Amplitude analyses of meson decays at Belle

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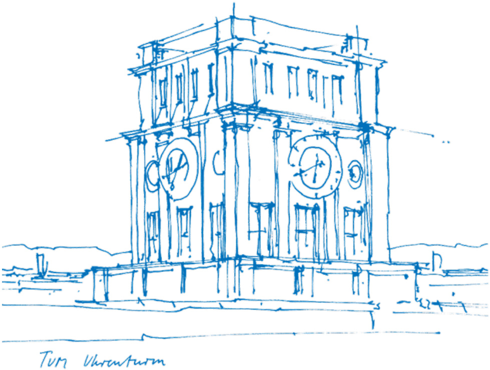

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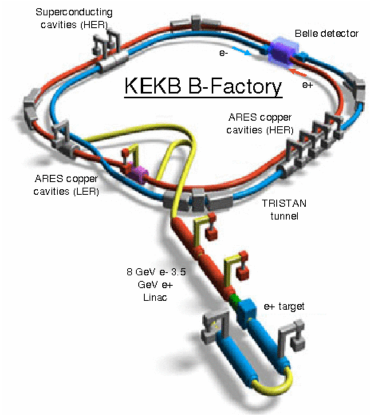

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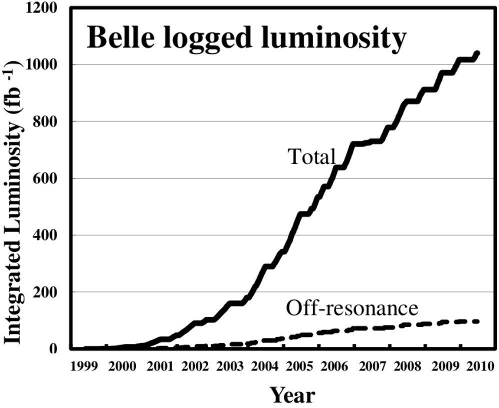
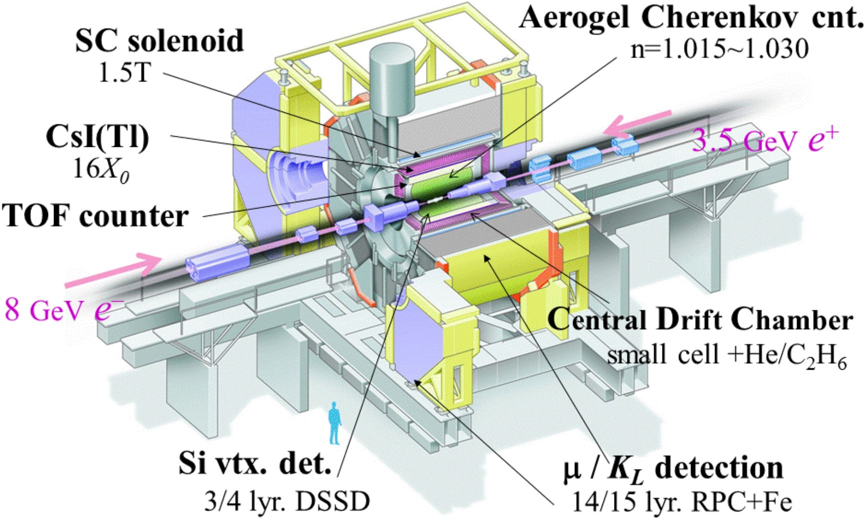
Over a decade of operation, Belle collected approx. $1\,\mathrm{ab}^{-1}$ of integrated luminonsity.



$\Upsilon(4S)$	$711.0\,\mathrm{fb}^{-1}$
Off resonance	$122.4\,\mathrm{fb}^{-1}$
$\Upsilon(5S)$	$121.4\,\mathrm{fb}^{-1}$
$\Upsilon(2S)$	$24.9\,\mathrm{fb}^{-1}$
$\Upsilon(1S)$	$5.7\,\mathrm{fb}^{-1}$
$\Upsilon(3S)$	$2.9\,\mathrm{fb}^{-1}$



SC solenoid
1.5T
CsI(Tl)
$16X_0$
TOF counter
8 GeV e^-
Aerogel Cherenkov cnt.
n=1.015~1.030
3.5 GeV e^+
Central Drift Chamber
small cell +He/C_2H_6
Si vtx. det.
3/4 lyr. DSSD
μ / K_L detection
14/15 lyr. RPC+Fe

I will focus on techniques used rather than specific results:

- "Classic" Dalitz-plot analysis of B/D decays to spinless final-state particles

  $D^0 \to K^0_S\pi^+\pi^-,\ B^0 \to K^0_S\pi^+\pi^-,\ \overline{D}^0\pi^+\pi^-;\ B^+ \to K^+\pi^+\pi^-,\ K^+K^+K^-$
- Variable-initial-mass Dalitz analysis:

  $B^+ \to (c\bar{c}) + K^+\pi^+\pi^-$
- Dalitz plot analysis of B decays to spinfull final-state particles

  $\overline{B}^0 \to J/\psi K^-\pi^+,\ \psi' K^-\pi^+,\ \chi_{c1}K^-\pi^+,\ D^{*+}\omega\pi^-$
- Amplitude analysis of

  $e^+e^- \to \Upsilon(nS)\pi^+\pi^-,\ \Upsilon(nS)\pi^0\pi^0,\ J/\psi\pi^+\pi^-$
- τ decay

  $\tau^- \to \nu_\tau\pi^-\pi^0,\ \nu_\tau K^0_S\pi^-,\ \nu_\tau K^0_S K^0_S\pi^-\pi^0$

PRL 121, 261801 (2018) / PRD 98, 112012 (2018) (924 fb^{-1} on and off resonance)

For study of $\sin 2\beta$ and $\cos 2\beta$ in $B^0 \rightarrow D^{(*)}h^0$, amplitude analysis of

$$D^0 \rightarrow K^0_S\pi^+\pi^-$$

Events reconstructed by detection of “good” K^0_S and pions consistent with D^0

and D^0 and $\pi^\pm$ consistent with being from $D^{*\pm} \longrightarrow$ tags $D^0/\overline{D}^0$

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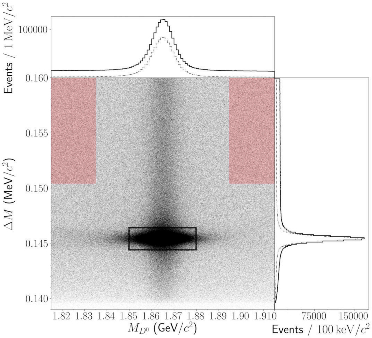

1.3×10^6 events in signal region (black box)

with 94 % purity
background includes wrongly tagged D^0

$D^0 \rightarrow K_S^0\pi^+\pi^-$

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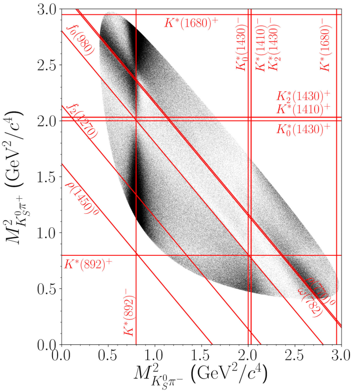

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For study of $\sin 2\beta$ and $\cos 2\beta$ in $\mathrm{B}^0 \rightarrow \mathrm{D}^{(*)}h^0$, amplitude analysis of

$$\mathrm{D}^0 \rightarrow \mathrm{K}^0_\mathrm{S}\pi^+\pi^-$$

with decay model combining conventional isobar with K matrix and LASS for S waves

Unbinnned maximum-likelihood fit to data of:

$$P(\mathrm{data}|\vec{\lambda}) = \prod_i f_\mathrm{sig} P_\mathrm{sig}(\vec{\tau}_i) + (1 - f_\mathrm{sig}) f_{\not{D}^*} P_{\not{D}^*}(\vec{\tau}) + (1 - f_\mathrm{sig})(1 - f_{\not{D}^*}) P_\mathrm{bg}(\vec{\tau})$$

with f_sig and $f_{\not{D}^*}$ fixed from fit to M_D–ΔM fit. (All P normalized.)

$$P_{\not{D}^*} = (1 - f_\mathrm{w.t.}) P_\mathrm{sig}(\vec{\tau}) + f_\mathrm{w.t} P_\mathrm{sig}(\vec{\tau}')$$

with $f_\mathrm{w.t.} = (49.2 \pm 7.5)\,\%$ the wrong-tag fraction—fixed from fit to ΔM side band.

$\vec{\tau}' =$ CP conjugated $\vec{\tau} \Rightarrow M(\mathrm{K}^0_\mathrm{S}\pi^+) \leftrightarrow M(\mathrm{K}^0_\mathrm{S}\pi^-)$.

Background is fixed from fit to M_D–ΔM sidebands:

$$P_{\text{bg}} = \text{Pol}_6(\vec{\tau}) + \sum_r a_r \left| A_r(m_r^2) \right|^2$$

with A_r Breit-Wigner line-shapes for $K^*(892)$, $K^*(1410)$, $K^*(1680)$, $\rho(770)$.

$D^0 \to K^0_S\pi^+\pi^-$: Signal

$$P_{\text{sig}}(\vec{\tau}) \propto \text{acceptance}(\vec{\tau}) \times \left| \sum_R \alpha_R A_R^{L\neq 0}(\vec{\tau}) + A_S^{\pi\pi}(\vec{\tau}) + A_S^{K\pi}(\vec{\tau}) \right|^2 \quad \text{(normalized)}$$

acceptance is parameterized in $\{m^2_{K\pi^-},\ \cos\theta_K\}$, with

$$\cos\theta_K \equiv -\hat{p}_D \cdot \hat{p}_K \quad \text{in } K^0_S\pi^- \text{ r.f.}$$

from large MC sample

$D^0 \rightarrow K^0_S\pi^+\pi^-$: $L\neq0$ Isobar Signal Model

$$A_R^{L\neq0} = F_D^{(L)}(\vec{\tau})\cdot F_R^{(L)}(\vec{\tau})\cdot\Omega^{(L)}(\vec{\tau})\cdot T_R(m_2^2)$$

$F \equiv$ Blatt-Weisskopf factors with radial par. 5 GeV^{-1} for D; 1.5 GeV^{-1} for all R.

$\Omega^{(L)} \equiv$ spin amplitudes in Zemach formalism

$T \equiv$ relativistic Breit-Wigner lineshapes with mass-dependent widths

Resonances:

$\pi^+\pi^-$ states:

$$\rho(770),\ \omega(782),\ f_2(1270),\ \rho(1450)$$

Cabibbo-favored $K^0_S\pi^-$ states:

$$K^*(892)^-,\ K^*(1410)^-,\ K^*_2(1430)^-,\ K^*(1680)^-$$

Cabibbo-suppressed $K^0_S\pi^+$ states:

$$K^*(892)^+,\ K^*(1410)^+,\ K^*_2(1430)^+$$

All masses and widths fixed, except for $K^*(892)$

$D^0 \to K^0_S \pi^+ \pi^-$: $\pi\pi$ S-wave Model

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A K matrix is used for the $\pi\pi$ S wave:

$$A_S^{\pi\pi}(\vec{\tau}) = \left[\mathbb{1} - iK(m^2_{\pi\pi})\,\rho(m^2_{\pi\pi})\right]^{-1}_{(\pi\pi),X} P_X(m^2_{\pi\pi})$$

where $X = \pi\pi,\ K\overline{K},\ \pi\pi\pi\pi,\ \eta\eta,\ \eta\eta'$

$$K_{ij}(s) = \left(f_{ij}^{\text{scat}} \frac{1\,\text{GeV} - s_0^{\text{scat}}}{s - s_0^{\text{scat}}} + \sum_\alpha \frac{g_i^\alpha g_j^\alpha}{m_\alpha^2 - s} \right) f_{\text{A0}}(s)$$

first term describes slowly varying smooth part of amplitude.
second term describes physical poles at m_α with couplings to the channels, g_i^α

all multiplied by Adler zero factor

$$F_{\text{A0}}(s) \equiv \frac{1\,\text{GeV} - s_{\text{A0}}}{s - s_{\text{A0}}} \left(s - s_{\text{A}} \frac{m_\pi^2}{2} \right)$$

to suppress kinematic singularity at $\pi\pi$ threshold.

$D^0 \to K^0_S\pi^+\pi^-$: $\pi\pi$ S-wave Model

A K matrix is used for the $\pi\pi$ S wave:

$$A_{\mathrm{S}}^{\pi\pi}(\vec{\tau}) = \left[\mathbb{1} - iK(m^2_{\pi\pi})\,\rho(m^2_{\pi\pi})\right]^{-1}_{(\pi\pi),X} P_X(m^2_{\pi\pi})$$

where $X = \pi\pi,\ \mathrm{K}\overline{\mathrm{K}},\ \pi\pi\pi\pi,\ \eta\eta,\ \eta\eta'$

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and P vector mimicks K-matrix structure

$$P_j(s) = f_{1j}^{\mathrm{prod}} \frac{1\,\mathrm{GeV} - s_0^{\mathrm{prod}}}{s - s_0^{\mathrm{prod}}} + \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - s}$$

first term describes slowly varying production
second term describes production of channels via complex couplings, β_α

A K matrix is used for the $\pi\pi$ S wave:

$$A_S^{\pi\pi}(\vec{\tau}) = \left[\mathbb{1} - iK(m_{\pi\pi}^2)\,\rho(m_{\pi\pi}^2)\right]^{-1}_{(\pi\pi),X} P_X(m_{\pi\pi}^2)$$

where $X = \pi\pi,\ \mathrm{K\overline{K}},\ \pi\pi\pi\pi,\ \eta\eta,\ \eta\eta'$

$$K_{ij}(s) = \left(f_{ij}^{\text{scat}} \frac{1\,\text{GeV} - s_0^{\text{scat}}}{s - s_0^{\text{scat}}} + \sum_\alpha \frac{g_i^\alpha g_j^\alpha}{m_\alpha^2 - s} \right) f_{A0}(s)$$

and P vector mimicks K-matrix structure

$$P_j(s) = f_{1j}^{\text{prod}} \frac{1\,\text{GeV} - s_0^{\text{prod}}}{s - s_0^{\text{prod}}} + \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - s}$$

All K-matrix parameters are fixed to results of
Aubert (BaBar) PRD78, 034023 (2008) and Anisovich & Sarantsev EPJA16 229 (2003)

P-vector parameters (f_{1j}^{prod}, β_α) free in fit

$D^0 \to K^0_S\pi^+\pi^-$: $K\pi$ S-wave Model

The LASS parameterization (NuclPhys B296 394, 1988) is used for the $K^0_S\pi$ S wave:

it describes rapid phase motion from the resonance $K^*_0(1430)$
and slow phase motion of nonresonant component

$$A_S^{K\pi}(\vec{\tau}) = \alpha_{nr} e^{i \arg T_{nr}} \sin[\arg\alpha_{nr} + \arg T_{nr}] + \alpha_R e^{i \arg T_R} \sin[\arg\alpha_R + \arg T_R] e^{i2 \arg T_{nr}}$$

α_R and α_{nr} are complex-valued amplitudes

$$T_{nr} \equiv (1 + \tfrac{1}{2}arq^2 - iaq)^{-1}$$

$T_R \equiv$ relativistic Breit-Wigner with mass-dependent width

T_R describes the $K^*_0(1430)$ resonance
(q is momentum of spectator pion in the $K\pi$ r.f.; a and r real pars)

All parameters are free in fit; But only one set of parameters is used for both Cabibbo-favored $K^*_0(1430)^-$ and Cabibbo-suppressed $K^*_0(1430)^+$

An unbinned maximum-likelihood fit is performed,
but the goodness of fit is calculated by binning the data:

$$\chi^2/\text{ndf} = 1.05$$

result is worsened by

- adding more resonances
- replacing K-matrix or LASS by isobars
- freeing massses and widths
- using more complicated line shapes (e.g. Gounaris-Sakurai)

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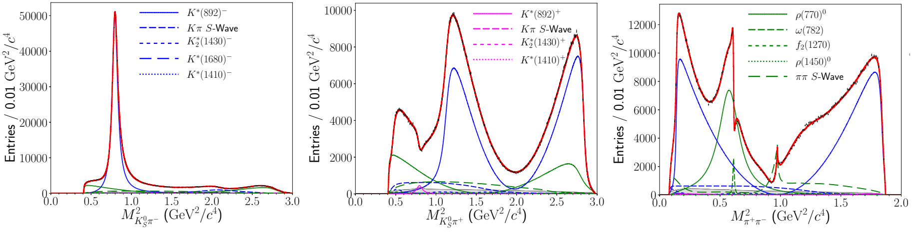

$D^0 \rightarrow K_S^0\pi^+\pi^-$: Results

An unbinned maximum-likelihood fit is performed,
but the goodness of fit is calculated by binning the data:

$$\chi^2/\text{ndf} = 1.05$$

Resonance	Amplitude	Phase (deg)	Fit Fraction (%)
$K_S^0\rho(770)^0$	1 (fixed)	0 (fixed)	20.4
$K_S^0\omega(782)$	0.0388 ± 0.0005	120.7 ± 0.7	0.5
$K_S^0 f_2(1270)$	1.43 ± 0.03	-36.3 ± 1.1	0.8
$K_S^0\rho(1450)^0$	2.85 ± 0.10	102.1 ± 1.9	0.6
$K^*(892)^-\pi^+$	1.720 ± 0.006	136.8 ± 0.2	59.9
$K_2^*(1430)^-\pi^+$	1.27 ± 0.02	-44.1 ± 0.8	1.3
$K^*(1680)^-\pi^+$	3.31 ± 0.20	-118.2 ± 3.1	0.5
$K^*(1410)^-\pi^+$	0.29 ± 0.03	99.4 ± 5.5	0.1
$K^*(892)^+\pi^-$	0.164 ± 0.003	-42.2 ± 0.9	0.6
$K_2^*(1430)^+\pi^-$	0.10 ± 0.01	-89.6 ± 7.6	< 0.1
$K^*(1410)^+\pi^-$	0.21 ± 0.02	150.2 ± 5.3	< 0.1

Phys. Rev. D 83, 032005 (2011) (492 fb^{-1} on-resonance)

Amplitude analysis of $K^+\pi^+\pi^-$ produced in

$$B^+ \to J/\psi K^+\pi^+\pi^- \quad (\text{and } B^+ \to \psi' K^+\pi^+\pi^-)$$

Events reconstructed by detection of “good” charmonium and hadrons with

$$M_{\rm bc} \equiv \sqrt{\frac{s}{4} - \left(\sum_i \vec{p}_i\right)^2} > 5.27\,\text{GeV} \quad \text{and} \quad |\Delta E| \equiv \left|\frac{\sqrt{s}}{2} - \sum_i E_i\right| < 0.2\,\text{Gev}$$

Phys. Rev. D 83, 032005 (2011) (492 fb^{-1} on-resonance)

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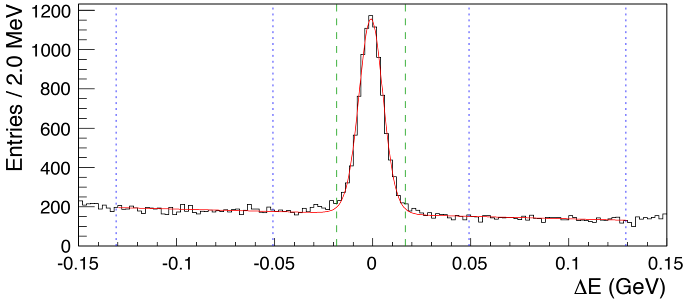


10 594 signal-region events, 12 913 side-band events

Phys. Rev. D 83, 032005 (2011) (492 fb^{-1} on-resonance)

Amplitude analysis of $K^+\pi^+\pi^-$ produced in

$$B^+ \rightarrow J/\psi K^+\pi^+\pi^- \quad (\text{and } B^+ \rightarrow \psi' K^+\pi^+\pi^-)$$

with an isobar model for three-body resonances R_3 and two-body resonances R_2:

$$R_3 \rightarrow aR_2 \quad \text{and} \quad R_2 \rightarrow bc \quad \text{with} \quad a, b, c = \text{FSP's}$$

Unbinned maximum-likelihood fit to data of:

$$P(\text{data}|\vec{\lambda}) = \prod_i f_{\text{bg}} P_{\text{bg}}(\vec{\tau}_i) + (1 - f_{\text{bg}}) P_{\text{sig}}(\vec{\tau}_i|\vec{\lambda})$$

with f_{bg} fixed from fit to ΔE distribution. (All P normalized.)

$$\vec{\tau} = \{m^2_{K\pi\pi}, m^2_{K\pi}, m^2_{\pi\pi}\}\{\underbrace{m^2_{K\pi\pi}}_{m^2_3}, m^2_{K\pi}, m^2_{\pi\pi}\}\{\underbrace{m^2_{K\pi\pi}}_{m^2_3}, \underbrace{m^2_{K\pi}, m^2_{\pi\pi}}_{m^2_2}\}$$

Background is fixed from fit to ΔE sidebands:

$$P_{\text{bg}} = C_5(m^2_{K\pi\pi}) \times C_1(m^2_{K\pi}) \times C_2(m^2_{\pi\pi}) + \exp\left(m^2_{K\pi\pi}\right) \sum_r a_r \left|A_r(m^2_r)\right|^2$$

with

$$C_n(x) \equiv n\text{'th-order Chebyshev series} \equiv \sum_{i=0}^{n} a_i T_i(x)$$

and

r	$\lvert A_r\rvert^2$
$K^*(892)$	$\left\lvert\text{Breit-Wigner}(m^2_{K\pi})\right\rvert^2$
$\rho(770)$	$\left\lvert\text{Breit-Wigner}(m^2_{\pi\pi})\right\rvert^2$
D^0	$\text{Gaus}(m^2_{K\pi})$
K^0_S	$\text{Gaus}(m^2_{\pi\pi})$

all normalized to kinematically allowed phsp total background

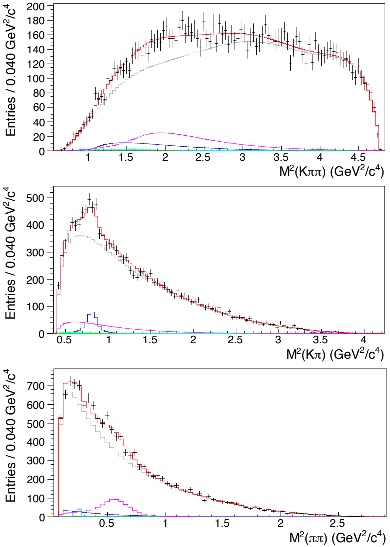
parametric background

$K^*(892)$

D^0

$\rho(770)$

K^0_S



$$P_{\text{sig}}(\vec{\tau}|\vec{\lambda}) = \text{acceptance}(\vec{\tau}) \times \text{phsp-density}(\vec{\tau}) \times s(\vec{\tau}|\vec{\lambda})$$

acceptance & phase-space density taken from MC
acceptance in $(0.15\,\text{GeV}^2)^3$ bins
density in $(0.02\,\text{GeV}^2)^3$ bins

$B^+ \to J/\psi + K^+\pi^+\pi^-$: Signal

$$P_{\text{sig}}(\vec{\tau}|\vec{\lambda}) = \text{acceptance}(\vec{\tau}) \times \text{phsp-density}(\vec{\tau}) \times s(\vec{\tau}|\vec{\lambda})$$

$$s(\vec{\tau}|\vec{\lambda}) = |\alpha_3^{\text{nr}}|^2 + \sum_{J_3^P} \left| \sum_{R_3 \text{ with } J_3^P} \sum_{R_2} \alpha_{R_3 \to R_2} \cdot \Omega_{J_3 \to J_2}(\vec{\tau}) \cdot T_{R_3}(m_3^2) \cdot T_{R_2}(m_2^2) \right|^2$$

α are fitted amplitude variables

$\Omega_{J_3 \to J_2}(\vec{\tau})$ from Filippini, Fontana, Rotondi (PRD51, 2247 [1995])

$T_{R_3}(m_3^2) =$ Constant-width Rel. Breit Wigner
$T_{R_2}(m_2^2) =$ Mass-dep.-width Rel. Breit Wigner (radial par $= 1.5\,\text{GeV}^{-1}$)

Two-body resonances: $\rho(770)$, ω, $f_0(980)$, $f_2(1270)$, $K^*(892)$, $K^*(1430)$

All with fixed masses and widths. (Varied within uncertainties for syst. unc.)

Data features prominent $K_1(1270)$ peak—

start with basic model of $K_1(1270) \rightarrow K^*(892)\pi$ and $K_1(1270) \rightarrow K\rho$

then add channels successively until reasonably good fit achieved:

$B^+ \rightarrow J/\psi + K^+\pi^+\pi^-$: Results

TABLE V. Fitted parameters of the signal function for $B^+ \to J/\psi K^+\pi^+\pi^-$, along with the corresponding decay fractions.

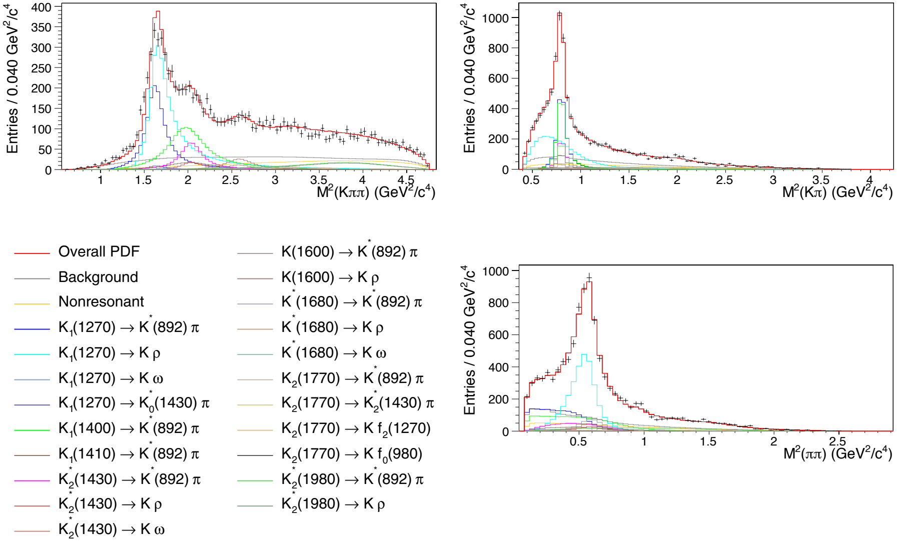
J_1	Submode	Modulus	Phase (radians)	Decay fraction
	Nonresonant $K^+\pi^+\pi^-$	1.0 (fixed)	0 (fixed)	$0.152 \pm 0.013 \pm 0.028$
1^+	$K_1(1270) \to K^*(892)\pi$	$0.962 \pm 0.058 \pm 0.176$	0 (fixed)	$0.232 \pm 0.017 \pm 0.058$
	$K_1(1270) \to K\rho$	$1.813 \pm 0.090 \pm 0.243$	$-0.764 \pm 0.069 \pm 0.127$	$0.383 \pm 0.016 \pm 0.036$
	$K_1(1270) \to K\omega$	$0.198 \pm 0.036 \pm 0.041$	$1.09 \pm 0.18 \pm 0.18$	$0.0045 \pm 0.0017 \pm 0.0014$
	$K_1(1270) \to K_0^*(1430)\pi$	$0.95 \pm 0.16 \pm 0.24$	$2.83 \pm 0.18 \pm 0.18$	$0.0157 \pm 0.0052 \pm 0.0049$
	$K_1(1400) \to K^*(892)\pi$	$0.894 \pm 0.066 \pm 0.125$	$-2.300 \pm 0.044 \pm 0.078$	$0.223 \pm 0.026 \pm 0.036$
1^-	$K^*(1410) \to K^*(892)\pi$	$0.516 \pm 0.090 \pm 0.103$	0 (fixed)	$0.047 \pm 0.016 \pm 0.015$
2^+	$K_2^*(1430) \to K^*(892)\pi$	$0.663 \pm 0.051 \pm 0.085$	0 (fixed)	$0.088 \pm 0.011 \pm 0.011$
	$K_2^*(1430) \to K\rho$	0.371 (fixed)	$-1.12 \pm 0.22 \pm 0.29$	0.0233 (fixed)
	$K_2^*(1430) \to K\omega$	0.040 (fixed)	$0.58 \pm 0.51 \pm 0.27$	0.00036 (fixed)
	$K_2^*(1980) \to K^*(892)\pi$	$0.775 \pm 0.054 \pm 0.118$	$-1.59 \pm 0.15 \pm 0.14$	$0.0739 \pm 0.0073 \pm 0.0095$
	$K_2^*(1980) \to K\rho$	$0.660 \pm 0.048 \pm 0.101$	$0.86 \pm 0.22 \pm 0.21$	$0.0613 \pm 0.0058 \pm 0.0059$
2^-	$K(1600) \to K^*(892)\pi$	$0.131 \pm 0.021 \pm 0.024$	0 (fixed)	$0.0187 \pm 0.0058 \pm 0.0050$
	$K(1600) \to K\rho$	$0.193 \pm 0.017 \pm 0.029$	$-0.27 \pm 0.27 \pm 0.18$	$0.0424 \pm 0.0062 \pm 0.0110$
	$K_2(1770) \to K^*(892)\pi$	$0.122 \pm 0.021 \pm 0.026$	$2.22 \pm 0.49 \pm 0.37$	$0.0164 \pm 0.0055 \pm 0.0061$
	$K_2(1770) \to K_2^*(1430)\pi$	$0.286 \pm 0.043 \pm 0.044$	$1.78 \pm 0.39 \pm 0.24$	$0.0100 \pm 0.0028 \pm 0.0020$
	$K_2(1770) \to Kf_2(1270)$	$0.444 \pm 0.069 \pm 0.077$	$2.30 \pm 0.37 \pm 0.32$	$0.0124 \pm 0.0033 \pm 0.0022$
	$K_2(1770) \to Kf_0(980)$	$0.113 \pm 0.029 \pm 0.024$	$1.83 \pm 0.45 \pm 0.53$	$0.0034 \pm 0.0017 \pm 0.0011$

All masses and widths fixed. Unbinned fit; binned g.o.f. check: $\chi^2/\mathrm{ndf} = 1.26$

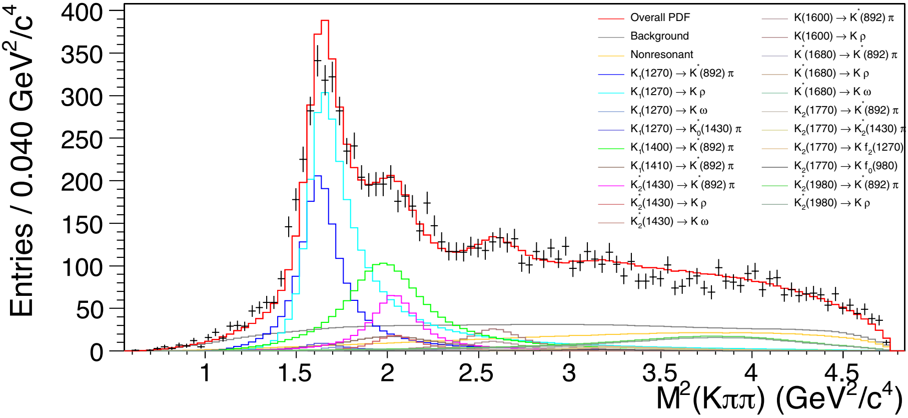
2nd fit with freed mass and width for $K_1(1270)$:

$$M_{K_1(1270)} = (1248.1 \pm 3.3 \pm 1.4)\ \text{MeV} \quad \text{and} \quad \Gamma_{K_1(1270)} = (119.5 \pm 5.2 \pm 6.7)\ \text{MeV}$$

$B^+ \rightarrow J/\psi + K^+\pi^+\pi^-$: Results



$B^+ \to J/\psi + K^+\pi^+\pi^-$: Results



Phys. Rev. D 90, 112009 (2014) (711 fb^{-1} on resonance)

Amplitude analysis of

$$\overline{B}^0 \to J/\psi K^- \pi^+$$

for Z_c spectroscopy.

Again, events reconstructed by detection of "good" J/ψ and hadrons with

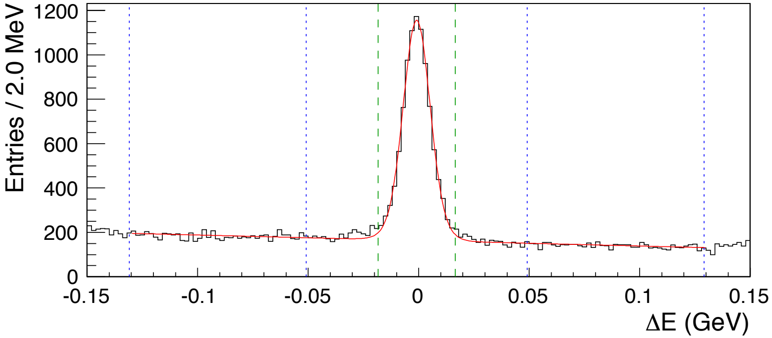
M_{bc} within 7 MeV of B

Phys. Rev. D 90, 112009 (2014) (711 fb^{-1} on resonance)

Amplitude analysis of

$$\overline{B}^0 \to J/\psi K^- \pi^+$$

for Z_c spectroscopy.



31 774 signal-region events, $(94.4 \pm 0.6)\,\%$ purity

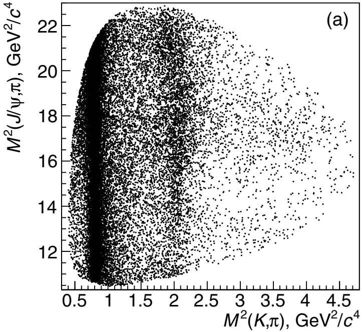
Phys. Rev. D 90, 112009 (2014) (711 fb^{-1} on resonance)

Amplitude analysis of

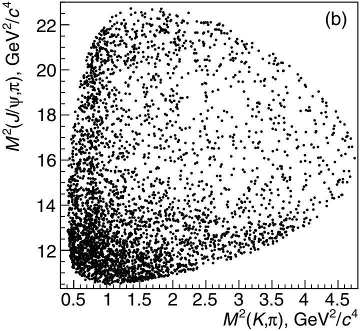
$$\overline{B}^0 \to J/\psi K^- \pi^+$$

for Z_c spectroscopy.

signal



sideband



Phys. Rev. D 90, 112009 (2014) (711 fb^{-1} on resonance)

Amplitude analysis of

$$\overline{B}^0 \to J/\psi K^- \pi^+$$

for Z_c spectroscopy.

with an isobar-model analysis & freed-isobar (model-independent) check

Unbinned maximum-likelihood fit to data of:

$$P(\text{data}) = \prod_i f_{\text{bg}} P_{\text{bg}}(\vec{\tau}_i) + (1 - f_{\text{bg}}) P_{\text{sig}}(\vec{\tau}_i)$$

with f_{bg} fixed from fit to ΔE distribution.
Both $P(\vec{\tau})$ normalized by detector-simulated MC—accounting for acceptance.

$$\vec{\tau} = \{m^2_{K\pi}, m^2_{J/\psi\pi}, \theta_{J/\psi}, \phi\}$$

$$\theta_{J/\psi} \equiv \text{angle}(\vec{p}_{\ell^+}, \vec{p}_{K\pi}) \text{ in } J/\psi \text{ r.f.} \quad \text{and} \quad \phi \equiv \text{angle}(\hat{n}_{\ell^+\ell^-}, \hat{n}_{K\pi}) \text{ in } \overline{B}^0 \text{ r.f.}$$

Background is fixed from fit to ΔE sidebands:

$$P_{\mathrm{bg}} = \left(B(m^2_{\mathrm{K}\pi}, m^2_{\mathrm{J}/\psi\,\pi}) + \sum_r \left| A_r(m^2_{\mathrm{K}\pi}) \right|^2 \mathrm{Pol}_4^{(r)}(m^2_{\mathrm{J}/\psi\,\pi}) \right) \times \mathrm{Pol}_2(\cos\theta_{\mathrm{J}/\psi})\ \mathrm{Pol}_2(\phi)$$

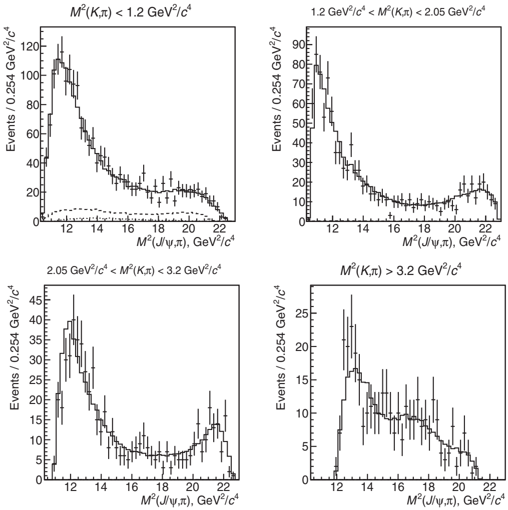
$B(m^2_{\mathrm{K}\pi}, m^2_{\mathrm{J}/\psi\,\pi})$ is a smooth function of the masses:

$$B = \left(\alpha_1 \exp\left(-\beta_1 m^2_{\mathrm{K}\pi} \right) + \alpha_2 \exp\left(-\beta_2 m^2_{\mathrm{J}/\psi\,\pi} \right) \right) \times \mathrm{Pol}_5(m^2_{\mathrm{K}\pi}, m^2_{\mathrm{J}/\psi\,\pi})$$

and

r	$\lvert A_r \rvert^2$
$\mathrm{K}^*(892)$	$\left\lvert \text{Breit-Wigner}(m^2_{\mathrm{K}\pi}) \right\rvert^2$
K^0_S with π seen as K	$\mathrm{Gaus}\left(m^2_{\mathrm{K}\pi} \lvert \mu(m^2_{\mathrm{J}/\psi\,\pi}) \right)$

$\overline{B}^0 \to J/\psi K^- \pi^+$: Background



$$P_{\mathrm{sig}}(\vec{\tau}) = \sum_{\zeta=-1,1} \left| \sum_R \sum_\lambda \alpha^R_\lambda \cdot \Omega^{j_R}_{\lambda\zeta}(\vec{\tau}) \cdot T_R(m^2_R) \cdot F^{(L_\mathrm{B})}_\mathrm{B} F^{(L_R)}_R \left(\frac{q_\mathrm{B}}{m_\mathrm{B}}\right)^{L_\mathrm{B}} \left(\frac{q_R}{m_\mathrm{B}}\right)^{L_R} \right|^2$$

α are fitted amplitude variables; F are Blatt-Weisskopf barrier factors;
q are breakup momenta; L orbital angular momenta of decays

spin amplitudes given in helicity formalism

$$\Omega^j_{\lambda\zeta}(\vec{\tau}) = d^j_{0\lambda}\left(\theta^{(\mathrm{J}/\psi\pi)}_{\mathrm{K}\pi}\right) \cdot e^{i\lambda\phi} \cdot d^1_{\lambda\zeta}\left(\theta_{\mathrm{J}/\psi}\right) \cdot e^{i\zeta\alpha}$$

for resonances in $\mathrm{J}/\psi\pi$. $d^j_{0\lambda}(\theta^{(\mathrm{J}/\psi\pi)}_{\mathrm{K}\pi}) \to d^j_{\lambda 0}(\theta^{(\mathrm{K}\pi)}_{\mathrm{J}/\psi\pi})$ for resonances in $\mathrm{K}\pi$.

T are all relativistic Breit-Wigner lineshapes with mass-dependent widths.

The model includes resonances in $K\pi$

$$K_0^*(800),\ K^*(892),\ K^*(1410),\ K_0^*(1430),\ K_2^*(1430),$$
$$K^*(1680),\ K_3^*(1780),\ K_0^*(1950),\ K_2^*(1980),\ K_4^*(2045)$$

all with fixed masses and widths.
Though $K_0^*(800)$ mass and width are fixed to a fit without new $Z_c(4200)^+$.

And resonances in $J/\psi\pi$

$$Z_c(4430)^+ \quad \text{and} \quad Z_c(4200)^+$$

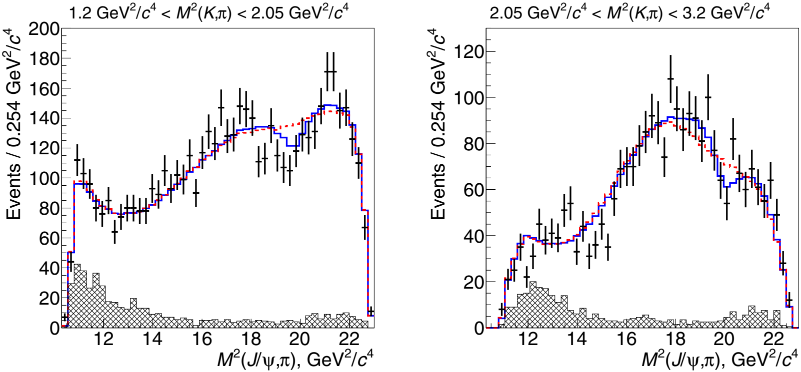
The former seen by Belle in $\psi(2S)\pi$ and confirmed by LHCb.
The latter a newly seen states!

For the Z_c states parity conservation applies:

$$\alpha_\lambda^Z = -\mathcal{P}_Z\ (-)^{J_Z}\ \alpha_{-\lambda}^Z$$

The masses and widths are left free,
but Gaussian priors are placed on those of the $Z_c(4430)^+$ from previous measurement

$$M = 4485^{+36}_{-25}\ \text{MeV} \quad \text{and} \quad \Gamma = 200^{+49}_{-58}\ \text{MeV}$$



with and without the $Z_c(4430)^+$ (both without the $Z_c(4200)^+$)

Seen with stat. significance of 5.1 σ (4.0 σ with syst.) $\to$ new decay channel

Several J^P hypotheses were tried for the $Z_c(4200)^+$

J^P	0^-	1^-	1^+	2^-	2^+
Mass, MeV/c^2	4318 ± 48	4315 ± 40	4196^{+31}_{-29}	4209 ± 14	4203 ± 24
Width, MeV	720 ± 254	220 ± 80	370 ± 70	64 ± 18	121 ± 53
Significance (Wilks)	3.9σ	2.3σ	8.2σ	3.9σ	1.9σ

$\overline{\mathrm{B}}^0 \to \mathrm{J}/\psi \mathrm{K}^- \pi^+$: Results

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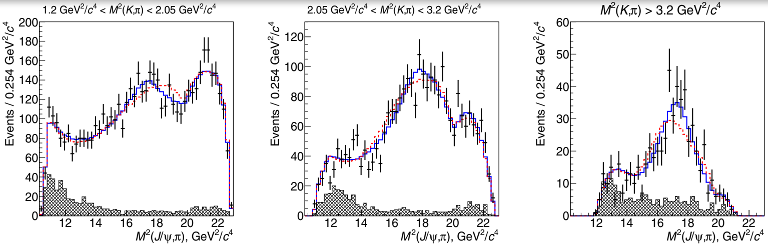
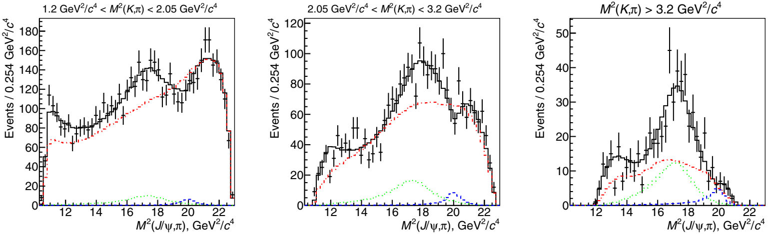


with and without the $J^P = 1^+$ $\mathrm{Z_c}(4200)^+$ (both with the $\mathrm{Z_c}(4430)^+$)



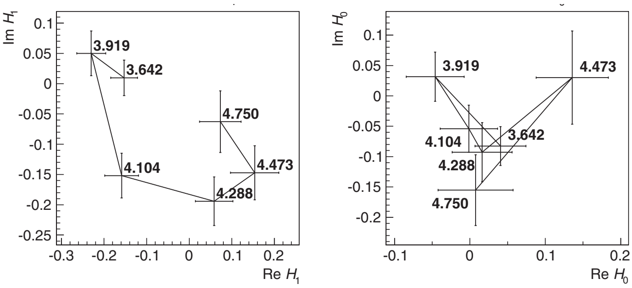
$\mathrm{K}\pi$ resonances, $\mathrm{Z_c}(4200)$, $\mathrm{Z_c}(4430)$

Full model altered:
Breit Wigner of $J^P{=}1^+$ $Z_c(4200)^+$ $\longrightarrow$ complex-valued step functions

two 6-step step-functions: one for $\lambda = 0$ and one for $|\lambda| = 1$

bin bounderies based on model-dependent fit results:

$$\{M - 2\Gamma,\ M - \Gamma,\ M - \frac{1}{2}\Gamma,\ M,\ M + \frac{1}{2}\Gamma,\ M + \Gamma,\ M + 2\Gamma\}$$



Phys. Rev. D 91, 072003 (2015) ($121.4\,\mathrm{fb}^{-1}$ on $\Upsilon(5S)$ resonance)

Amplitude analysis of

$$e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^- \quad n = 1, 2, 3$$

for $Z_b^{\pm}$ spectroscopy.

Events reconstructed via $e^+e^- \rightarrow \mu^+\mu^-\pi^+\pi^-$

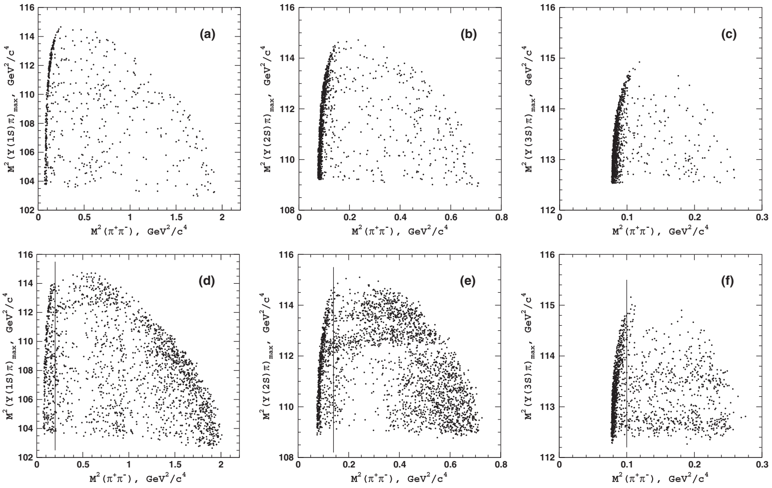
$$m_{\mu\mu}^2 \sim m_{\Upsilon}^2 \quad \text{and} \quad m_{\mathrm{miss}}^2 \equiv (\sqrt{s} - E_{\pi\pi})^2 - |\vec{p}_{\pi\pi}|^2 \sim m_{\Upsilon}^2$$

Unbinned maximum-likelihood fit to data of

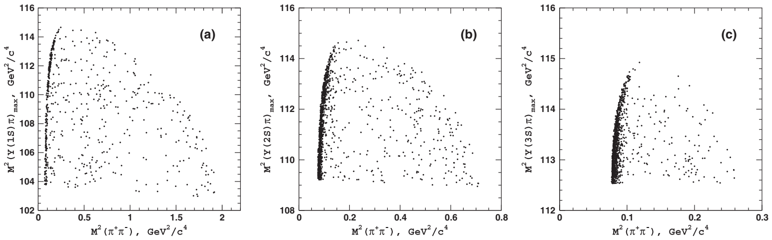
$$P(\text{data}) = \prod_i f_{\mathrm{sig}} P_{\mathrm{sig}}(\vec{\tau}_i) + (1 - f_{\mathrm{sig}}) P_{\mathrm{bg}}(\vec{\tau}_i)$$

with f_{sig} fixed from fit to m_{miss}^2 distribution.

background is learned from m_Υ side bands:



background is learned from m_Υ side bands:



Remaining background is parameterized as flat in phase space with additional component exponential in $m^2_{\pi\pi}$

$$P_{\text{sig}}(\vec{\tau}) \propto \left| \alpha^{(1)}_{\text{n.r.}} + \alpha^{(2)}_{\text{n.r.}} m^2_{\pi\pi} + \sum_R \alpha_R \cdot \Omega_R(\vec{\tau}) \cdot T_R(\vec{\tau}) \right|^2$$

convolved with resolution

normalized with detector-simulated MC $\to$ acceptance accounted for

$\Omega_R(\vec{\tau})$ = Lorentz-invariant spin amplitudes

In $\pi\pi$:

$$\sigma,\ f_0(980),\ f_2(1270)$$

All Breit-Wigner, but f_0 as Flatte ($M_\sigma = 600\,\text{MeV}$, $\Gamma_\sigma = 400\,\text{MeV}$)

all shape parameters fixed

In $\Upsilon(nS)\pi$:

$$Z_b(10610)^\pm,\ Z_b(10650)^\pm$$

Both Breit-Wigner

all parameters free

Several J^P hypotheses are tested for the $Z_b^{\pm}$:

$Z_b(10610)$	$Z_b(10650)$ 1^+	1^-	2^+	2^-
1^+	0(0)	60(33)	42(33)	77(63)
1^-	226(47)	264(73)	224(68)	277(106)
2^+	205(33)	235(104)	207(87)	223(128)
2^-	289(99)	319(111)	321(110)	304(125)

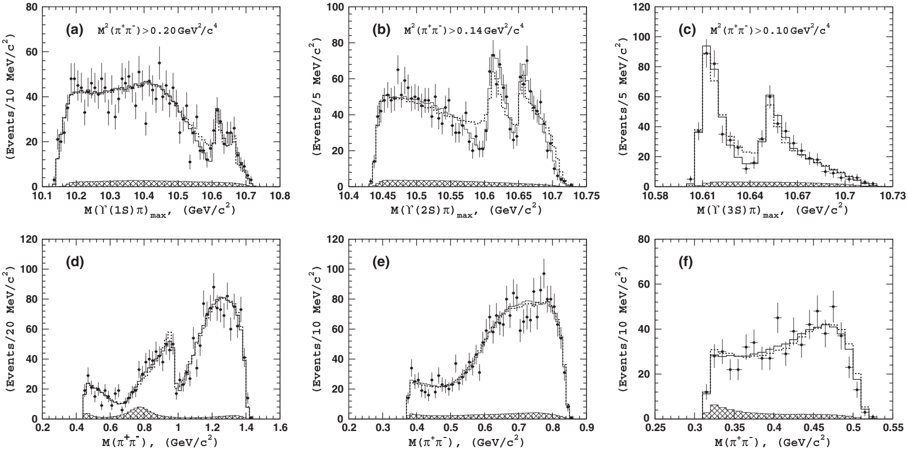
$$\Delta \equiv \log P(\text{both } J^P = 1^+) - \log P \text{ for } \Upsilon(2S)\pi\pi \ (\Upsilon(3S)\pi\pi)$$

For $\Upsilon(1S)\pi\pi$, J^P assumed the same for both Z_b:

$$\Delta(1^-) = 64 \quad \Delta(2^+) = 41 \quad \Delta(2^-) = 59$$

$J^P = 1^+$ favored over other configurations by over 6σ

$e^+e^- \to \Upsilon(nS)\pi^+\pi^-$: Results



solid = both $J^P = 1^+$ dotted = both $J^P = 2^+$

Parameter	$\Upsilon(1S)\pi^+\pi^-$	$\Upsilon(2S)\pi^+\pi^-$	$\Upsilon(3S)\pi^+\pi^-$
$f_{Z_b^\mp(10610)\pi^\pm}$, %	$4.8 \pm 1.2^{+1.5}_{-0.3}$	$18.1 \pm 3.1^{+4.2}_{-0.3}$	$30.0 \pm 6.3^{+5.4}_{-7.1}$
$Z_b(10610)$ mass, MeV/c^2	$10608.5 \pm 3.4^{+3.7}_{-1.4}$	$10608.1 \pm 1.2^{+1.5}_{-0.2}$	$10607.4 \pm 1.5^{+0.8}_{-0.2}$
$Z_b(10610)$ width, MeV	$18.5 \pm 5.3^{+6.1}_{-2.3}$	$20.8 \pm 2.5^{+0.3}_{-2.1}$	$18.7 \pm 3.4^{+2.5}_{-1.3}$
$f_{Z_b^\mp(10650)\pi^\pm}$, %	$0.87 \pm 0.32^{+0.16}_{-0.12}$	$4.05 \pm 1.2^{+0.95}_{-0.15}$	$13.3 \pm 3.6^{+2.6}_{-1.4}$
$Z_b(10650)$ mass, MeV/c^2	$10656.7 \pm 5.0^{+1.1}_{-3.1}$	$10650.7 \pm 1.5^{+0.5}_{-0.2}$	$10651.2 \pm 1.0^{+0.4}_{-0.3}$
$Z_b(10650)$ width, MeV	$12.1^{+11.3+2.7}_{-4.8-0.6}$	$14.2 \pm 3.7^{+0.9}_{-0.4}$	$9.3 \pm 2.2^{+0.3}_{-0.5}$
ϕ_Z, degrees	$67 \pm 36^{+24}_{-52}$	$-10 \pm 13^{+34}_{-12}$	$-5 \pm 22^{+15}_{-33}$
$c_{Z_b(10650)}/c_{Z_b(10610)}$	$0.40 \pm 0.12^{+0.05}_{-0.11}$	$0.53 \pm 0.07^{+0.32}_{-0.11}$	$0.69 \pm 0.09^{+0.18}_{-0.07}$
$f_{\Upsilon(nS)f_2(1270)}$, %	$14.6 \pm 1.5^{+6.3}_{-0.7}$	$4.09 \pm 1.0^{+0.33}_{-1.0}$	–
$f_{\Upsilon(nS)(\pi^+\pi^-)_S}$, %	$86.5 \pm 3.2^{+3.3}_{-4.9}$	$101.0 \pm 4.2^{+6.5}_{-3.5}$	$44.0 \pm 6.2^{+1.8}_{-4.3}$
$f_{\Upsilon(nS)f_0(980)}$, %	$6.9 \pm 1.6^{+0.8}_{-2.8}$	–	–

for both $J^P = 1^+$

Summary

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Belle has a history of amplitude analyses:

- in B and D decays to light pseudoscalar mesons
- in B decays to flavorless mesons and open and closed charm states
- in $e^+e^- \rightarrow$ quarkonia $+ \pi\pi$ production
- in hadronic τ decays
- in time-dependent B decays
- in baryonic B decays

Belle II will provide a rich data set for further amplitude analyses

- this large data set will allow new techniques
- this large data set will *require* new techniques