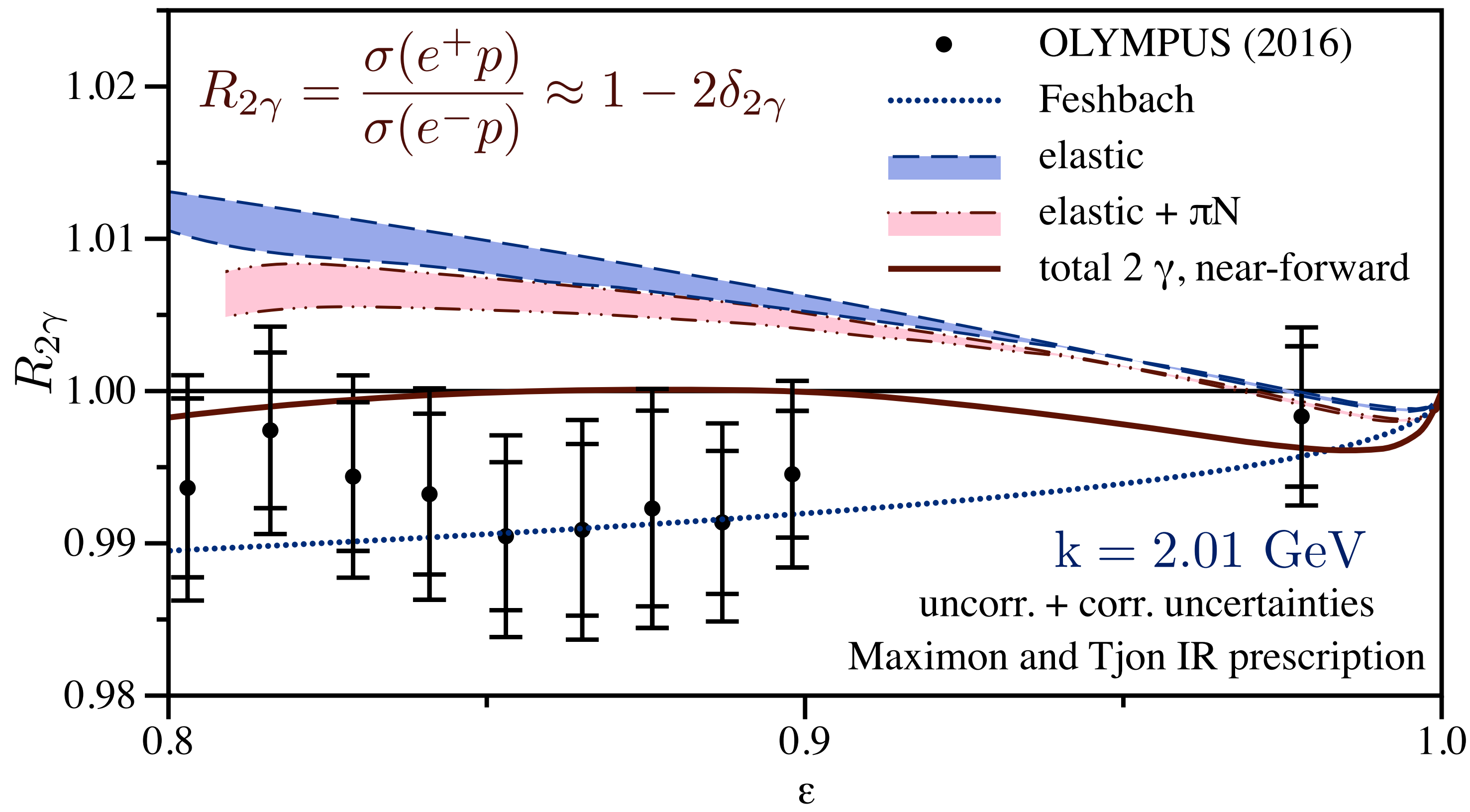
data

Jlab/Hall A

Polarization

data

2γ-exchange: comparison with data



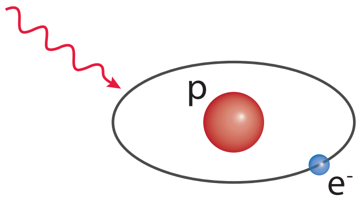
near-forward 2γ agree with data

multi-particle 2γ, e.g. ππN, is important

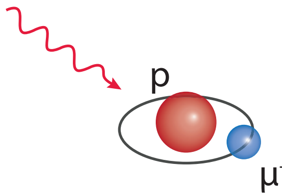
Tomalak, Pasquini, Vdh (2017)

Pasquini, Vdh, Ann.Rev.Nucl.Part.Sci (2018)

TPE in atomic spectroscopy

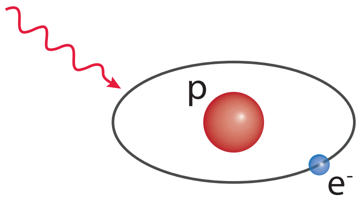


H spectroscopy

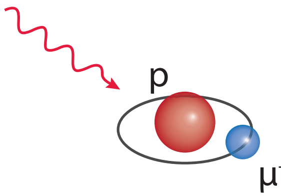


μp spectroscopy

TPE in atomic spectroscopy

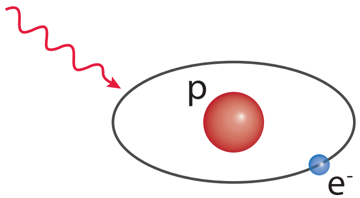


H spectroscopy

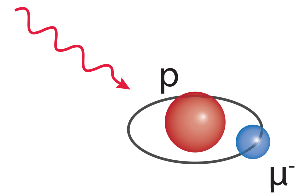


μp spectroscopy

TPE in atomic spectroscopy



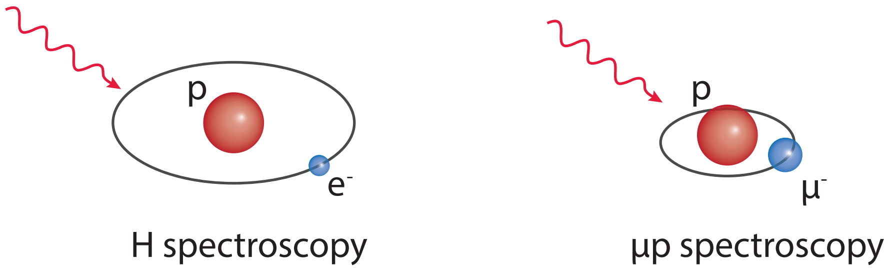
H spectroscopy



μp spectroscopy

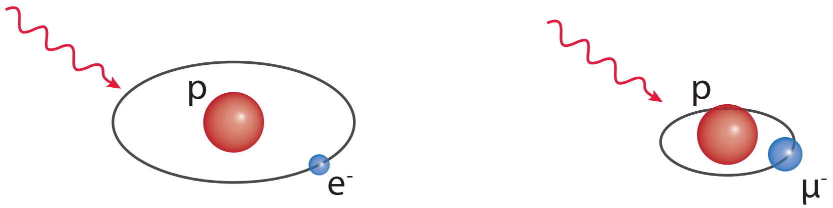
- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS

TPE in atomic spectroscopy



- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS
- TPE not calculable (at present) from first principles, *viz*. QCD

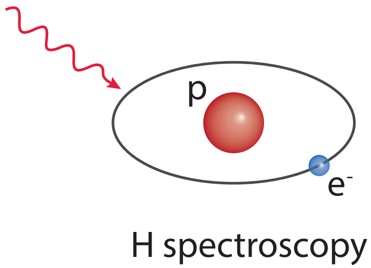
TPE in atomic spectroscopy

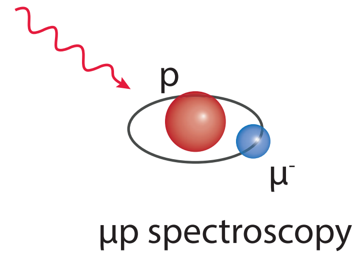


H spectroscopy μp spectroscopy

- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS
- TPE not calculable (at present) from first principles, *viz*. QCD
- fortunately, forward calculation is sufficient

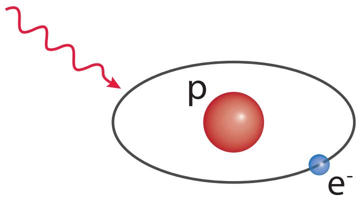
TPE in atomic spectroscopy



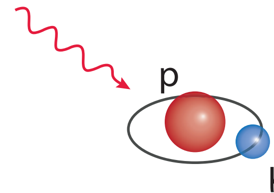


- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS
- TPE not calculable (at present) from first principles, *viz*. QCD
- fortunately, forward calculation is sufficient
- off-forward is suppressed by an extra α in HFS and $\alpha \log \alpha$ in the Lamb shift

TPE in atomic spectroscopy



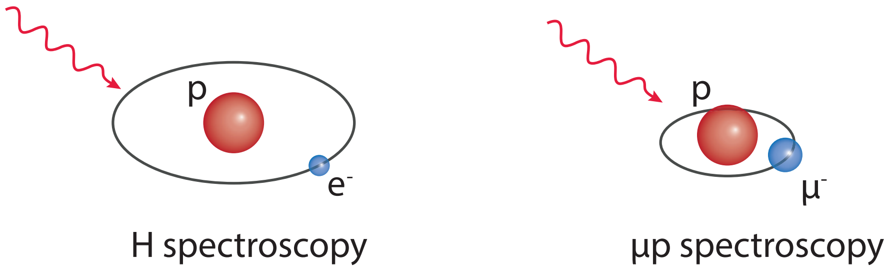
H spectroscopy



μp spectroscopy

- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS
- TPE not calculable (at present) from first principles, *viz*. QCD
- fortunately, forward calculation is sufficient
- off-forward is suppressed by an extra α in HFS and $\alpha \log \alpha$ in the Lamb shift
- forward calculations can be done by dispersive data-driven approaches
  - Lamb shift: Pachucki (1999), Carlson & Vanderhaeghen (2011), more refs below
  - HFS: Martynenko et al (2003); Carlson, Nazaryan & Griffioen (2008, 2011)

TPE in atomic spectroscopy

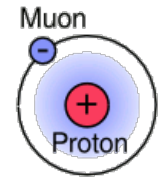


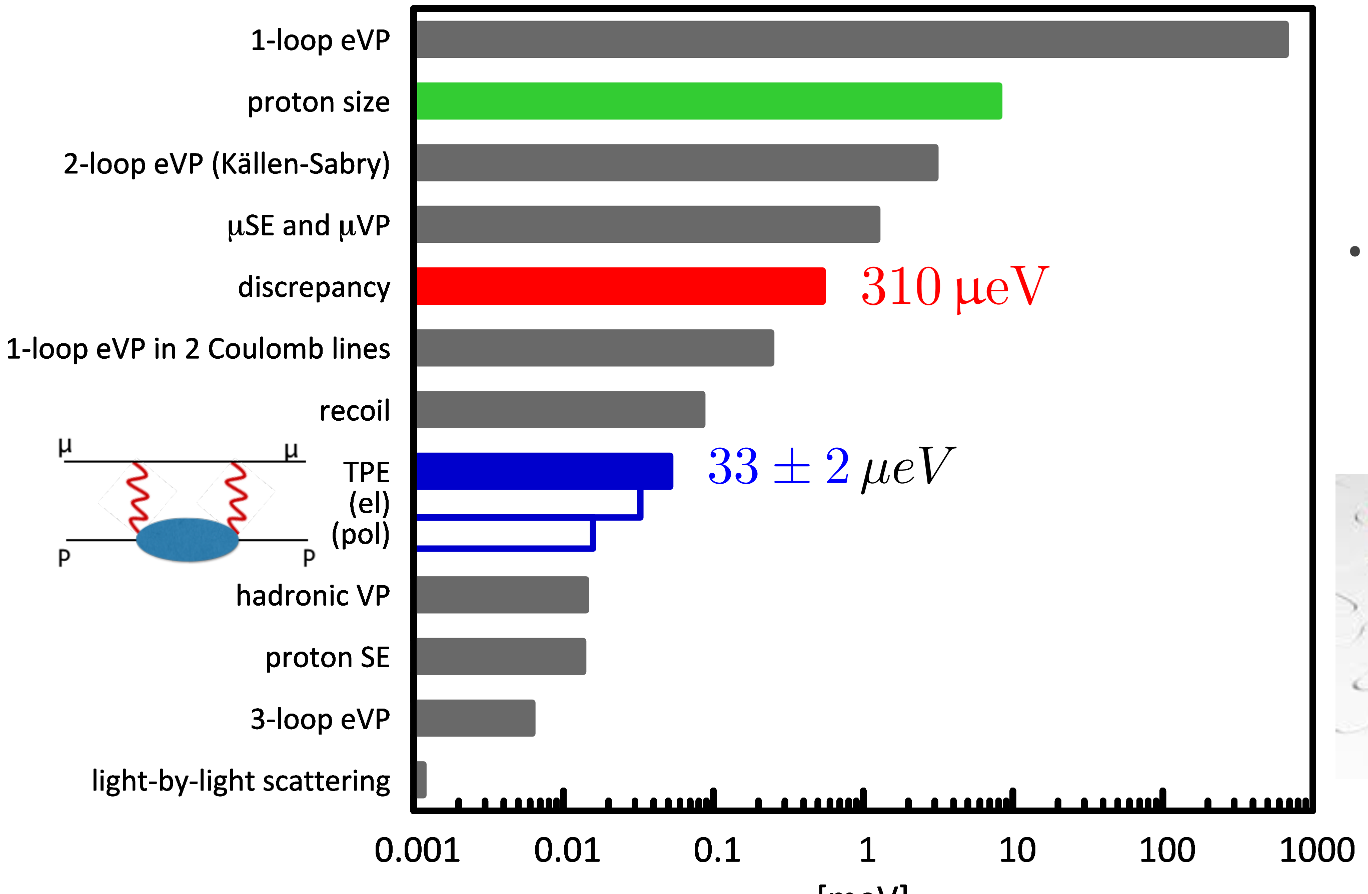
- TPE is relevant at the current level of precision, especially in muonic hydrogen, and hydrogen HFS
- TPE not calculable (at present) from first principles, *viz*. QCD
- fortunately, forward calculation is sufficient
- off-forward is suppressed by an extra α in HFS and $\alpha \log \alpha$ in the Lamb shift
- forward calculations can be done by dispersive data-driven approaches
  - Lamb shift: Pachucki (1999), Carlson & Vanderhaeghen (2011), more refs below
  - HFS: Martynenko et al (2003); Carlson, Nazaryan & Griffioen (2008, 2011)
- chiral perturbation theory calculations are available as well
  - Lamb shift: Nevado & Pineda (2008); Alarcon, Lensky, VP (2014)
  - HFS: Hagelstein et al (2017, 2019)

Example: Lamb shift budget in muonic H

$$\Delta E_{\mathrm{LS}}^{\mathrm{th}} = 206.0668(25) - 5.2275(10)\,(R_E/\mathrm{fm})^2$$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).





theory uncertainty:
$2.5\ \mu eV$

- **The uncertainty of the proton-radius extraction is dominated by the proton polarizability contribution**

Lamb shift: hadronic corrections summary

polarizability correction

on 2S level in μH in μeV

dispersive estimates

HBChPT

HBChPT
+ dispersive

BChPT

(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\text{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\text{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\text{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
[b] Taken from Ref. [12]

[9] K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
[10] A. P. Martynenko, Phys. Atom. Nucl. **69**, 1309 (2006).
[11] D. Nevado and A. Pineda, Phys. Rev. C **77**, 035202 (2008).
[12] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).
[13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A **48**, 120 (2012).
[14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, **052501** (2013).

[LO-BχPT] Alarcon, Lensky, Pascalutsa, EPJC (2014) 74:2852

- elastic contribution on 2S level: ΔE_{2S} = -23 μeV
- inelastic contribution: Carlson, Vdh (2011) + Birse, McGovern (2012)

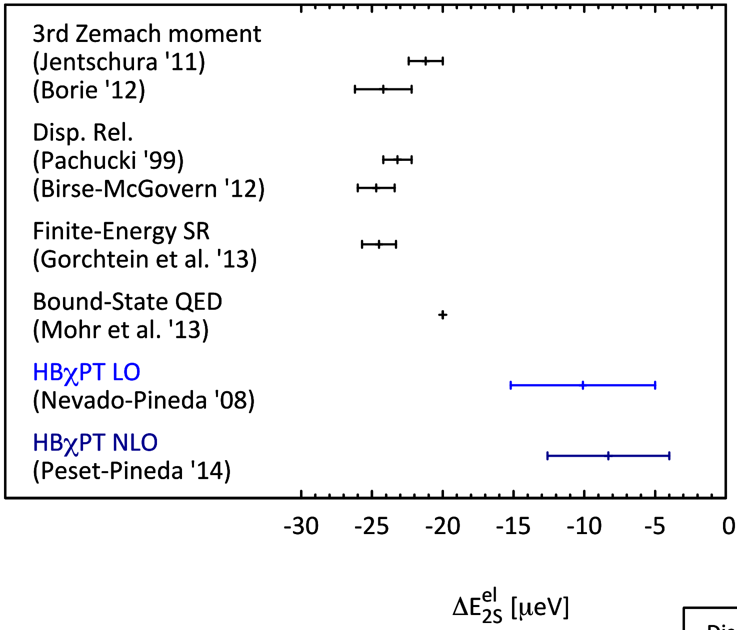
total hadronic correction on Lamb shift

$\Delta E_{TPE}(2P - 2S) = (33 \pm 2)$ μeV

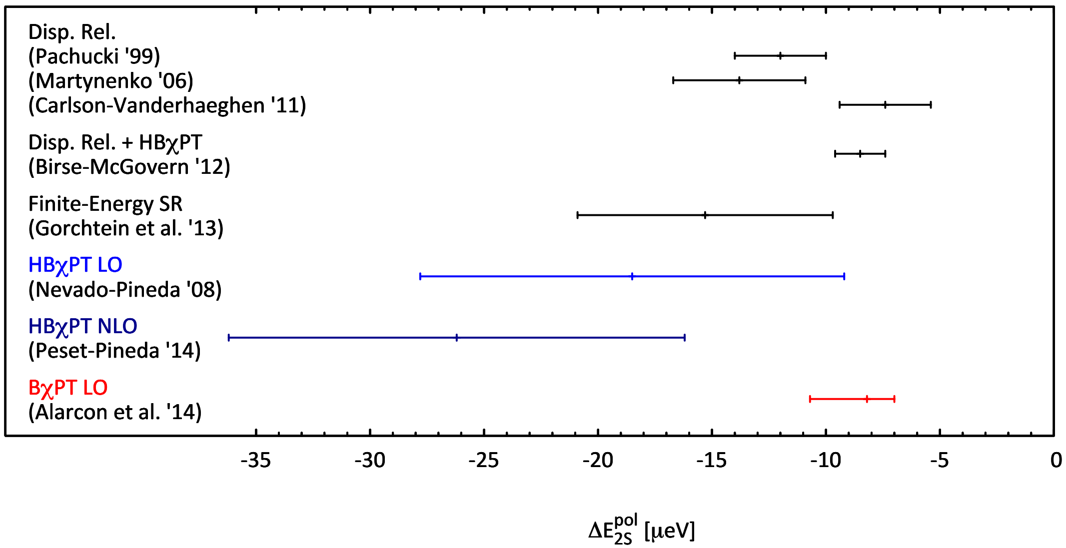
...or about 10% of needed correction

Lamb shift: hadronic corrections summary

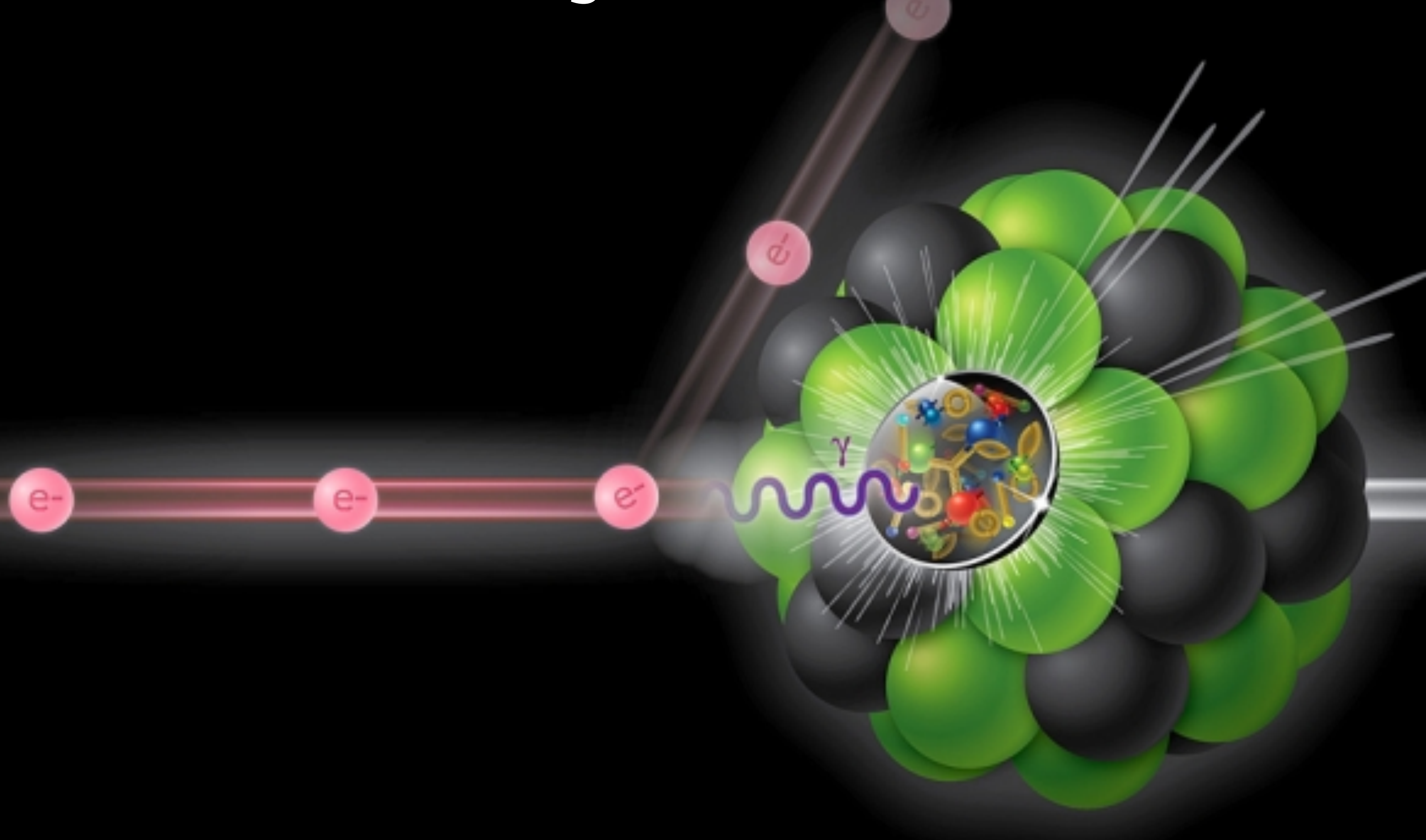
TPE elastic correction:



TPE polarizability correction

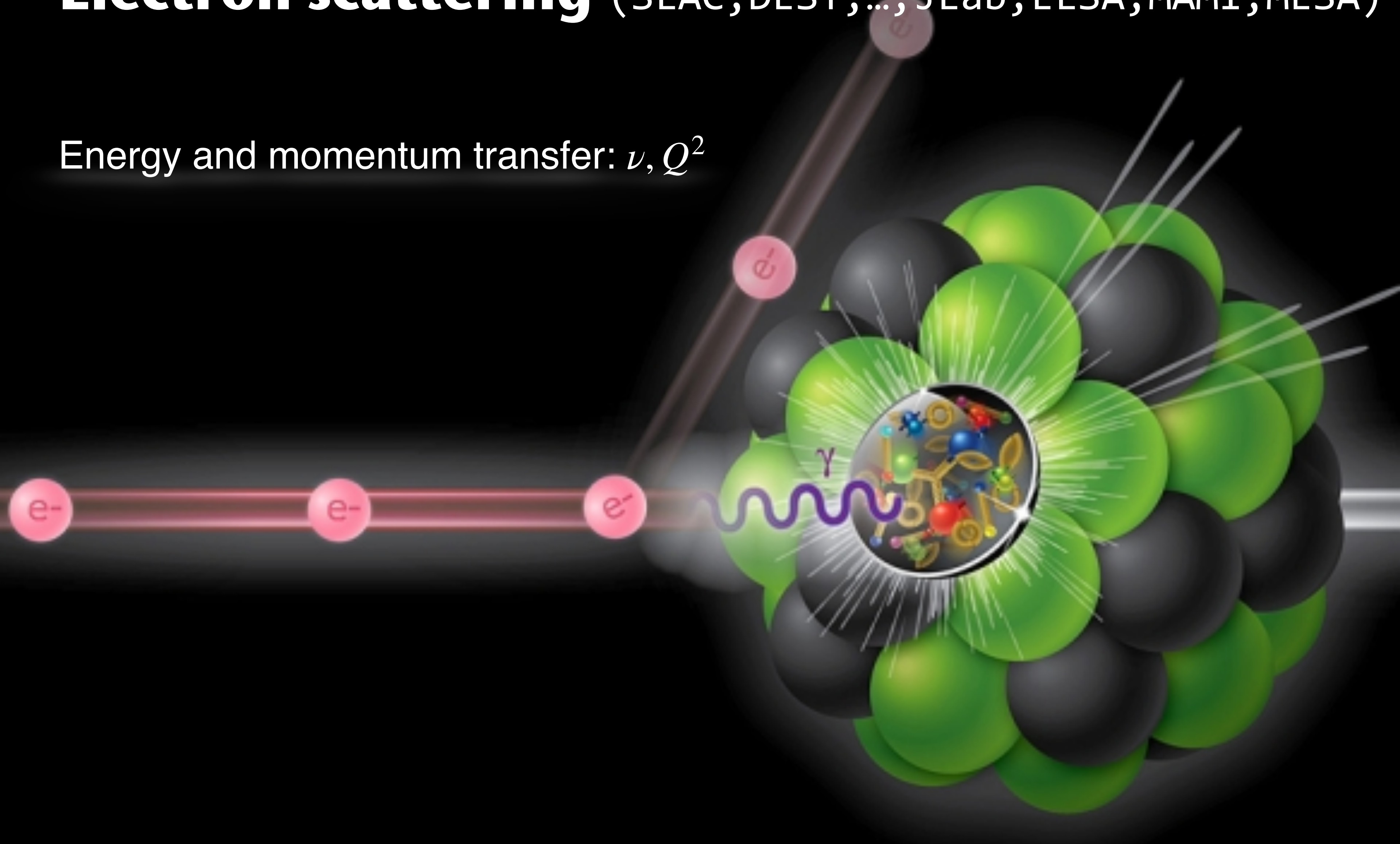


Electron scattering (SLAC,DESY,…,JLab,ELSA,MAMI,MESA)



Electron scattering (SLAC,DESY,…,JLab,ELSA,MAMI,MESA)

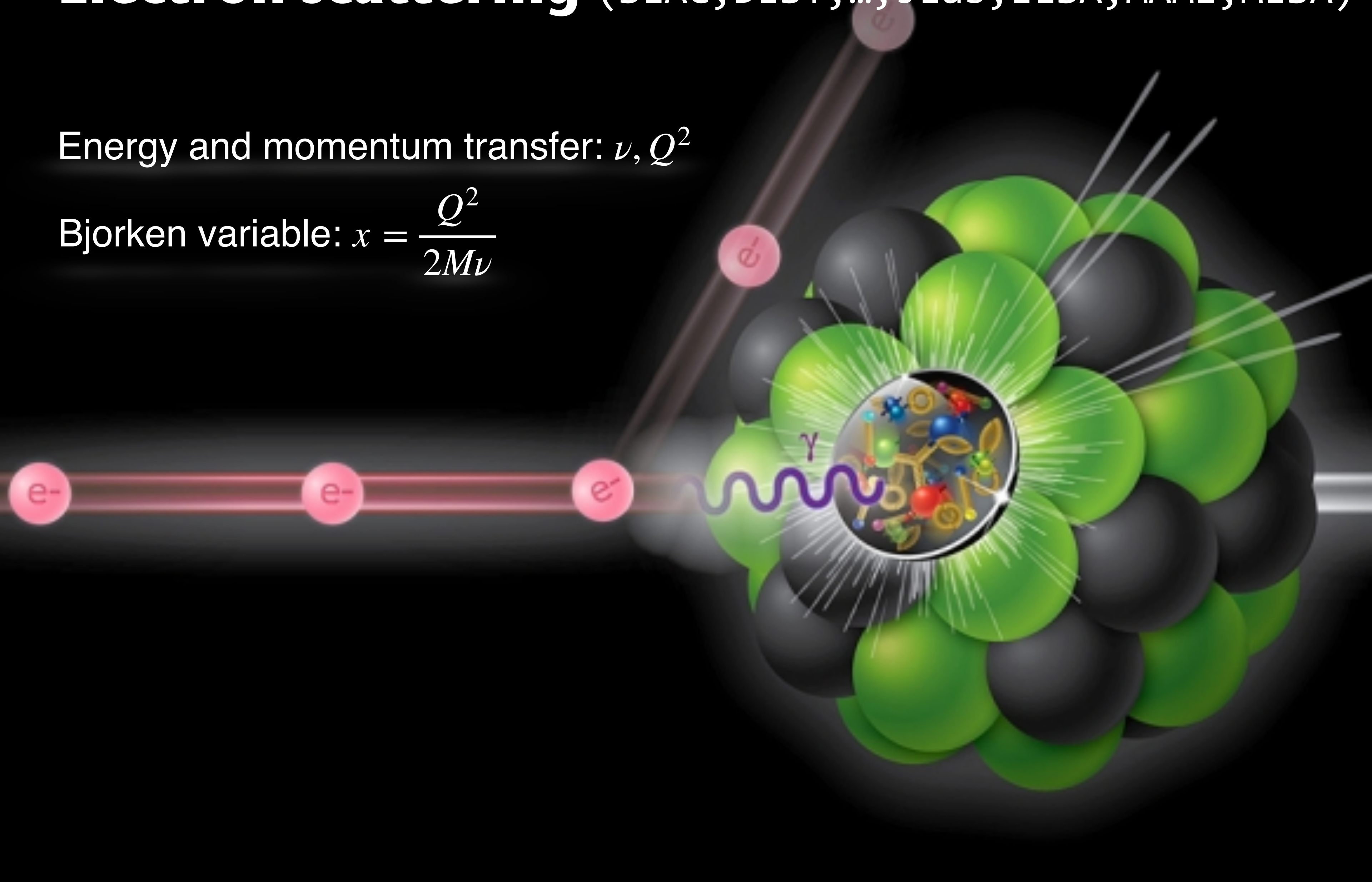
Energy and momentum transfer: ν, Q^2



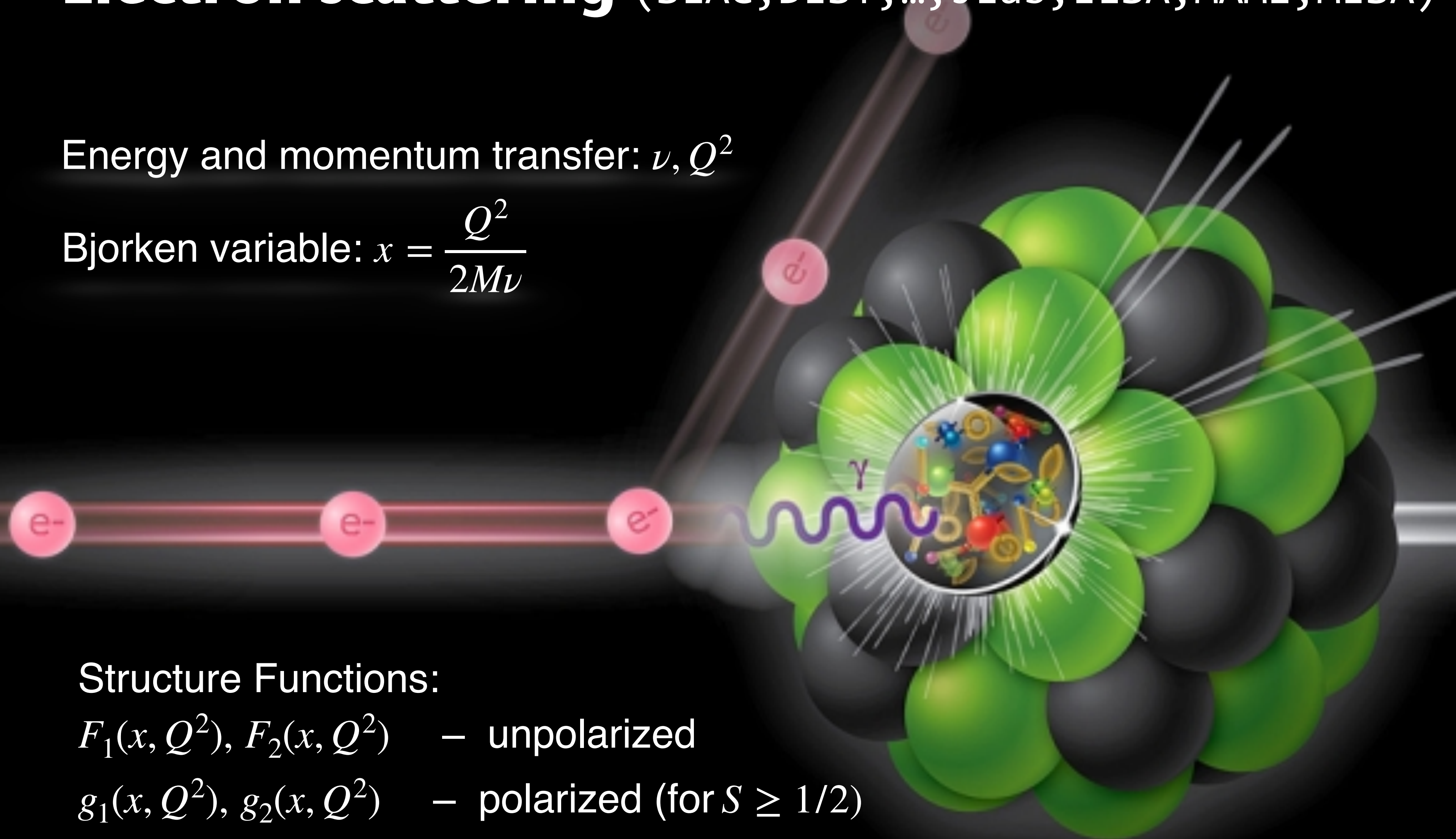
Electron scattering (SLAC,DESY,…,JLab,ELSA,MAMI,MESA)

Energy and momentum transfer: ν, Q^2

Bjorken variable: $x = \dfrac{Q^2}{2M\nu}$



Electron scattering (SLAC,DESY,…,JLab,ELSA,MAMI,MESA)



Energy and momentum transfer: ν, Q^2

Bjorken variable: $x = \dfrac{Q^2}{2M\nu}$

Structure Functions:

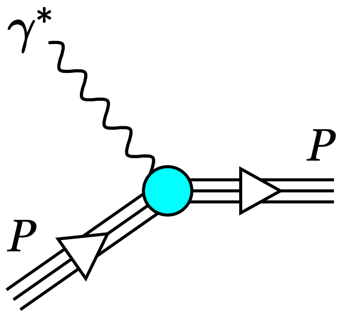
$F_1(x, Q^2), F_2(x, Q^2)$ – unpolarized

$g_1(x, Q^2), g_2(x, Q^2)$ – polarized (for $S \geq 1/2$)

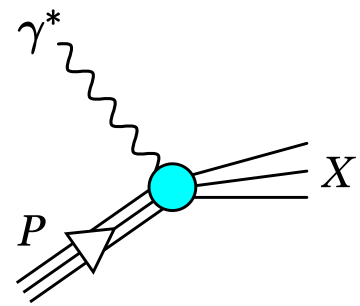
$b_{1,2,3,4}(x, Q^2)$ – tensor (for $S \geq 1$)

Structure functions

Elastic $(x = 1)$

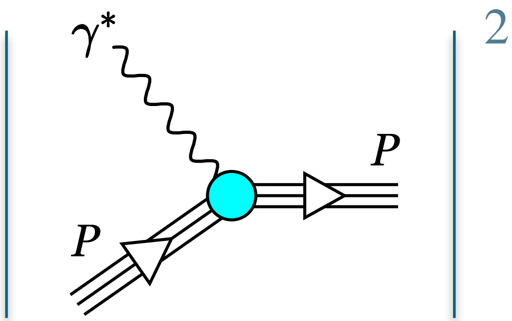
$\Bigg|$  $\Bigg|^2$

Inelastic $(0 < x \leq x_0 < 1)$

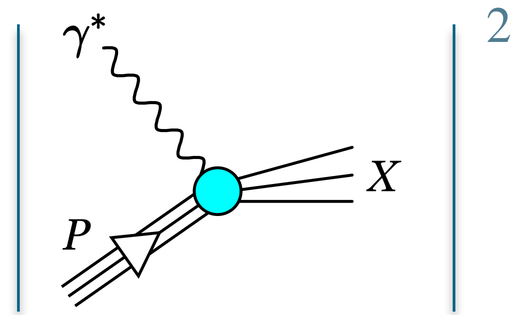
$\Bigg|$  $\Bigg|^2$

Structure functions

Elastic $(x = 1)$



Inelastic $(0 < x \leq x_0 < 1)$



$$F_1^{\mathrm{el}}(x,Q^2) = \frac{1}{2} G_M^2(Q^2)\ \delta(1-x)$$

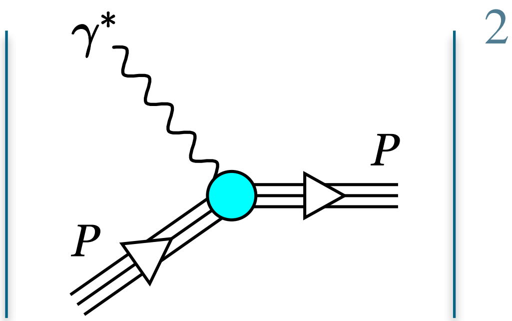
$$F_2^{\mathrm{el}}(x,Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\ \delta(1-x)$$

$$g_1^{\mathrm{el}}(x,Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1+\tau)} G_M(Q^2)\ \delta(1-x)$$

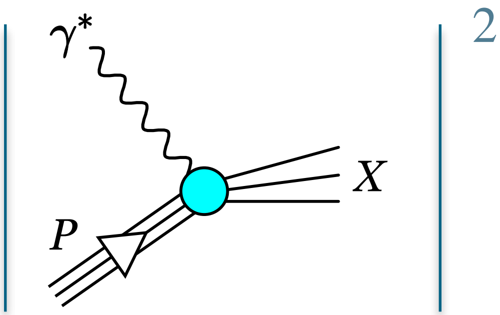
$$g_2^{\mathrm{el}}(x,Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1+\tau)} \tau G_M(Q^2)\ \delta(1-x)$$

Structure functions

Elastic $(x = 1)$



Inelastic $(0 < x \leq x_0 < 1)$



$$F_1^{\rm el}(x, Q^2) = \frac{1}{2} G_M^2(Q^2)\; \delta(1-x)$$

$$F_2^{\rm el}(x, Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\; \delta(1-x)$$

$$g_1^{\rm el}(x, Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1+\tau)} G_M(Q^2)\; \delta(1-x)$$

$$g_2^{\rm el}(x, Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1+\tau)} \tau G_M(Q^2)\; \delta(1-x)$$

$$F_1(x, Q^2) \sim \sigma_T(\nu, Q^2)$$

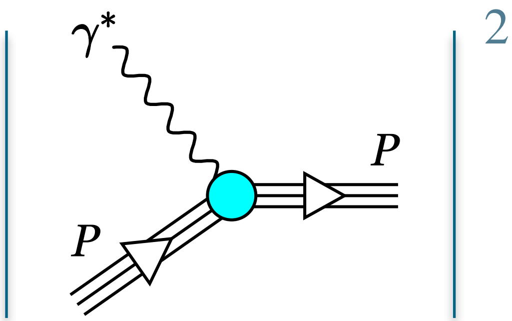
$$F_2(x, Q^2) \sim (\sigma_T + \sigma_L)(\nu, Q^2)$$

$$g_1(x, Q^2) \sim [(Q/\nu)\sigma_{LT} + \sigma_{TT}](\nu, Q^2)$$

$$g_2(x, Q^2) \sim [(\nu/Q)\sigma_{LT} - \sigma_{TT}](\nu, Q^2)$$

Structure functions

Elastic $(x = 1)$



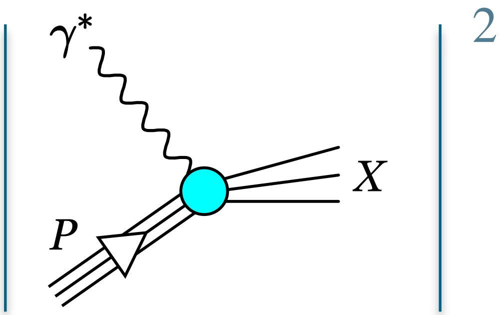
$$F_1^{\rm el}(x,Q^2) = \frac{1}{2} G_M^2(Q^2)\ \delta(1-x)$$

$$F_2^{\rm el}(x,Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\ \delta(1-x)$$

$$g_1^{\rm el}(x,Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1+\tau)} G_M(Q^2)\ \delta(1-x)$$

$$g_2^{\rm el}(x,Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1+\tau)} \tau G_M(Q^2)\ \delta(1-x)$$

Inelastic $(0 < x \le x_0 < 1)$



$$F_1(x,Q^2) \sim \sigma_T(\nu, Q^2)$$

$$F_2(x,Q^2) \sim (\sigma_T + \sigma_L)(\nu, Q^2)$$

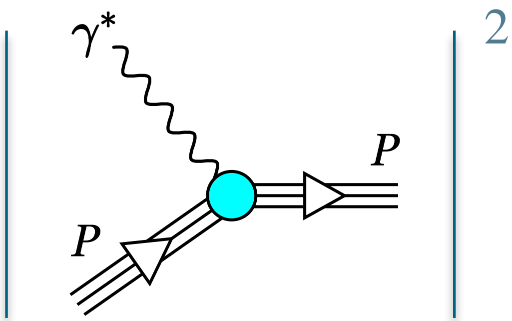
$$g_1(x,Q^2) \sim [(Q/\nu)\sigma_{LT} + \sigma_{TT}](\nu, Q^2)$$

$$g_2(x,Q^2) \sim [(\nu/Q)\sigma_{LT} - \sigma_{TT}](\nu, Q^2)$$

with G_E electric and G_M magnetic Form Factors,

Structure functions

Elastic $(x = 1)$



$$F_1^{\rm el}(x,Q^2) = \frac{1}{2} G_M^2(Q^2)\ \delta(1-x)$$

$$F_2^{\rm el}(x,Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\ \delta(1-x)$$

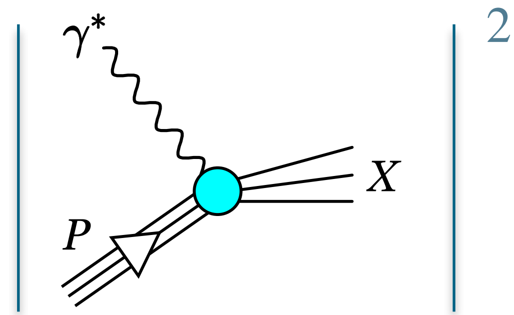
$$g_1^{\rm el}(x,Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1+\tau)} G_M(Q^2)\ \delta(1-x)$$

$$g_2^{\rm el}(x,Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1+\tau)} \tau G_M(Q^2)\ \delta(1-x)$$

with G_E electric and G_M magnetic Form Factors,

$$\tau = Q^2/4M^2$$

Inelastic $(0 < x \le x_0 < 1)$



$$F_1(x,Q^2) \sim \sigma_T(\nu, Q^2)$$

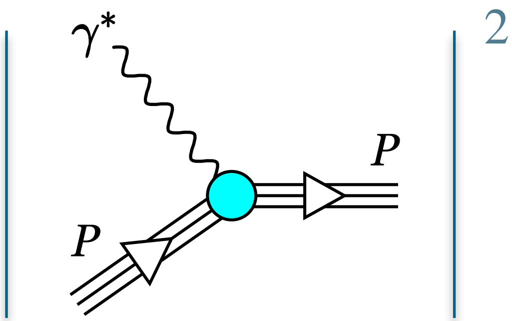
$$F_2(x,Q^2) \sim (\sigma_T + \sigma_L)(\nu, Q^2)$$

$$g_1(x,Q^2) \sim [(Q/\nu)\sigma_{LT} + \sigma_{TT}](\nu, Q^2)$$

$$g_2(x,Q^2) \sim [(\nu/Q)\sigma_{LT} - \sigma_{TT}](\nu, Q^2)$$

Structure functions

Elastic $(x = 1)$



$$F_1^{\text{el}}(x,Q^2) = \frac{1}{2}G_M^2(Q^2)\ \delta(1-x)$$

$$F_2^{\text{el}}(x,Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\ \delta(1-x)$$

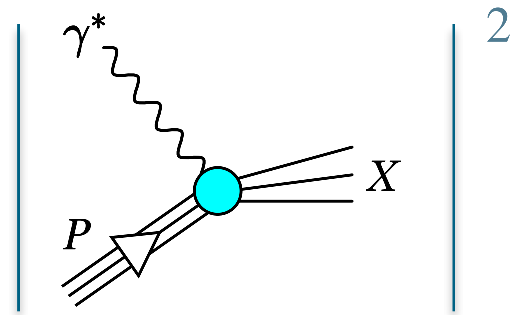
$$g_1^{\text{el}}(x,Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1+\tau)} G_M(Q^2)\ \delta(1-x)$$

$$g_2^{\text{el}}(x,Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1+\tau)} \tau G_M(Q^2)\ \delta(1-x)$$

with G_E electric and G_M magnetic Form Factors,

$\tau = Q^2/4M^2$

Inelastic $(0 < x \leq x_0 < 1)$



$$F_1(x,Q^2) \sim \sigma_T(\nu,Q^2)$$

$$F_2(x,Q^2) \sim (\sigma_T + \sigma_L)(\nu,Q^2)$$

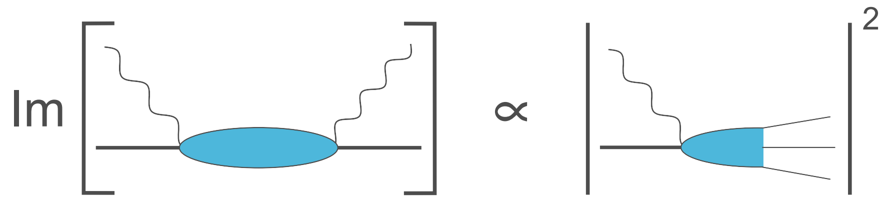
$$g_1(x,Q^2) \sim [(Q/\nu)\sigma_{LT} + \sigma_{TT}](\nu,Q^2)$$

$$g_2(x,Q^2) \sim [(\nu/Q)\sigma_{LT} - \sigma_{TT}](\nu,Q^2)$$

with total photoabsorption cross sections $\sigma(\nu, Q^2)$

Relation to forward Compton scattering

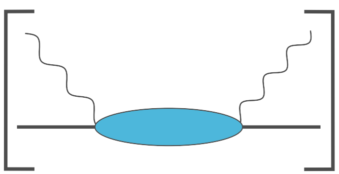
- Optical theorem:



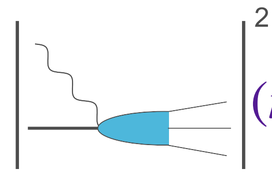
- Causality (analyticity, Cauchy formula):

$$\boxed{f(w) = \frac{1}{2\pi i}\oint_C \frac{f(z)\,dz}{z-w}}$$

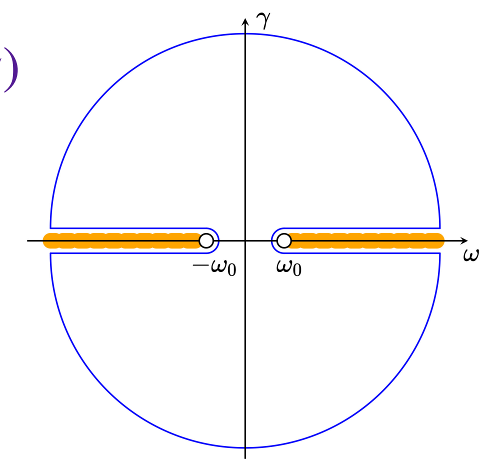
for any interior pt. w of C



$$(\nu, Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu' \, \frac{\nu'}{\nu'^2-\nu^2}$$



$$(\nu', Q^2)$$



Polarizabilities

Sum Rules

eg: GDH, Baldin, Schwinger

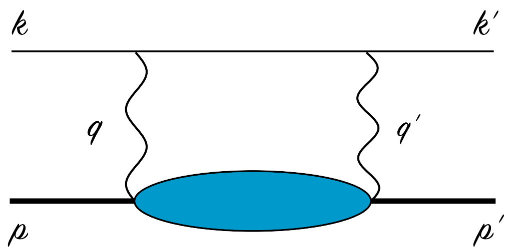
Structure functions

Structure Effects through 2γ

F. Hagelstein, R. Miskimen and V. Pascalutsa, Prog. Part. Nucl. Phys. **88** (2016) 29-97

- proton-structure effects at subleading orders arise through multi-photon processes

off-forward
two-photon exchange (2γ)



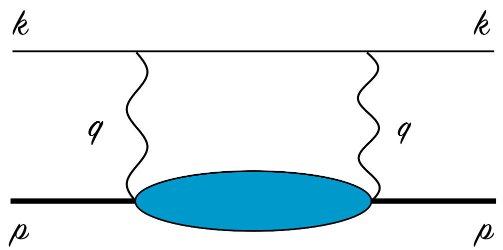
$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\nu, Q^2)$$
$$- \frac{1}{M}\gamma^{\mu\nu\alpha} q_\alpha\, S_1(\nu, Q^2) - \frac{1}{M^2}\left(\gamma^{\mu\nu} q^2 + q^\mu \gamma^{\nu\alpha} q_\alpha - q^\nu \gamma^{\mu\alpha} q_\alpha\right) S_2(\nu, Q^2)$$

Structure Effects through 2γ

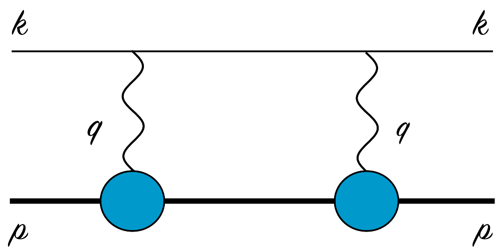
F. Hagelstein, R. Miskimen and V. Pascalutsa, Prog. Part. Nucl. Phys. **88** (2016) 29-97

- proton-structure effects at subleading orders arise through multi-photon processes

forward
two-photon exchange (2γ)



polarizability contribution



elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

CS Amplitudes & Structure Functions

<u>optical theorem:</u>
unitarity

$$\boxed{\mathrm{Im}\, T_1(\nu, Q^2)} = \frac{4\pi^2 Z^2 \alpha}{M} f_1(x, Q^2)$$

$$\mathrm{Im}\, T_2(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha}{\nu} f_2(x, Q^2)$$

$$\mathrm{Im}\, S_1(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha}{\nu} g_1(x, Q^2)$$

$$\mathrm{Im}\, S_2(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha M}{\nu^2} g_2(x, Q^2)$$

+

<u>dispersion relations:</u>
analyticity, crossing symmetries

$$T_i(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \boxed{\mathrm{Im}\, T_i(\nu', Q^2)}}{\nu'^{\,2} - \nu^2 - i0^+}$$

$$S_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \, \mathrm{Im}\, S_1(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+}$$

$$\nu S_2(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^{\,2} \, \mathrm{Im}\, S_2(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+}$$

$$T_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \, \mathrm{Im}\, T_1(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+} = \frac{8\pi Z^2 \alpha}{M} \int_0^1 \frac{\mathrm{d}x}{x} \, \frac{f_1(x, Q^2)}{1 - x^2 (\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$T_2(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \, \mathrm{Im}\, T_2(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+} = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 \mathrm{d}x \, \frac{f_2(x, Q^2)}{1 - x^2 (\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$S_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \, \mathrm{Im}\, S_1(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+} = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 \mathrm{d}x \, \frac{g_1(x, Q^2)}{1 - x^2 (\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$\nu S_2(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^2 \, \mathrm{Im}\, S_2(\nu', Q^2)}{\nu'^{\,2} - \nu^2 - i0^+} = \frac{16\pi Z^2 \alpha M^2}{Q^2} \int_0^1 \mathrm{d}x \, \frac{g_2(x, Q^2)}{1 - x^2 (\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

with
$\nu_{\mathrm{el}} = Q^2/2M$

2γ in Lamb Shift

wave function at the origin

$$\boxed{\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}}$$

<u>dispersion relation & optical theorem:</u>

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

2γ in Lamb Shift

wave function at the origin

$$\boxed{\Delta E(nS) = 8\pi\alpha m\,\phi_n^2\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}}$$

dispersion relation & optical theorem:

$$T_1(\nu,Q^2) = \boxed{T_1(0,Q^2)} + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

- *data-driven* dispersive calculations:

low-energy expansion:

$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior:

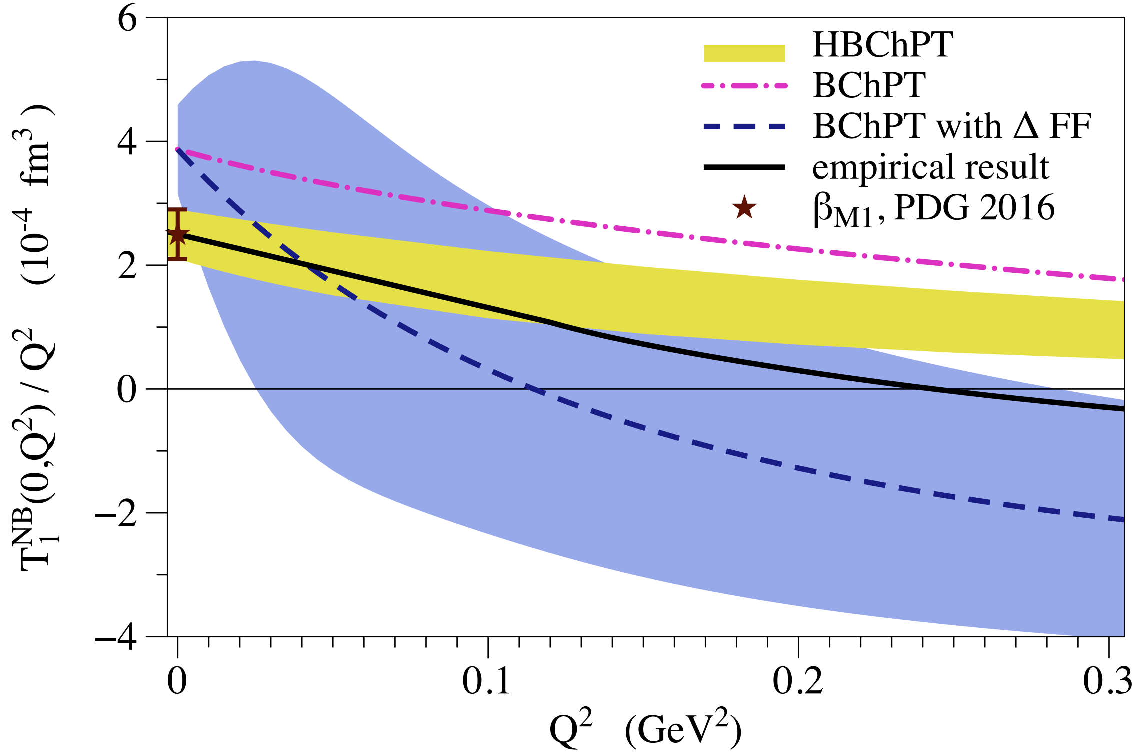
$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\,Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

Caution:
in the dispersive approach
the $T_1(0,Q^2)$ subtraction function
is modelled!

Pinning down the subtraction function

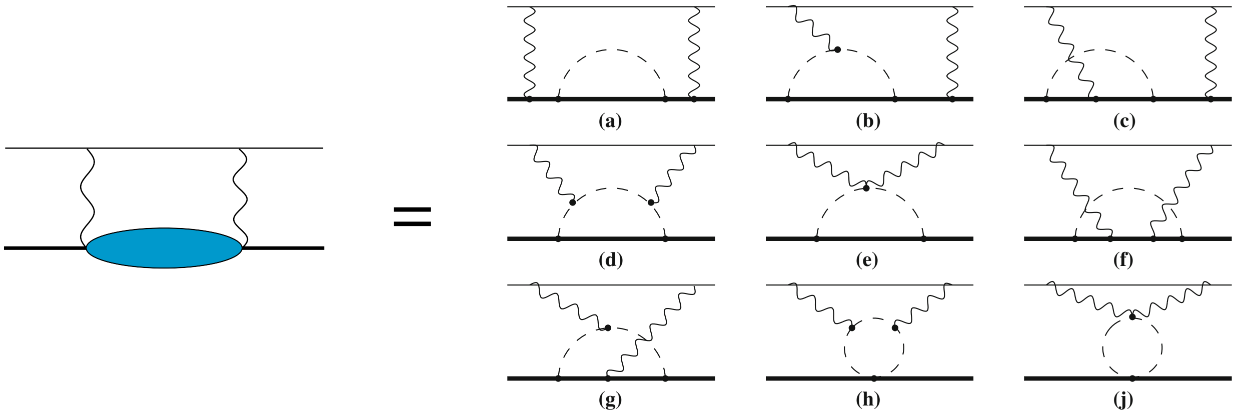
$$T_1^{NB}(0,Q^2) = \beta_{M1}\,Q^2 + \left(\frac{1}{6}\beta_{M2} + 2\beta'_{M1} + \alpha_{\rm em} b_{3,0} + \frac{1}{(2M)^2}\beta_{M1}\right) Q^4 + \mathcal{O}(Q^6)$$

Subtraction function: Q² dependence



Lensky,Hagelstein,VP & Vdh (2018)

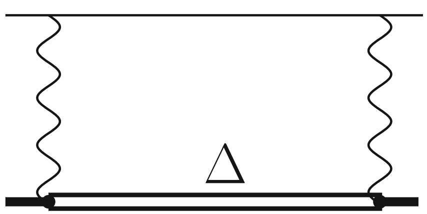
Chiral Perturbation Theory of the Lamb Shift



J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

- LO BChPT prediction:

$$E_{\mathrm{LS}}^{\langle \mathrm{LO}\rangle\ \mathrm{pol.}}(\mu\mathrm{H}) = 8^{+3}_{-1}\ \mu\mathrm{eV}$$



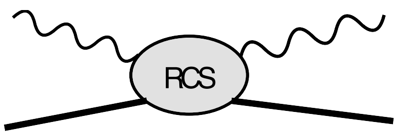
V. Lensky, F. Hagelstein, V. Pascalutsa,
M. Vanderhaeghen, Phys. Rev. D **97** (2018) 074012

- Δ prediction:

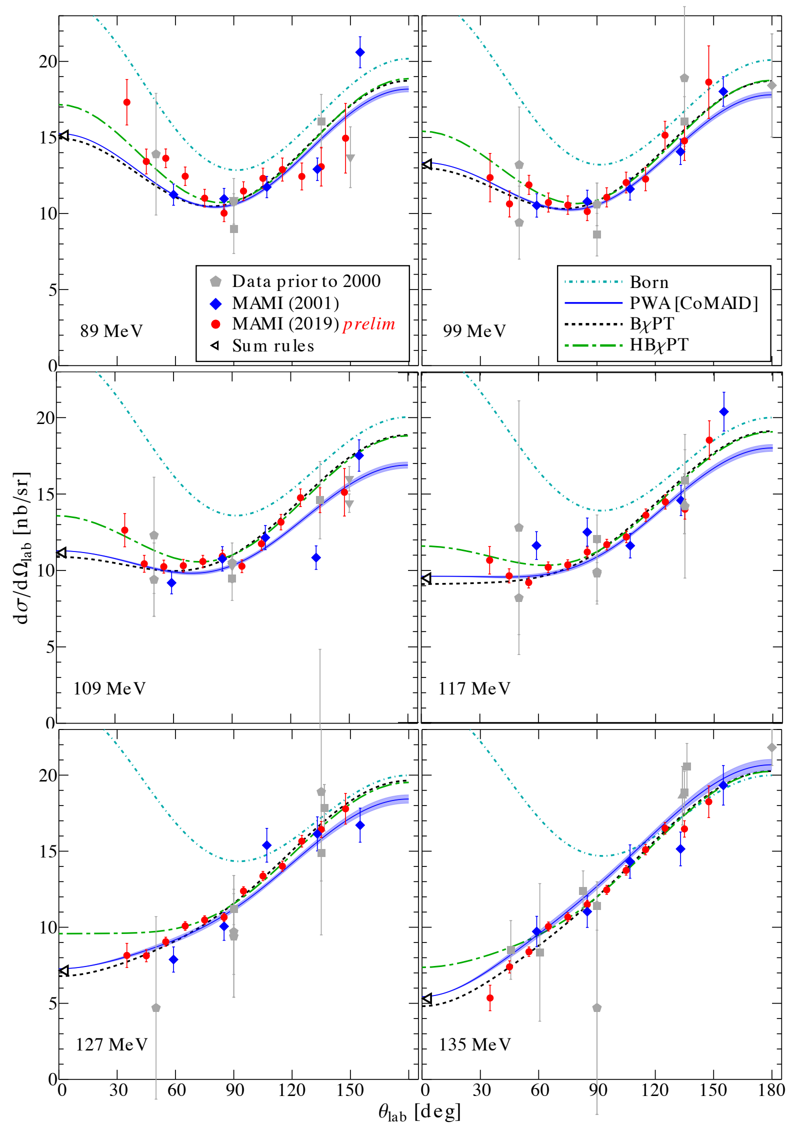
$$E_{\mathrm{LS}}^{\langle \Delta\text{-exch.}\rangle\ \mathrm{pol.}}(\mu\mathrm{H}) = -1.0(1.0)\ \mu\mathrm{eV}$$

New data on proton Compton scattering

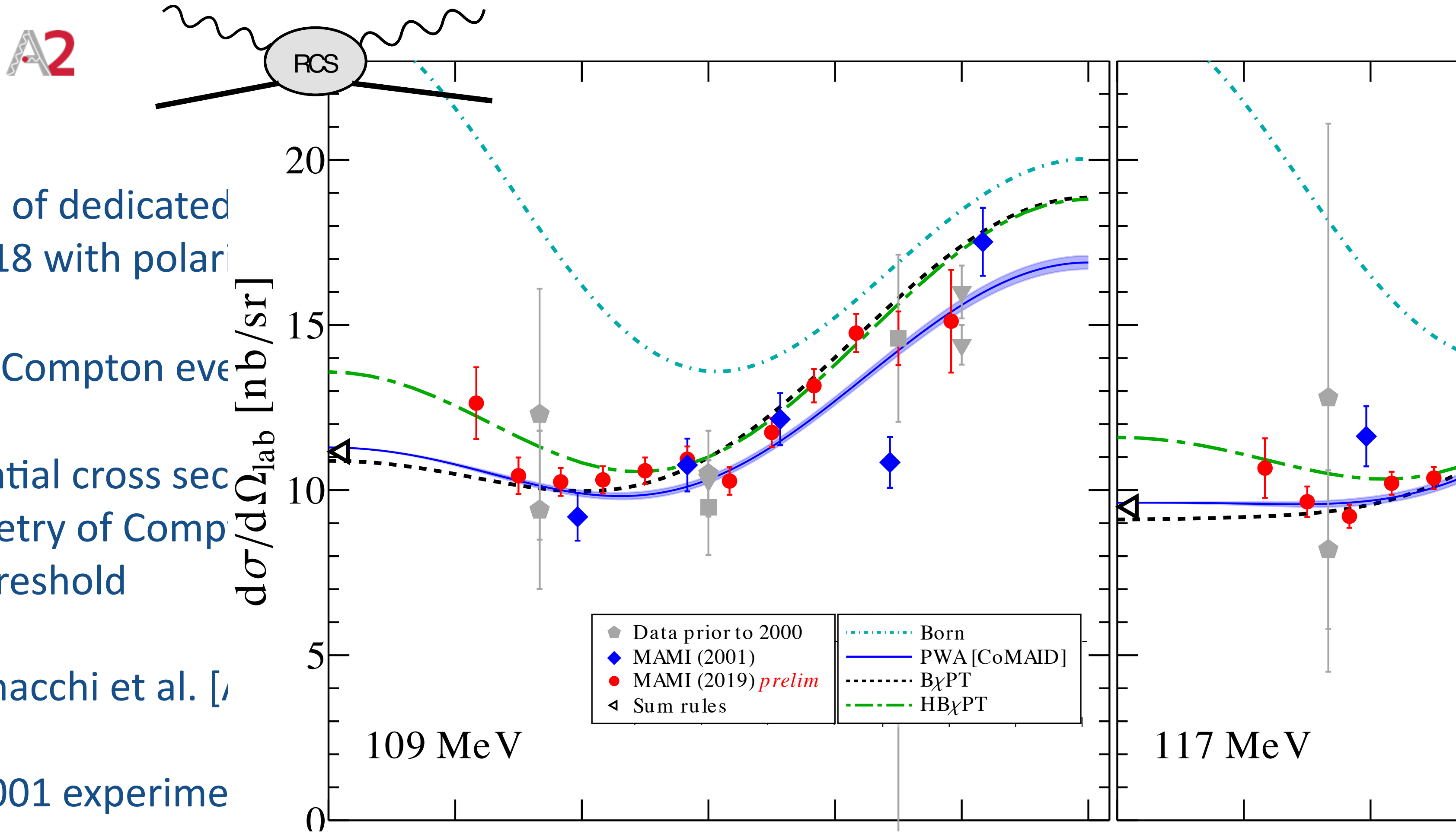
A2



- 60 days of dedicated data-taking in 2017 and 2018 with polarized beam
- $> 10^6$ Compton events, tagger upgrade
- differential cross sections and beam asymmetry of Compton scattering below pion threshold
- E. Mornacchi et al. [A2 Coll.] **(red points)**
- TAPS 2001 experiment **(blue points)**



New data on proton Compton scattering



- 60 days of dedicated
  and 2018 with polari
- $> 10^6$ Compton eve
- differential cross sec
  asymmetry of Comp
  pion threshold
- E. Mornacchi et al. [
- TAPS 2001 experime

2γ (in Lamb Shift) from ChEFT

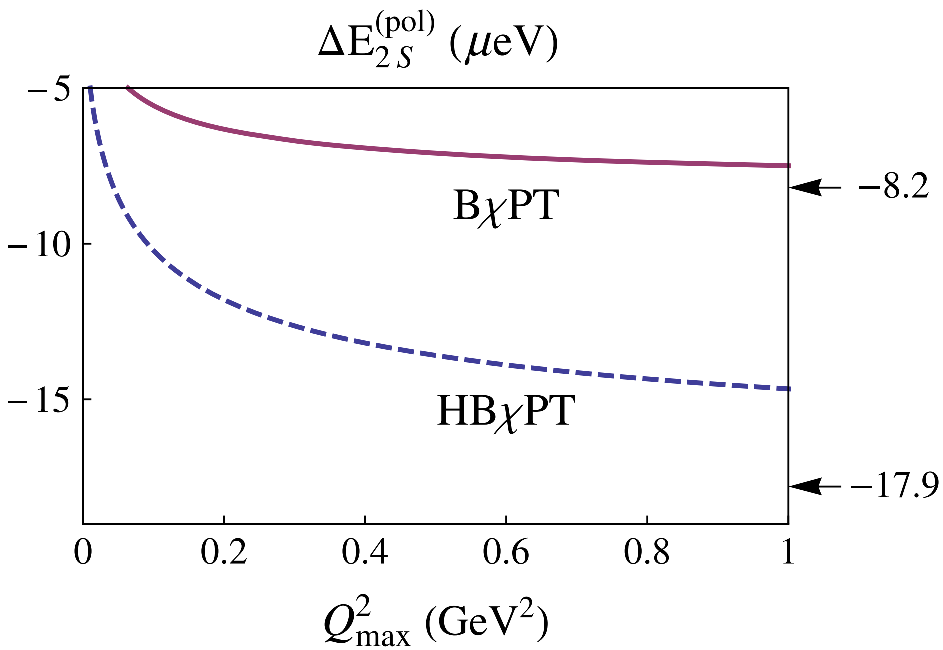


Fig. 4 The polarizability effect on the 2S-level shift in μH computed in HBχPT and BχPT as a function of the ultraviolet cutoff Q_{max}. The *arrows* on the *right* indicate the asymptotic ($Q_{max} \to \infty$) values

J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

- contributions from above $Q_{max} > m_\rho = 775$ MeV
  - HBChPT: at least 25%
  - BCHPT: less than 15%

Assuming BChPT is working, it should be best applicable to atomic systems, where the energies are very small !

2γ in HFS

$$\frac{E_{\mathrm{HFS}}(nS)}{E_F(nS)} = \frac{4m}{1+\kappa}\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\frac{1}{Q^4-4m^2\nu^2}\left\{\frac{(2Q^2-\nu^2)}{Q^2}S_1(\nu,Q^2)+\frac{3\nu}{M}S_2(\nu,Q^2)\right\}$$

with $\nu_{\mathrm{el}} = Q^2/2M$

$$S_1(\nu,Q^2) = S_1^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{2\pi\alpha}{M}F_2^2(Q^2) + \frac{16\pi\alpha M}{Q^2}\int_0^{x_0}\mathrm{d}x\,\frac{g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_1}$$

$$\nu S_2(\nu,Q^2) = \nu S_2^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{64\pi\alpha M^4\nu^2}{Q^6}\int_0^{x_0}\mathrm{d}x\,\frac{x^2 g_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_2}$$

Δ_1, Δ_2 → Δ_{Pol}

using dispersion relation & optical theorem

- (non-Born) polarizability + (Born) elastic 2γ contributions
- S_1 and S_2 fulfil unsubtracted dispersion relations

Polarizability Effect on the HFS

with $v = \sqrt{1 + {}^{1}\!/\!_{\tau}}, v_l = \sqrt{1 + {}^{1}\!/\!_{\tau_l}}, \tau_l = {}^{Q^2}\!/\!_{4m^2}$ and $\tau = {}^{Q^2}\!/\!_{4M^2}$

$$\Delta_{\mathrm{pol}} = \frac{\alpha m}{2\pi(1+\kappa)M}\left[\Delta_1 + \Delta_2\right]$$

$$\Delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\frac{5+4v_l}{(v_l+1)^2}\boxed{\left[4I_1(Q^2) + F_2^2(Q^2)\right]} - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\left\{\frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)}\left[4 + \frac{1}{1+\sqrt{1+x^2\tau^{-1}}} + \frac{1}{v_l+1}\right]\right\}\right)$$

$$\Delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left\{\frac{1}{v_l+\sqrt{1+x^2\tau^{-1}}} - \frac{1}{v_l+1}\right\}$$

$$I_1(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0}\mathrm{d}x\, g_1(x,Q^2)$$

$$I_1^{\mathrm{non-pol}}(Q^2) = I_A^{\mathrm{non-pol}}(Q^2) = -\frac{1}{4}F_2^2(Q^2)$$

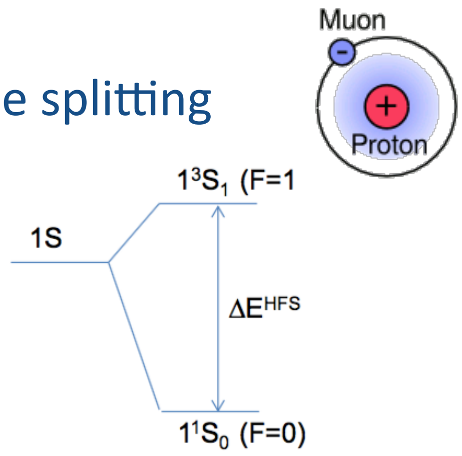
$I_1(Q^2)$ is not a pure polarizability

- proton-polarizability effect on the HFS is completely *constrained by empirical information*
- a ChPT calculation will put the reliability of dispersive calculations (and ChPT) to the test

Hyperfine splitting in muonic H

- spin polarizabilities are of similar importance for the hyperfine splitting

Measurements of the μH ground-state HFS planned by
1) CREMA, 2) FAMU, 3) J-PARC/Riken-RAL collaborations

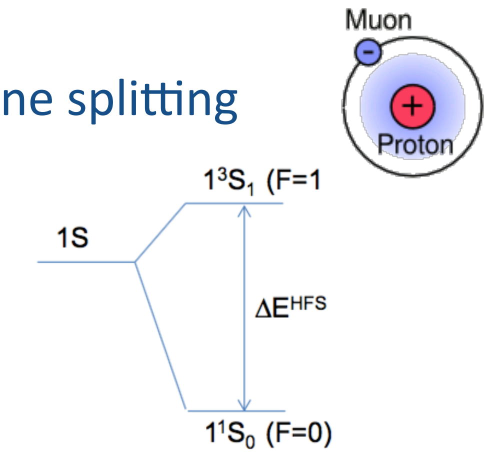


F : total angular momentum

Hyperfine splitting in muonic H

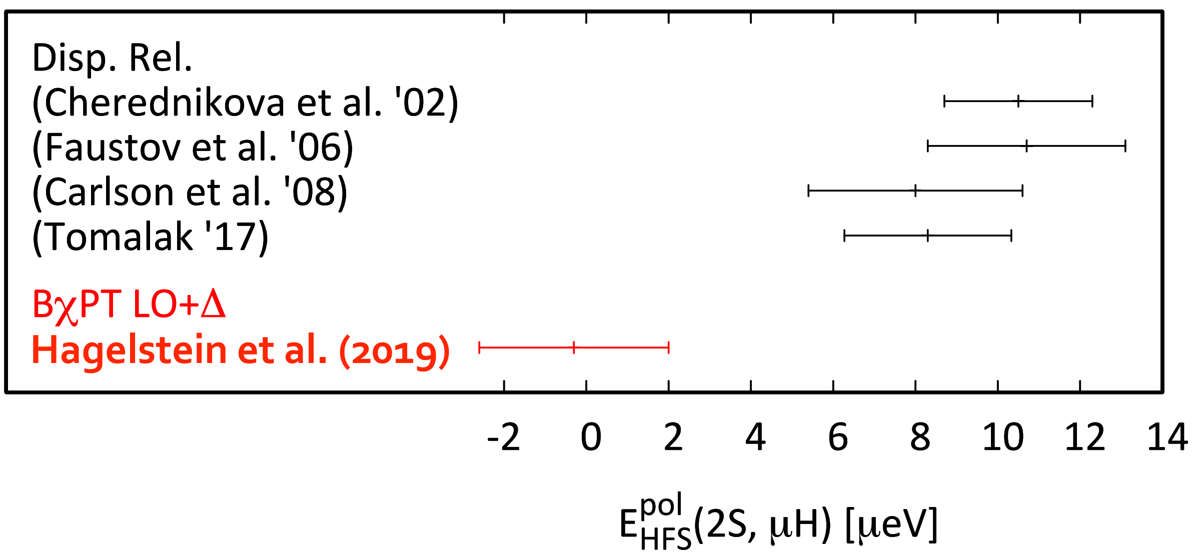
- spin polarizabilities are of similar importance for the hyperfine splitting

Measurements of the μH ground-state HFS planned by
1) CREMA, 2) FAMU, 3) J-PARC/Riken-RAL collaborations



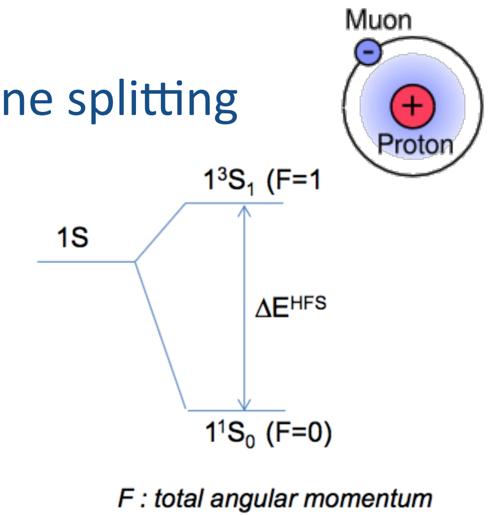
F : total angular momentum

currently, disagreement between data-driven evaluations and chiral perturbation theory



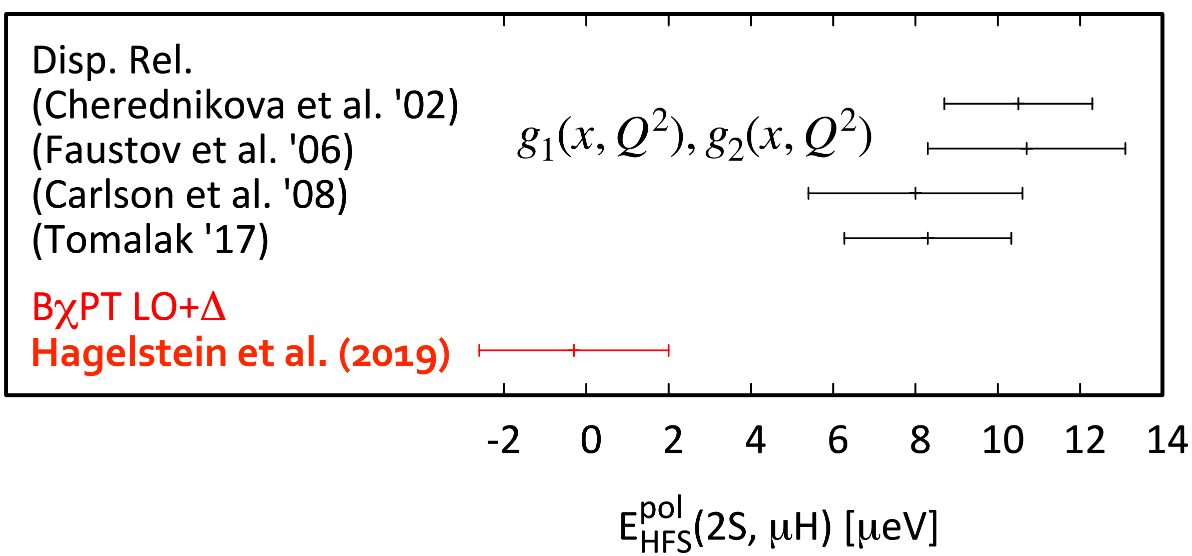
Hyperfine splitting in muonic H

- spin polarizabilities are of similar importance for the hyperfine splitting



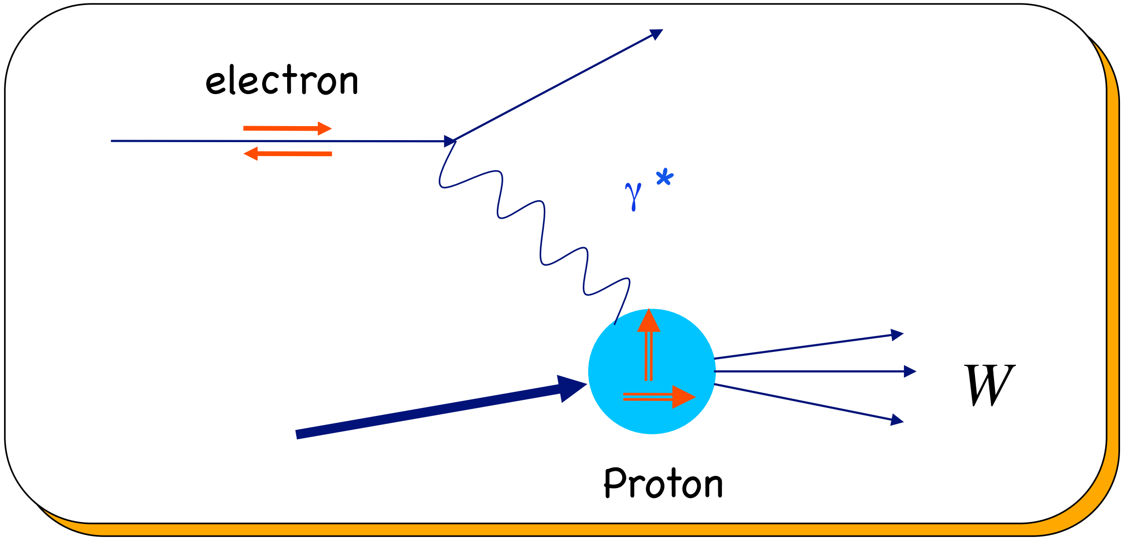
Measurements of the μH ground-state HFS planned by
1) CREMA, 2) FAMU, 3) J-PARC/Riken-RAL collaborations

currently, disagreement between data-driven evaluations and chiral perturbation theory



- Questions to empirical parametrizations of nucleon structure functions (next slides)

Inclusive cross section



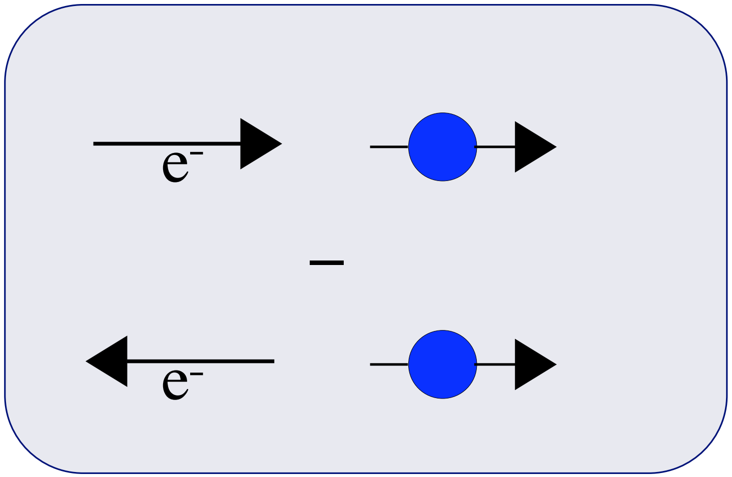
When we add spin degrees of freedom to the target and beam, 2 Additonal SF needed.

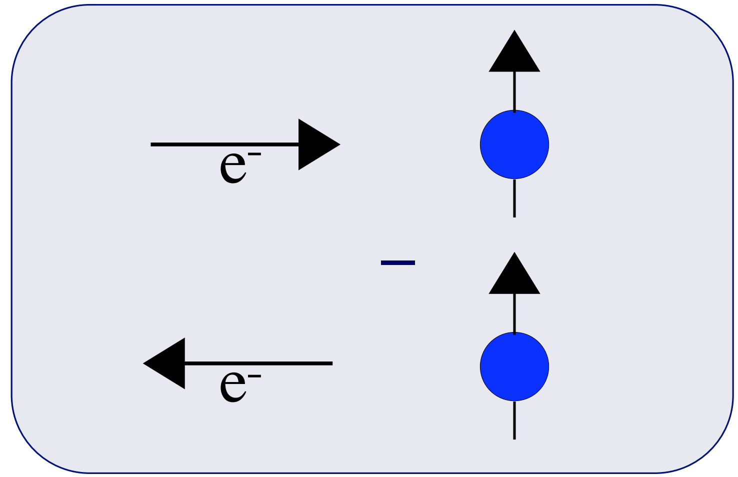
$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right]$$

$$+\gamma\, g_1(x,Q^2) + \delta\, g_2(x,Q^2)$$

Inclusive **Polarized** Cross Section

Access polarized structure functions



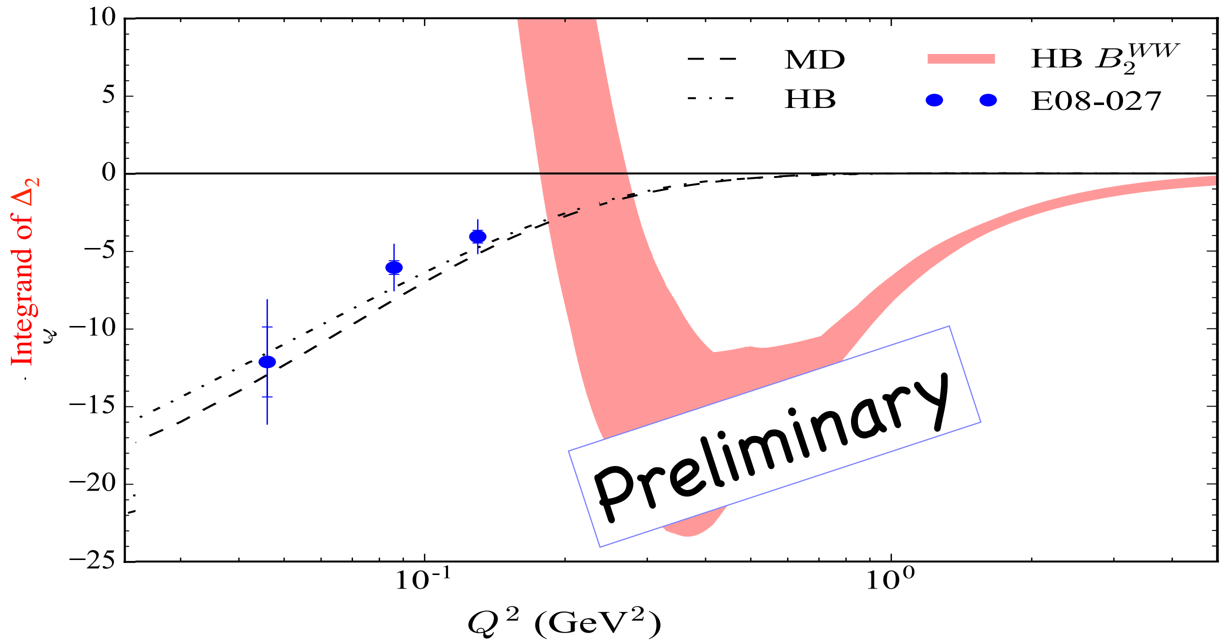


$$\frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\left[(E + E'\cos\theta)\, g_1 - 2Mx g_2\right]$$

$$\frac{d^2\sigma^{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\sin\theta\left[g_1 + \frac{2ME}{\nu} g_2\right]$$

Results from Hall A @ JLab [K. Slifer et al]

g_2 contribution to Hyperfine Structure



good agreement with the MAID and *most recent* Hall B models

200% difference from Hall B 2007 model used in PRA78, 02251

- Carlson, Nazaryan & Griffioen (2008, 2011)

Results from Hall A @ JLab [K. Slifer et al]

- How do new models compare with previous publications?

Term	Q^2 (GeV^2)	MAID	Hall B	HB 2007
Δ_2	(0,0.05)	-0.87	-0.80	-0.23
	(0.05,20)	-1.26	-1.16	-0.33
	(20,∞)	0.00	0.00	0.00
Total Δ_2		-2.13	-1.96	-0.56

Phys.Rev.A.78.022517

Carlson, Nazaryan & Griffioen (2008, 2011)

Results from Hall A @ JLab [K. Slifer et al]

- How do new models compare with previous publications?

Term	Q^2 (GeV^2)	MAID	Hall B	HB 2007
Δ_2	(0,0.05)	-0.87	-0.80	-0.23
	(0.05,20)	-1.26	-1.16	-0.33
	(20,∞)	0.00	0.00	0.00
Total Δ_2		-2.13	-1.96	-0.56

Phys.Rev.A.78.022517

Carlson, Nazaryan & Griffioen (2008, 2011)

- bigger cancellation between Δ_1 and Δ_2, hence better agreement with ChPT.

Conclusions on TPE

Conclusions on TPE

- Lepton scattering: model-dependent, for review see **Pasquini & Vanderhaeghen, Ann.Rev.Nucl.Part.Sci (2018)**

- Hydrogen spectroscopy: dispersive calculations vs. ChPT agree in the Lamb shift and might soon agree in HFS — *stay tuned!*

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_F(nS)$$

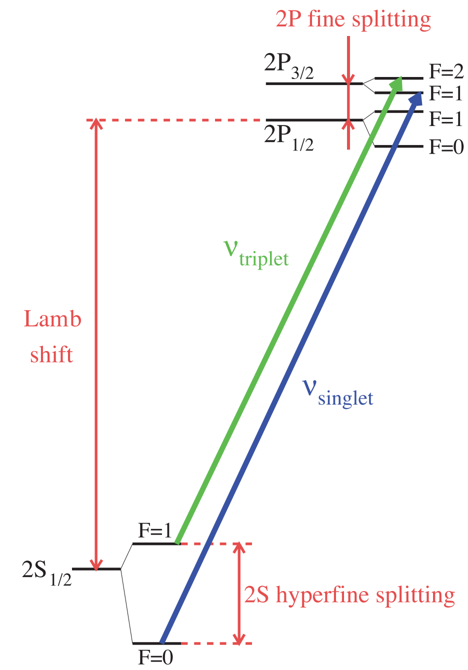
with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \boxed{\Delta_{\mathrm{pol}}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\,\mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

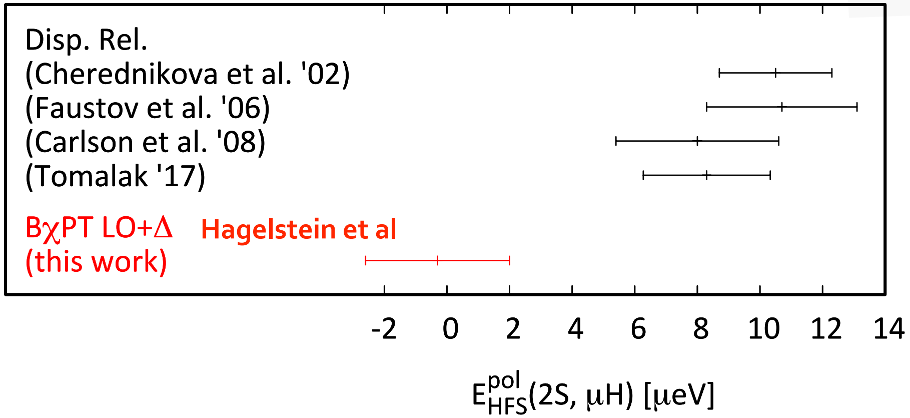
- Very precise input for the 2γ polarizability effect needed to find the μH ground-state HFS transition in experiment
- Zemach radius involves magnetic properties of the proton

HFS theory status

$$\Delta E_{\rm HFS}(1S) = [1 + \Delta_{\rm QED} + \Delta_{\rm weak+hVP} + \underbrace{\Delta_{\rm Zemach} + \Delta_{\rm recoil} + \Delta_{\rm pol}}_{\Delta_{\rm TPE}}]\Delta E_0^{\rm HFS}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

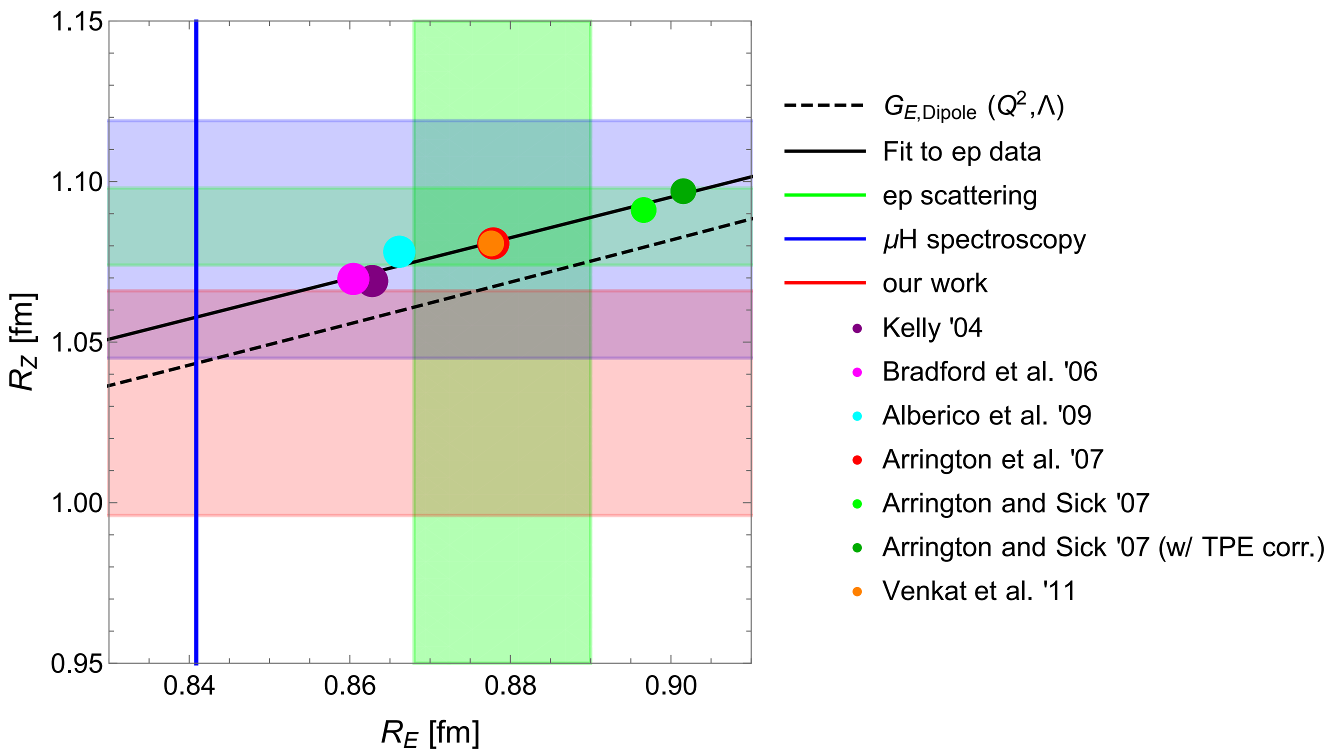
	μp		
	Magnitude	Uncertainty	
$\Delta E_0^{\rm HFS}$	182.443 meV	0.1×10^{-6}	
$\Delta_{\rm QED}$	1.1×10^{-3}	1×10^{-6}	
$\Delta_{\rm weak+hVP}$	2×10^{-5}	2×10^{-6}	
$\Delta_{\rm Zemach}$	7.5×10^{-3}	7.5×10^{-5}	$\leftarrow G_E(Q^2), G_M(Q^2)$
$\Delta_{\rm recoil}$	1.7×10^{-3}	10^{-6}	$\leftarrow G_E, G_M, F_1, F_2$
$\Delta_{\rm pol}$	4.6×10^{-4}	8×10^{-5}	$\leftarrow g_1(x,Q^2), g_2(x,Q^2)$



Polarizability correction is fully expressed in terms of spin structure functions (no subtractions),
yet their poor knowledge leads disagreement with ChPT !

Zemach radius vs the rms charge radius

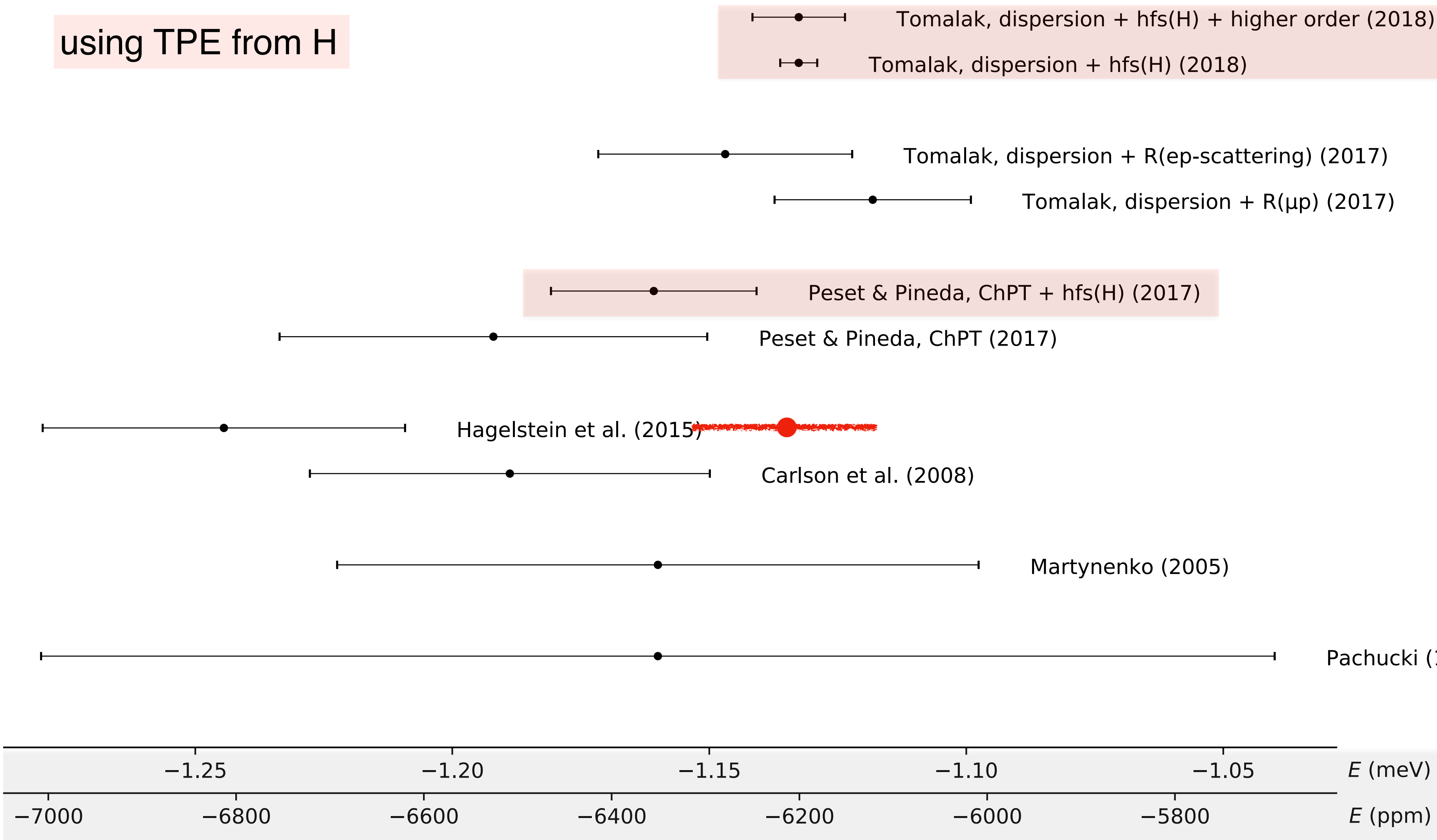
Hagelstein et al, in prep.

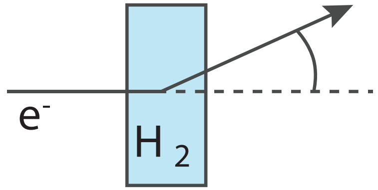


An extraction of Zemach radius from muonic H hfs should be consistent with the charge radius extraction from muH Lamb shift?!

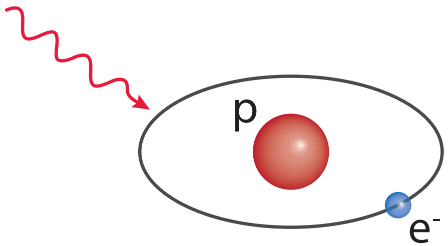
Recent values of the TPE

using TPE from H

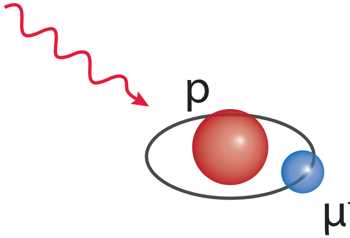




e^--p scattering

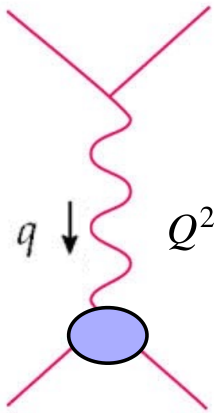


H spectroscopy



μp spectroscopy

Radius from elastic e-p scattering



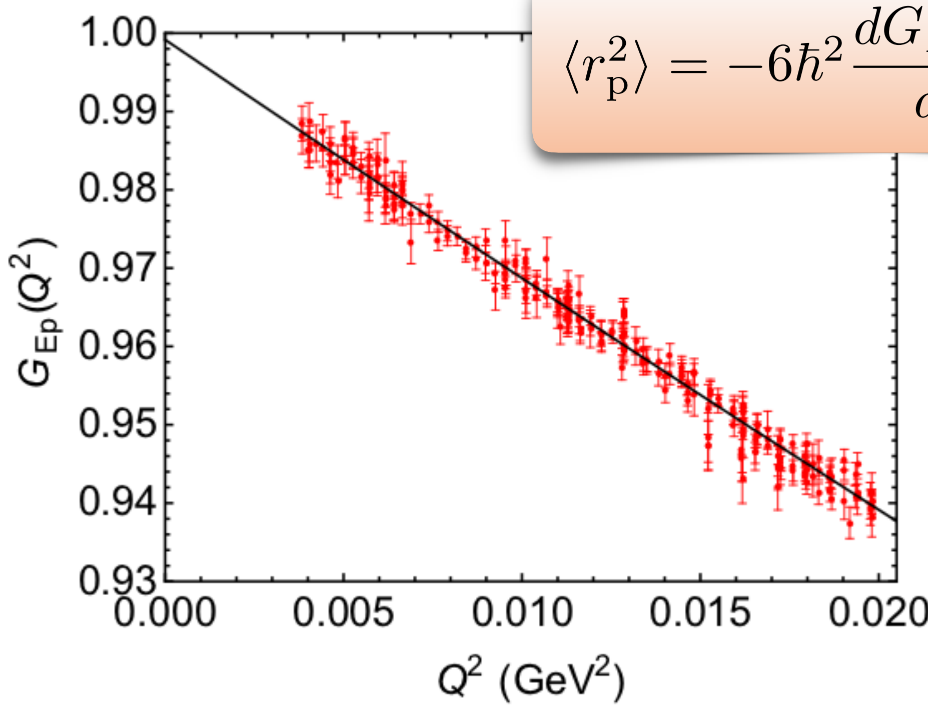
$Q^2 = -q^2 > 0$

$G_E(Q^2), G_M(Q^2)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{(1+\tau)}\left(\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right)$$

with $\tau = Q^2/4M_p^2, \quad \varepsilon \lesssim 1$

$$\langle r_{\rm p}^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{\rm Q^2=0}$$



Caveat:
Radius extraction involves extrapolation to 0

data points: J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).

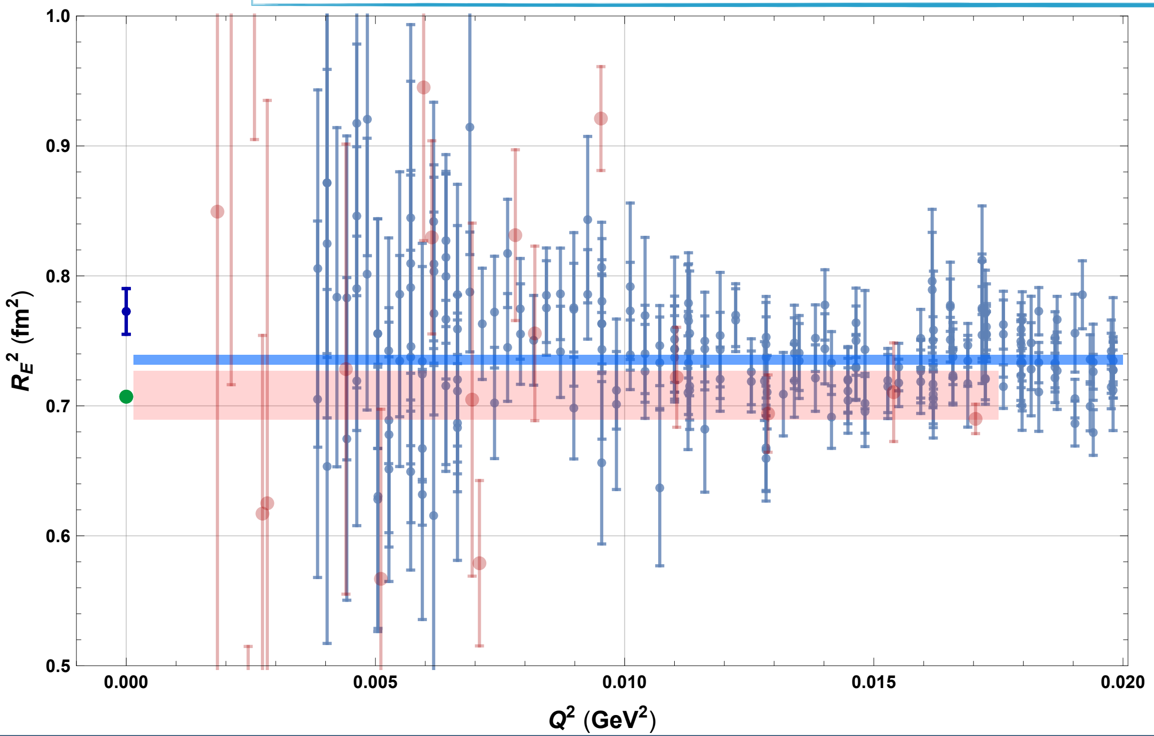
Lower bound directly from e-p data

$$R_E^2(Q^2) = -\frac{6}{Q^2}\log G_E(Q^2)\ _{Q^2=0} \rightarrow R_E^2$$

This function sets a lower bound:

$$\boxed{R_E^2(Q^2) \leq R_E^2\,, \quad \text{for } Q^2 \geq 0}$$

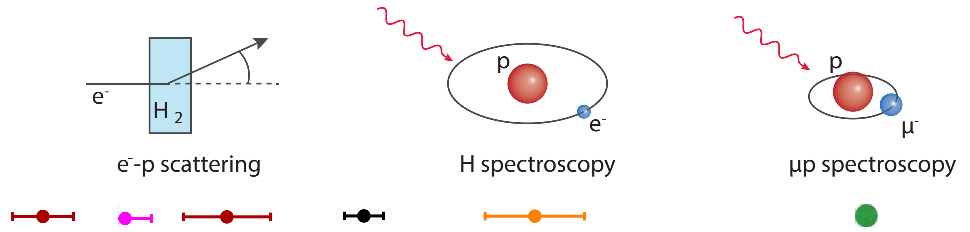
Hagelstein & VP, Phys. Lett. B (2019).

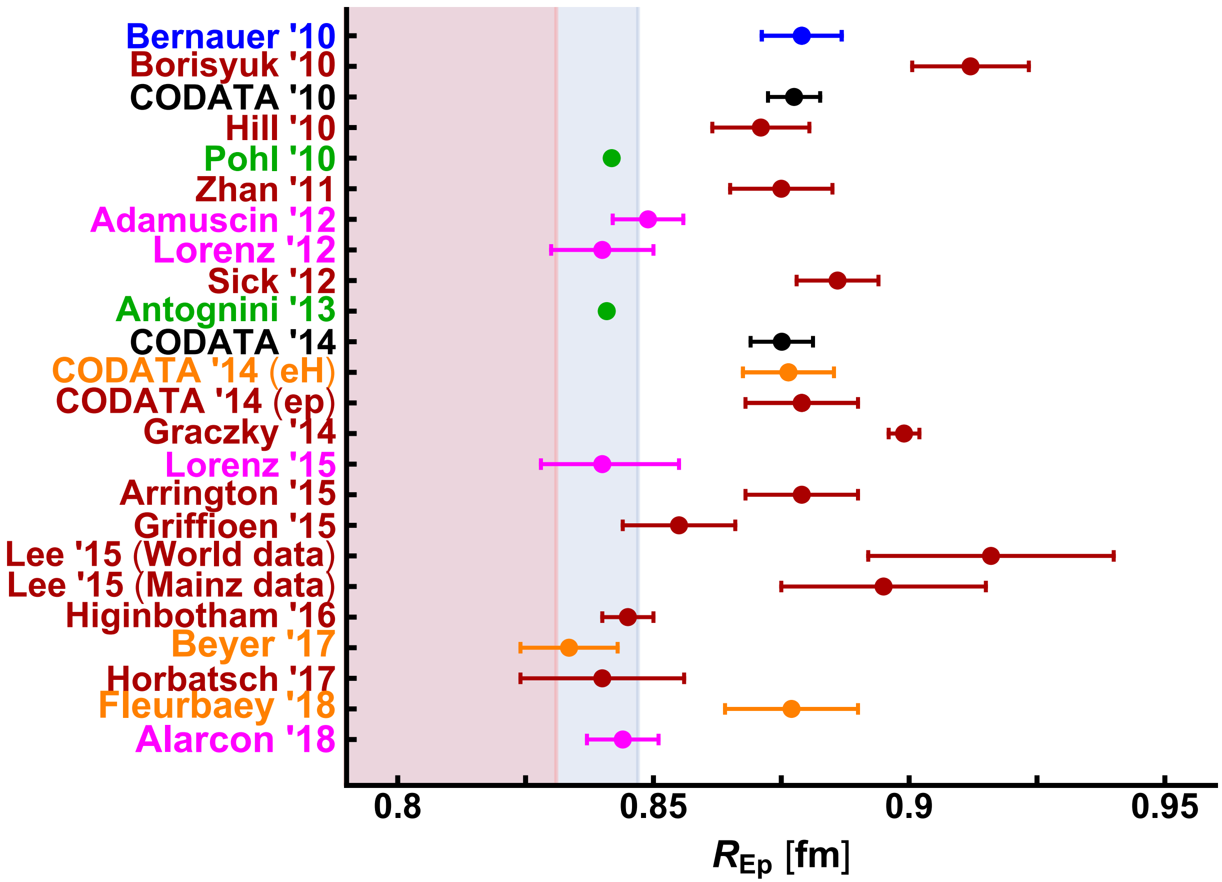


Data points from A1 Coll.:
Bernauer et al (2010)
Mihovilovic et al (2017)

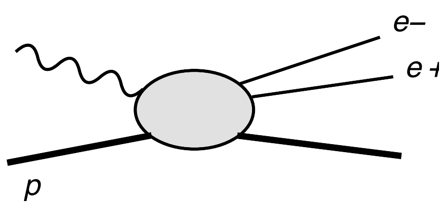
No extrapolation required

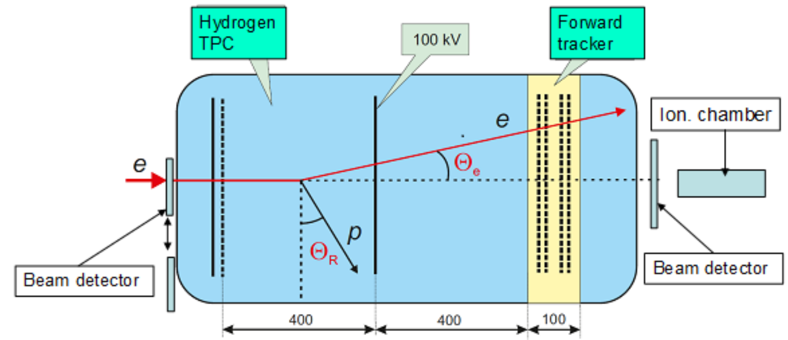
Various extractions





Plans for new proton-radius experiment in A2@MAMI





Measured quantities:

Recoil energy T_R

Recoil angle Θ_R

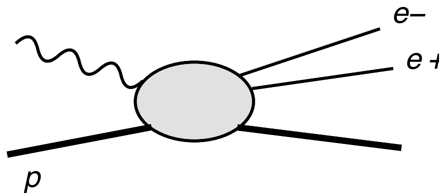
Vertex Z coordinate

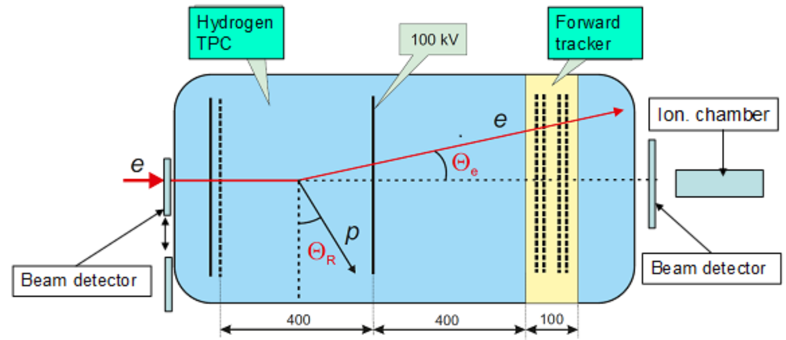
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1 + \frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

Plans for new proton-radius experiment in A2@MAMI





Measured quantities:

Recoil energy T_R

Recoil angle Θ_R

Vertex Z coordinate

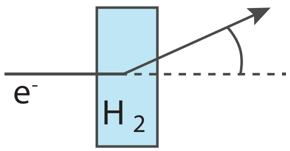
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1+\frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

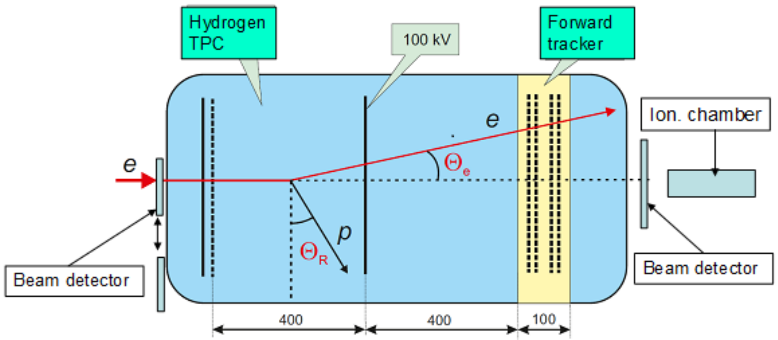
$$-t = 2MT_R$$

- New high-pressure time projection chamber (TPC) is built by the PNPI group (St. Petersburg) for use at A2

Plans for new proton-radius experiment in A2@MAMI



e⁻-p scattering



Measured quantities:
Recoil energy T_R
Recoil angle Θ_R
Vertex Z coordinate
E scattering angle Θ_e

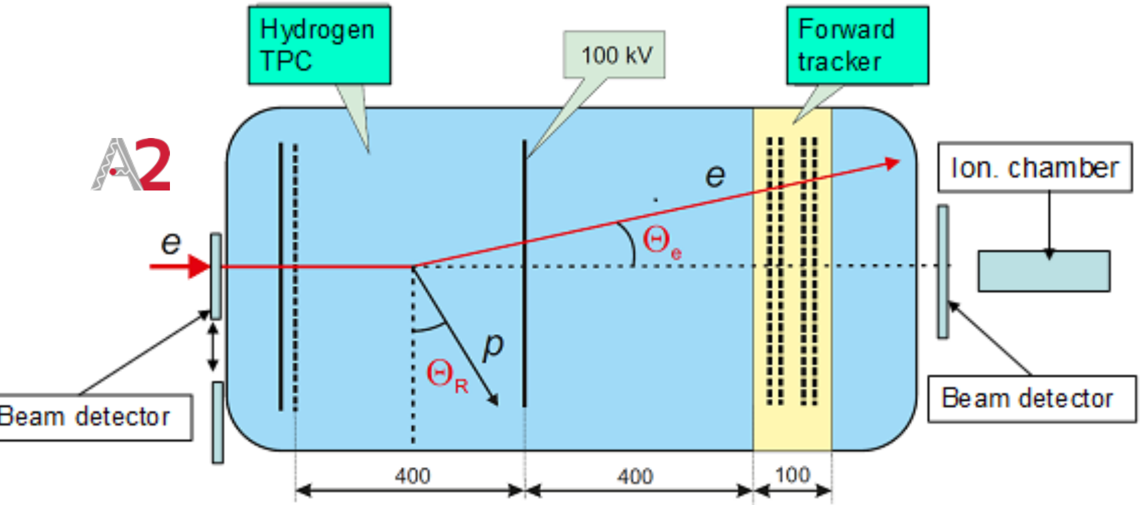
$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1+\frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

- New high-pressure time projection chamber (TPC) is built by the PNPI group (St. Petersburg) for use at A2

- First experiment: elastic *ep* scattering

Plans for new proton-radius experiment in A2@MAMI



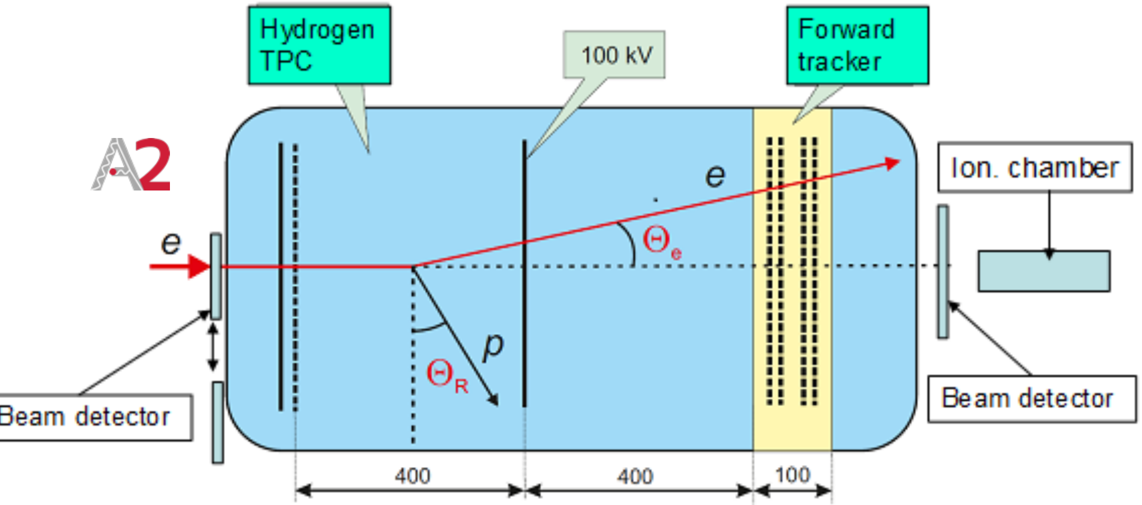
Measured quantities:

Recoil energy T_R
Recoil angle Θ_R
Vertex Z coordinate
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2 \frac{\vartheta}{2}}{1 + \frac{2\varepsilon_e}{M} \sin^2 \frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

Plans for new proton-radius experiment in A2@MAMI



Measured quantities:

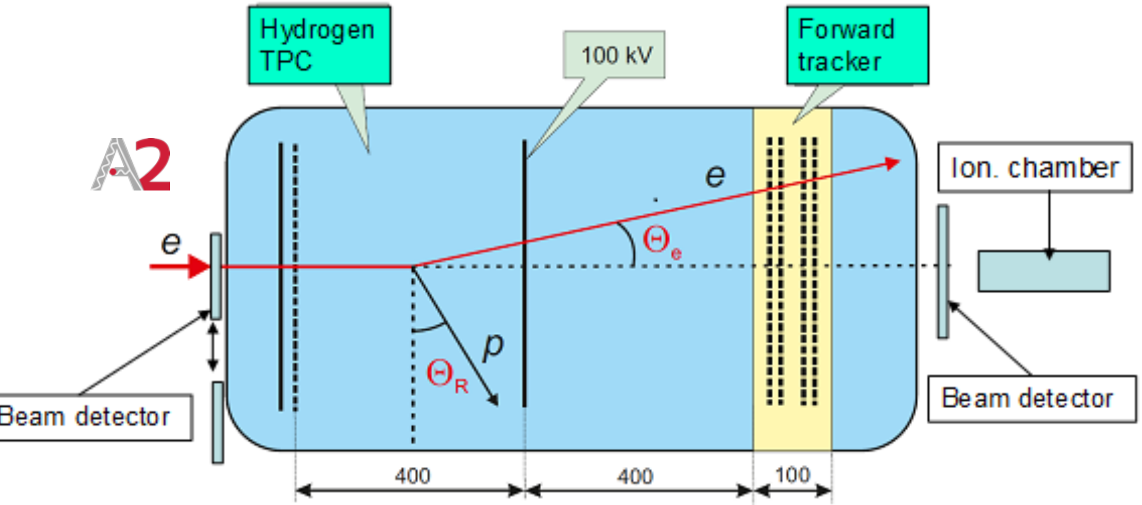
Recoil energy T_R
Recoil angle Θ_R
Vertex Z coordinate
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1+\frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

- Detection of both scattered electron and recoil proton — first "overdetermined kinematics" experiment — reducing systematic uncertainties (radiative corrections).

Plans for new proton-radius experiment in A2@MAMI



Measured quantities:

Recoil energy T_R

Recoil angle Θ_R

Vertex Z coordinate

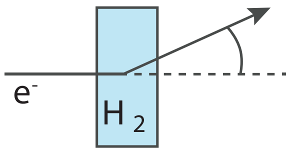
E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2 \frac{\vartheta}{2}}{1 + \frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

- Detection of both scattered electron and recoil proton — first "overdetermined kinematics" experiment — reducing systematic uncertainties (radiative corrections).
- Successful test runs performed.

Plans for new proton-radius experiment in A2@MAMI

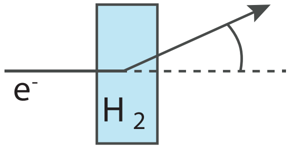


e^--p scattering

- New high-pressure time projection chamber (TPC) is built by the PNPI group (St. Petersburg) for use at A2
- First experiment: elastic *ep* scattering
- Detection of both scattered electron and recoil proton — first "overdetermined kinematics" experiment — reducing systematic uncertainties (radiative corrections).
- Successful test runs performed.

- Assembling and commissioning of the hydrogen TPC in the A2 Hall
- Construction of e-beam line and beam monitoring system.
- Data taking in 2022.
- Obtain form factor data at Q^2 between 0.001 and 0.02 GeV^2.

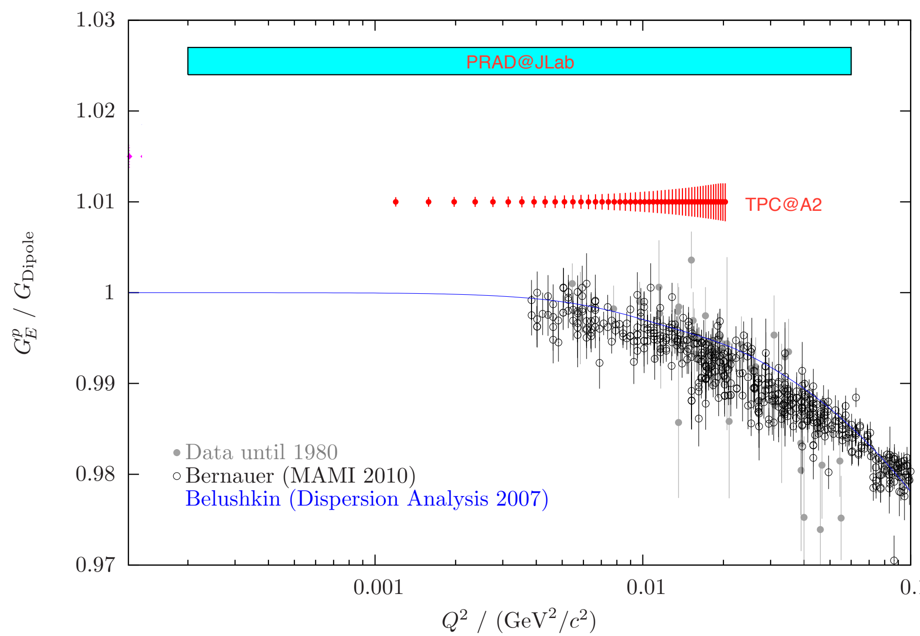
Plans for new proton-radius experiment in A2@MAMI



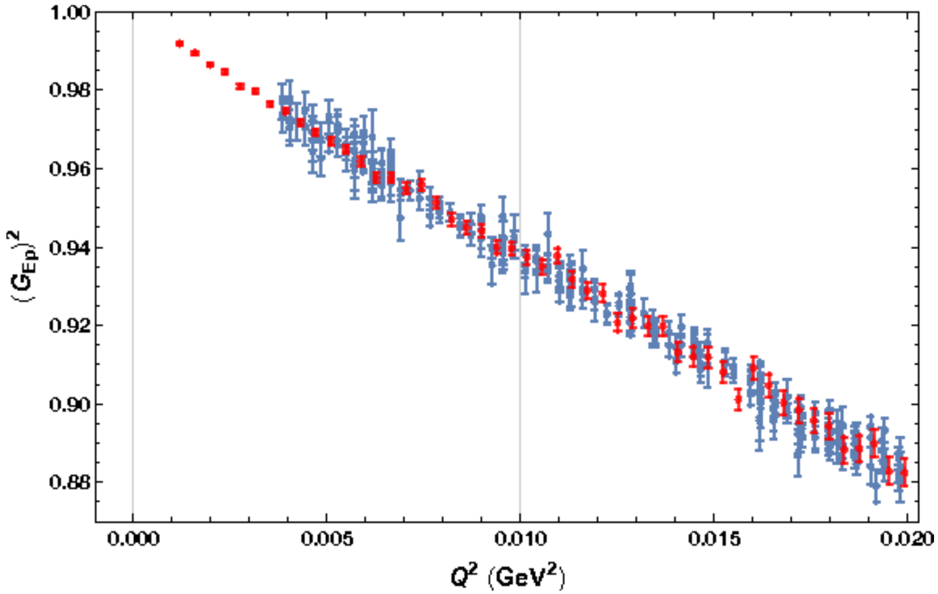
e⁻-p scattering

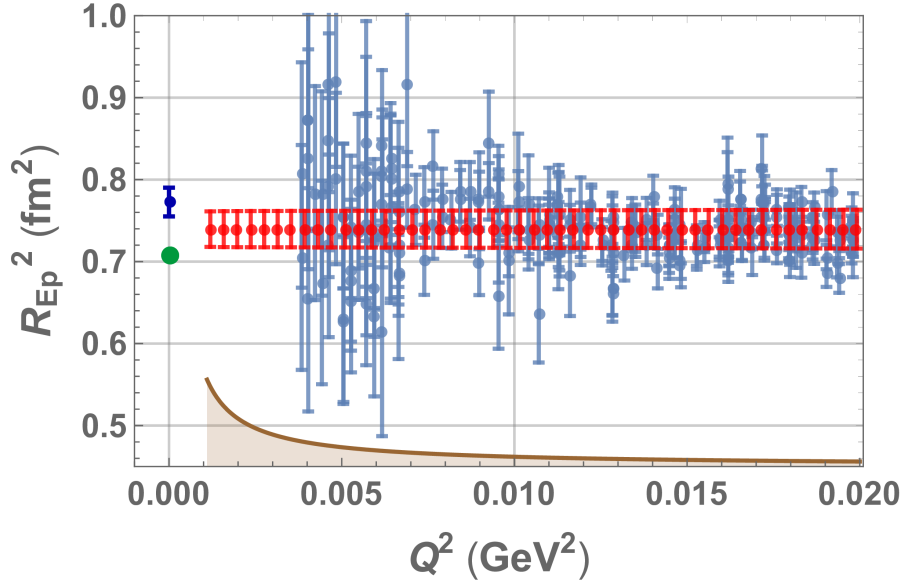
- New high-pressure time projection chamber (TPC) is built by the PNPI group (St. Petersburg) for use at A2
- First experiment: elastic *ep* scattering
- Detection of both scattered electron and recoil proton — first "overdetermined kinematics" experiment — reducing systematic uncertainties (radiative corrections).
- Successful test runs performed.

- Assembling and commissioning of the hydrogen TPC in the A2 Hall
- Construction of e-beam line and beam monitoring system.
- Data taking in 2022.
- Obtain form factor data at Q^2 between 0.001 and 0.02 GeV^2.

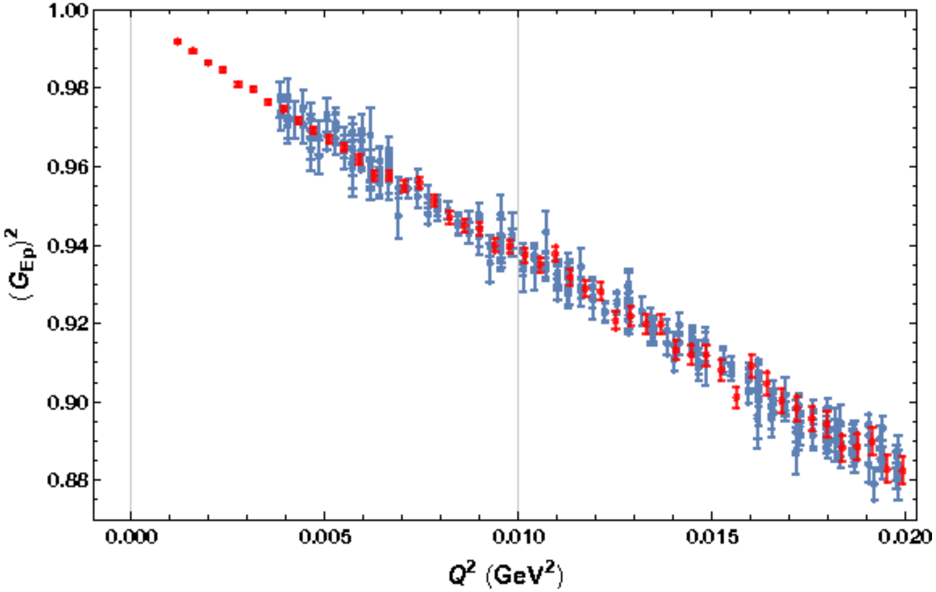


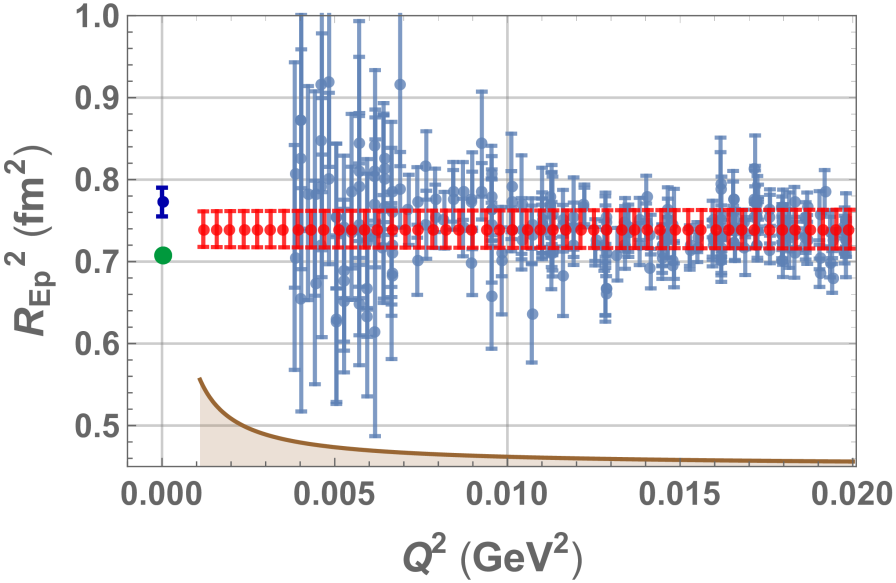
Projected data for TPC@A2 [V. Sokhoyan et al] in comparison with Bernauer et al (A1 Coll)





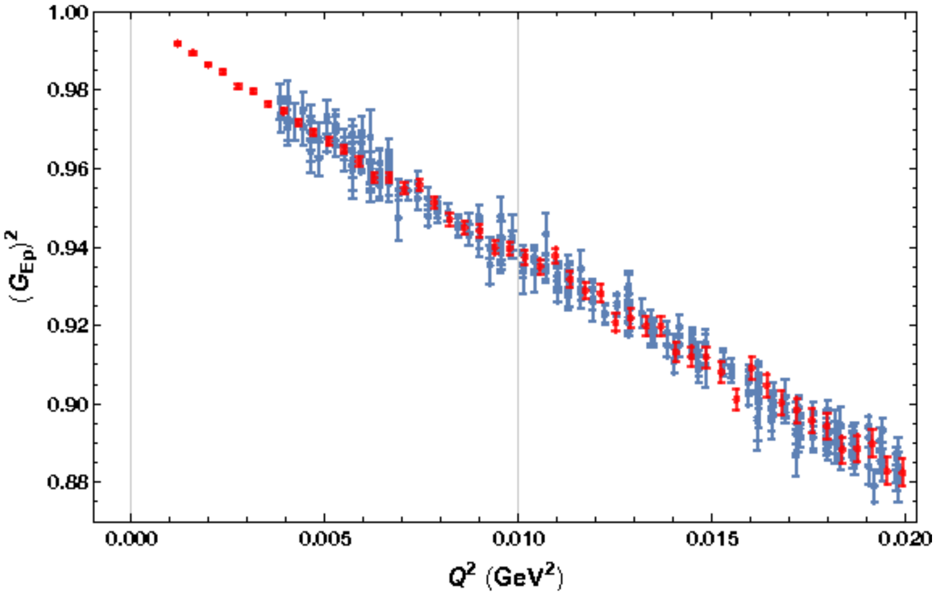
Projected data for TPC@A2 [V. Sokhoyan et al] in comparison with Bernauer et al (A1 Coll)

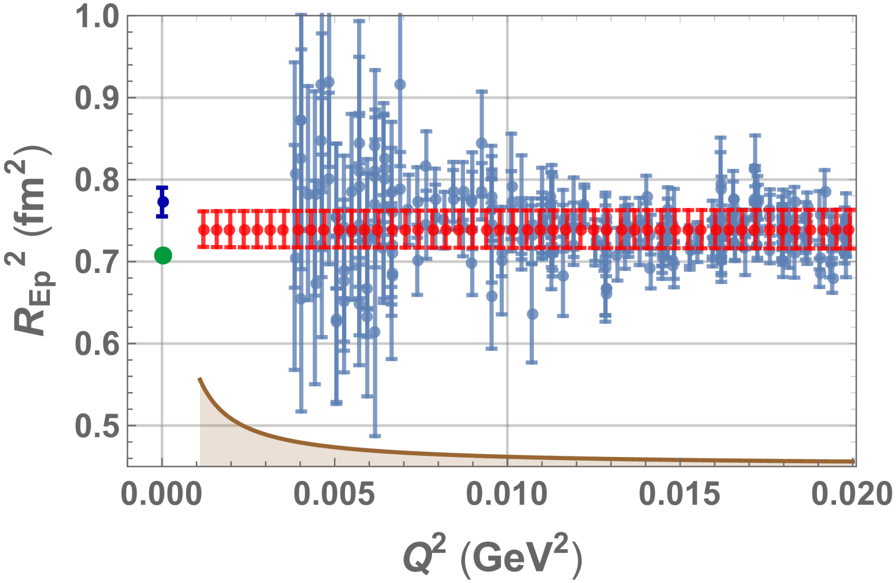




- This is happening within the next funding period of SFB 1044

Projected data for TPC@A2 [V. Sokhoyan et al] in comparison with Bernauer et al (A1 Coll)





- This is happening within the next funding period of SFB 1044

- **Join and/or support!**