Partial-Wave Analysis at COMPASS

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for the COMPASS Collaboration

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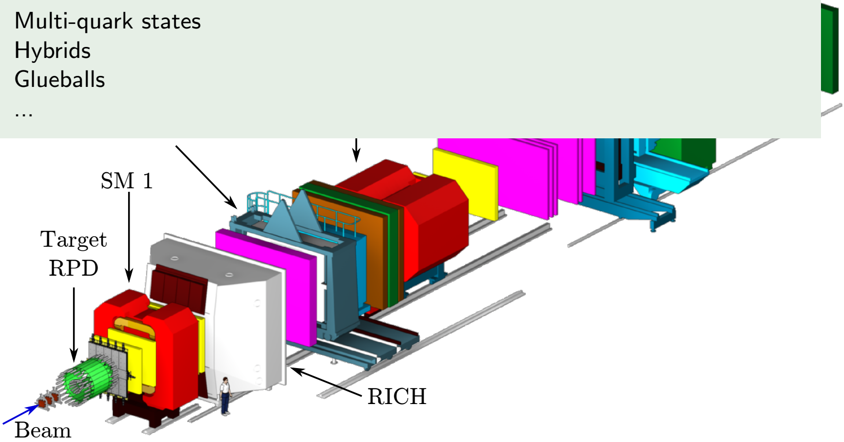
September 6, 2019
International Workshop on Partial Wave Analyses and Advanced Tools for Hadron Spectroscopy

COMPASS Setup for Hadron Beams

[NIMA 779 (2015) 69]

(see talk by B. Ketzer on Monday)

- Explore light-meson spectrum for $m \lesssim 3\,\mathrm{GeV}/c^2$
- High-precision measurement of known states
- Search for new forms of matter:
  - Multi-quark states
  - Hybrids
  - Glueballs
  - ...

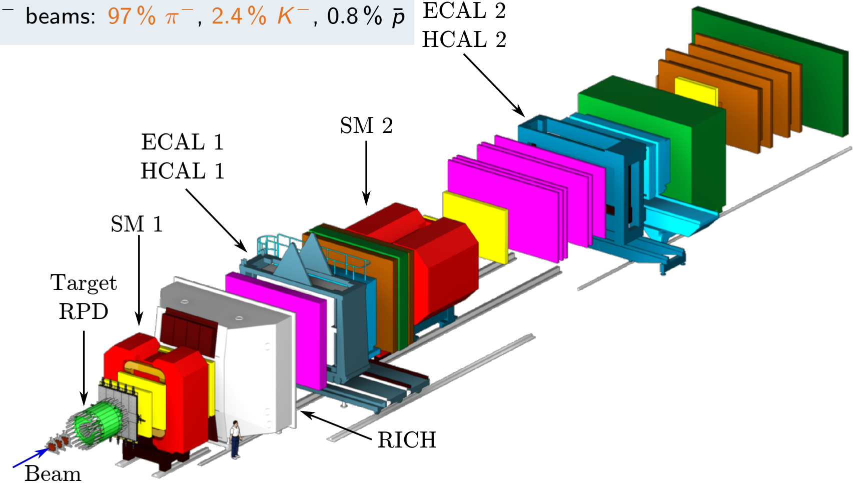


COMPASS Setup for Hadron Beams

[NIMA 779 (2015) 69]

M2 beam line

- Located at CERN (SPS)
- 190 GeV/c secondary hadron beams
  - h^- beams: 97 % π^-, 2.4 % K^-, 0.8 % $\bar{p}$

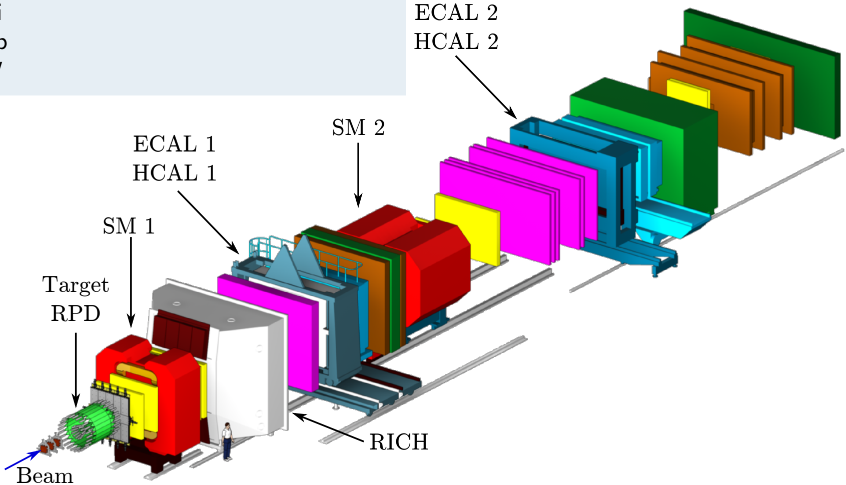


COMPASS Setup for Hadron Beams

[NIMA 779 (2015) 69]

Target

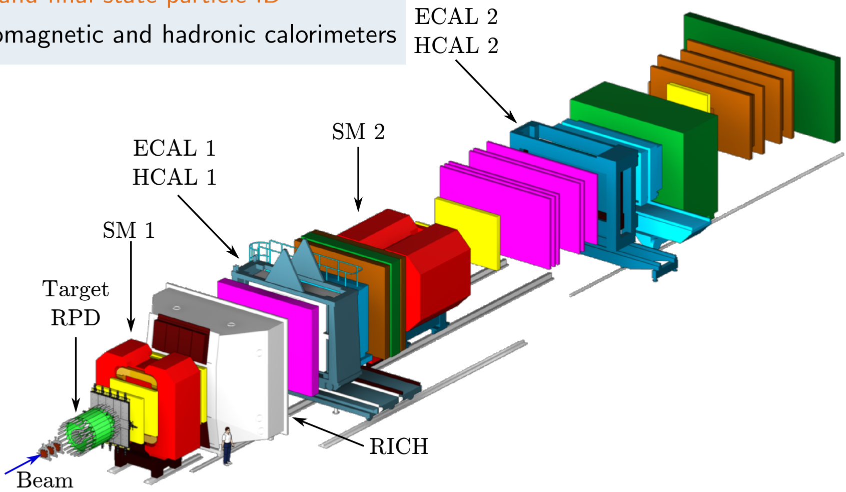
- Various targets:
  - ℓH_2
  - Ni
  - Pb
  - W



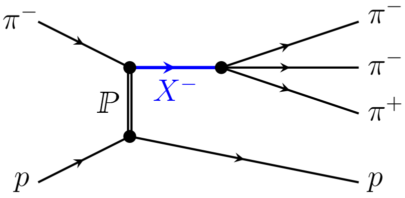
COMPASS Setup for Hadron Beams

COMPASS spectrometer

- Two-stage magnetic spectrometer
- Beam and final-state particle ID
- Electromagnetic and hadronic calorimeters



Diffractive Production



- Diffractive production in high-energy scattering
- Light mesons appear as intermediate states X^-
- Observed in decays into quasi-stable particles:
  - $\pi^-\pi^-\pi^+$ final state: Access to a_J and π_J states
  - $K^-\pi^-\pi^+$ final state: Access to all K and K^* states

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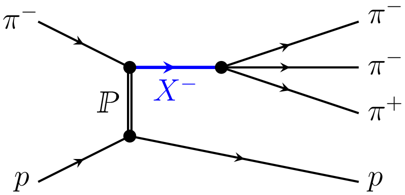

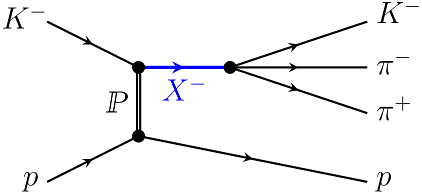

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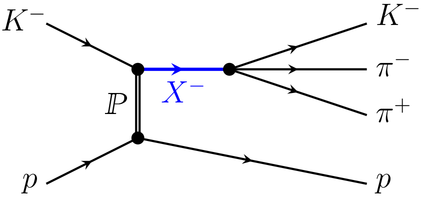
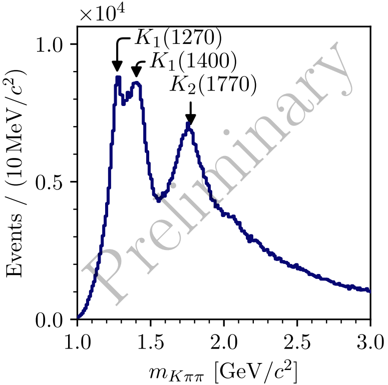
Kinematic Distributions





- Rich spectrum of overlapping and interfering X^-
  - Dominant well known states
  - States with lower intensity are "hidden"

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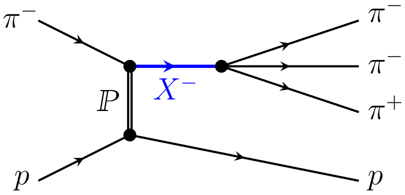

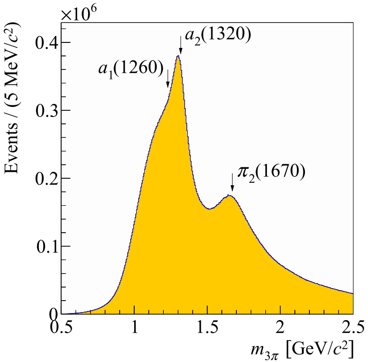

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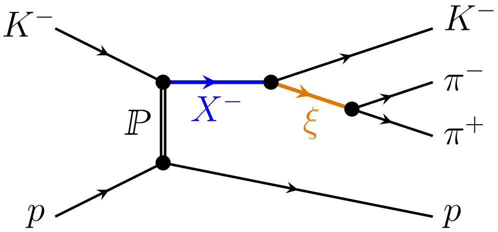
Partial-Wave Decomposition

Isobar Model



- Partial wave $a = J^{P(C)} M^{\varepsilon} \xi^0 b^- L$ at fixed invariant mass of X^- system
  - ➥ Calculate 5D decay phase-space distribution $\Psi(\tau)$ of final state
- Total intensity distribution: Coherent sum of partial-wave amplitudes

$$\mathcal{I}(\tau) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a \, \psi_a(\tau) \right|^2$$

- Perform maximum-likelihood fit in cells of $(m_{K\pi\pi}, t')$
  - ➥ Extract $m_{K\pi\pi}$ and t' dependence of transition amplitudes $\mathcal{T}_a$

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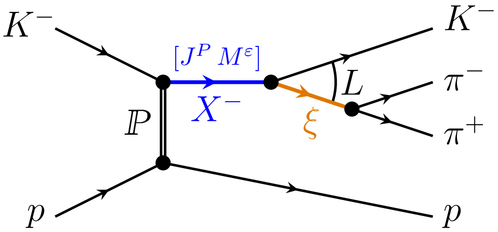

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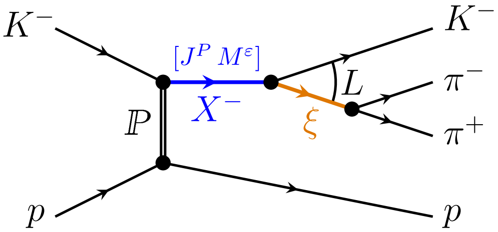

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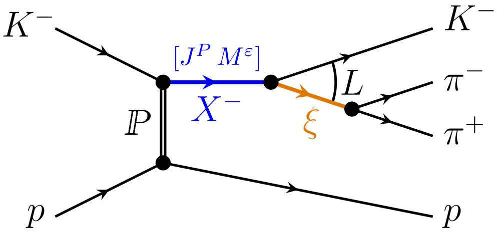

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Wave-Set Selection

$$\mathcal{I}(\tau) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

Challenge: Find the "best" set of waves that describes the data

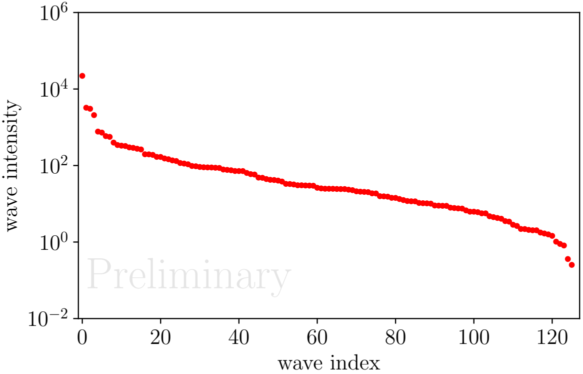
- If the wave set is too large
  - ➥ Starting to describe statistical fluctuations
- If waves that contribute to the data are missing
  - ➥ Intensity can be wrongly attributed to other waves
  - ➥ Model leakage

Wave-Set Selection

Infer wave set from data

- Systematically construct large set of allowed partial waves
  ➥ "Wave pool"
- Fit wave pool to data
  - Impose penalty on $|\mathcal{T}_a|^2 \Rightarrow$ regularization
  - Suppress insignificant waves
- Select waves that significantly contribute to data
  ➥ "Best" subset of waves that describe the data

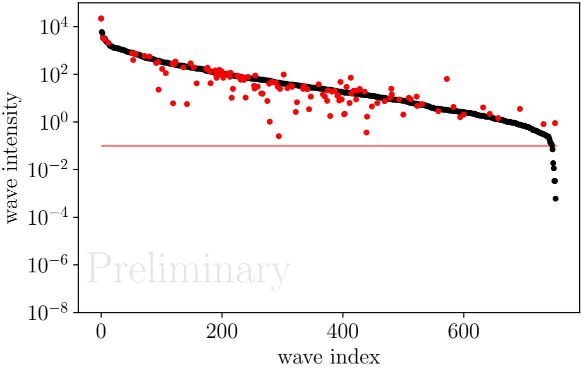
Wave-Set Selection



- $\pi^-\pi^-\pi^+$ Monte Carlo mock data set with 126 partial waves
- Fitting wave pool of 753 waves
  - ➥ Massive overfitting
  - ➥ Almost all waves pick up intensity

Courtesy F. Kaspar, TUM

Wave-Set Selection



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Wave-Set Selection

Regularization: LASSO

$$\ln \mathcal{L}_{\text{fit}} = \ln \mathcal{L}_{\text{extended}} + \sum_{a}^{\text{waves}} \ln \mathcal{L}_{\text{reg}}(|T_a|; \{c_{\text{para}}\})$$

LASSO/L1 regularization[1]

$$\ln \mathcal{L}_{\text{reg}}(|T_a|; \lambda) = -\lambda |T_a|$$

- Maximum at $|T_a| = 0$
- Well established[2]
- "Smoothing" at $|T_a| = 0$

$$|T_a| \rightarrow \sqrt{|T_a|^2 + \varepsilon}$$

[1] Robert Tibshirani. "Regression Shrinkage and Selection via the Lasso". In: Journal of the Royal Statistical Society. Series B 58.1 (1996)
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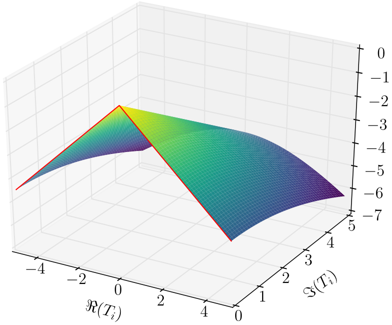

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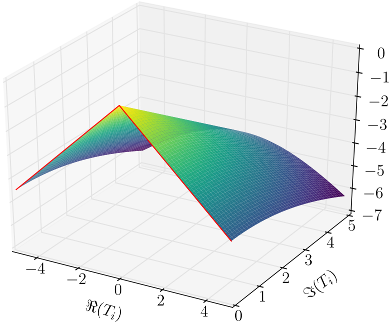

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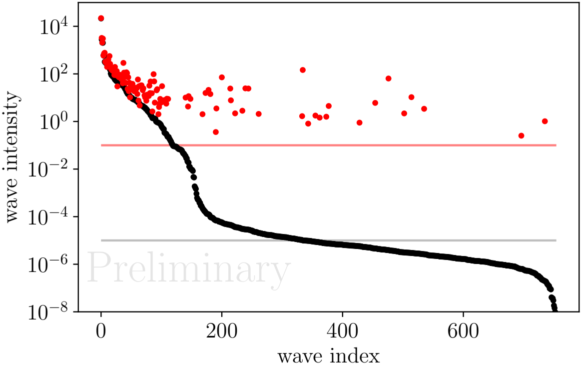


$\lambda = 0.3$

$\varepsilon = 10^{-5}$

- Bias also on large transition amplitudes
- Some additional waves
- Some waves missing

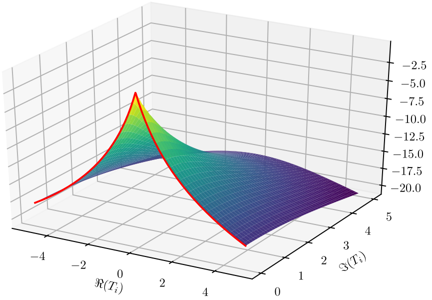
Courtesy F. Kaspar, TUM

Generalized Pareto[1]

$$\ln \mathcal{L}_{\mathrm{reg}}(|T_a|;\Gamma,\zeta) = -\frac{1}{\zeta}\ln\left[1+\zeta\frac{|\mathcal{T}_a|}{\Gamma}\right]$$

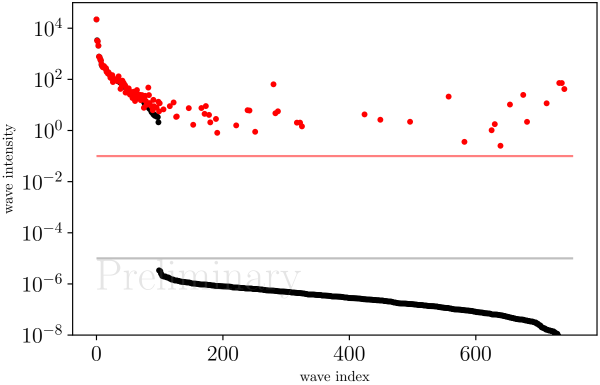
- Wave intensities spread over orders of magnitudes
- Use logarithmic prior
  - ➥ Heavy-tailed
  - ➥ Less bias on large waves
- LASSO-like for $|T_a| \to 0$
- "Smoothing" at $|\mathcal{T}_a| = 0$

$$|\mathcal{T}_a| \to \sqrt{|\mathcal{T}_a|^2+\varepsilon}$$



[1] Artin Armagan, David B. Dunson, and Jaeyong Lee. "Generalized double Pareto shrinkage". In: Statistica Sinica (2013). doi: 10.5705/ss.2011.048.

Wave-Set Selection
Regularization: Generalized Pareto



$\zeta = 0.5$

$\Gamma = 0.1$

$\varepsilon = 10^{-5}$

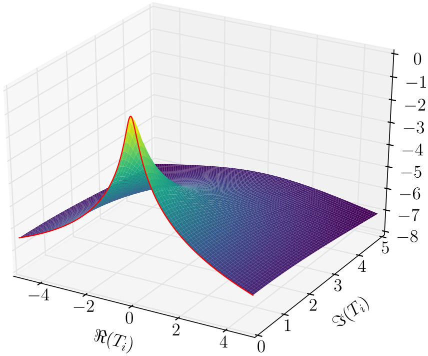
- Less bias on large transition amplitudes
- Clear kink in intensity distribution to smoothing scale $\Rightarrow$ Selection
- Less additional waves
- Some small waves missing

Courtesy F. Kaspar, TUM

"Cauchy"

$$\ln \mathcal{L}_{\mathrm{reg}}(|T_a|;\Gamma) = -\ln\left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2}\right]$$

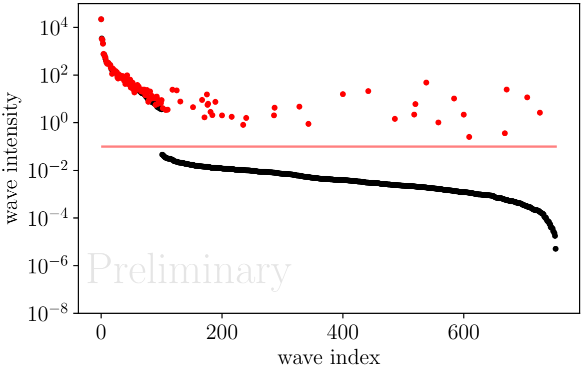
- Logarithmic prior
- L2-like for $|T_a| \to 0$



Wave-Set Selection

Regularization: Cauchy

$\Gamma = 0.2$



- Less bias on large transition amplitudes
- Clear kink in intensity distribution
- Few additional waves
- Few small waves missing

Courtesy F. Kaspar, TUM

Wave-Set Selection

For the $K^-\pi^-\pi^+$ Final State

Wave pool

- Spin $J \leq 7$
- Angular momentum $L \leq 7$
- Positive naturality of exchange particle
- 12 isobars
  - $[K\pi]_S^{K\pi}$, $[K\pi]_S^{K\eta}$, $K^*(892)$, $K^*(1680)$, $K_2^*(1430)$, $K_3^*(1780)$
  - $[\pi\pi]_S$, $f_0(980)$, $f_0(1500)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$

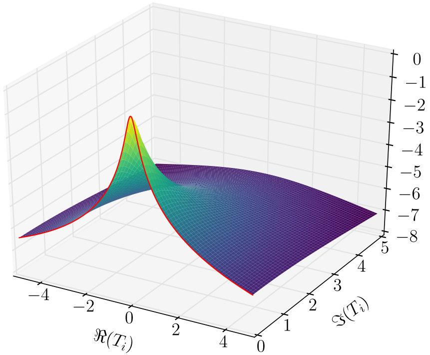
$\Rightarrow$ "Wave pool" of 596 waves

Regularization

$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|;\Gamma) = -\ln\left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2}\right]$$

- Use Cauchy regularization
- Scale of $|\mathcal{T}_a|$ depends on experimental acceptance
  - Apply penalty on expected number $\bar{N}_a$ of observed events

$$\Gamma_a = \frac{\Gamma}{\sqrt{\bar{\eta}_a}} \Rightarrow \frac{|\mathcal{T}_a|^2}{\Gamma_a^2} = \frac{\bar{N}_a}{\Gamma^2}$$



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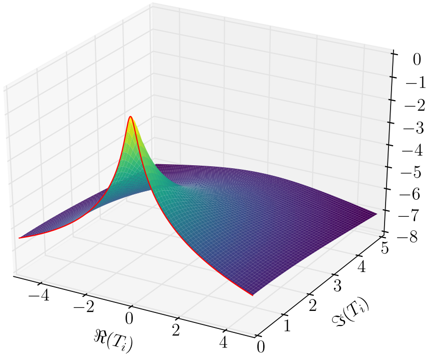

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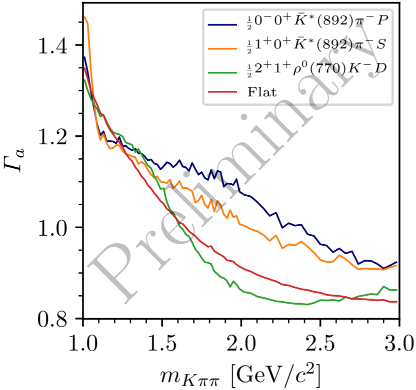
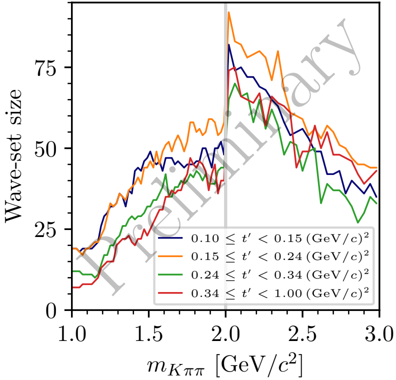


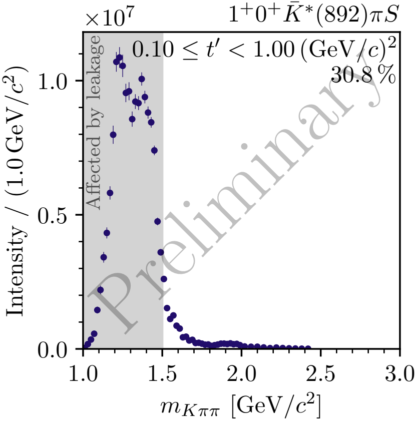
Wave-set size

- 5 to 90 waves per $(m_{K\pi\pi}, t')$ cell
- Larger wave set for larger binning in $m_{K\pi\pi}$
- Larger wave set in t' bins with more events



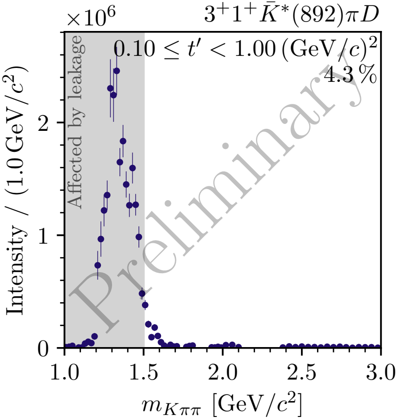
$1^+ 0^+ K^*(892)\, \pi\, S$

- Dominant signal
- $K_1(1270)$, $K_1(1400)$ double peak



Leakage Effect

- Unexpected low-mass enhancement in $3^+ 1^+ K^*(892)\,\pi\,D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Loss of orthogonality taking acceptance into account
- Limited acceptance due to limited kinematic range of final-state PID
- Only a small sub-set of partial waves affected



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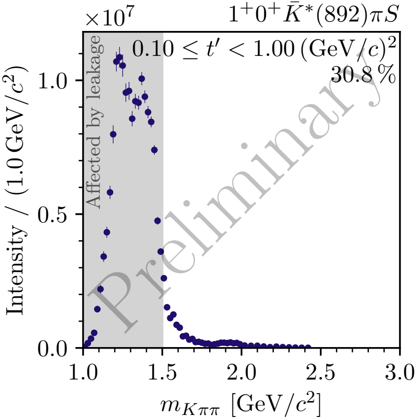

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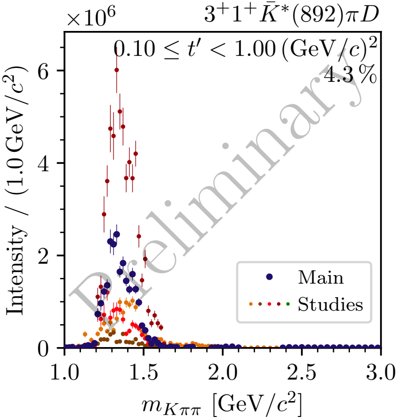

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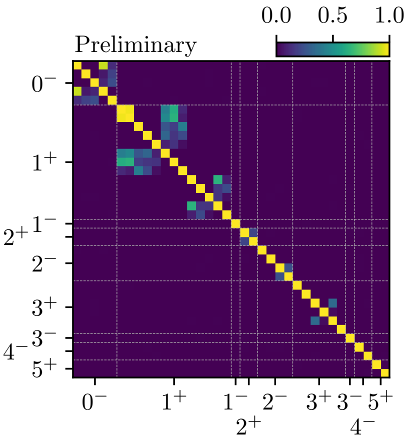

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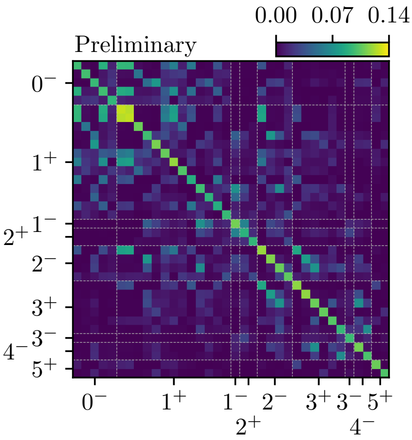

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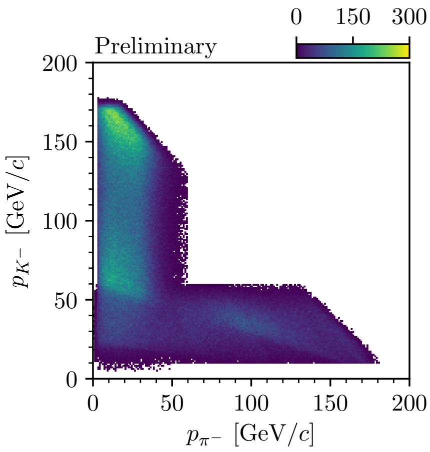

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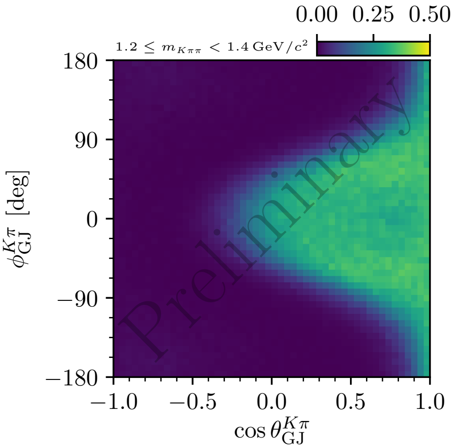
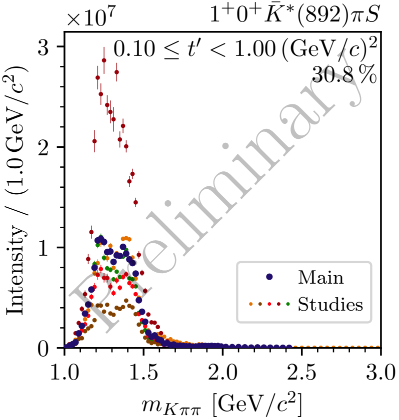
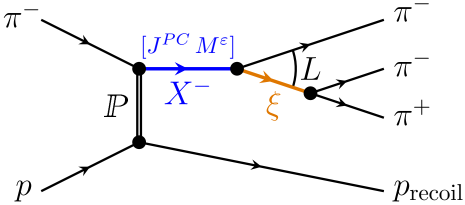
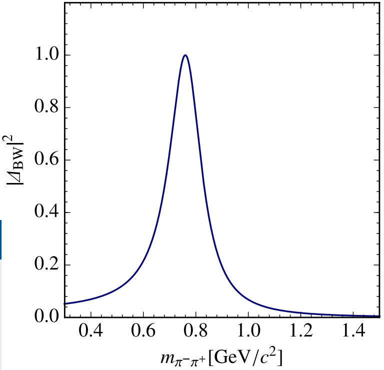


Leakage Effect

- Unexpected low-mass enhancement in $3^+\,1^+\,K^*(892)\,\pi\,D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Loss of orthogonality taking acceptance into account
- Limited acceptance due to limited kinematic range of final-state PID
- Only a small sub-set of partial waves affected

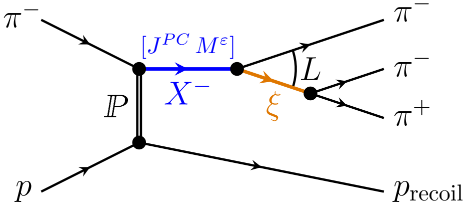


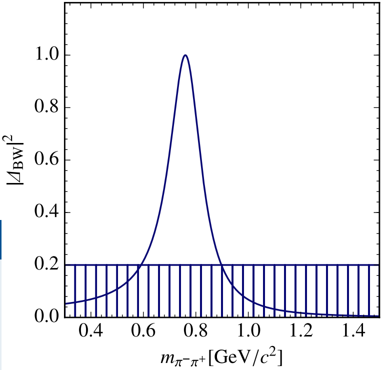




Extract isobar amplitudes from data

- Replace model with step-like isobars
- Extract binned shape in mass-independent fit





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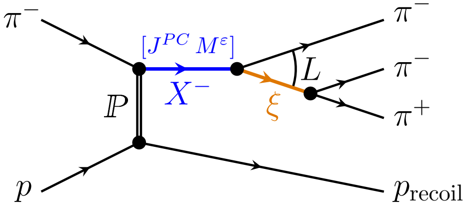
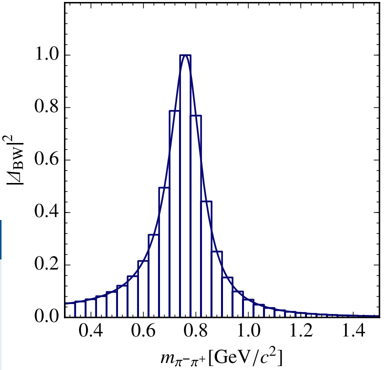
Freed-Isobar Analysis





Extract isobar amplitudes from data

- Replace model with step-like isobars
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- 12 dominant waves with freed isobars
  - About 75 % of total intensity
- Replace 16 of 88 fixed-isobar waves
  - 72 small waves with fixed isobars
  - Stabilize the fit
- Extract $\pi^-\pi^+$ amplitude
  - for 1100 $(m_{3\pi}, t')$ cells
  - for different J^{PC} of $\pi^-\pi^-\pi^+$ system
- 5 "zero modes" (see talk by F. Krinner on Thursday)
  - Mathematical ambiguities in the model
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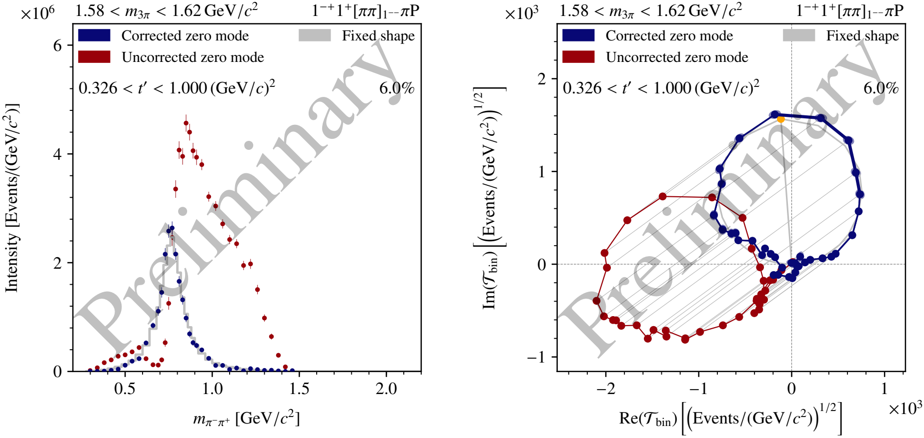
Freed-Isobar Analysis

Spin-Exotic $1^{-+}\,1^{+}\,[\pi\pi]_{1^{--}}\,\pi\,P$ Wave in $\pi^{-}\pi^{-}\pi^{+}$



- Resolve zero mode: $\rho(770)$ Breit Wigner
- Similar to fixed-isobar shape
- Isobar model: Valid assumption

- Observed deviations?
  - Final state interactions
  - Non-resonant contributions
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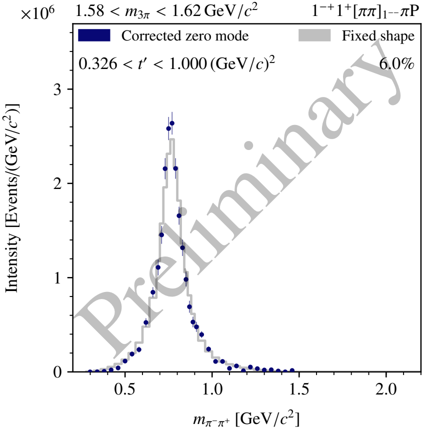

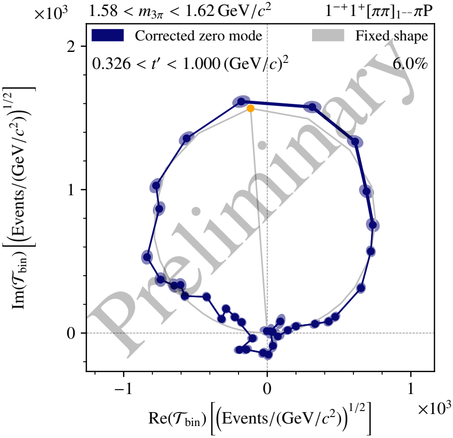

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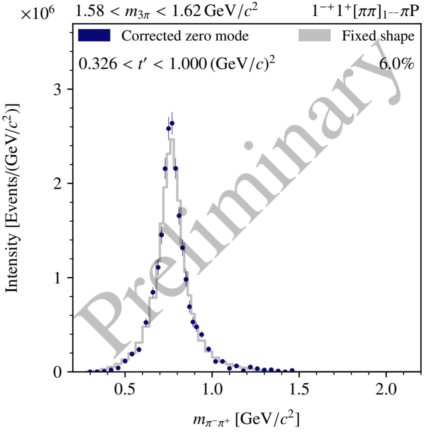

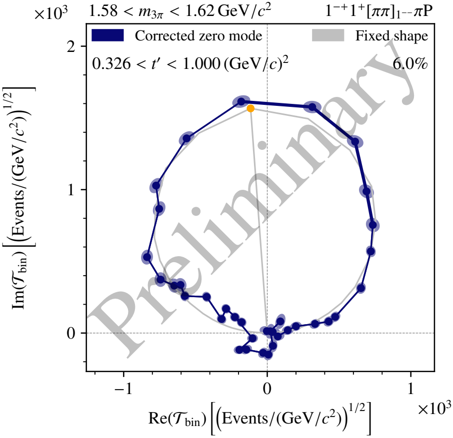

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Summary

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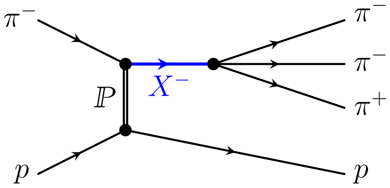
- Regularization-based wave-set selection
- Study of leakage effects
- Freed-isobar analysis to study sub-system amplitudes



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  - $K^-\pi^-\pi^+$
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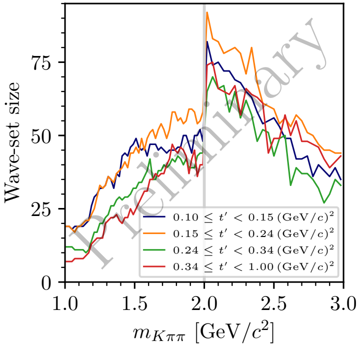

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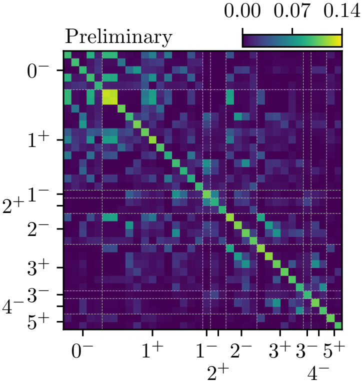

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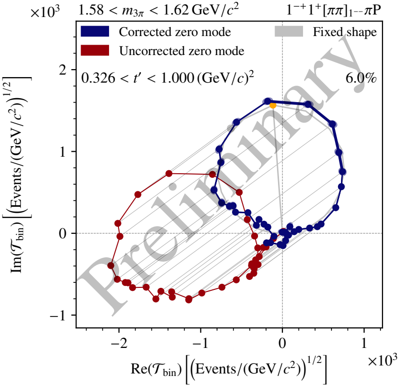

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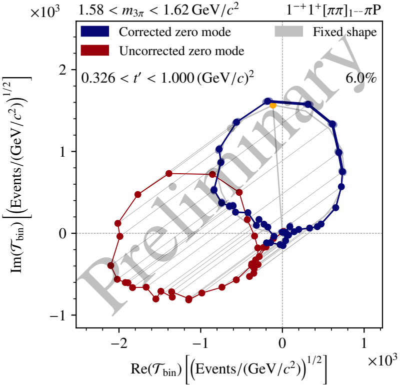

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Backup

Outline