AMPLITUDE ANALYSIS AND POLE INTERPRETATION: THE $P_C(4312)$ CASE

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AMPLITUDE ANALYSIS: BOTTOM-TOP APPROACH

- Build the minimally-biased theory (model) with the correct physical restrictions
- Fit the experimental data and perform an error analysis
- Analytically continue the amplitude to the complex plane and the unphysical Riemann sheets
- Hunt and study poles. Two aspects:
    - Are they poles of the model only or are they also poles of the data?
    - Can we make a model-independent interpretation of the nature of the singularity?

UNCERTAINTIES ANALYSIS: BOOTSTRAP

Alessandro will elaborate on this later

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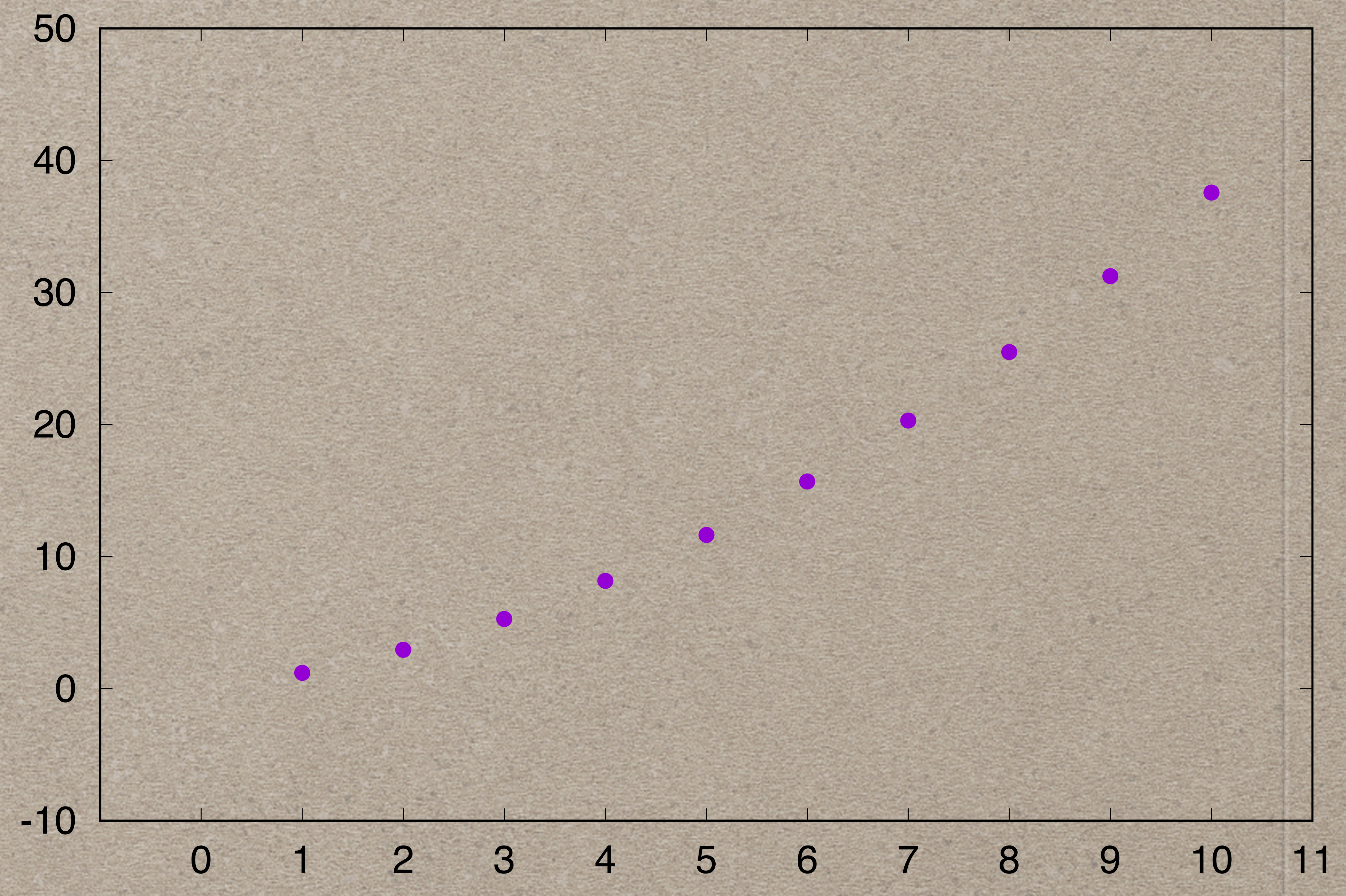
- Take the data with errors



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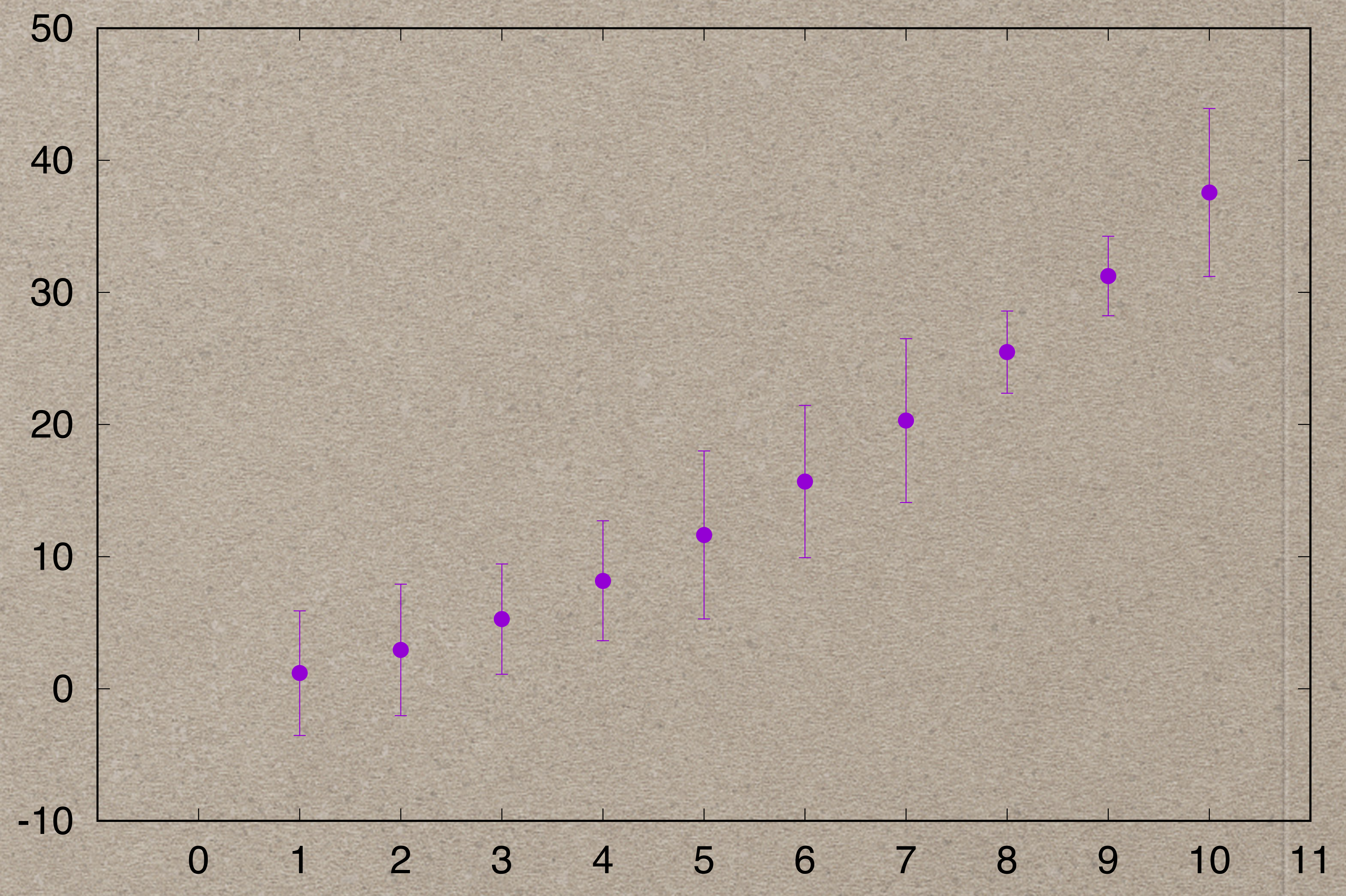

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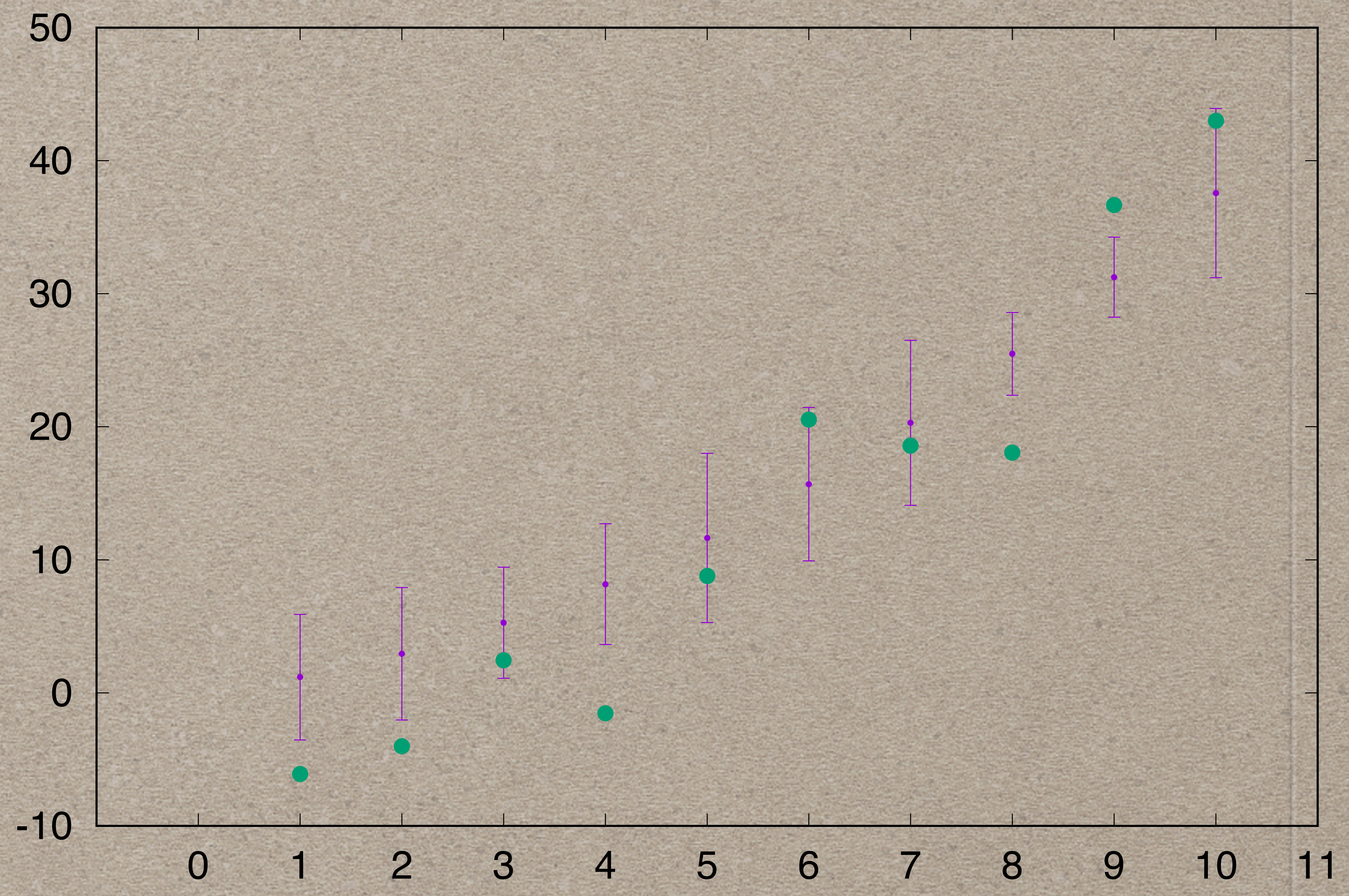
- Take the data with errors
- Randomize according to uncertainties (generate pseudodata)



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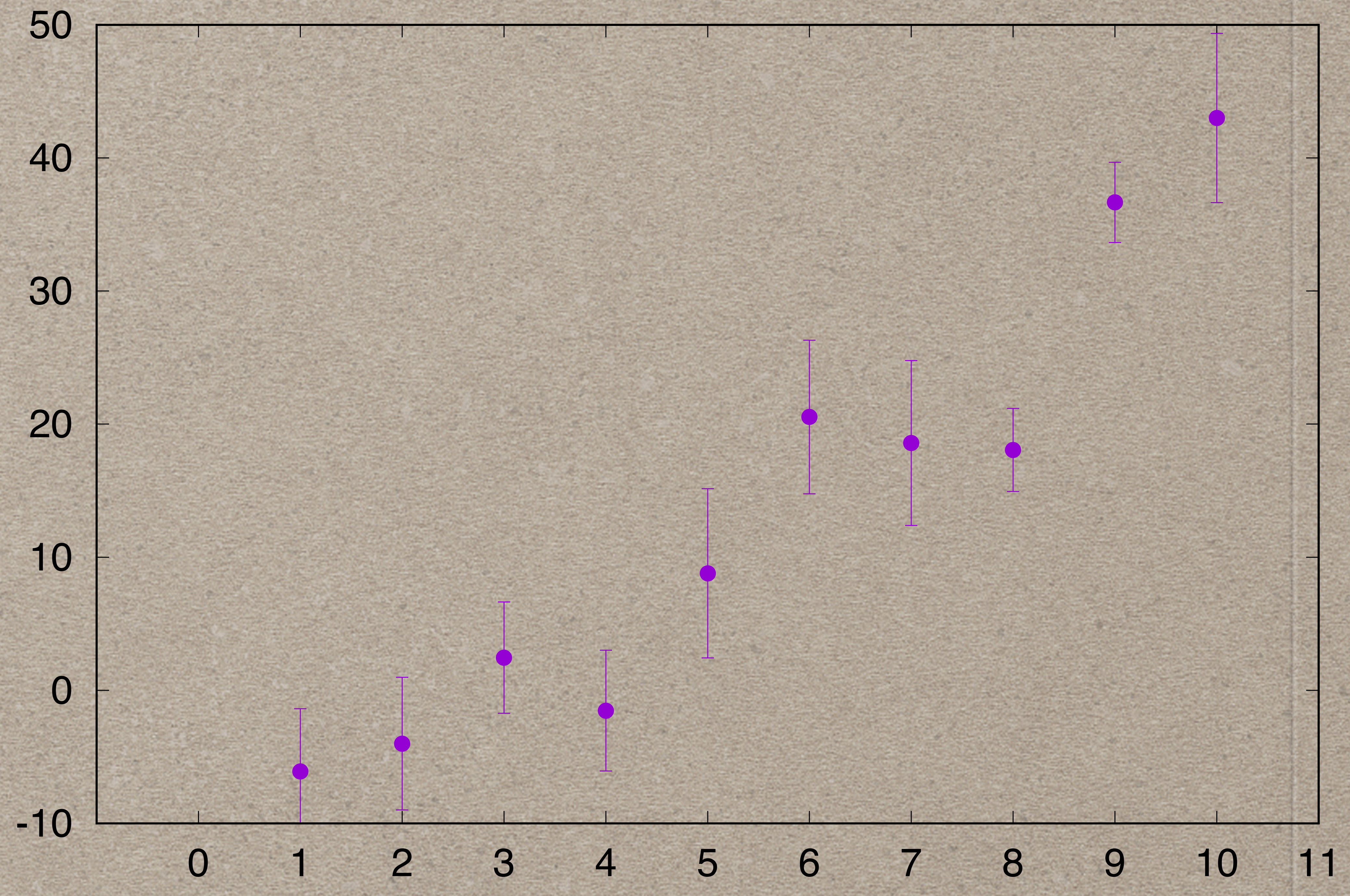

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- Take the data with errors
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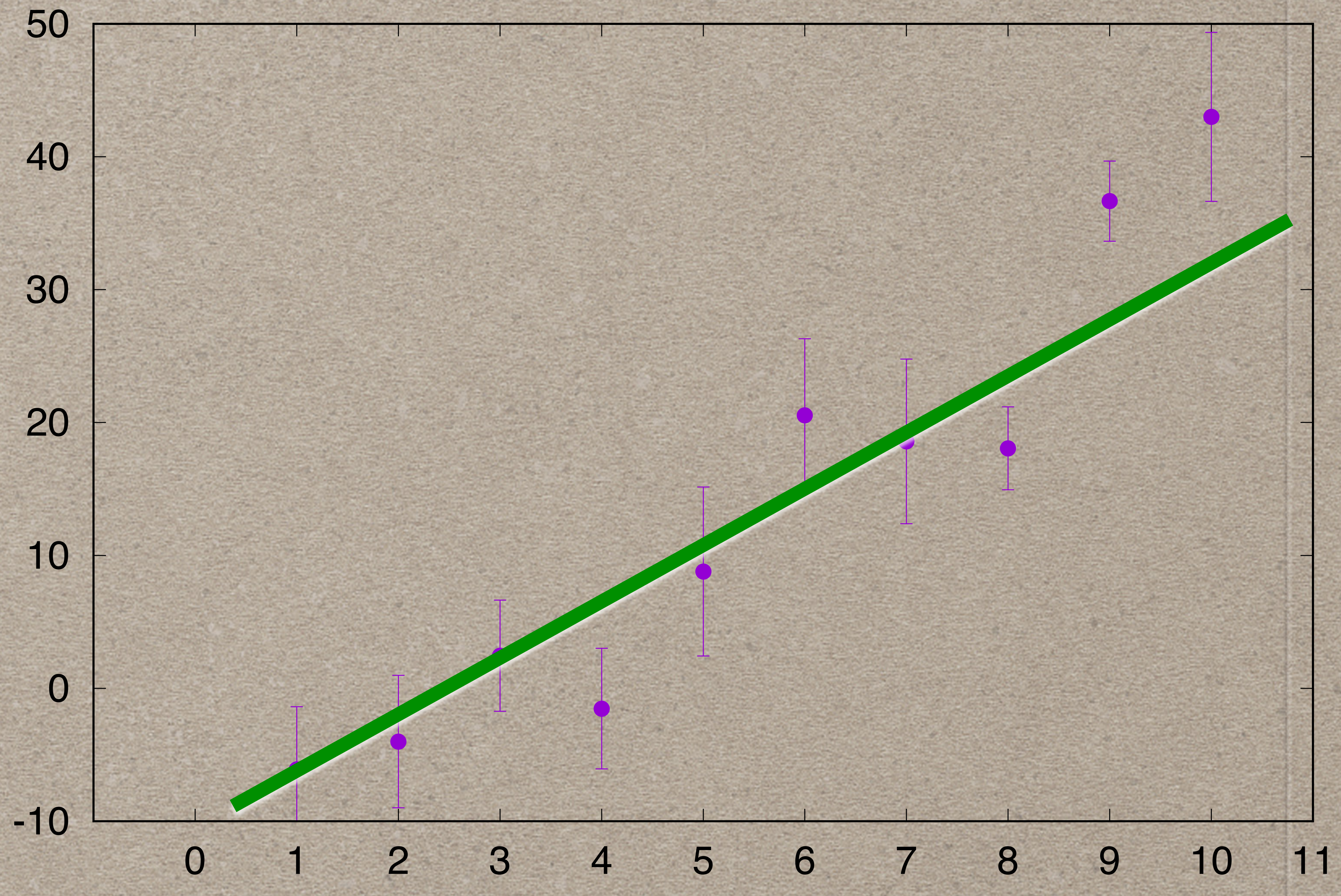

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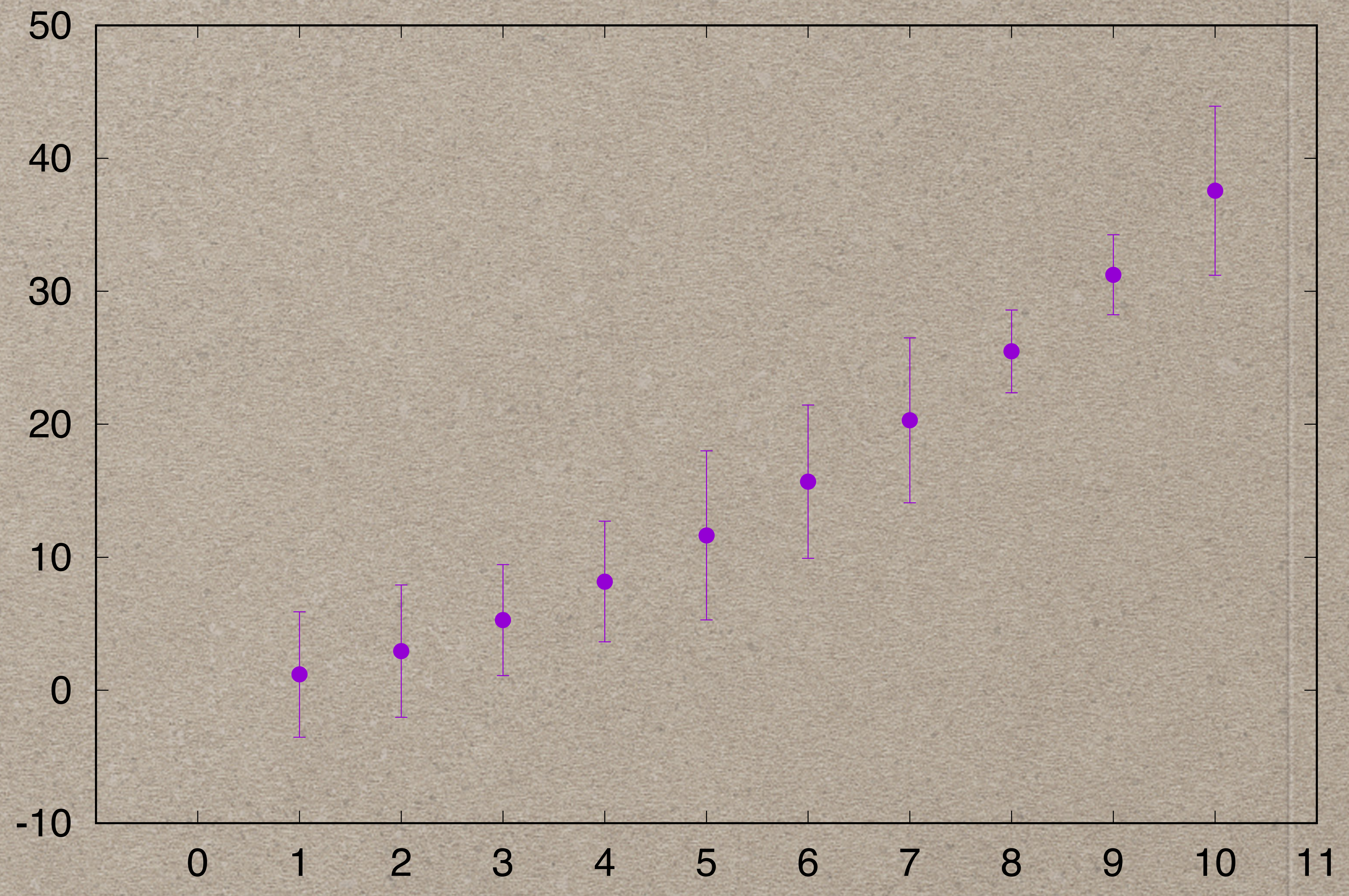

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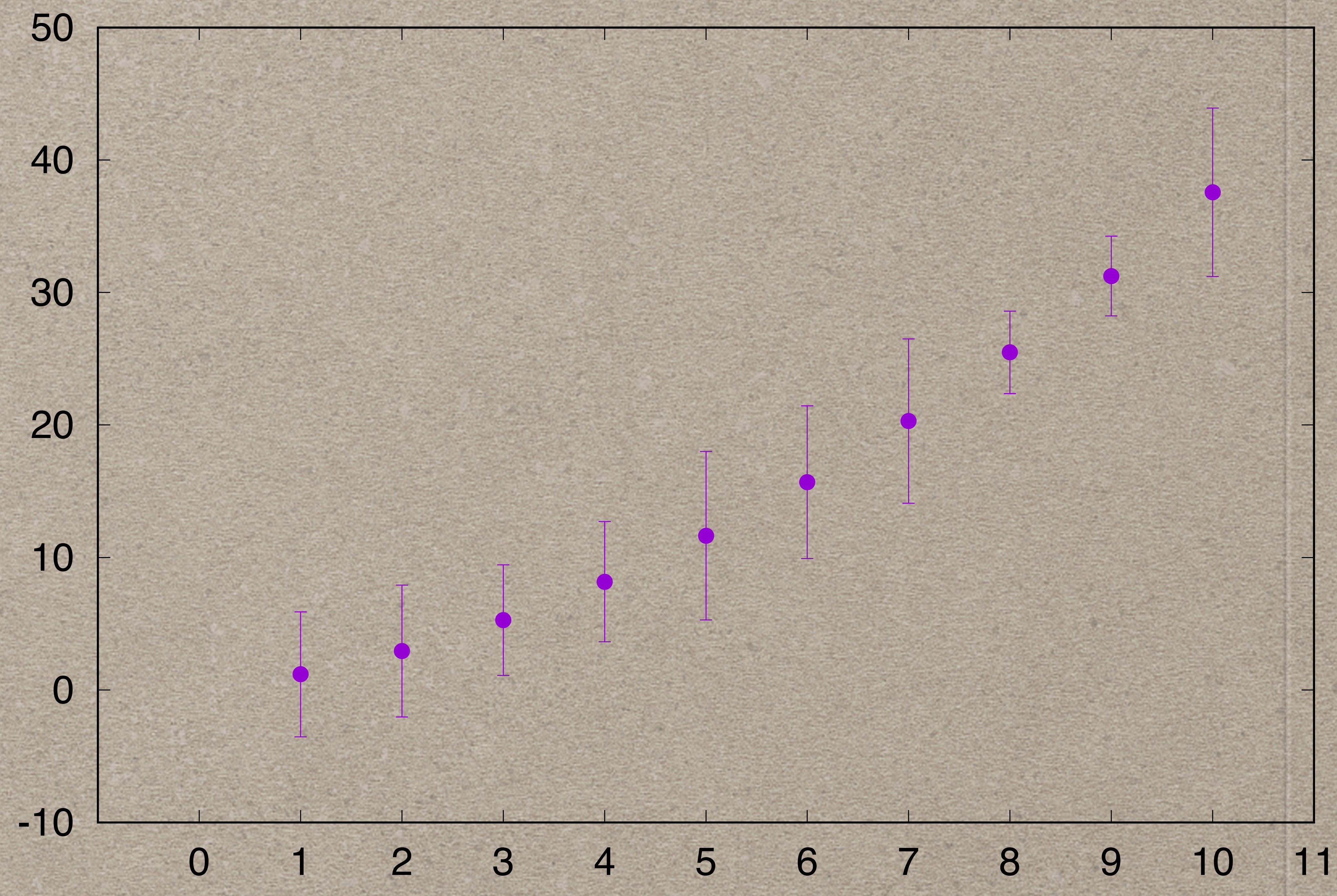

You end up with N sets of parameters,and you can perform statistics on them and compute derivative quantities (poles, observables) propagating in full the errors

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NATURE OF $P_c(4312)$

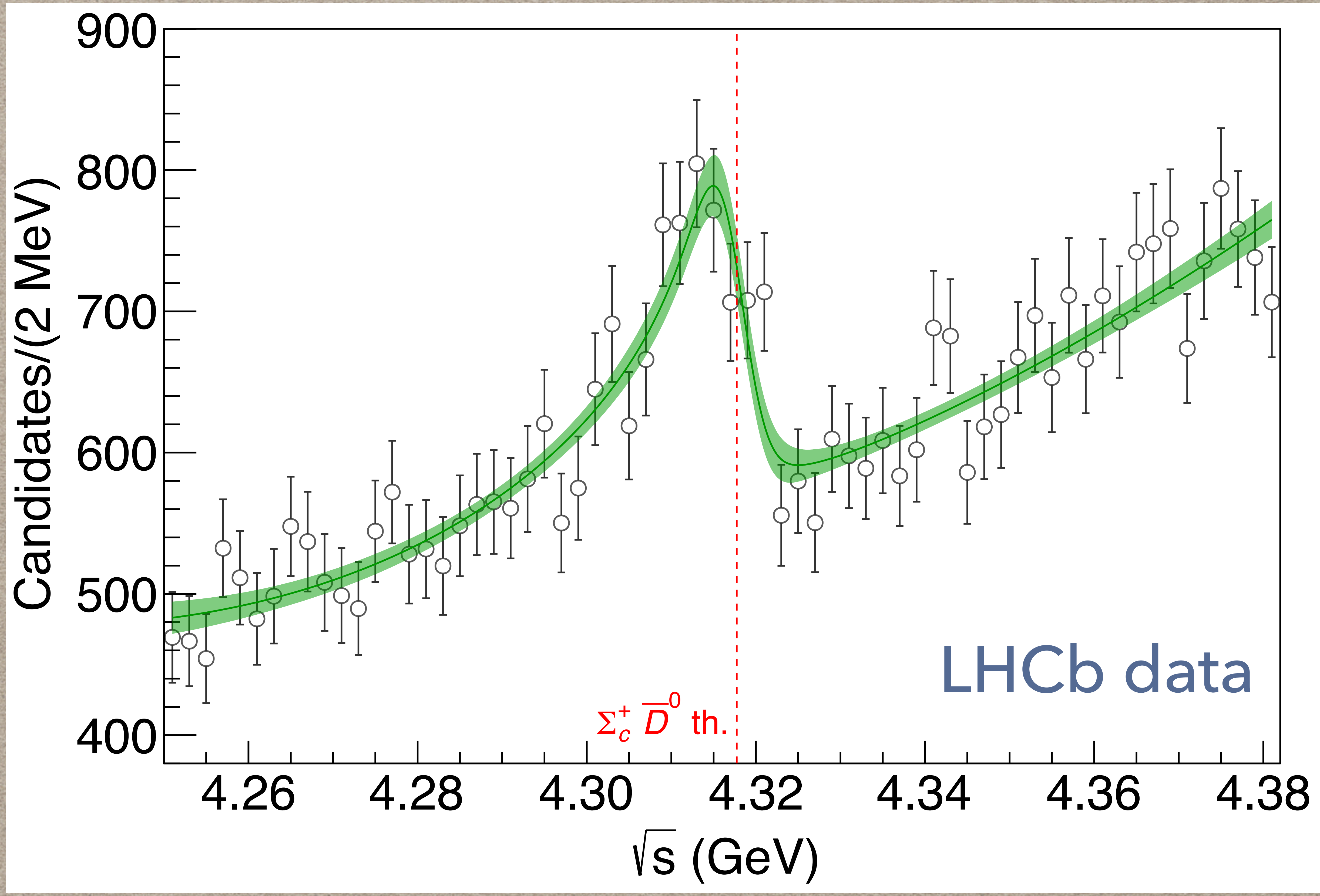
Close to a threshold

Triangle singularity

Compact pentaquark

Molecule

Virtual state



LHCb, PRL 122 (2019) 222001

Λ_b^0

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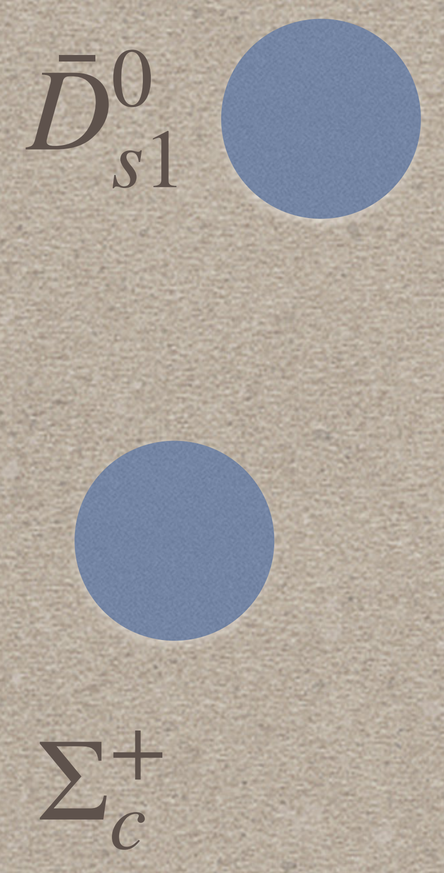
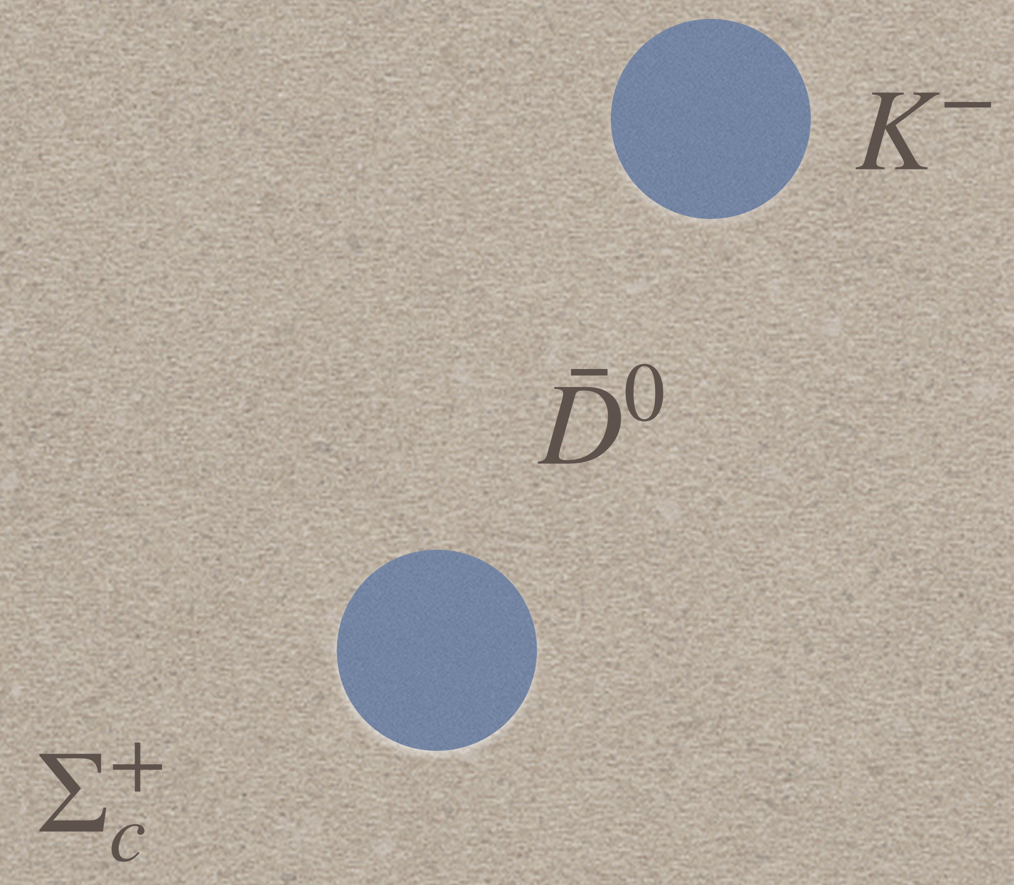
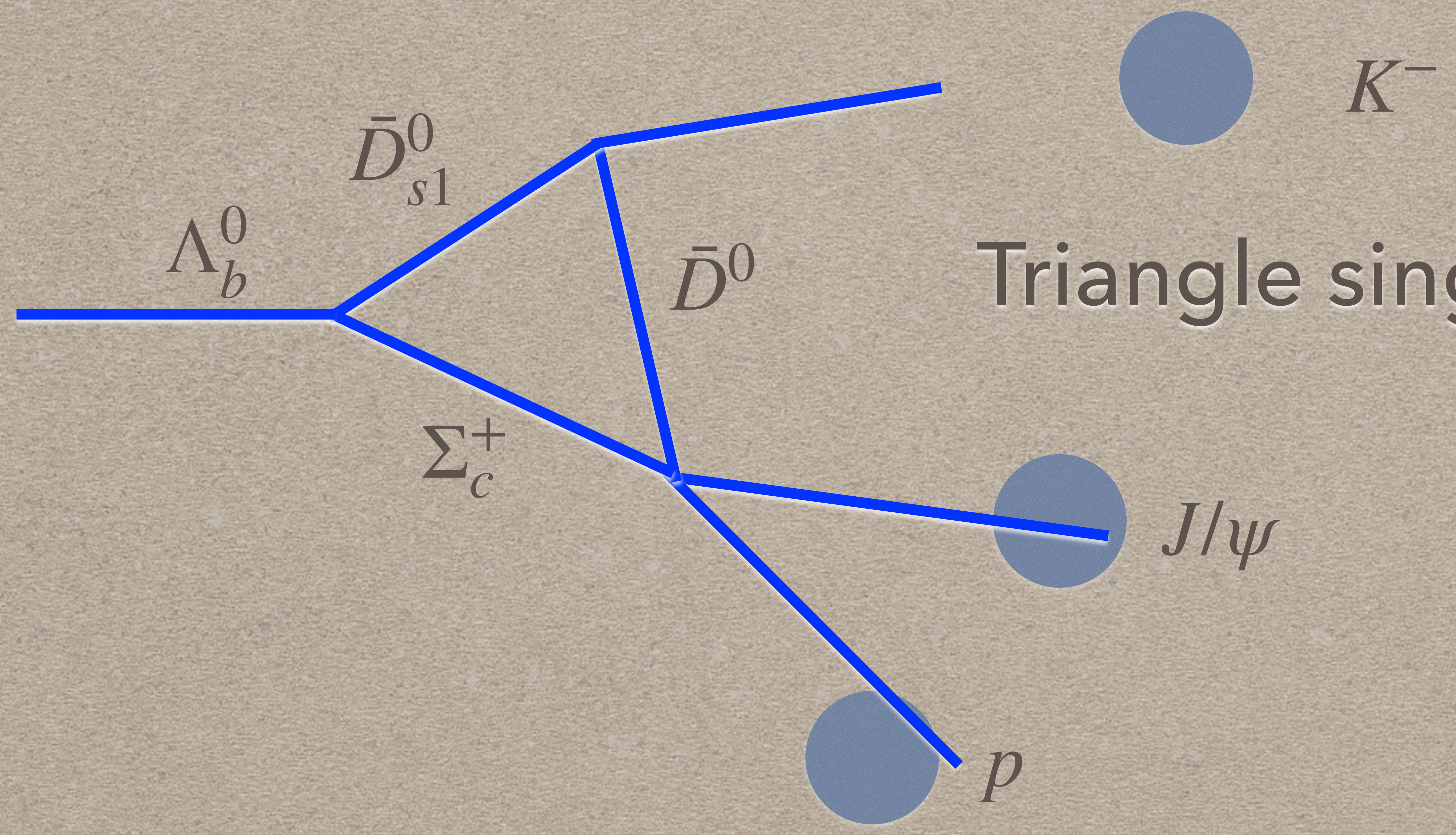
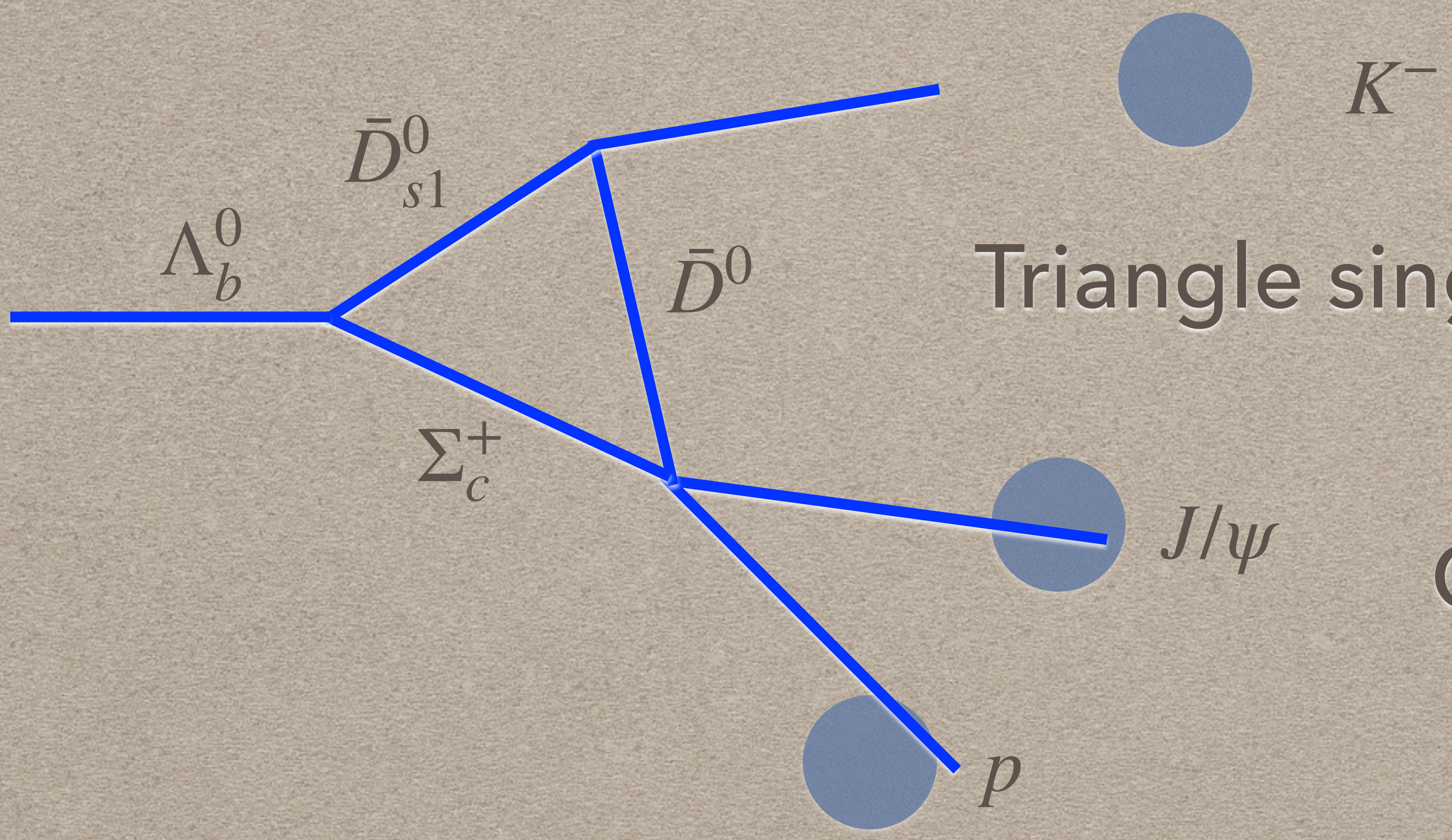

$\bar{D}_{s1}^0$
Σ_c^+


K^-
$\bar{D}^0$
Σ_c^+


K^-
J/ψ
p



Triangle singularity



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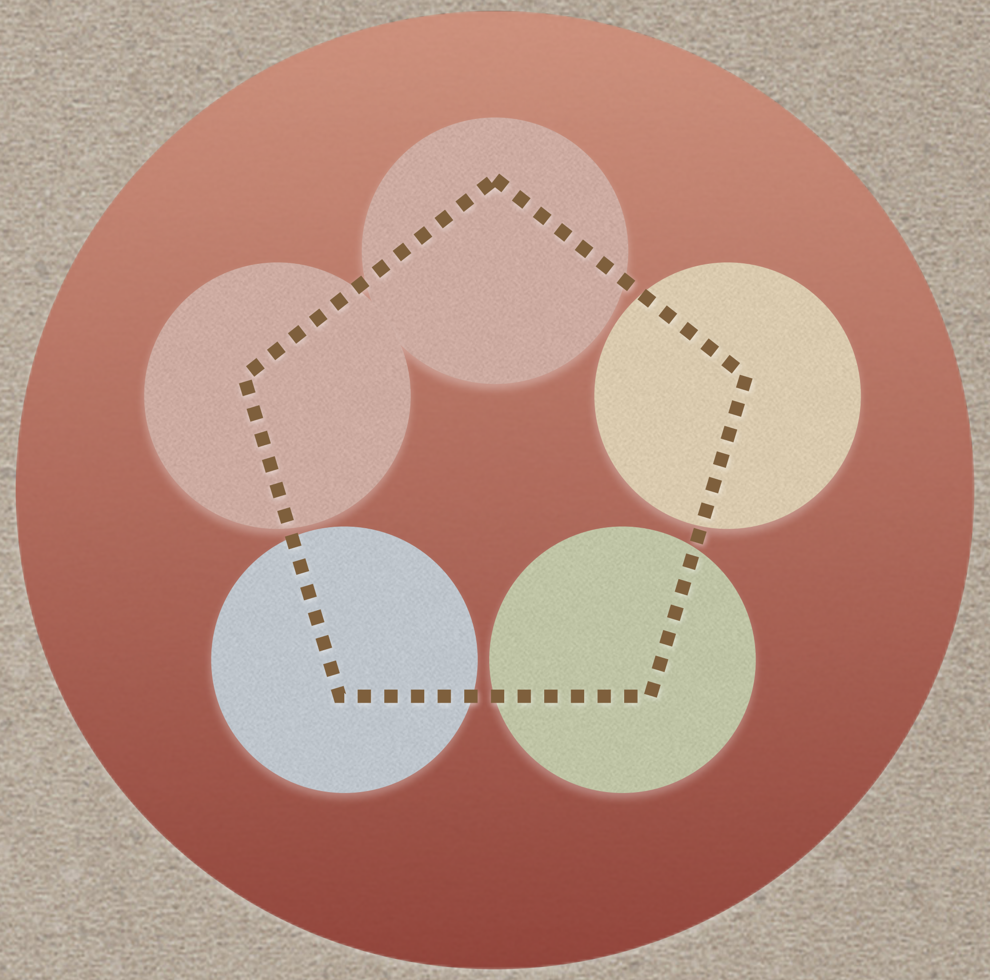

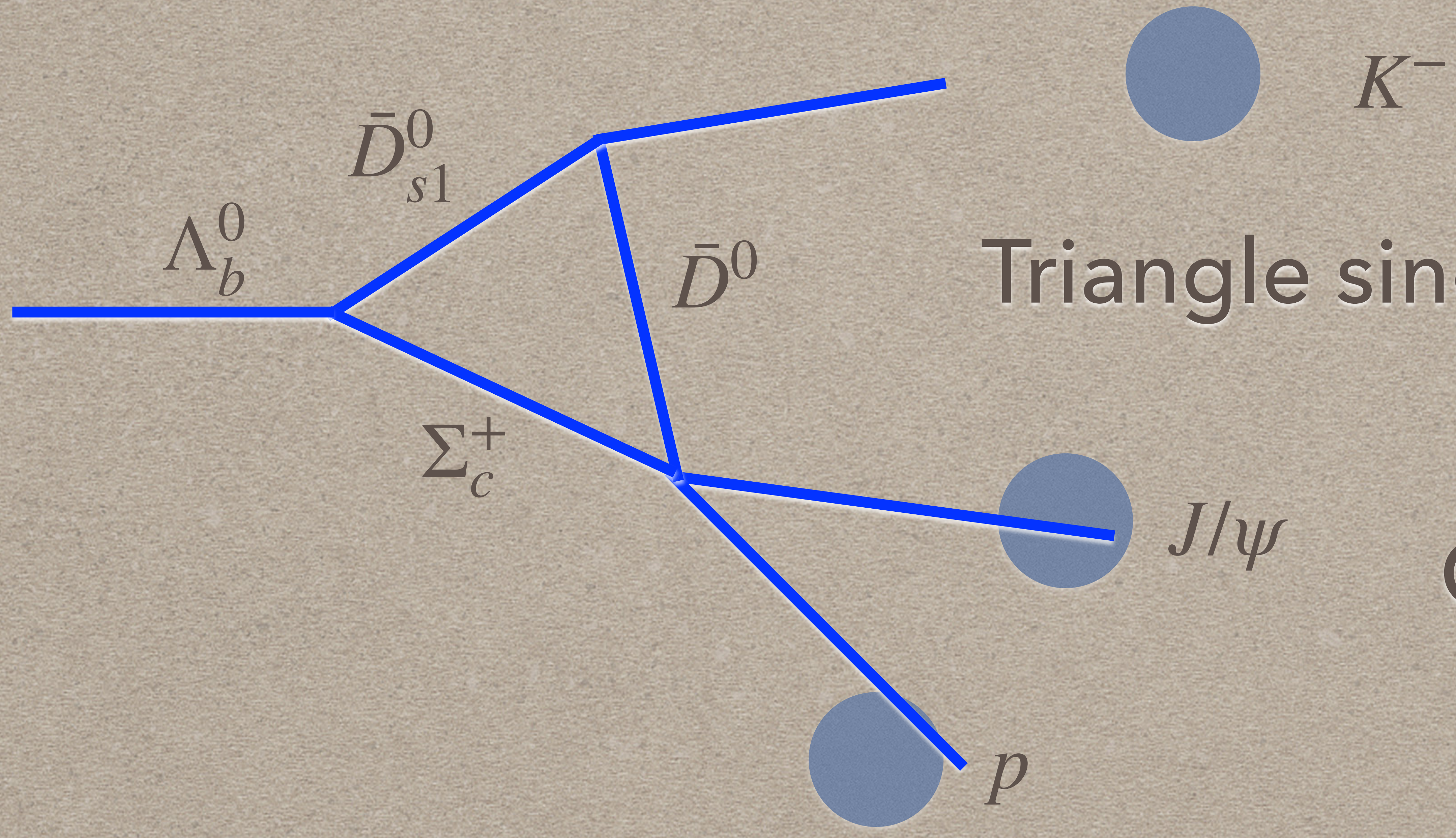

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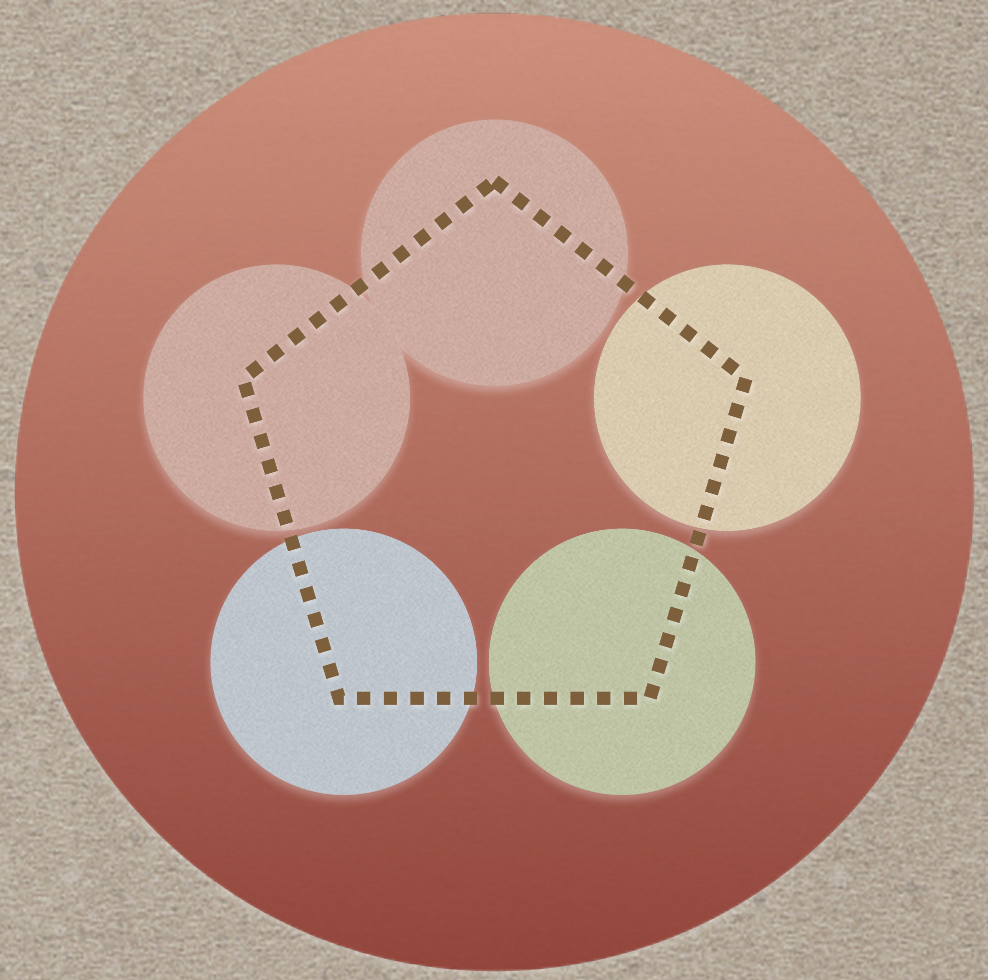
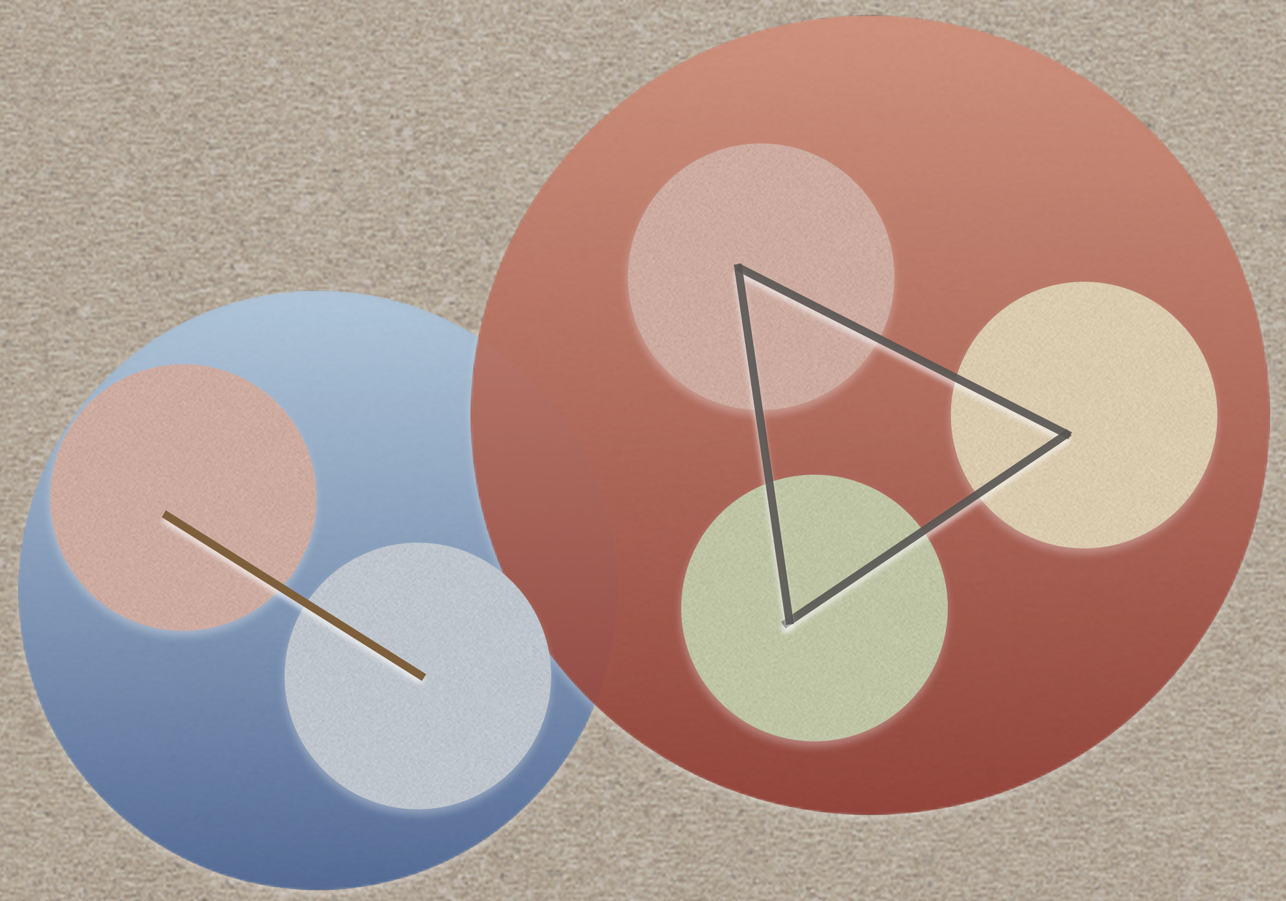
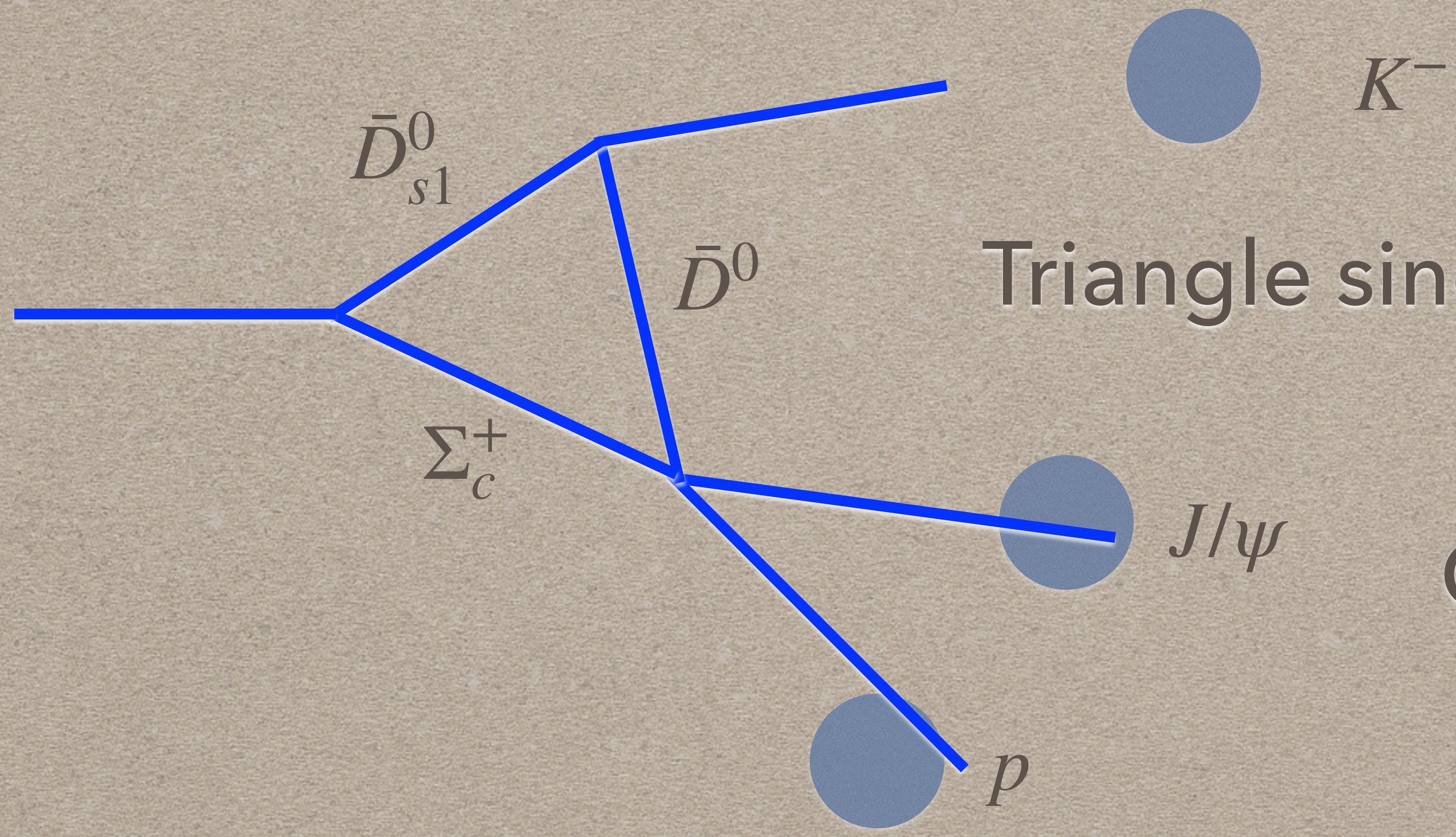




Molecule



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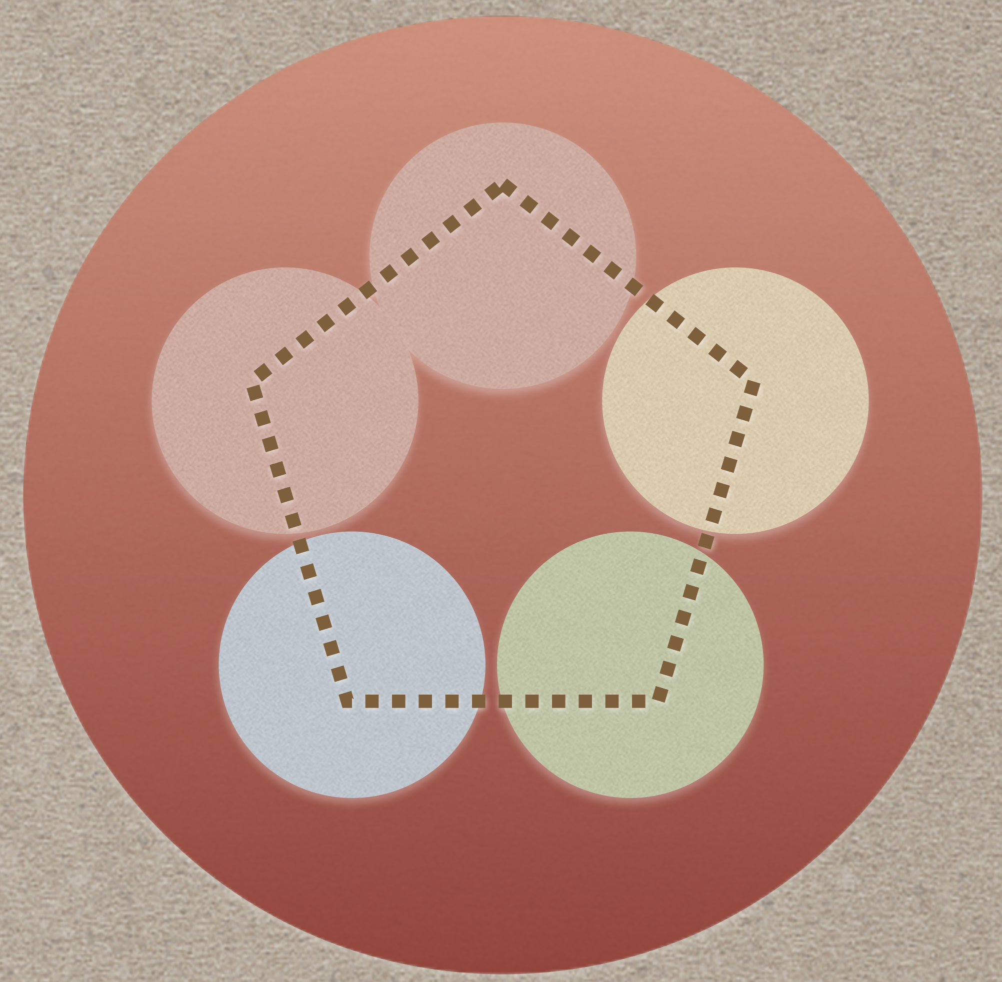

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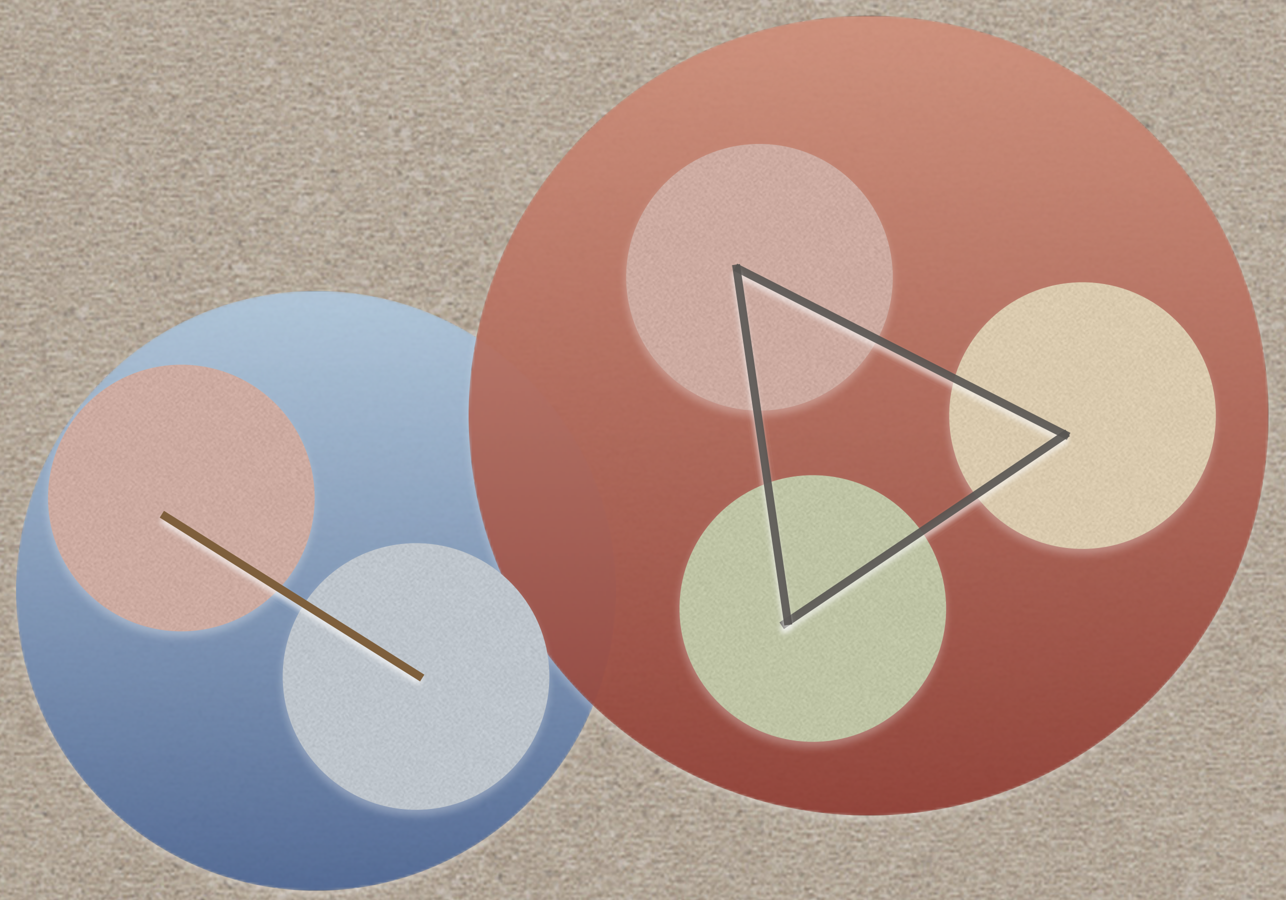

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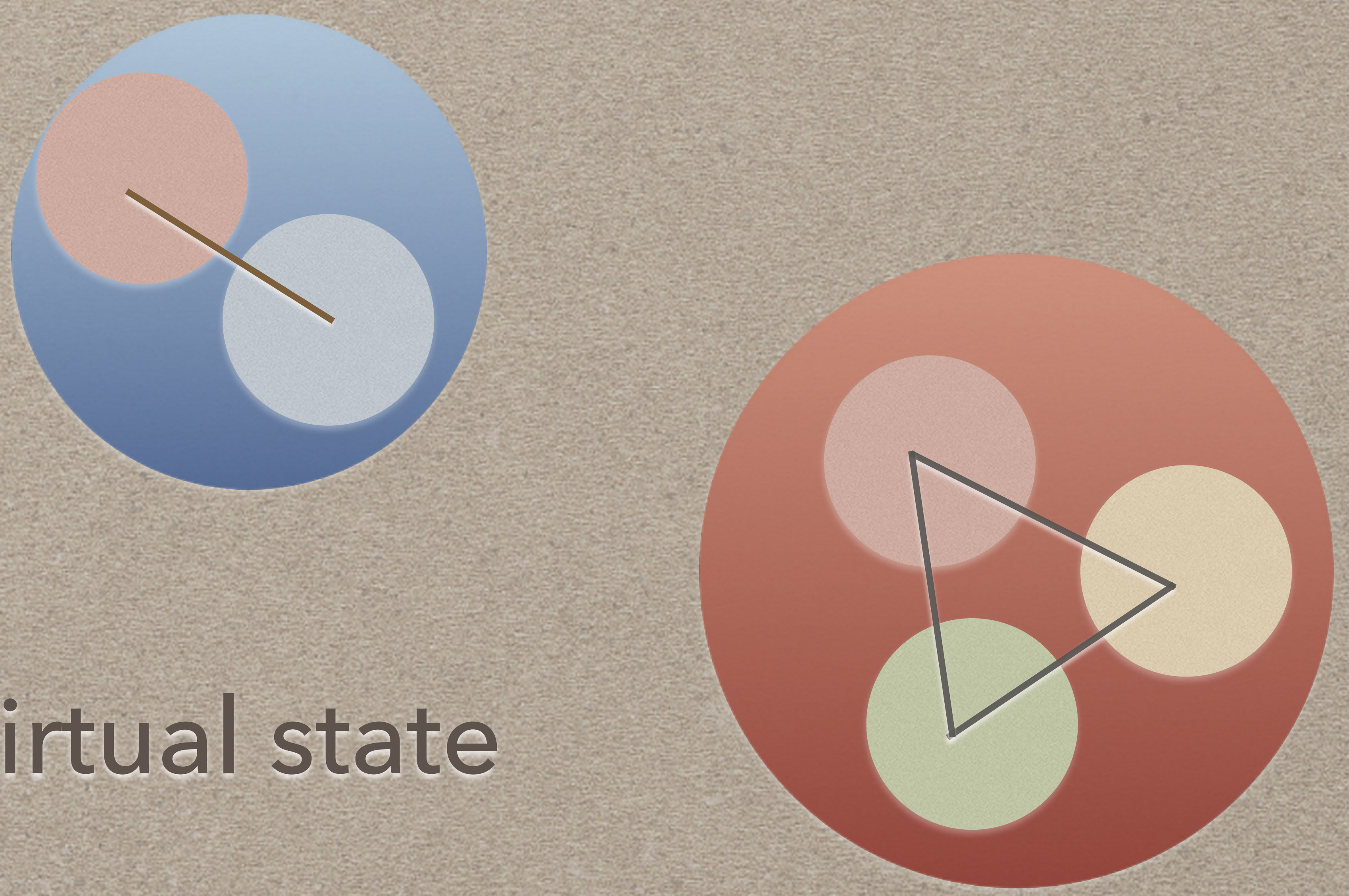

Virtual state

VIRTUAL STATE?

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The virtual state is actually a very well-established physics case

Consider Nucleon-Nucleon interaction:

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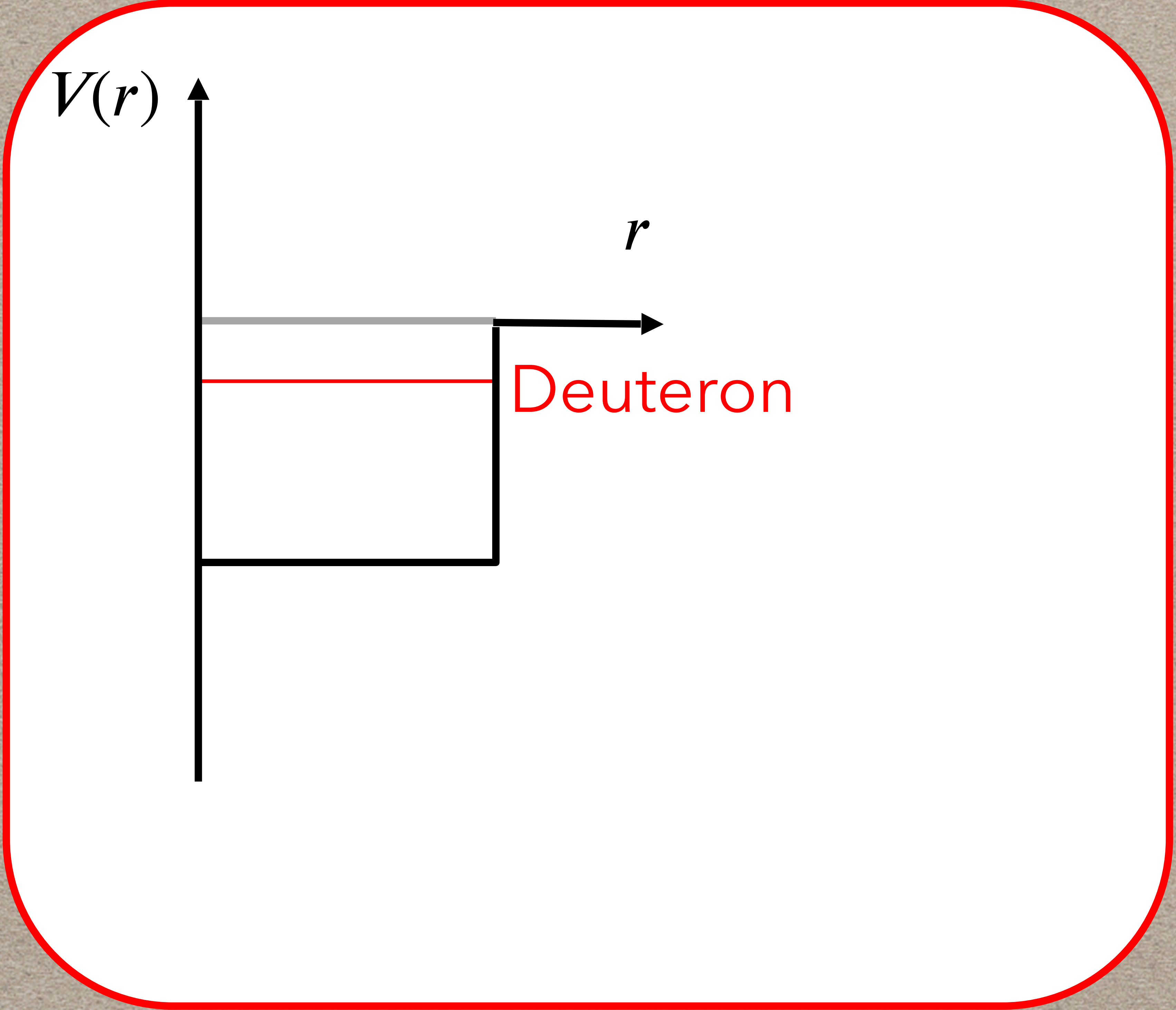
nn scattering never generates a bound state, but generates a signal that can be seen in the scattering lengths

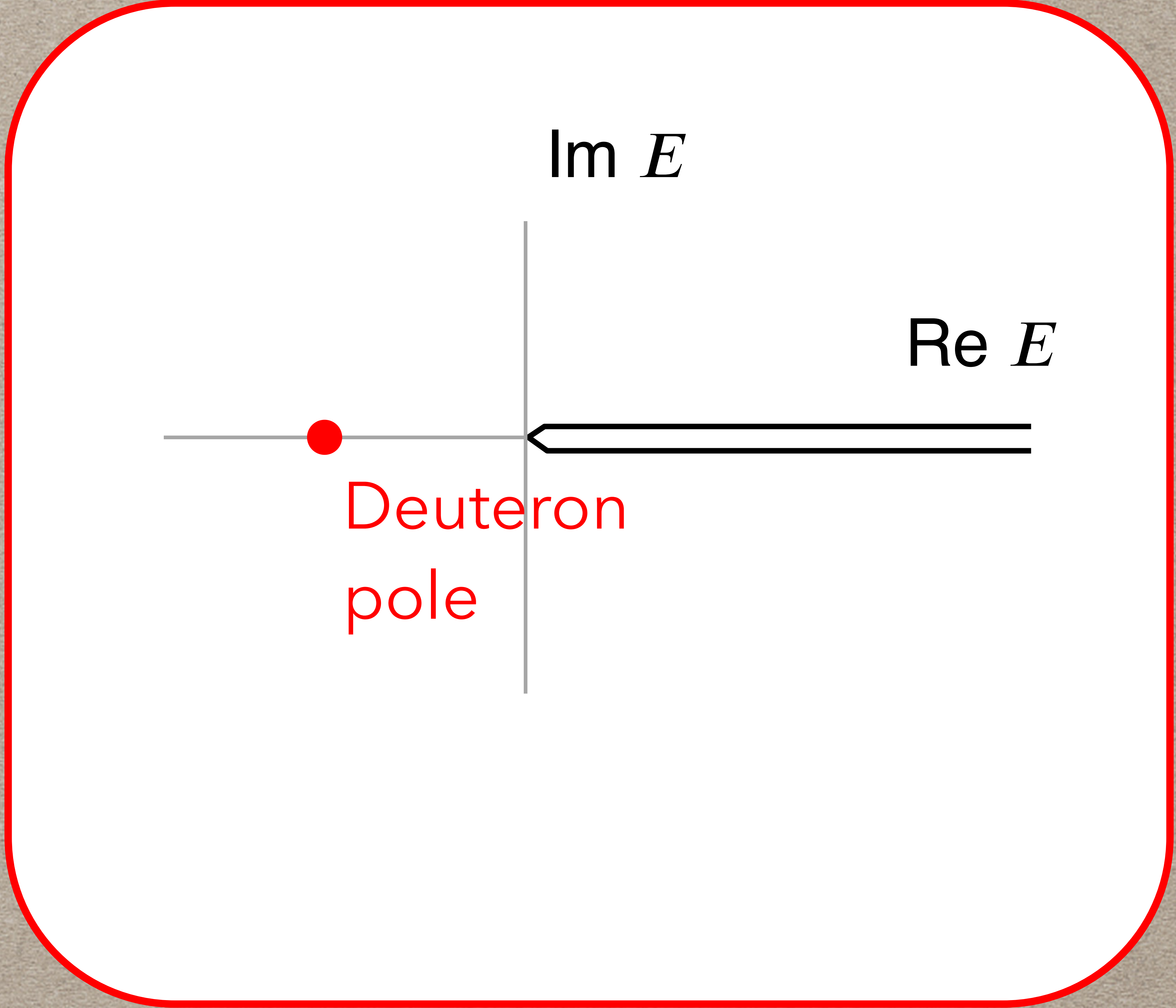
The interaction is strong enough to generate pole but not to bind the system

BOUND AND VIRTUAL STATES

Example from *pn* scattering

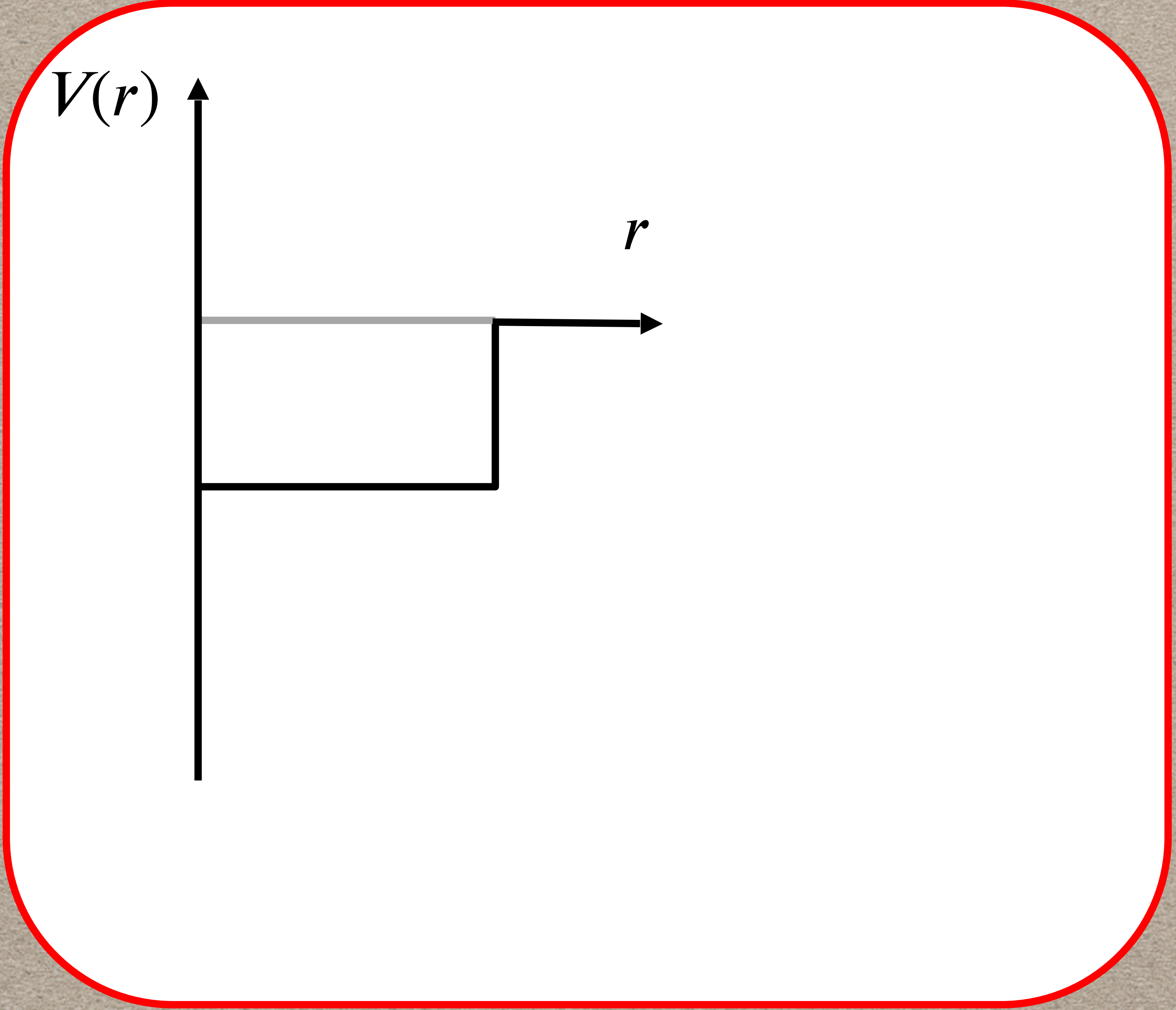
Bound state on the real axis I sheet (deuteron)

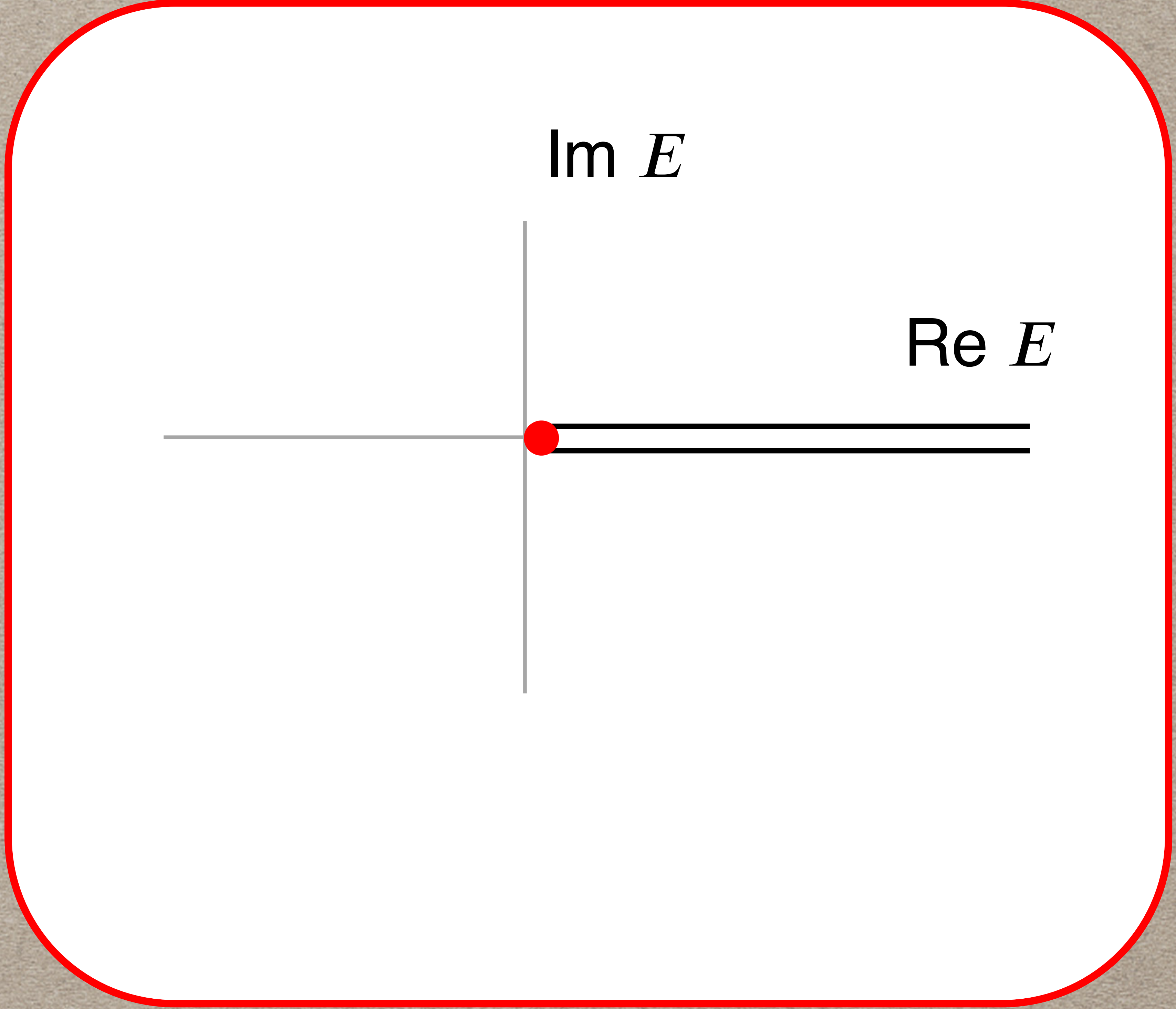




BOUND AND VIRTUAL STATES

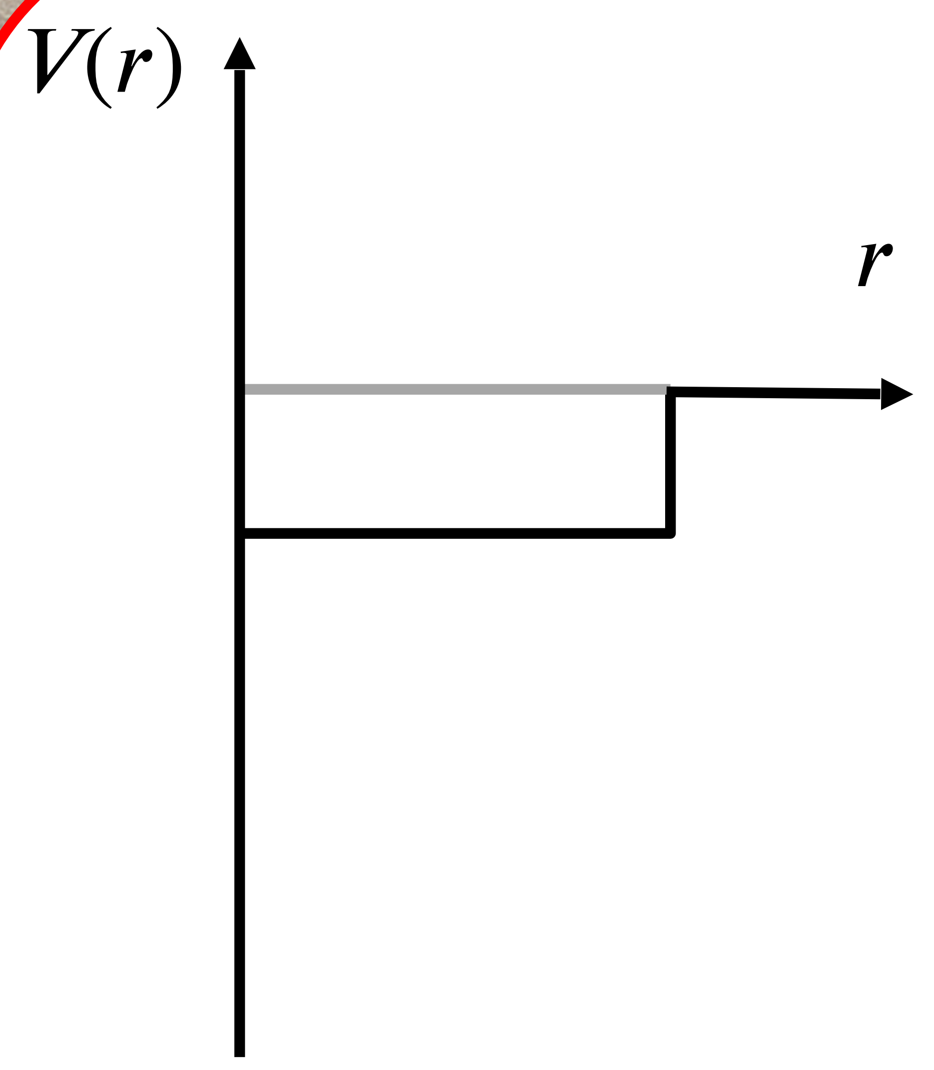
Decreasing the potential strength,
the pole reaches threshold

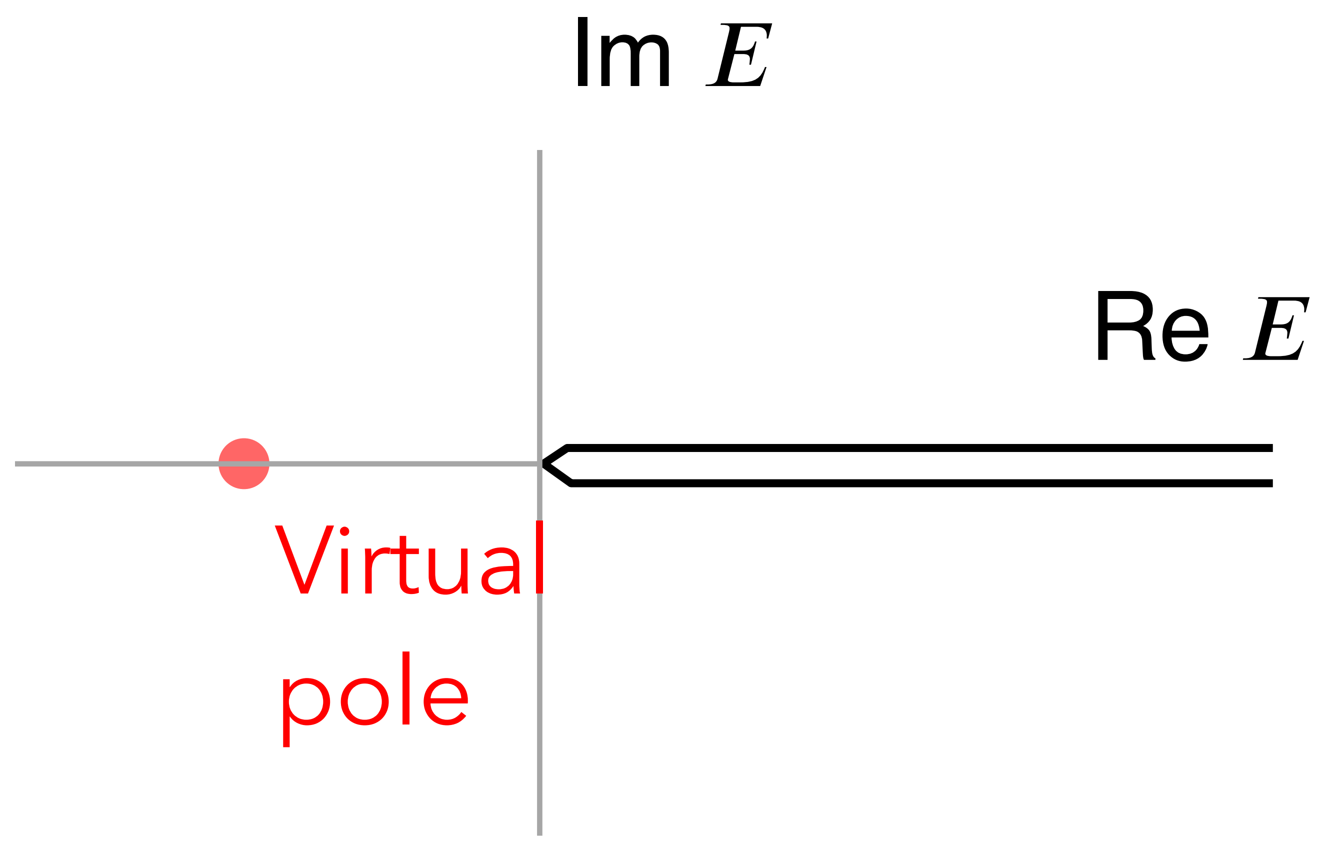




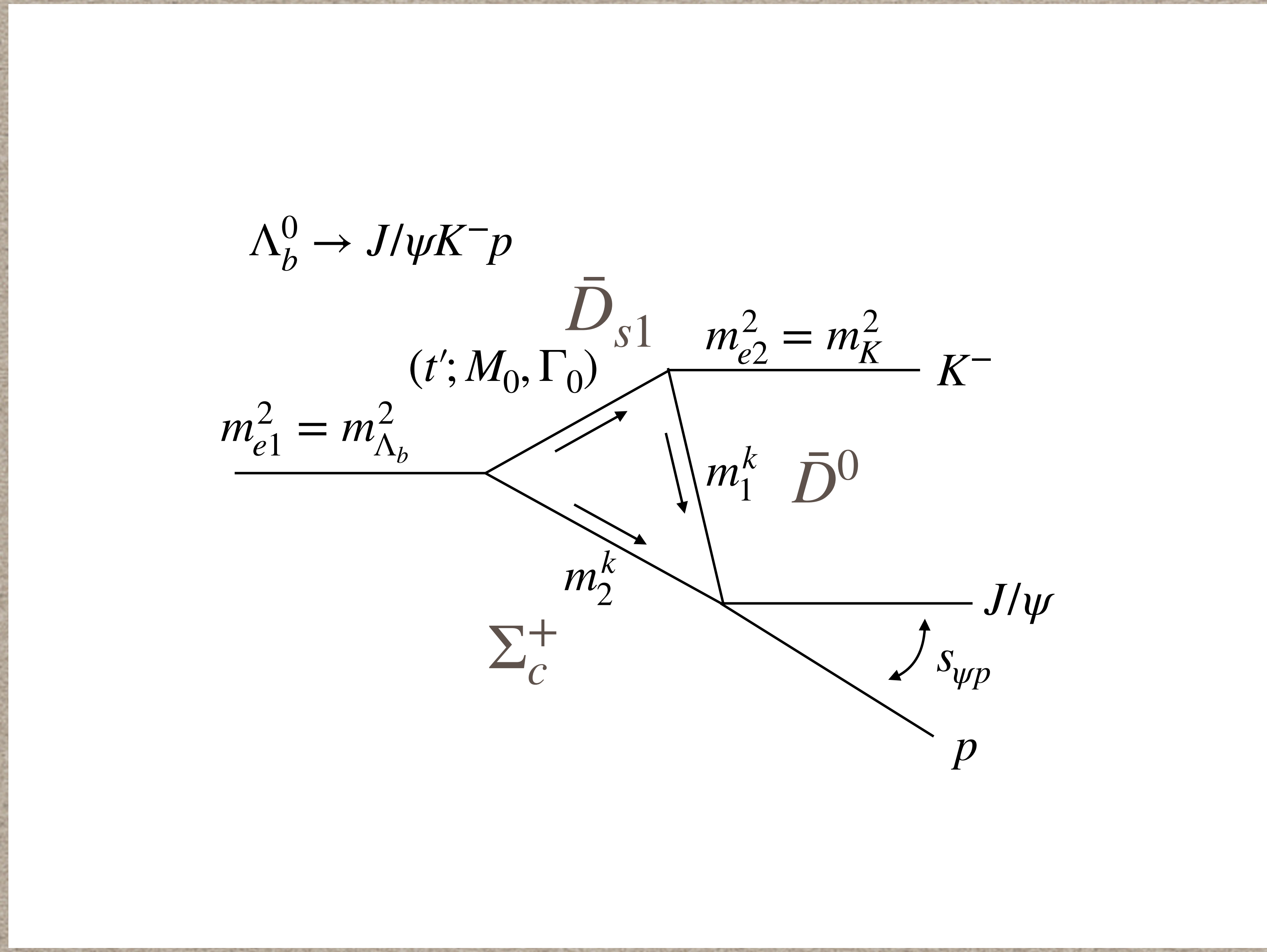
BOUND AND VIRTUAL STATES

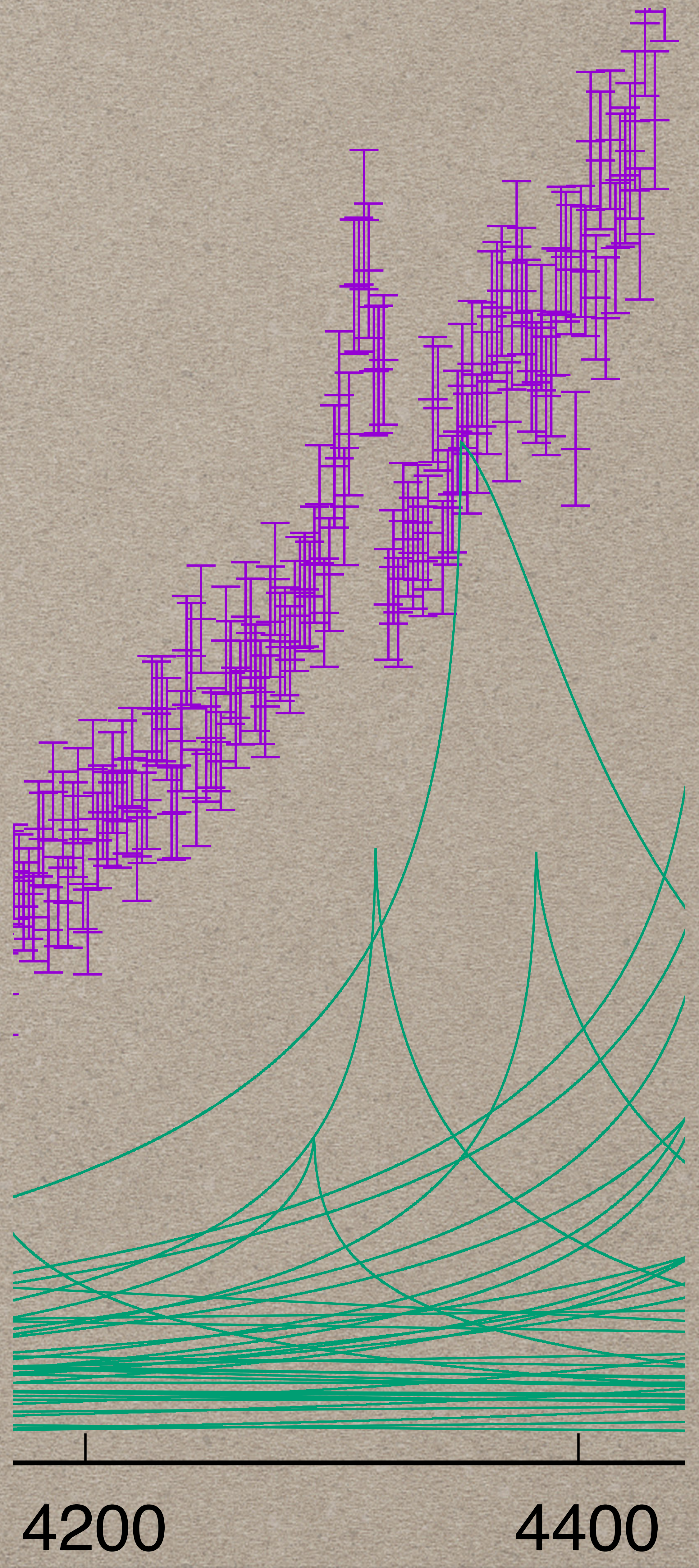
The pole jumps on the II sheet,
it becomes a virtual state





TRIANGLE SINGULARITY





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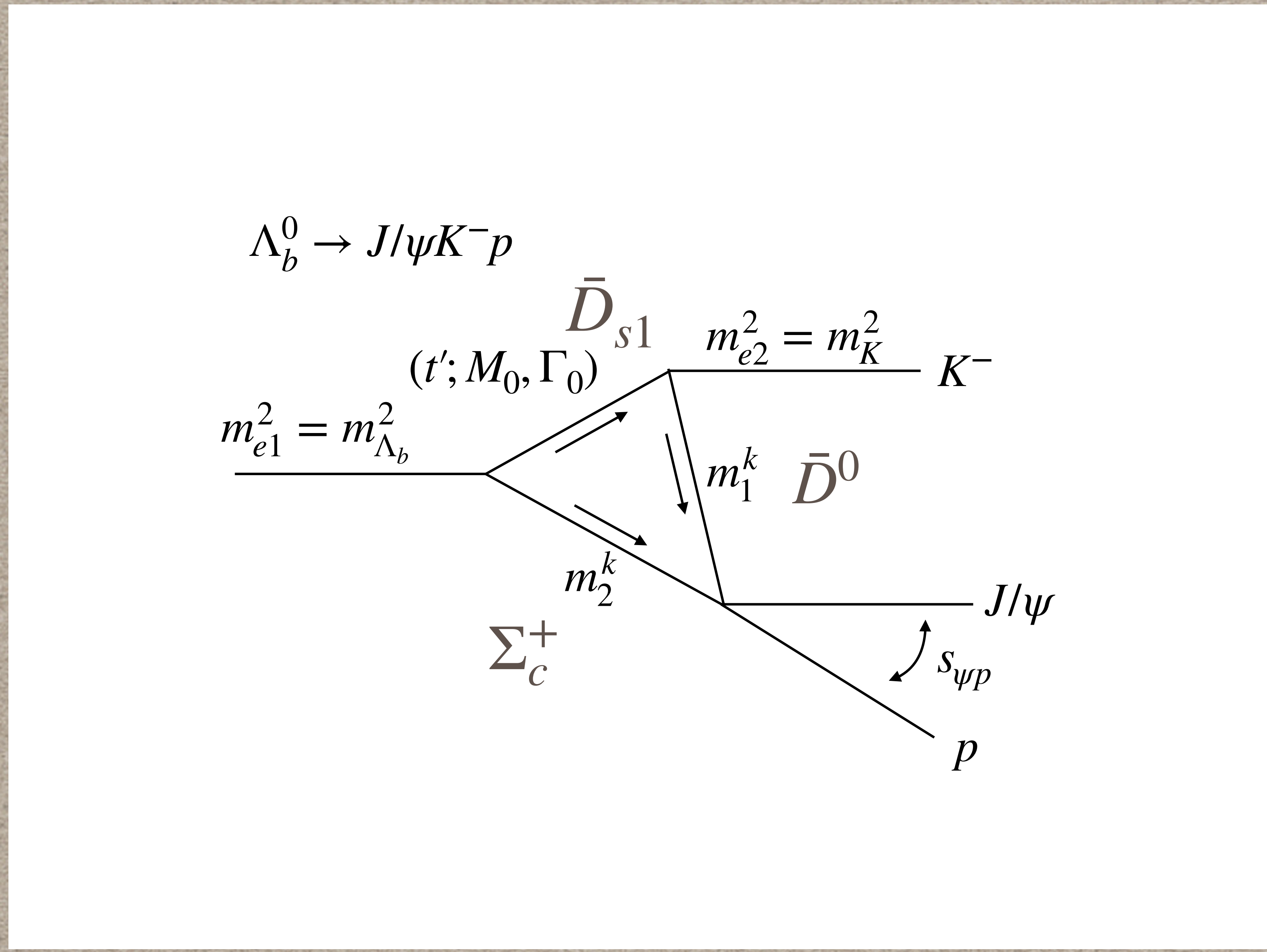

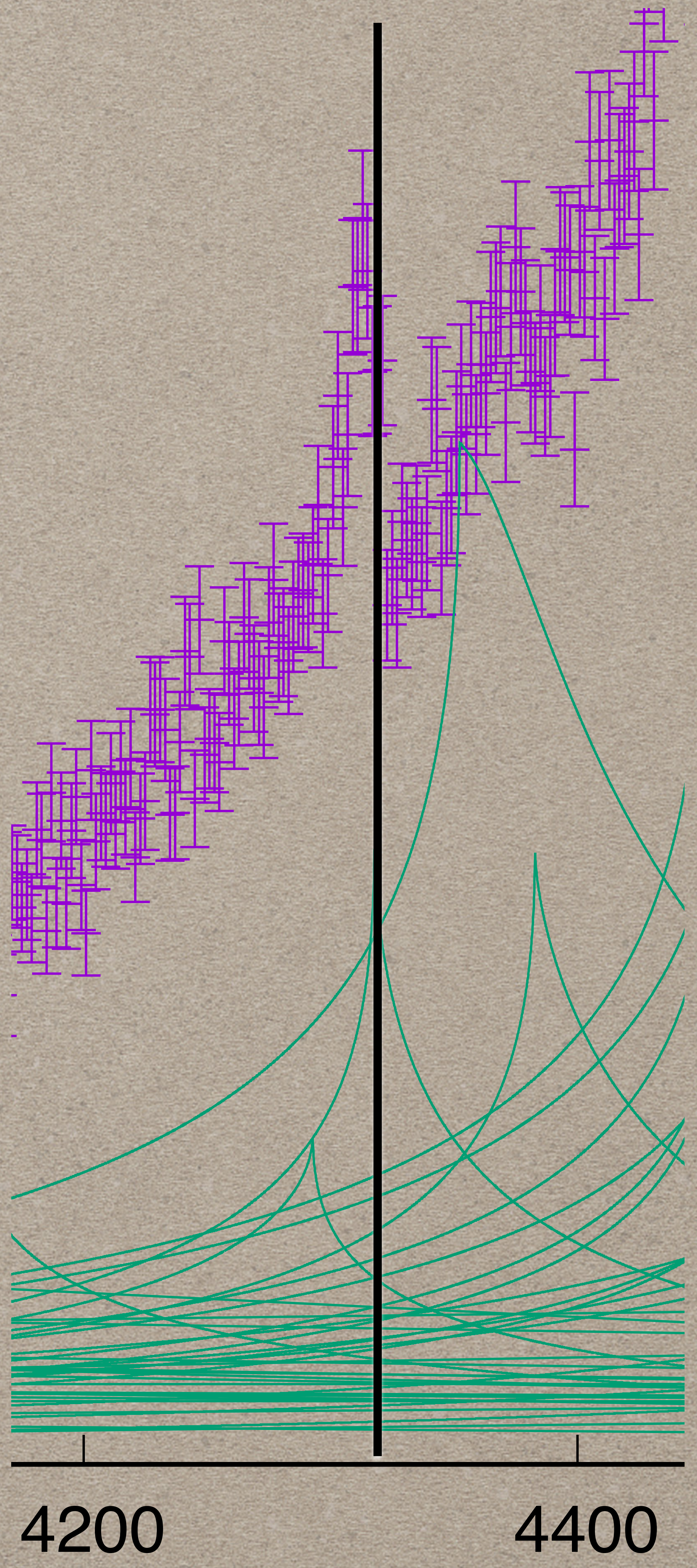

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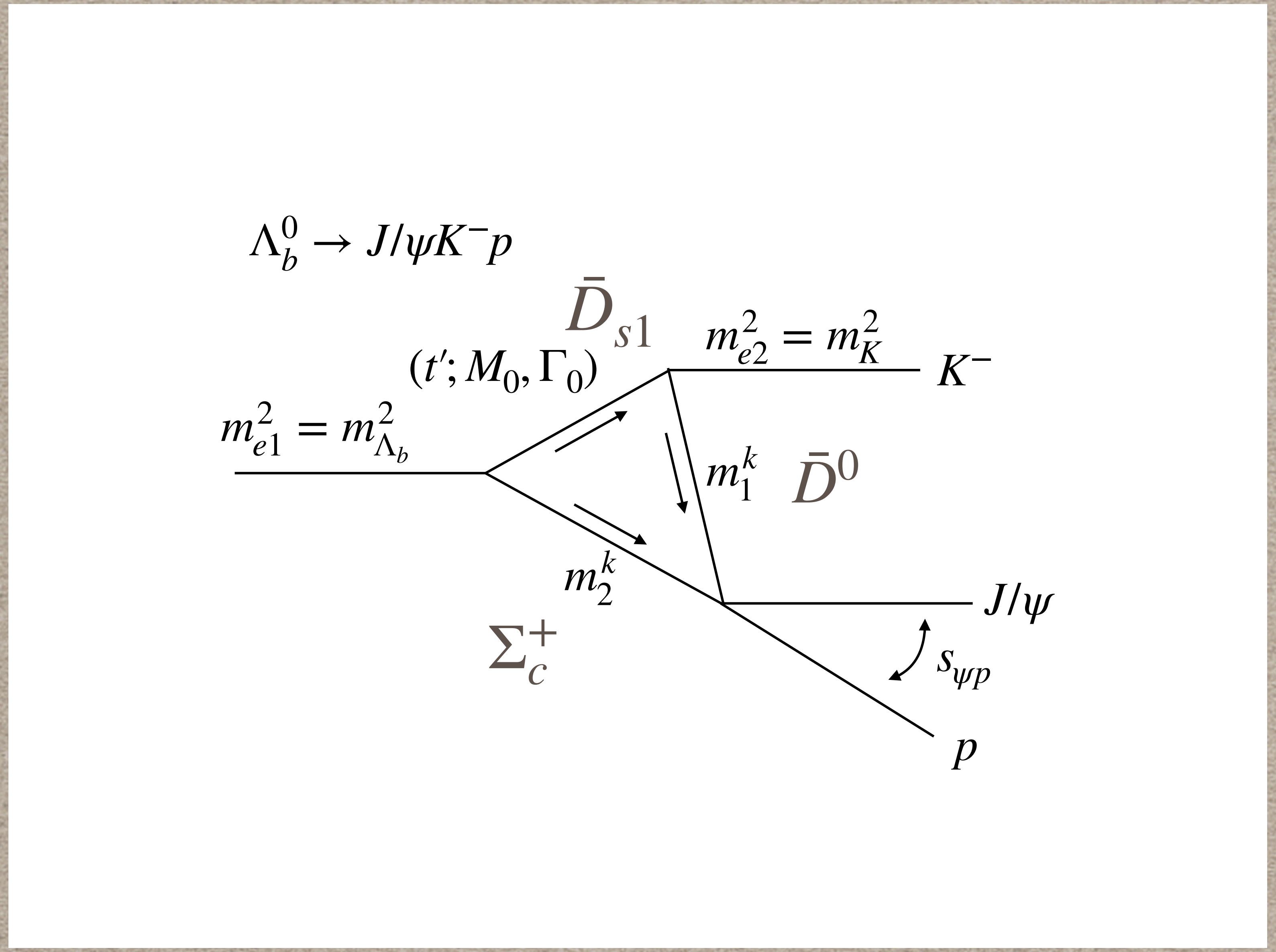
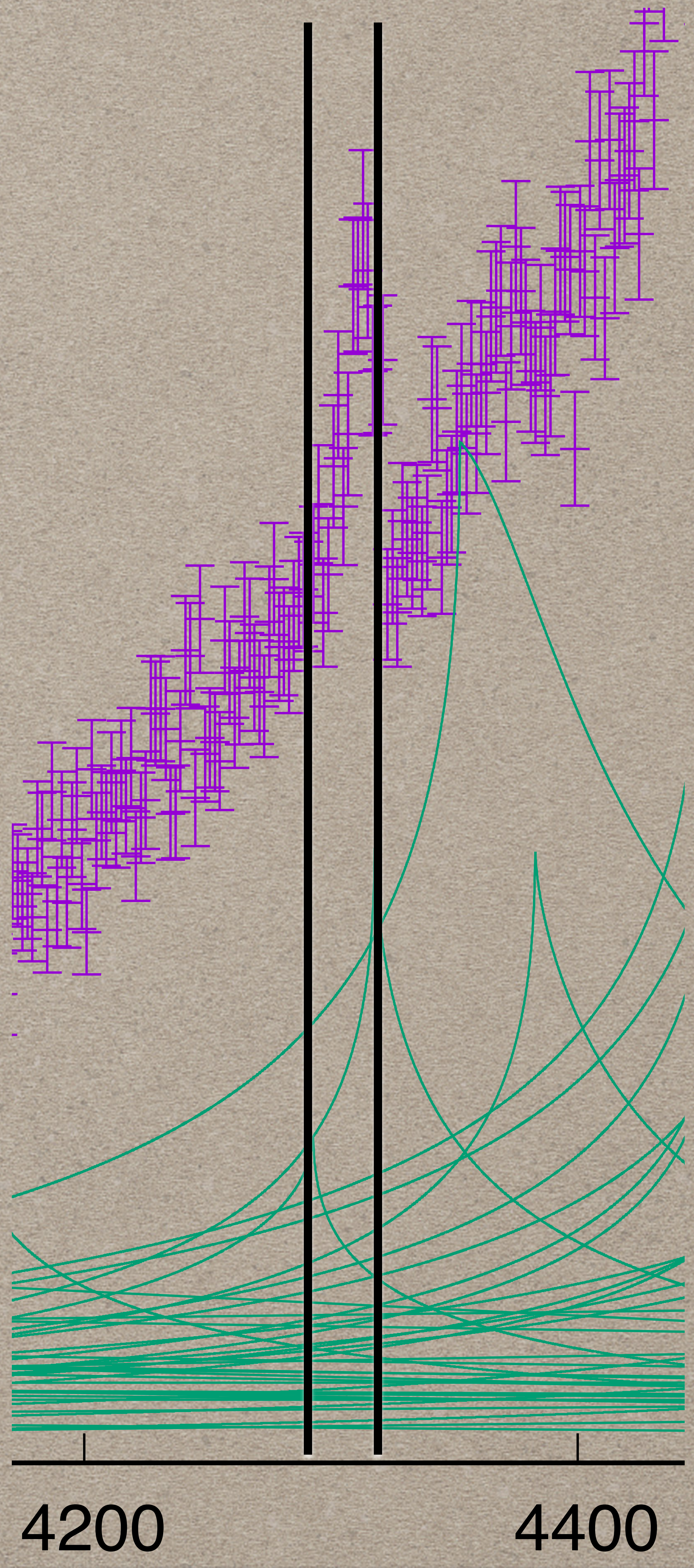


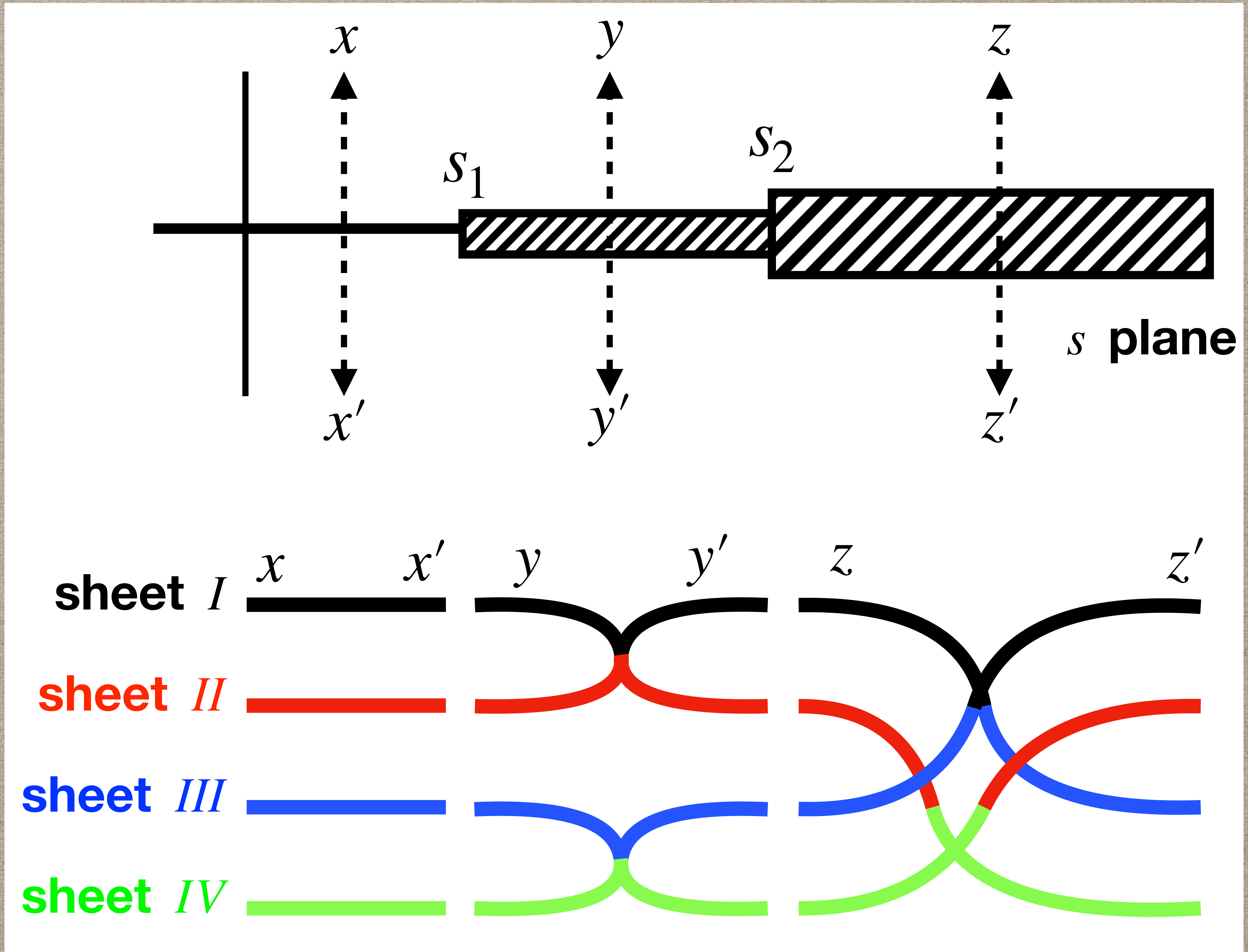


Triangle singularities do not generate poles but the phase motion is the same as for a pole (i.e. the Argand plot is going to be the same)

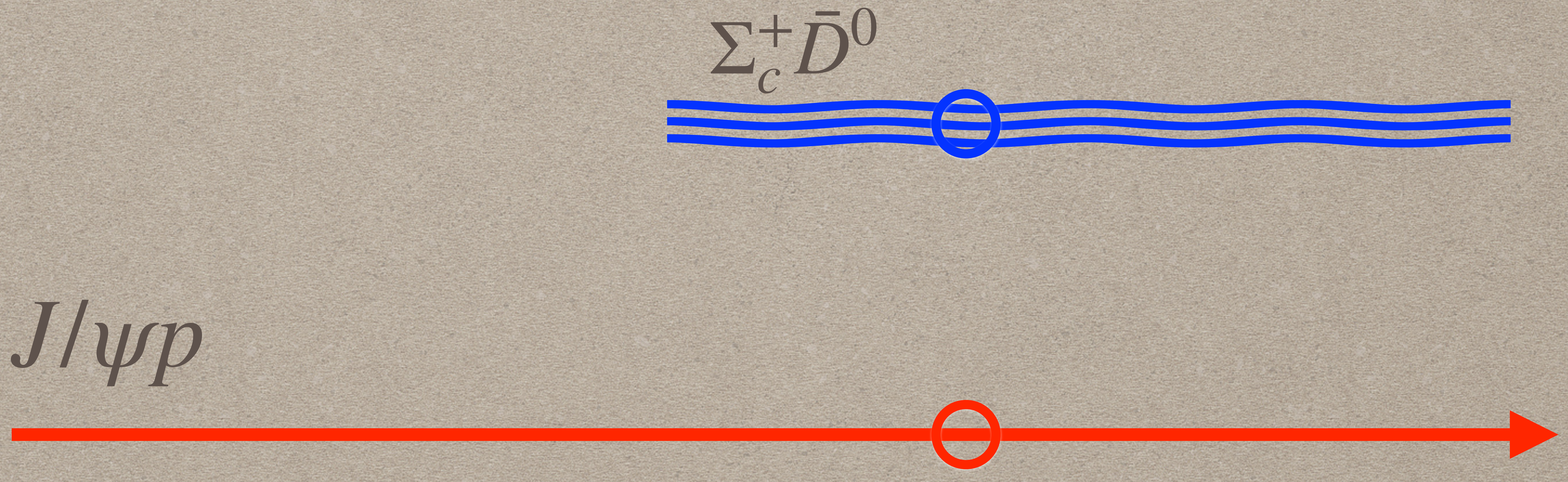
[Remember Bernhard's talk]

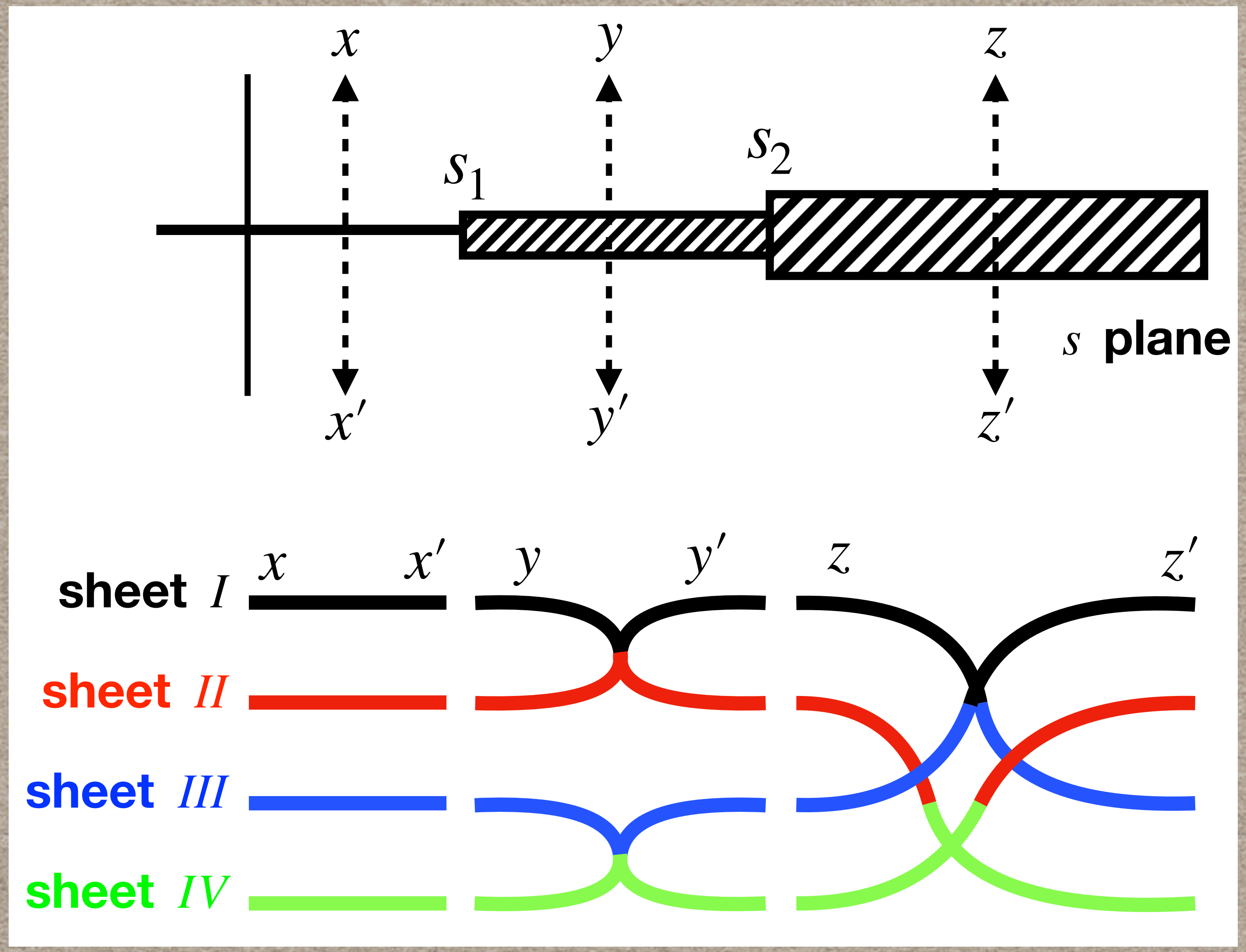
RIEMANN SHEETS STRUCTURE

First threshold is $J/\psi\ p$ channel and the second is $\Sigma_c^+ \bar{D}^0$



COMPACT PENTAQUARK





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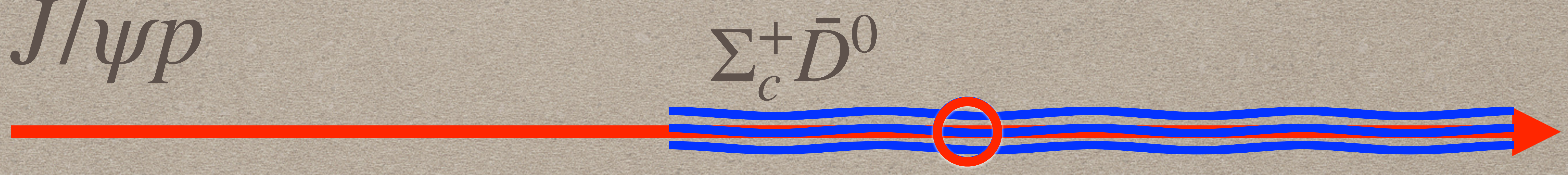

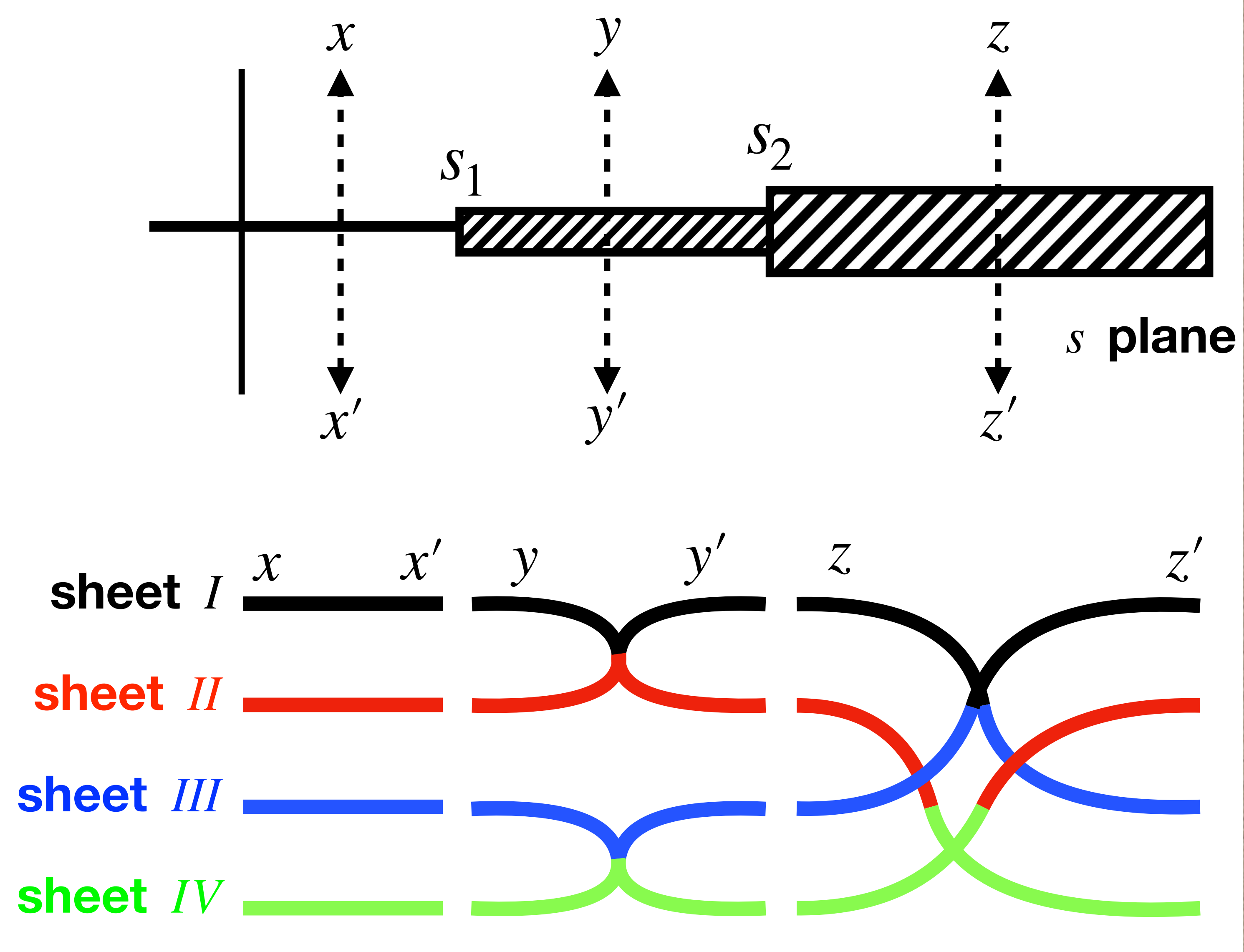

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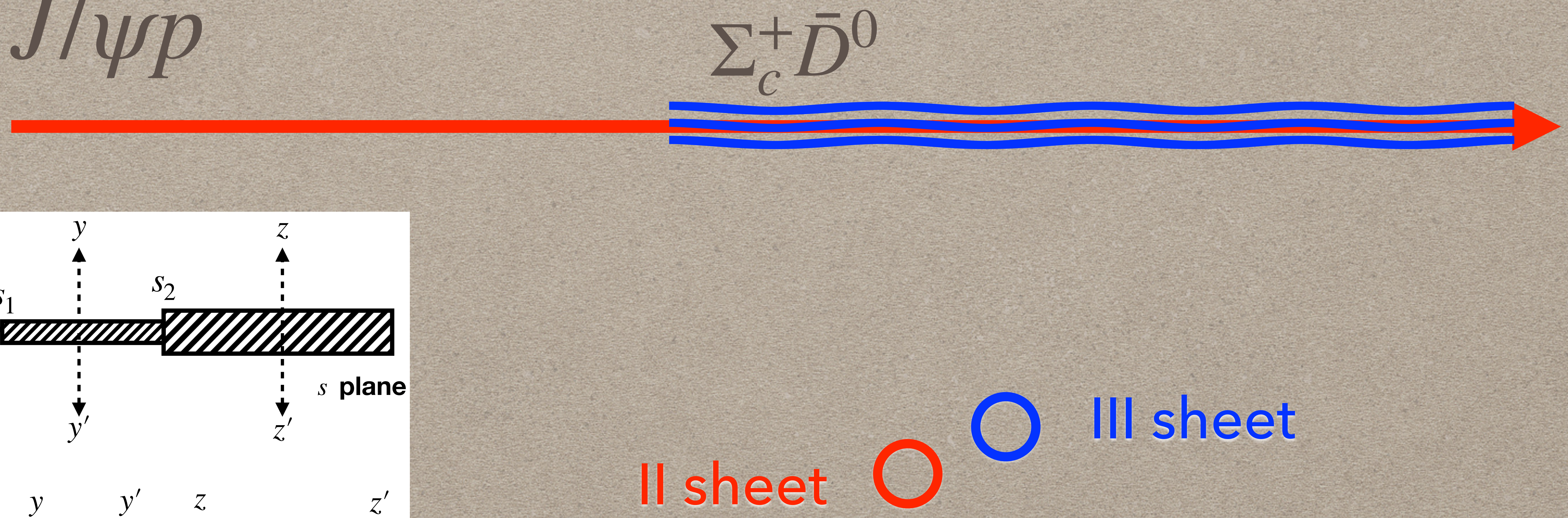
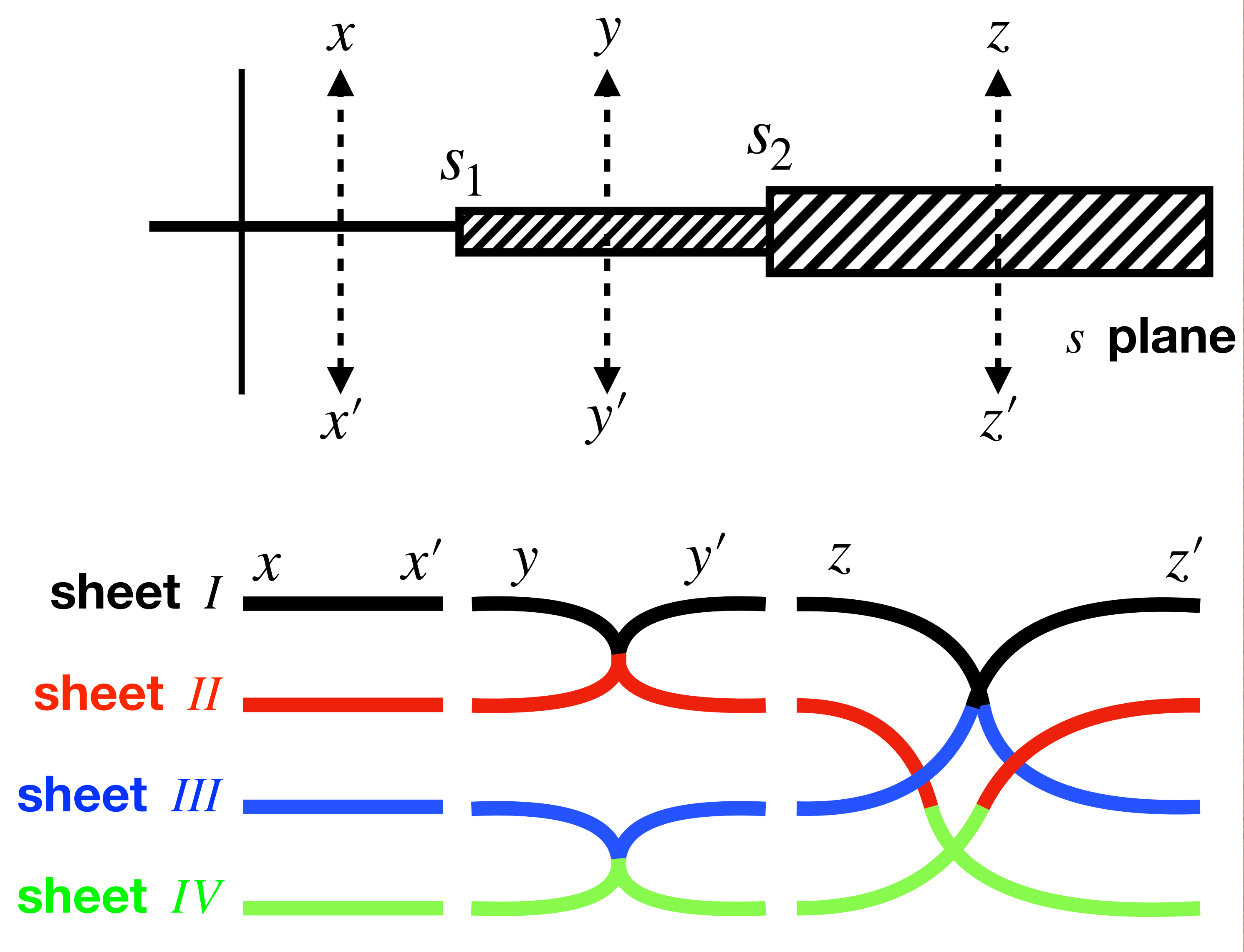
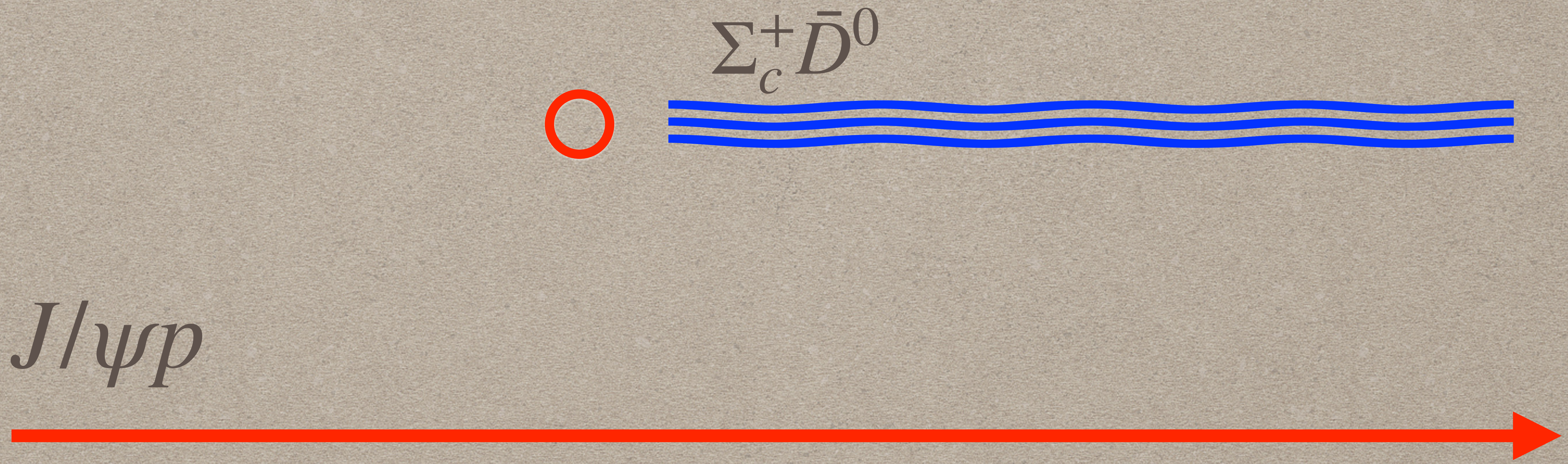
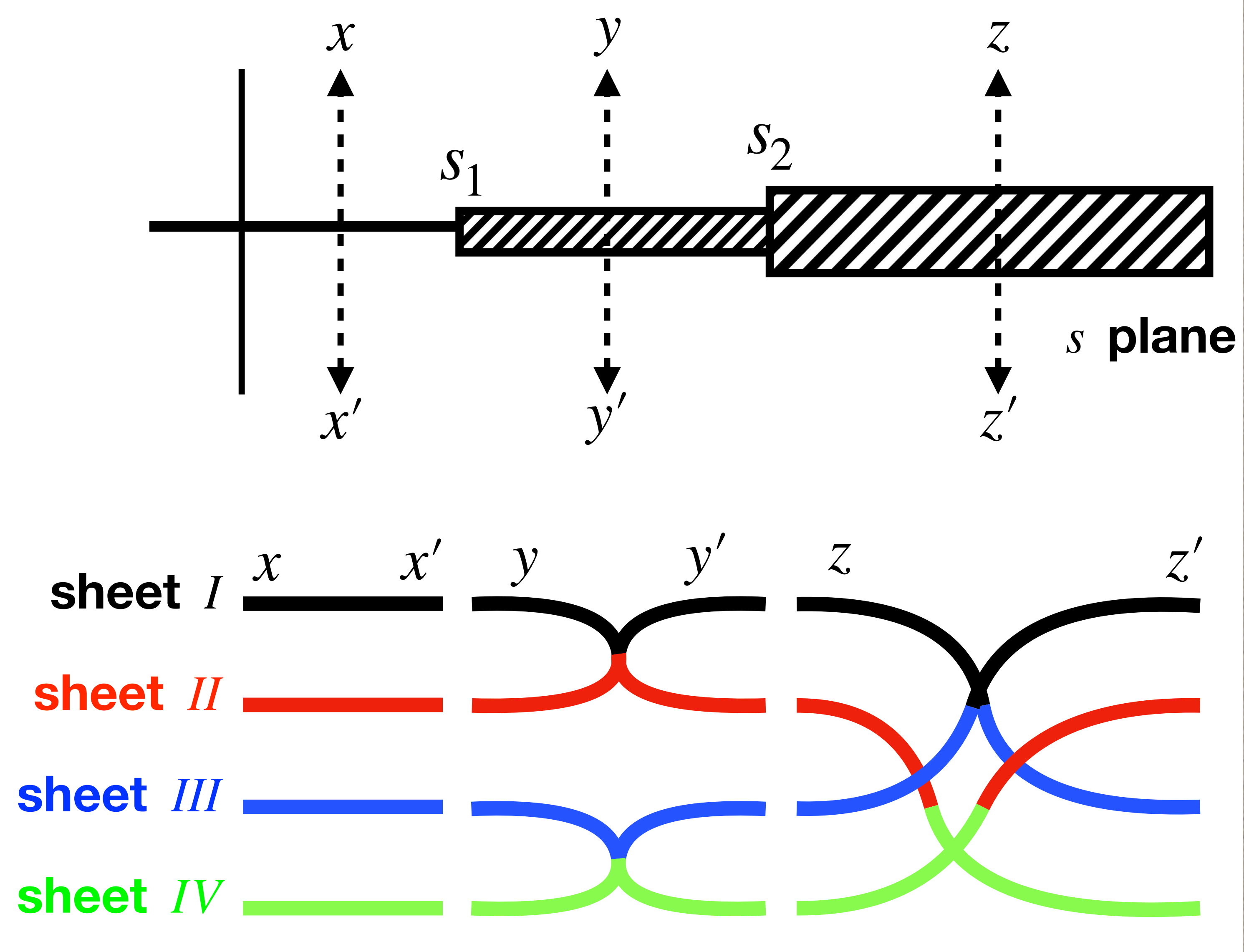




MOLECULE

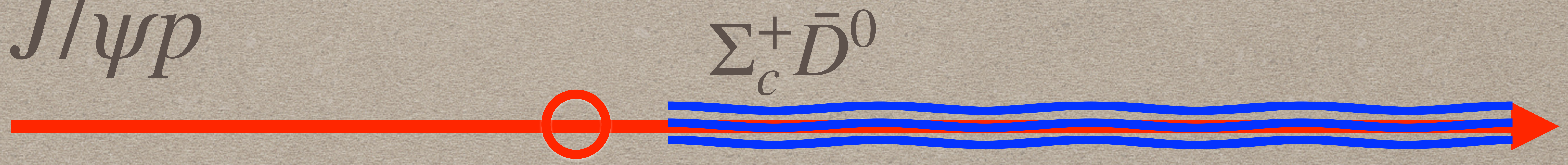


$\Sigma_c^+ \bar{D}^0$
$J/\psi p$



x
y
z
s_1
s_2
s plane
x'
y'
z'
sheet I
sheet II
sheet III
sheet IV

MOLECULE



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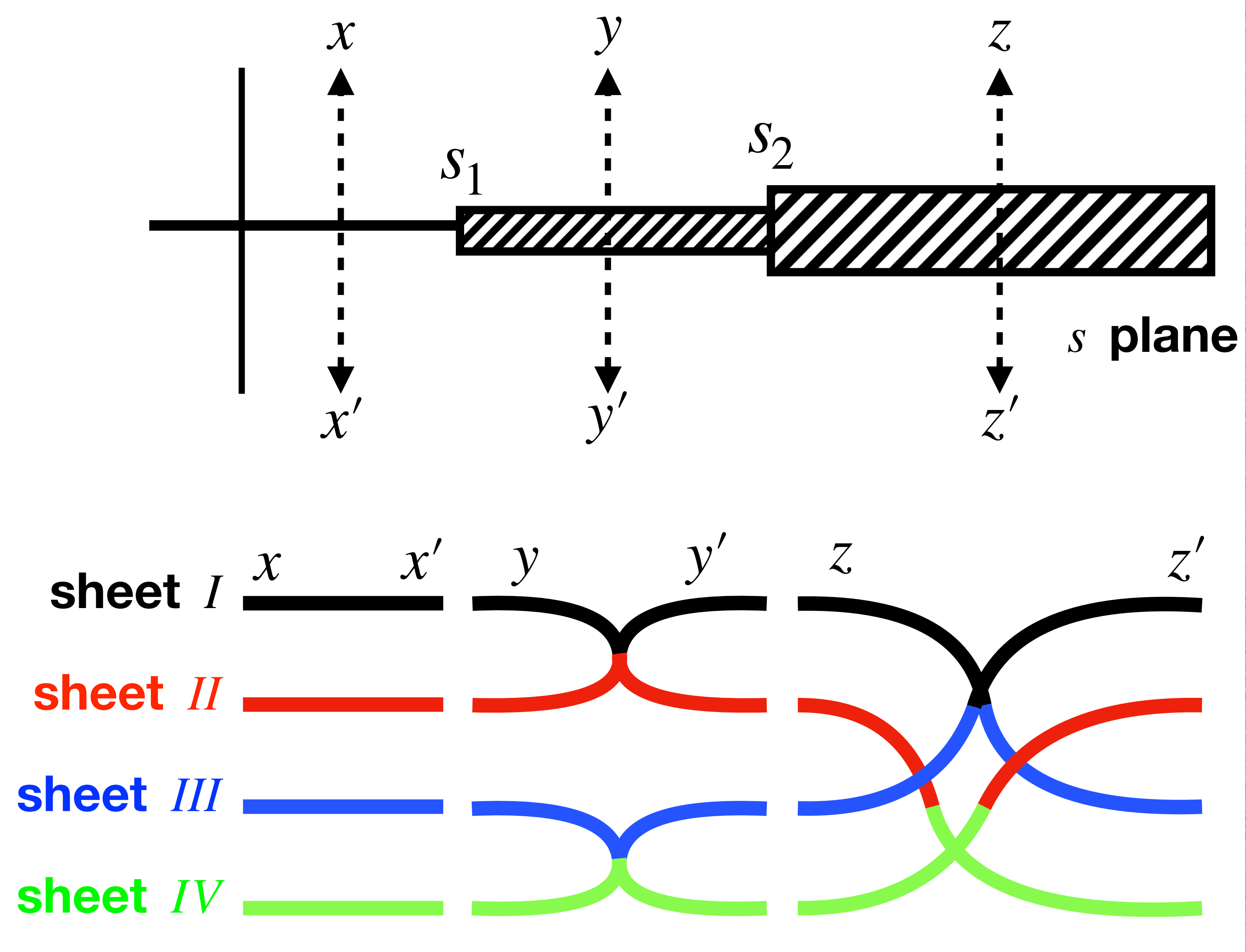

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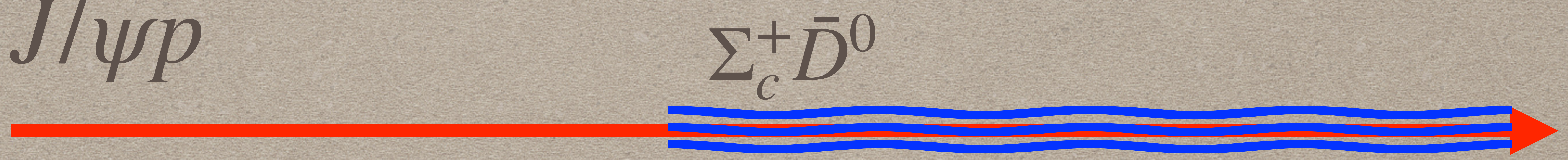
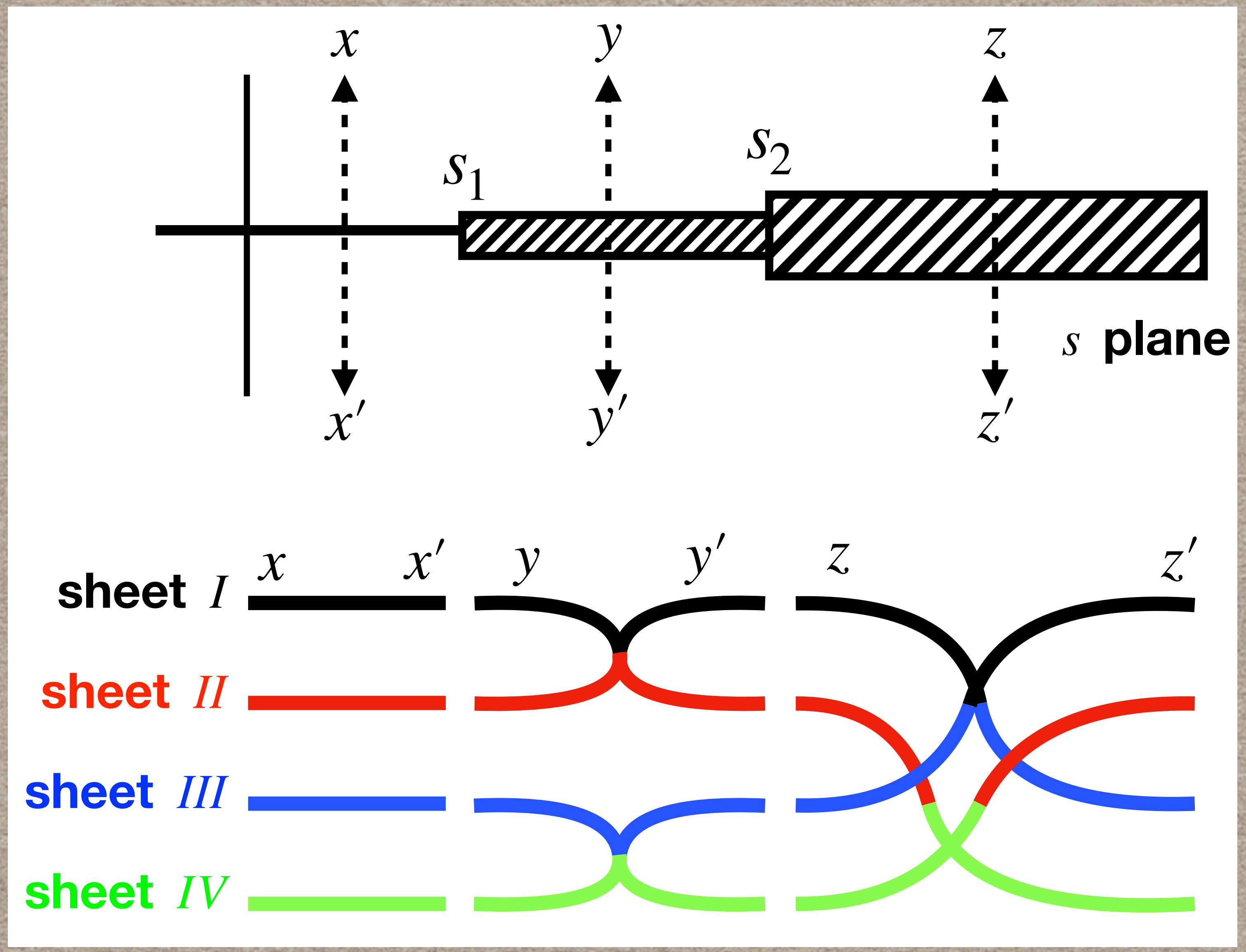
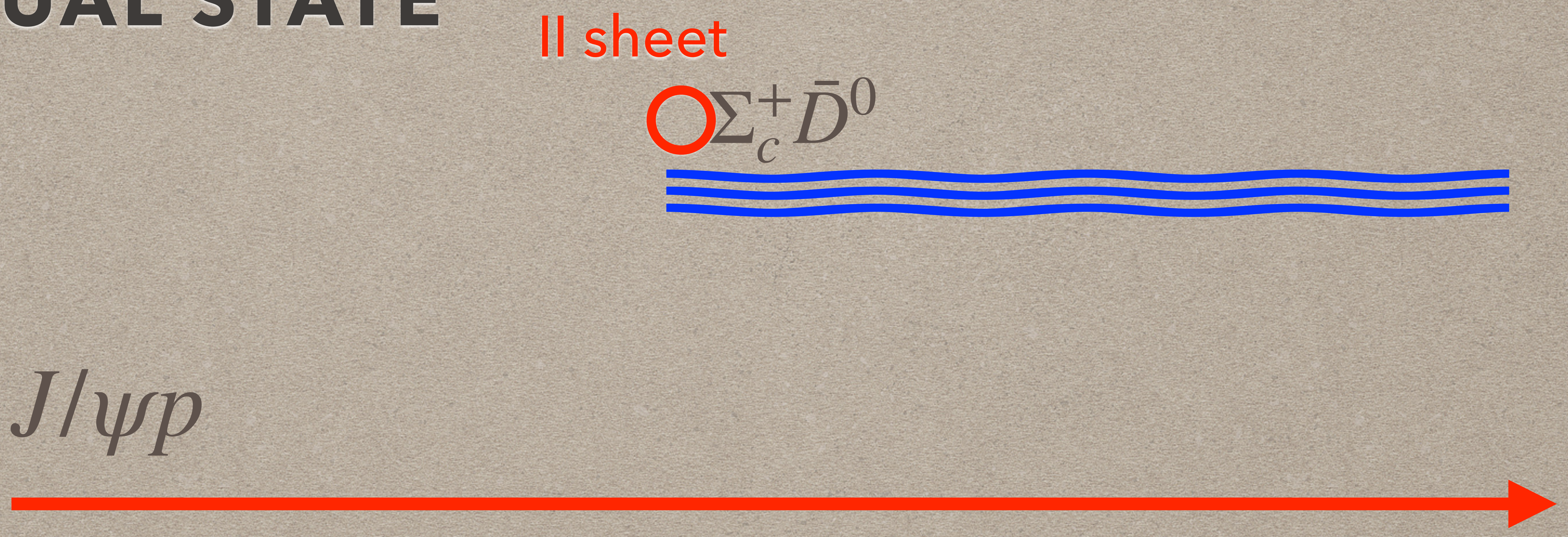


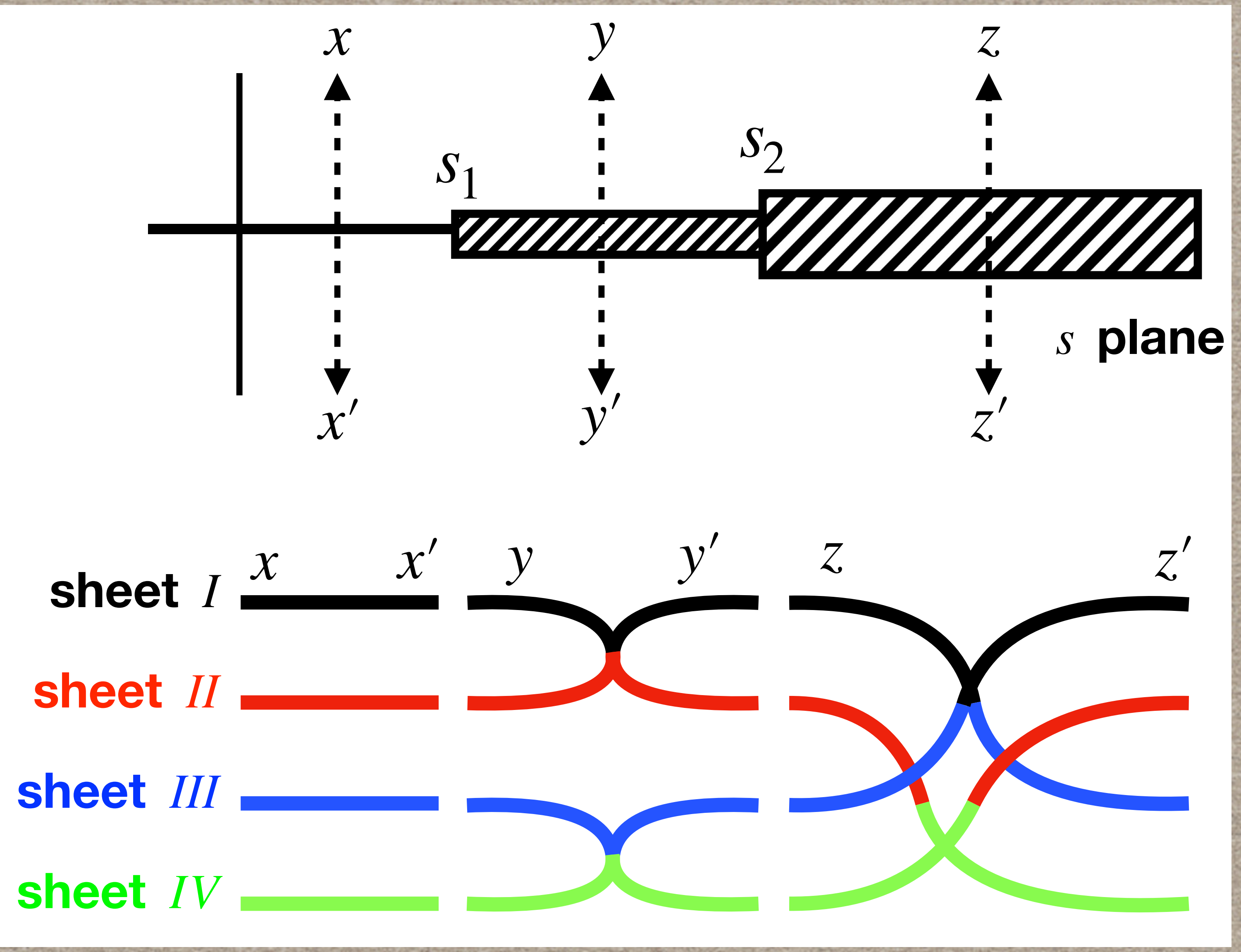
II sheet

Either nothing on the III sheet or shadow pole



VIRTUAL STATE





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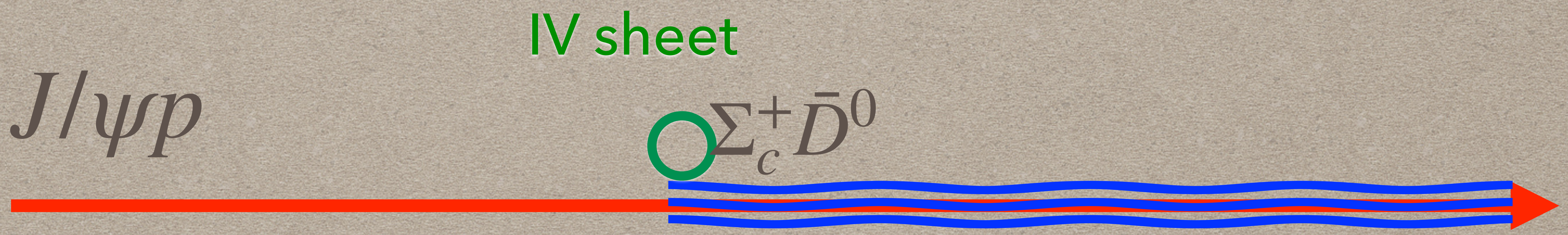

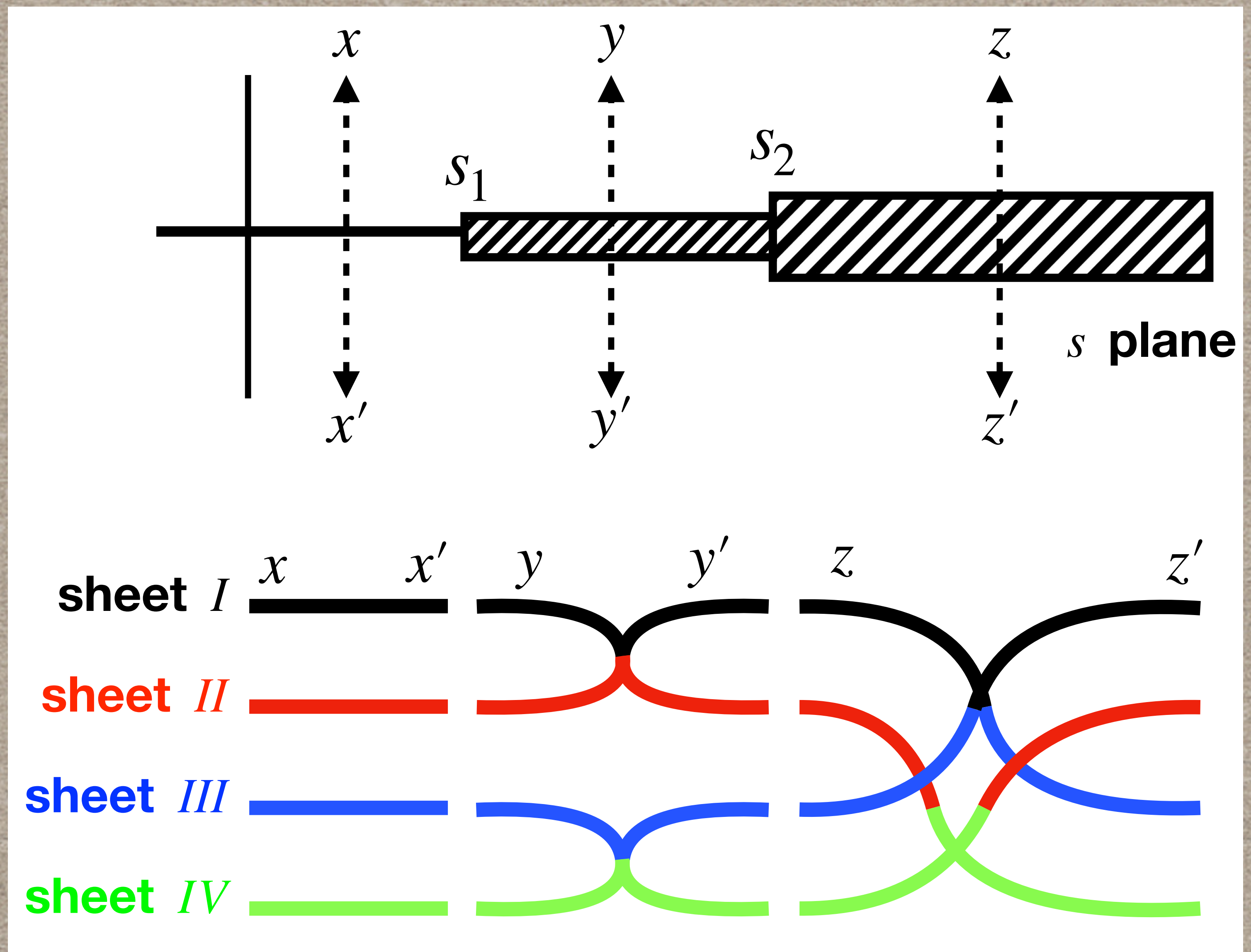

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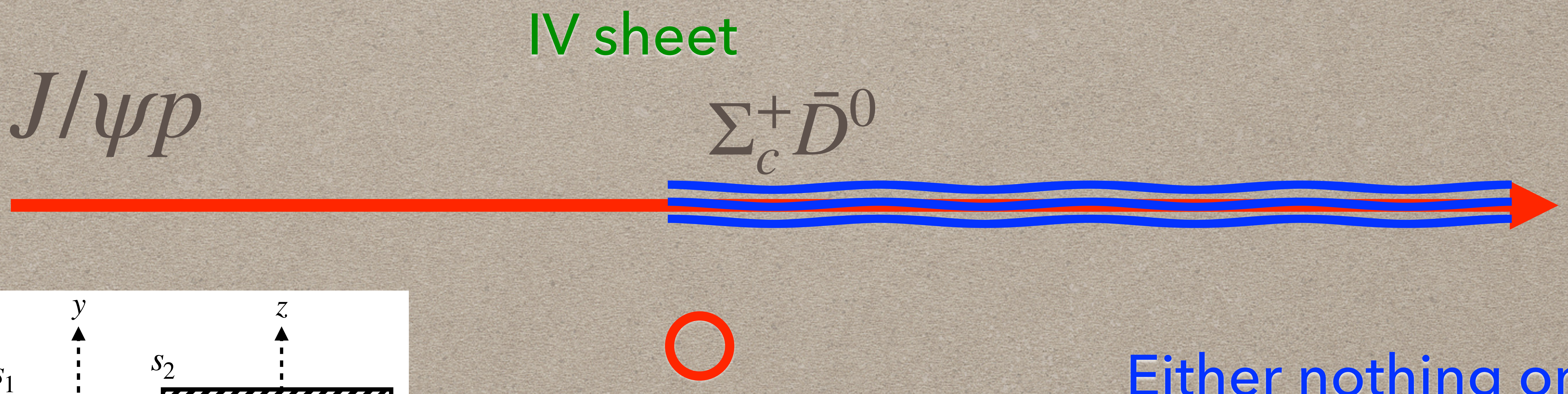

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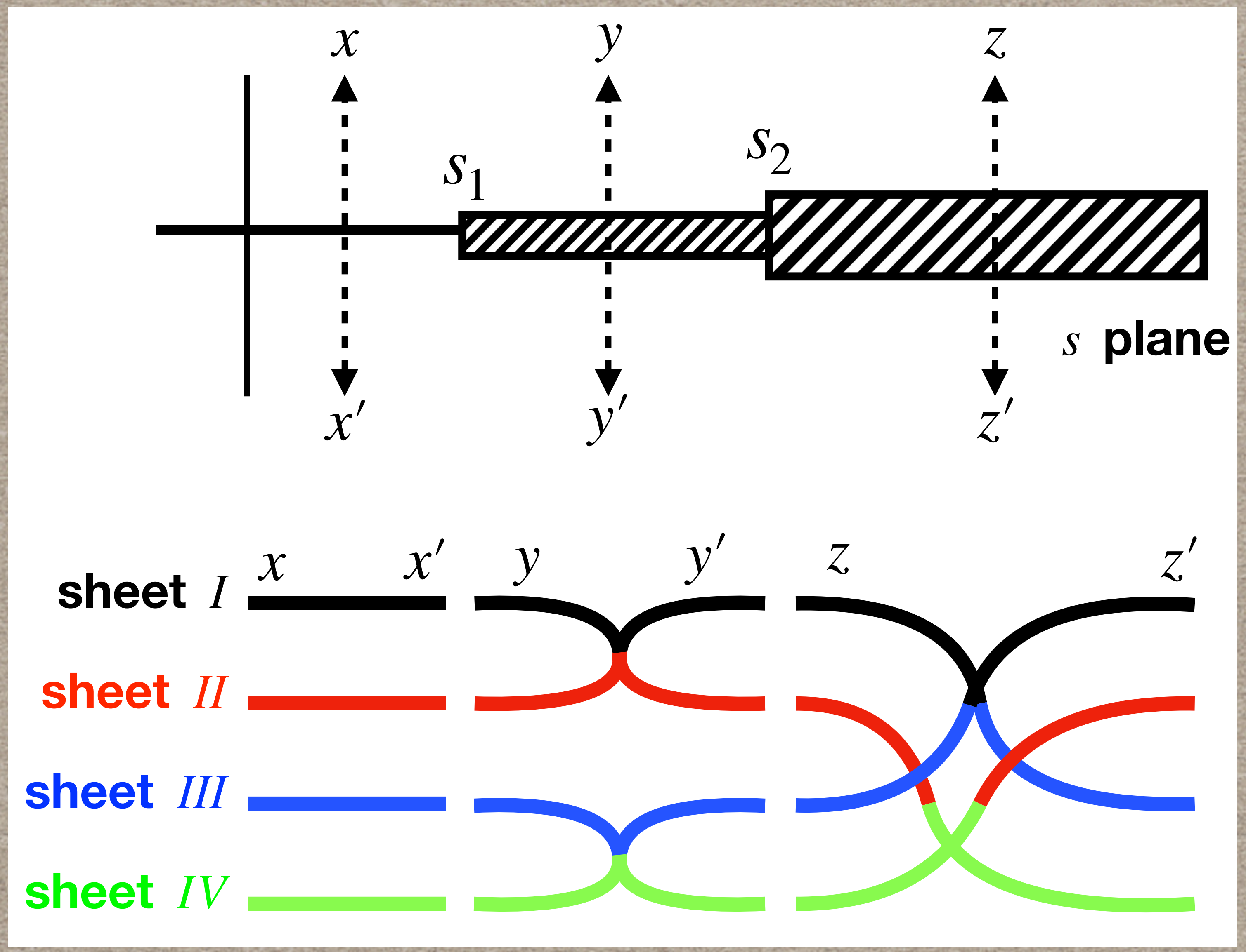

ANALYSIS OF THE $P_c(4312)$ SIGNAL

- Build a theory in the near threshold region
- Analyze the three datasets provided by LHCb $P_c(4312)$
- 66 experimental data
- Experimental resolution incorporated
- Error analysis through bootstrap

JPAC, PRL 123 (2019) 092001

NEAR-THRESHOLD THEORY

Hypotheses:

- Only one partial wave contributes to the signal
- The threshold drives the physics (testable)
- Other effects are absorbed in the parameters (testable)

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Caveat:

- We fit the $J/\psi\, p$ projection (no info on quantum numbers)

NEAR-THRESHOLD THEORY

$$\frac{dN}{d\sqrt{s}} = \rho(s)\left[\,|\,F(s)\,|^2 + B(s)\right]$$

$$\left(T^{-1}\right)_{ij} = M_{ij} - ik_i\delta_{ij}$$

$$M_{ij}(s) = m_{ij} - c_{ij}s$$

The matrix elements M_{ij} are singularity free near threshold and can be expanded in a Taylor series

$$F(s) = P_1(s)T_{11}(s) \qquad B(s) = b_0 + b_1 s \qquad P_1(s) = p_0 + p_1 s$$

Frazer & Hendry, PR 134 (1964) B1307

AMPLITUDE IN THE NEAR THRESHOLD REGION

$$\frac{dN}{d\sqrt{s}} = \rho(s)\Big[\,|F(s)|^2 + B(s)\Big]$$

$$B(s) = b_0 + b_1 s$$

$$F(s) = (p_0 + p_1 s)\frac{[m_{22} - c_{22}s - ik_2]}{[m_{22} - c_{22}s - ik_2][m_{11} - c_{11}s - ik_1] - m_{12}^2}$$

AMPLITUDE IN THE NEAR THRESHOLD REGION

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Production, hyperons and effects due to other (far) singularities

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Channel coupling

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Channel coupling

Effective range approximation if $c_{ij}=0$

Under the effective range approximation only poles in the II and IV sheet can happen

When $c_{ij}\neq 0$, poles can appear in any sheet (no threshold domination hypothesis)

FIT RESULTS

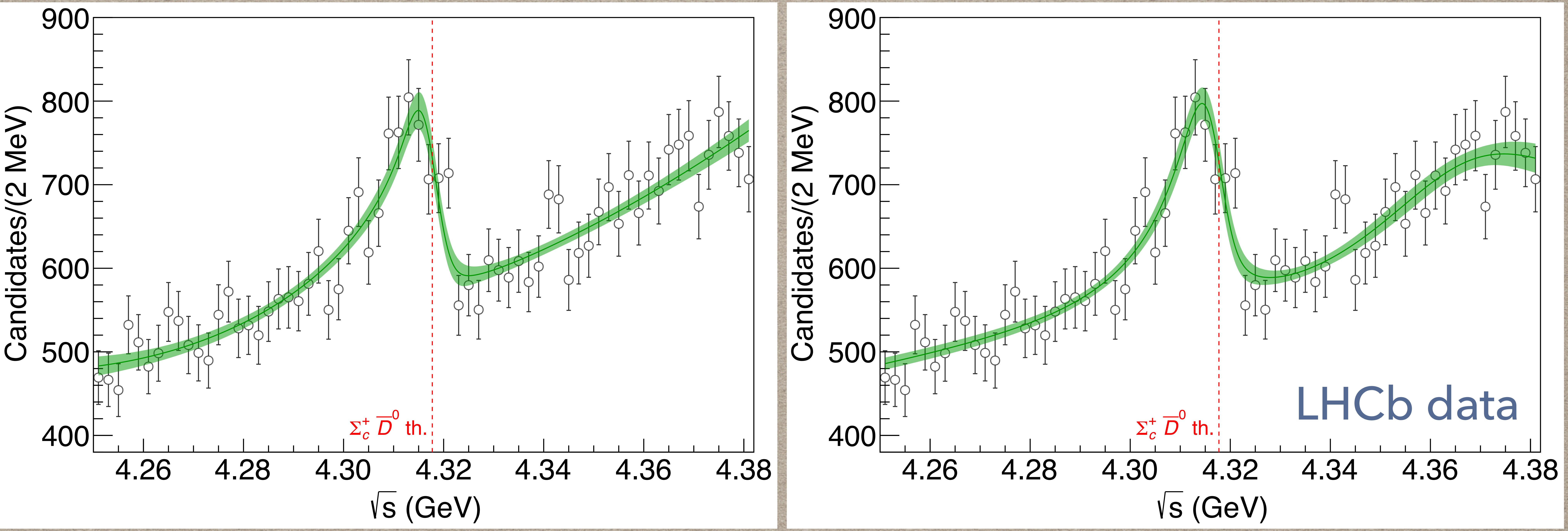
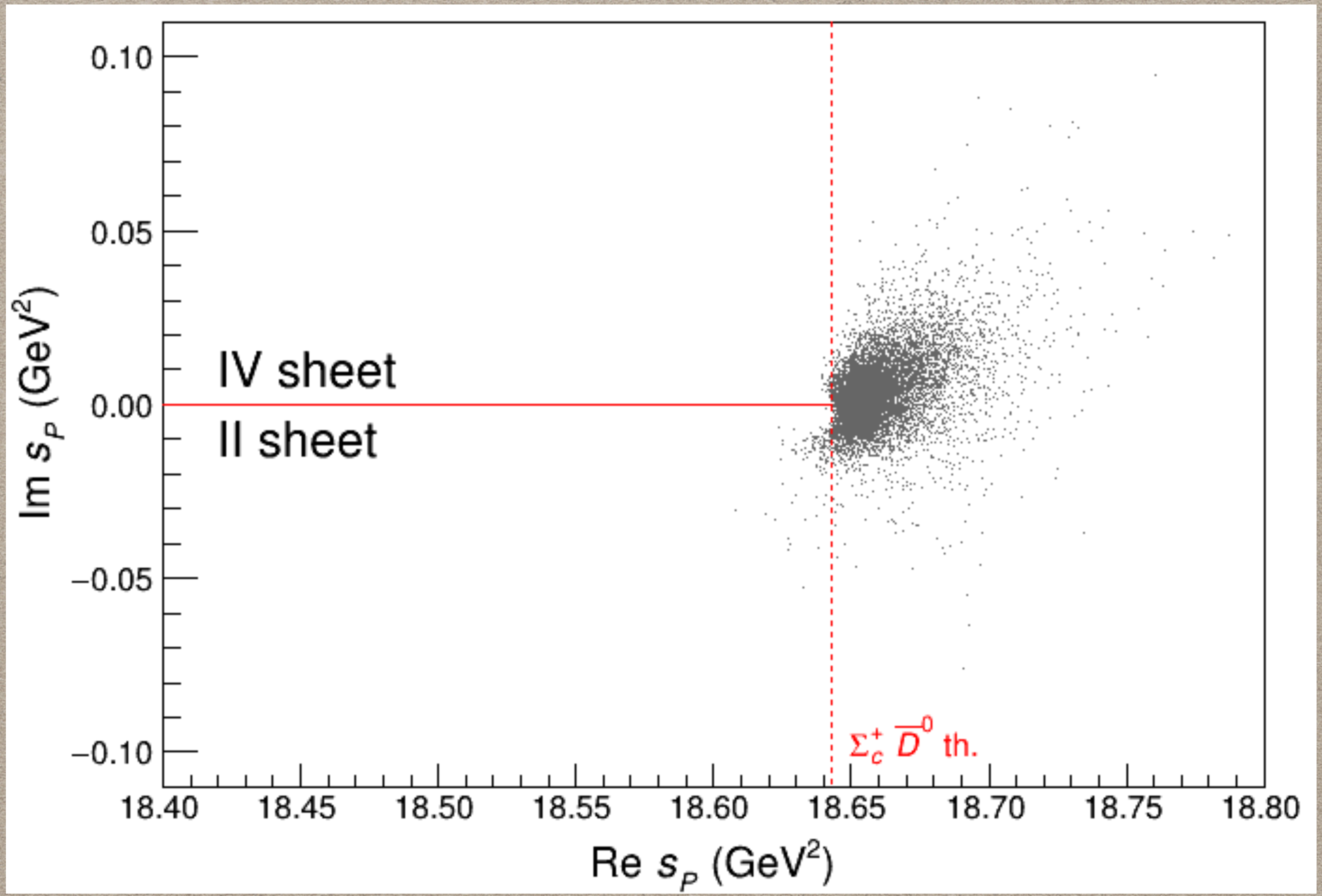
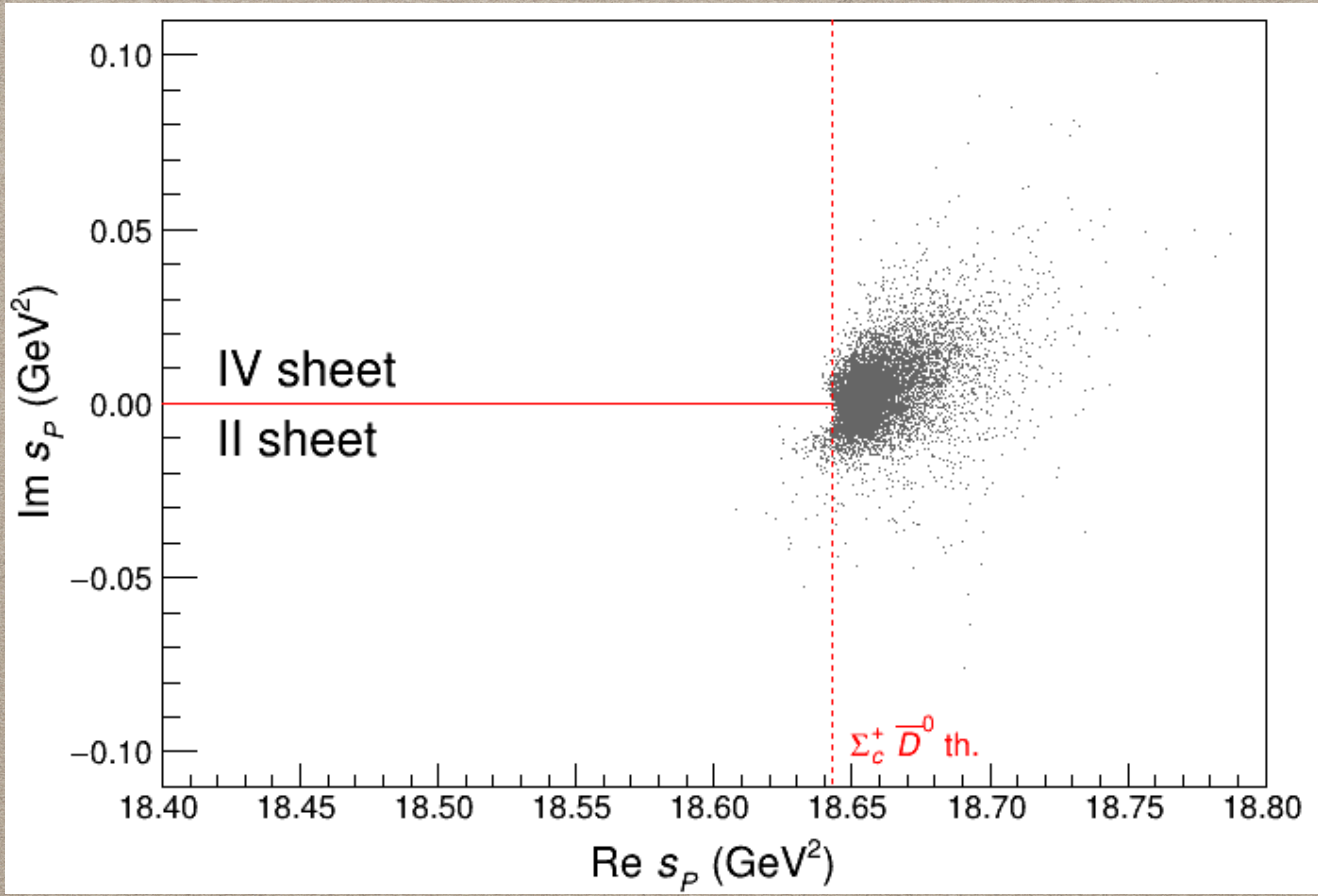


FIG. 1. Fits to the $\cos\theta_{P_c}$-weighted $J/\psi\, p$ mass distribution from LHCb [9] according to cases A (left) and B (right). The amplitude of case A is expressed in the scattering length approximation, *i.e.* $c_{ij} = 0$ in Eq. (3), and is able to describe either bound (molecular) or virtual states. The amplitude of case B is given in the effective range approximation, *i.e.* finite c_{ii}, and extends the description to genuine pentaquark states. The solid line and green band show the result of the fit and the 1σ confidence level provided by the bootstrap analysis, respectively.

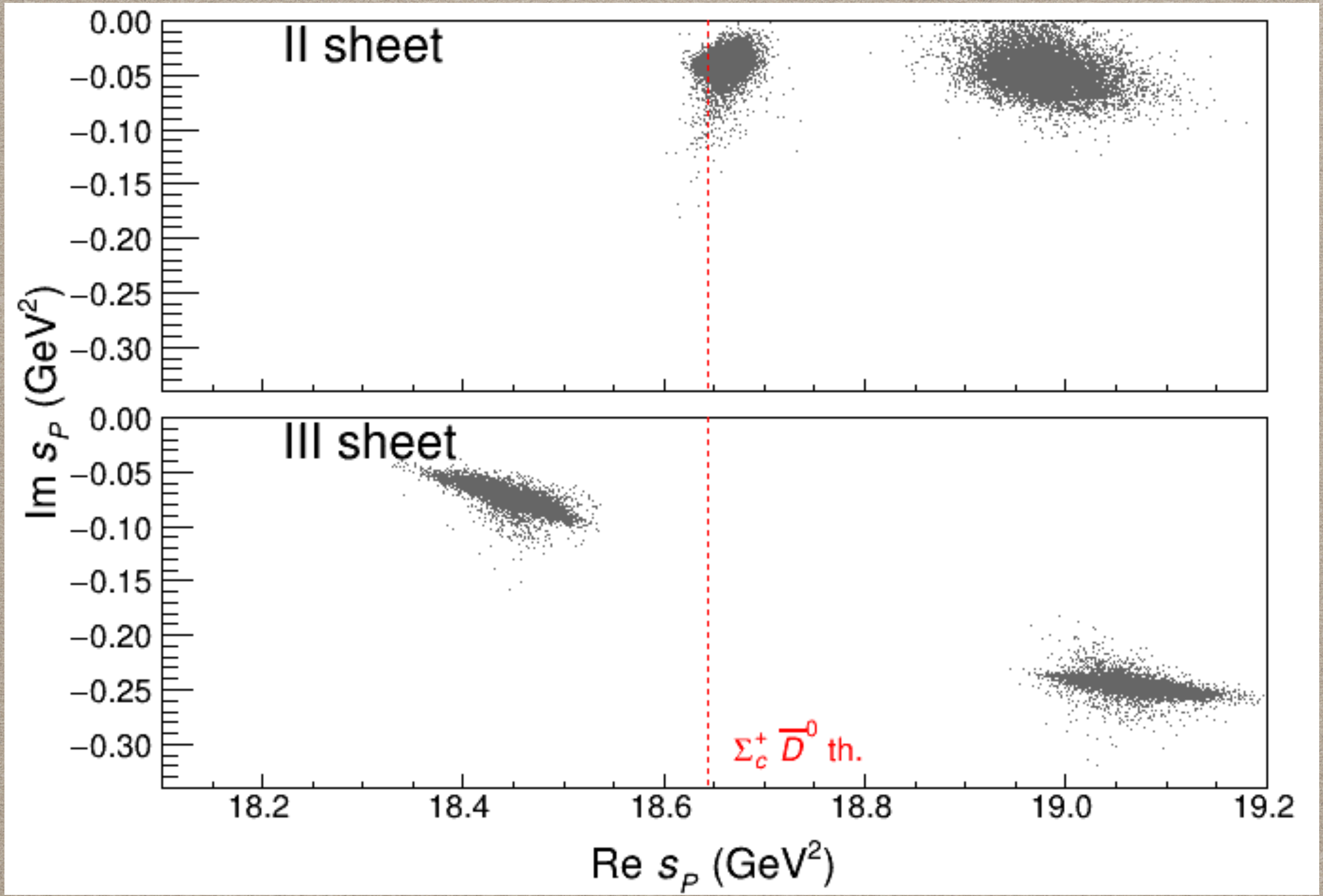
POLE MOVEMENT: $P_C(4312)$



POLE MOVEMENT: $P_C(4312)$



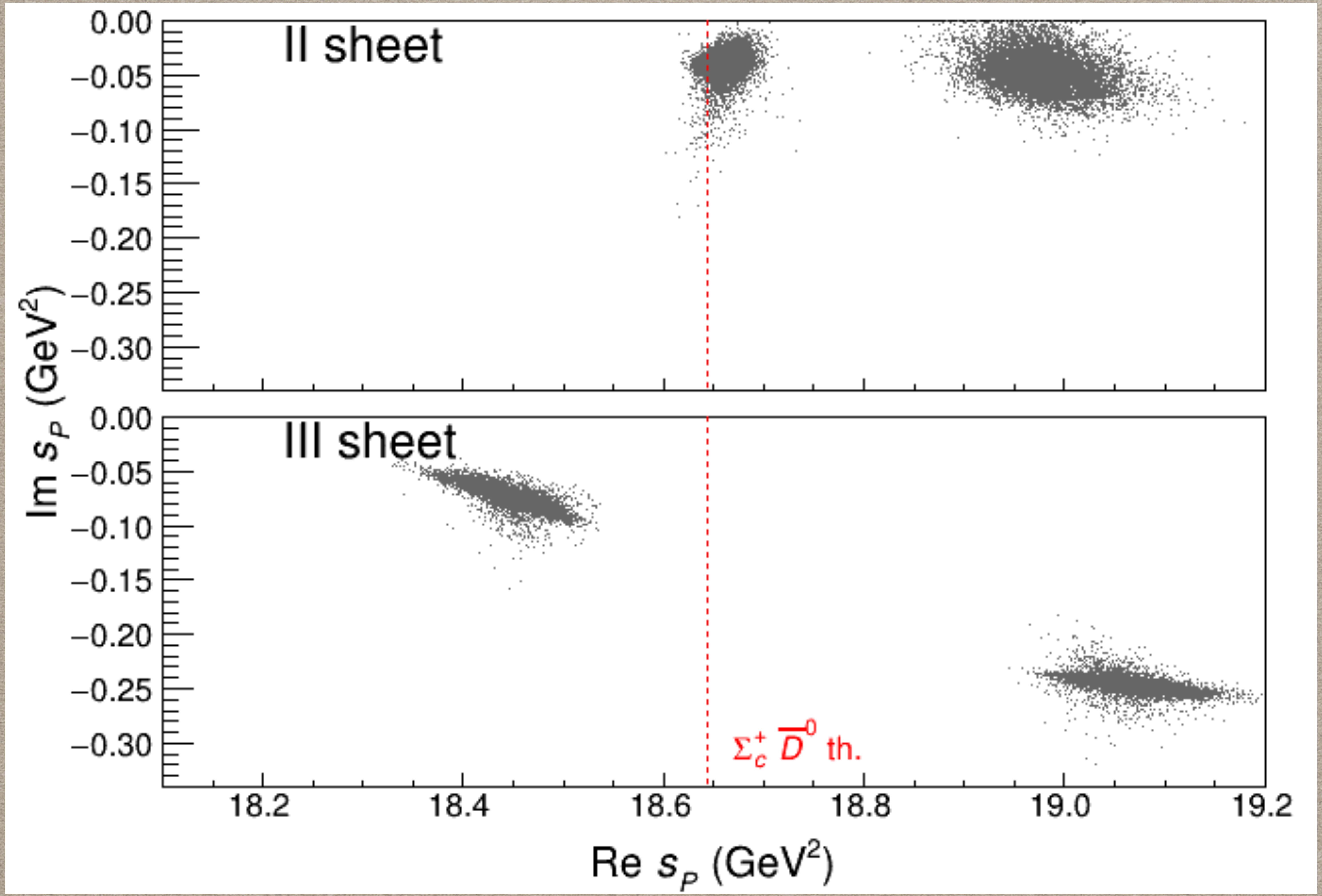
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

$$\frac{[m_{22} - c_{22}s - ik_2]}{[m_{22} - c_{22}s - ik_2][m_{11} - c_{11}s - ik_1] - m_{12}^2}$$

POLE MOVEMENT: C≠0



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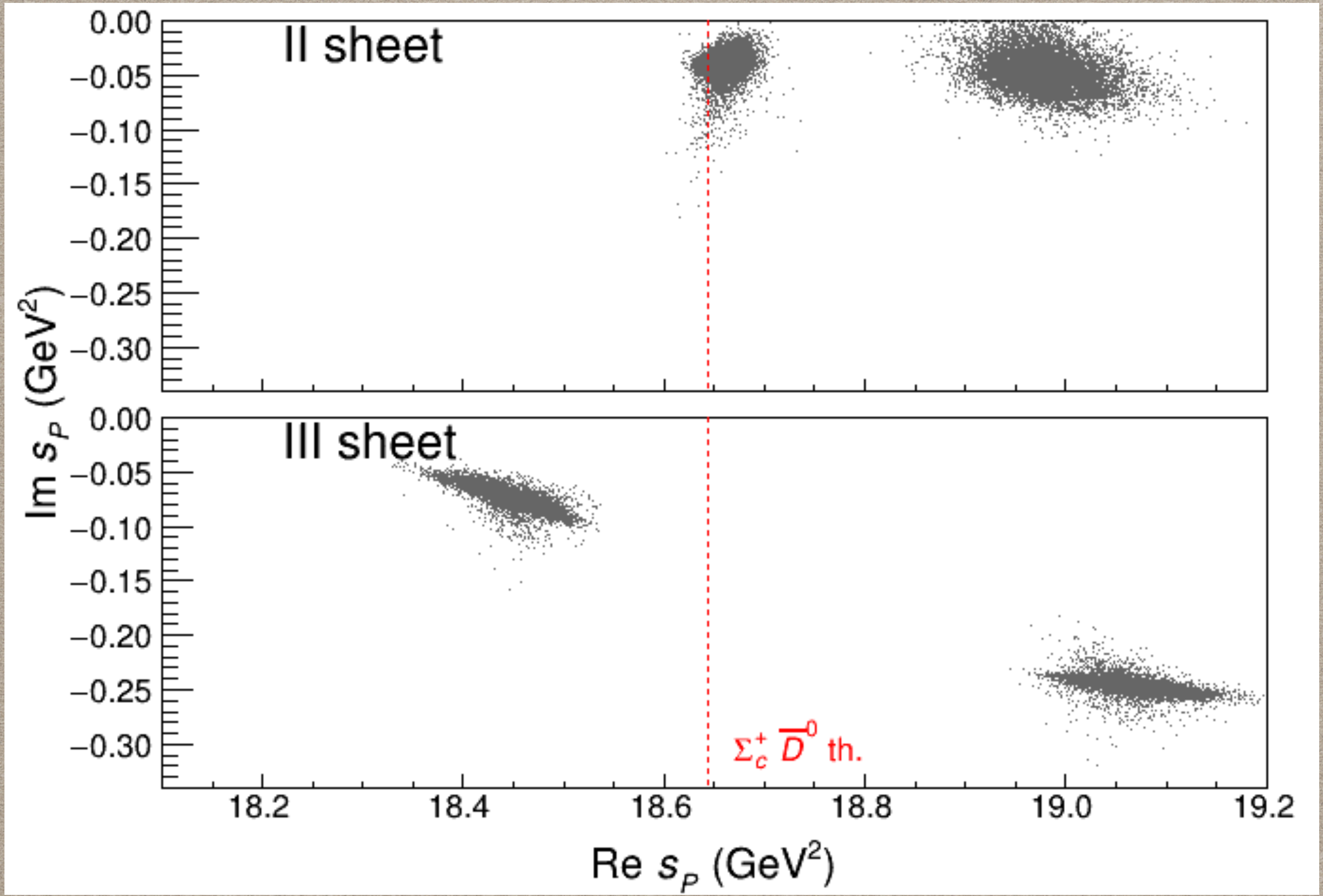
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

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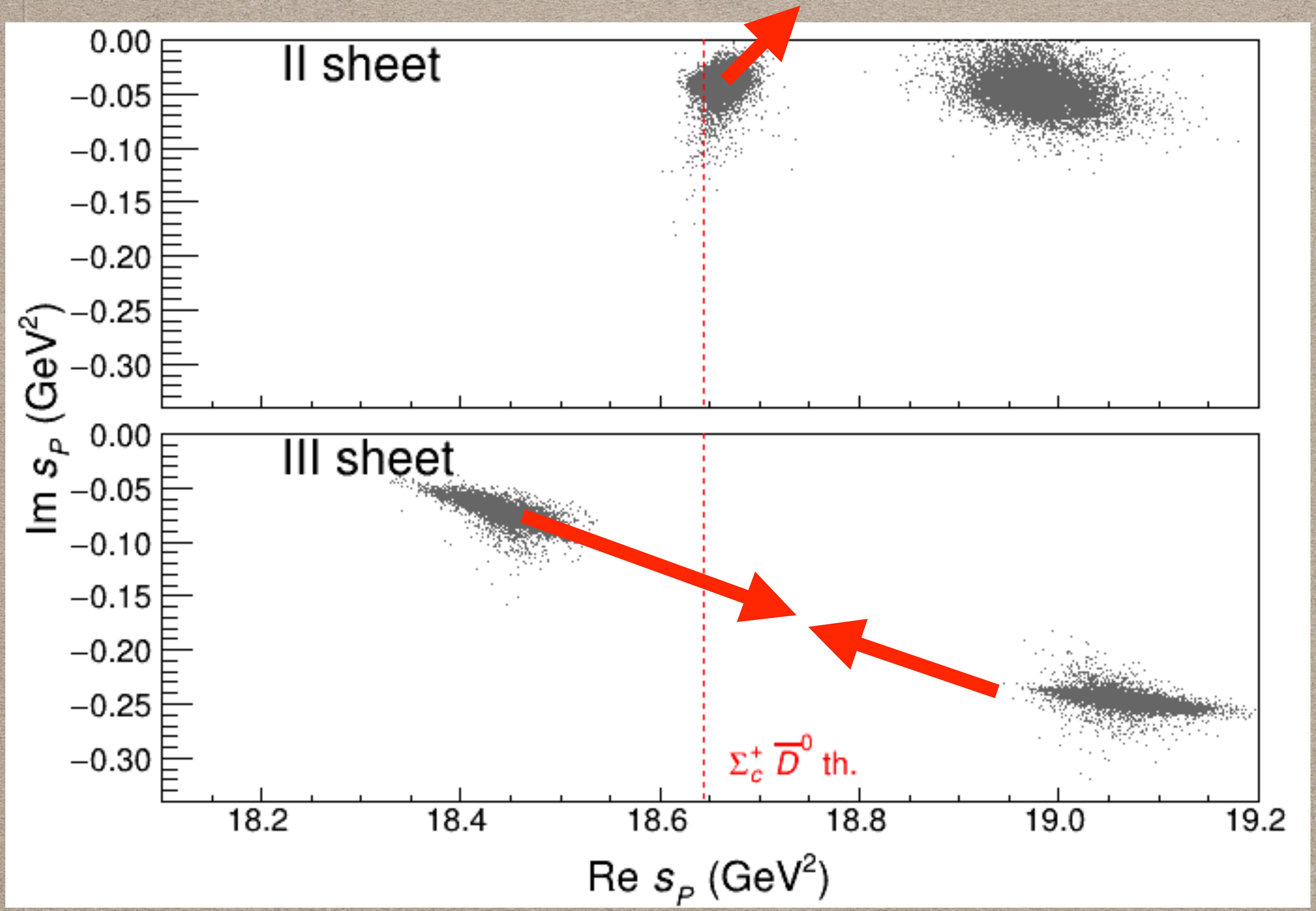
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

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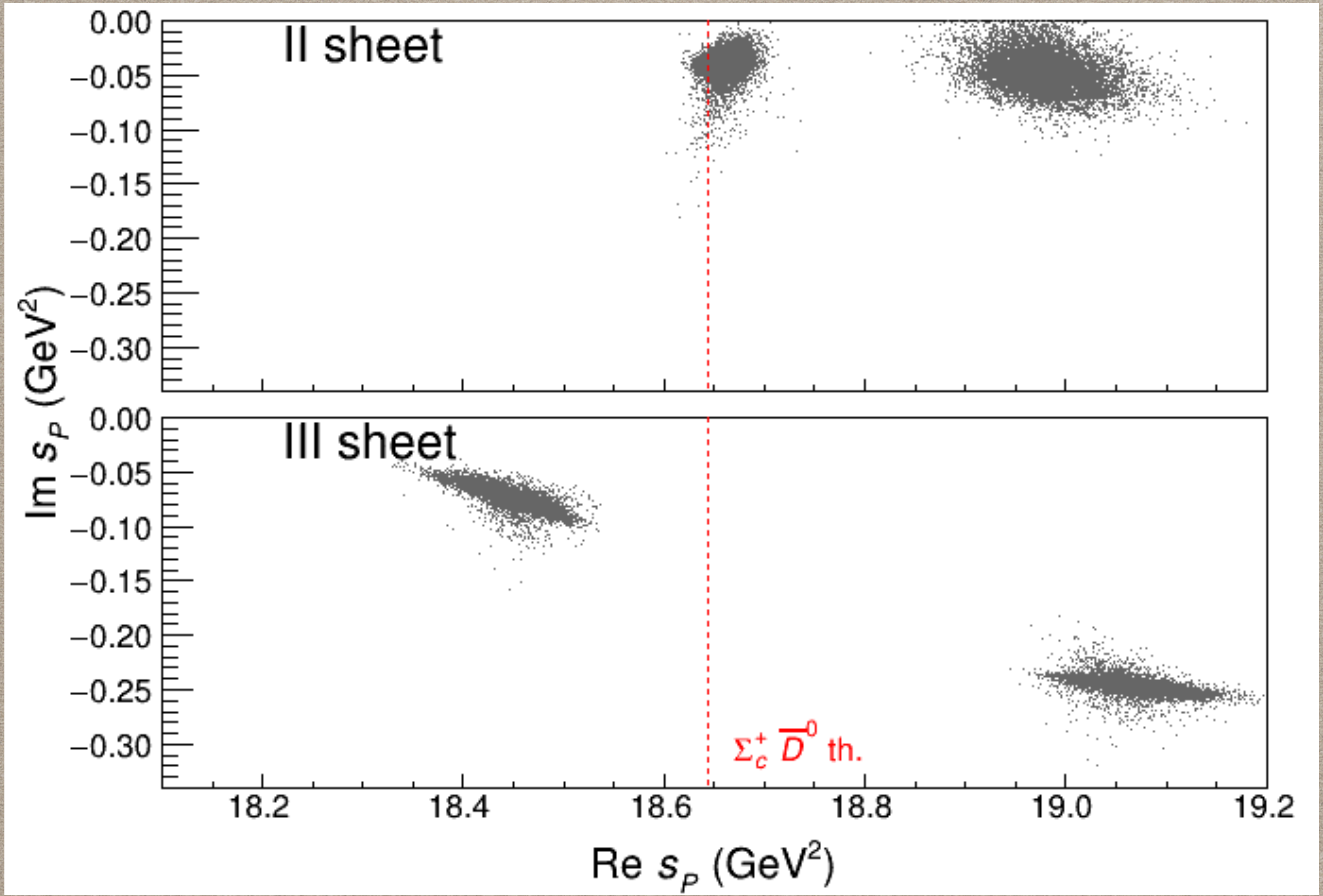

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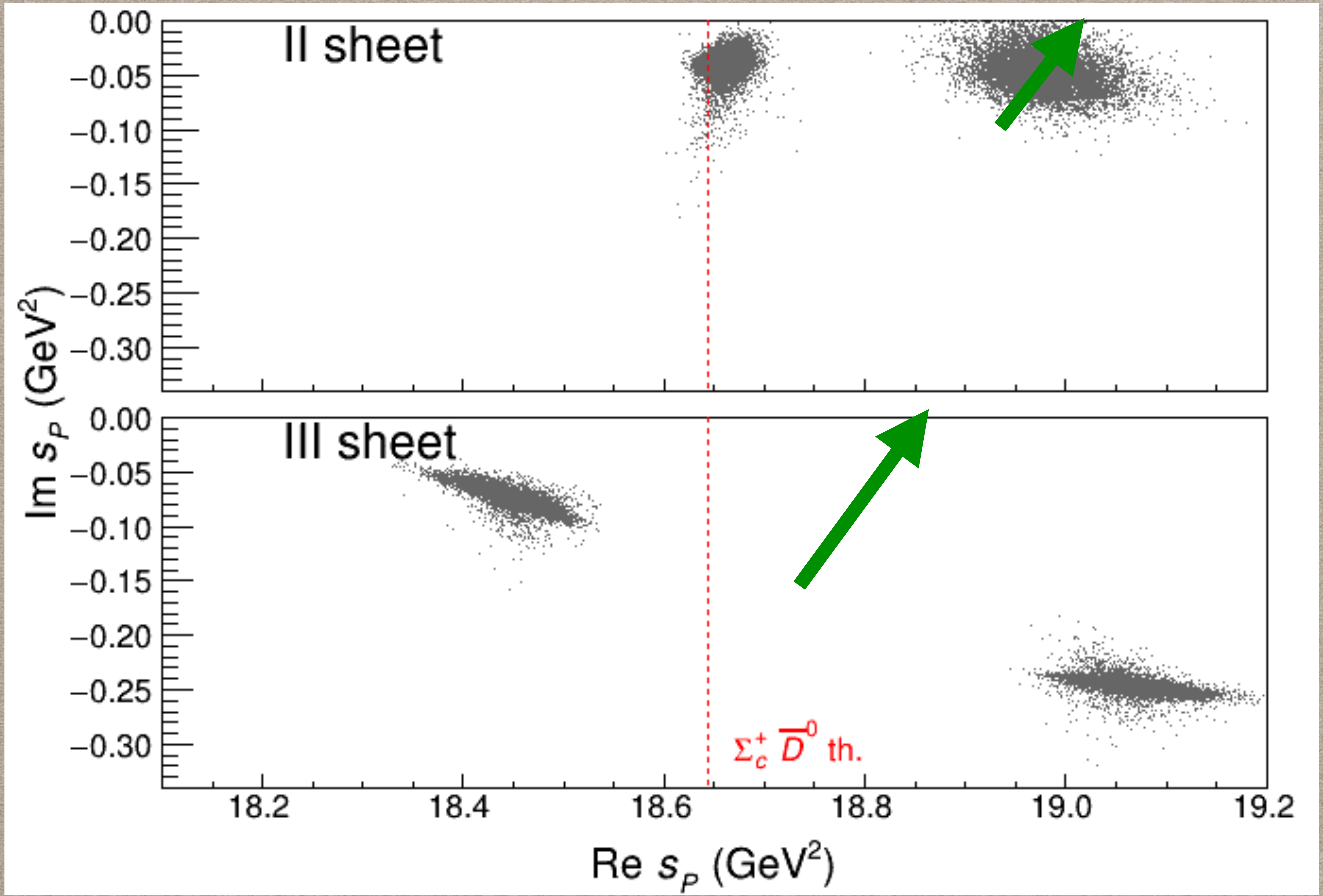
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

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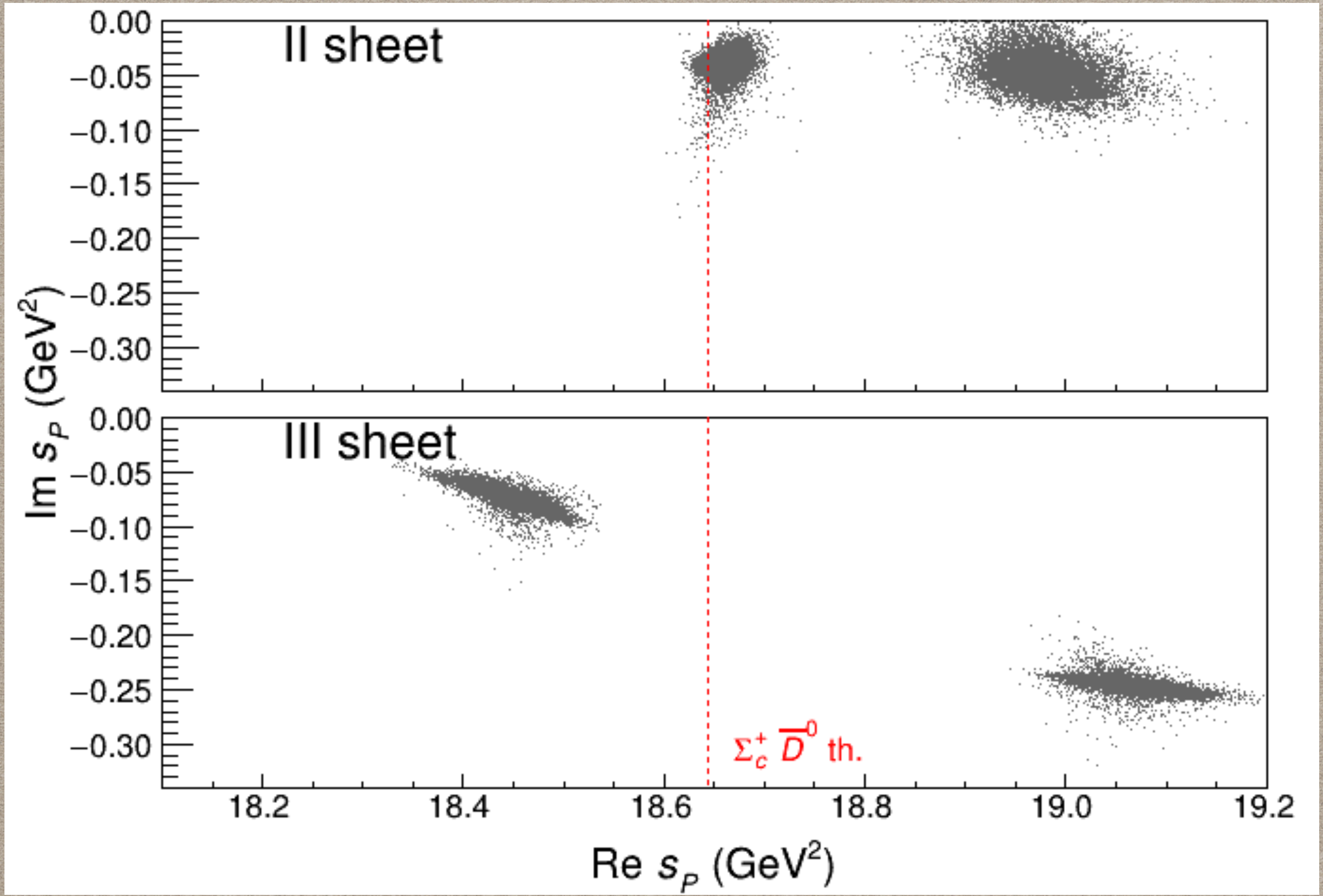
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

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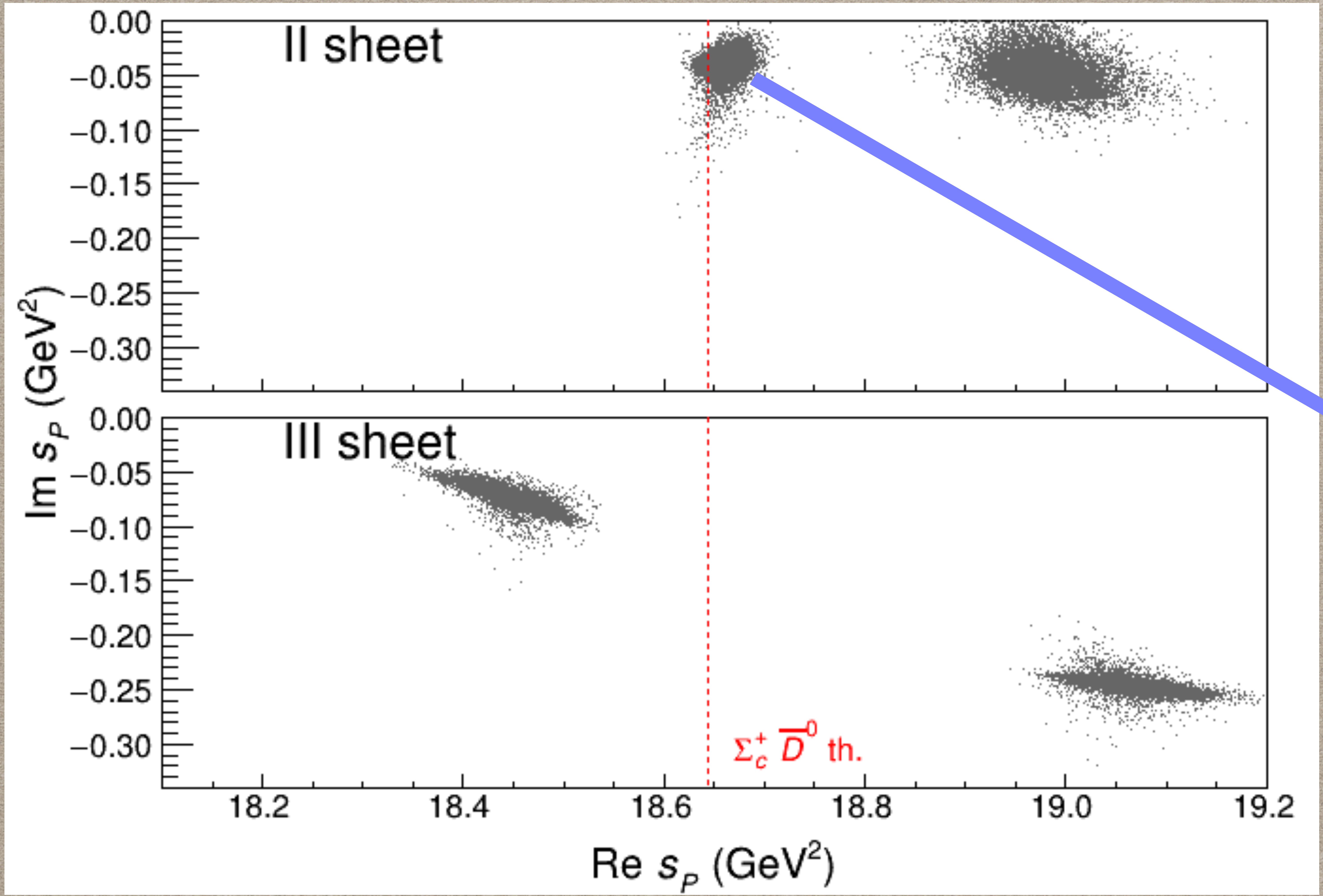
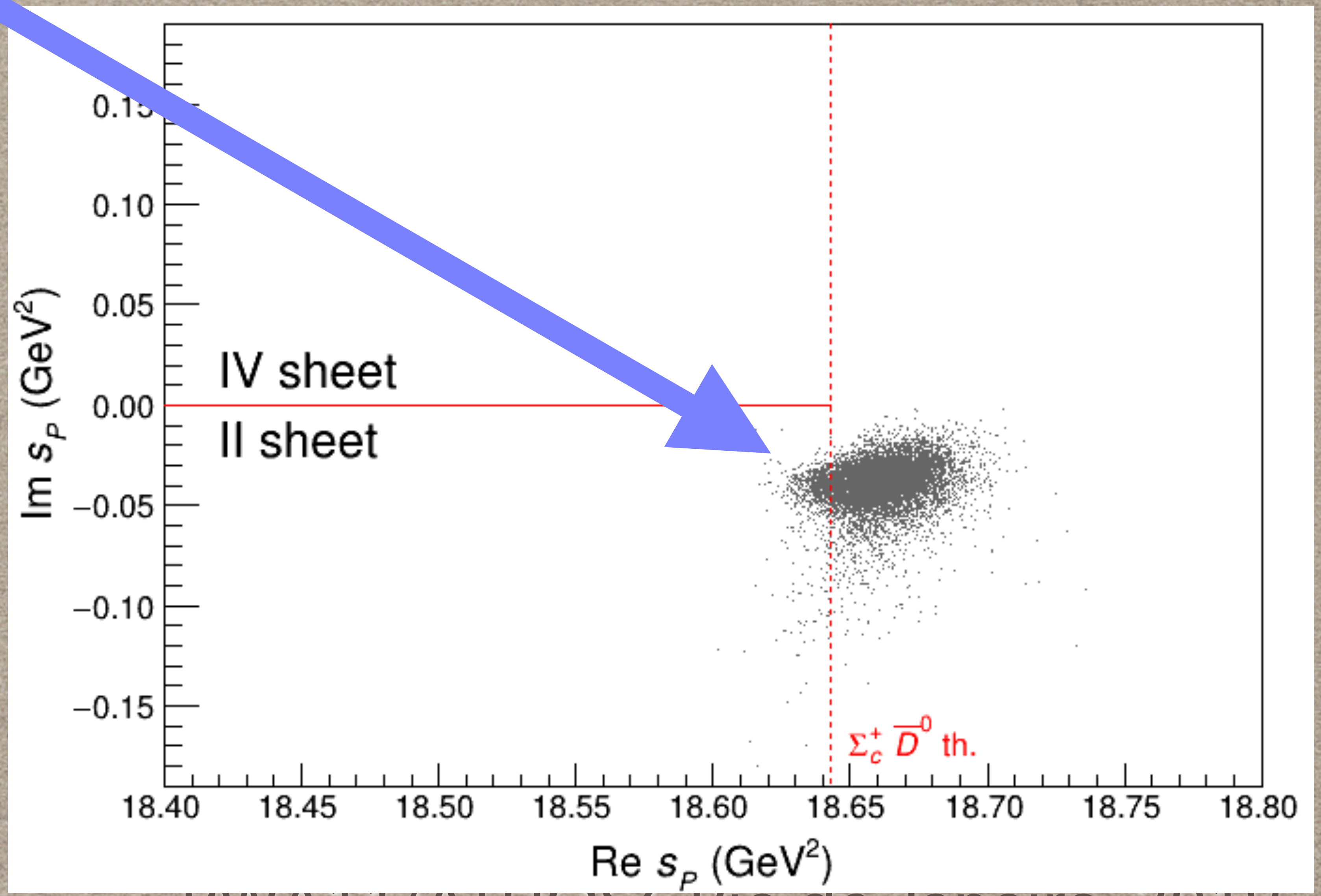
POLE MOVEMENT: C≠0



When m_{12}=0 both channels decouple and

$$\frac{[m_{22} - c_{22}s - ik_2]}{[m_{22} - c_{22}s - ik_2][m_{11} - c_{11}s - ik_1] - m_{12}^2}$$



CONCLUSIONS

- In a near-threshold region we can build a minimally biased approach
- We can study pole stability against changes in the parameters compatible with the experimental uncertainties through bootstrap (more in Alessandro's talk)
- We can study pole motion, getting insight in the nature of the signal **without any prior model if the situation is simple enough**
- The $P_c(4312)$ is a very suitable test case. Pole obtained :
  - [M= 4319.7(1.6) MeV; Γ=−0.8(2.4) MeV; M= 4319.8(1.5) MeV; Γ= 9.2(2.9) MeV]
- The favored interpretation based on pole motion is **Virtual state**

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