Dispersive constraints on amplitude determination

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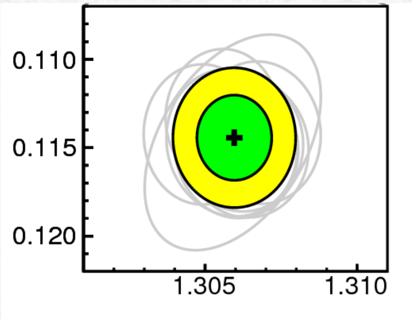
PWA/ATHOS, Rio de Janeiro, September 5th, 2019

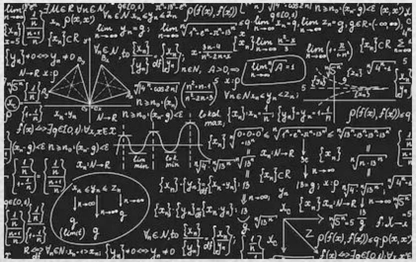
In life and physics you need a bit of luck....

The flowchart

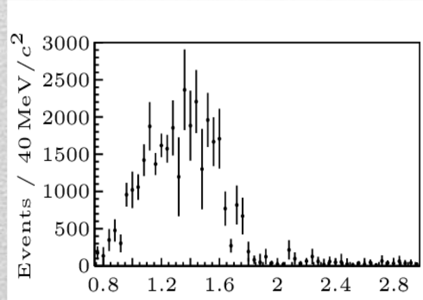
Less predictive power ✗
Some physical interpretation ✗
Minimally biased ✓



3) You extract physics

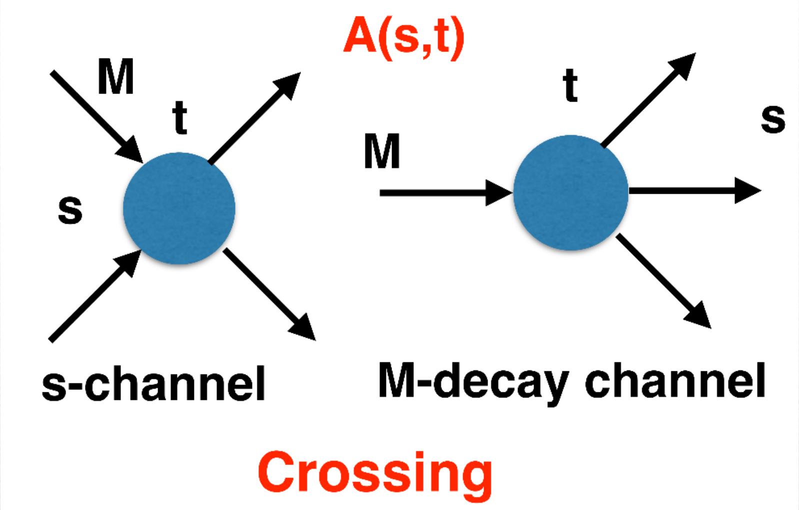


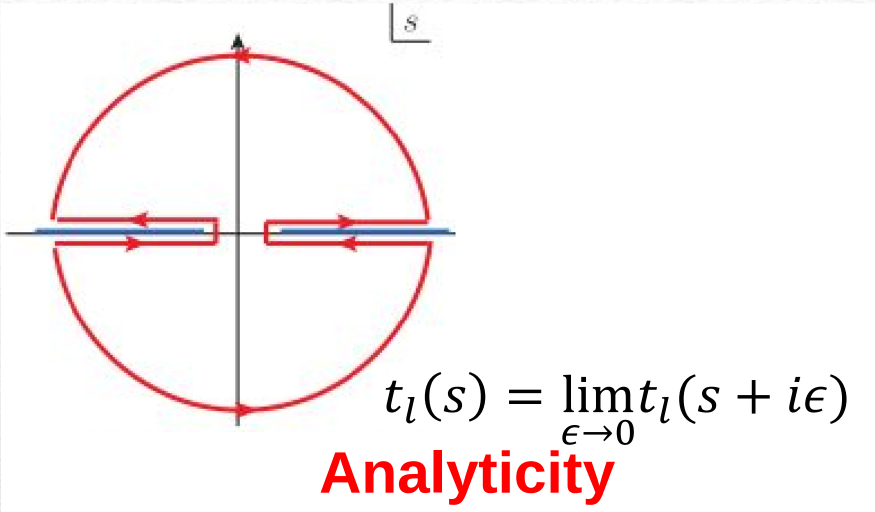
2) You choose a set of generic amplitudes



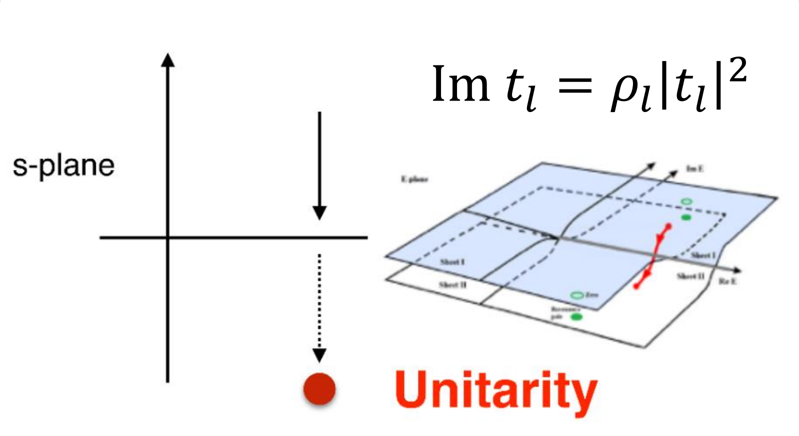
1) You start with data

S-Matrix principles





$$t_l(s) = \lim_{\epsilon\to 0} t_l(s+i\epsilon)$$



$$\mathrm{Im}\, t_l = \rho_l |t_l|^2$$

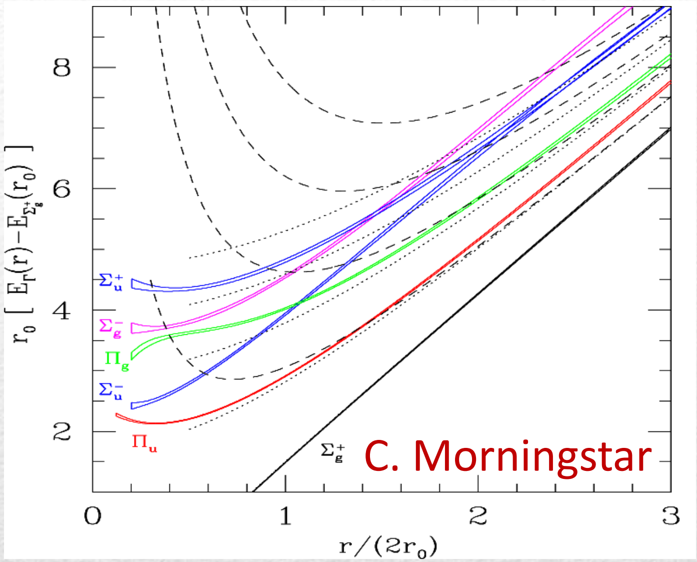
+ Lorentz, discrete & global symmetries

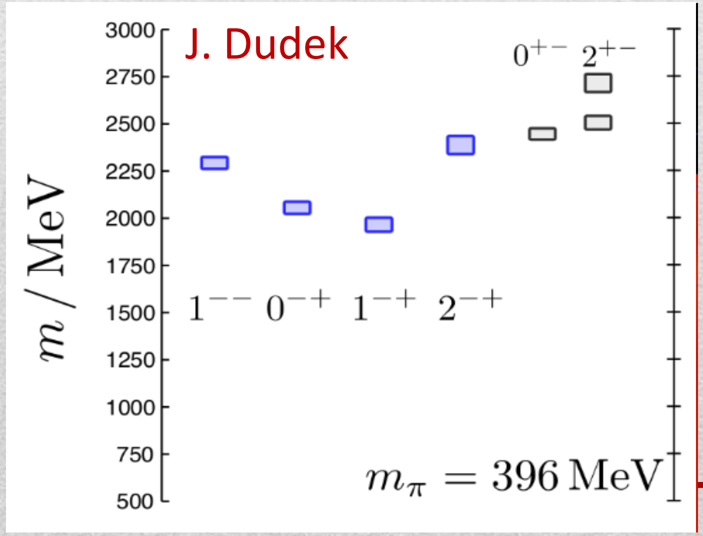
These are constraints the amplitudes have to satisfy, but do not fix the dynamics

They can be imposed with an increasing amount of rigor, to extract robust physics information

The «background» phenomena can be effectively parameterized in a controlled way

Learning about confinement





If quarks were infinitely heavy, gluonic field is confined in a string

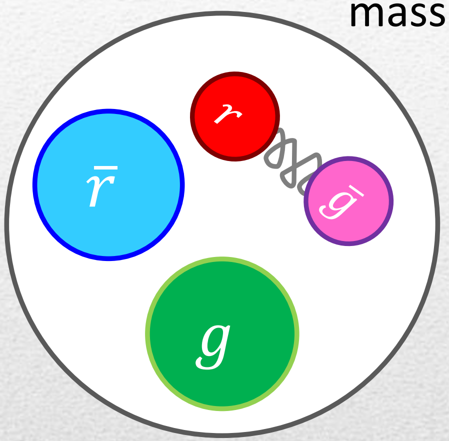
What is a constituent gluon?

- vibration of the string?
  → degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, (\mathbf{0}, 1, \mathbf{2})^{+-}$
- excitation of a quasiparticle!
  → degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$
- excitation of a quasiparticle?
  → degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$

says Lattice QCD...

What about real data?

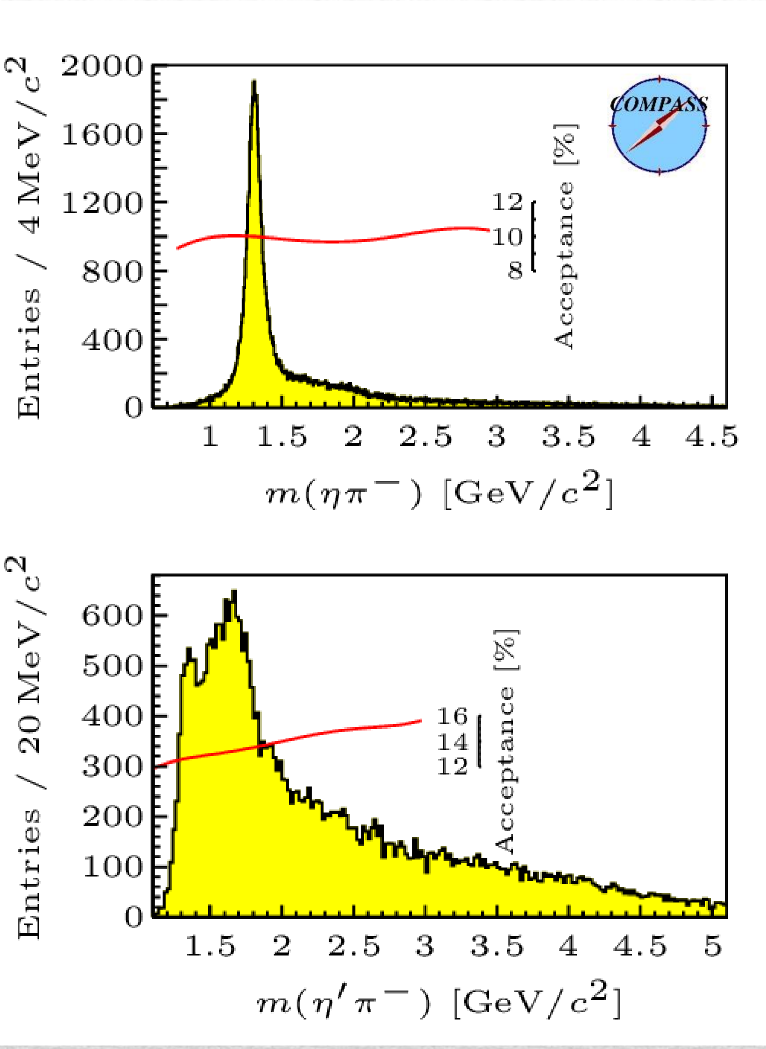
Hybrid hunting



Excited gluon,
$J^{PC} = 1^{+-}$
mass ~ 1.0−1.5 GeV

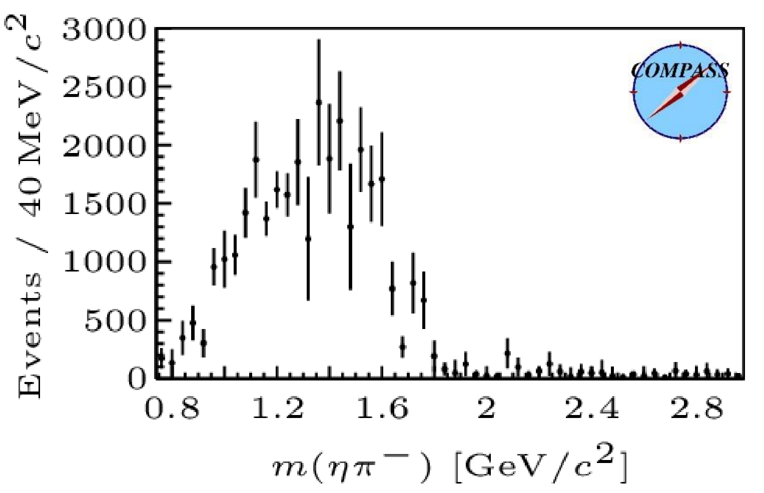
Look for a π_1 state with $\boldsymbol{J^{PC} = 1^{-+}}$

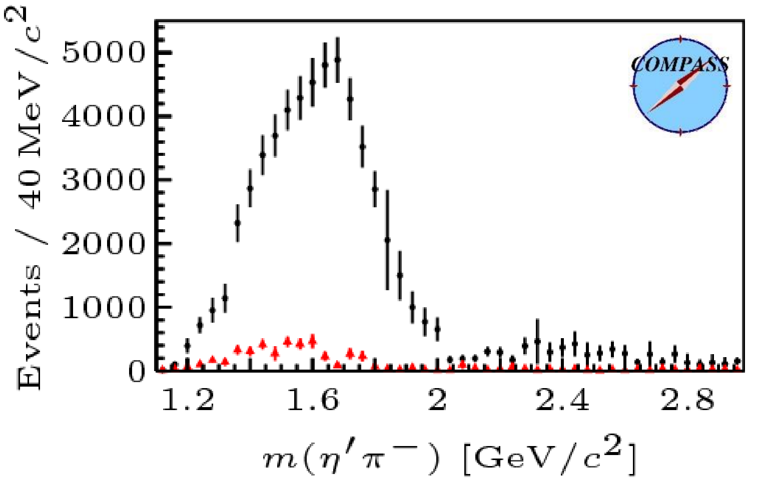
$$\text{decaying into} \begin{cases} \eta\,\pi \text{ and } \eta'\pi \\ \rho\,\pi \to 3\pi \\ b_1\pi \to 5\pi \end{cases}$$



Small signal in data

Two hybrid states???





$\pi_1(1400)$ $I^G(J^{PC})$ = $1^-(1^{-+})$

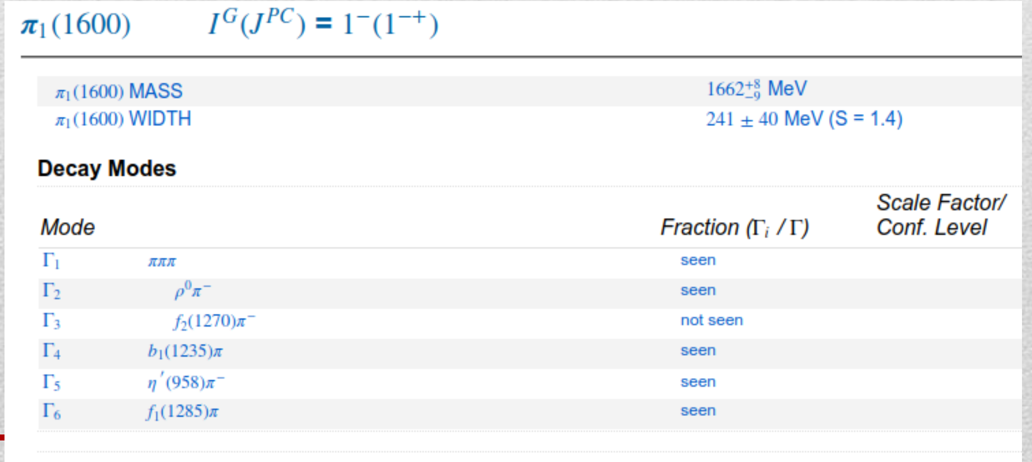
See also the mini-review under non- $q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level
Γ_1	$\eta\pi^0$	seen	
Γ_2	$\eta\pi^-$	seen	
Γ_3	$\eta'\pi$		

Neither lattice nor models predict two 1^{-+} states in this region!



$\pi_1(1600)$ $I^G(J^{PC})$ = $1^-(1^{-+})$

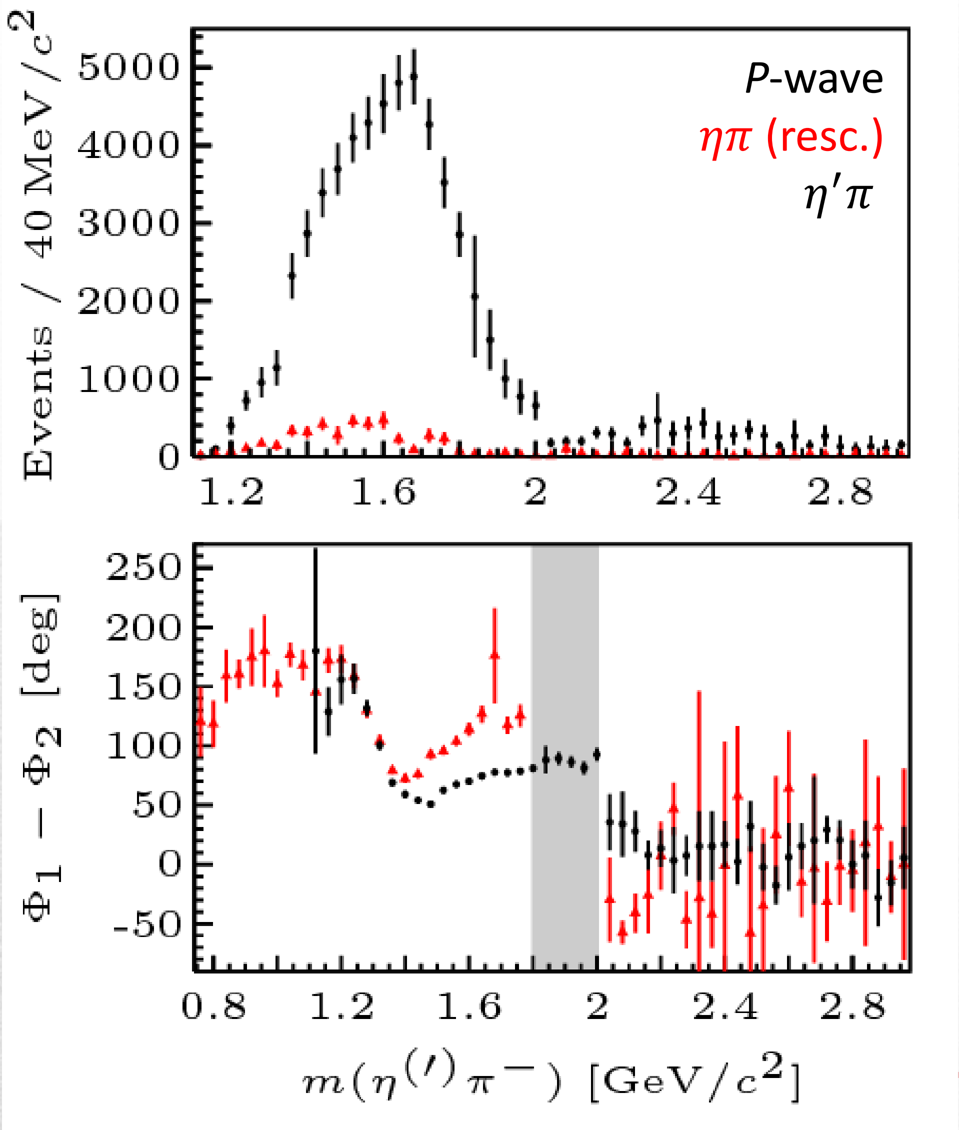
$\pi_1(1600)$ MASS	1662^{+8}_{-9} MeV
$\pi_1(1600)$ WIDTH	241 ± 40 MeV (S = 1.4)

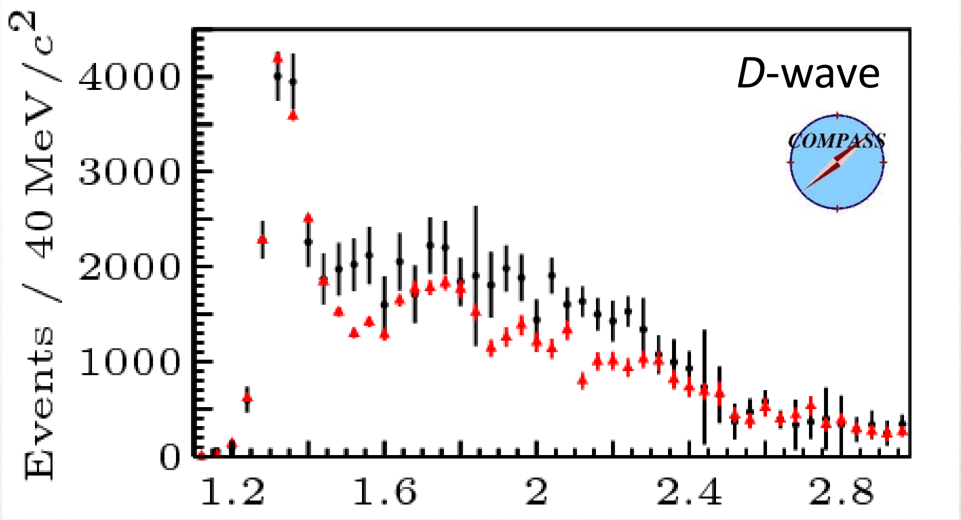
Decay Modes

	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level
Γ_1	$\pi\pi\pi$	seen	
Γ_2	$\rho^0\pi^-$	seen	
Γ_3	$f_2(1270)\pi^-$	not seen	
Γ_4	$b_1(1235)\pi$	seen	
Γ_5	$\eta'(958)\pi^-$	seen	
Γ_6	$f_1(1285)\pi$	seen	

Data

COMPASS, PLB740, 303-311





A sharp drop appears at 2 GeV in *P*-wave intensity and phase

No convincing physical motivation for it

It affects the position of the $a_2'(1700)$

We decided to fit up to 2 GeV only

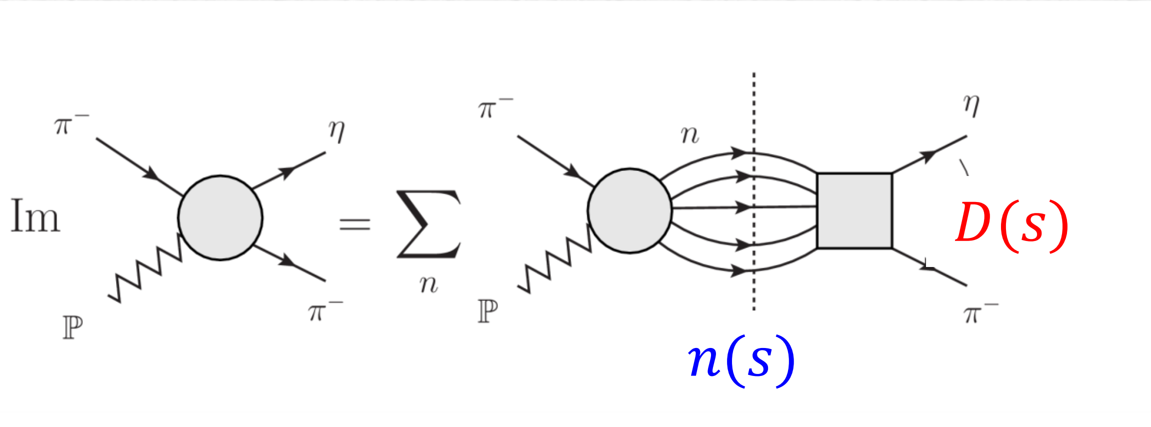
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the N/D method

Jackura, Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB

Rodas, AP *et al.* (JPAC), PRL

$$a(s) = \frac{n(s)}{D(s)}$$



The $n(s)$ → background physics, process-dependent, smooth

The $D(s)$ contains all the Final State Interactions constrained by unitarity → universal

Coupled channel: the model

Two channels, $i,k=\eta\pi,\eta'\pi$

Two waves, $J=P,D$

37 fit parameters

$$D^J_{ki}(s)=\left[K^J(s)^{-1}\right]_{ki}-\frac{s}{\pi}\int_{s_k}^{\infty}ds'\frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}$$

$$K^J_{ki}(s)=\sum_R\frac{g_k^{(R)}g_i^{(R)}}{m_R^2-s}+c^J_{ki}+d^J_{ki}\,s$$

1 *K*-matrix pole for the P-wave
2 *K*-matrix poles for the D-wave

$$\rho N^J_{ki}(s')=\delta_{ki}\frac{\lambda^{J+1/2}\left(s',m^2_{\eta^{(\prime)}},m^2_\pi\right)}{(s'+s_R)^{2J+1+\alpha}}$$

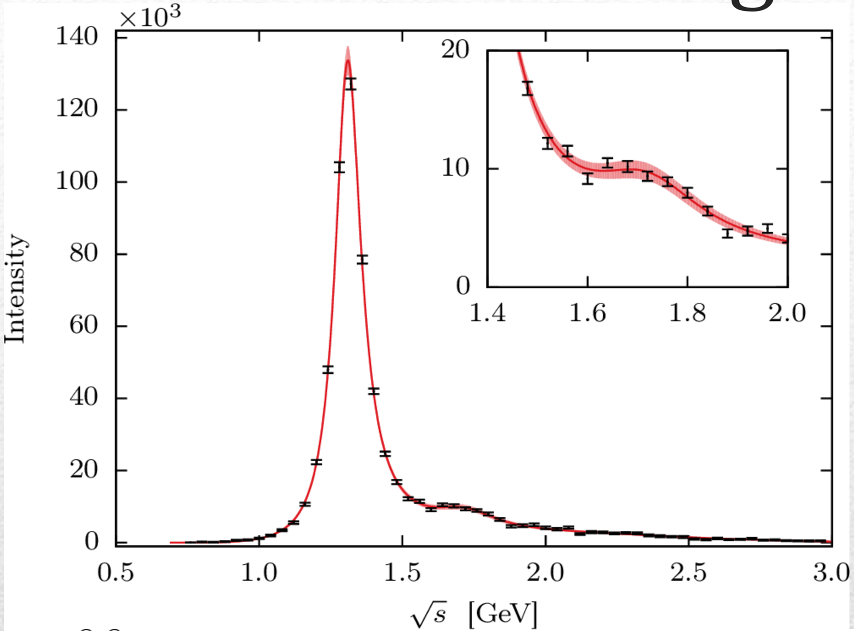
$$n^J_k(s)=\sum_{n=0}^{3}a_n^{J,k}\,T_n\left(\frac{s}{s+s_0}\right)$$

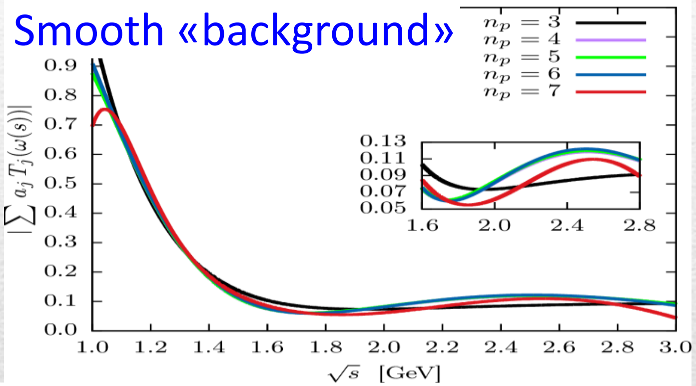
Left-hand scale (Blatt-Weisskopf radius) $s_R=s_0=1\ \mathrm{GeV}^2$
$\alpha=2$, 3rd order polynomial for $n^J_k(s)$

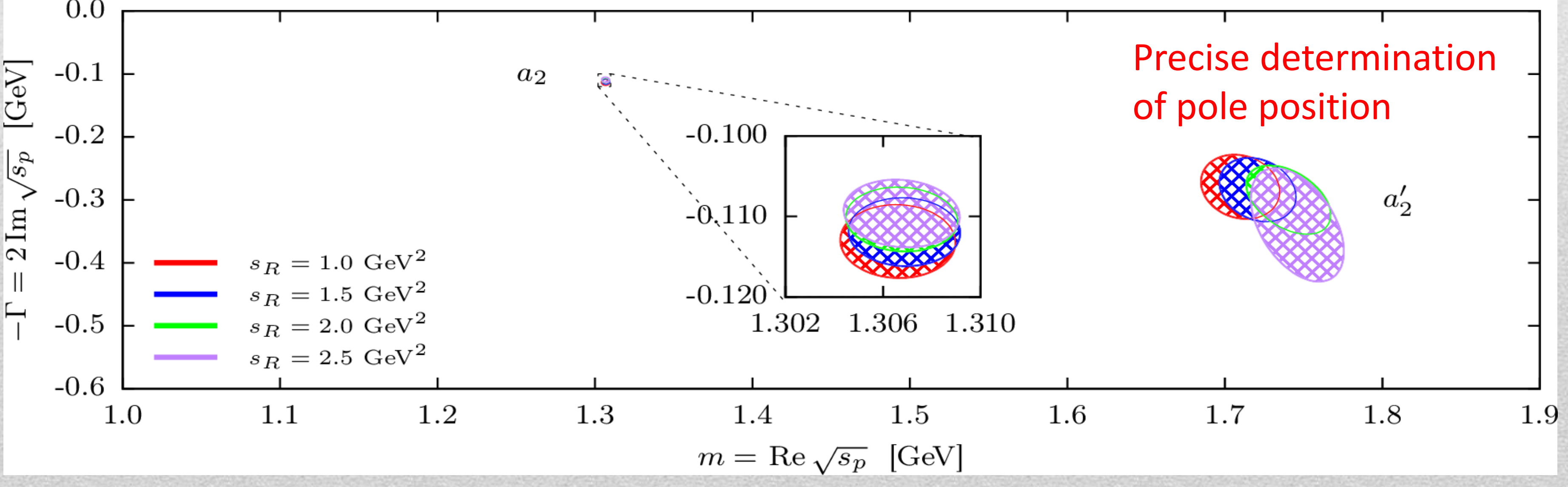
Benchmark: single channel $\eta\pi$

Test against the D-wave $\eta\pi$ data, where the a_2 and the a_2' show up

A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472



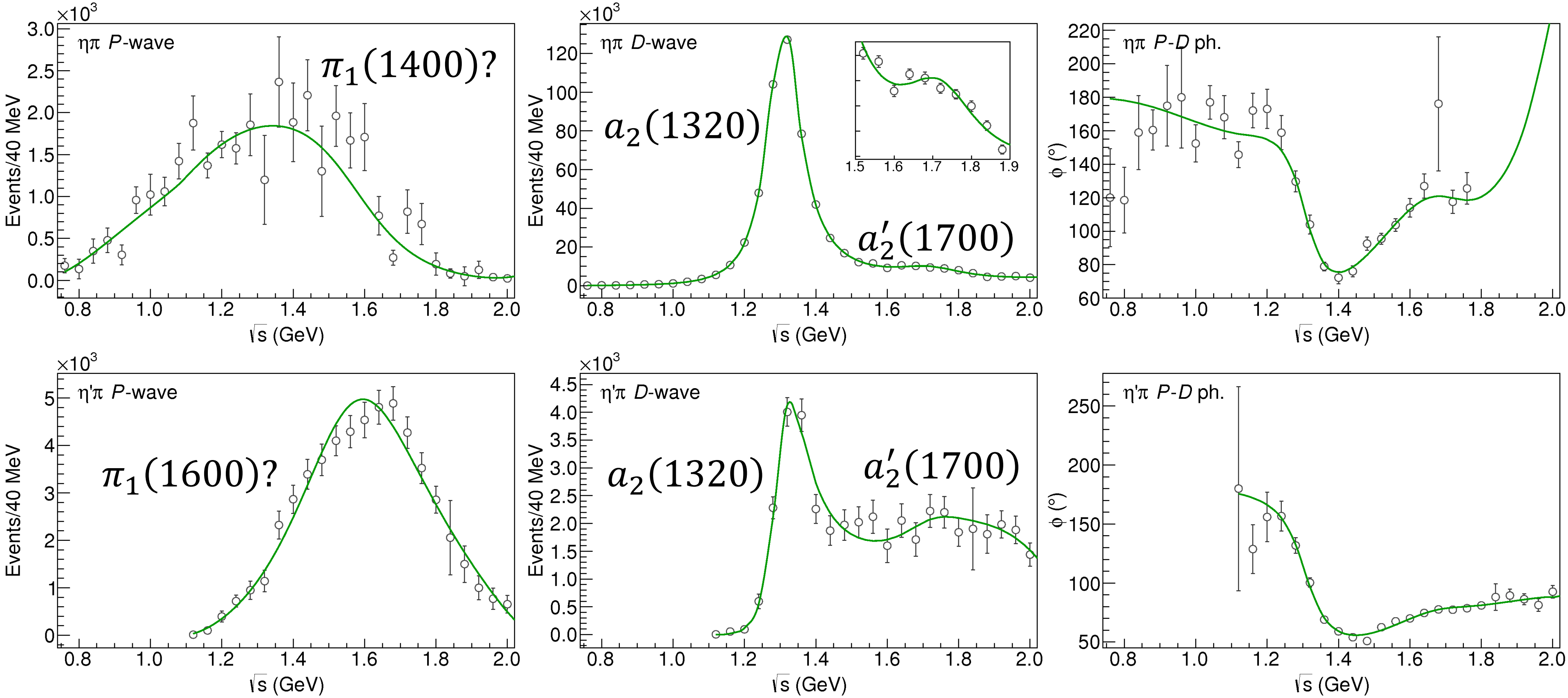

Smooth «background»


Precise determination of pole position

Fit to $\eta^{(\prime)}\pi$

$\chi^2 \sim 1.3$

(data)

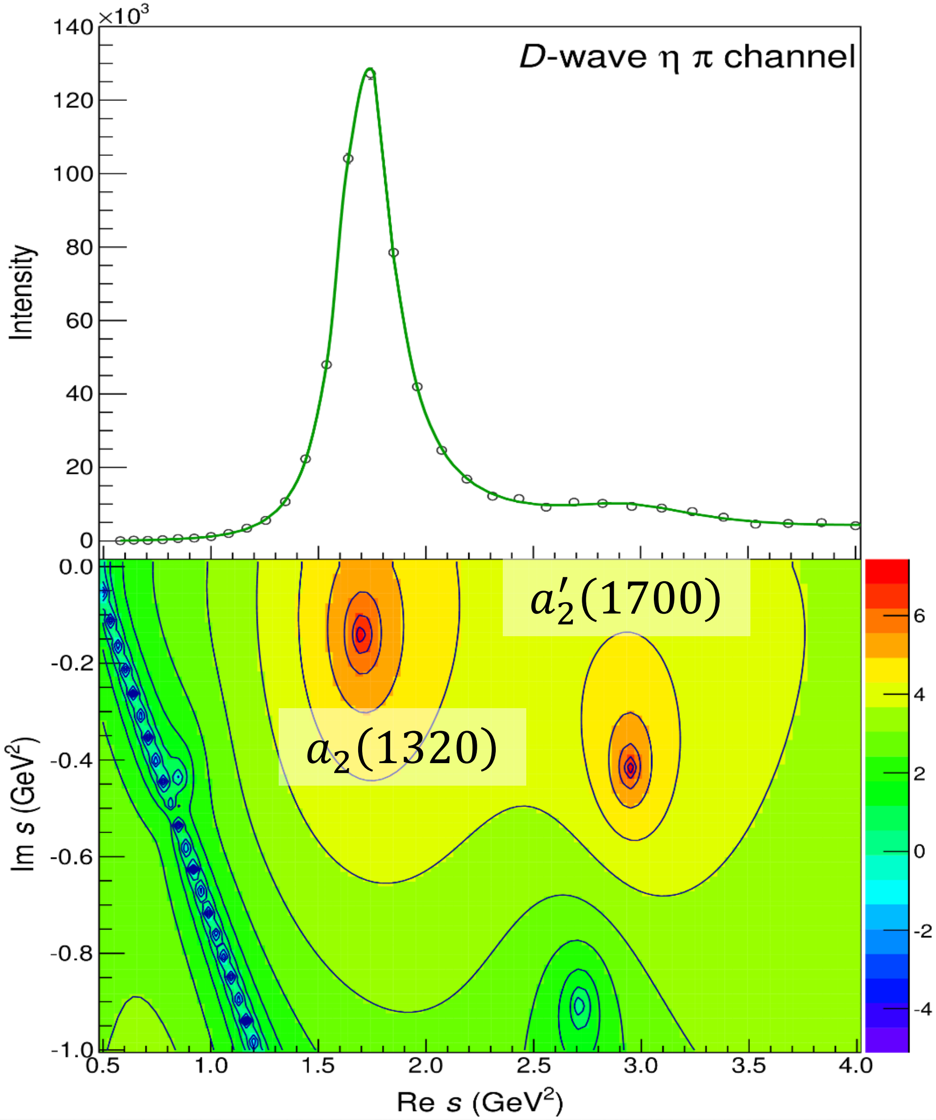


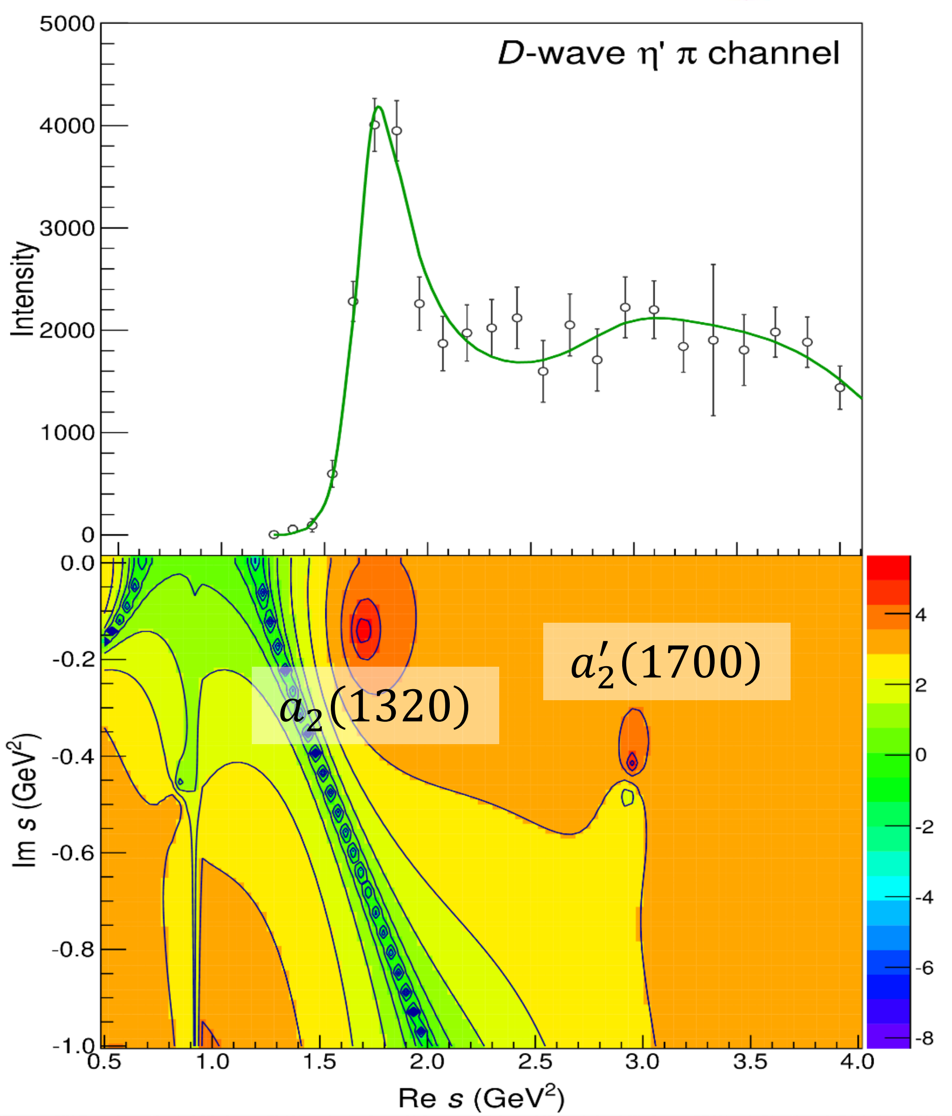
$J^{PC} = 1^{-+}$

$J^{PC} = 2^{++}$

Pole hunting

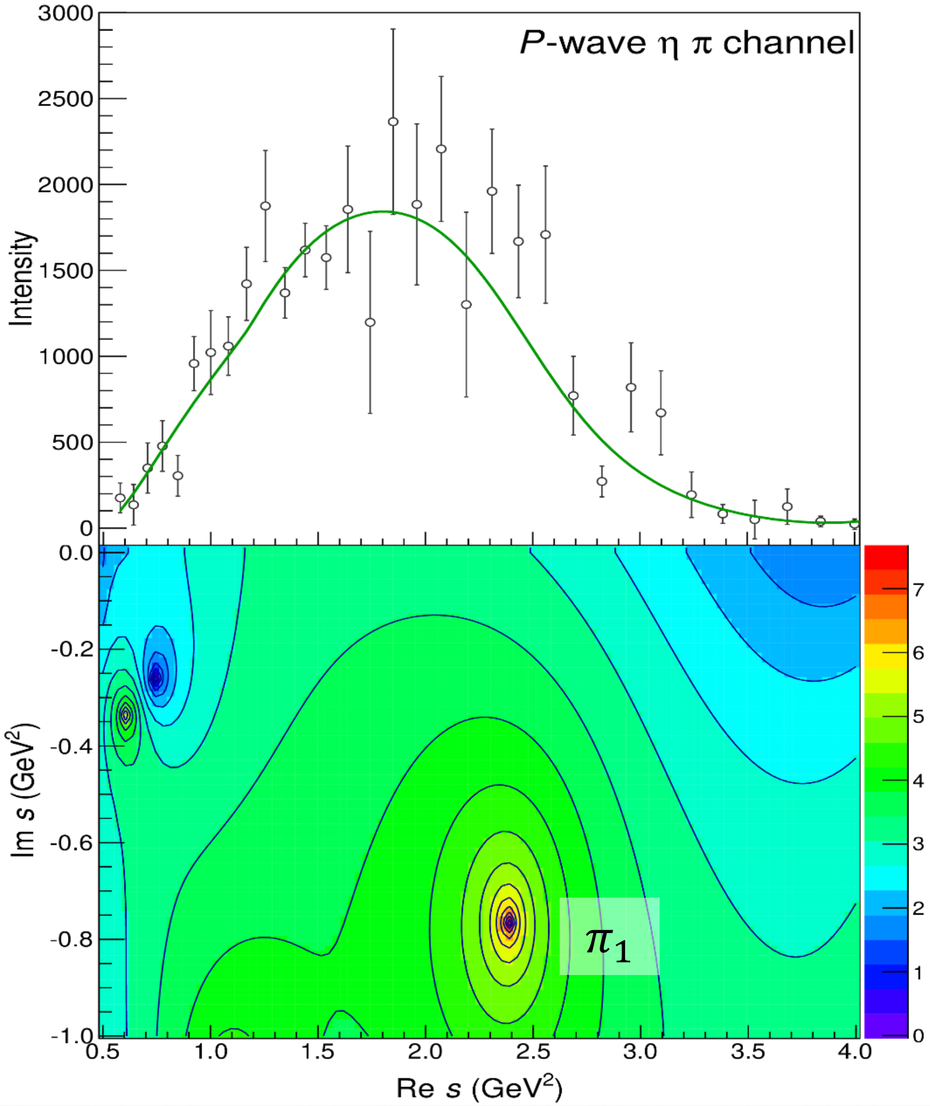
(COMPASS data)

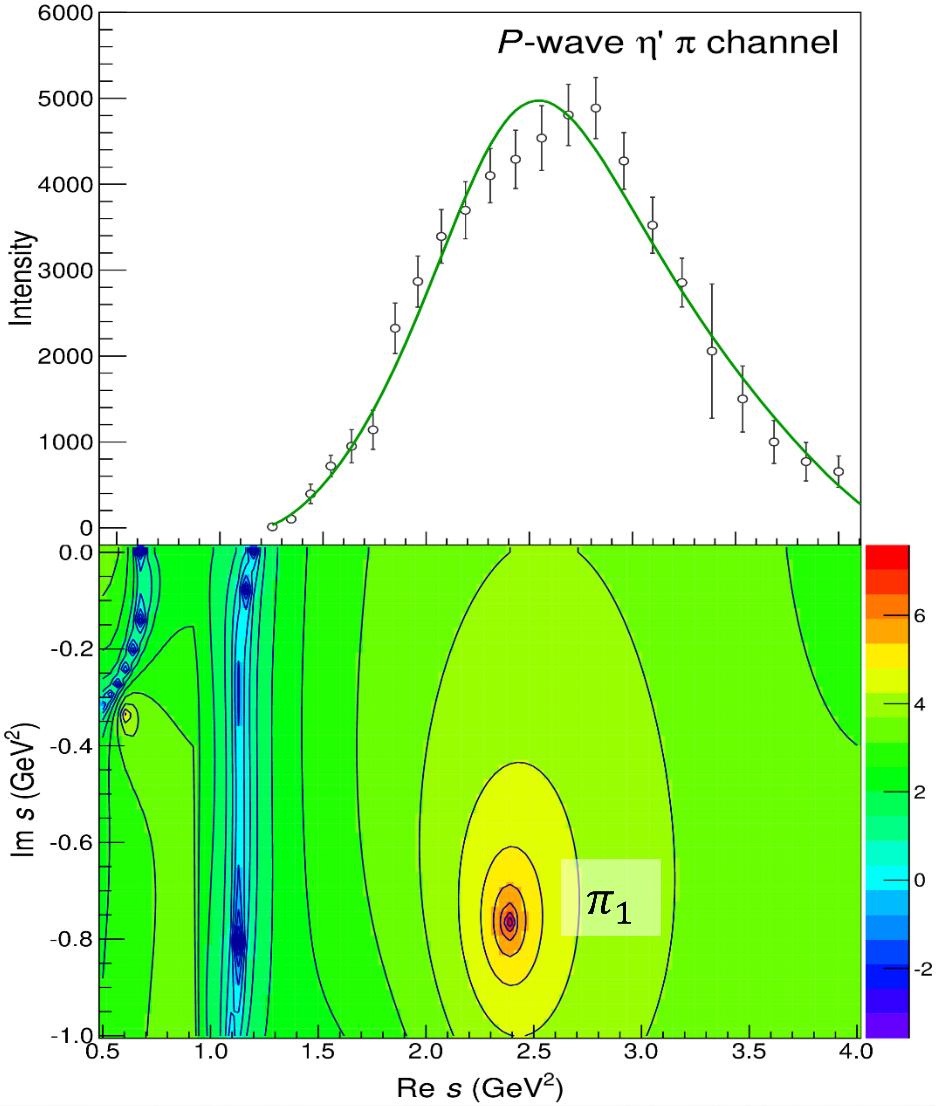




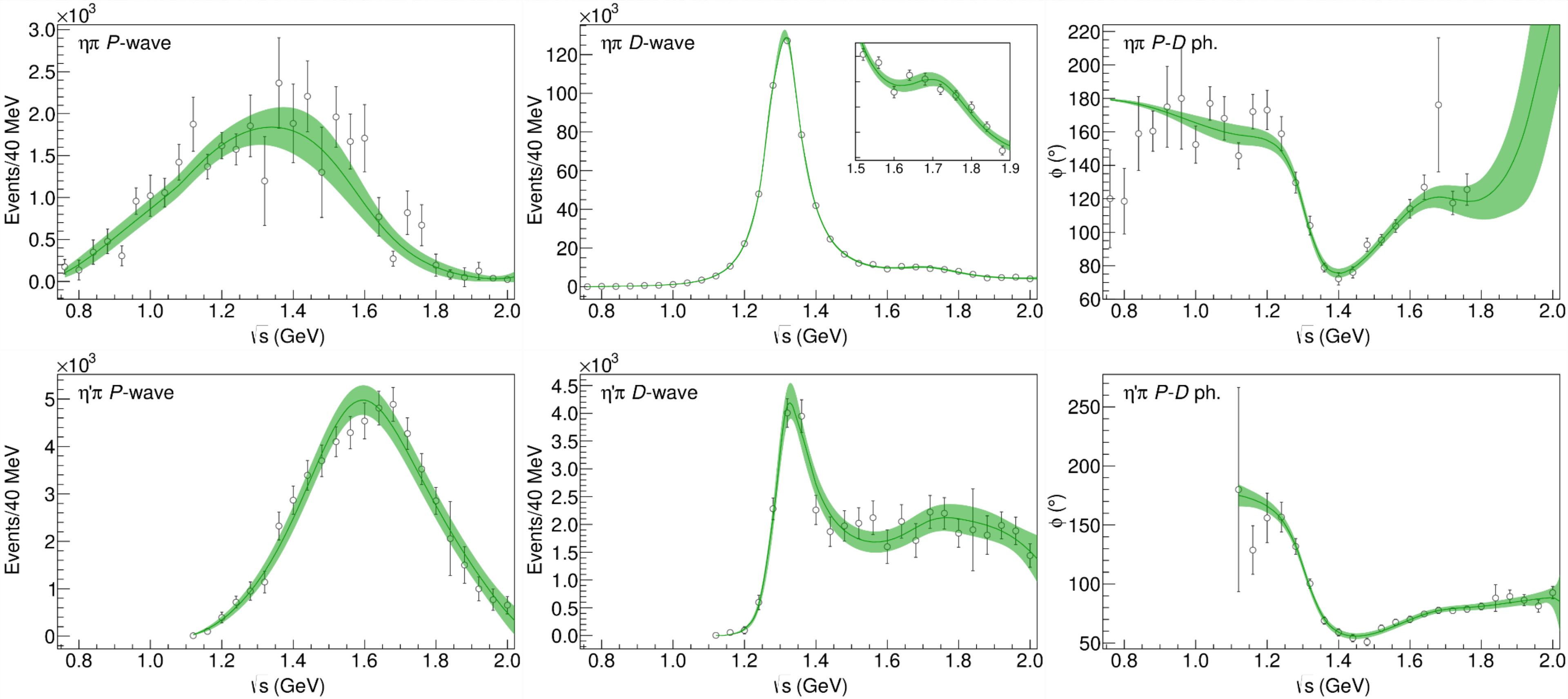
Pole hunting

(COMPASS data)





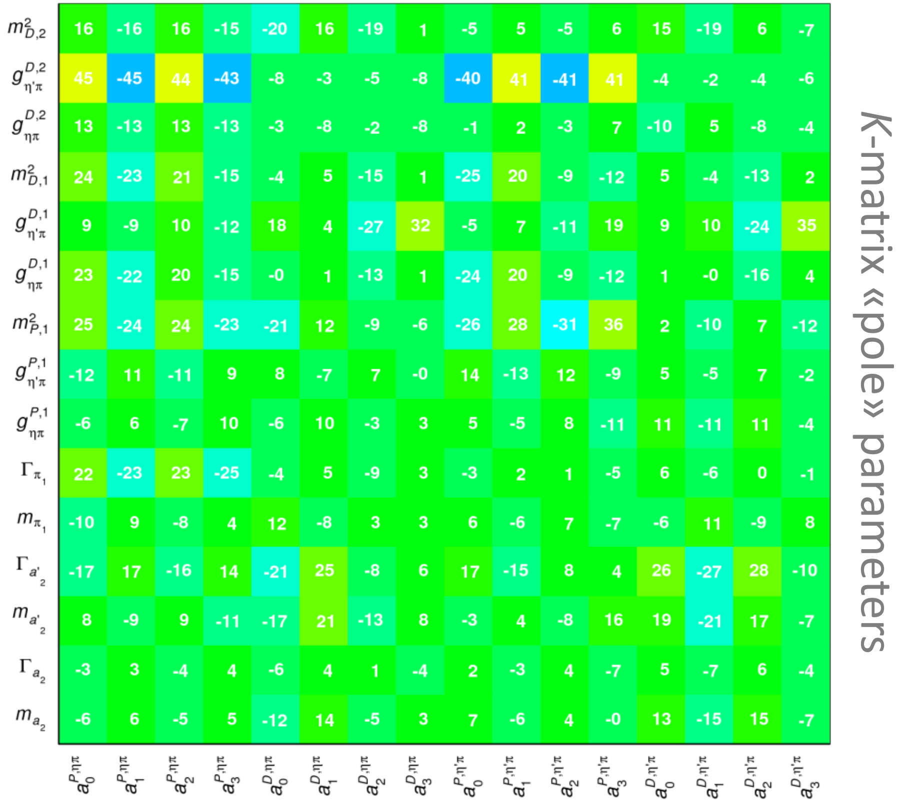
Statistical Bootstrap



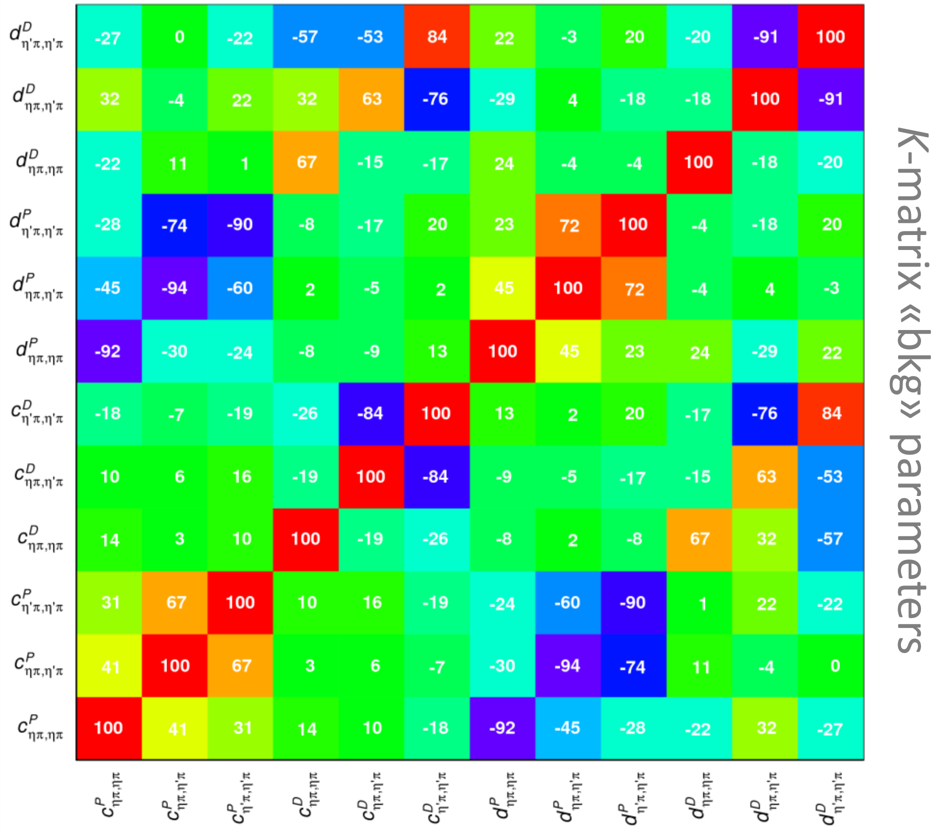
Correlations

Denominator parameters uncorrelated with the numerator ones ✓

Denominator parameters uncorrelated between *P*- and *D*-wave ✓

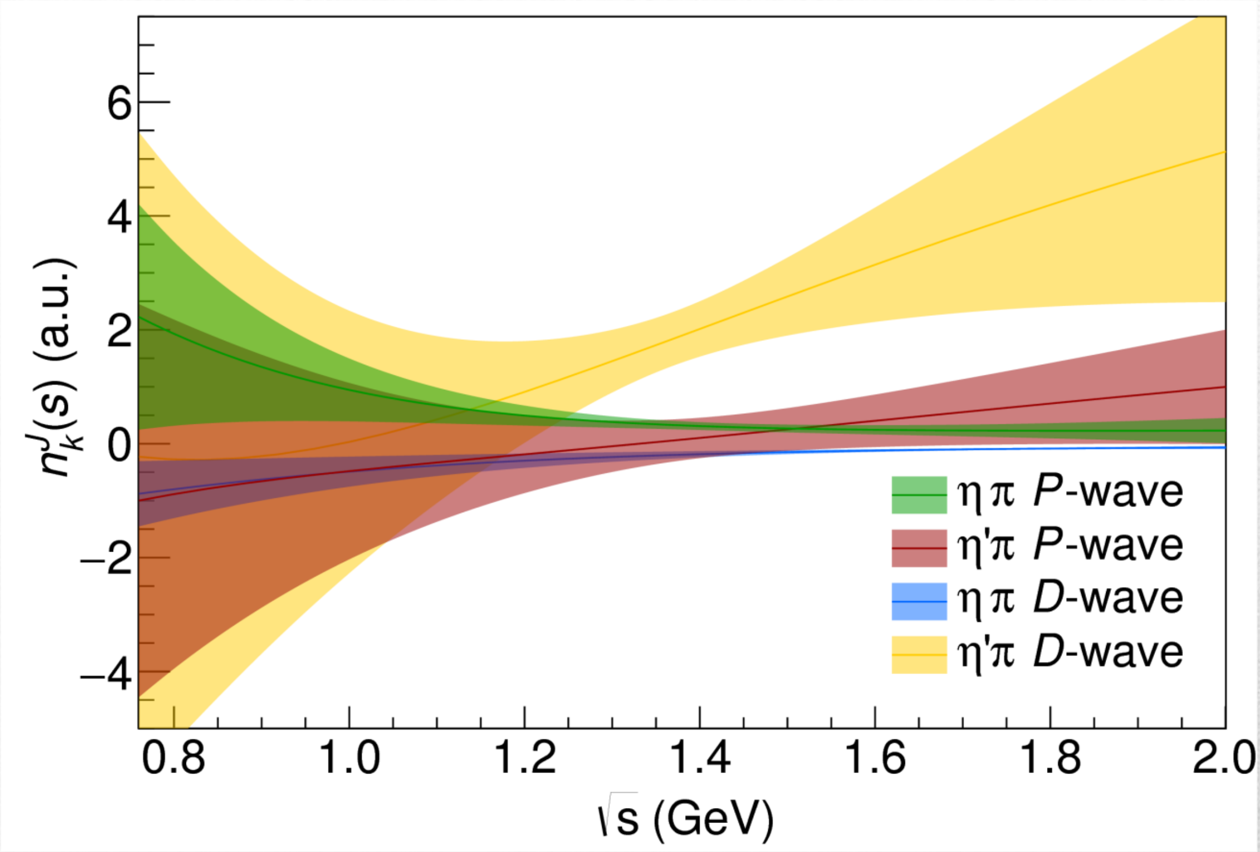

Production (numerator) parameters

	$a_0^{P,\eta\pi}$	$a_1^{P,\eta\pi}$	$a_2^{P,\eta\pi}$	$a_3^{P,\eta\pi}$	$a_0^{D,\eta\pi}$	$a_1^{D,\eta\pi}$	$a_2^{D,\eta\pi}$	$a_3^{D,\eta\pi}$	$a_0^{P,\eta'\pi}$	$a_1^{P,\eta'\pi}$	$a_2^{P,\eta'\pi}$	$a_3^{P,\eta'\pi}$	$a_0^{D,\eta'\pi}$	$a_1^{D,\eta'\pi}$	$a_2^{D,\eta'\pi}$	$a_3^{D,\eta'\pi}$
$m^2_{D,2}$	16	-16	16	-15	-20	16	-19	1	-5	5	-5	6	15	-19	6	-7
$g^{D,2}_{\eta'\pi}$	45	-45	44	-43	-8	-3	-5	-8	-40	41	-41	41	-4	-2	-4	-6
$g^{D,2}_{\eta\pi}$	13	-13	13	-13	-3	-8	-2	-8	-1	2	-3	7	-10	5	-8	-4
$m^2_{D,1}$	24	-23	21	-15	-4	5	-15	1	-25	20	-9	-12	5	-4	-13	2
$g^{D,1}_{\eta'\pi}$	9	-9	10	-12	18	4	-27	32	-5	7	-11	19	9	10	-24	35
$g^{D,1}_{\eta\pi}$	23	-22	20	-15	-0	1	-13	1	-24	20	-9	-12	1	-0	-16	4
$m^2_{P,1}$	25	-24	24	-23	-21	12	-9	-6	-26	28	-31	36	2	-10	7	-12
$g^{P,1}_{\eta'\pi}$	-12	11	-11	9	8	-7	7	-0	14	-13	12	-9	5	-5	7	-2
$g^{P,1}_{\eta\pi}$	-6	6	-7	10	-6	10	-3	3	5	-5	8	-11	11	-11	11	-4
Γ_{π_1}	22	-23	23	-25	-4	5	-9	3	-3	2	1	-5	6	-6	0	-1
m_{π_1}	-10	9	-8	4	12	-8	3	3	6	-6	7	-7	-6	11	-9	8
$\Gamma_{a'_2}$	-17	17	-16	14	-21	25	-8	6	17	-15	8	4	26	-27	28	-10
$m_{a'_2}$	8	-9	9	-11	-17	21	-13	8	-3	4	-8	16	19	-21	17	-7
Γ_{a_2}	-3	3	-4	4	-6	4	1	-4	2	-3	4	-7	5	-7	6	-4
m_{a_2}	-6	6	-5	5	-12	14	-5	3	7	-6	4	-0	13	-15	15	-7


K-matrix «bkg» parameters

	$c^P_{\eta\pi,\eta\pi}$	$c^P_{\eta\pi,\eta'\pi}$	$c^P_{\eta'\pi,\eta'\pi}$	$c^D_{\eta\pi,\eta\pi}$	$c^D_{\eta\pi,\eta'\pi}$	$c^D_{\eta'\pi,\eta'\pi}$	$d^P_{\eta\pi,\eta\pi}$	$d^P_{\eta\pi,\eta'\pi}$	$d^P_{\eta'\pi,\eta'\pi}$	$d^D_{\eta\pi,\eta\pi}$	$d^D_{\eta\pi,\eta'\pi}$	$d^D_{\eta'\pi,\eta'\pi}$
$d^D_{\eta'\pi,\eta'\pi}$	-27	0	-22	-57	-53	84	22	-3	20	-20	-91	100
$d^D_{\eta\pi,\eta'\pi}$	32	-4	22	32	63	-76	-29	4	-18	-18	100	-91
$d^D_{\eta\pi,\eta\pi}$	-22	11	1	67	-15	-17	24	-4	-4	100	-18	-20
$d^P_{\eta'\pi,\eta'\pi}$	-28	-74	-90	-8	-17	20	23	72	100	-4	-18	20
$d^P_{\eta\pi,\eta'\pi}$	-45	-94	-60	2	-5	2	45	100	72	-4	4	-3
$d^P_{\eta\pi,\eta\pi}$	-92	-30	-24	-8	-9	13	100	45	23	24	-29	22
$c^D_{\eta'\pi,\eta'\pi}$	-18	-7	-19	-26	-84	100	13	2	20	-17	-76	84
$c^D_{\eta\pi,\eta'\pi}$	10	6	16	-19	100	-84	-9	-5	-17	-15	63	-53
$c^D_{\eta\pi,\eta\pi}$	14	3	10	100	-19	-26	-8	2	-8	67	32	-57
$c^P_{\eta'\pi,\eta'\pi}$	31	67	100	10	16	-19	-24	-60	-90	1	22	-22
$c^P_{\eta\pi,\eta'\pi}$	41	100	67	3	6	-7	-30	-94	-74	11	-4	0
$c^P_{\eta\pi,\eta\pi}$	100	41	31	14	10	-18	-92	-45	-28	-22	32	-27

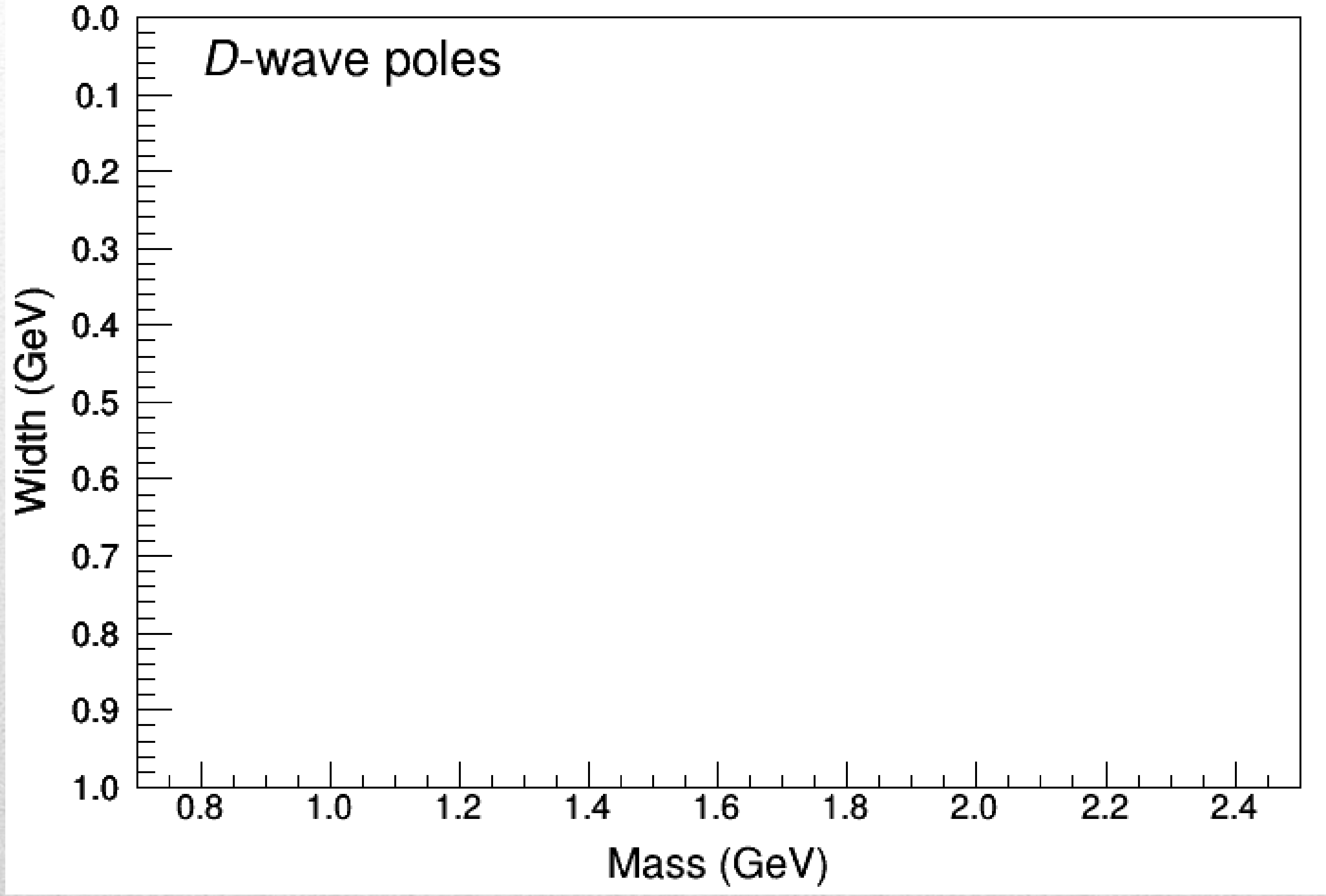
Polynomial in the numerator



The numerator should be smooth and have variation milder that the typical resonance width

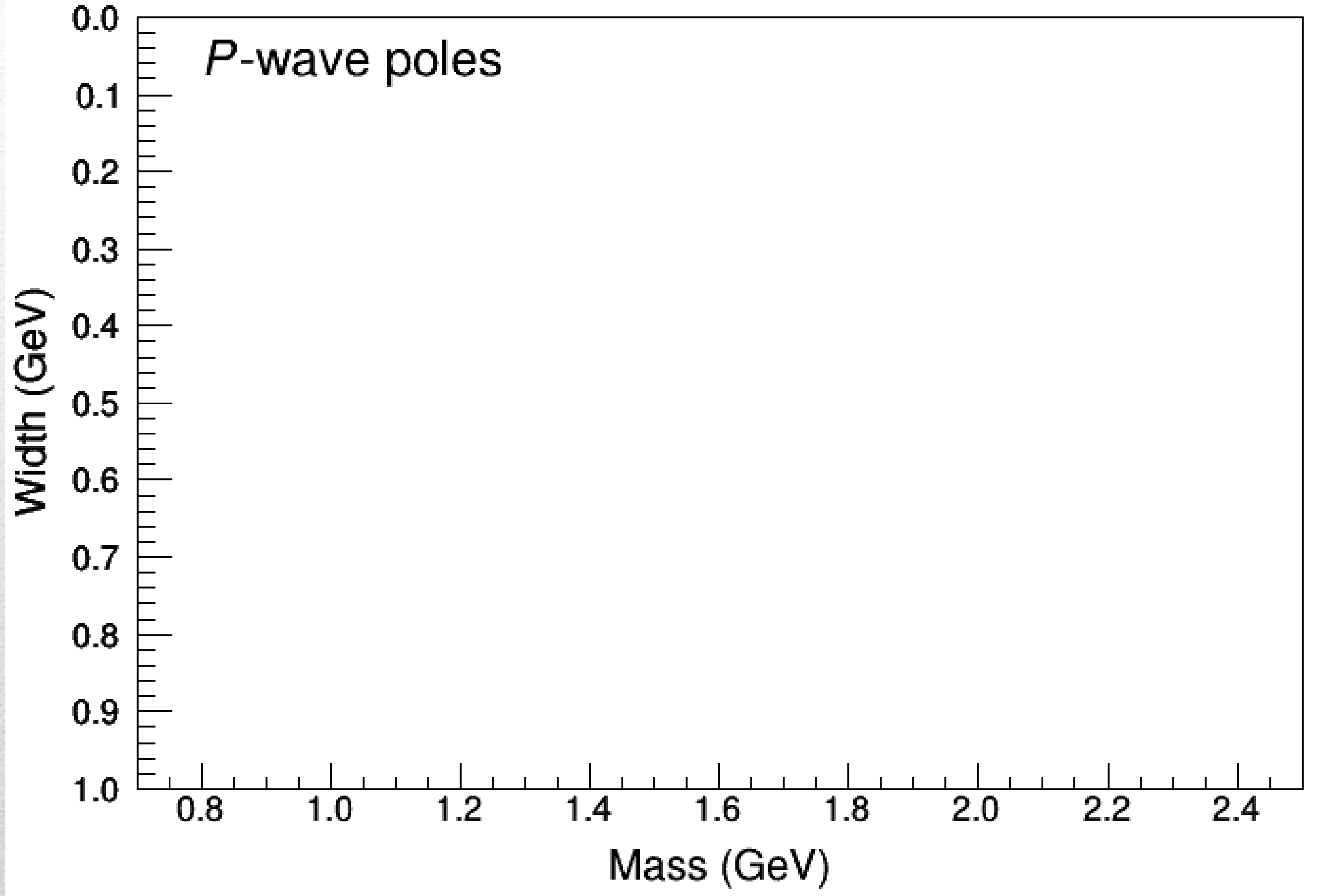
This happens indeed

Statistical Bootstrap



For each fit, we search poles: two clusters in *D*-wave: $a_2(1320)$ and $a_2'(1700)$

Statistical Bootstrap



Only one stable cluster in P-wave: a single π_1

Systematic studies

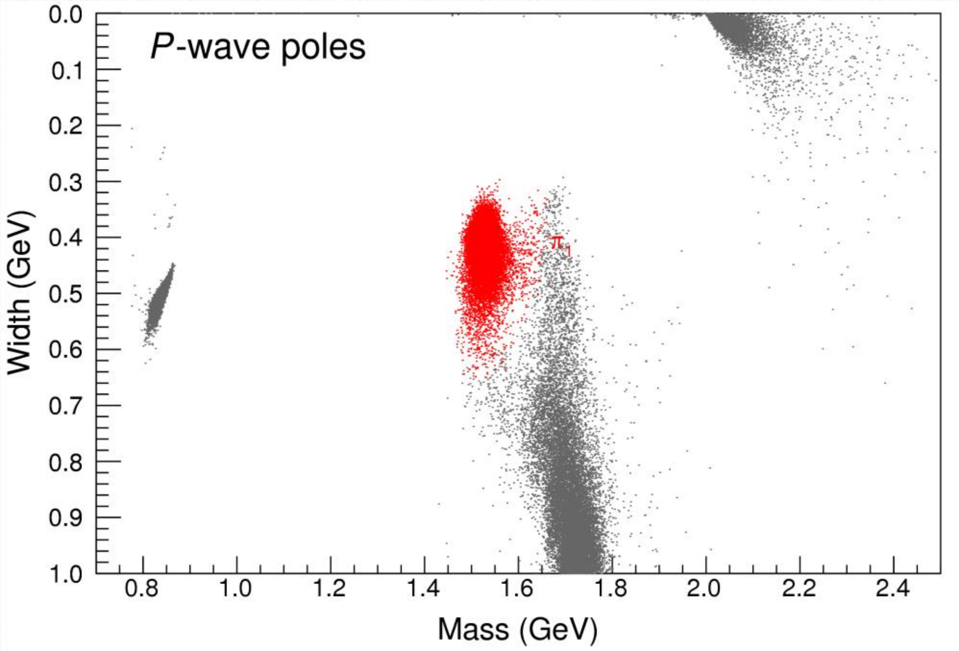
- Change of functional form and parameters in the denominator

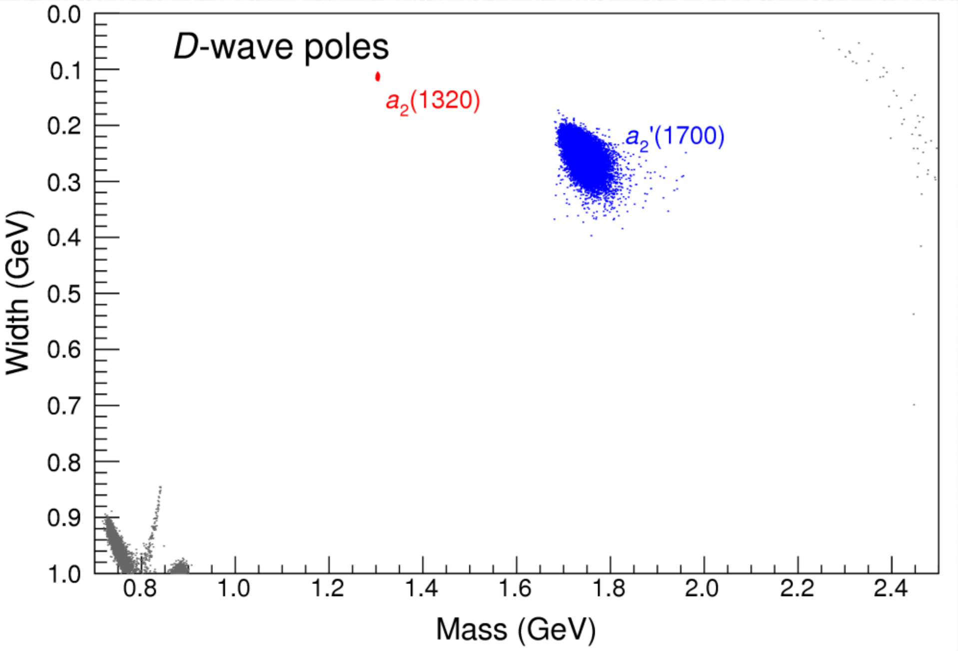
$$\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{\lambda^{J+1/2}\left(s', m_{\eta^{(\prime)}}^2, m_\pi^2\right)}{(s'+s_R)^{2J+1+\alpha}}$$

  - Default: $s_R = 1\ \mathrm{GeV}^2$. We try $s_R = 0.8, 1.8\ \mathrm{GeV}^2$
  - Default: $\alpha = 2$. We try $\alpha = 1$
  - We also try a different function: $\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{Q_J(z_{s'})}{s'^{\alpha}\lambda^{1/2}(s', m_{\eta^{(\prime)}}, m_\pi)}$ with $\alpha = 2, 1.5, 1$

- Change of parameters in the numerator
  - Default: $t_{\text{eff}} = -0.1\ \mathrm{GeV}^2$. We try$t_{\text{eff}} = -0.5\ \mathrm{GeV}^2$
  - Default: 3rd order polynomial. We try 4th

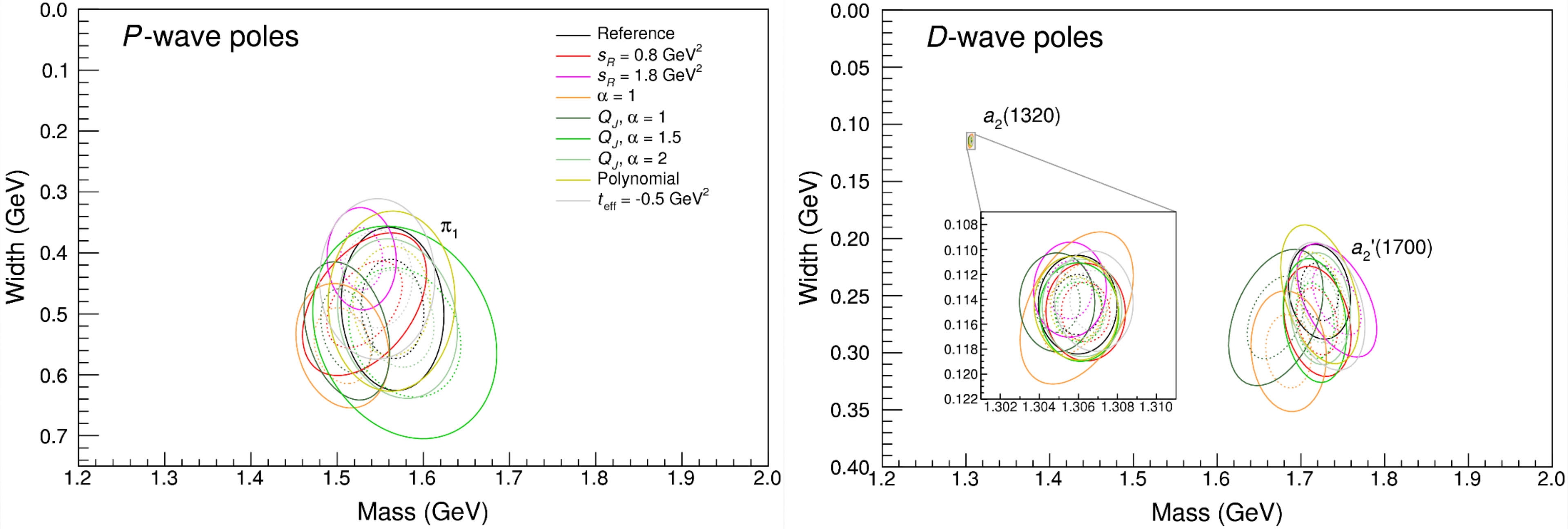
Bootstrap for $s_R = 1.8\ \text{GeV}^2$





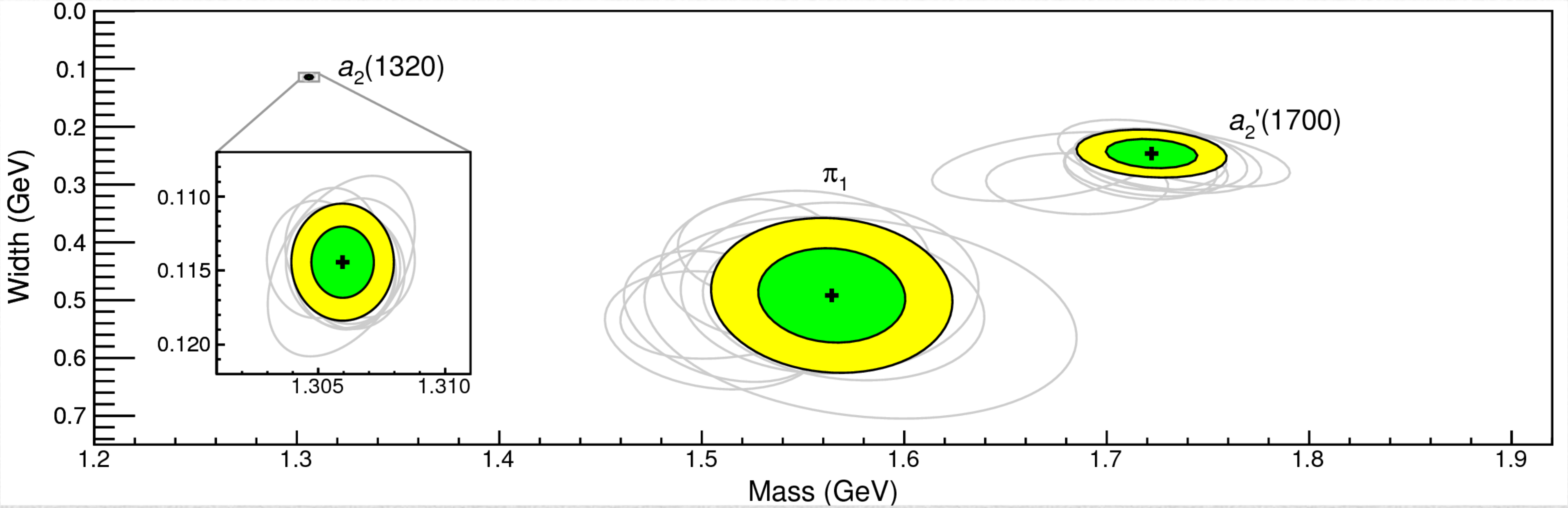
Our skepticism about a second pole in the relevant region is confirmed:
It is unstable and not trustable

Systematic studies



For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

Final results



Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

Agreement with theory is restored

That’s the most rigorous extraction of an exotic meson available so far!

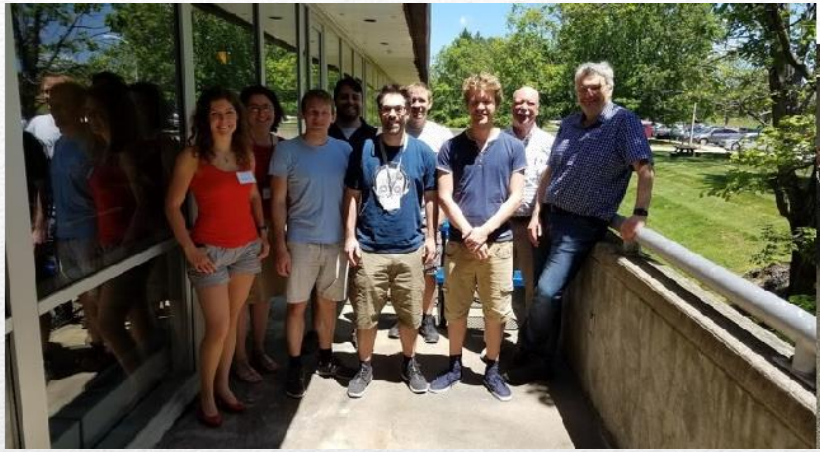
Conclusions

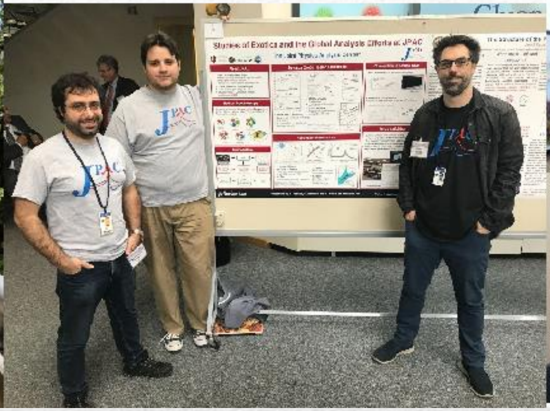
Bottom-up approaches are important!

- They allow us to get the most out of high statistics data!
- The study of analytic stuctrures offer insights ino the nature of resonances
- Dispersive methods can improve the rigour and robustness in the extraction of the spectrum

Thank you!

Joint Physics Analysis Center

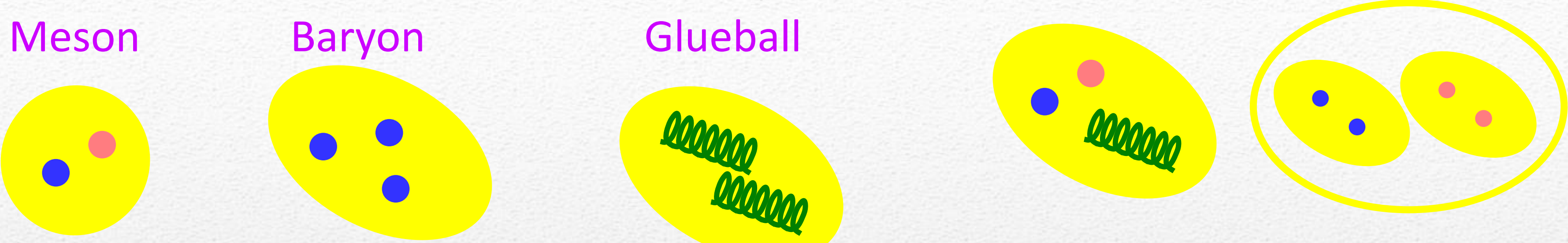


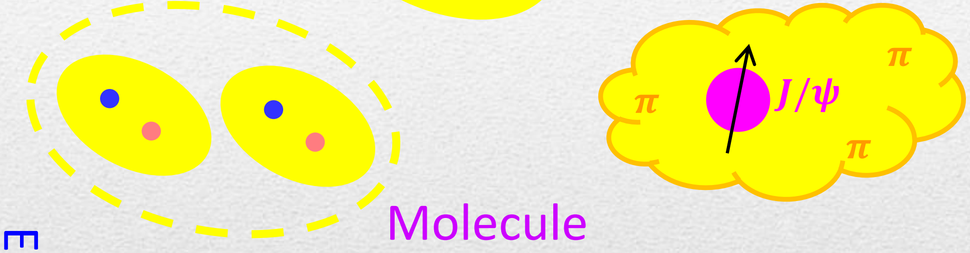


JPAC

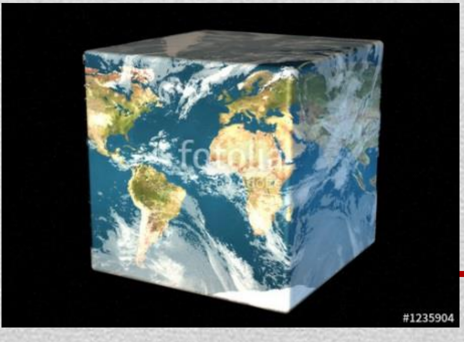
BACKUP

Hadron Spectroscopy





Experiment



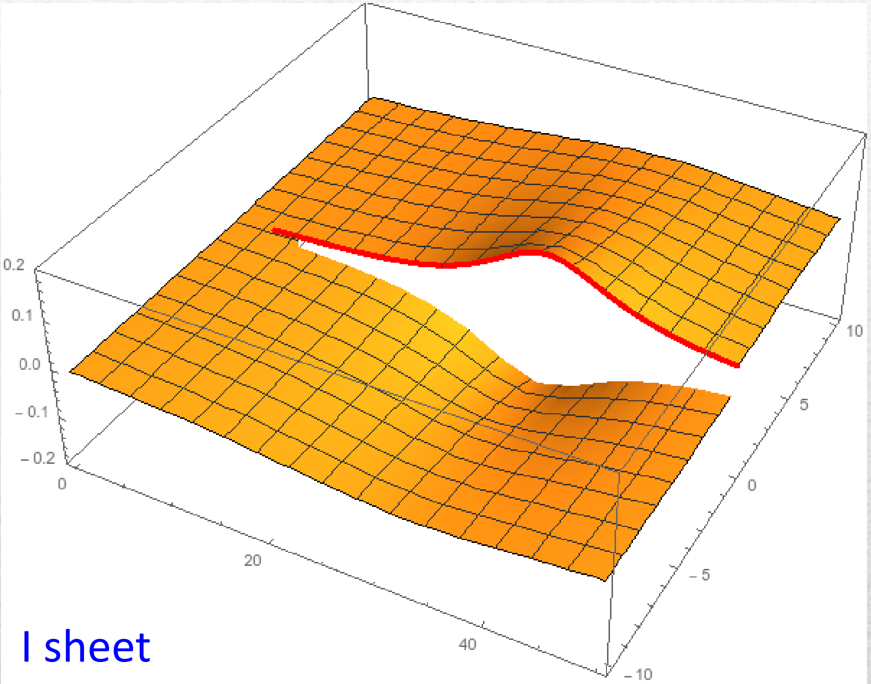
Lattice QCD

Data → Amplitude analysis → Properties, Model building

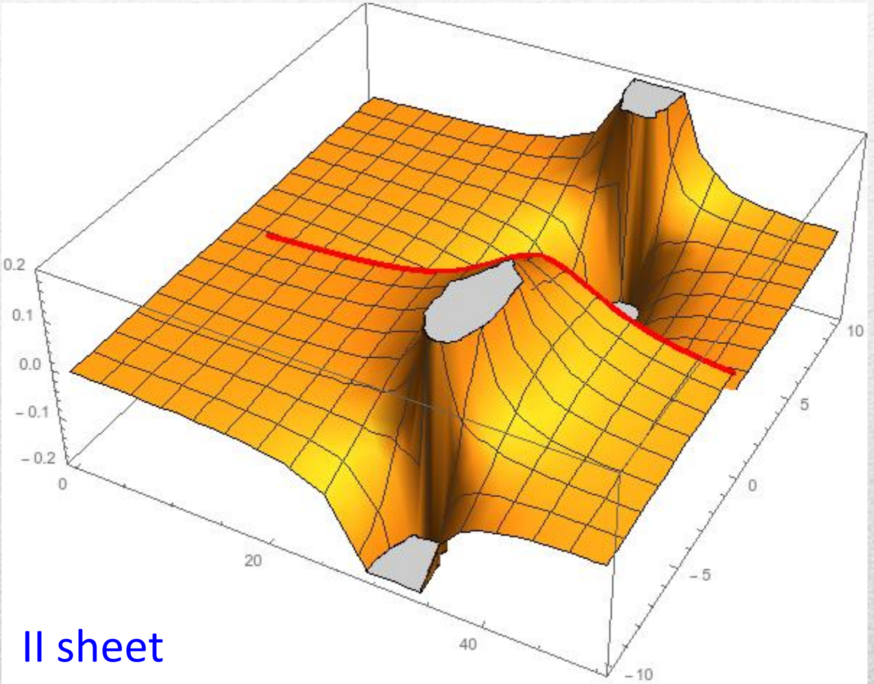
Interpretations on the spectrum leads to understanding fundamental laws of nature

Unitarity & Pole hunting

Unitarity creates a branch cut on the real axis, two sheets continuosly connected



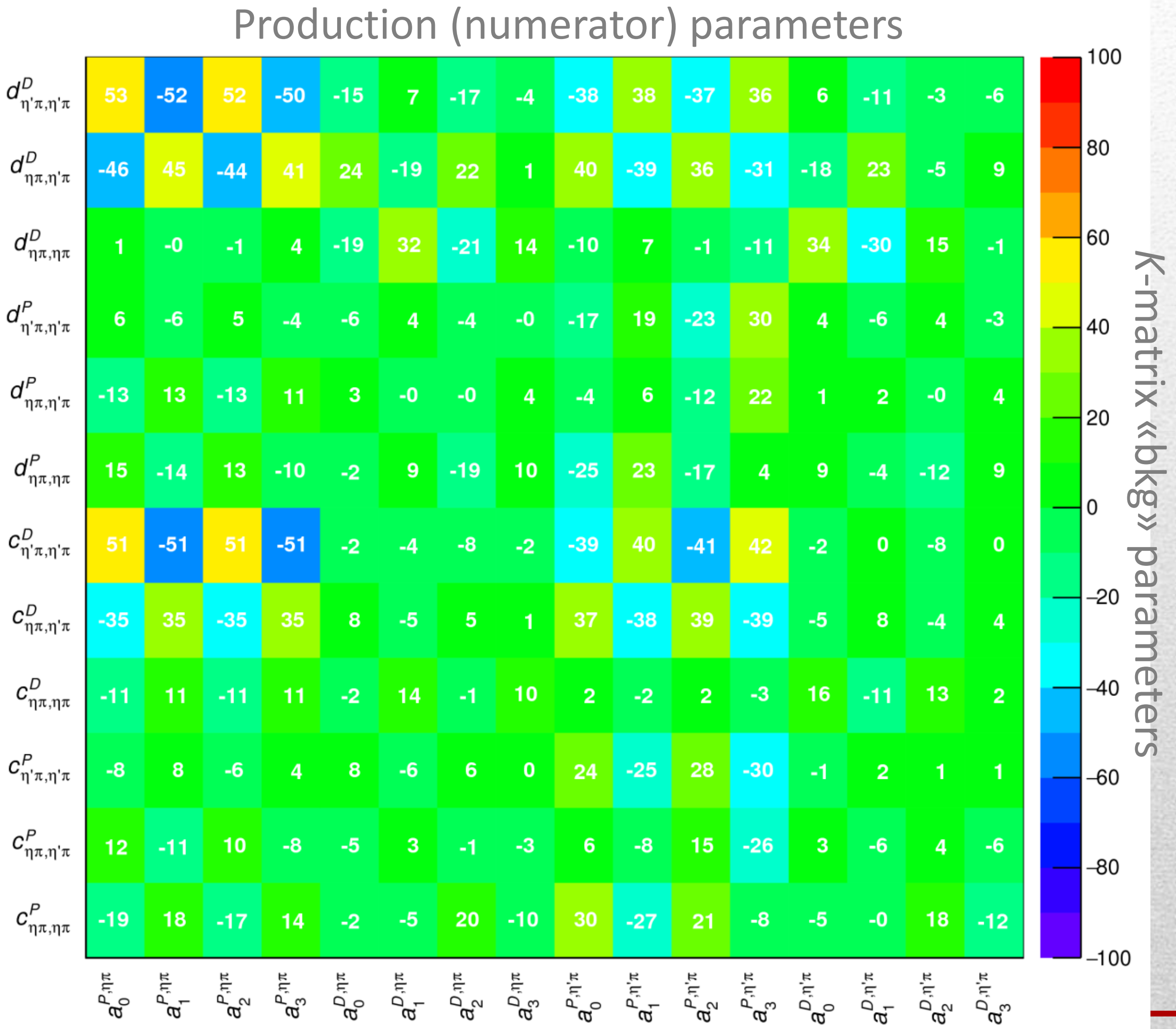
I sheet



II sheet

Finding resonances means writing analytic amplitude, and hunting for poles in the complex plane

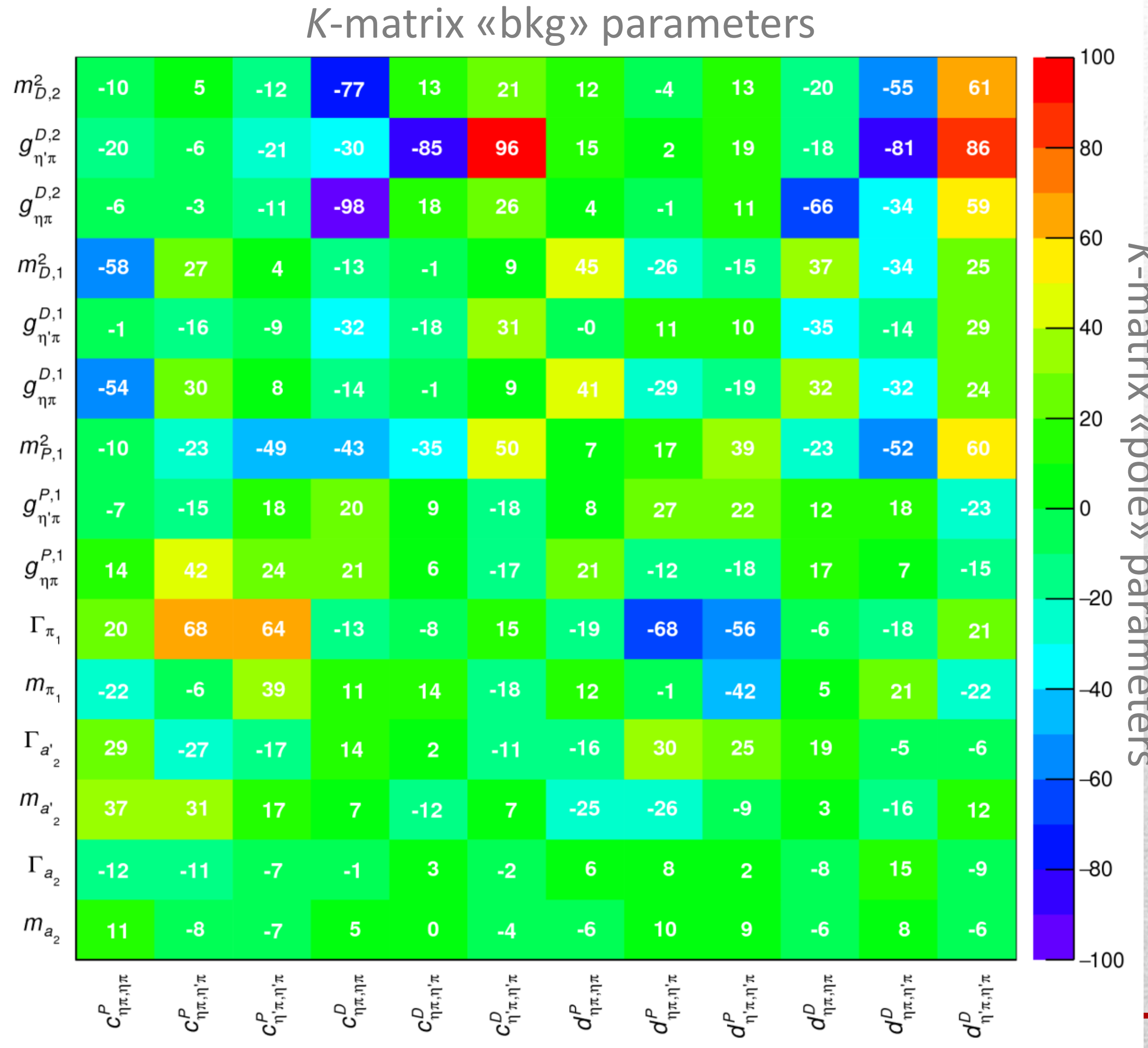
Correlations



	$a_0^{P,\eta\pi}$	$a_1^{P,\eta\pi}$	$a_2^{P,\eta\pi}$	$a_3^{P,\eta\pi}$	$a_0^{D,\eta\pi}$	$a_1^{D,\eta\pi}$	$a_2^{D,\eta\pi}$	$a_3^{D,\eta\pi}$	$a_0^{P,\eta'\pi}$	$a_1^{P,\eta'\pi}$	$a_2^{P,\eta'\pi}$	$a_3^{P,\eta'\pi}$	$a_0^{D,\eta'\pi}$	$a_1^{D,\eta'\pi}$	$a_2^{D,\eta'\pi}$	$a_3^{D,\eta'\pi}$
$d^D_{\eta'\pi,\eta'\pi}$	53	-52	52	-50	-15	7	-17	-4	-38	38	-37	36	6	-11	-3	-6
$d^D_{\eta\pi,\eta'\pi}$	-46	45	-44	41	24	-19	22	1	40	-39	36	-31	-18	23	-5	9
$d^D_{\eta\pi,\eta\pi}$	1	-0	-1	4	-19	32	-21	14	-10	7	-1	-11	34	-30	15	-1
$d^P_{\eta'\pi,\eta'\pi}$	6	-6	5	-4	-6	4	-4	-0	-17	19	-23	30	4	-6	4	-3
$d^P_{\eta\pi,\eta'\pi}$	-13	13	-13	11	3	-0	-0	4	-4	6	-12	22	1	2	-0	4
$d^P_{\eta\pi,\eta\pi}$	15	-14	13	-10	-2	9	-19	10	-25	23	-17	4	9	-4	-12	9
$c^D_{\eta'\pi,\eta'\pi}$	51	-51	51	-51	-2	-4	-8	-2	-39	40	-41	42	-2	0	-8	0
$c^D_{\eta\pi,\eta'\pi}$	-35	35	-35	35	8	-5	5	1	37	-38	39	-39	-5	8	-4	4
$c^D_{\eta\pi,\eta\pi}$	-11	11	-11	11	-2	14	-1	10	2	-2	2	-3	16	-11	13	2
$c^P_{\eta'\pi,\eta'\pi}$	-8	8	-6	4	8	-6	6	0	24	-25	28	-30	-1	2	1	1
$c^P_{\eta\pi,\eta'\pi}$	12	-11	10	-8	-5	3	-1	-3	6	-8	15	-26	3	-6	4	-6
$c^P_{\eta\pi,\eta\pi}$	-19	18	-17	14	-2	-5	20	-10	30	-27	21	-8	-5	-0	18	-12

Denominator parameters not very correlated with the numerator ones ✓

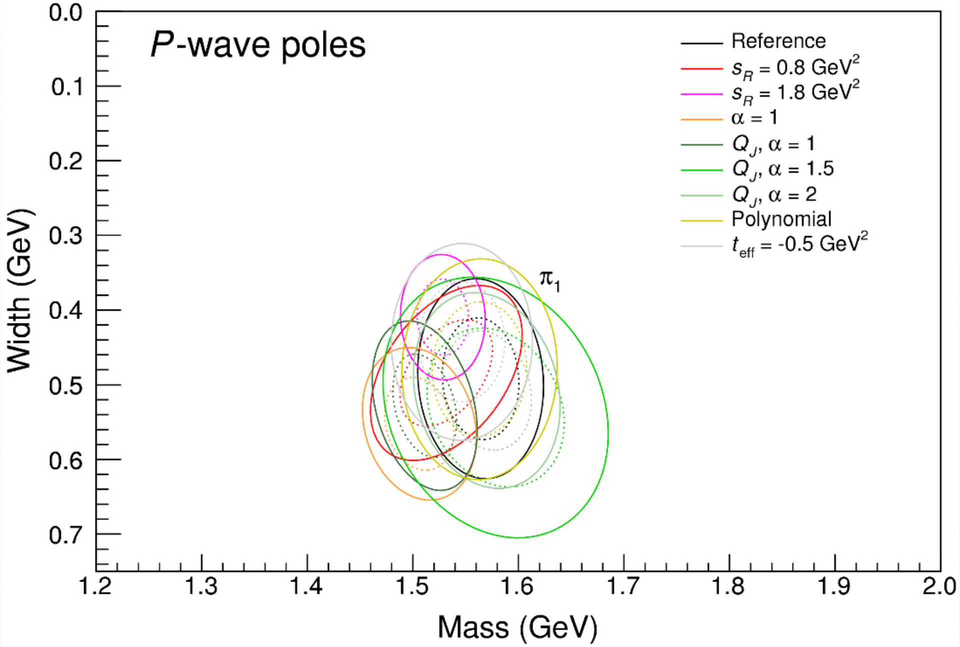
Correlations

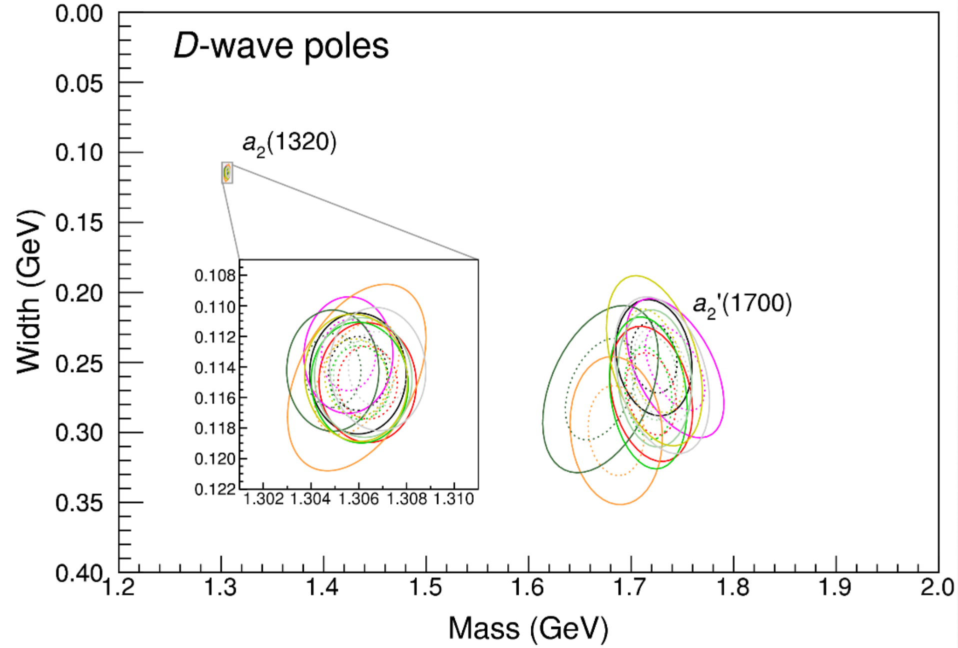


	$c^P_{\eta\pi,\eta\pi}$	$c^P_{\eta\pi,\eta'\pi}$	$c^P_{\eta'\pi,\eta'\pi}$	$c^D_{\eta\pi,\eta\pi}$	$c^D_{\eta\pi,\eta'\pi}$	$c^D_{\eta'\pi,\eta'\pi}$	$d^P_{\eta\pi,\eta\pi}$	$d^P_{\eta\pi,\eta'\pi}$	$d^P_{\eta'\pi,\eta'\pi}$	$d^D_{\eta\pi,\eta\pi}$	$d^D_{\eta\pi,\eta'\pi}$	$d^D_{\eta'\pi,\eta'\pi}$
$m^2_{D,2}$	-10	5	-12	-77	13	21	12	-4	13	-20	-55	61
$g^{D,2}_{\eta'\pi}$	-20	-6	-21	-30	-85	96	15	2	19	-18	-81	86
$g^{D,2}_{\eta\pi}$	-6	-3	-11	-98	18	26	4	-1	11	-66	-34	59
$m^2_{D,1}$	-58	27	4	-13	-1	9	45	-26	-15	37	-34	25
$g^{D,1}_{\eta'\pi}$	-1	-16	-9	-32	-18	31	-0	11	10	-35	-14	29
$g^{D,1}_{\eta\pi}$	-54	30	8	-14	-1	9	41	-29	-19	32	-32	24
$m^2_{P,1}$	-10	-23	-49	-43	-35	50	7	17	39	-23	-52	60
$g^{P,1}_{\eta'\pi}$	-7	-15	18	20	9	-18	8	27	22	12	18	-23
$g^{P,1}_{\eta\pi}$	14	42	24	21	6	-17	21	-12	-18	17	7	-15
Γ_{π_1}	20	68	64	-13	-8	15	-19	-68	-56	-6	-18	21
m_{π_1}	-22	-6	39	11	14	-18	12	-1	-42	5	21	-22
$\Gamma_{a'_2}$	29	-27	-17	14	2	-11	-16	30	25	19	-5	-6
$m_{a'_2}$	37	31	17	7	-12	7	-25	-26	-9	3	-16	12
Γ_{a_2}	-12	-11	-7	-1	3	-2	6	8	2	-8	15	-9
m_{a_2}	11	-8	-7	5	0	-4	-6	10	9	-6	8	-6

Denominator parameters uncorrelated between *P*- and *D*-wave ✓

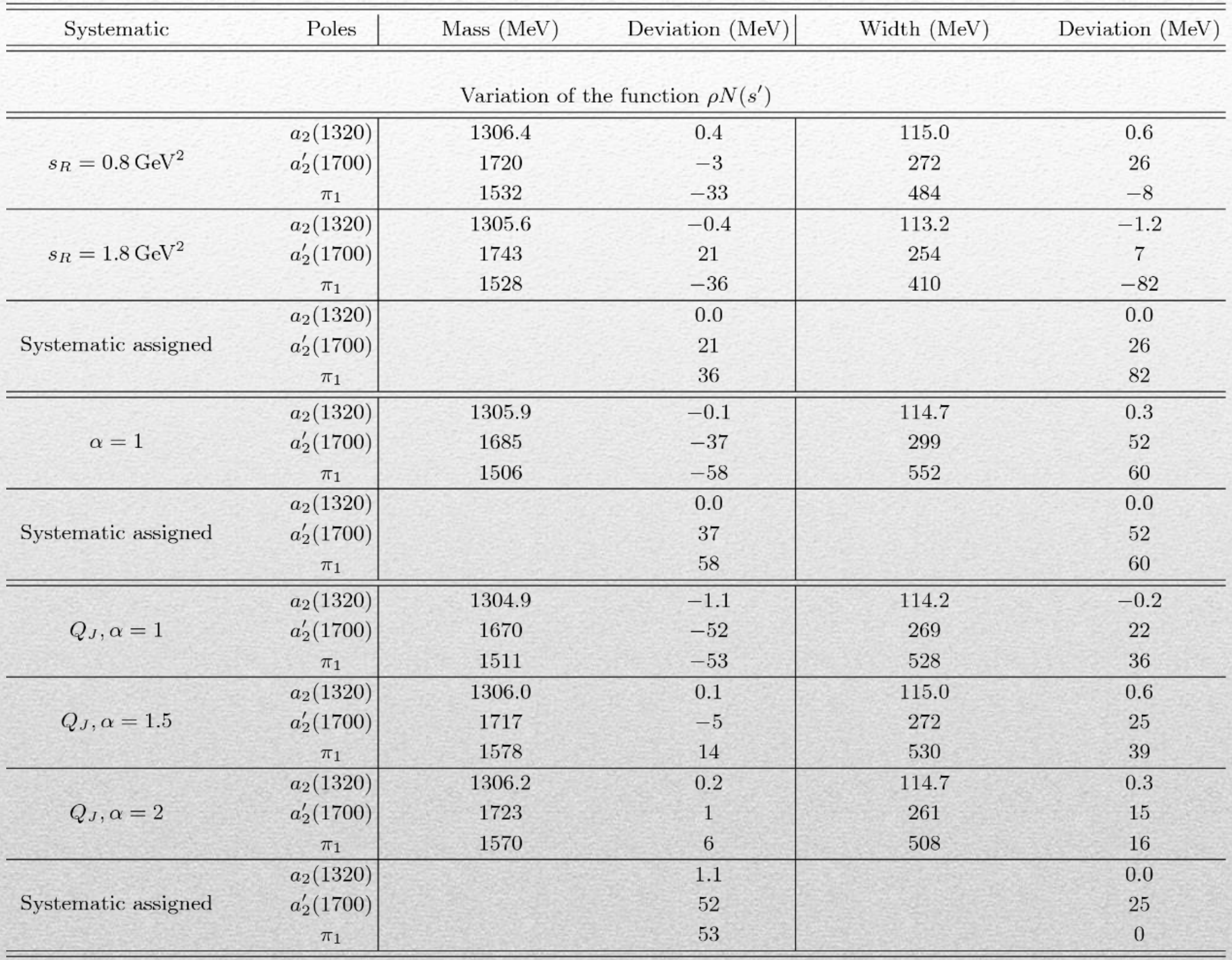
Systematic studies





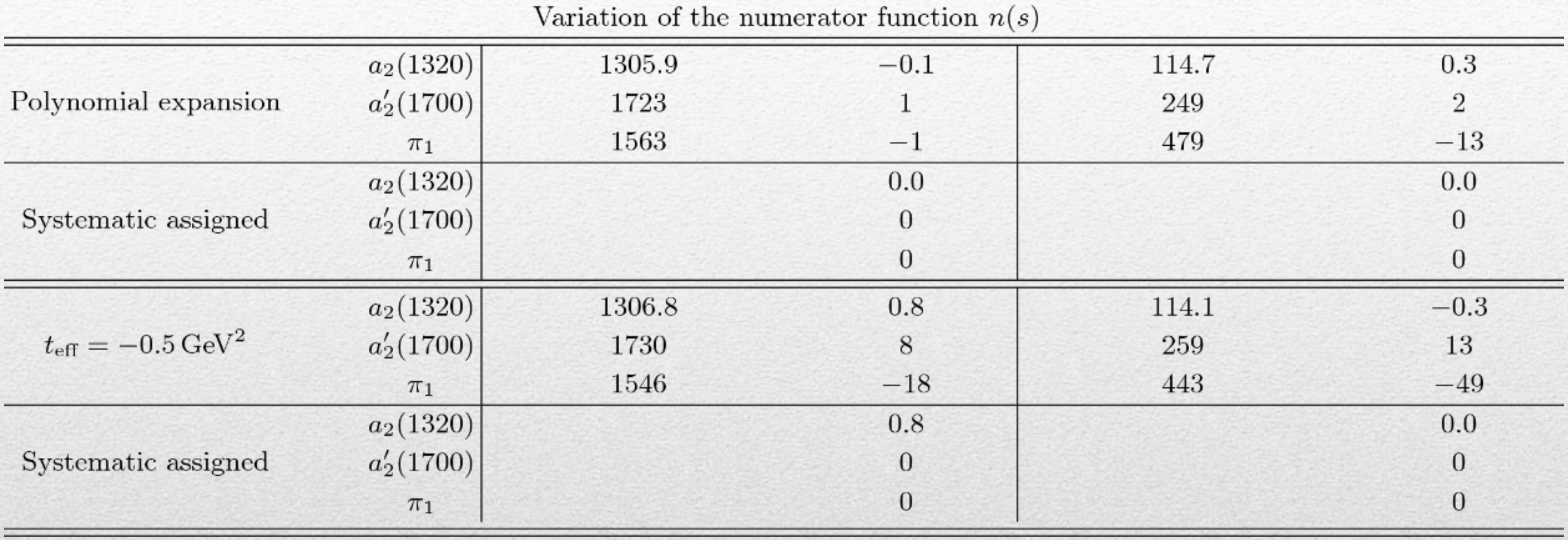
For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

Systematic studies



Systematic	Poles	Mass (MeV)	Deviation (MeV)	Width (MeV)	Deviation (MeV)
Variation of the function $\rho N(s')$					
$s_R = 0.8\,\mathrm{GeV}^2$	$a_2(1320)$	1306.4	0.4	115.0	0.6
	$a_2'(1700)$	1720	-3	272	26
	π_1	1532	-33	484	-8
$s_R = 1.8\,\mathrm{GeV}^2$	$a_2(1320)$	1305.6	-0.4	113.2	-1.2
	$a_2'(1700)$	1743	21	254	7
	π_1	1528	-36	410	-82
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		21		26
	π_1		36		82
$\alpha = 1$	$a_2(1320)$	1305.9	-0.1	114.7	0.3
	$a_2'(1700)$	1685	-37	299	52
	π_1	1506	-58	552	60
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		37		52
	π_1		58		60
$Q_J, \alpha = 1$	$a_2(1320)$	1304.9	-1.1	114.2	-0.2
	$a_2'(1700)$	1670	-52	269	22
	π_1	1511	-53	528	36
$Q_J, \alpha = 1.5$	$a_2(1320)$	1306.0	0.1	115.0	0.6
	$a_2'(1700)$	1717	-5	272	25
	π_1	1578	14	530	39
$Q_J, \alpha = 2$	$a_2(1320)$	1306.2	0.2	114.7	0.3
	$a_2'(1700)$	1723	1	261	15
	π_1	1570	6	508	16
Systematic assigned	$a_2(1320)$		1.1		0.0
	$a_2'(1700)$		52		25
	π_1		53		0

Systematic studies



Variation of the numerator function $n(s)$

Polynomial expansion	$a_2(1320)$	1305.9	-0.1	114.7	0.3
	$a_2'(1700)$	1723	1	249	2
	π_1	1563	-1	479	-13
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		0		0
	π_1		0		0
$t_{\text{eff}} = -0.5\,\text{GeV}^2$	$a_2(1320)$	1306.8	0.8	114.1	-0.3
	$a_2'(1700)$	1730	8	259	13
	π_1	1546	-18	443	-49
Systematic assigned	$a_2(1320)$		0.8		0.0
	$a_2'(1700)$		0		0
	π_1		0		0