Hadron Spectrum and Form Factors in Continuum QCD

International Workshop on Partial Wave Analyses and Advanced Tools for Hadron Spectroscopy, PWA11/ATHOS6

Centro Brasileiro de Pesquisas Físicas

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FAPESP
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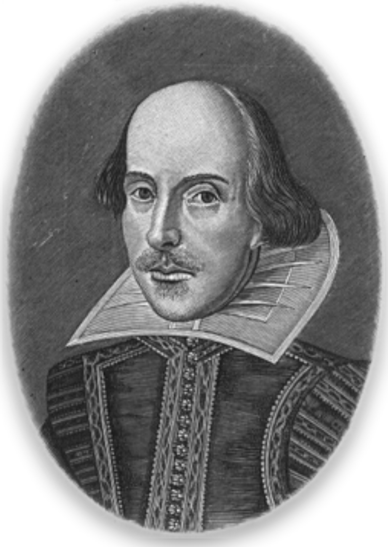
Much Excitement About Nothing?

adapted freely from William Shakespeare



- ***A central goal of Nuclear Physics: understand the properties of hadrons in terms of the elementary excitations in Quantum Chromodynamics (QCD): quarks and gluons.***
- ***Why excited states?***
- ***Spectroscopy is a successful time-honored tool in the history of Physics!***
- ***Observation of the hadron mass spectrum as well as of elastic and transition form factors can be used to study the long-range behavior of the strong QCD interaction.***

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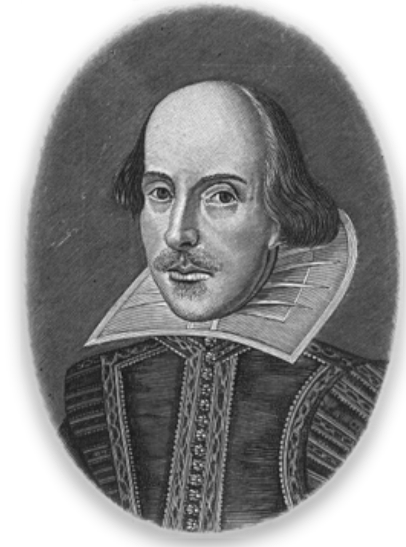

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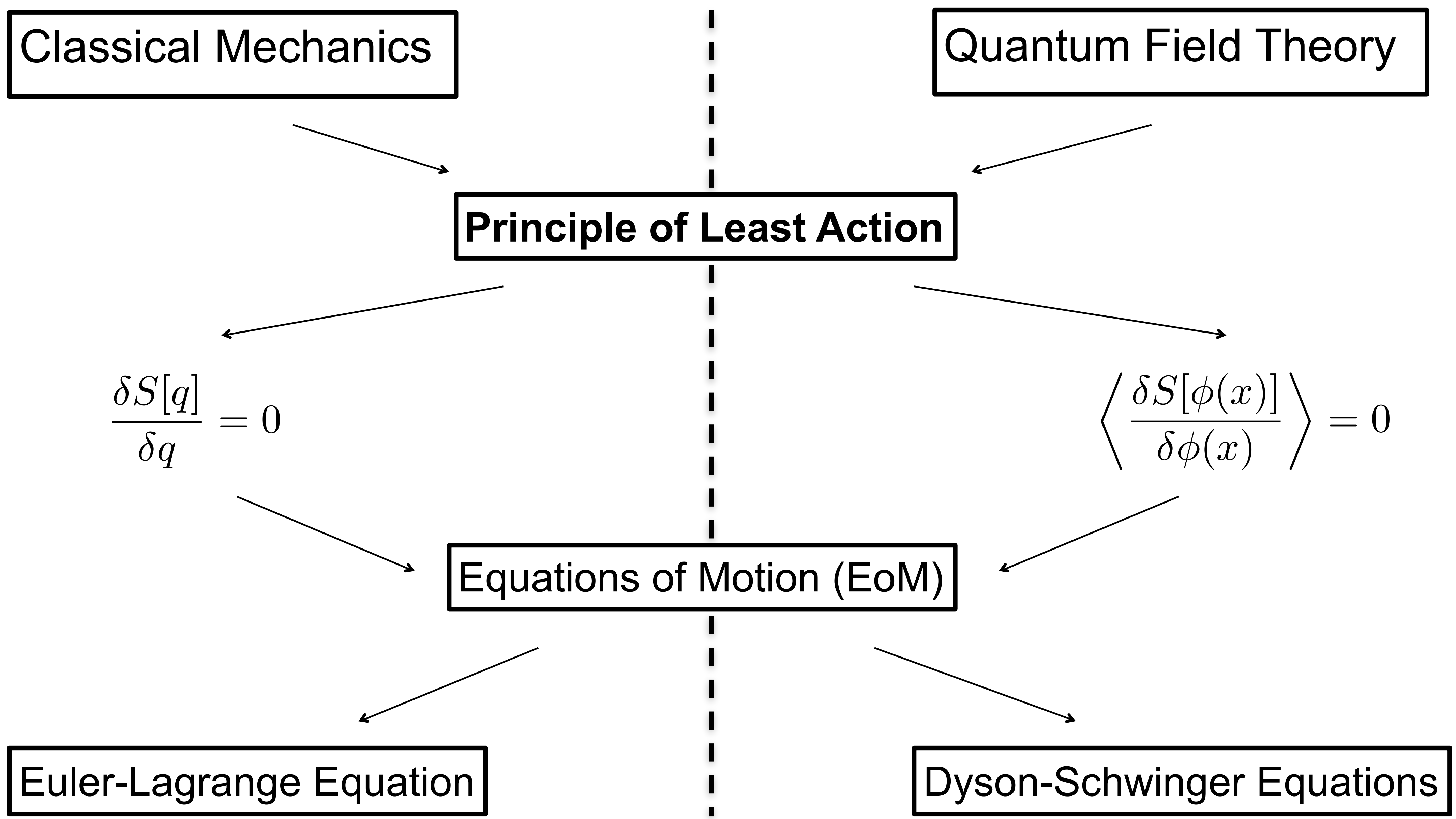
Elastic and transition form factors

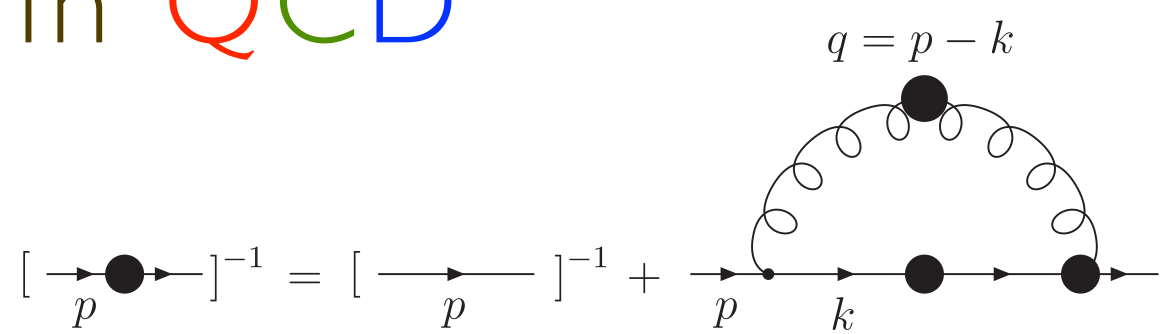
Probe the excited nucleon structures at perturbative and nonperturbative QCD scales

Distinctive information on the roles played by DCSB and confinement in QCD

NONPERTURBATIVE CONTINUUM TOOLS FOR QCD



Quark-Gap Equation in QCD



The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

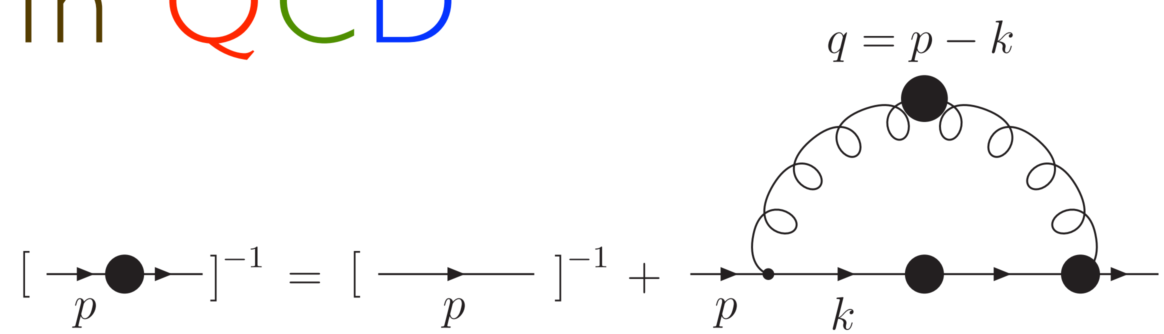
$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

Each satisfies it's own DSE !

Quark-Gap Equation in QCD



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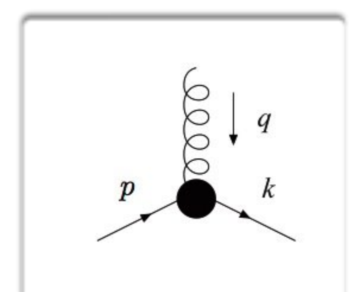

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$D_{\mu\nu}$: dressed-gluon propagator

$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

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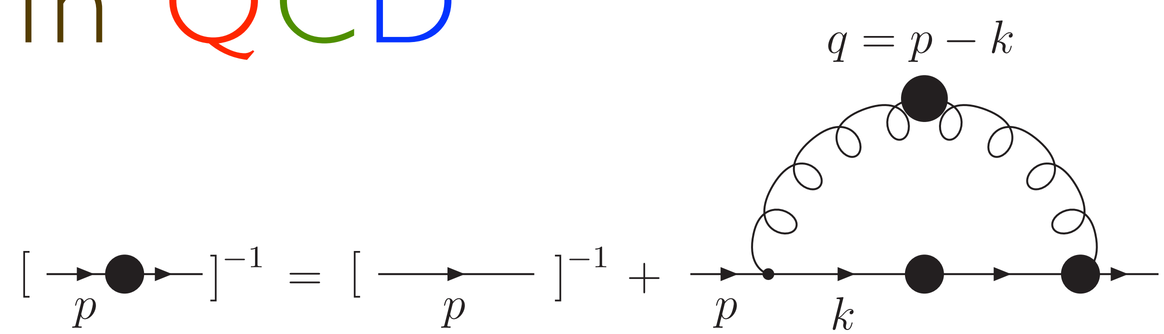

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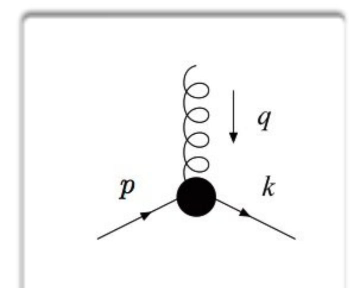

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Z_2 : quark wave function renormalization constant

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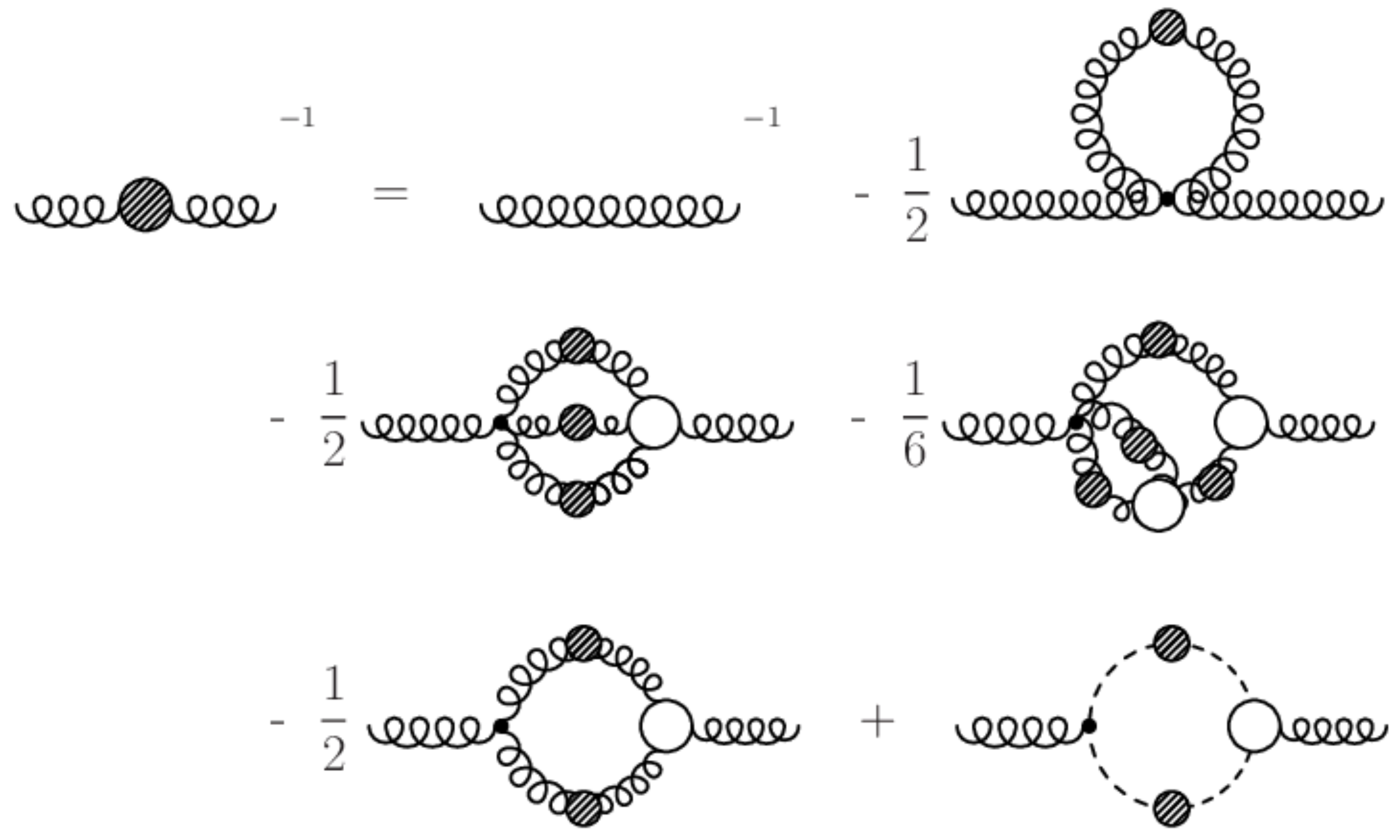
Each satisfies it's own DSE !

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

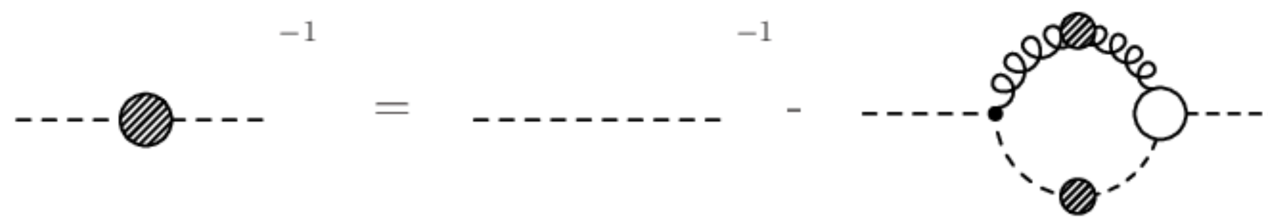
where ζ is the renormalization point.

QCD's Dyson-Schwinger Equations

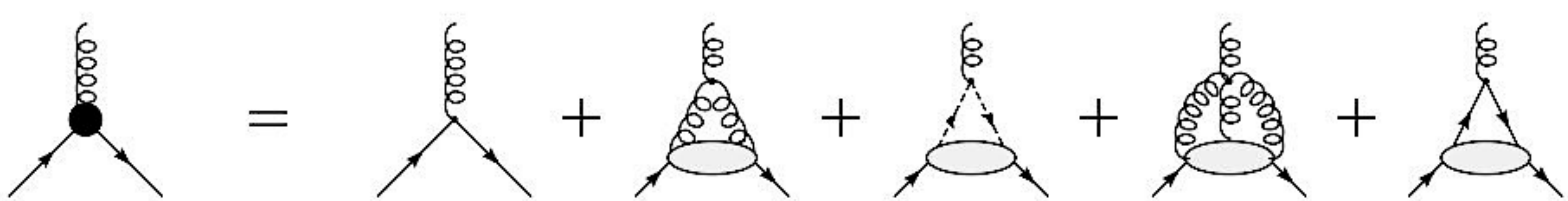
Gluon



Ghost



Quark-Gluon Vertex



Truncation schemes and symmetries

- The Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

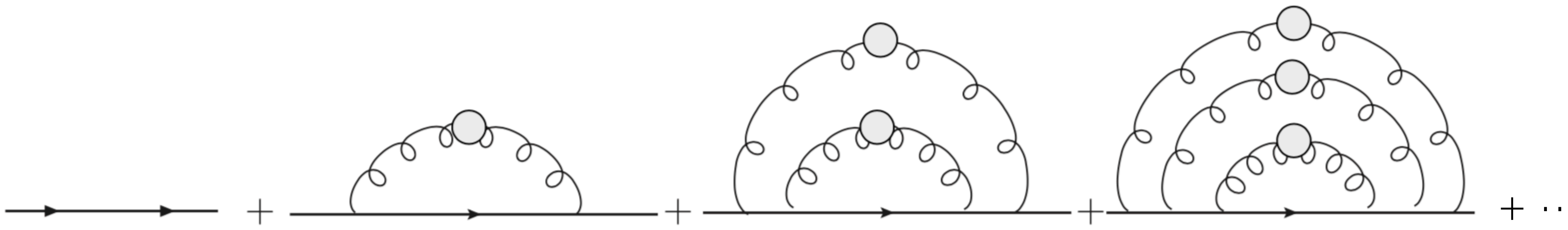
- Kernel of the equation for the quark self-energy involves:
  $D_{\mu\nu}(k)$ – dressed-gluon propagator and $\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex.

- Coupling between equations ***requires*** a truncation!

- Truncation must preserve essential symmetries of QCD, in particular chiral symmetry.

- Axialvector-Ward-Takahashi identity must be satisfied $\Rightarrow$ in chiral limit, the pion is massless.

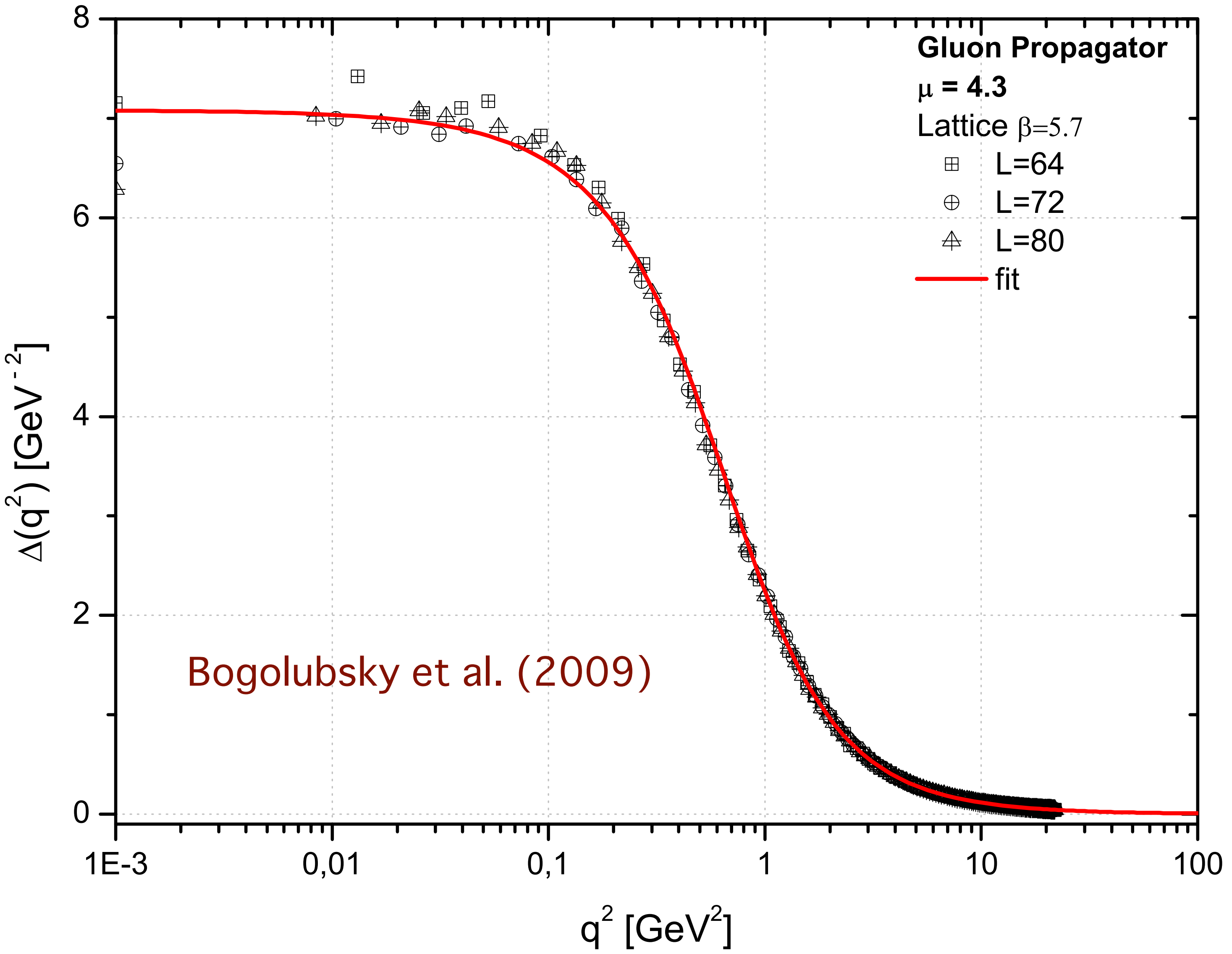
Leading truncation: *rainbow*



- Only bare structure of the quark-gluon vertex is kept.
- Leads to planar diagrams (large N_c limit).
- Effective model dressing accounts for both, gluon and vertex.
- Nonperturbative dressing controls the integral kernel's strength.
- Sufficient strength $\Longrightarrow$ *Dynamical Chiral Symmetry Breaking*

  $\Longrightarrow$ **Constituent quark mass**

Lattice QCD gluon dressing function

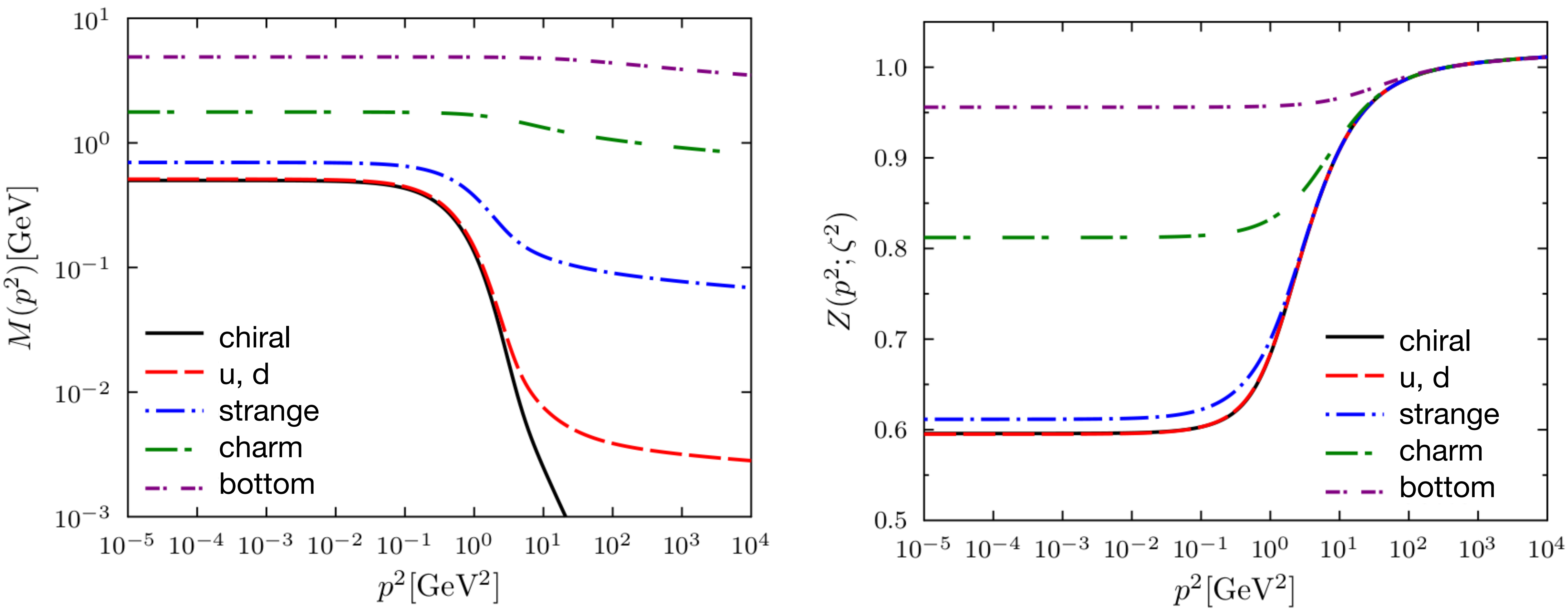


Landau gauge:

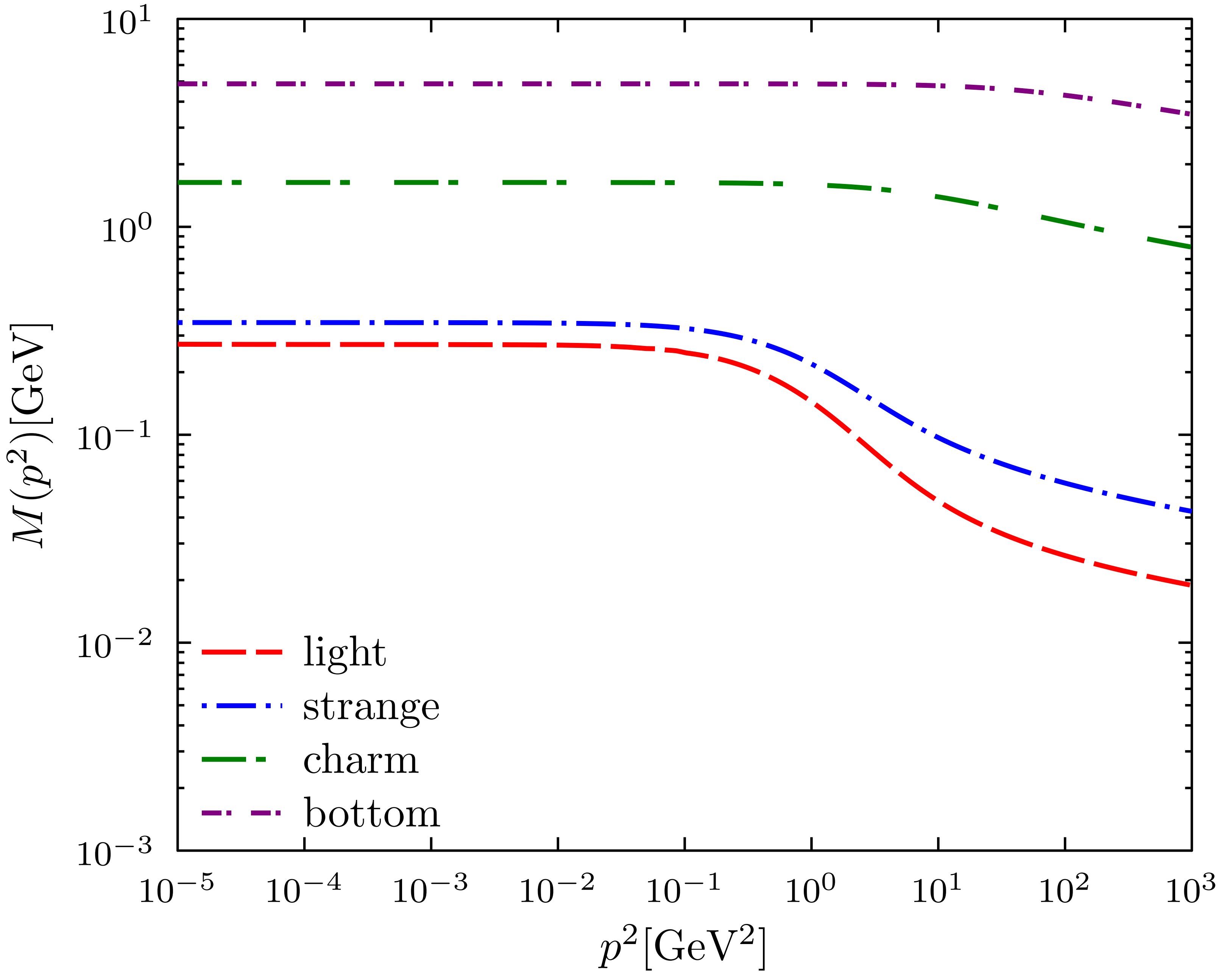
$$\Delta_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Delta(q^2)$$

Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: *infrared massive fixed point; ultraviolet massless propagator.*

Mass & Wave Renormalization Functions

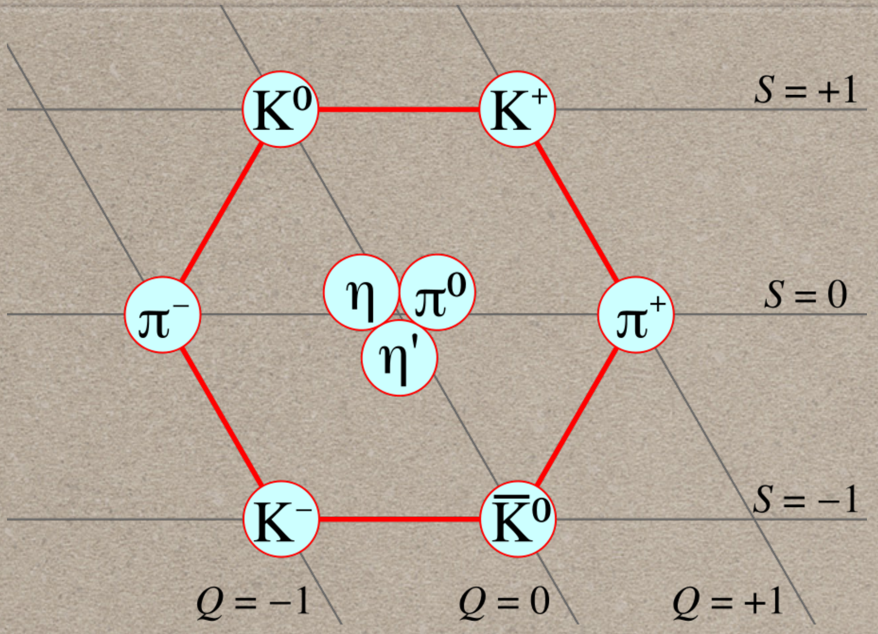


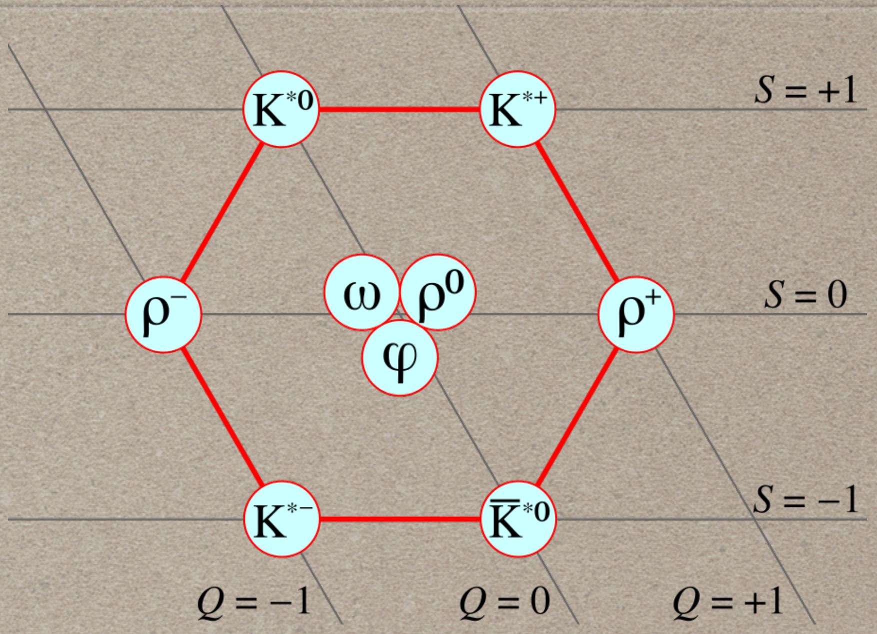
Solving the Quark-Gap Equation with full Quark-Gluon Vertex and Gluon- and Ghost Dressing Functions from Lattice QCD



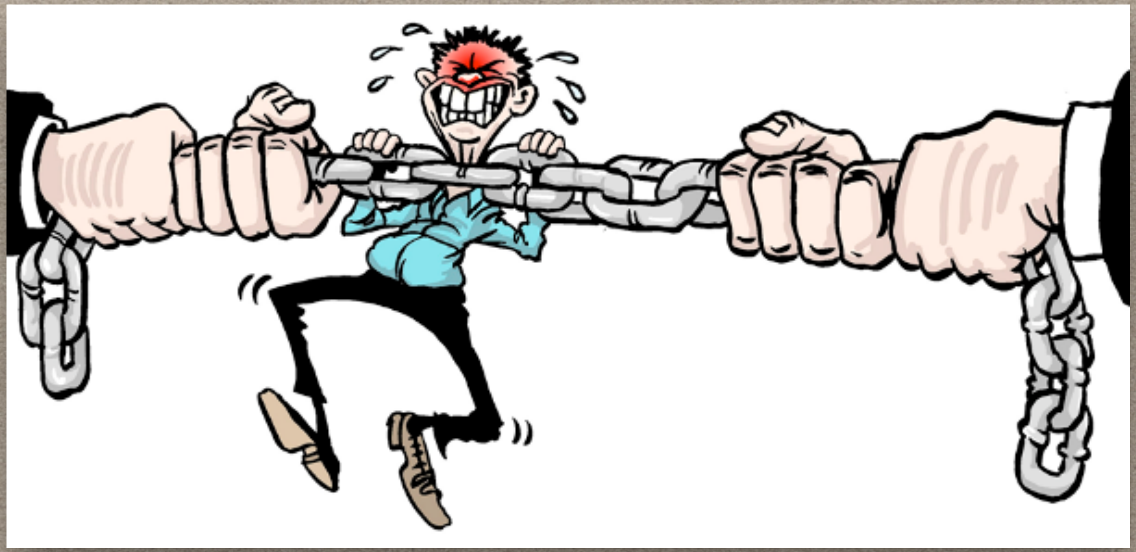
F. Serna, C. Chen, B.E., PRD 99 (2019)

L. Albino, A. Bashir, L. Gutiérrez, B.E., E. Rojas, PRD (2019)

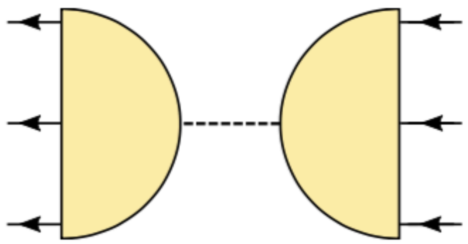




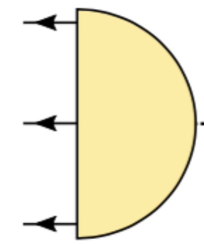
Meson Bound States



Everything we need is encoded in *n-point Green functions*



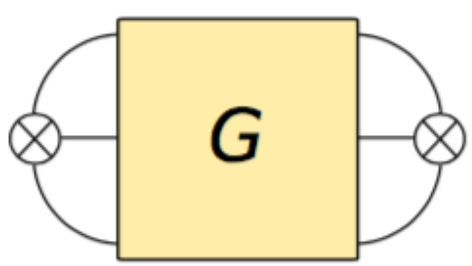
$P^2 \longrightarrow -m_\lambda{}^2$



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} \simeq \sum_\lambda \frac{\mathbf{\Psi}^\lambda_{\alpha\beta\gamma}\,\bar{\mathbf{\Psi}}^\lambda_{\alpha'\beta'\gamma'}}{P^2 + m_\lambda^2}$$

$$\mathbf{\Psi}^\lambda_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\, |\lambda\rangle$$

Spectral decomposition: extract gauge-invariant baryon poles from gauged-fixed quark n-point functions. $\Longrightarrow$ Residue at pole: Bethe-Salpeter/Faddeev wave function, contains all information about meson/baryon.



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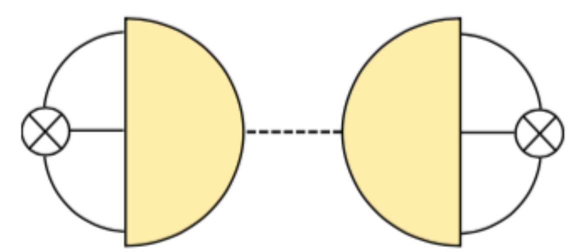
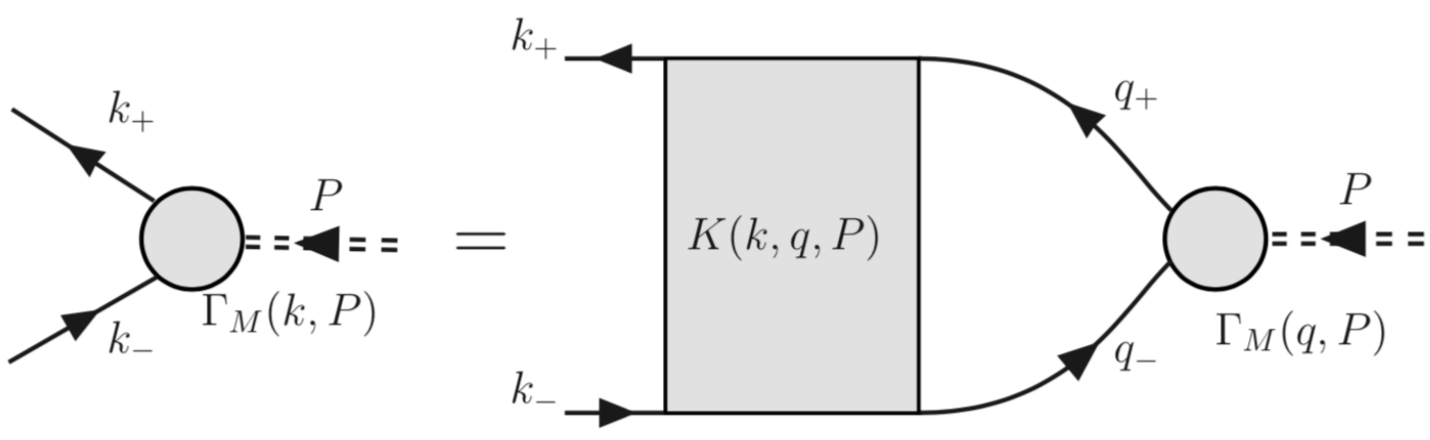


$\sim \exp(-m_N t)$

Lattice: extract baryon poles from gauge-invariant correlators $\Longrightarrow$ Exponential Euclidean time decay

Bethe-Salpeter Equations for QCD Bound States



$$\Gamma(P,p) = \int \frac{d^4k}{(2\pi)^4} \, K(P,p,k) \, S(k - \tfrac{P}{2}) \, \Gamma(P,k) \, S(k + \tfrac{P}{2})$$

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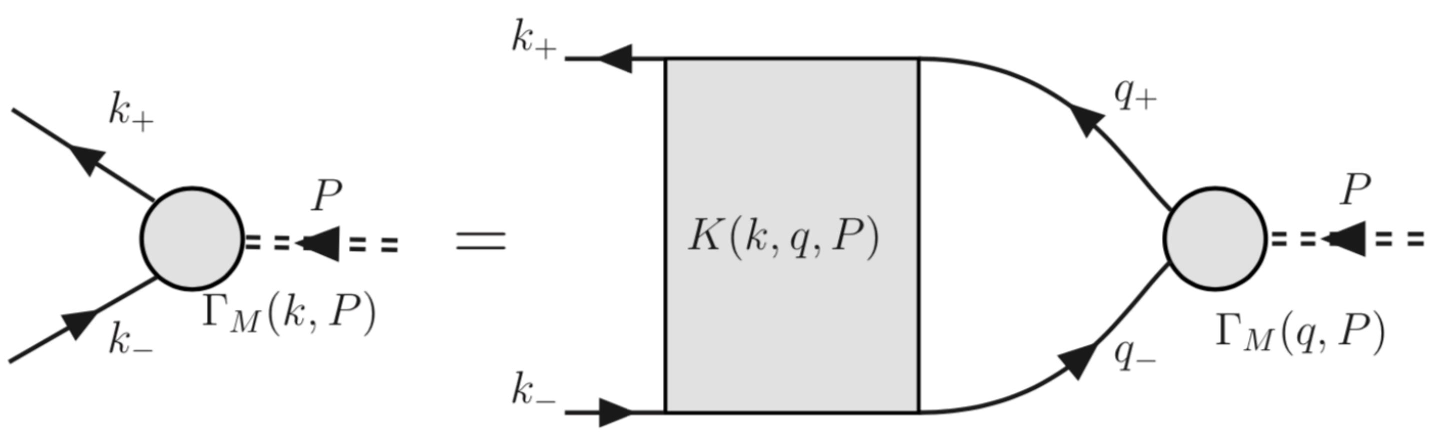

Nonperturbative QCD based ansatz for interaction kernel

$$\Gamma(P,p) = \int \frac{d^4k}{(2\pi)^4} K(P,p,k)\, S(k-\tfrac{P}{2})\, \Gamma(P,k)\, S(k+\tfrac{P}{2})$$

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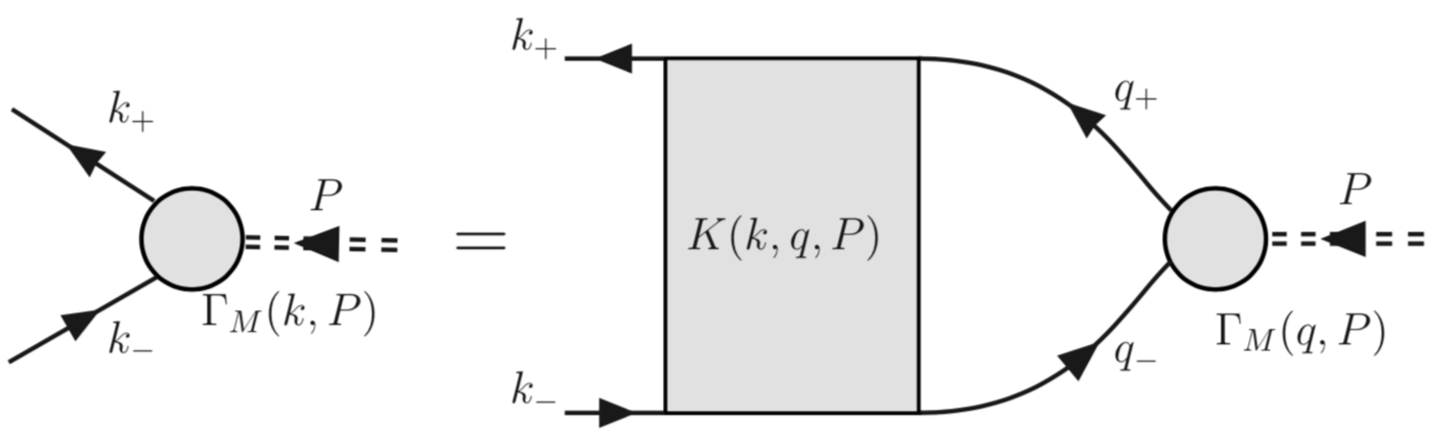

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General solution for Poincaré invariant pseudoscalar Bethe-Salpeter Amplitude

$$\begin{aligned}\Gamma^{f\bar{g}}_{\rm ps}(k;P) \quad &= \quad \gamma_5 \Big[iE^{f\bar{g}}_{\rm ps}(k;P) + \gamma\cdot P F^{f\bar{g}}_{\rm ps}(k;P) \\ &+ \quad \gamma\cdot P P\cdot k G^{f\bar{g}}_{\rm ps}(k;P) + P_\alpha \sigma_{\alpha\beta} k_\beta H^{f\bar{g}}_{\rm ps}(k;P)\Big]\end{aligned}$$

Beyond the Quark Model

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) + F_2(P,p) i\not{P} + F_3(P,p) pP i\not{p} + F_4(P,p)[\not{p}, \not{P}]\right]$$

s-wave

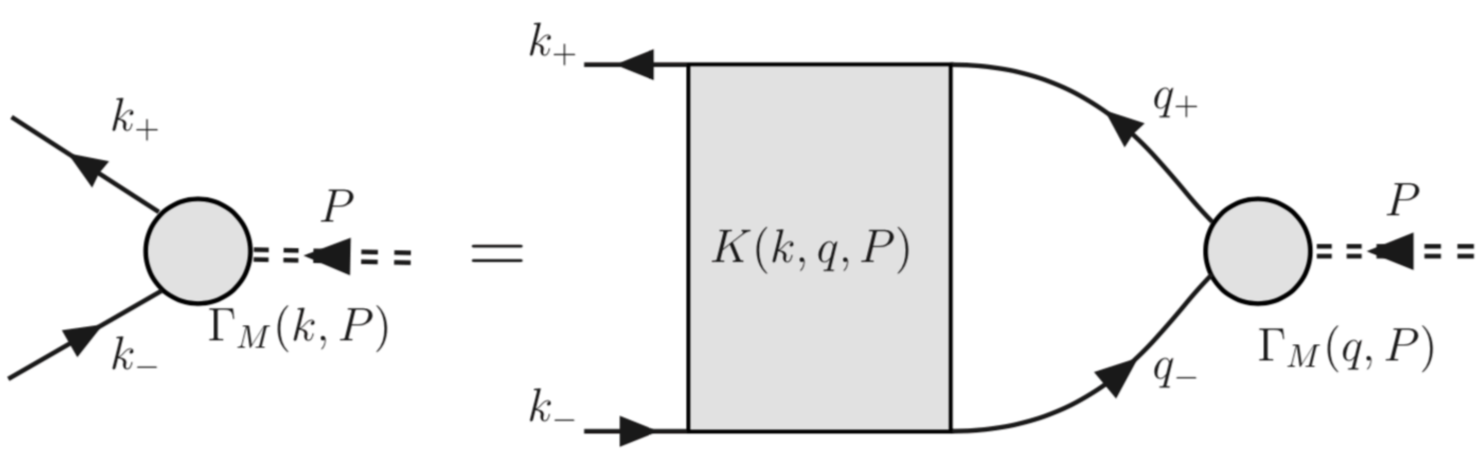
p-wave

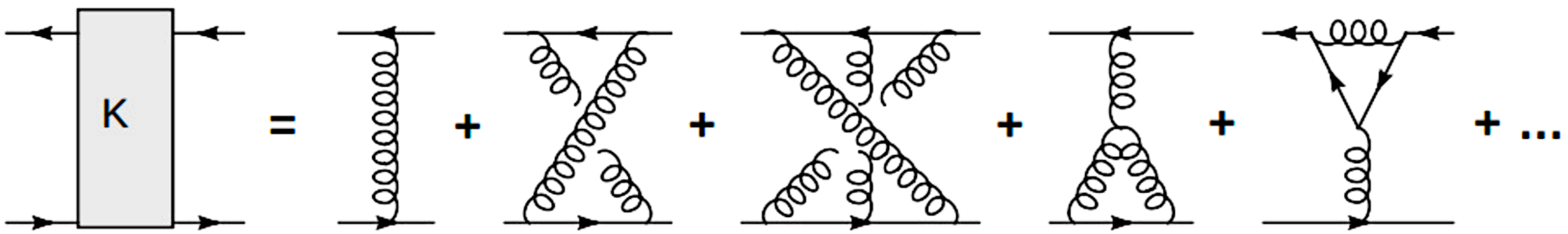
$$\require{cancel}\cancel{P : (-1)^{L+1}}$$

Llewellyn-Smith, Annals Phys. 53 (1969) 521–558

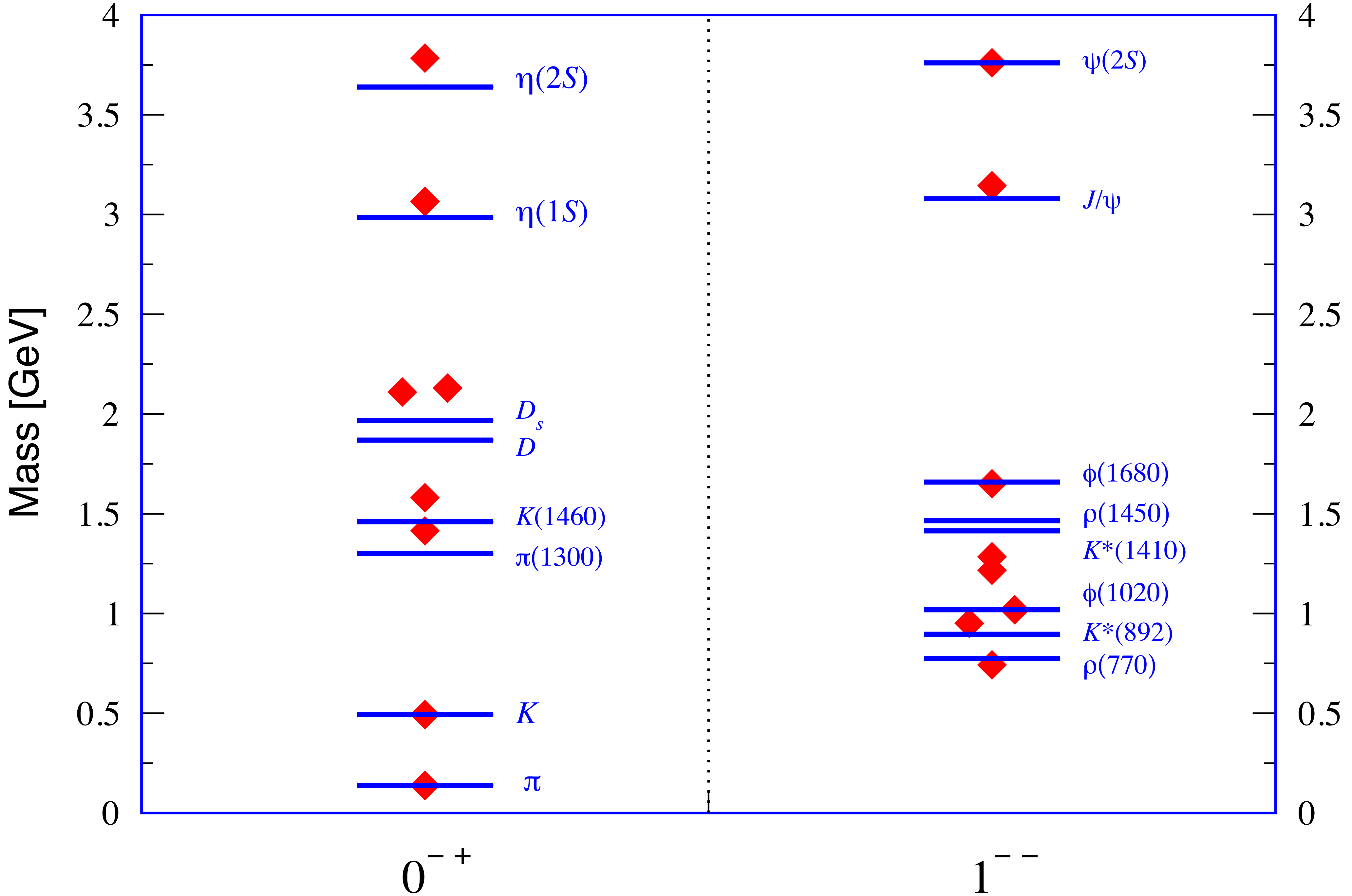
Bethe-Salpeter Equations for QCD Bound States

$$\left[\Gamma_M^{f\bar{g}}(k;P)\right]_{AB} = \int \frac{d^4q}{(2\pi)^4} \left[K^{f\bar{g}}(k,q;P)\right]_{AC,DB} \left[S_f(q_+)\Gamma_M^{f\bar{g}}(q;P)S_{\bar{g}}(q_-)\right]_{CD}$$





Pseudoscalar- and Vector-Meson Spectroscopy

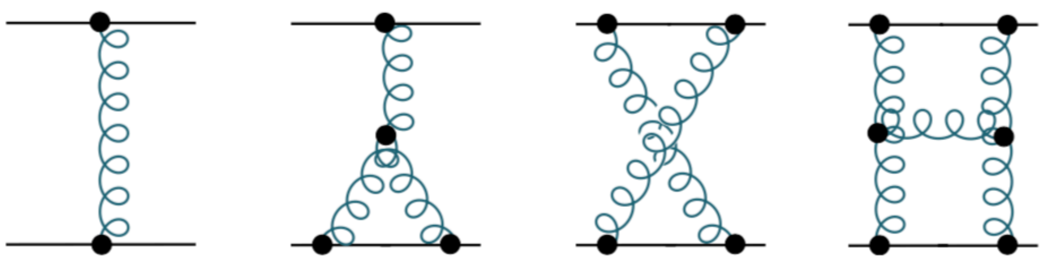


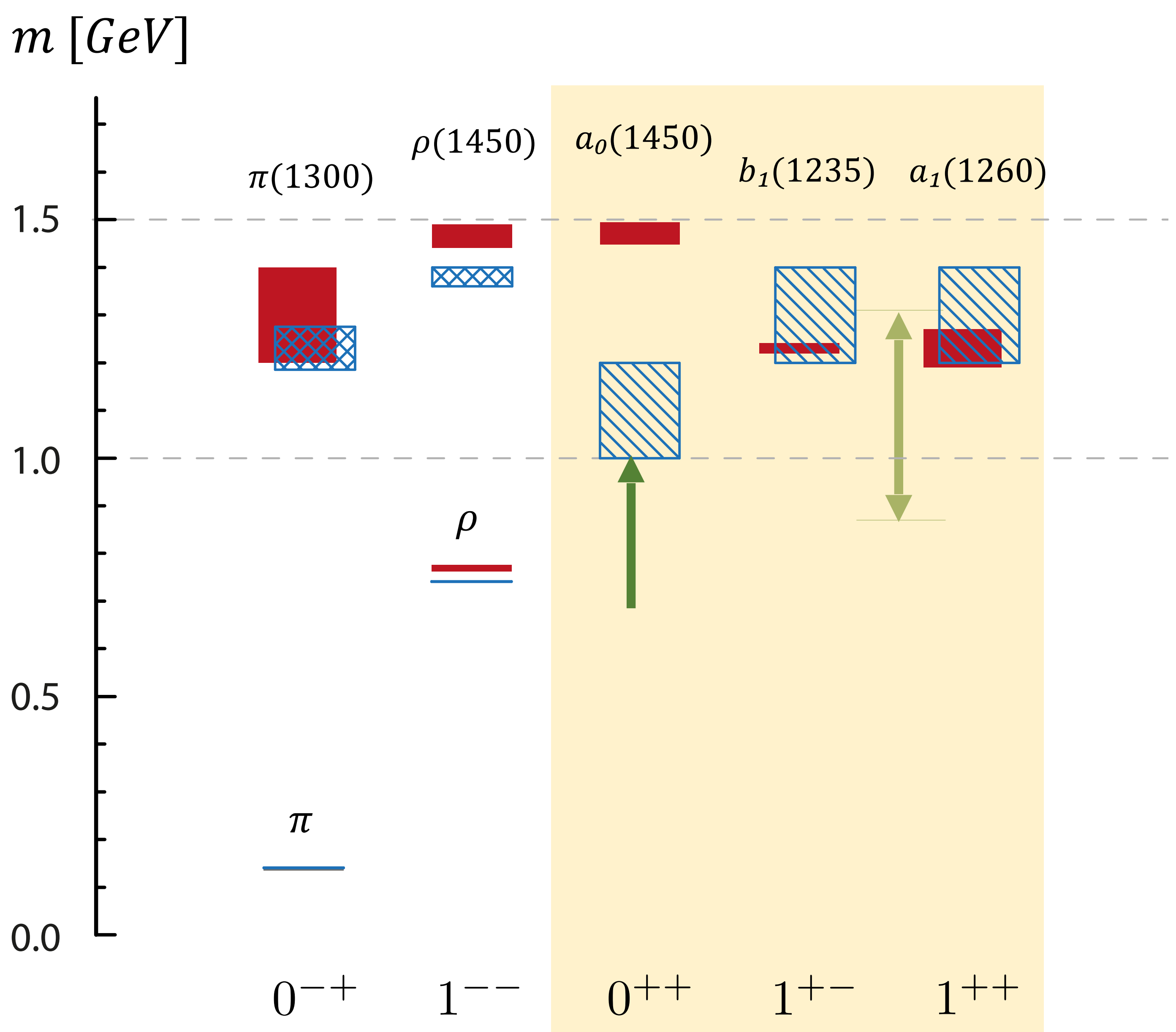
E. Rojas, B. E. & J. P. B. C. de Melo, PRD (2014)

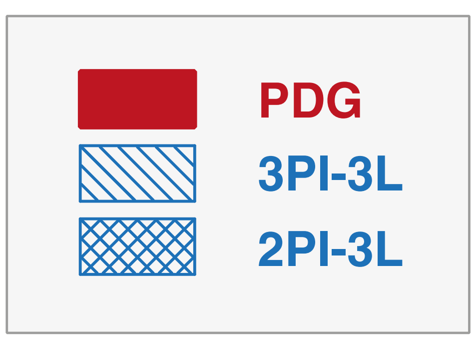
F. Mojica, C. Vera, E. Rojas & B. E., PRD (2017)

Beyond Rainbow-Ladder Truncation

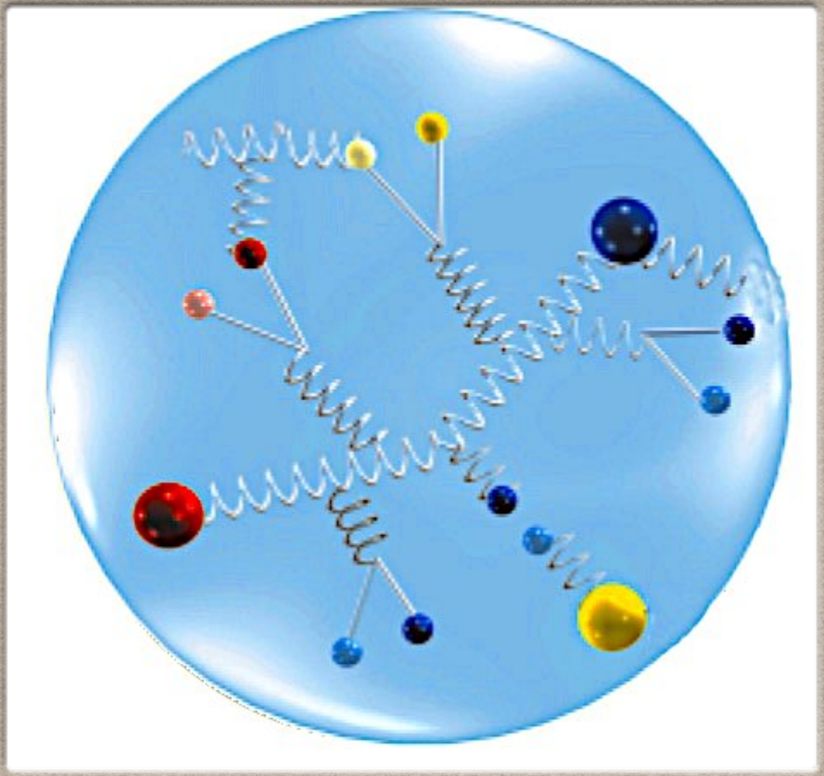
- Corrects $\rho - a_1$ splitting and degeneracy of axial-vectors.
- Pushes lightest $q\bar{q}$ scalar above 1 GeV.







R. Williams and C. Fischer, PRL 103 (2009)
H. Sanchis-Alepuz, R. Williams PLB 749 (2015)

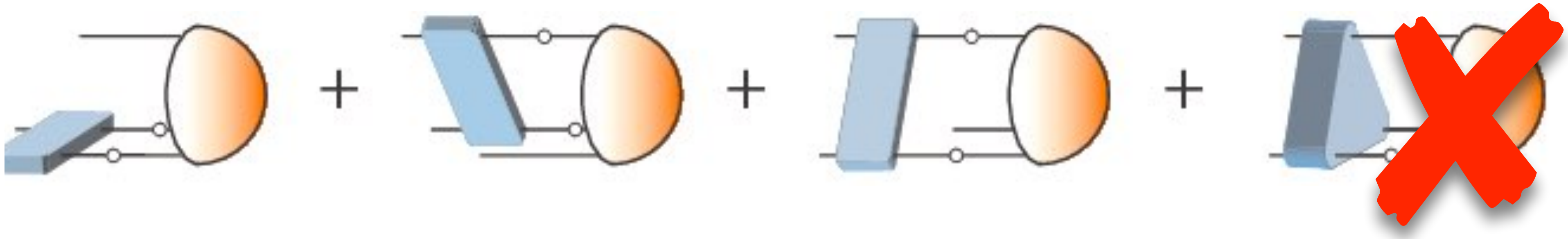


Baryon Spectrum

SU(3): $3 \otimes 3 = \bar{3} \oplus 6$

Covariant Fadeev Equation

- The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a color-singlet baryon.



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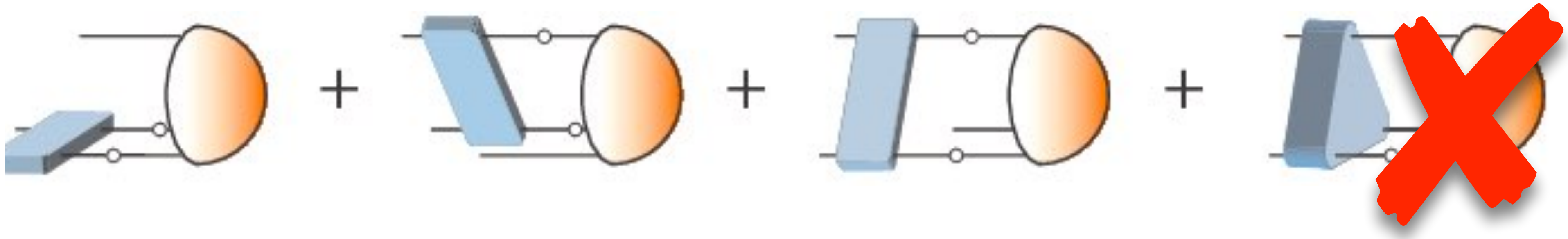

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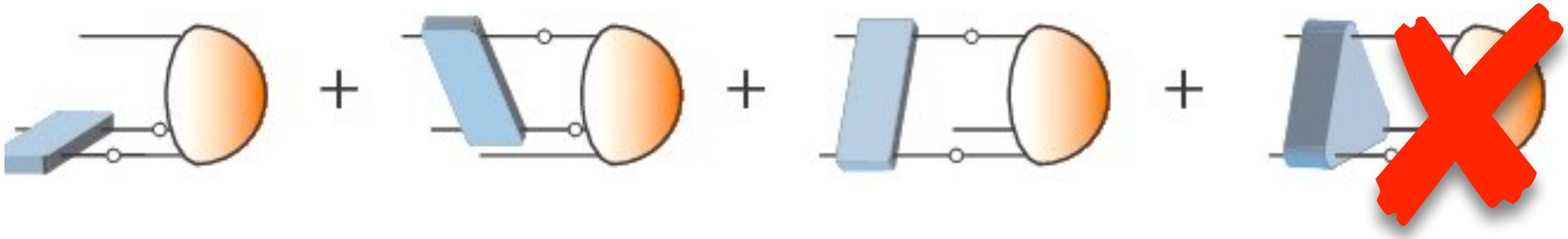

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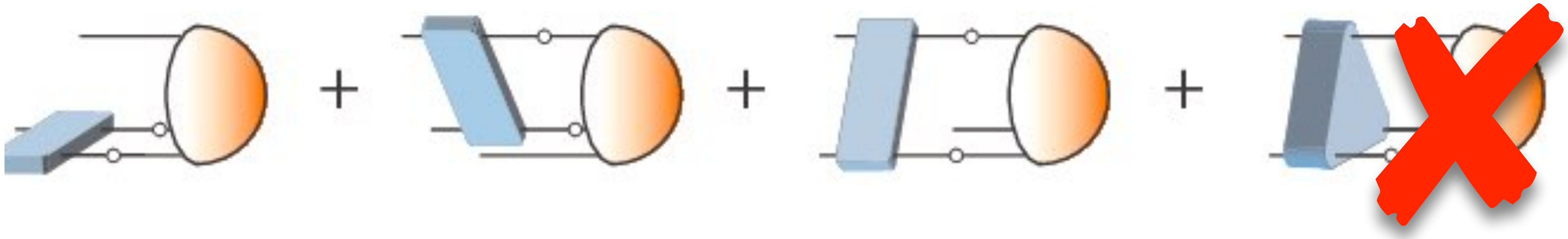
- Scalar and axialvector diquarks: dominant right parity.



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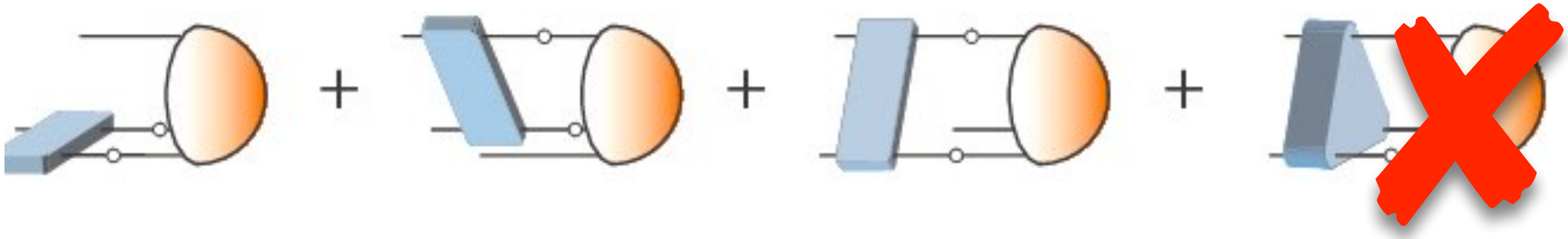

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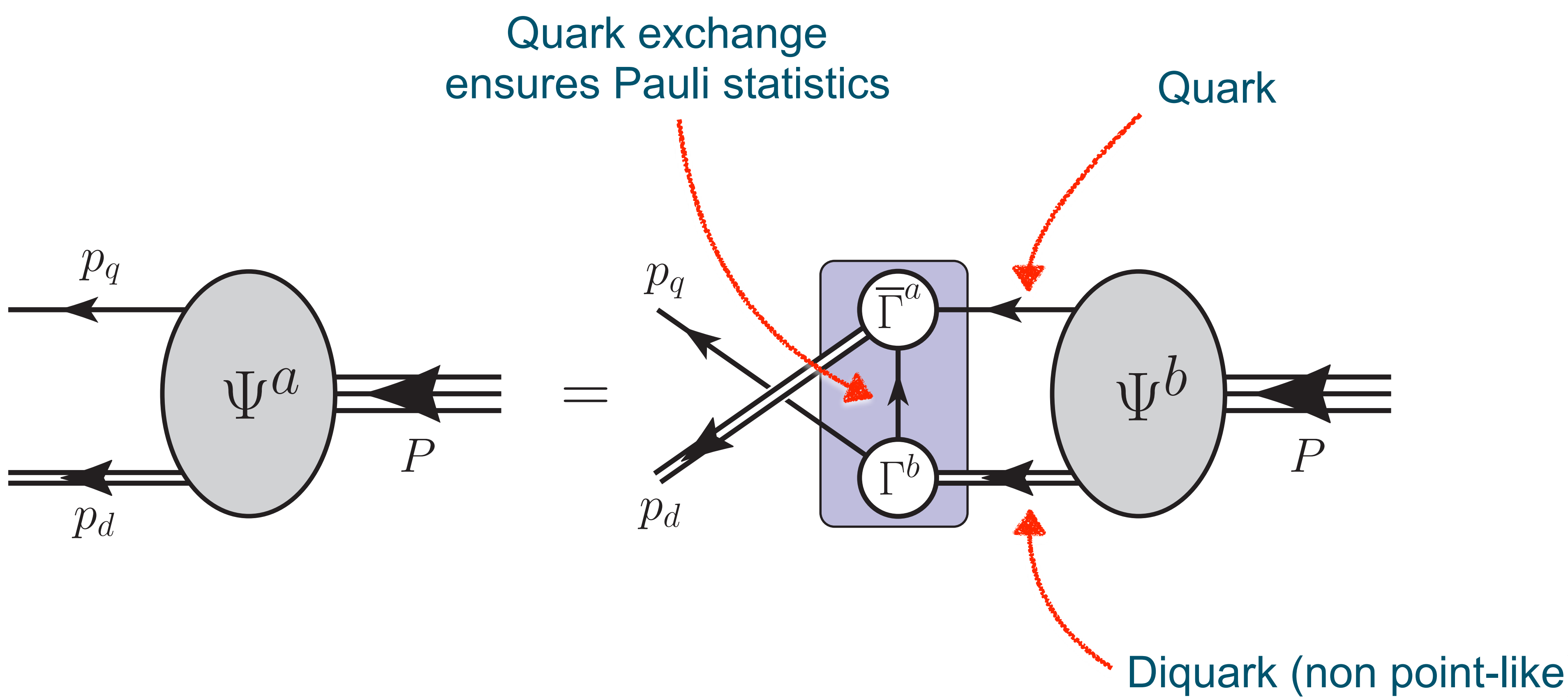
- Pseudoscalar and vector diquarks: initially neglected, **now included**.

Covariant Fadeev Equation



R.T. Cahill, C.D. Roberts, J. Praschifka (1989)

M. Oettel, L. von Smekal, R. Alkofer (2001)

I.C. Cloët, G. Eichmann, **B. E.**, T. Klähn and C.D. Roberts (2009)

G. Eichmann, C. Fischer, H. Sanchis-Alepuz (2016)

Linear homogeneous matrix equation yields Poincaré covariant Faddeev amplitude (wave function) that describes relative motion of quark-diquark within nucleon.

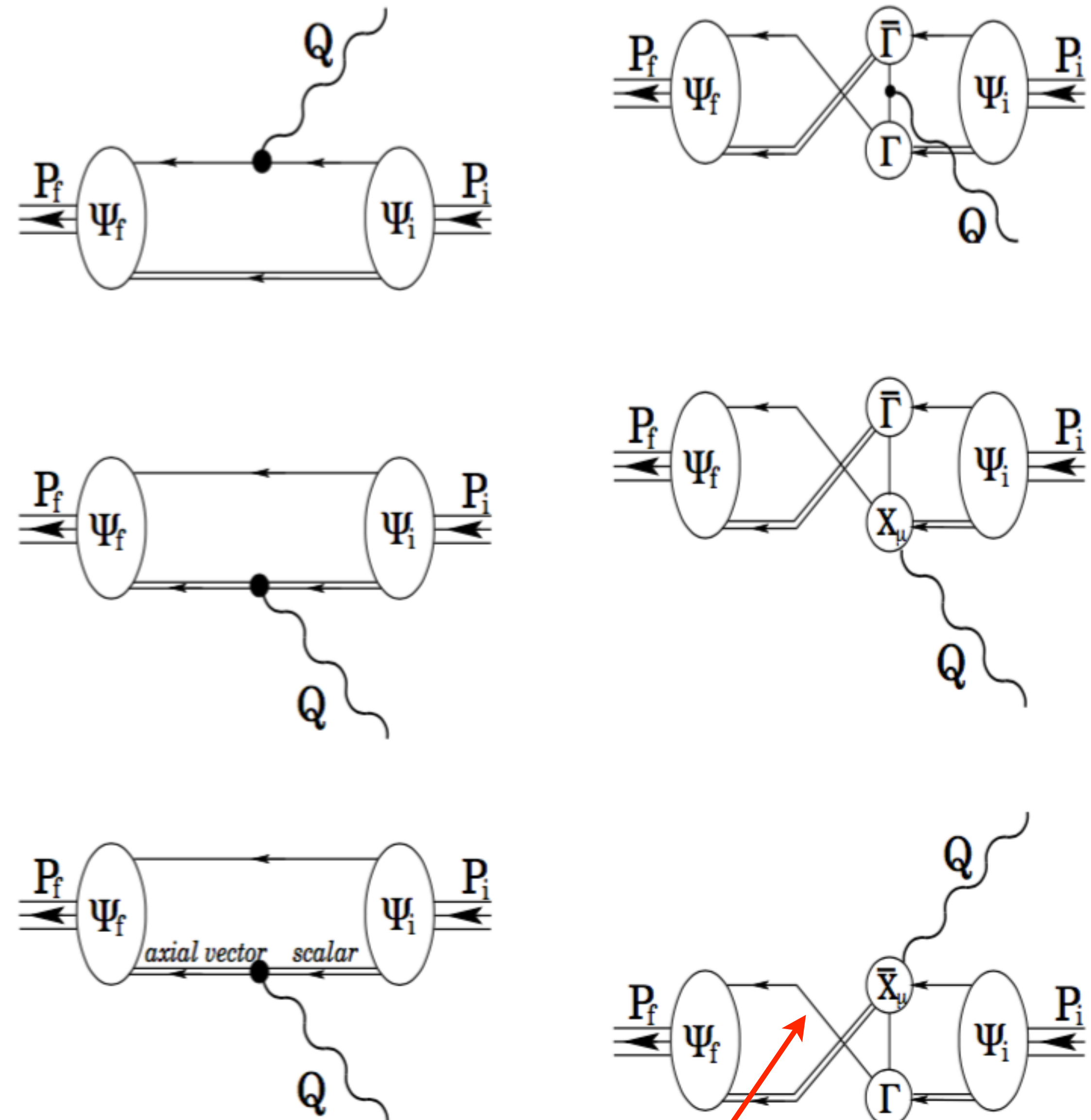
Nucleon Electromagnetic Form Factors

- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Takahashi identities (EM gauge invariance).
- Coupling of the photon to the dressed quark.
- Coupling of the photon to the dressed diquark:
  Elastic & induced transitions
- Exchange and seagull terms.

$$J_\mu(P', P) = ie\,\bar{u}(P')\,\Lambda_\mu(q, P)\,u(P)\,,$$
$$= ie\,\bar{u}(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u(P)\,.$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\ G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

$$\mu_n = \kappa_n = G_M^n(0)\,,\ \mu_p = 1 + \kappa_p = G_M^p(0)$$



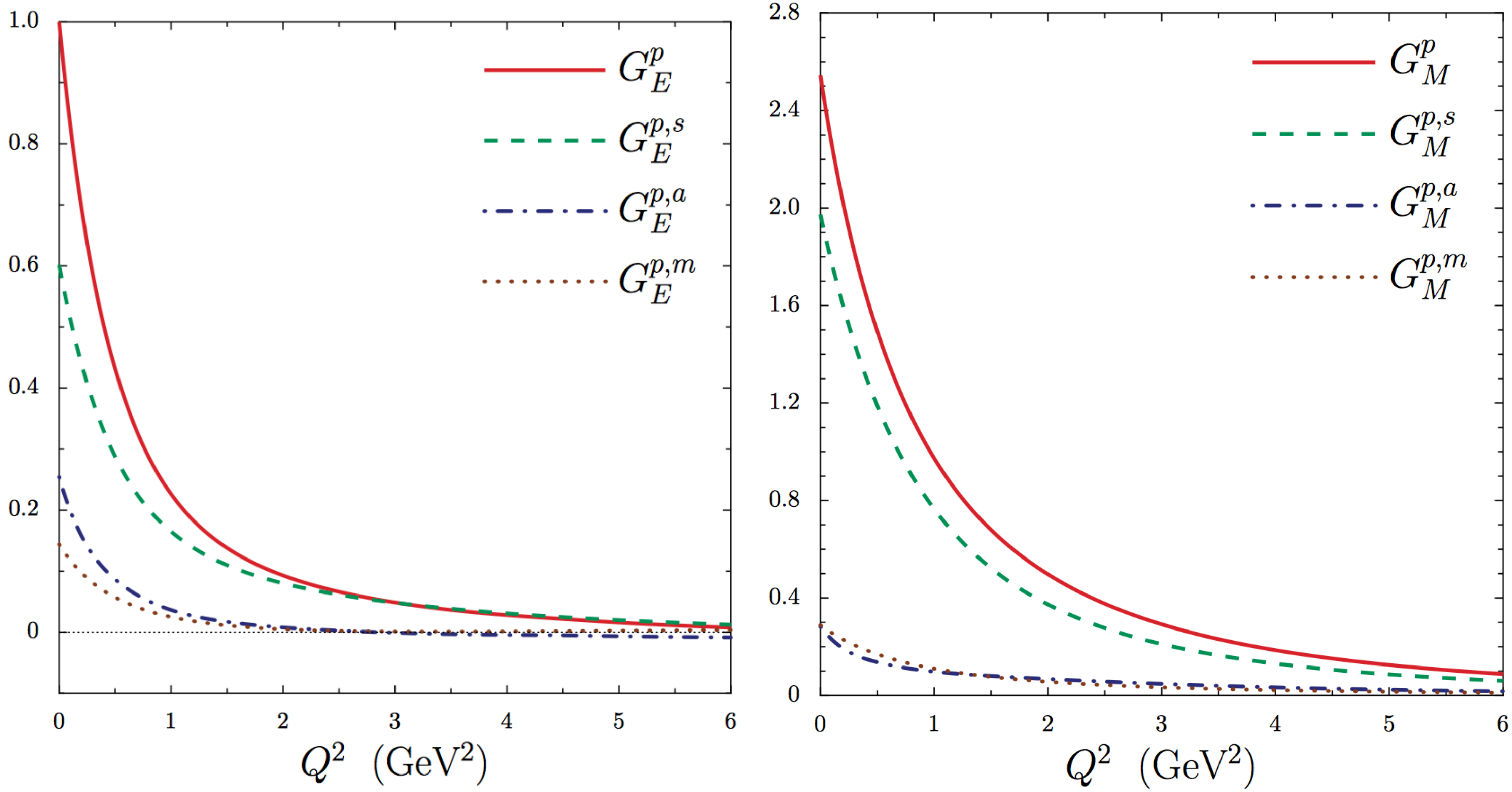
Dressed quark propagator solutions of QCD's Dyson-Schwinger equations.

⇒ momentum dependence !

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

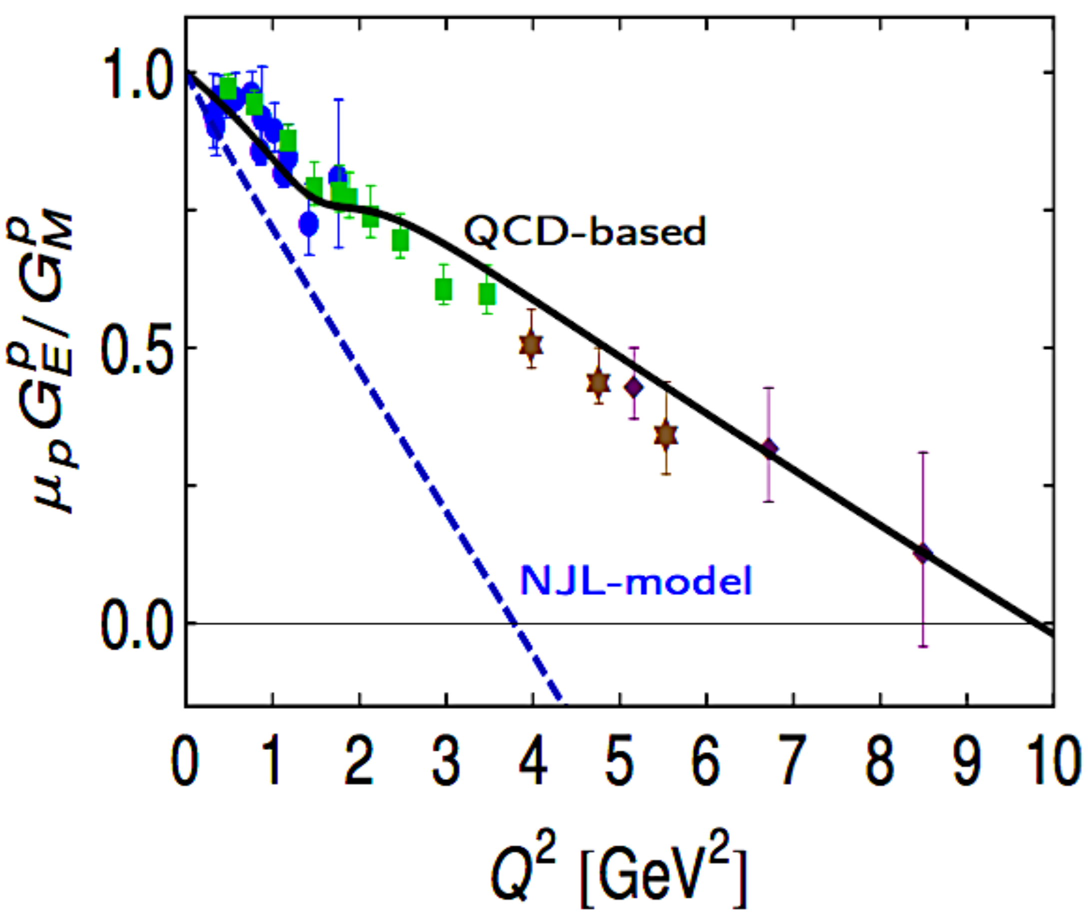
Proton's Sachs Electric and Magnetic Form Factors

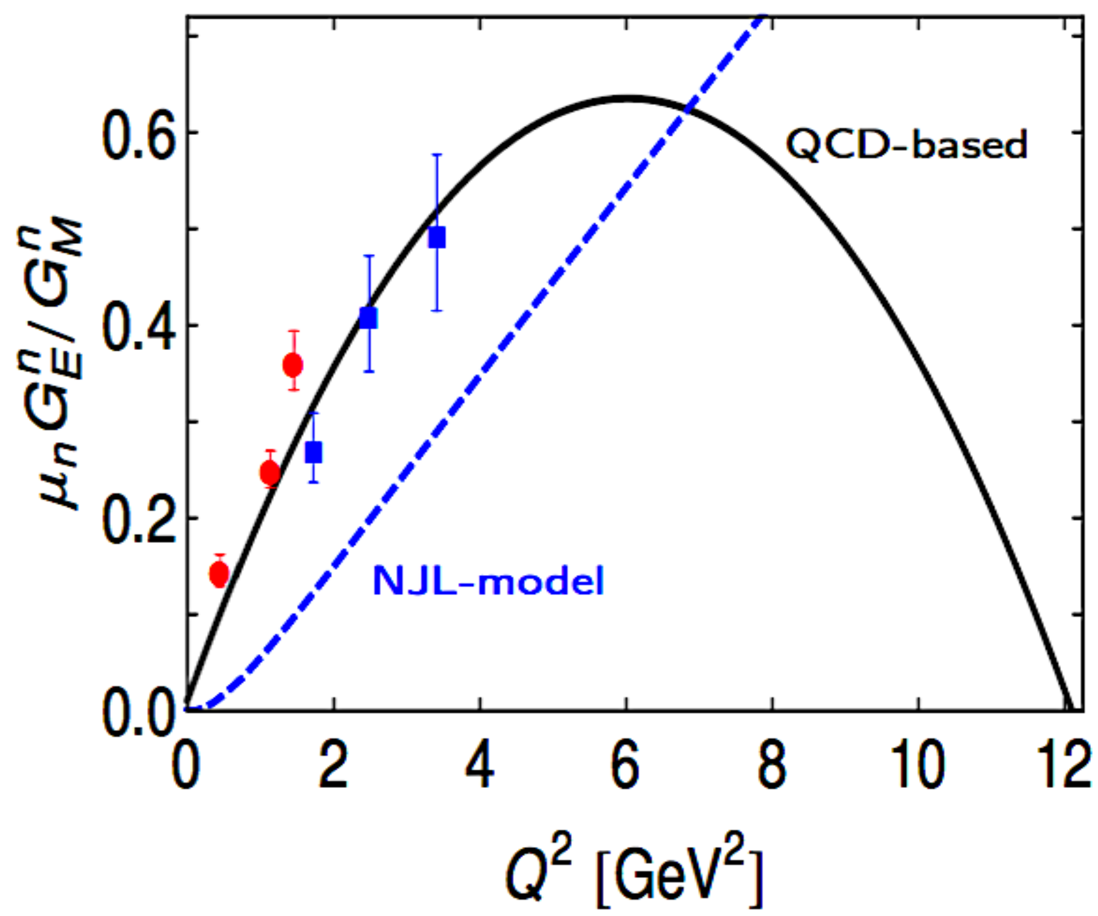
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)\,,\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



I.C. Cloët, G. Eichmann, B. E., T. Klähn and C.D. Roberts, Few Body Syst. 46 (2009)

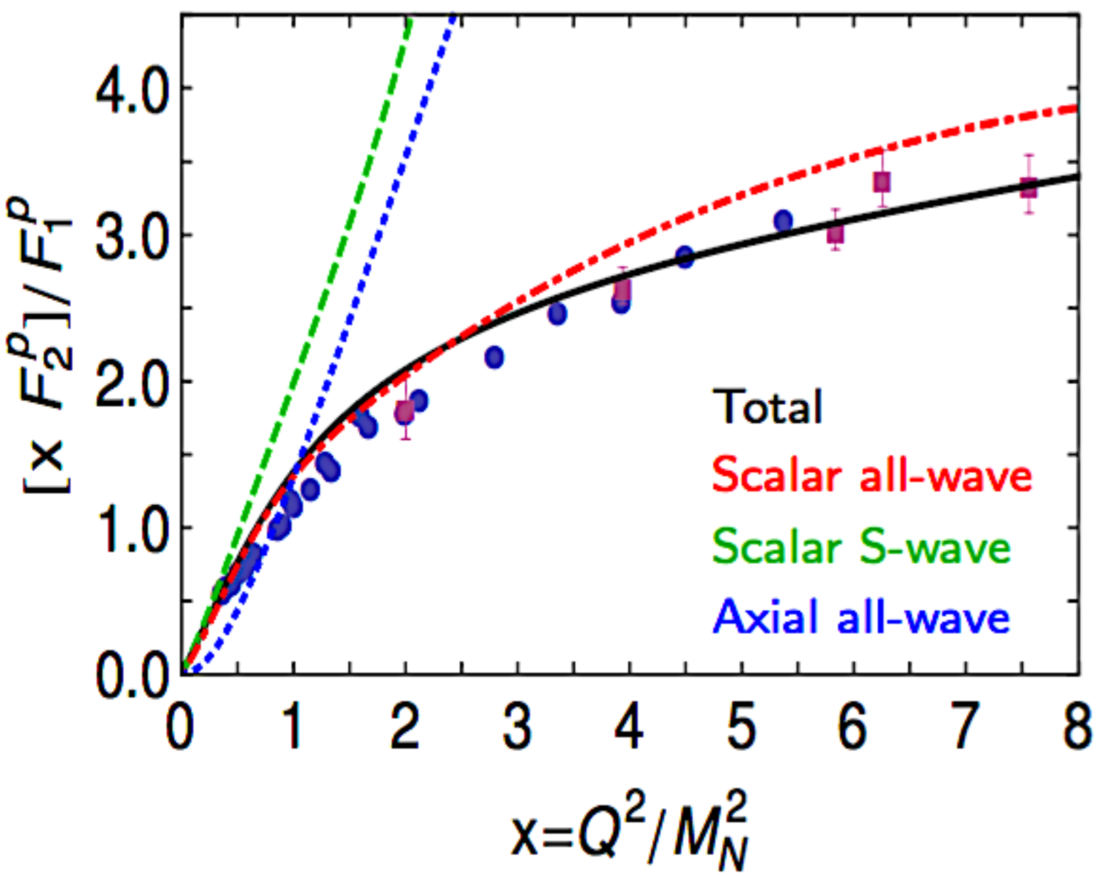
Electric Sachs form factors: mass function dependence

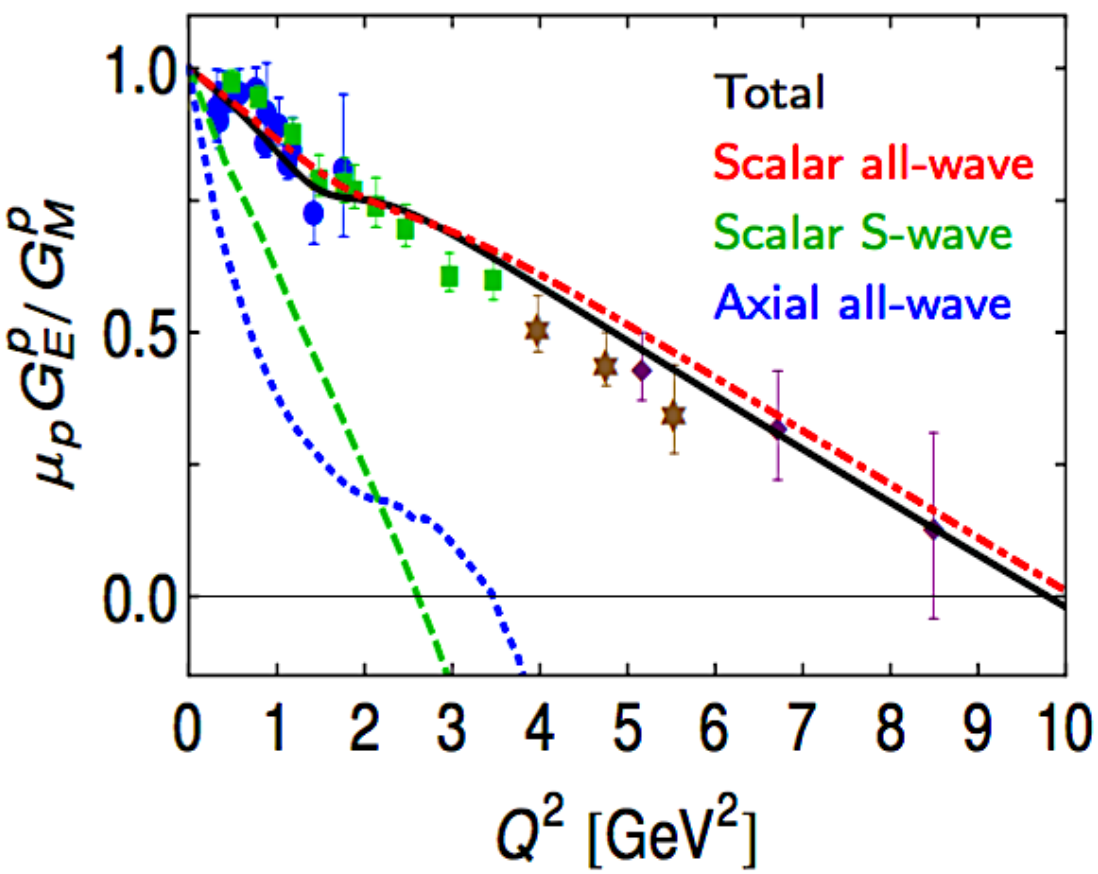




- Both CI and QCD-based frameworks predict a zero crossing in $\mu_p \frac{G_E^p}{G_M^p}$.
- The possible existence and location of the zero in $\mu_p \frac{G_E^p}{G_M^p}$ is an indirect measure of the nature of the quark-quark interaction.

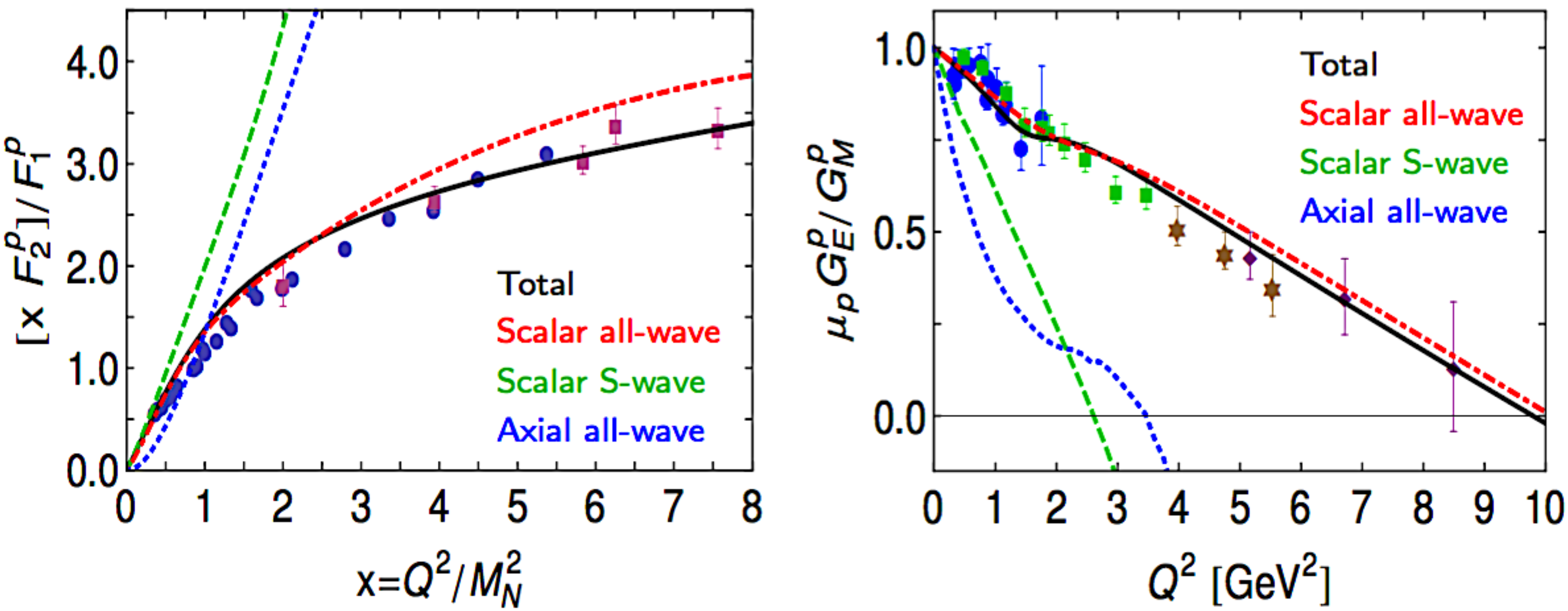
Scalar and axialvector diquark contributions



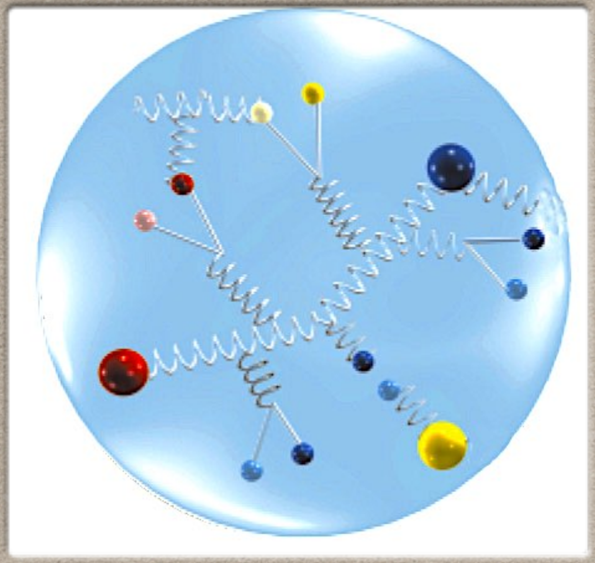


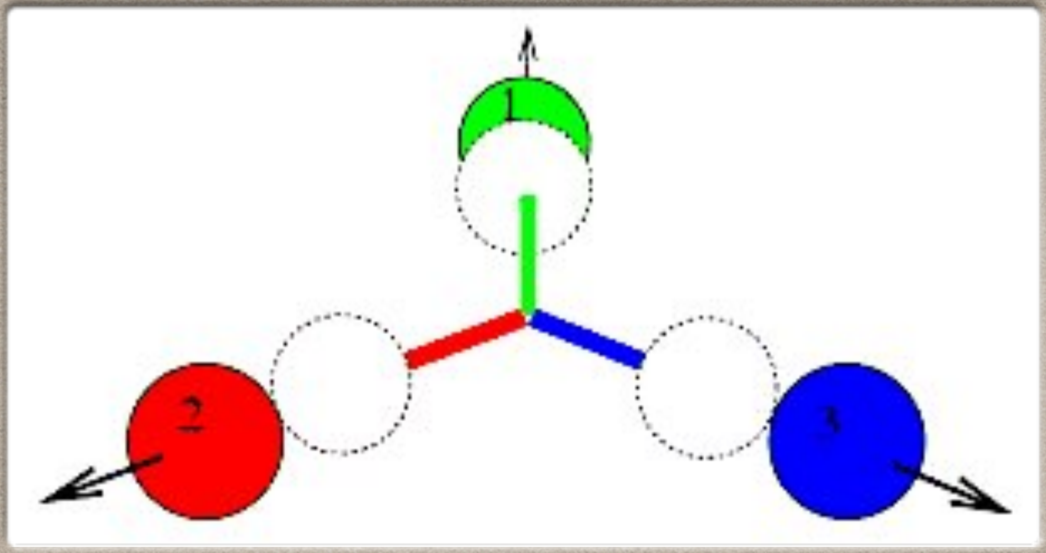
- Axialvector diquark contribution is not enough in order to explain the proton's electromagnetic ratios.
- *Scalar diquark contribution is dominant* and responsible of the Q^2-behavior of the the proton's e.m. ratios.
- Higher quark-diquark angular momentum components of the nucleon are critical in explaining the data.

Scalar and axialvector diquark contributions



The presence of higher orbital angular momentum components in the nucleon is an inescapable consequence of solving a realistic Poincaré-covariant Faddeev equation



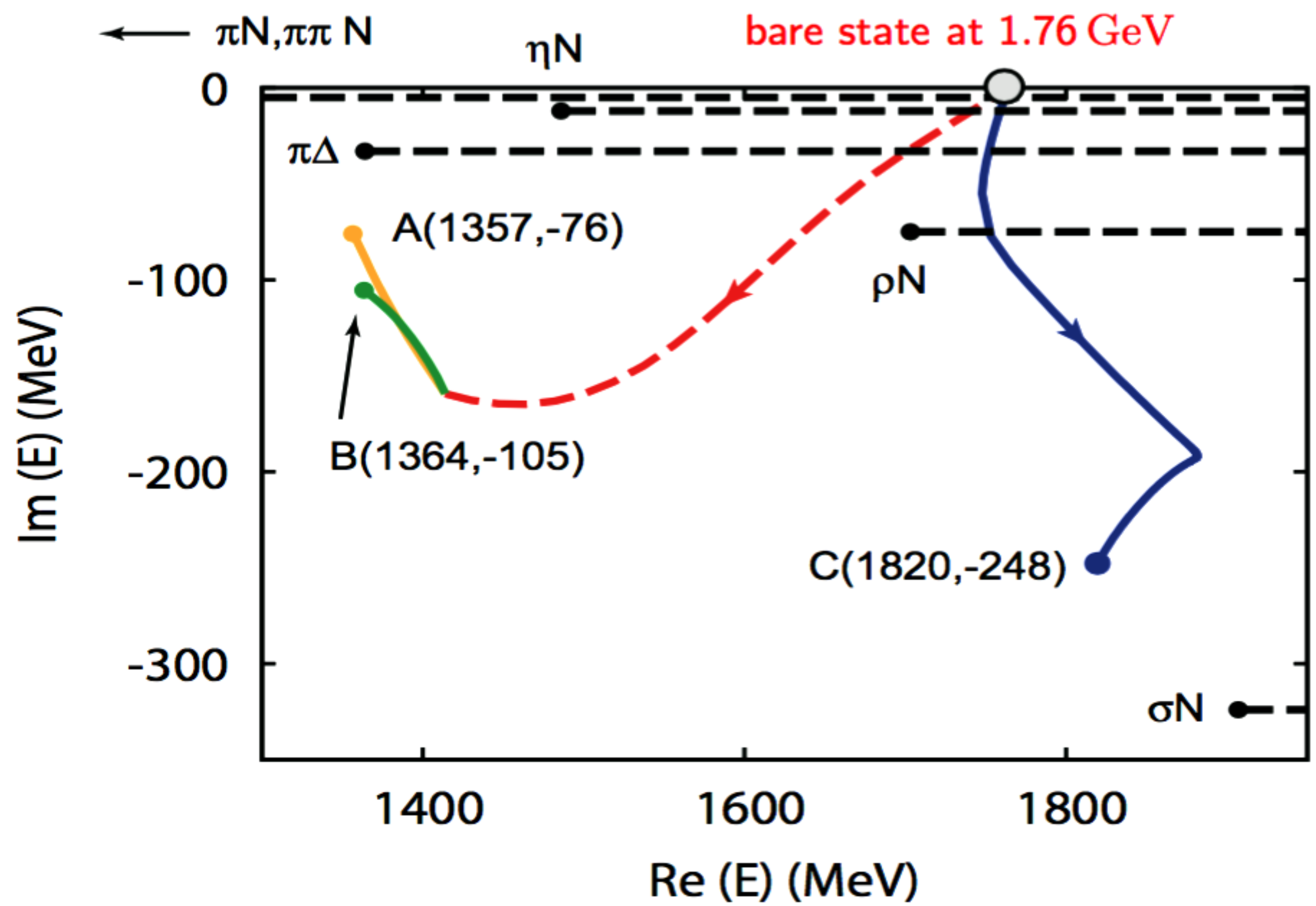


The Roper

PRL 104, 042302 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

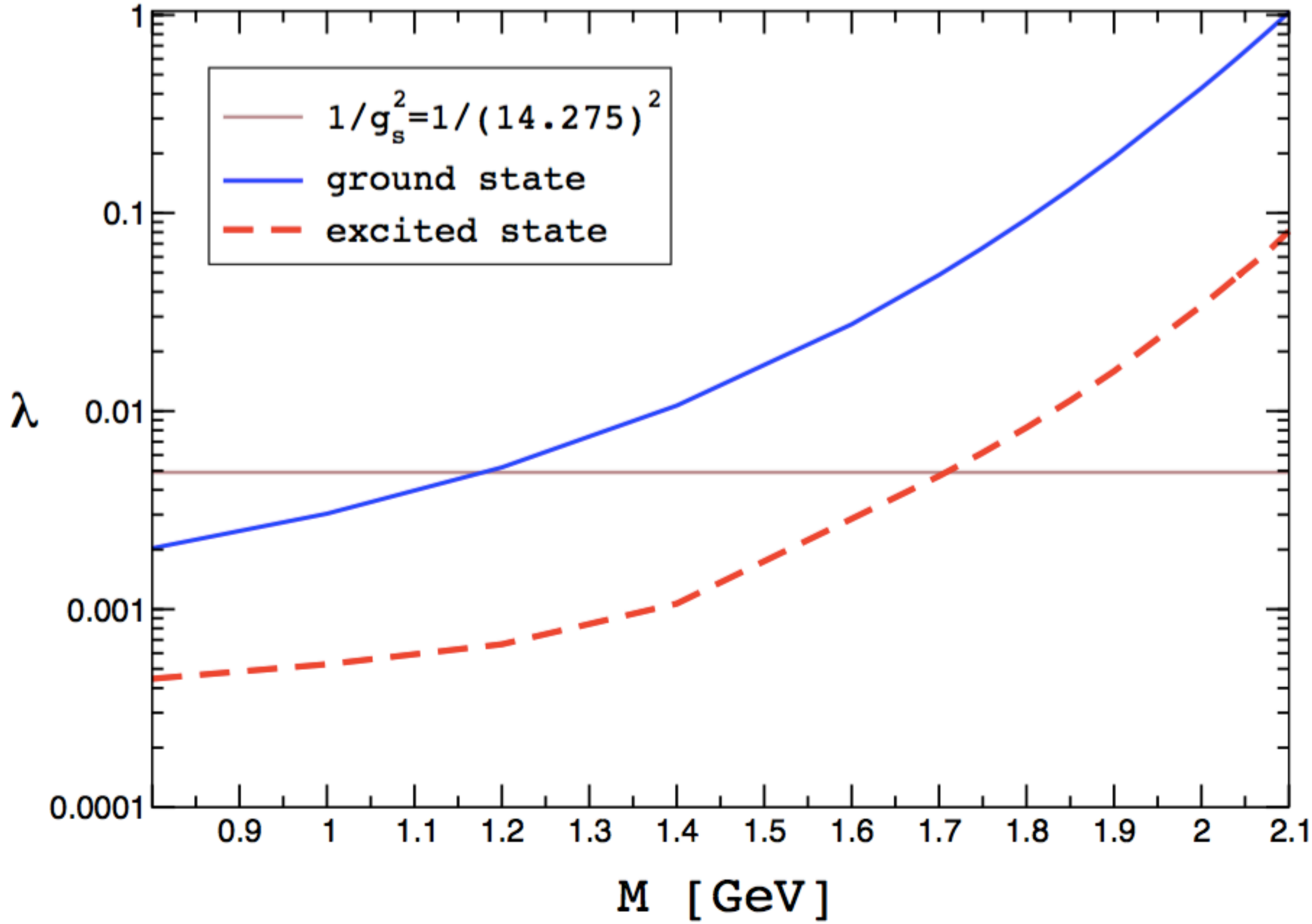
Disentangling the Dynamical Origin of P_{11} Nucleon Resonances

N. Suzuki,[1,2] B. Juliá-Díaz,[3,2] H. Kamano,[2] T.-S. H. Lee,[2,4] A. Matsuyama,[5,2] and T. Sato[1,2]



- EBAC examined dynamical origins of two poles associated with the *Roper resonance*.
- Both of them, together with the *next higher resonance in the P_{11} partial wave* have the same originating bare state.
- The meson cloud shields quark-core state and diminishes its mass considerably.

Ground and Radially Excited States of the Nucleon



Roper Quark-Core Mass

	$R^{DSE}_{q(qq)}$	$R^{DSE}_{q(qq)}$	R^{DSE}_{qqq}	$R^{Contact}_{core}$	R^{DCCM}_{bare}
mass [GeV]	1.73	1.45	1.50	1.72	1.76

DSE: Faddeev quark-diquark amplitude of 1st excited state with dressed quark propagators.

J. Segovia, B. El-Bennich, E. Rojas, I.C. Cloët, C.D. Roberts, S.-S. Xu, H.-S. Zhong, Phys. Rev. Lett. 115 (2015)

G. Eichmann, C. Fischer, H. Sanchis-Alepuz, Phys.Rev. D94 (2016)

DSE: Faddeev three-quark interaction amplitude of 1st excited state with dressed propagators.

G. Eichmann, C. Fischer, H. Sanchis-Alepuz, Phys.Rev. D94 (2016)

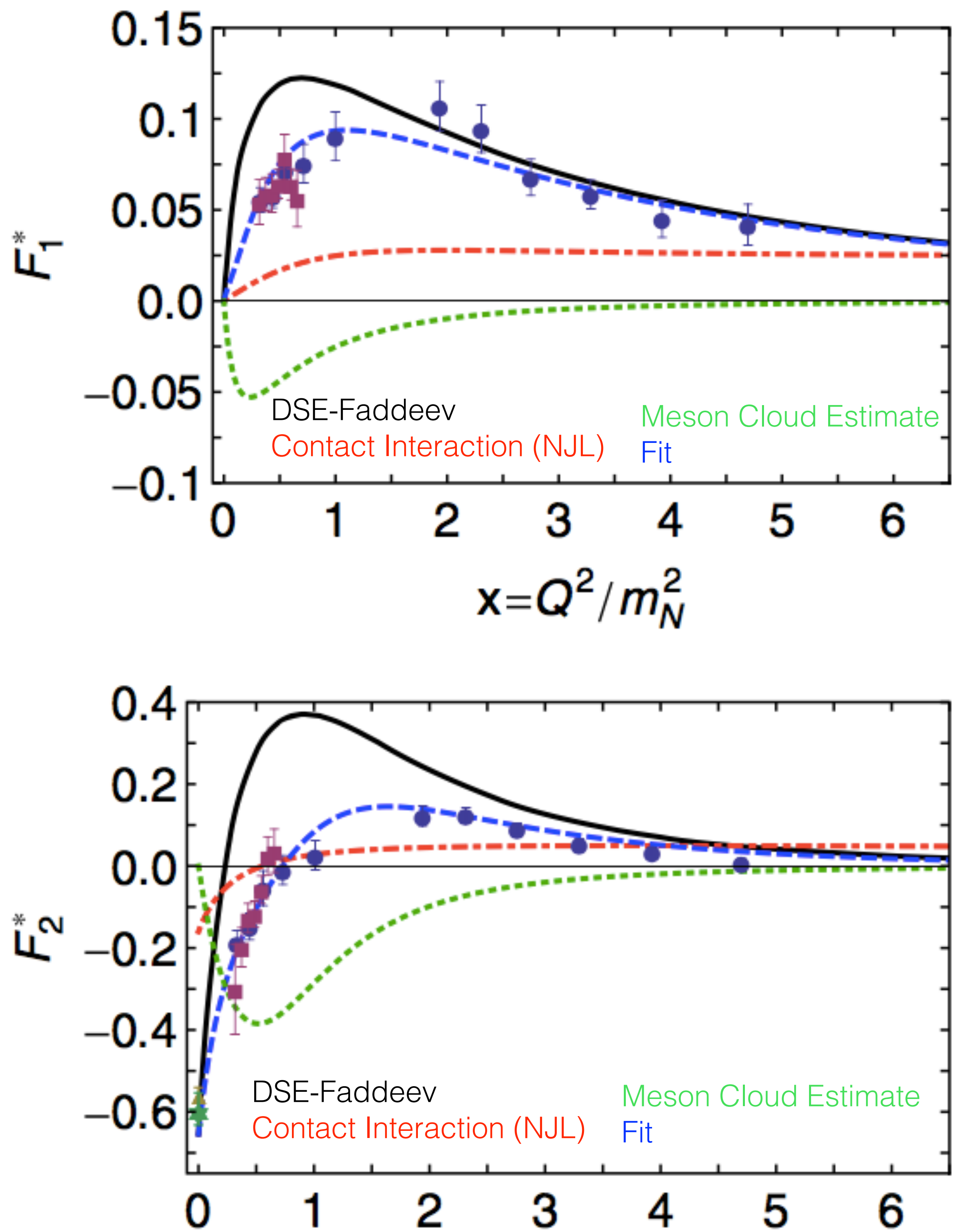
Contact : Faddeev amplitude of 1st excited state with contact interaction gap equation.

D.J. Wilson, I. C. Cloët, L. Chang, C.D. Roberts, Phys. Rev. C85 (2012)

DCCM : Dynamical Coupled Channel Model.

N. Suzuki, B. Julio-Díaz, H. Kamano, T.-S. H. Lee, A. Matsuyama, T. Sato, Phys. Rev. Lett. 104 (2010)

$\gamma\, p \to R^{+}$ Dirac and Pauli Transition Form Factors



- Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on $x > 2$.
- The mismatch between our prediction and the data on $x = 2$ is due to meson cloud contribution.
- The dotted-green curve is an inferred form of meson cloud contribution from the fit to the data.
- The contact-interaction prediction disagrees both quantitatively and qualitatively with the data.

J. Segovia, B.E., E. Rojas, I.C. Cloët, C.D. Roberts, S.-S. Xu, H.-S. Zhong, Phys. Rev. Lett. (2015)

Nucleon & Parity Partner

Including pseudoscalar and vector *diquarks* …

- The *Nucleon* and *Roper* remain dominated by scalar and axialvector diquark correlation.
- Both, the *Nucleon* and *Roper* are dominated by *S*-waves (~75% & 85%).
- However, while the $N^*(1535)$ and $N^*(1650)$ are still dominated by scalar and axialvector correlations, the Faddeev amplitude is dominated by *P*-waves (~70% & 85%).

	$N_{n=0}$	$N_{n=1}$	$N^*_{n=0}$	$N^*_{n=1}$
m^{DSE}	1.19	1.73	1.83	1.94
$m^{\text{expt.}}$	0.94	1.44	1.54	1.65
$m^{\text{DSE}} - m^{\text{expt.}}$	0.25	0.29	0.29	0.29

$$m^*_N(1535) > m^*_N(1440)$$

C. Chen, B.E., C. D.Roberts, S. M. Schmidt, J. Segovia and S. Wan, Phys. Rev. D **97** (2018)

Conclusions

- Continuous efforts in the combined approaches of DSE, BSE and Fadeev equations to hadron physics, in particular the physics of Jlab, BES, JPARC … and in future at FAIR & EIC.

- The leading truncation approaches are good enough to study simplest light mesons, baryons and quarkonia: *the light meson and quarkonia spectrum is well reproduced*.

- Poincaré covariance demands the presence of dressed-quark orbital angular momentum in the baryon ⇒ new insights on the G_E/G_M gained which lead to predictions about zero crossing.

- Explanation of exotics, heavy-flavored mesons, many excited states need corrections to truncation.

- Progress has been made and is underway: combining DSE and lattice QCD gluon & ghost propagators beyond RL truncation and include them in symmetry-preserving manner in bound-state equations.