New amplitudes for B and D three-body decay

Patricia C. Magalhães

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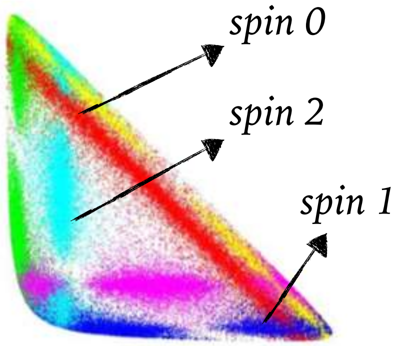
Motivation

- D and B three-body HADRONIC decay are dominated by resonances
  - spectroscopy



→ underling strong force behave

  - obtain meson-meson amplitudes up to high mass (including KK)

- CP-Violation
  - weak and strong phase

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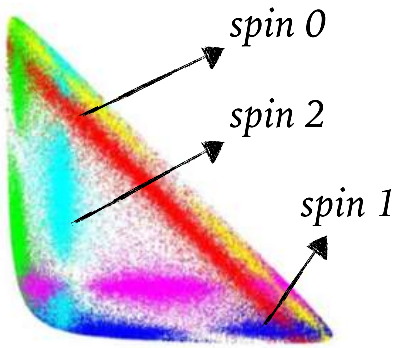

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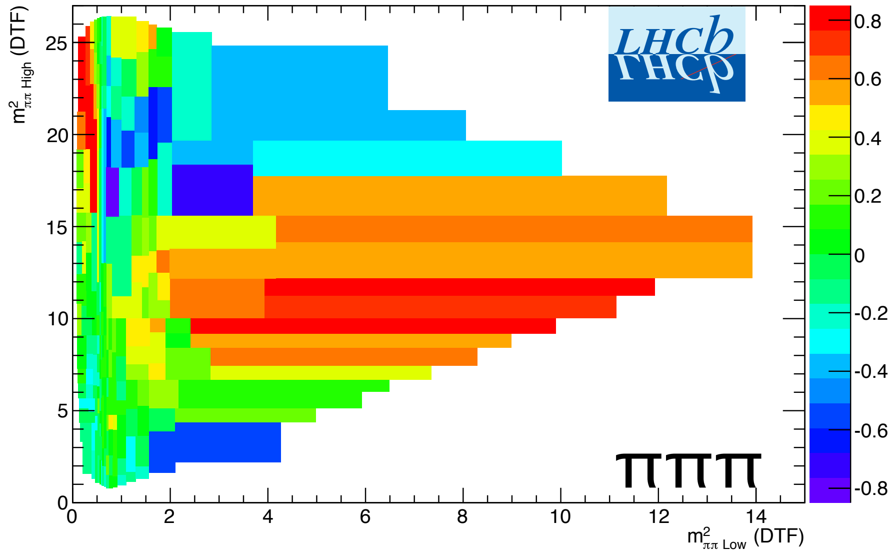
  - weak and strong phase
  - $B^{\pm} \to h^{\pm}h^{-}h^{+}$ massive localized direct CP asymmetry



LHCb PRD90 (2014) 112004

- dynamic effect

↘

Final state interactions play a massive role

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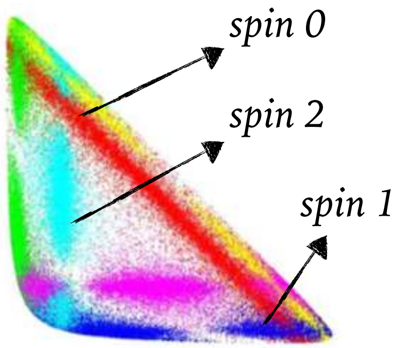

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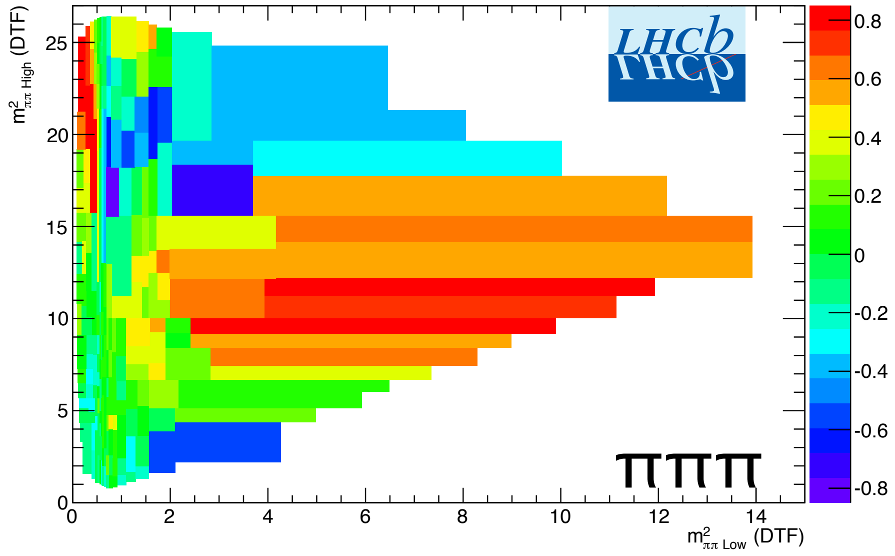


LHCb PRD90 (2014) 112004

- 1st observation in charm

LHCb $D^0(\bar{D^0}) \rightarrow h^- h^+$ mixing

↳ CPV on three-body?

- dynamic effect

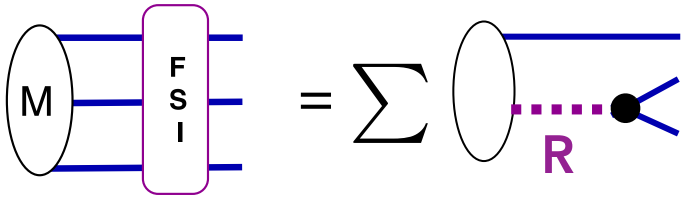
Final state interactions play a massive role

→ can lead to new physics

Motivation

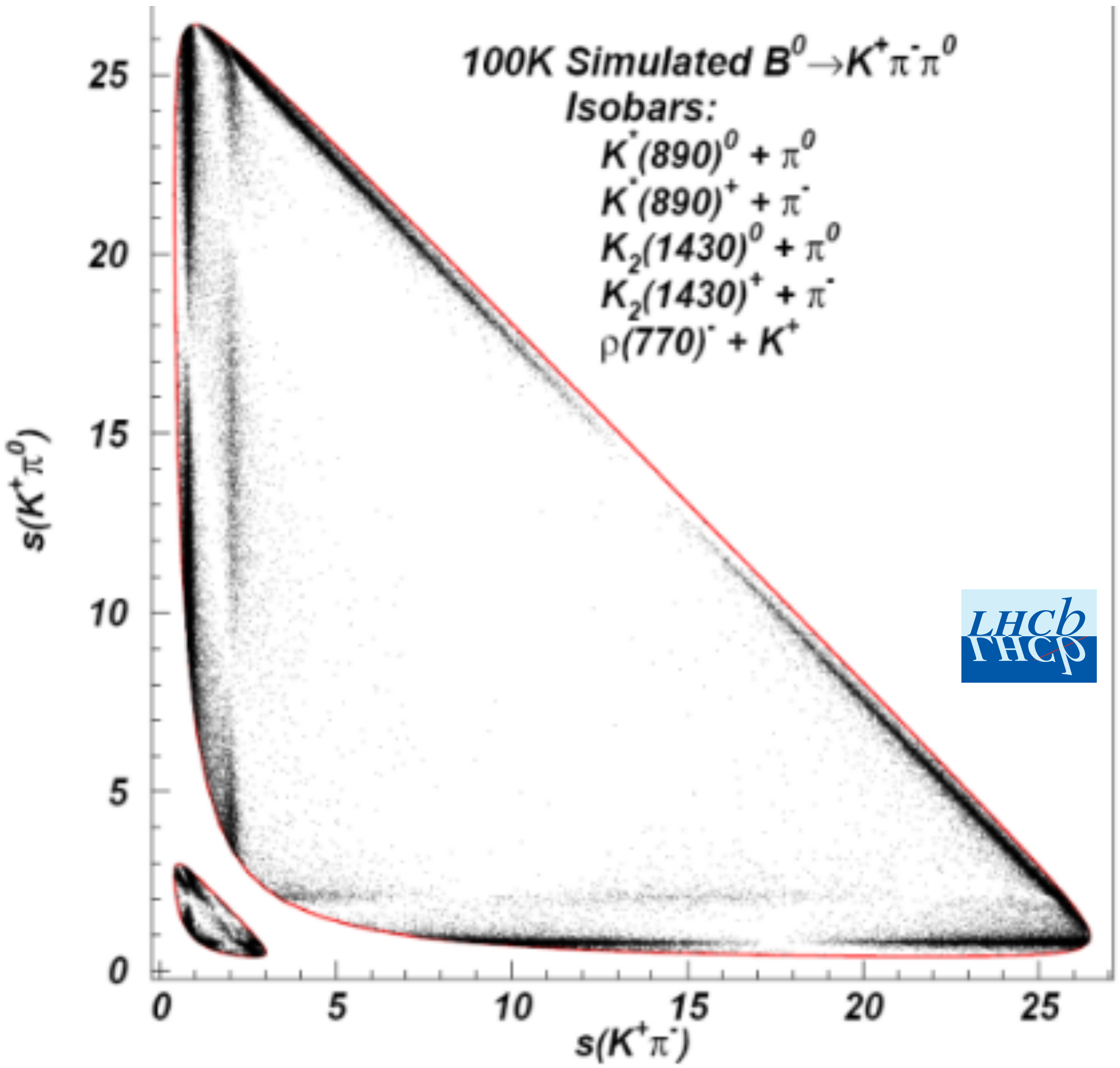
- new large data sample from LHCb ⟶ more to come from LHCb and Belle II

  ↳ simple models (isobar model with Breit-Wigners resonances) (2+1)



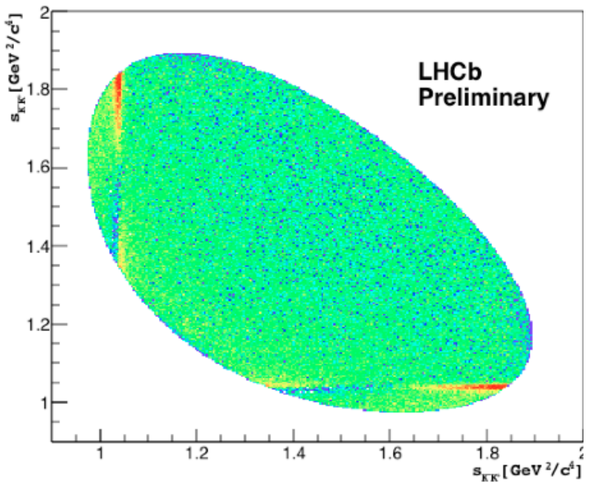
→ not enough to explain data anymore

- difference phase-space in D and B decays
  - ≠ scales!!! → still similar FSI
  - 3-body effects expected to be smaller in B
  - B phase-space → + FSI possibilities



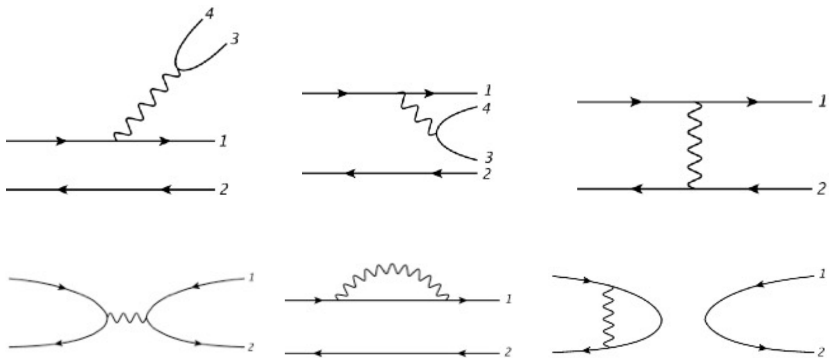
Dynamics of 3-body heavy decay

- ex: $D^+ \to K^- K^+ K^-$



primary vertex

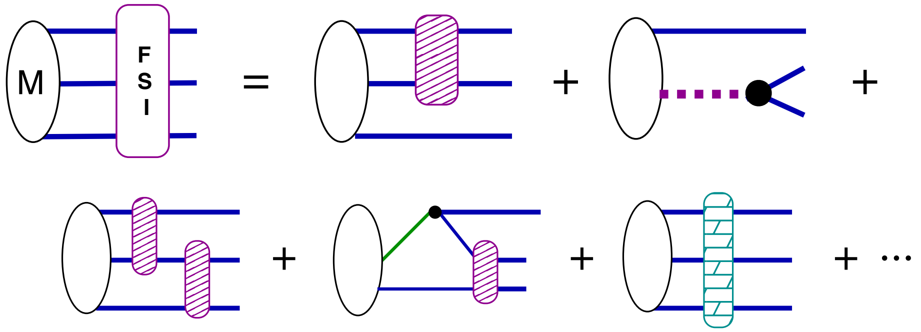
- weak -



QCD, CKM coupling and phase

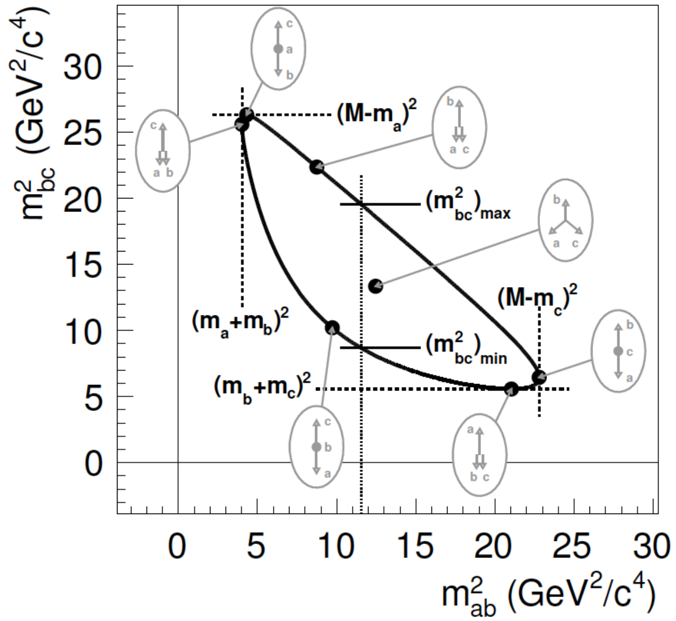
Final State Interactions

- strong -



(2+1) + 3-body interactions

Dalitz plot



$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12},s_{23})|^2$$

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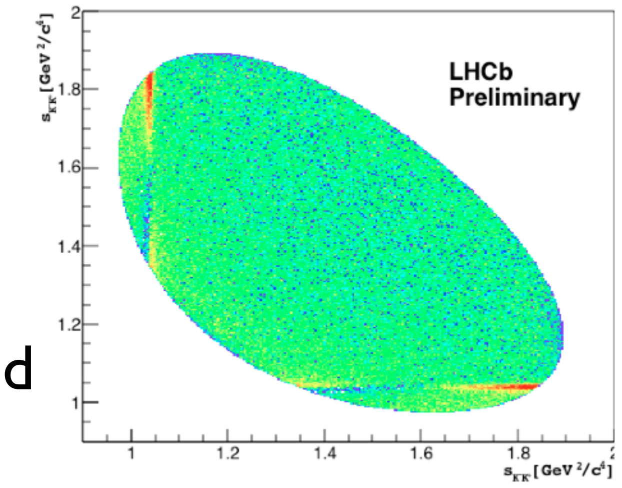
- ex: $D^+ \to K^- K^+ K^-$

hadronize

observed



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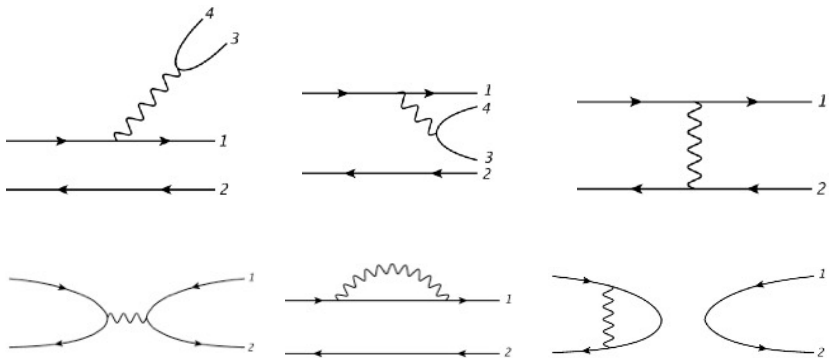

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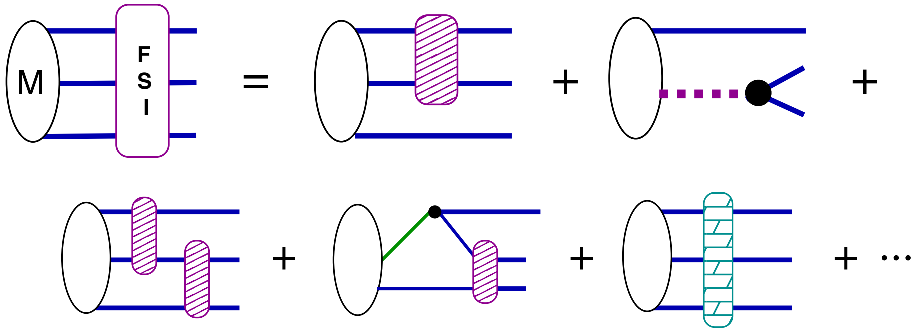

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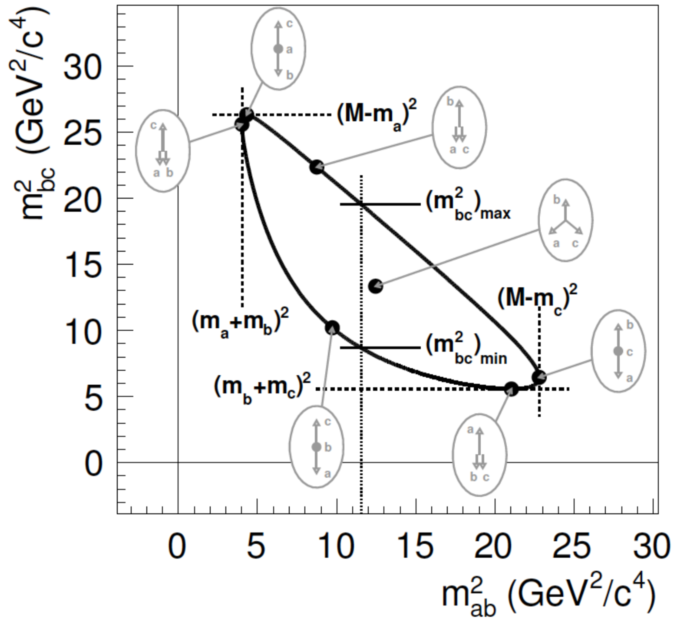

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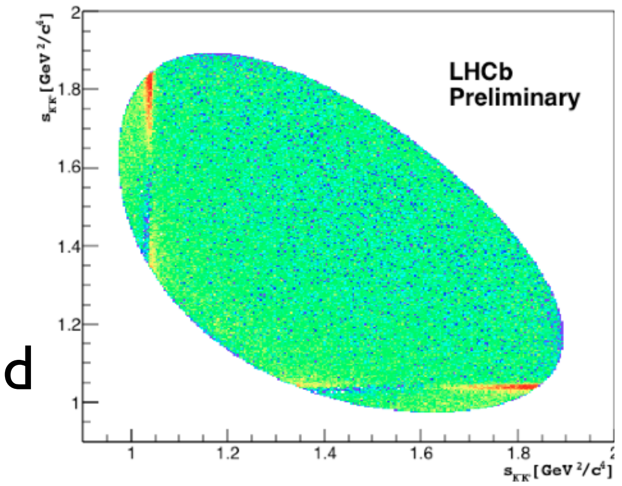

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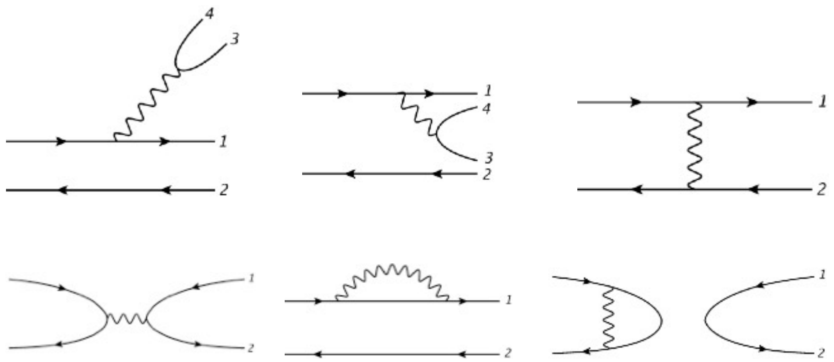

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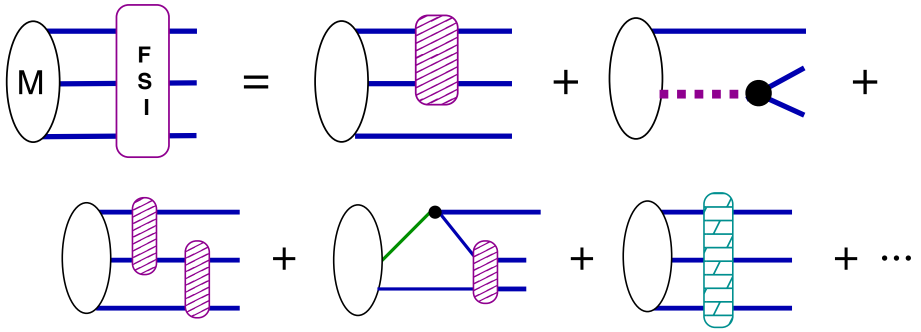

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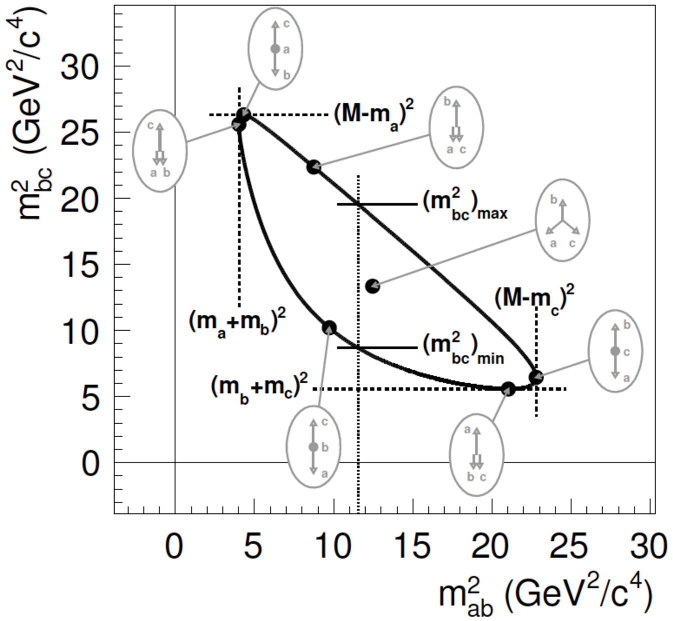
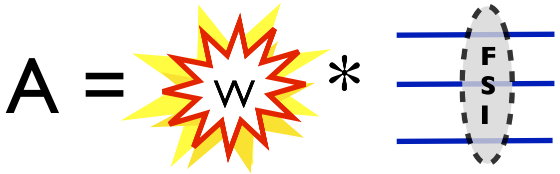


To extract information from data
we need an amplitude MODEL



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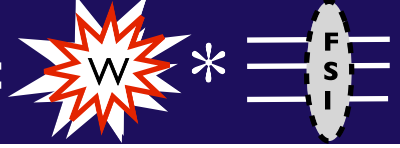
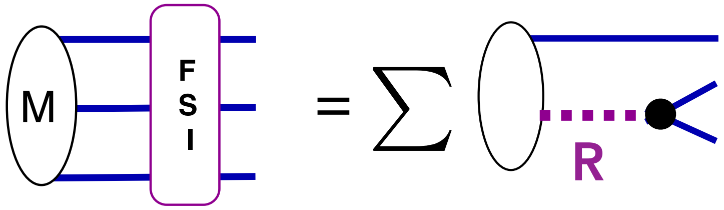
Models available A =



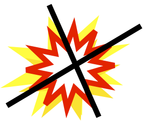
- isobar model: widely used by experimentalists

  - (2+1) → ignore the 3rd particle (bachelor)

  - aways intermediated by a resonance R →



$A = \sum c_k \, A_k$, + NR { non-resonant as constant or exponential!
each resonance as Breit-Wigner $\mathrm{BW}(s_{12}) = \frac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})},$


weak vertex is not considered explicitly

- warnings:

Models available

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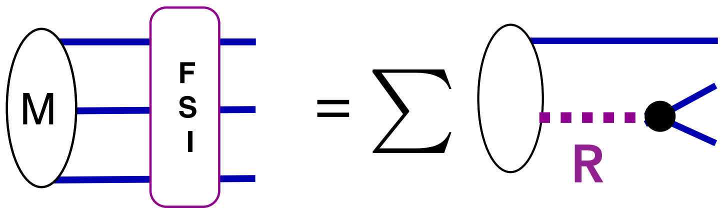

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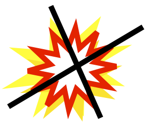
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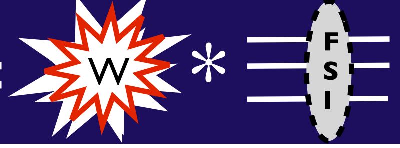
- warnings:

  - sum of BW violates two-body unitarity (2 res in the same channel);
  - do NOT include rescattering and coupled-channels;
  - free parameters are not connected with theory !

!

Models available

A = 

- movement to use better 2-body (unitarity) inputs in data analysis

  - “K-matrix" : ππ S-wave 5 coupled-channel modulated by a production amplitude

    ↳ used by Babar, LHCb, BES III- analyticity problems ! Anisovich PLB653(2007)

  - rescattering $\pi\pi \to KK$ contribution in LHCb $\begin{cases} B^\pm \to \pi^+\pi^-\pi^\pm \\ B^\pm \to K^-K^+\pi^\pm \end{cases}$ soon [arXiv:1905.09244]

    Pelaez, Yndurain PRD71(2005) 074016

    ↳ new parametrization Pelaez, and Rodas EPJ. C78 (2018) 11, 897

↳ Madrid Parametrization Pelaez’s talk

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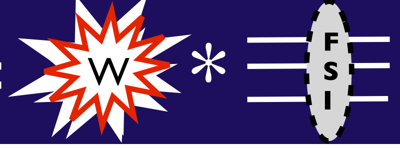

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- alternative ⟶ scalar and vector form factors using Dispersion Relation

  $< \pi\pi|0 >$ scalar Moussallam EPJ C 14, 111 (2000); Daub, Hanhart, and B. Kubis JHEP 02 (2016) 009.

  vector Hanhart, PL B715, 170 (2012). Dumm and Roig EPJ C 73, 2528 (2013).

  $< K\pi|0 >$ scalar Moussallam EPJ C 53, 401 (2008) Jamin, Oller and Pich, PRD 74, 074009 (2006)

  vector Boito, Escribano, and Jamin EPJ C 59, 821 (2009).

  ↳ Madrid Parametrization Pelaez’s talk

  - no data for KK

  $< KK|0 >$

  Fit from 3-body data PCM, Robilotta + LHCb JHEP 1904 (2019) 063

  extrapolate from unitarity model Albaladejo and Moussallam EPJ C 75, 488 (2015).

  quark model with isospin symmetry Bruch, Khodjamirian, and Kühn , EPJ C 39, 41 (2005)

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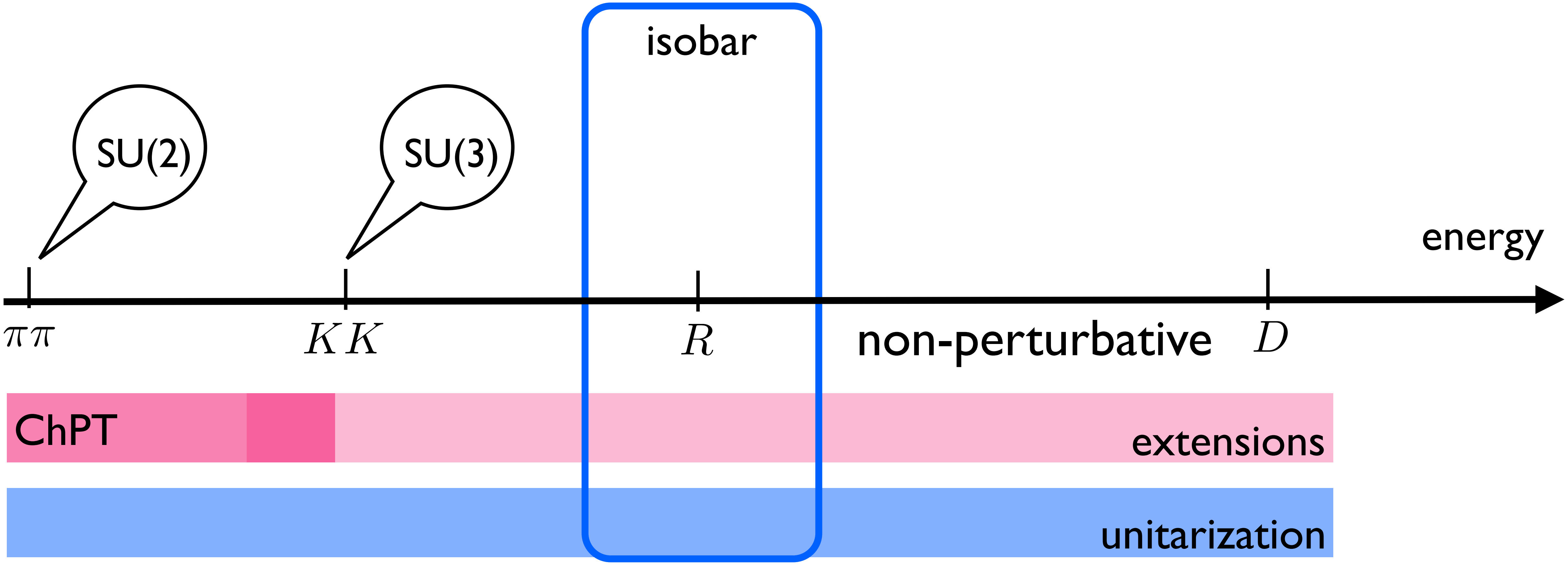
scale issue



- best theoretical $\pi\pi$, $K\pi$ scattering amplitude → constrained by data limited to low E
  ↳ no KK data/theory

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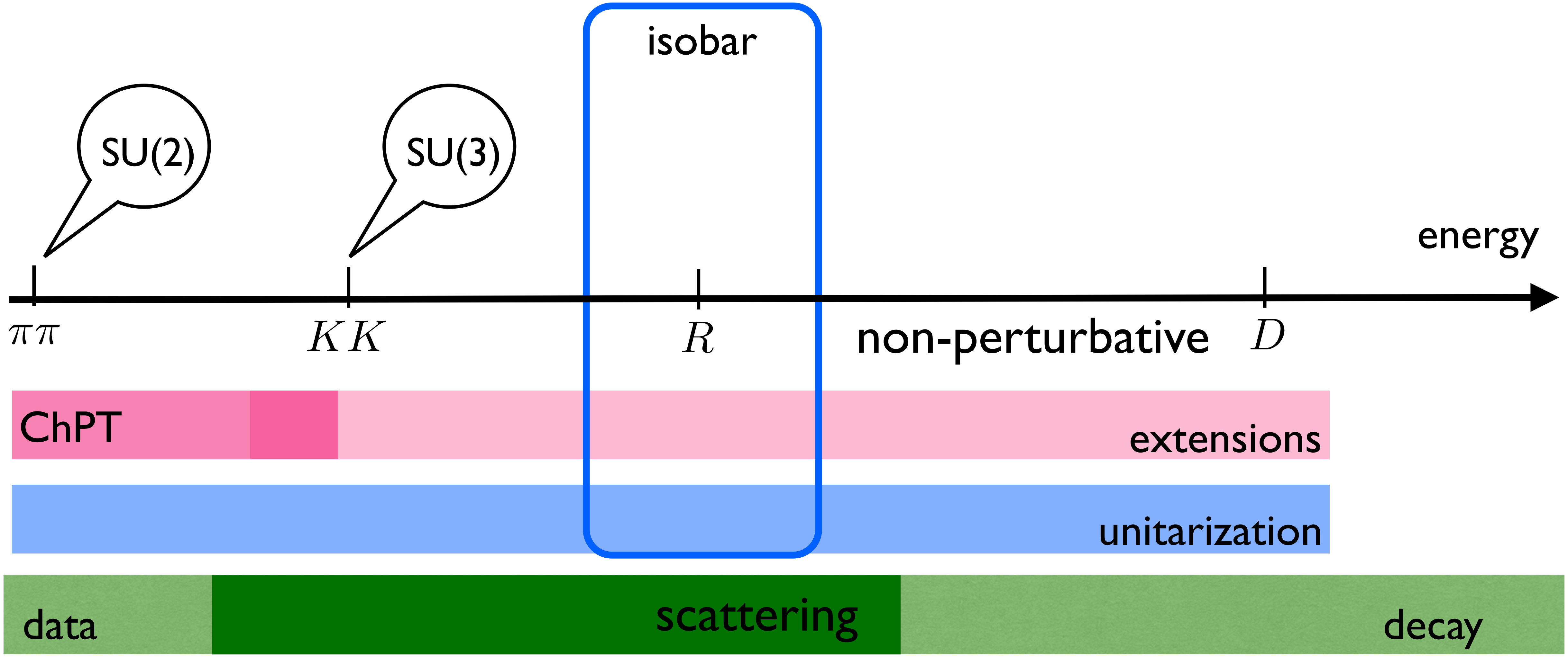

- best theoretical $\pi\pi,\ K\pi$ scattering amplitude → constrained by data
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  limited to low E

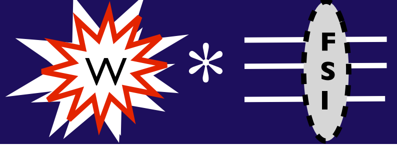
- we need non-perturbative meson-meson interactions up to…. B sector is far

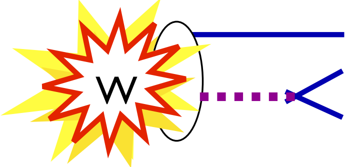
- extend 2-body amplitude theory validity

Ropertz, Kubis, Hanhart
EPJ Web Conf. 202 (2019) 06002

PCM, Robilotta
work in progress

Models available A =





- QCD factorization approach → factorize the quark currents

(2+1)

Keri’s talk

$$\mathcal{H}_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \Big[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \Big] + \text{ h.c. } ,$$

challenging for 3-body
not all FSI and 3-body NR
scale issue with charm
!

→ ex: $B^+ \to \pi^+\pi^-\pi^+$

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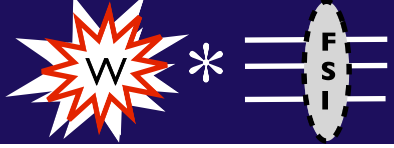
A = 

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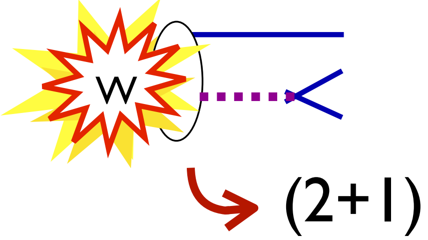

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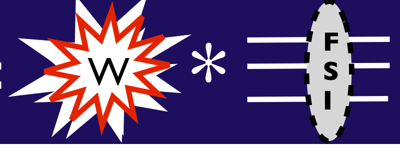
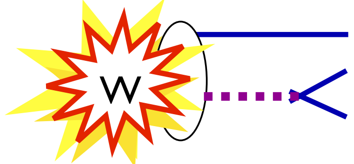
A = 



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- naive factorization
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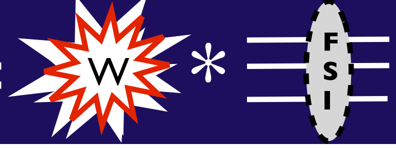
↳ parametrizations for B and D→3h Boito et al. PRD96 113003 (2017)

Models available

A = 

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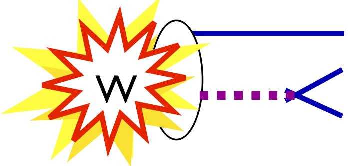

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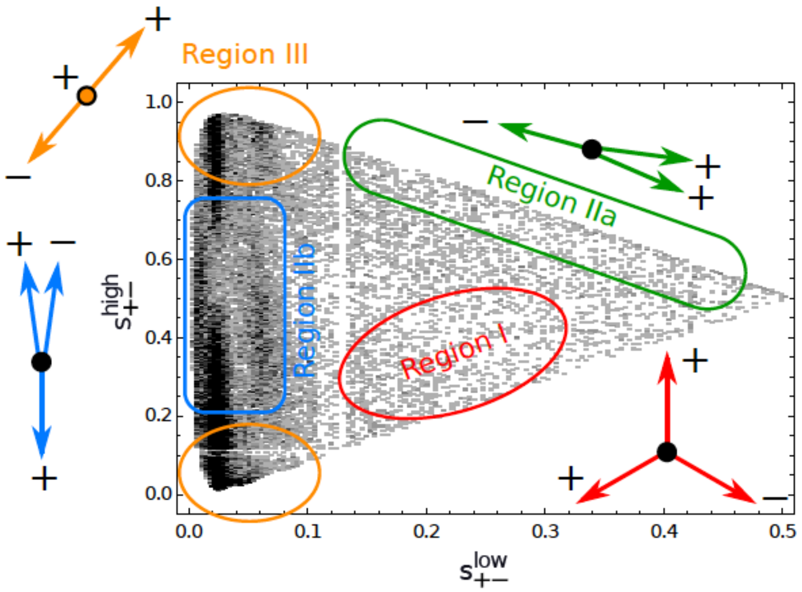
- modern QDC factorization: different in each region

↳ improvement over (2+1)

→ introduce new non-perturbative strong phase

Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)



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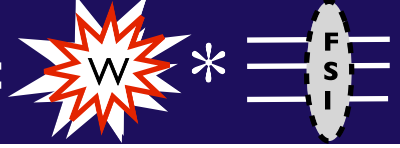
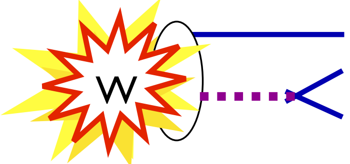




- QCDF predictions

Branching Fraction (tree dominated decays)

	Theory I	Theory II	Experiment
$B^- \to \pi^- \pi^0$	$5.43^{+0.06\,+1.45}_{-0.06\,-0.84}$ $(\star)$	$5.82^{+0.07\,+1.42}_{-0.06\,-1.35}$ $(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+ \pi^-$	$7.37^{+0.86\,+1.22}_{-0.69\,-0.97}$ $(\star)$	$5.70^{+0.70\,+1.16}_{-0.55\,-0.97}$ $(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0 \pi^0$	$0.33^{+0.11\,+0.42}_{-0.08\,-0.17}$	$0.63^{+0.12\,+0.64}_{-0.10\,-0.42}$	1.55 ± 0.19
		BELLE CKM 14:	0.90 ± 0.16
$B^- \to \pi^- \rho^0$	$8.68^{+0.42\,+2.71}_{-0.41\,-1.56}$ $(\star\star)$	$9.84^{+0.41\,+2.54}_{-0.40\,-2.52}$ $(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0 \rho^-$	$12.38^{+0.90\,+2.18}_{-0.77\,-1.41}$ $(\star)$	$12.13^{+0.85\,+2.23}_{-0.73\,-2.17}$ $(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+ \rho^-$	$17.80^{+0.62\,+1.76}_{-0.56\,-2.10}$ $(\star)$	$13.76^{+0.49\,+1.77}_{-0.44\,-2.18}$ $(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^- \rho^+$	$10.28^{+0.39\,+1.37}_{-0.39\,-1.42}$ $(\star\star)$	$8.14^{+0.34\,+1.35}_{-0.33\,-1.49}$ $(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm \rho^\mp$	$28.08^{+0.27\,+3.82}_{-0.19\,-3.50}$ $(\dagger)$	$21.90^{+0.20\,+3.06}_{-0.12\,-3.55}$ $(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0 \rho^0$	$0.52^{+0.04\,+1.11}_{-0.03\,-0.43}$	$1.49^{+0.07\,+1.77}_{-0.07\,-1.29}$	2.0 ± 0.5
$B^- \to \rho_L^- \rho_L^0$	$18.42^{+0.23\,+3.92}_{-0.21\,-2.55}$ $(\star\star)$	$19.06^{+0.24\,+4.59}_{-0.22\,-4.22}$ $(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho_L^+ \rho_L^-$	$25.98^{+0.85\,+2.93}_{-0.77\,-3.43}$ $(\star\star)$	$20.66^{+0.68\,+2.99}_{-0.62\,-3.75}$ $(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho_L^0 \rho_L^0$	$0.39^{+0.03\,+0.83}_{-0.03\,-0.36}$	$1.05^{+0.05\,+1.62}_{-0.04\,-1.04}$	$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05,\ A_0^{B\rho}(0) = 0.30 \pm 0.05,\ \lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15\,\text{GeV}$

Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03,\ A_0^{B\rho}(0) = 0.28 \pm 0.03,\ \lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}\,\text{GeV}$

→ good agreement for Br

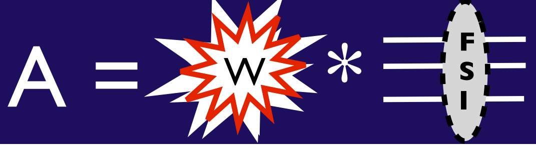
Acp (penguin dominante decays)

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^{*0}$	$1.36^{+0.25\,+0.60}_{-0.26\,-0.47}$	$1.49^{+0.27\,+0.69}_{-0.29\,-0.56}$	$0.27^{+0.05\,+3.18}_{-0.05\,-0.67}$	-3.8 ± 4.2
$\pi^0 K^{*-}$	$13.85^{+2.40\,+5.84}_{-2.70\,-5.86}$	$18.16^{+3.11\,+\ 7.79}_{-3.52\,-10.57}$	$-15.81^{+3.01\,+69.35}_{-2.83\,-15.39}$	-6 ± 24
$\pi^+ K^{*-}$	$11.18^{+2.00\,+\ 9.75}_{-2.15\,-10.62}$	$19.70^{+3.37\,+10.54}_{-3.80\,-11.42}$	$-23.07^{+4.35\,+86.20}_{-4.05\,-20.64}$	-23 ± 6
$\pi^0 \bar{K}^{*0}$	$-17.23^{+3.33\,+\ 7.59}_{-3.00\,-12.57}$	$-15.11^{+2.93\,+12.34}_{-2.65\,-10.64}$	$2.16^{+0.39\,+17.53}_{-0.42\,-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72\,+5.44}_{-0.67\,-4.30}$	$-1.54^{+0.45\,+4.60}_{-0.58\,-9.19}$	$7.26^{+1.21\,+12.78}_{-1.34\,-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38\,+3.38}_{-1.28\,-5.35}$	$-3.45^{+0.67\,+9.48}_{-0.59\,-4.95}$	$-1.02^{+0.19\,+4.32}_{-0.18\,-7.86}$	-5 ± 45
$\rho^- \bar{K}^0$	$0.38^{+0.07\,+0.16}_{-0.07\,-0.27}$	$0.22^{+0.04\,+0.19}_{-0.04\,-0.17}$	$0.30^{+0.06\,+2.28}_{-0.06\,-2.39}$	-12 ± 17
$\rho^0 K^-$	$-19.31^{+3.42\,+13.95}_{-3.61\,-\ 8.96}$	$-4.17^{+0.75\,+19.26}_{-0.80\,-19.52}$	$43.73^{+7.07\,+\ 44.00}_{-7.62\,-137.77}$	37 ± 11
$\rho^+ K^-$	$-5.13^{+0.95\,+6.38}_{-0.97\,-4.02}$	$1.50^{+0.29\,+\ 8.69}_{-0.27\,-10.36}$	$25.93^{+4.43\,+25.40}_{-4.90\,-75.63}$	20 ± 11
$\rho^0 \bar{K}^0$	$8.63^{+1.59\,+2.31}_{-1.65\,-1.69}$	$8.99^{+1.66\,+3.60}_{-1.71\,-7.44}$	$-\ 0.42^{+0.08\,+19.49}_{-0.08\,-\ 8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17^{+2.80\,+7.98}_{-2.96\,-5.39}$	$-5.67^{+0.96\,+10.86}_{-1.01\ \ -9.79}$	$17.80^{+3.15\,+19.51}_{-3.01\,-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75^{+1.62\,+4.78}_{-1.66\,-6.48}$	$-10.84^{+1.98\,+11.67}_{-2.09\,-\ 9.09}$	$-\ 2.43^{+0.46\,+\ 4.60}_{-0.42\,-19.43}$	-37 ± 37

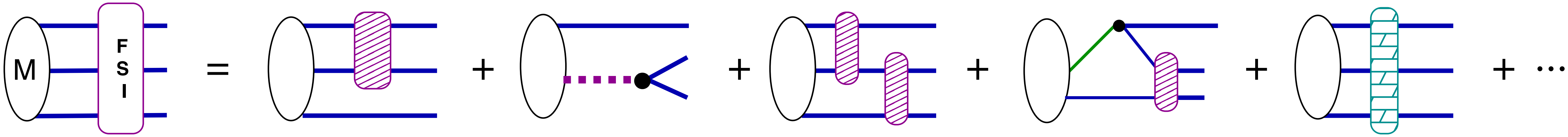
not good agreement for Acp ←

Beneke Seminar at "Future Challenges in Non-Leptonic B Decays", Bad Honnef, 2016

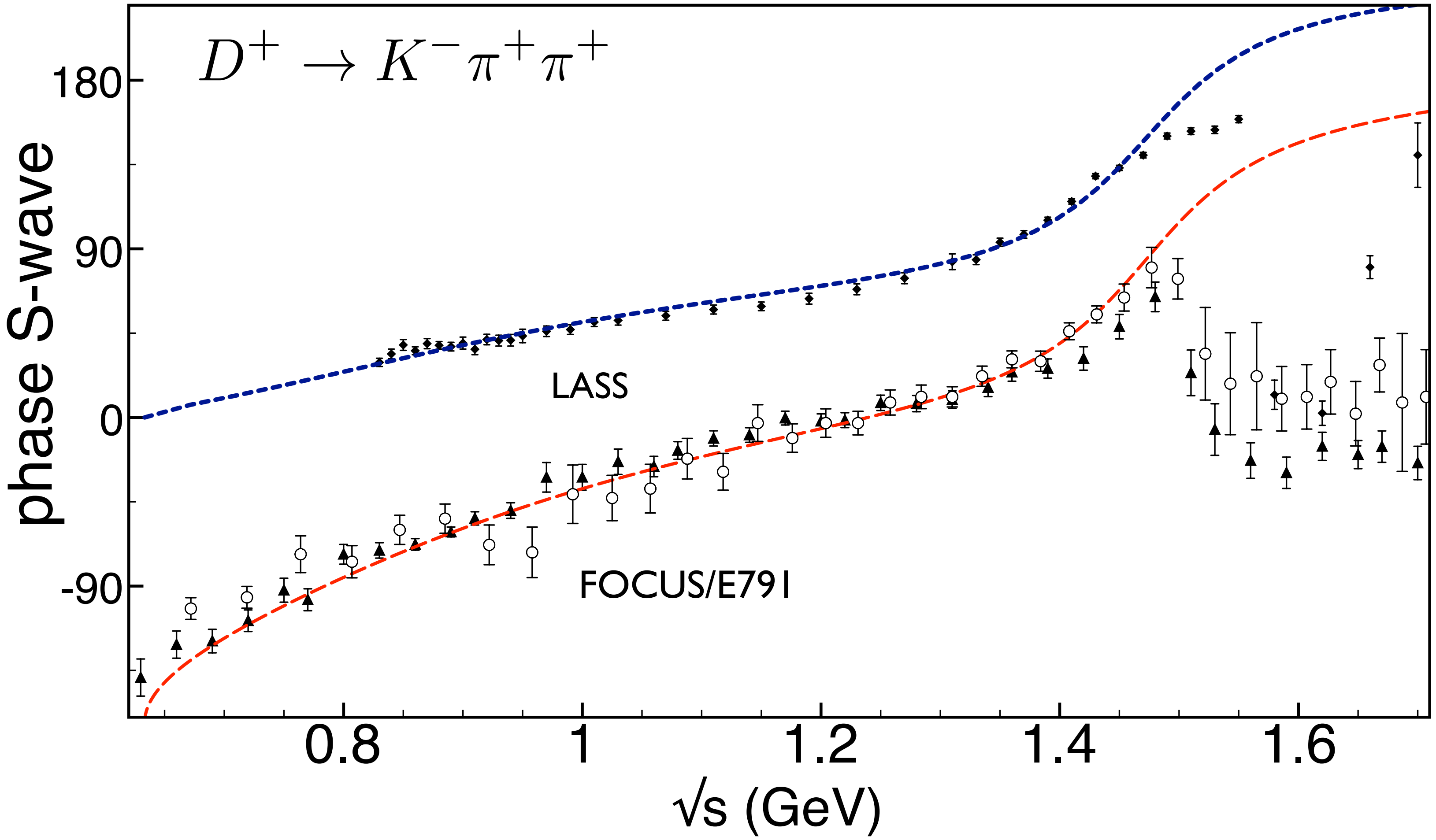
Models available



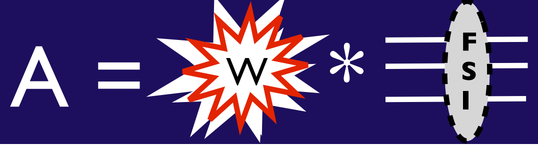
- Three-body FSI (beyond 2+1)



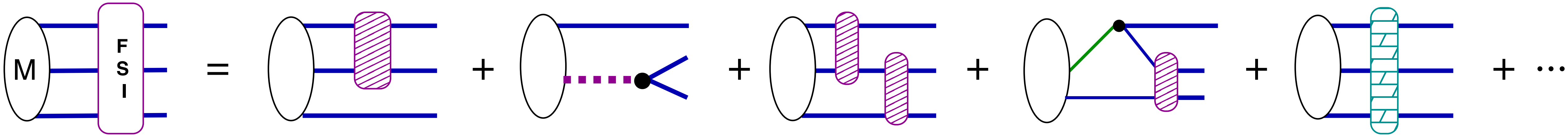
- shown to be relevant on charm sector



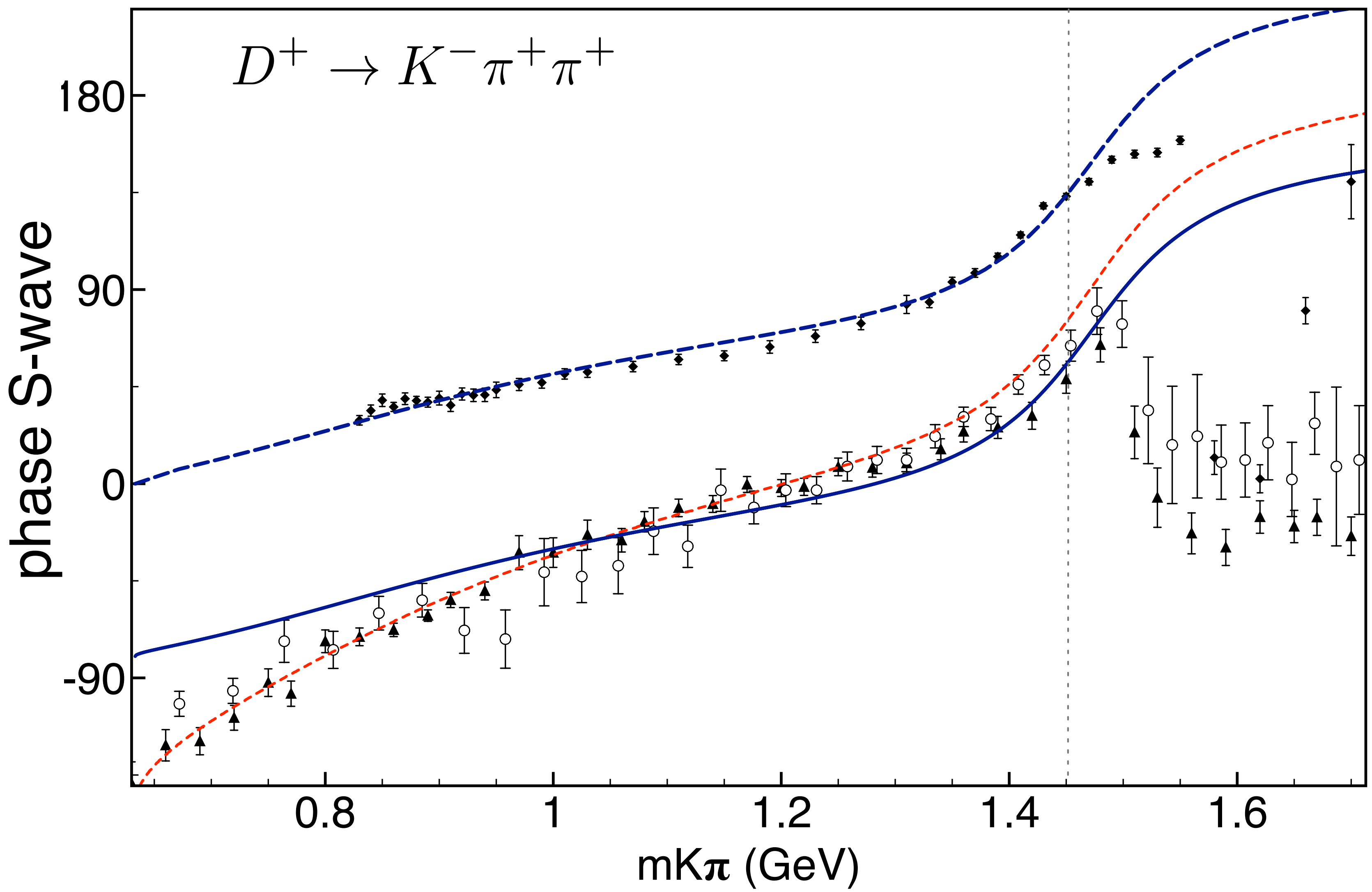
Models available



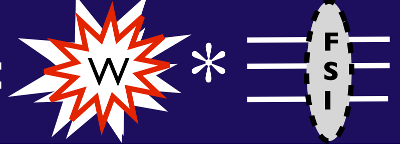
- Three-body FSI (beyond 2+1)



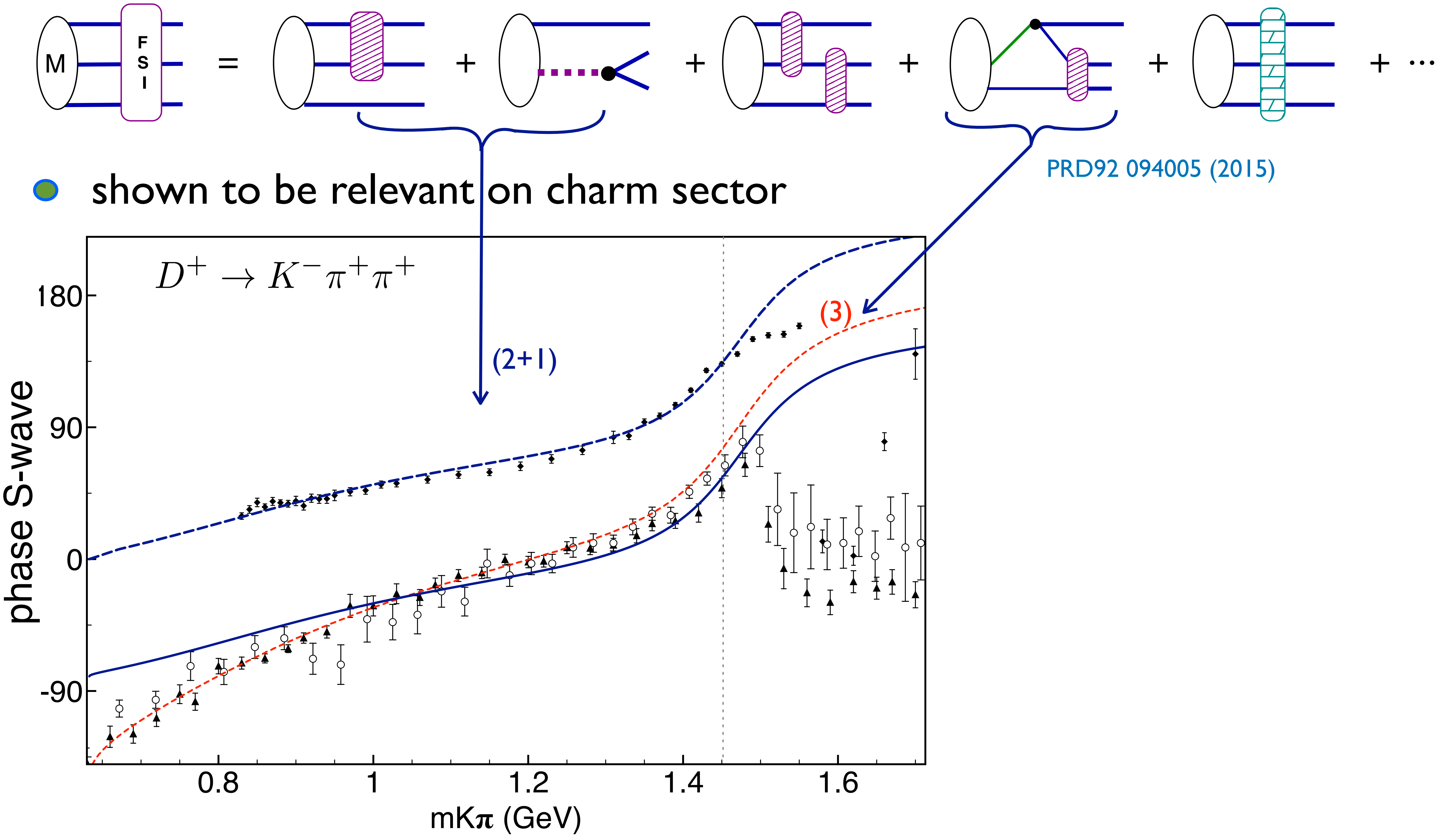
- shown to be relevant on charm sector



Models available

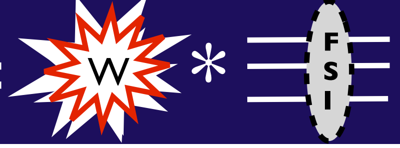
A = 

- Three-body FSI (beyond 2+1)



- shown to be relevant on charm sector

Models available

A = 

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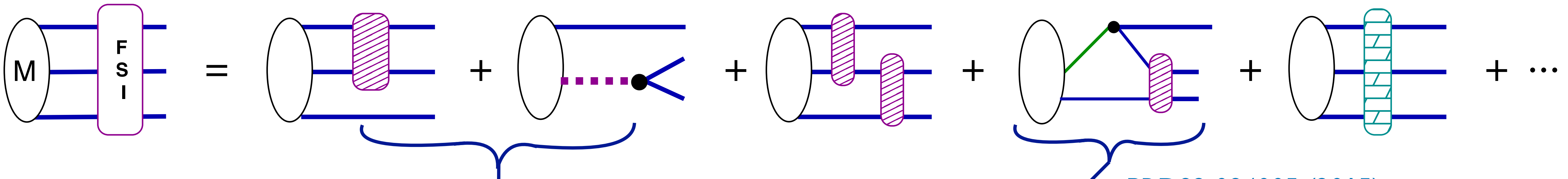
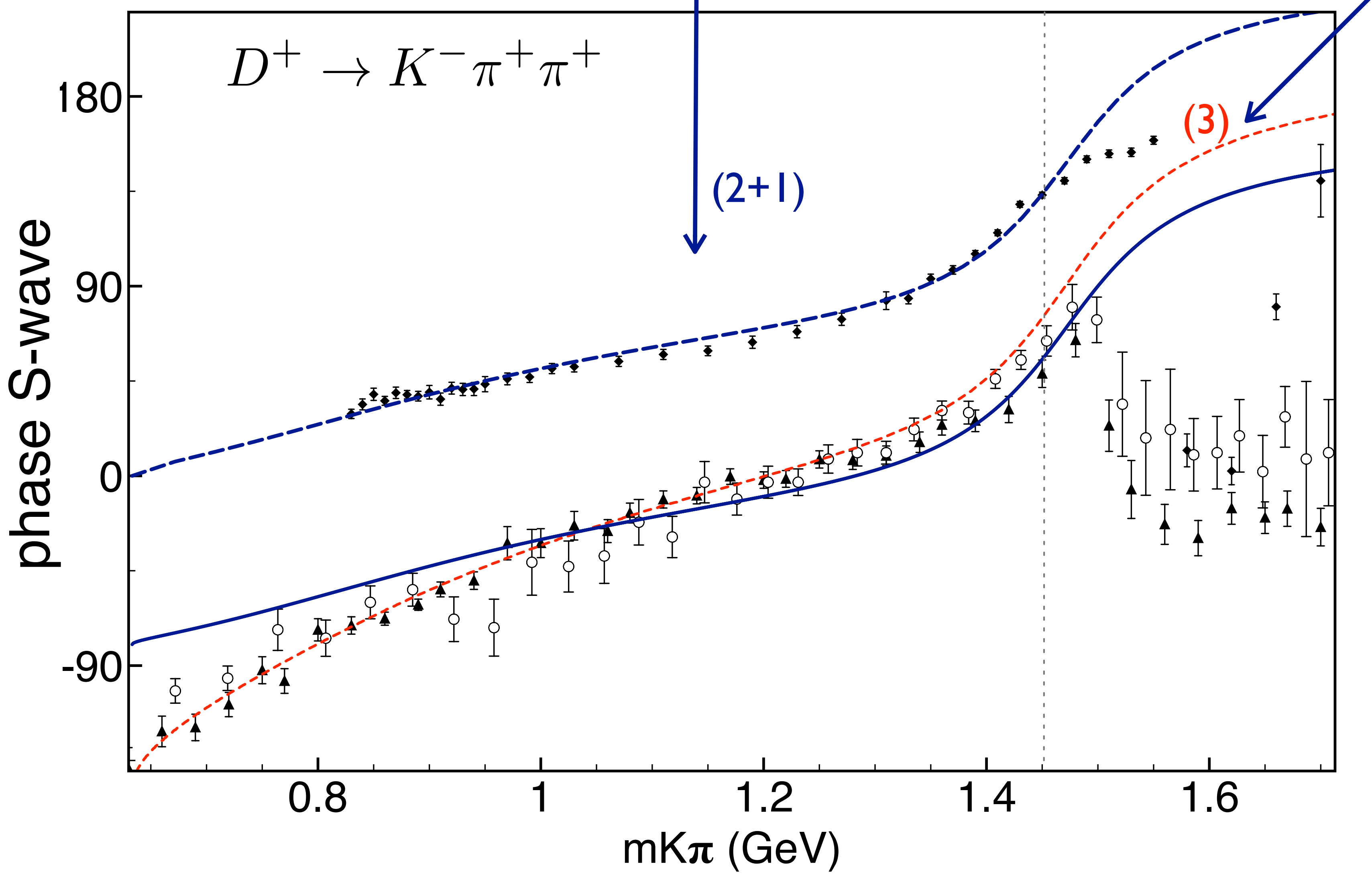
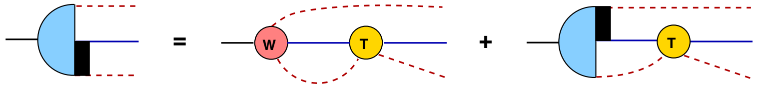


PRD92 094005 (2015)

- shown to be relevant on charm sector

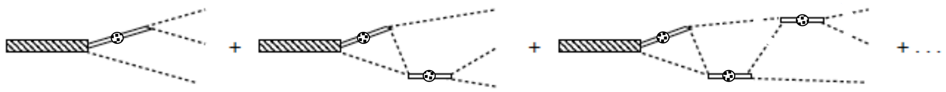


- Faddeev integral eq



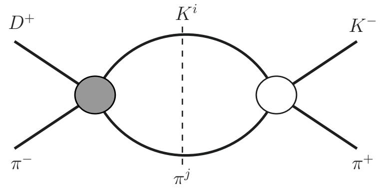
PCM et.al: PRD84 094001 (2011),

- triangle loops



S.Nakamura PRD93 014005 (2016)

- Khuri-Treiman



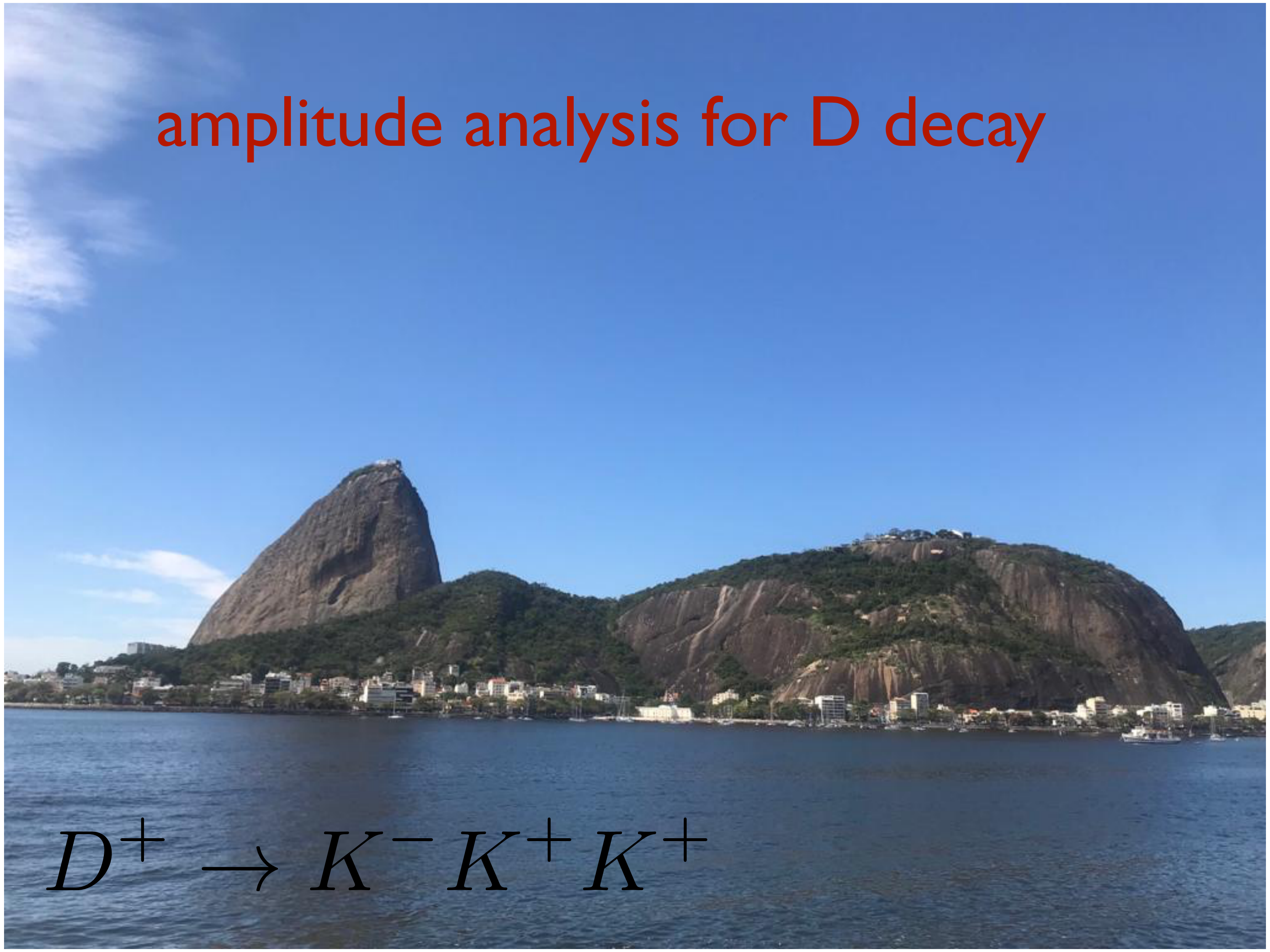
Niecknig, Kubis, JHEP10 142 (2015)

↳ most precise for E <1.2 GeV

↳ apply to $D^0 \to K^0\pi^+\pi^-$

Moussallam + Bonn

ex: multi meson model - $D^+ \to K^-K^+K^+$

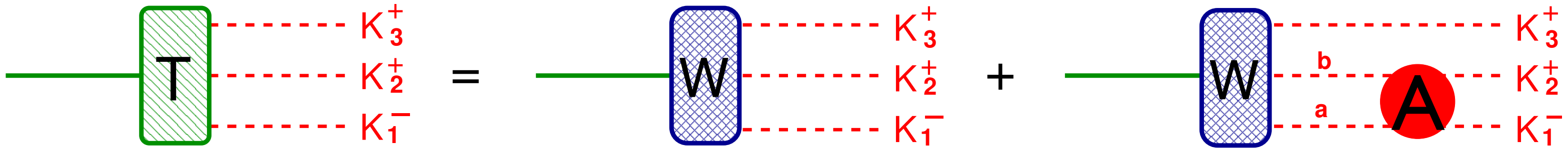


amplitude analysis for D decay

$D^+ \to K^-K^+K^+$

ex: multi meson model - $D^+ \to K^-K^+K^+$

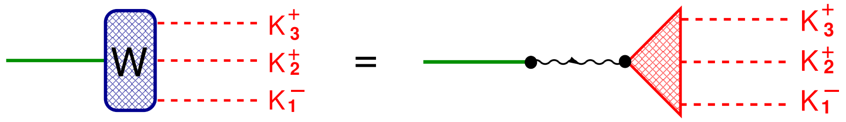
- Model for $D^+ \to K^-K^+K^-$



PCM, Aoude, dos Reis and Robilotta
PRD 98 056021 (2018)

→ A_{ab}^{JI} unitary scattering amplitude for $ab \to K^+K^-$

→ hypotheses that annihilation is dominant



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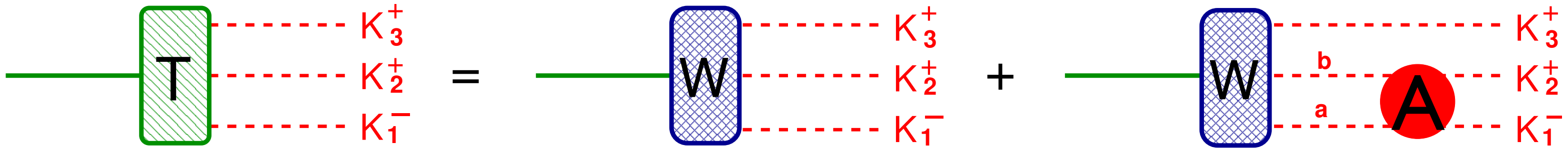

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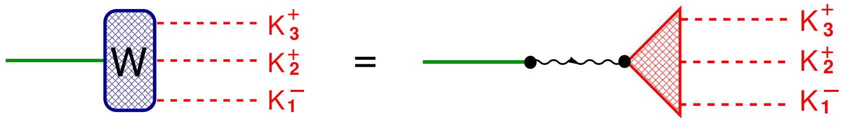


- separate the different energy scales:

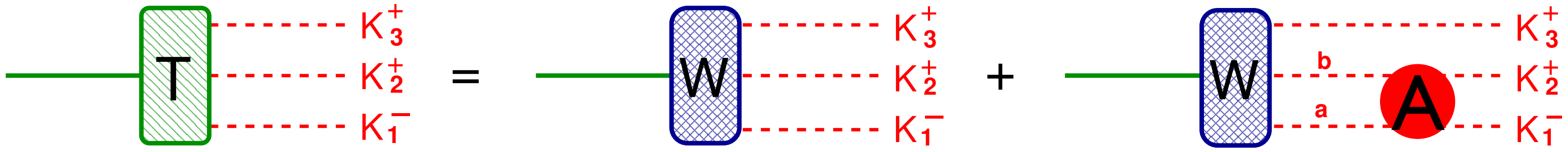
$$\mathcal{T} = \langle (KKK)^+|T|D^+\rangle = \underbrace{\langle (KKK)^+|A_\mu|0\rangle}_{\text{ChPT}} \; \langle 0|A^\mu|D^+\rangle .$$

$$\hookrightarrow \; -i\,G_F\,\sin^2\theta_C\,F_D\,P^\mu$$

→ parameters have physical meaning: resonance masses and coupling constants

ex: multi meson model - $D^+ \to K^- K^+ K^+$

- Model for $D^+ \to K^- K^+ K^-$



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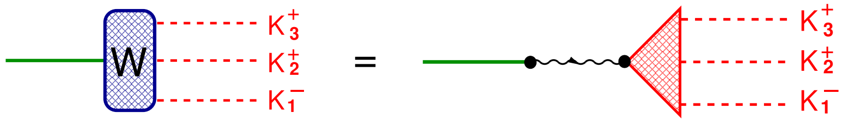

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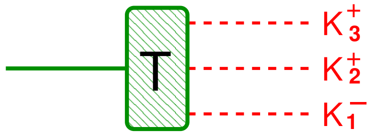
$$\mathcal{T} = \langle (KKK)^+ | T | D^+ \rangle = \underbrace{\langle (KKK)^+ | A_\mu | 0 \rangle}_{\text{ChPT}} \; \langle 0 | A^\mu | D^+ \rangle .$$

$$\hookrightarrow \; -i\,G_F\,\sin^2\theta_C\,F_D\,P^\mu$$

→ parameters have physical meaning: resonance masses and coupling constants

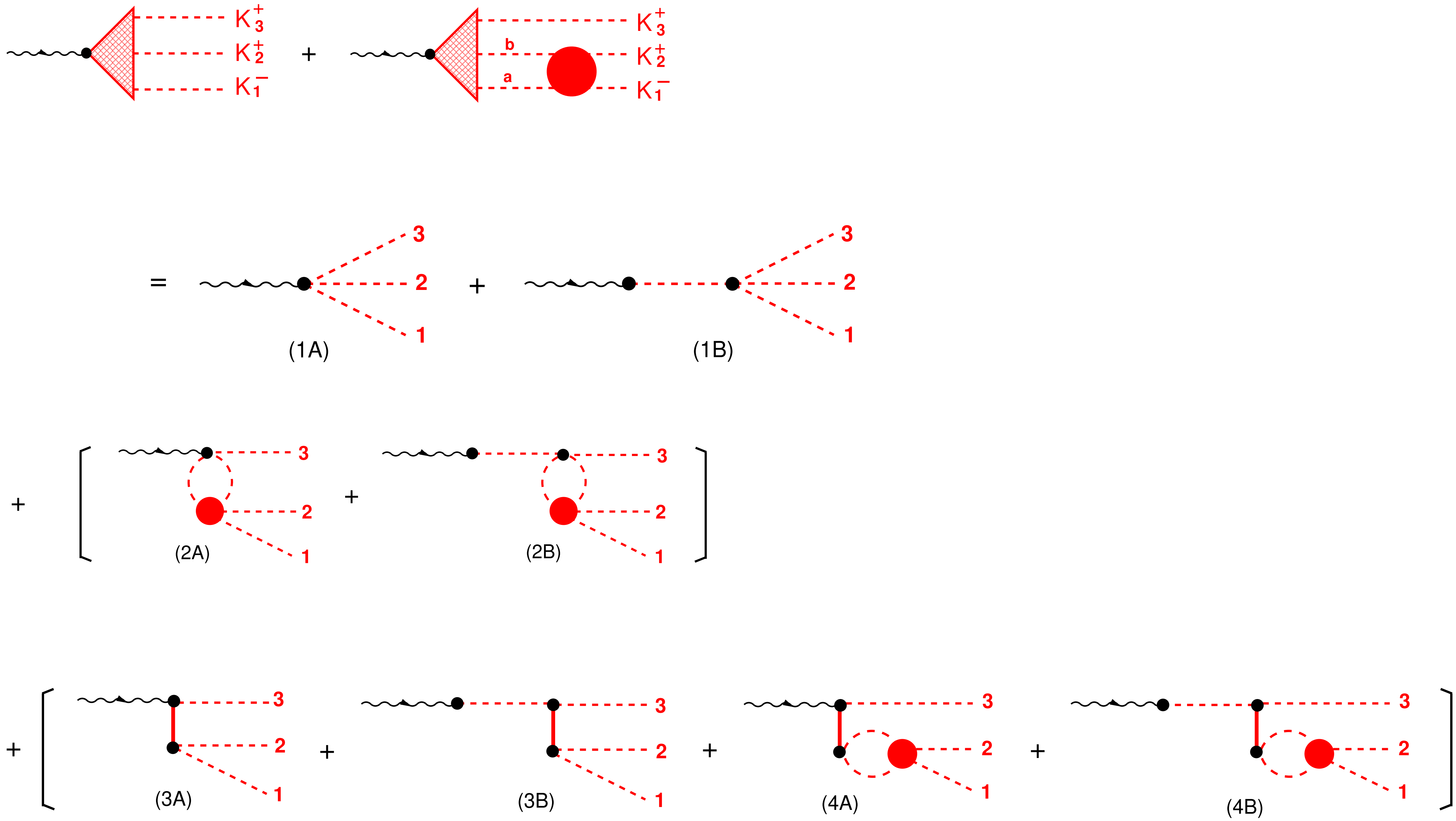
- alternative to isobar model in amplitude analysis



fitted to LHCb data → predict KK scattering amplitude

JHEP 1904 (2019) 063

Triple - M

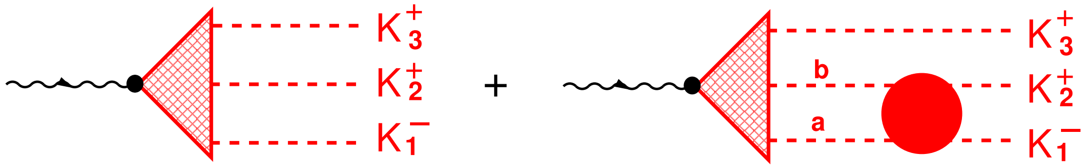


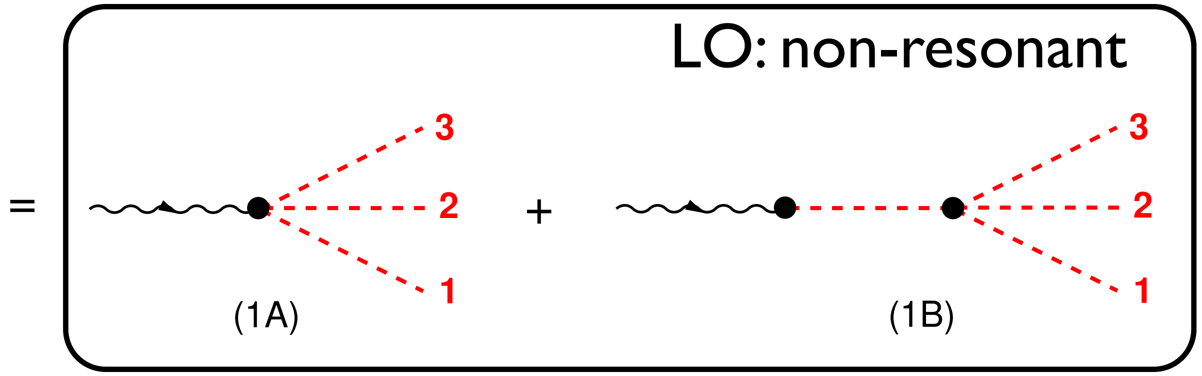
● $K\bar{K}$ coupled-channel unitary amplitude
$\pi\pi,\ \eta\eta,\ \pi\eta,\ \rho\pi$

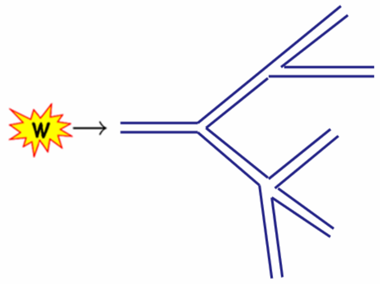
● isospin decomposition $[J, I = (0,1), (0,1)]$

p.magalhaes@bristol.ac.uk

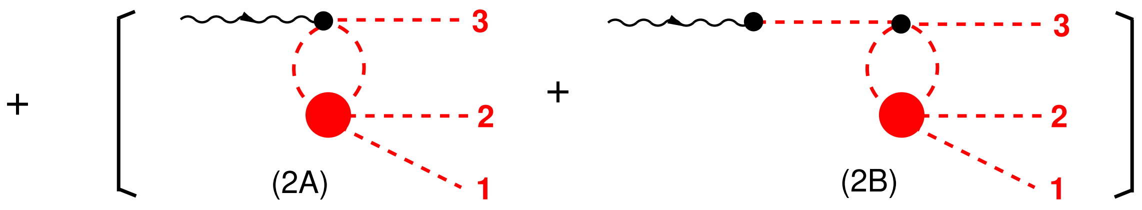
Triple - M

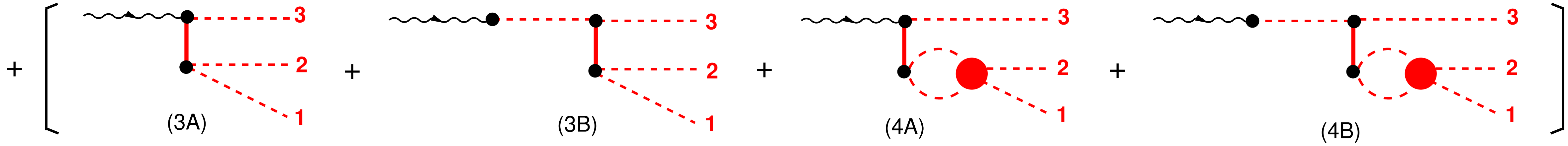



LO: non-resonant



Chiral symmetry





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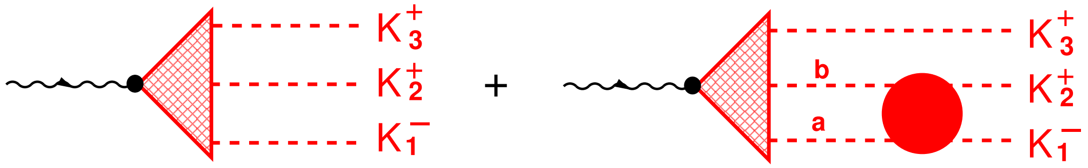
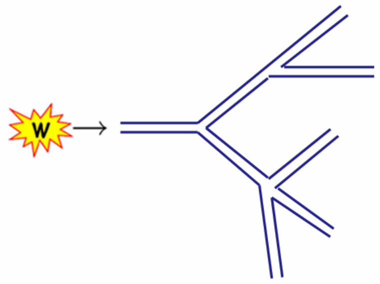
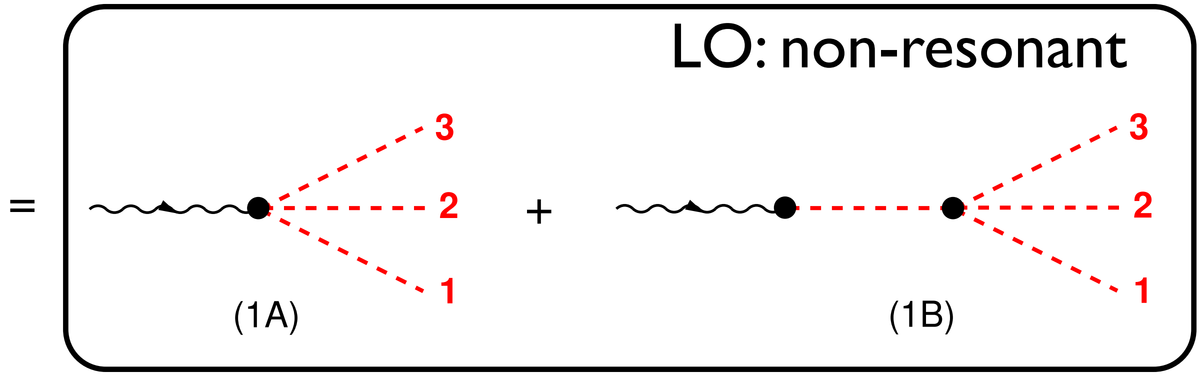
Triple - M





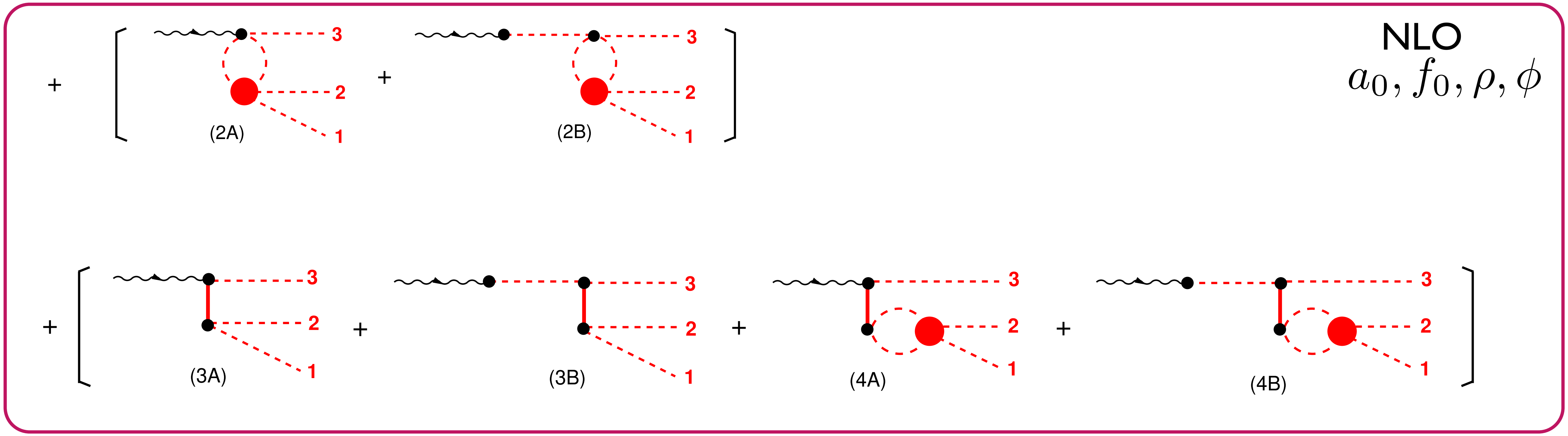
Chiral symmetry

LO: non-resonant



NLO

a_0, f_0, ρ, ϕ



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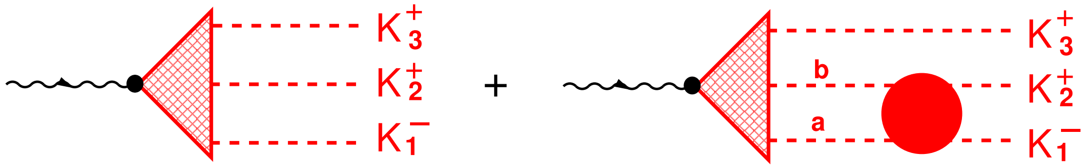
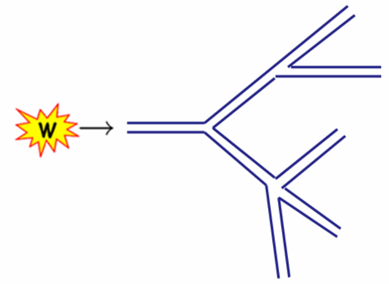
$\pi\pi,\ \eta\eta,\ \pi\eta,\ \rho\pi$

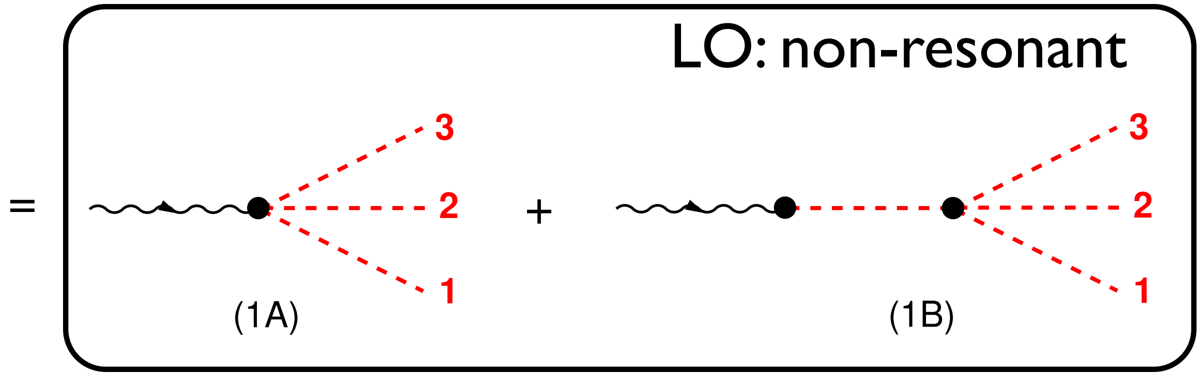
● isospin decomposition $[J, I = (0,1), (0,1)]$

Triple - M





Chiral symmetry



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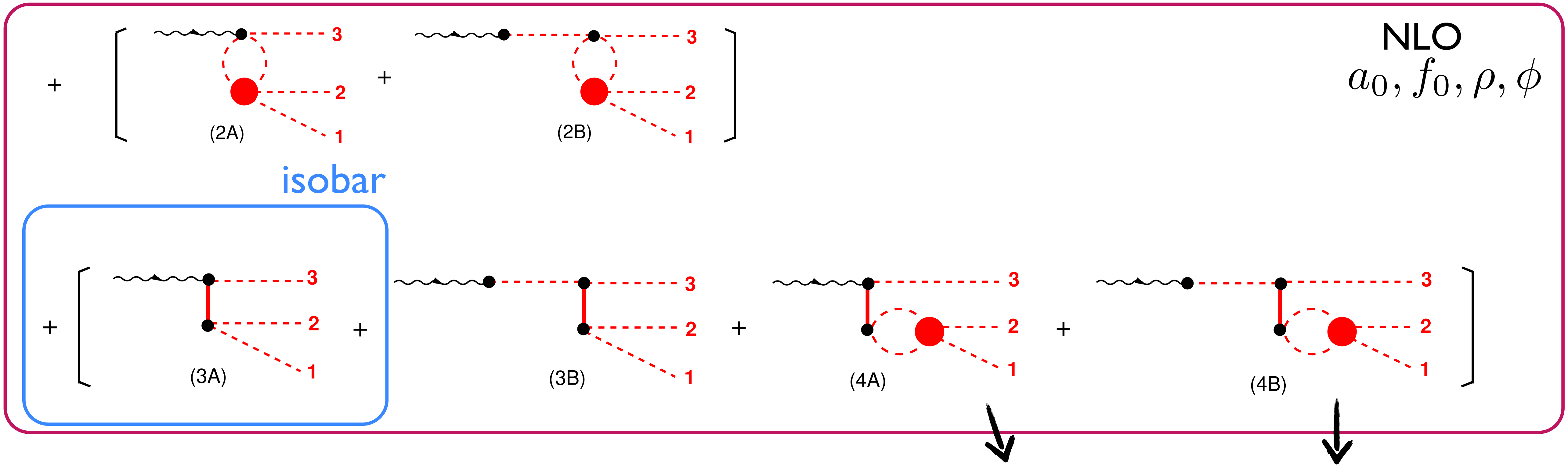
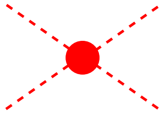


NLO

a_0, f_0, ρ, ϕ

isobar

width obtained through dynamics



$K\bar{K}$ coupled-channel unitary amplitude

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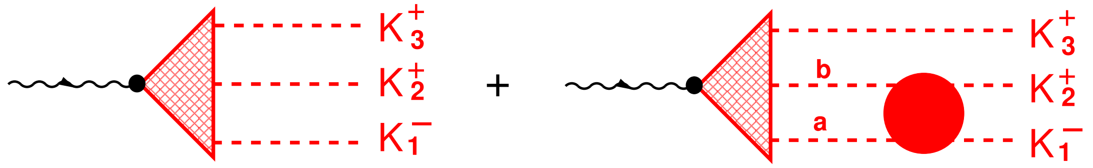
Triple M   LHCb fit



$$T^S = T^S_{NR} + T^{00} + T^{01}$$

$$T^P = T^P_{NR} + T^{11} + T^{10}$$

● χ^2/ndof = 1.12   (Isobar 1.14-1.6)

parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5$ MeV
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6$ MeV
m_{S_o}	$992.0^{+8.5}_{-7.5} \pm 8.6$ MeV
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1$ MeV
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51$ MeV
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9$ MeV
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3$ MeV
$\tilde{c}_d$	$-6.15^{+0.55}_{-0.54} \pm 0.19$ MeV
$\tilde{c}_m$	$-10.8^{+2.0}_{-1.5} \pm 0.4$ MeV

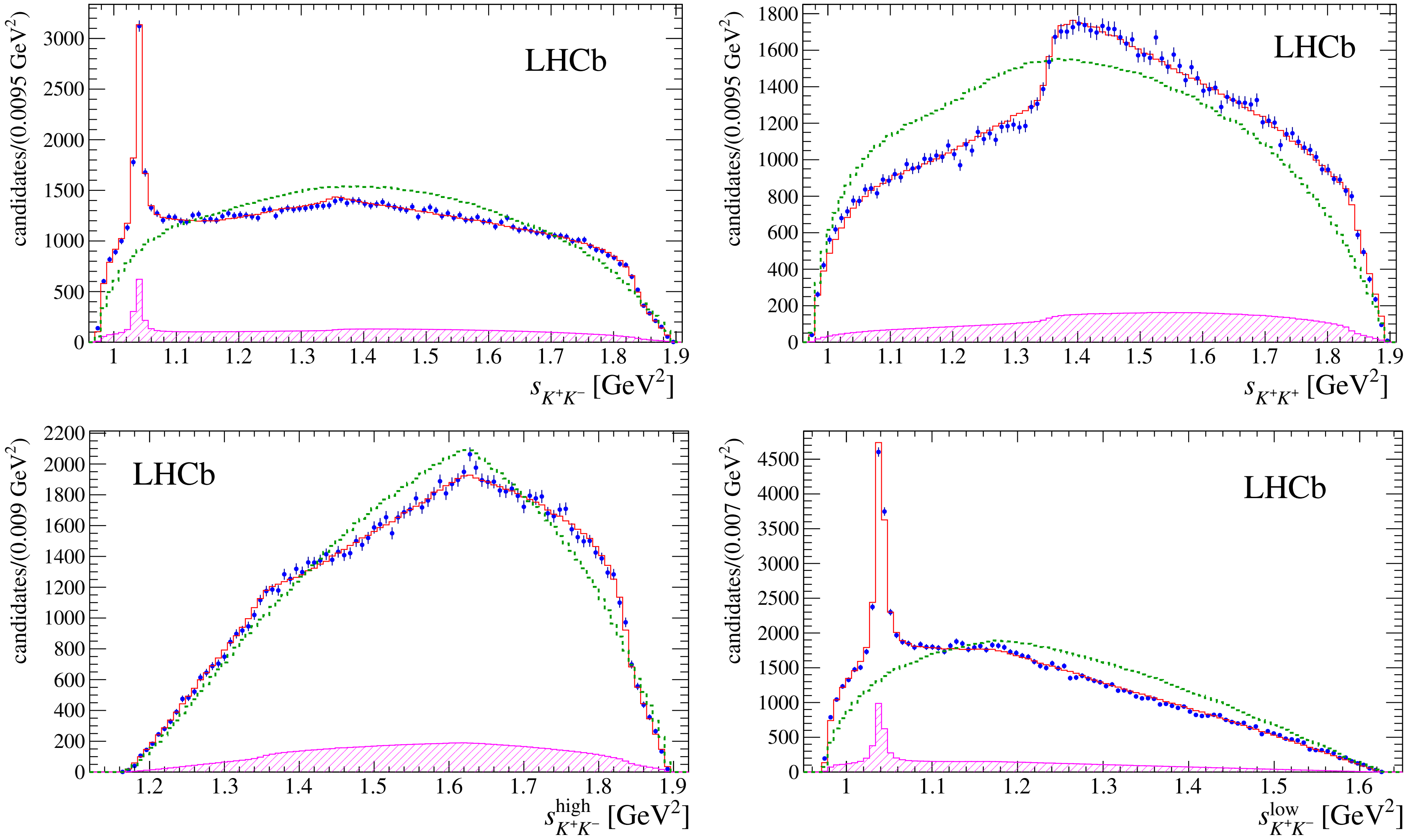
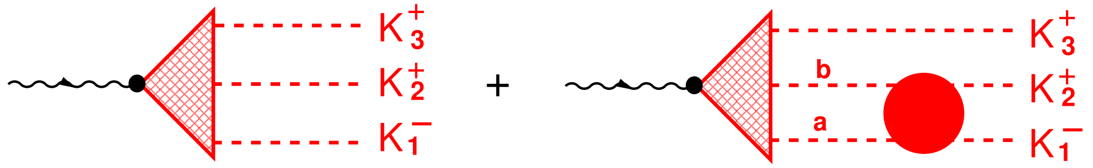


Figure 11. Projections of the Dalitz plot onto (top left) $s_{K^+K^-}$, (top right) $s_{K^+K^+}$, (bottom left) $s^{\mathrm{high}}_{K^+K^-}$ and (bottom right) $s^{\mathrm{low}}_{K^+K^-}$ axes, with the fit result with the Triple-M amplitude superimposed, whereas the dashed green line is the phase space distribution weighted by the efficiency. The magenta histogram represents the contribution from the background.

Triple M LHCb fit



$$T^S = T^S_{NR} + T^{00} + T^{01}$$

$$T^P = T^P_{NR} + T^{11} + T^{10}$$

● χ^2/ndof = 1.12 (Isobar 1.14-1.6)

→ parameters with physical meaning

parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5\,\mathrm{MeV}$
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6\,\mathrm{MeV}$
m_{S_o}	$992.0^{+8.5}_{-7.5} \pm 8.6\,\mathrm{MeV}$
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1\,\mathrm{MeV}$
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51\,\mathrm{MeV}$
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9\,\mathrm{MeV}$
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3\,\mathrm{MeV}$
$\tilde{c}_d$	$-6.15^{+0.55}_{-0.54} \pm 0.19\,\mathrm{MeV}$
$\tilde{c}_m$	$-10.8^{+2.0}_{-1.5} \pm 0.4\,\mathrm{MeV}$

→ can disentangle a_0 and f_0

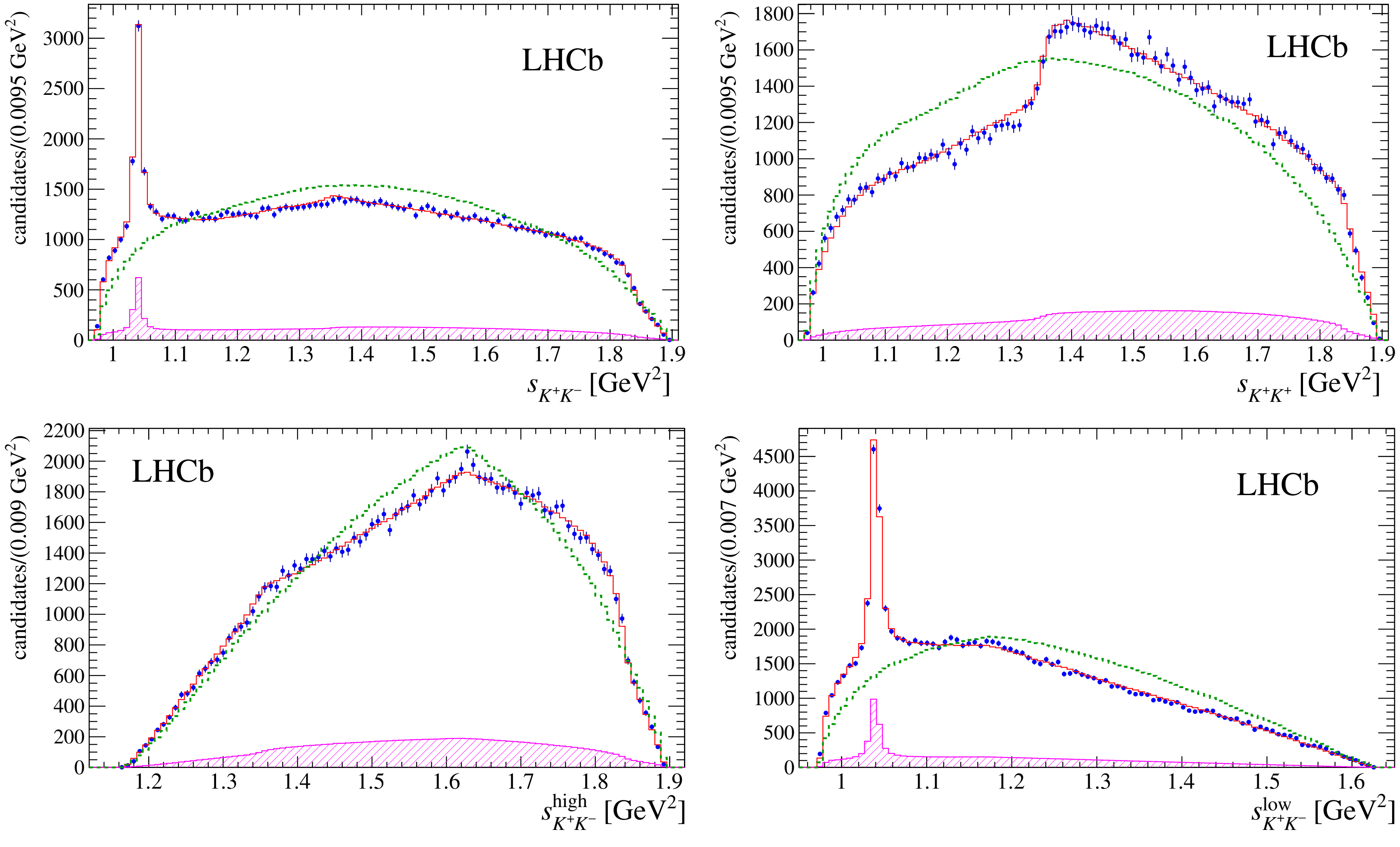
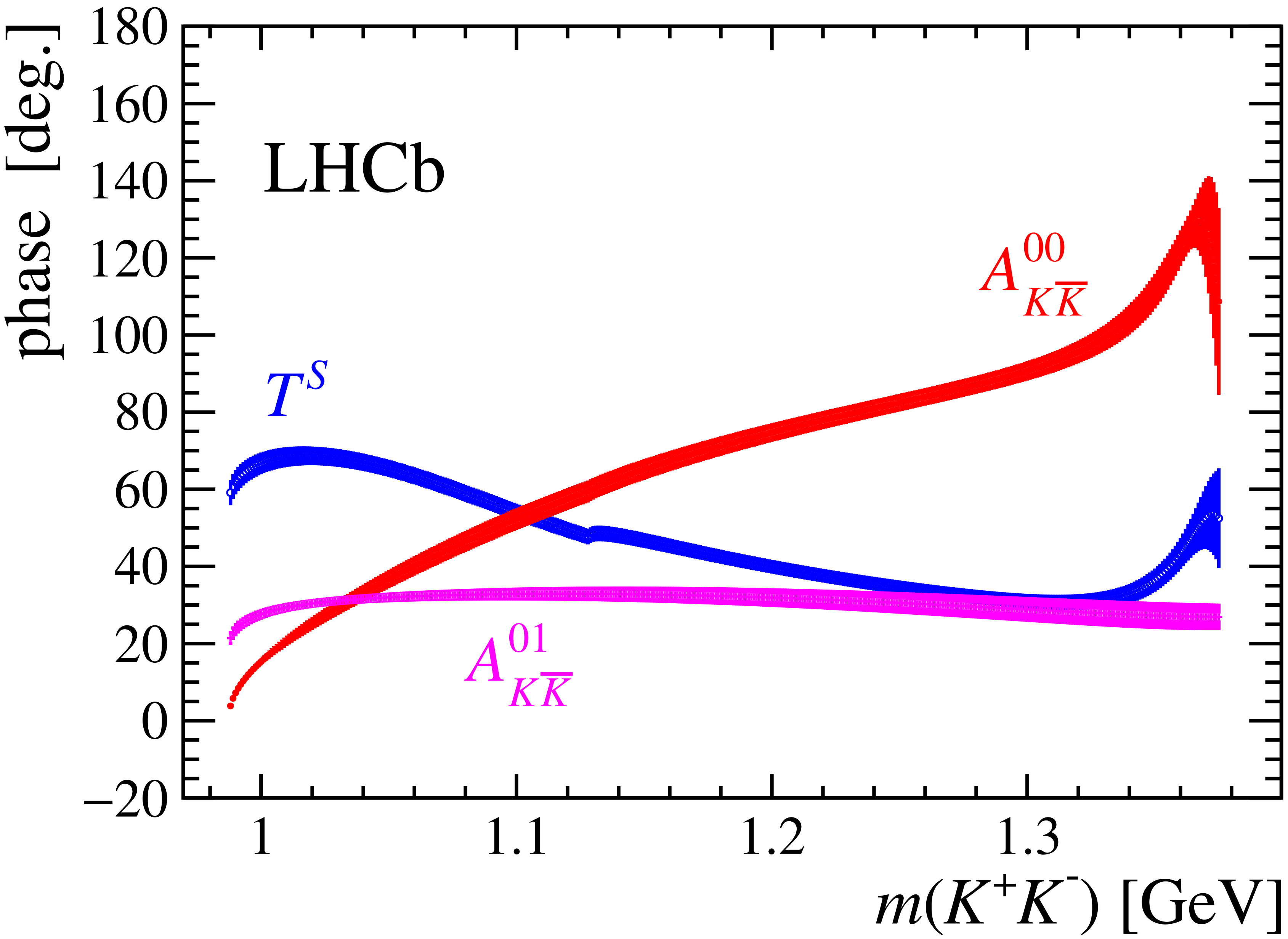


Figure 11. Projections of the Dalitz plot onto (top left) $s_{K^+K^-}$, (top right) $s_{K^+K^+}$, (bottom left) $s^{\mathrm{high}}_{K^+K^-}$ and (bottom right) $s^{\mathrm{low}}_{K^+K^-}$ axes, with the fit result with the Triple-M amplitude superimposed, whereas the dashed green line is the phase space distribution weighted by the efficiency. The magenta histogram represents the contribution from the background.

Triple M   LHCb fit S-wave

- intensity of each component is predict by theory
- 3-body amplitude $\neq$ from 2-body



→ couple channel structure cannot be ignored

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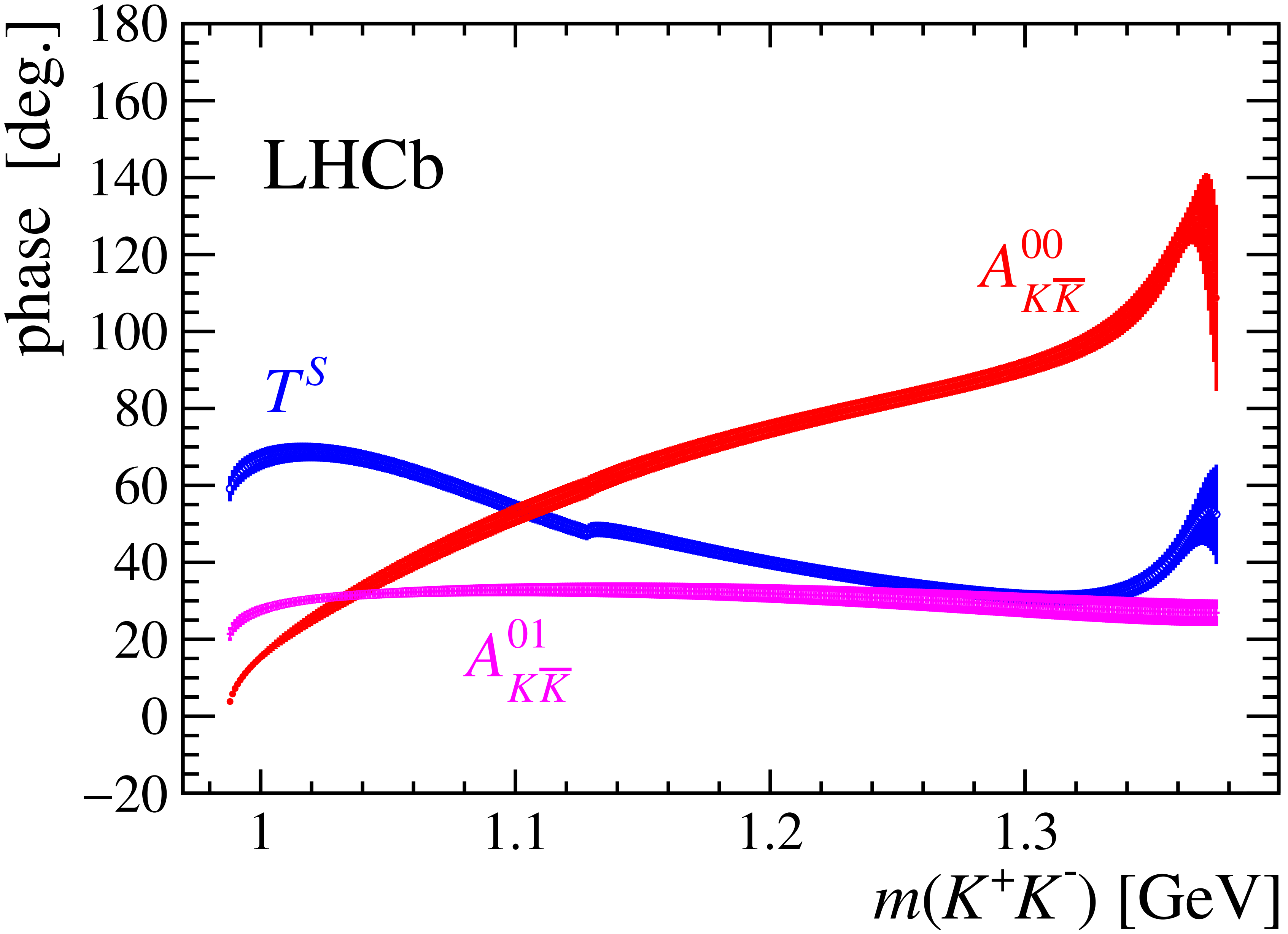
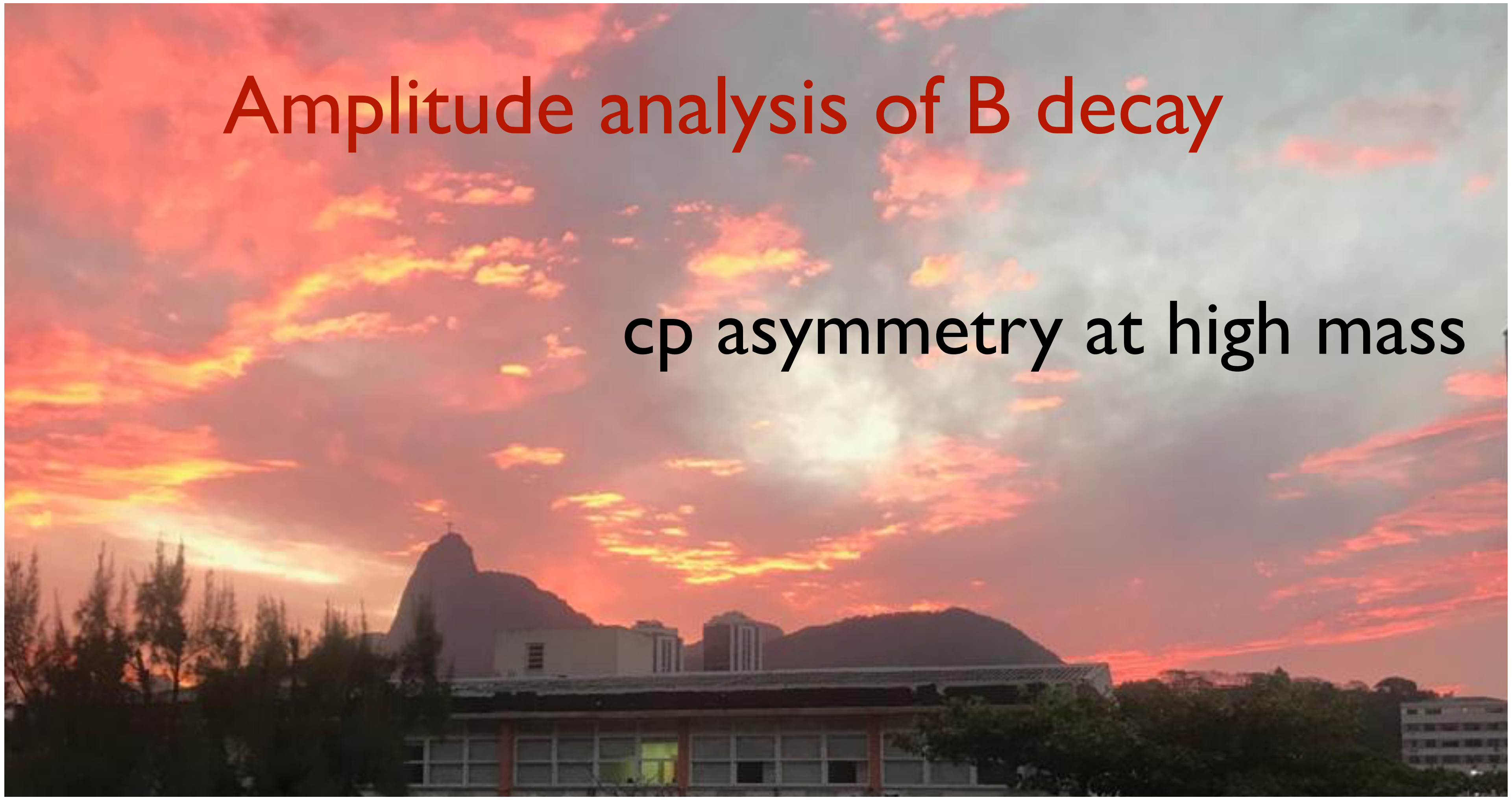


→ couple channel structure cannot be ignored

→ to extract 2-body need a full model

→ predict KK scattering amplitude to be used in other process

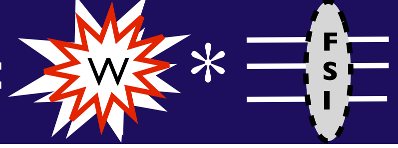
example for B decays

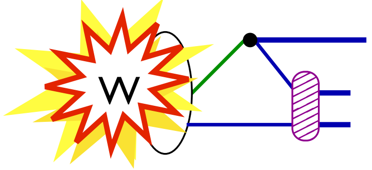


Amplitude analysis of B decay

cp asymmetry at high mass

Models available

A = 



- FSI on B decays

- CPV needs:

→ 2 interfering amplitudes

→ 2 ≠ strong phases $[\sin(\delta_1 - \delta_2) \neq 0]$

→ 2 ≠ weak phases $[\sin(\phi_1 - \phi_2) \neq 0]$

Models available

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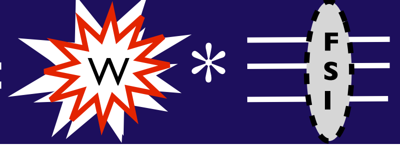

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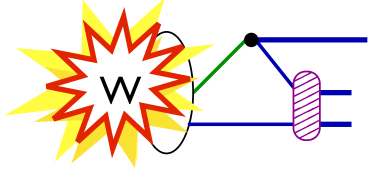

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hadronic FSI

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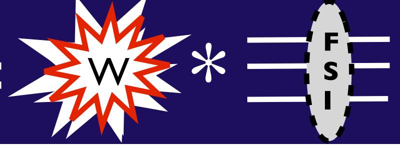
Models available A = 

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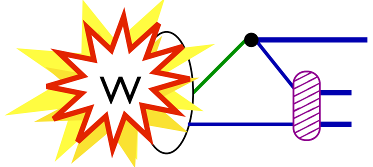

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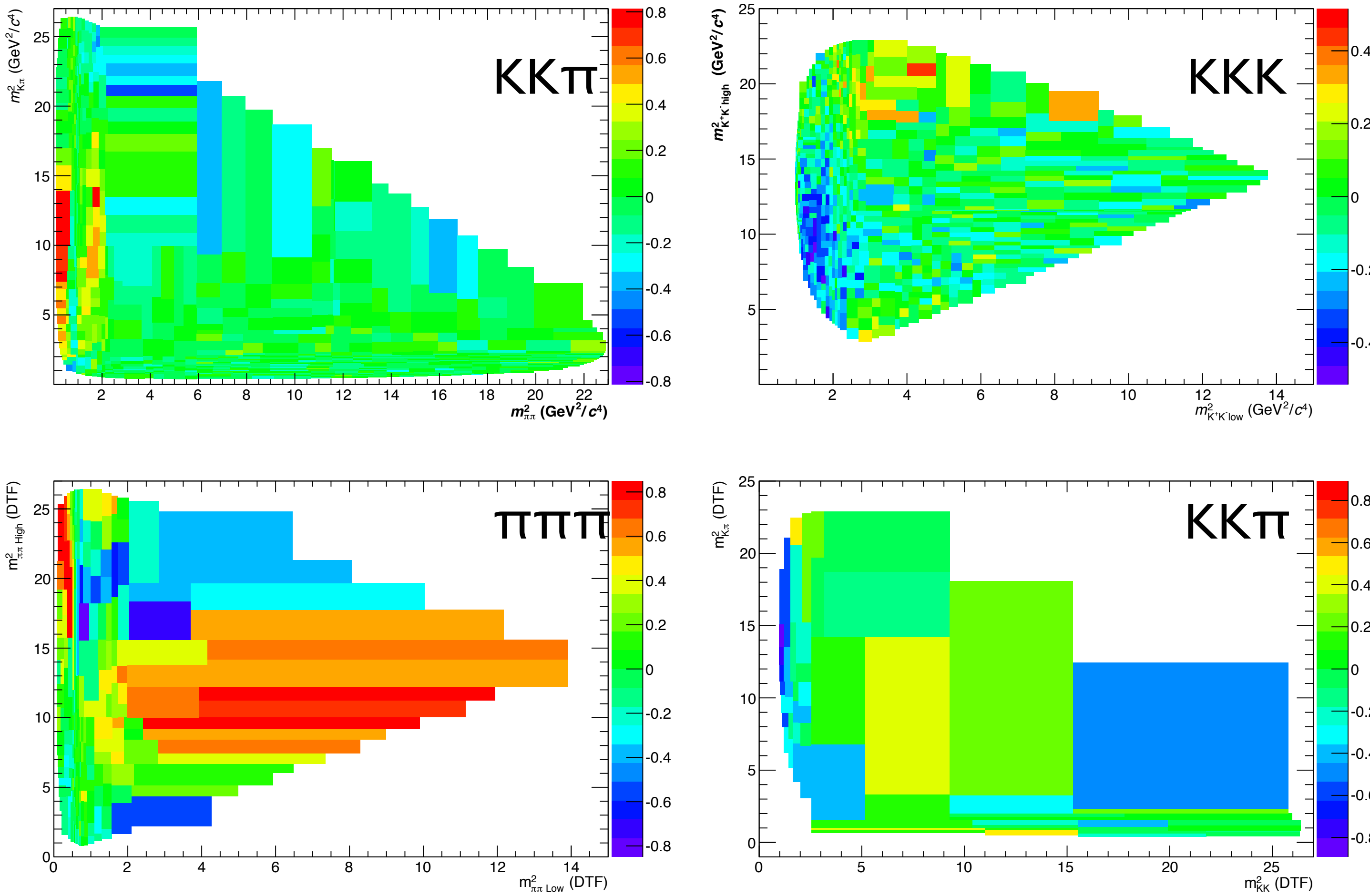
- $B^\pm \to h^\pm h^- h^+$ CP violation puzzle

↳ middle with no resonance
but have CPV

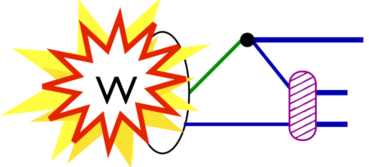
- $\neq$ mechanisms for low-energy CPV

ex: $B^\pm \to \pi^\pm \pi^- \pi^+$ Wen Bin talk



LHCb PRD90 (2014) 112004

rescattering as a CPV mechanism



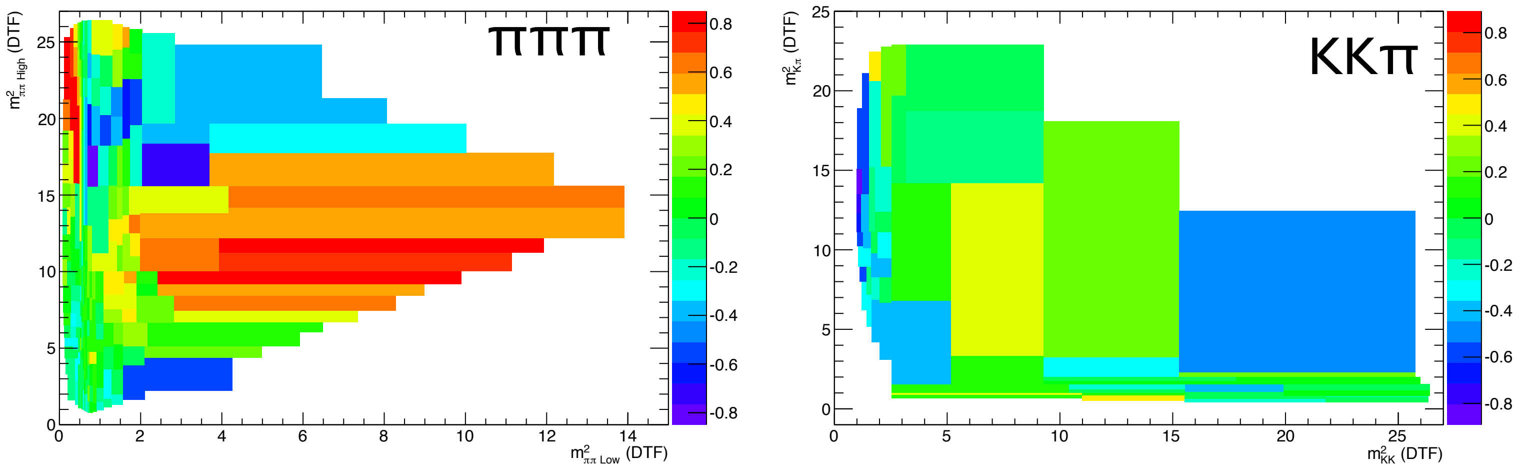
- rescattering $\pi\pi \to KK$

↳ CPV [1 - 2] GeV

Frederico, Bediaga, Lourenço
PRD89(2014)094013

↳ FSI as strong phase

Wolfenstein PRD43 (1991) 151



LHCb PRD90 (2014) 112004

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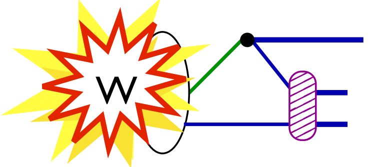

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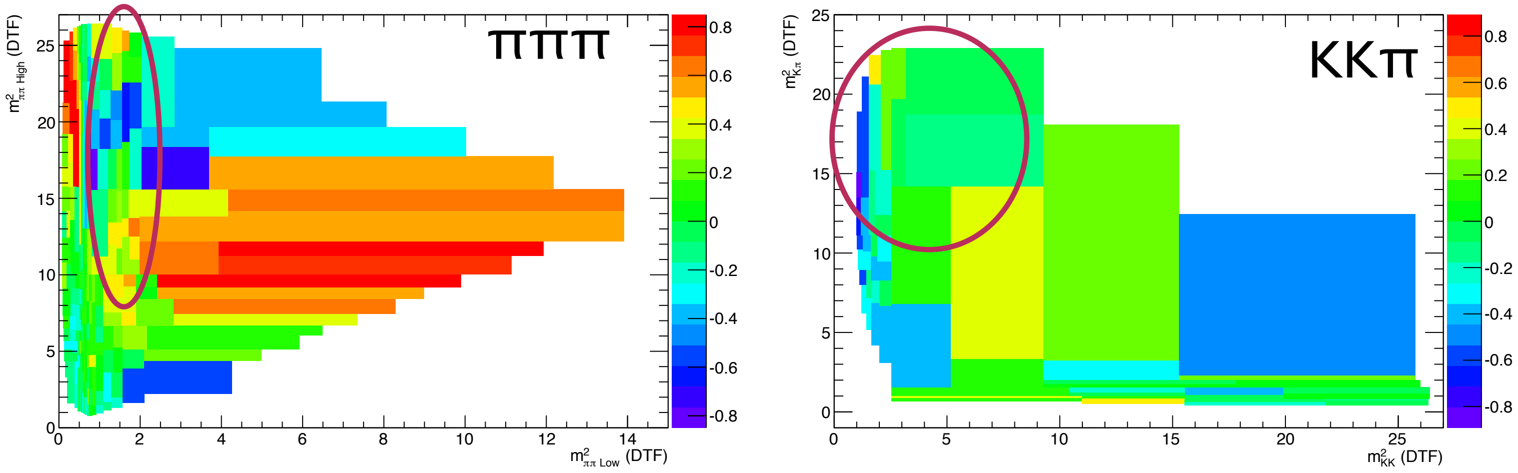

LHCb PRD90 (2014) 112004

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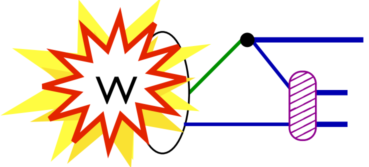

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PRD89(2014)094013

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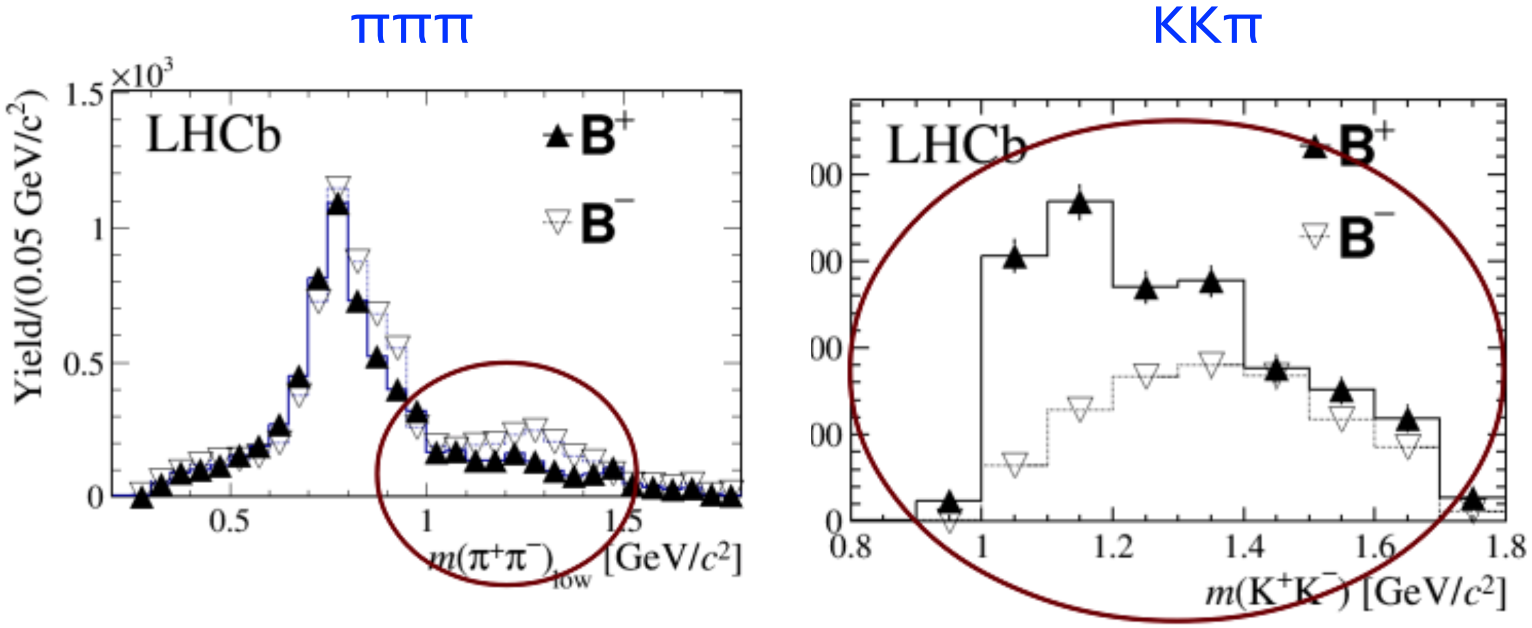
ππππ KKπ



LHCb PRD90 (2014) 112004

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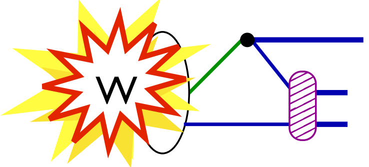

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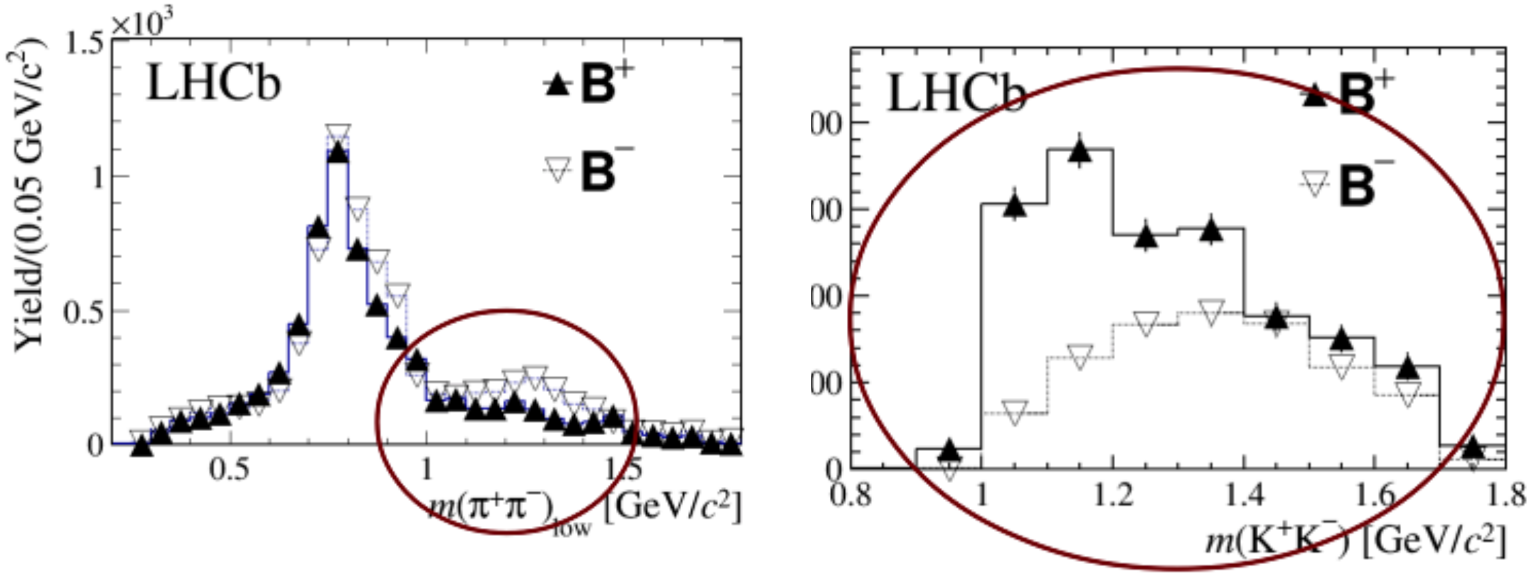


LHCb PRD90 (2014) 112004

- CPT must be preserved

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \overline{\Gamma}_{total}$

$$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$$

$$\overline{\Gamma}_{total} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \ldots$$

→ CPV in one channel should be compensated by another one with opposite sign

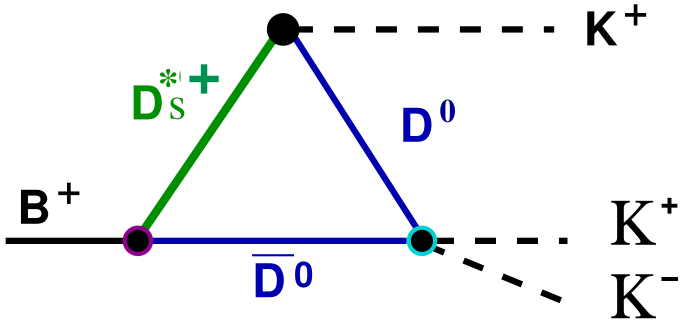
→ rescattering contribution for CPV confirmed by LHCb analysis Misha’s talk

charm rescattering contribution

- CPV at high mass?

→ charm intermediate processes as source of strong phase

PCM, I. Bediaga, T Frederico PLB 780 (2018) 357



- $D^0\bar{D}^0 \to K^+K^-$ phenomenological amplitude

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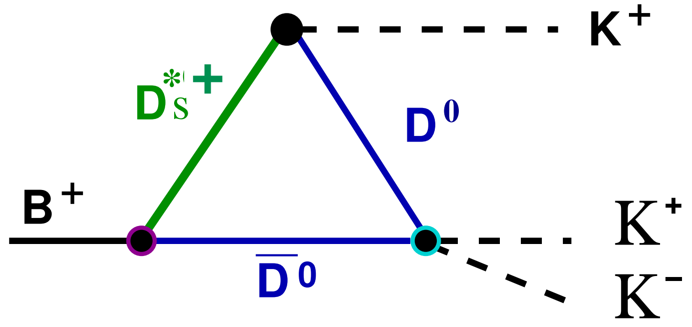

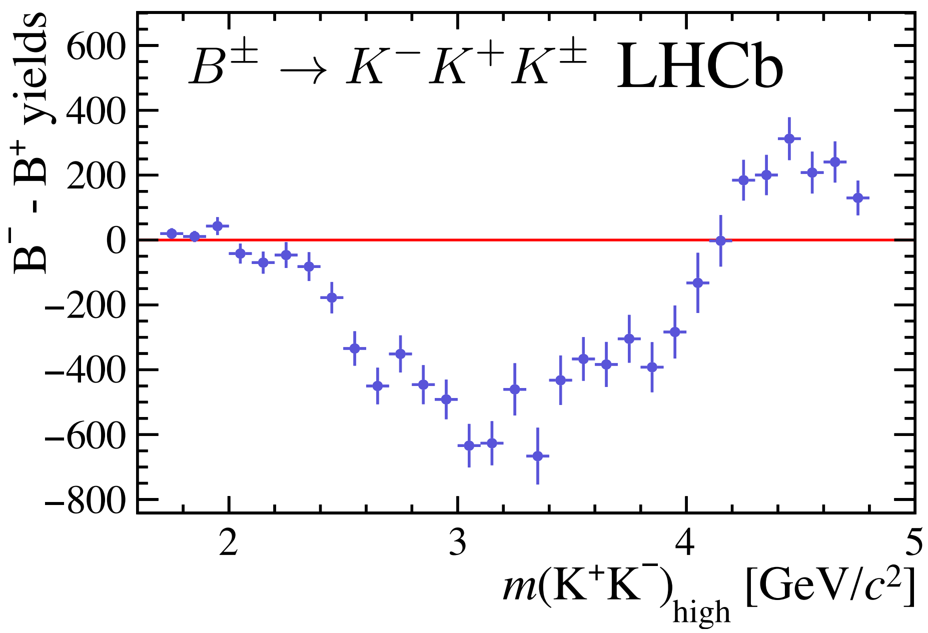

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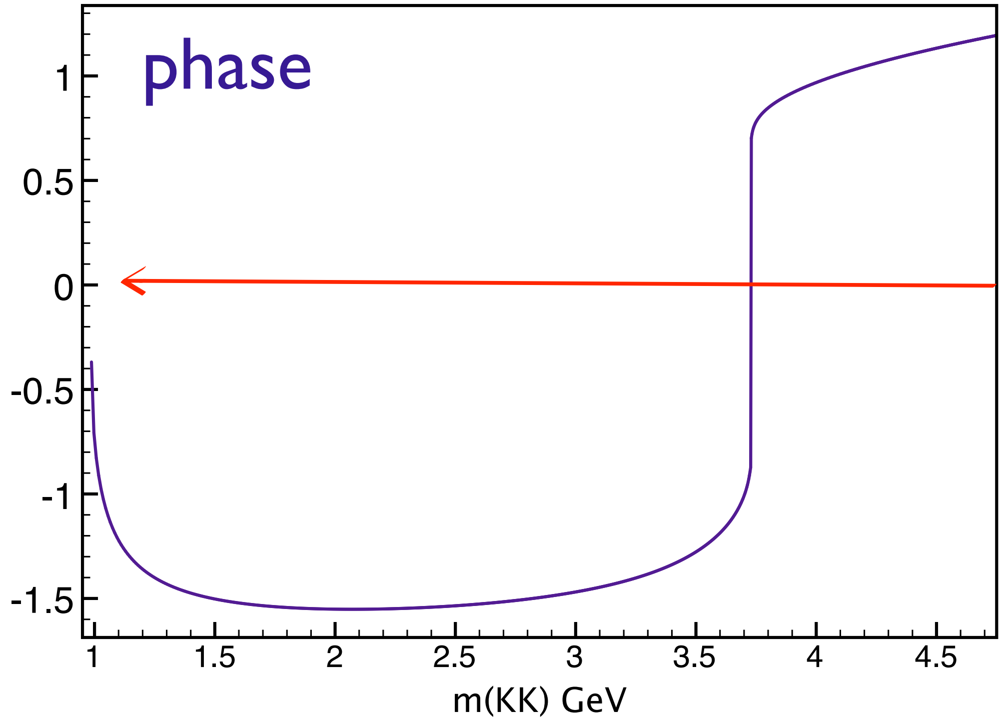

→ change signal in the same region as Acp data

→ charm loops can be a mechanism to generate CPV E ~ 14 GeV

charm rescattering contribution

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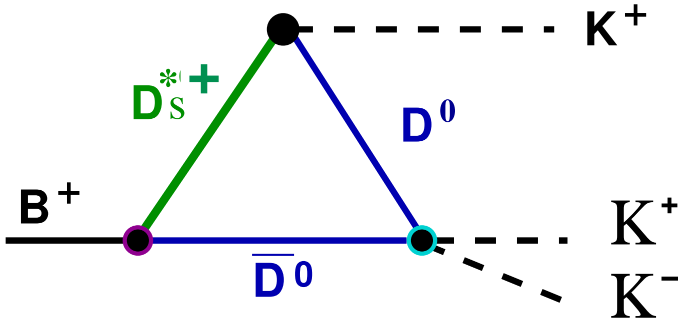
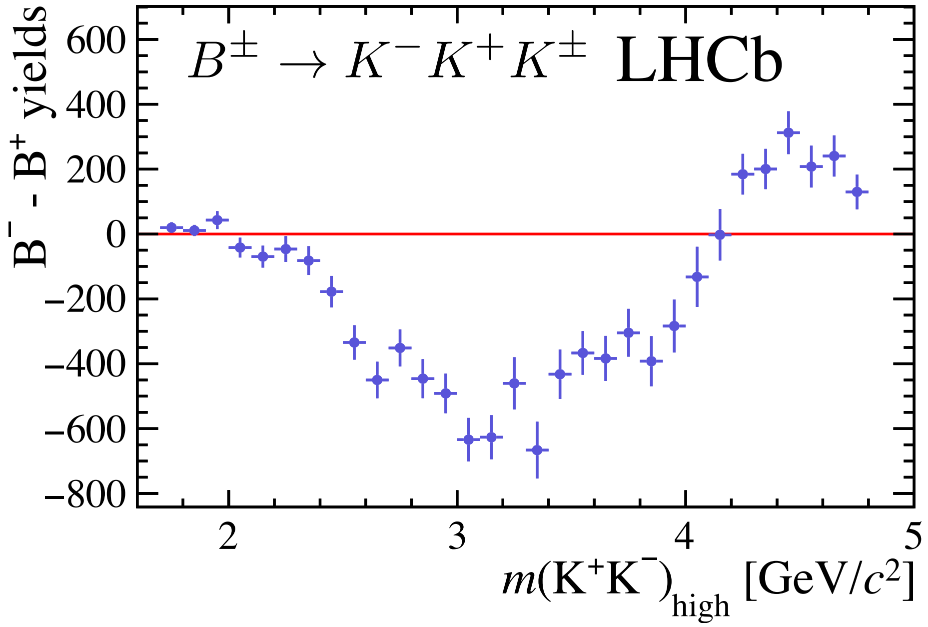
PCM, I. Bediaga, T Frederico PLB 780 (2018) 357



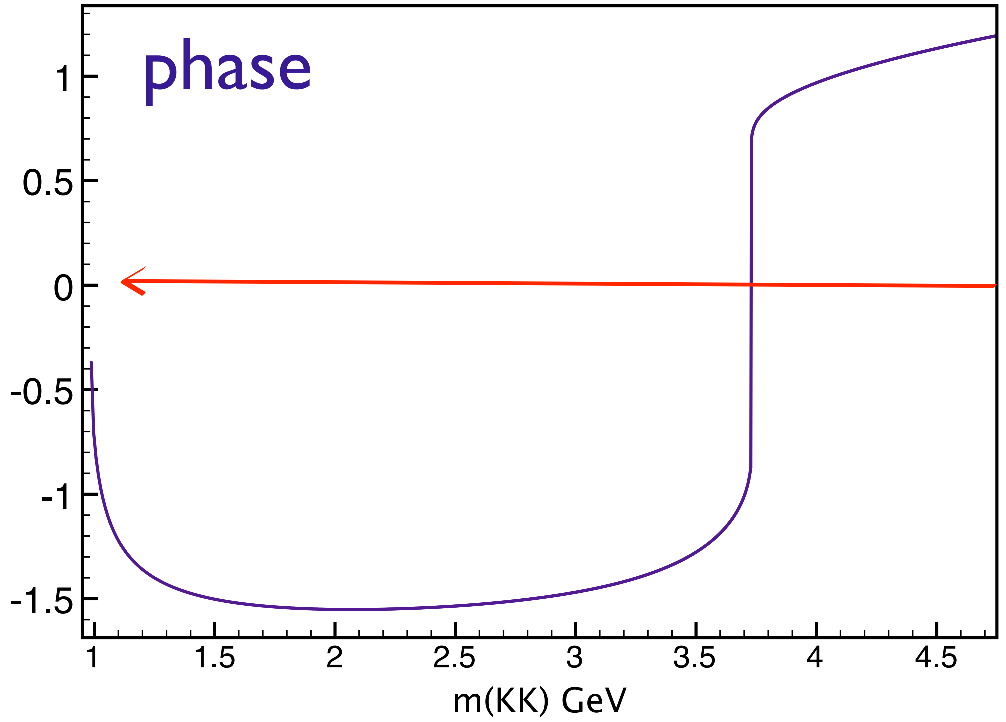


- $D^0 \bar{D}^0 \to K^+ K^-$ phenomenological amplitude

- charm FSI: $B \to 3h$, $B_c \to 3h$, $B \to K^* \mu\mu$, ...
  - $B_c^+ \to K^- K^+ \pi^+$

production mechanism

PCM, I. Bediaga, T Frederico
PLB 785 (2018) 581



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Final remarks

→ two-body unitary, coupled-channel description in mandatory

→ FSI play an important role in B/D hadronic decays

↳ B decays —> understand of CPV, low and high mass,

↳ D decays —> 3-body effects, extract 2-body information from data, CPV?

→ Triple M : theory/experimental joint work

→ models need to connect the weak and strong description

↳ QCDF and FSI

on going project…

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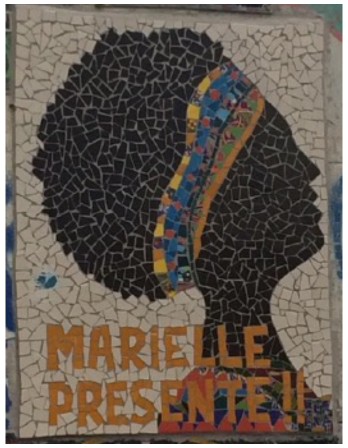


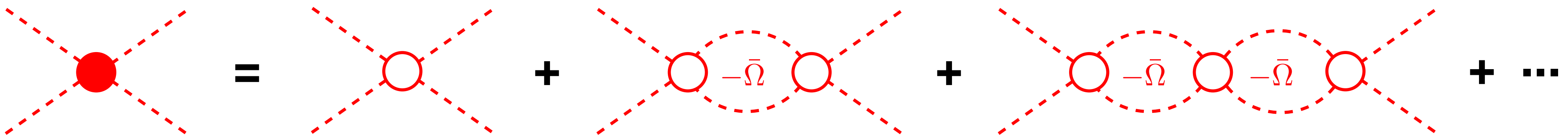
Thank you very much

p.magalhaes@bristol.ac.uk

Extra slides

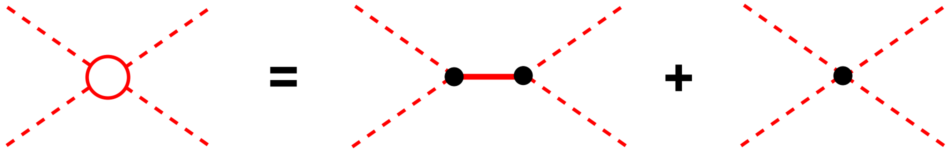
unitarized amplitude $P^a P^b \to P^c P^d$

- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]



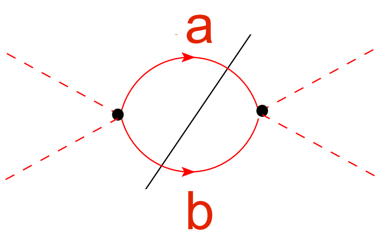
$$\boxed{\mathcal{A}^{JI}_{ab} = \frac{\mathcal{K}^{(JI)}_{ab\to cd}}{1+\bar\Omega_{ab}\,\mathcal{K}^{(JI)}_{ab\to cd}}}$$

  - kernel $\mathcal{K}^{(J,I)}_{ab\to cd}$



resonance (NLO) + contact (LO)

  - loops → K-matrix approximation: only on-shell



$$\{I_{ab};\, I^{\mu\nu}_{ab}\} = \int \frac{d^4\ell}{(2\pi)^4}\,\frac{\{1;\, \ell^\mu\,\ell^\nu\}}{D_a\,D_b}$$

$$D_a = (\ell+p/2)^2 - M_a^2 \qquad D_b = (\ell-p/2)^2 - M_b^2$$

→

$$\bar\Omega^S_{ab} = -\,\frac{i}{8\pi}\,\frac{Q_{ab}}{\sqrt{s}}\;\theta(s-(M_a+M_b)^2)$$

$$\bar\Omega^P_{aa} = -\,\frac{i}{6\pi}\,\frac{Q^3_{aa}}{\sqrt{s}}\;\theta(s-4\,M_a^2)$$

$$Q_{ab} = \frac{1}{2}\sqrt{s-2\,(M_a^2+M_b^2)+(M_a^2-M_b^2)^2/s}$$

- free parameters

  - masses:

$m_\rho\,,\; m_{a_0}\,,\; \boxed{m_{s0}\,,\; m_{s1}}$ $SU(3)$ singlet and octet

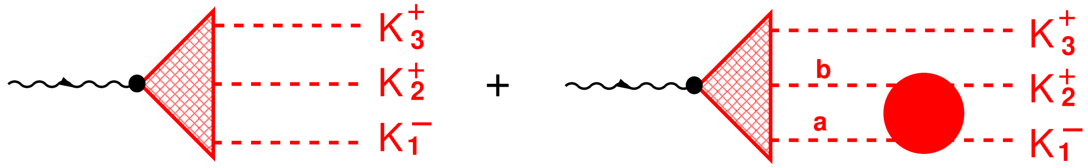
→ physical f_0 states are linear combination of $m_{s0}\,,\; m_{s1}$

  - coupling constants:

$g_\rho\,,\; g_\phi \qquad c_d\,,\; c_m\,,\; \tilde{c}_d\,,\; \tilde{c}_m$

vector scalar

Triple M LHCb fit



LHCb

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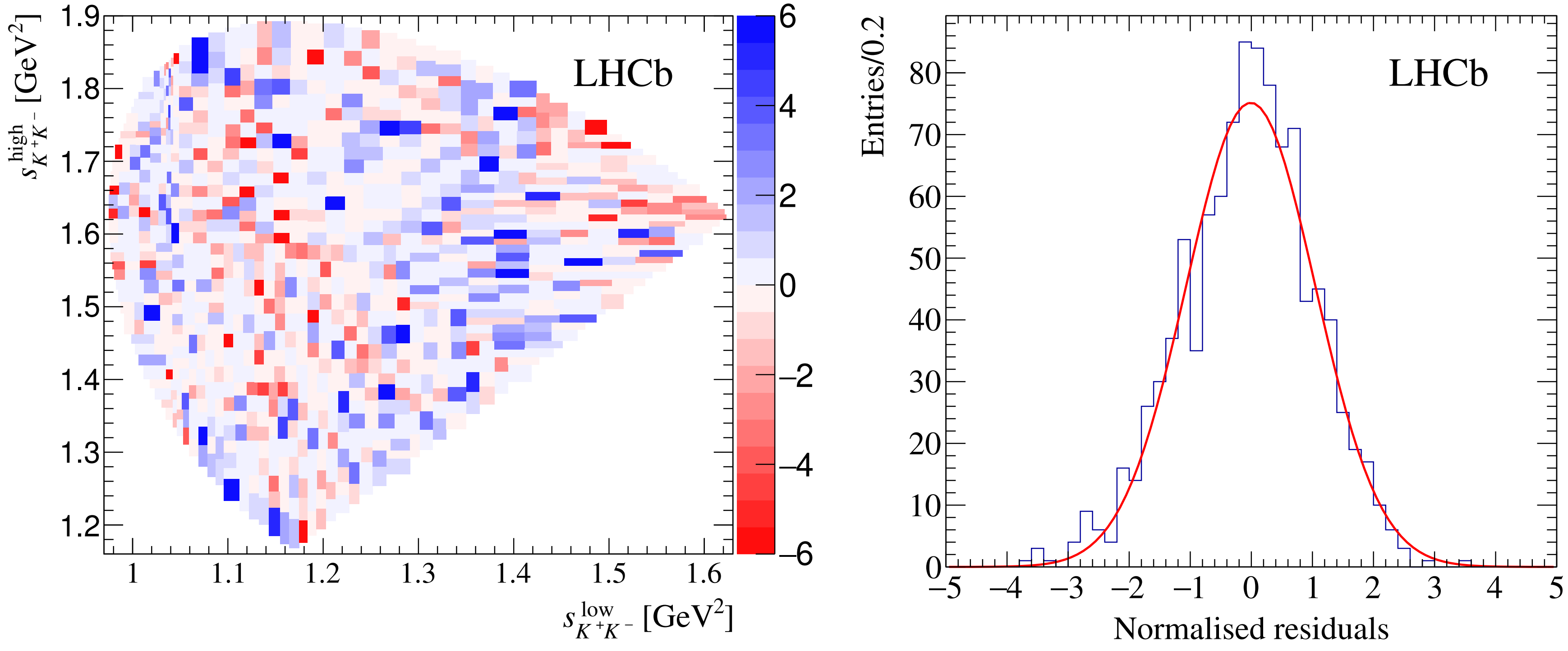


Figure 12. (left) Two-dimensional distribution of the normalised residuals for the Triple-M fit. (right) Distribution of normalised residuals of each bin.

KK scattering prediction

- S-wave, isospin 0 and I

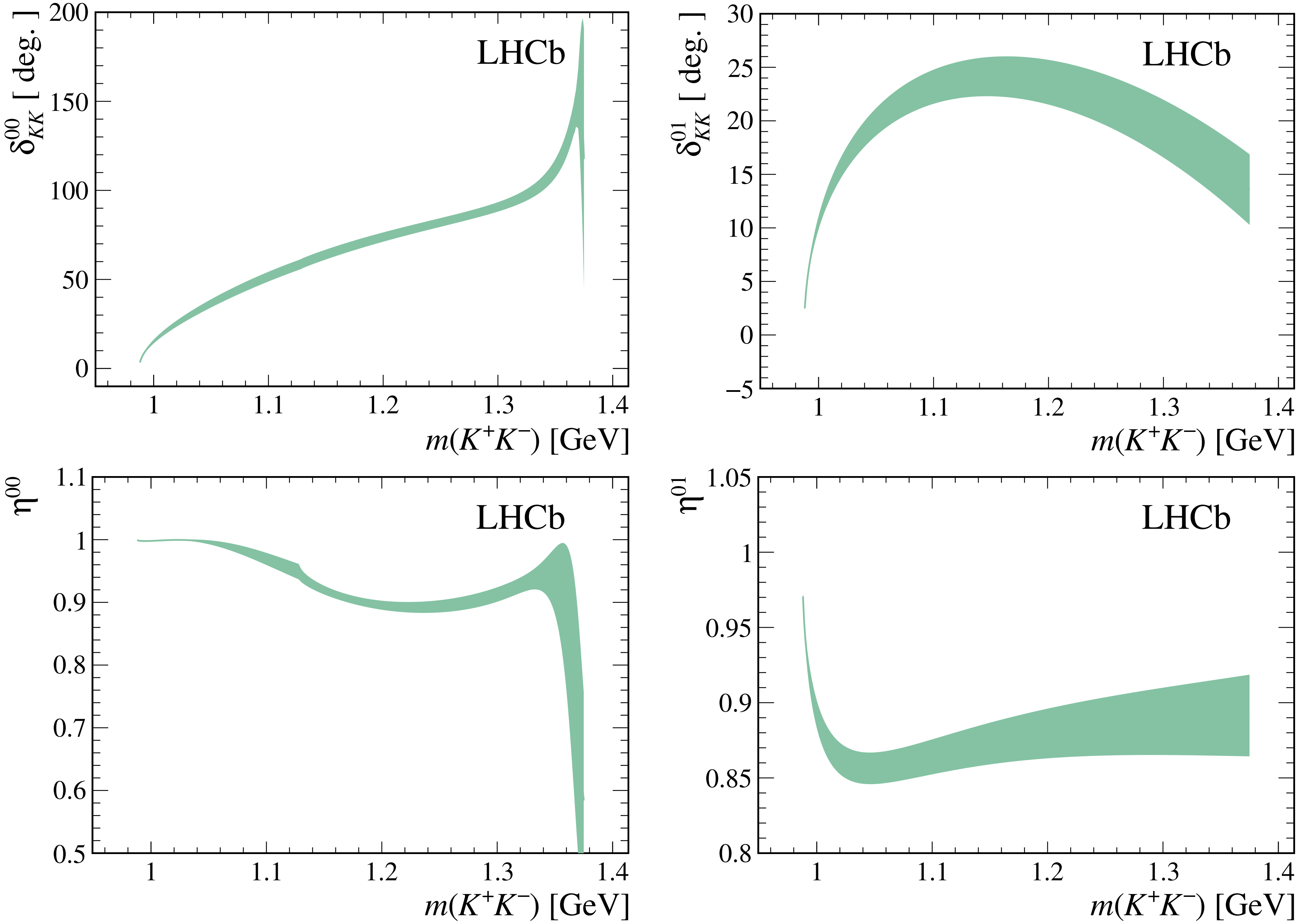
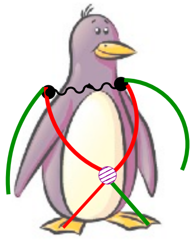
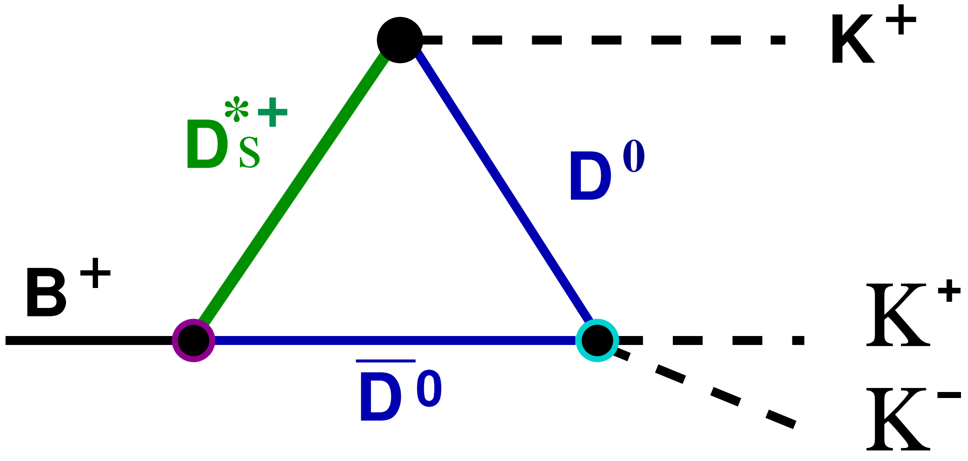


Figure 14. (top) Phase-shifts $\delta^{0I}_{K^+K^-}$ and (bottom) inelasticities η^{0I} as a function of the K^+K^- invariant mass, for both isospin states.

→ can be used in other process

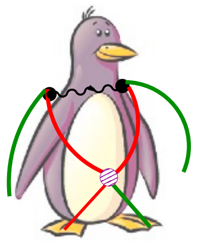
hadronic loop

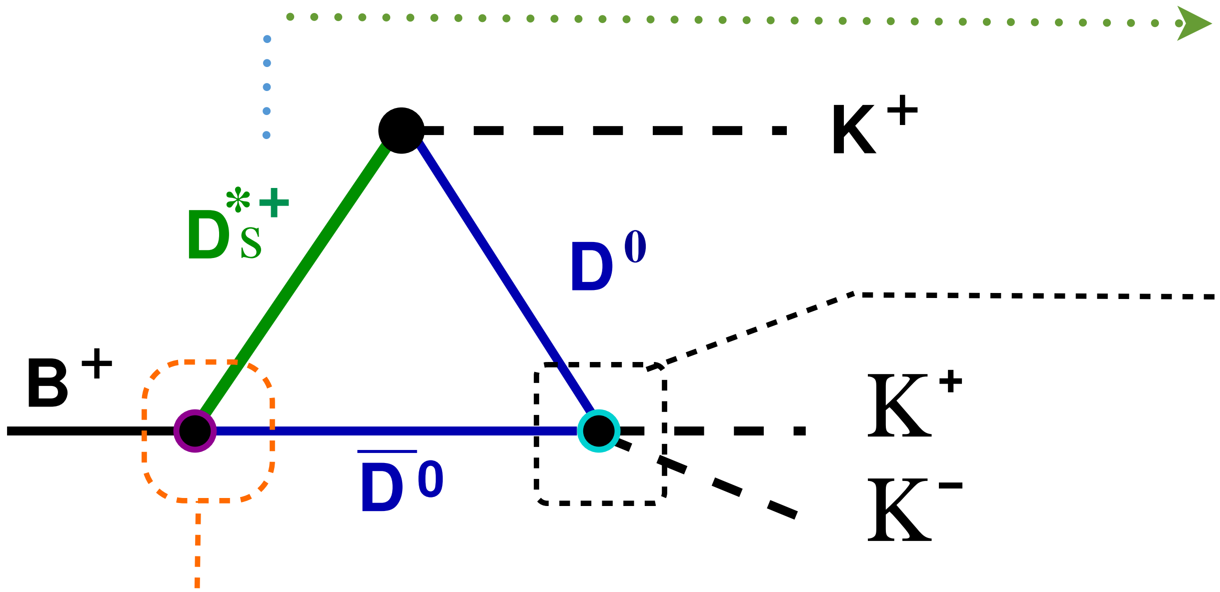




- $Br\left[B \to DD_s^*\right]$ ~1% → 1000 x $Br\left[B \to KKK\right]$

hadronic loop





form factor for $W^+ \to D^0 K^+$

$D^0 \bar{D}^0 \to K^+ K^-$ scattering amplitude

phenomenological:
S- matrix unitarity + Regge theory

weak transition $B^+ \to W^+ \bar{D}^0$ → $C_0 \times$ form factor to regulate

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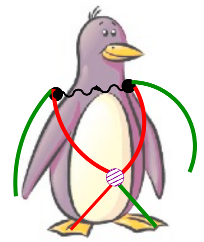

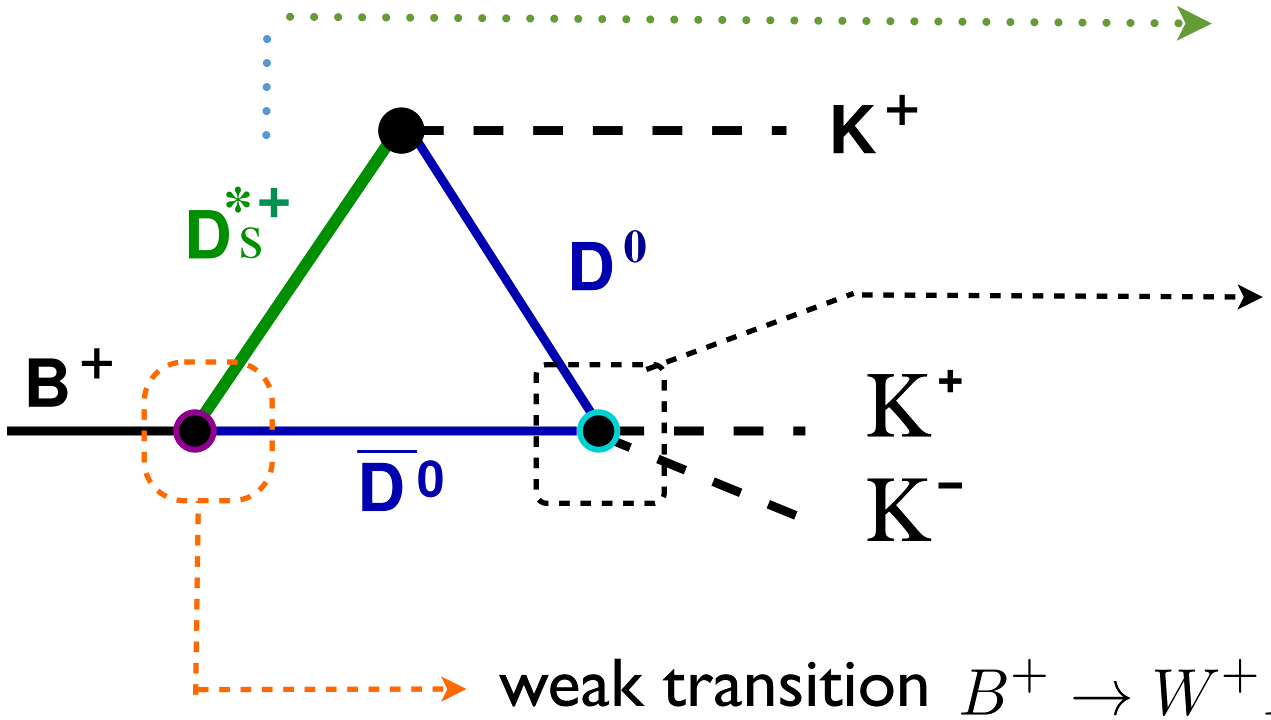

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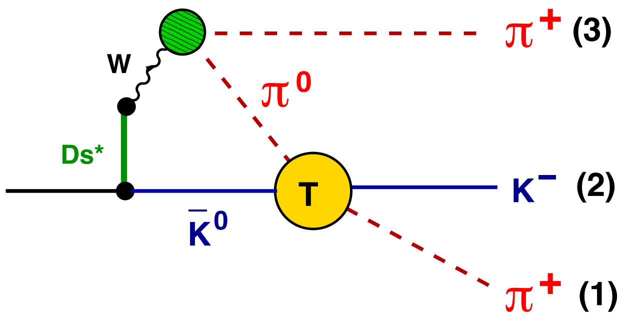
- $Br\,[B \to D D_s^*]$ ~1% → **1000 x** $Br\,[B \to KKK]$
- hadronic loop → three-body FSI - introduce new complex structures

- $B^+ \to \pi^+ \pi^- \pi^+$

PCM & I Bediaga
arXiv:1512.09284

- $D^+ \to \pi^+ K^- \pi^+$



PCM & M Robilotta
PRD 92 094005 (2015) [arXiv:1504.06346]
PCM et al
PRD 84 094001 (2011) [arXiv:1105.5120]

$D^0\bar{D}^0 \rightarrow K^+K^-$ scattering amplitude

- not well understand on literature

Donoghue *et al.*, *PRL* 77(1996)2178;
Suzuki,Wolfenstein, PRD 60 (1999)074019;
Falk et al. PRD 57,4290(1998);
Blok, Gronau, Rosner, PRL 78, 3999 (1997).

- important as FSI in B two-body decays

- phenomenological amplitude

Antunes, Bediaga, Frederico, PCM
ICHEP2016 - proceedings

  - unitarity of the S-matrix $S = \begin{pmatrix} \eta\, e^{2i\alpha} & \sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} \\ -\sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} & \eta\, e^{2i\beta} \end{pmatrix}$
  - inspired in the damping factor of the S matrix i.e. $\pi\pi \rightarrow KK$

$$\eta = \mathcal{N}\sqrt{s/s_{th}-1}/(s/s_{th})^{2.5}$$

KK: $e^{2i\alpha} = 1 - \dfrac{2ik_1}{\frac{c}{1-k_1/k_0} + ik_1}$, DD: $e^{2i\beta} = 1 - \dfrac{2ik}{\frac{1}{a} + ik}$

$$S_{\beta,\alpha} = \delta_{\beta,\alpha} + it_{\beta,\alpha}$$

$k = \sqrt{\frac{s-s_{th}}{4}}$, $k_1 = \sqrt{\frac{s-s_{th1}}{4}}$ and $k_0 = \sqrt{\frac{s_0-s_{th}}{4}}$

$$t_{\beta,\alpha} = \sqrt{1-\eta^2} e^{i(\alpha+\beta)}$$

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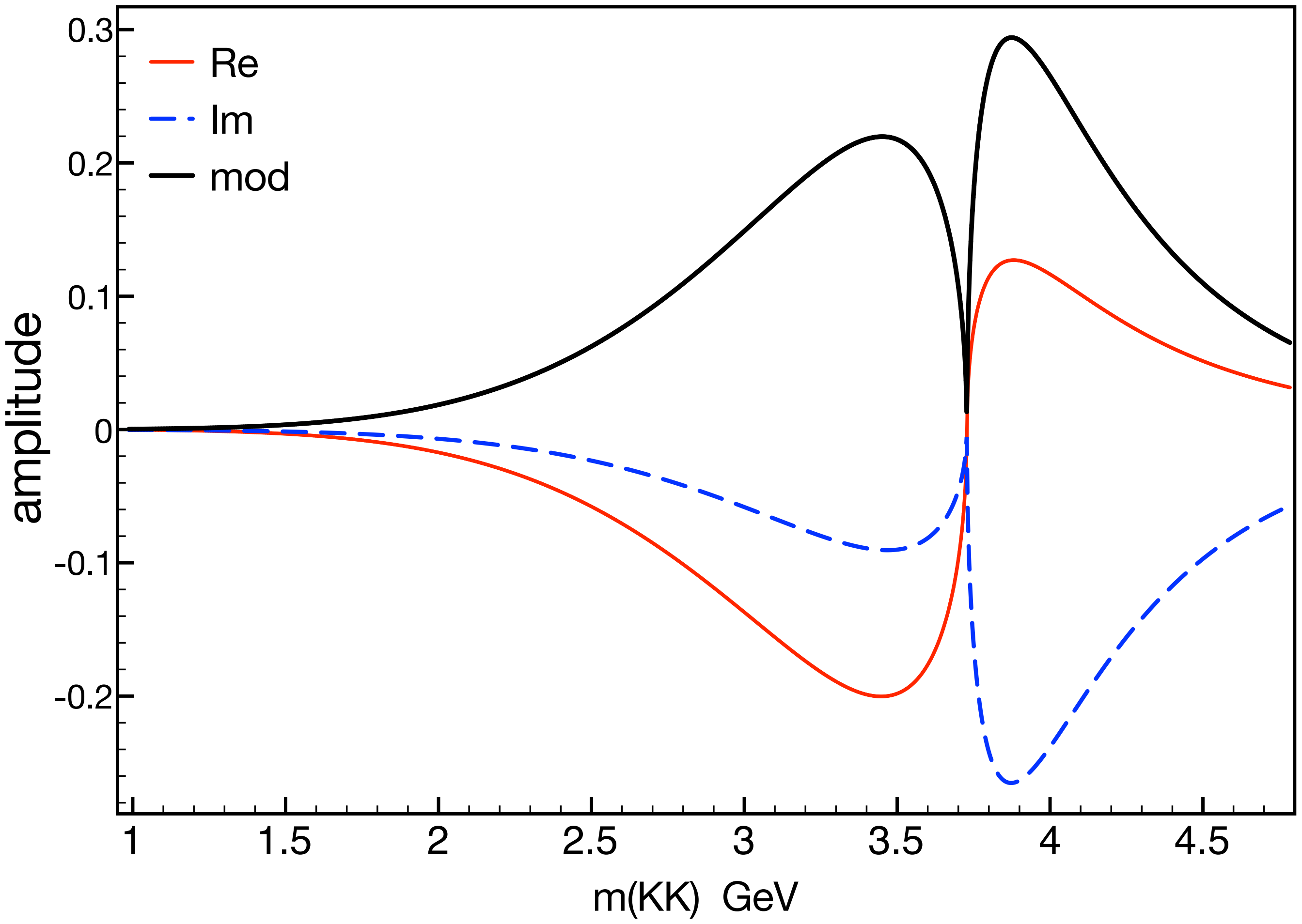
$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

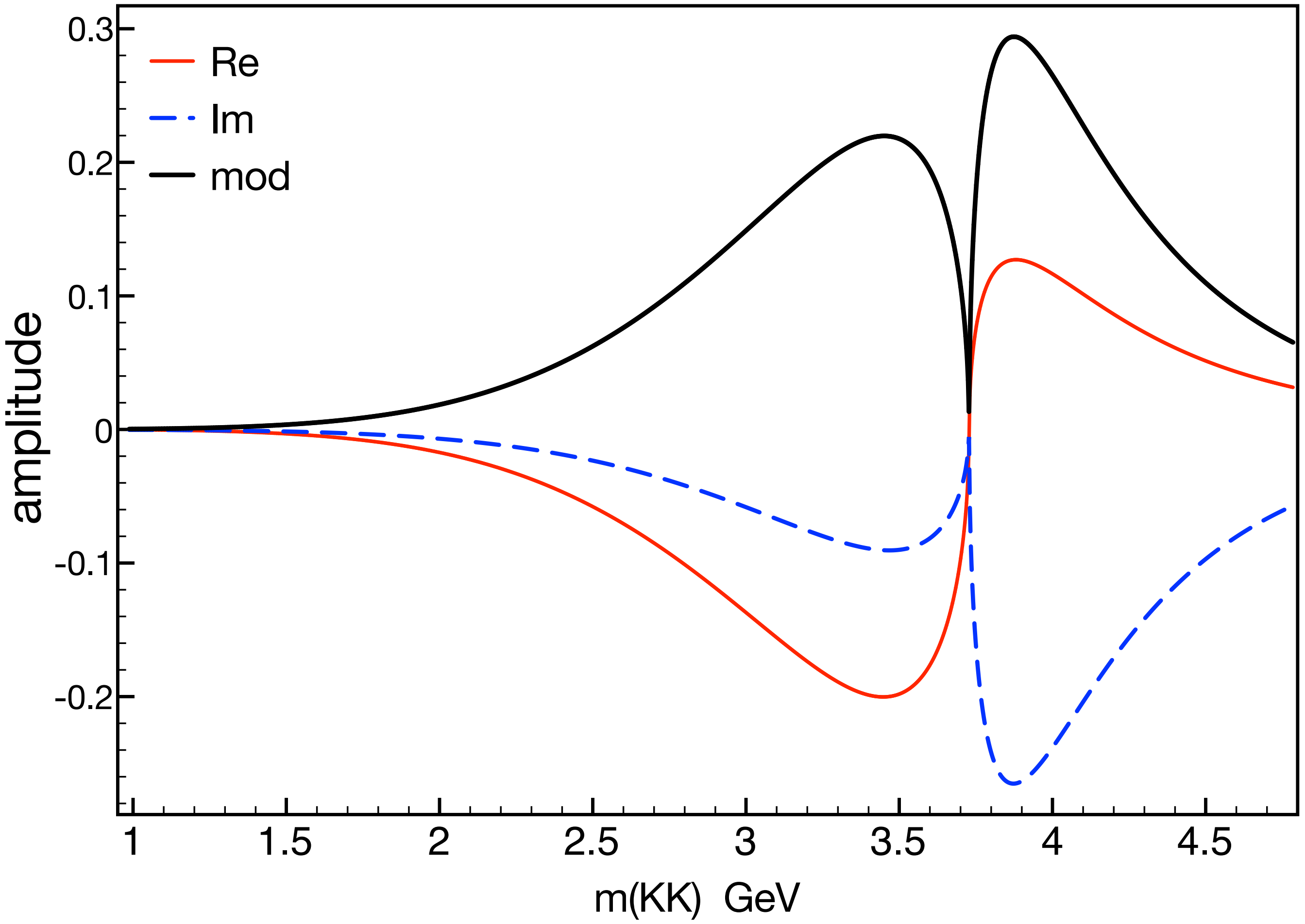
- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^{\alpha}}{s^{\alpha}_{th\,D\bar{D}}} \frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right) \left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}}, \; s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi} \left(\frac{m_0}{s-m_0}\right)^{\beta} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right) \left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}}, s \geq s_{th\,D\bar{D}}$$



$D^0\bar D^0 \to K^+K^-$ scattering amplitude

- $$T_{\bar D^0 D^0\to KK}(s) = \frac{s^{\alpha}}{s^{\alpha}_{th\,D\bar D}}\,\frac{2\kappa_2}{\sqrt{s_{th\,D\bar D}}}\left(\frac{s_{th\,D\bar D}}{s+s_{QCD}}\right)^{\xi+\alpha}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}},\; s<s_{th\,D\bar D}$$
  $$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar D}}}\left(\frac{s_{th\,D\bar D}}{s+s_{QCD}}\right)^{\xi}\left(\frac{m_0}{s-m_0}\right)^{\beta}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}},\; s\geq s_{th\,D\bar D}$$

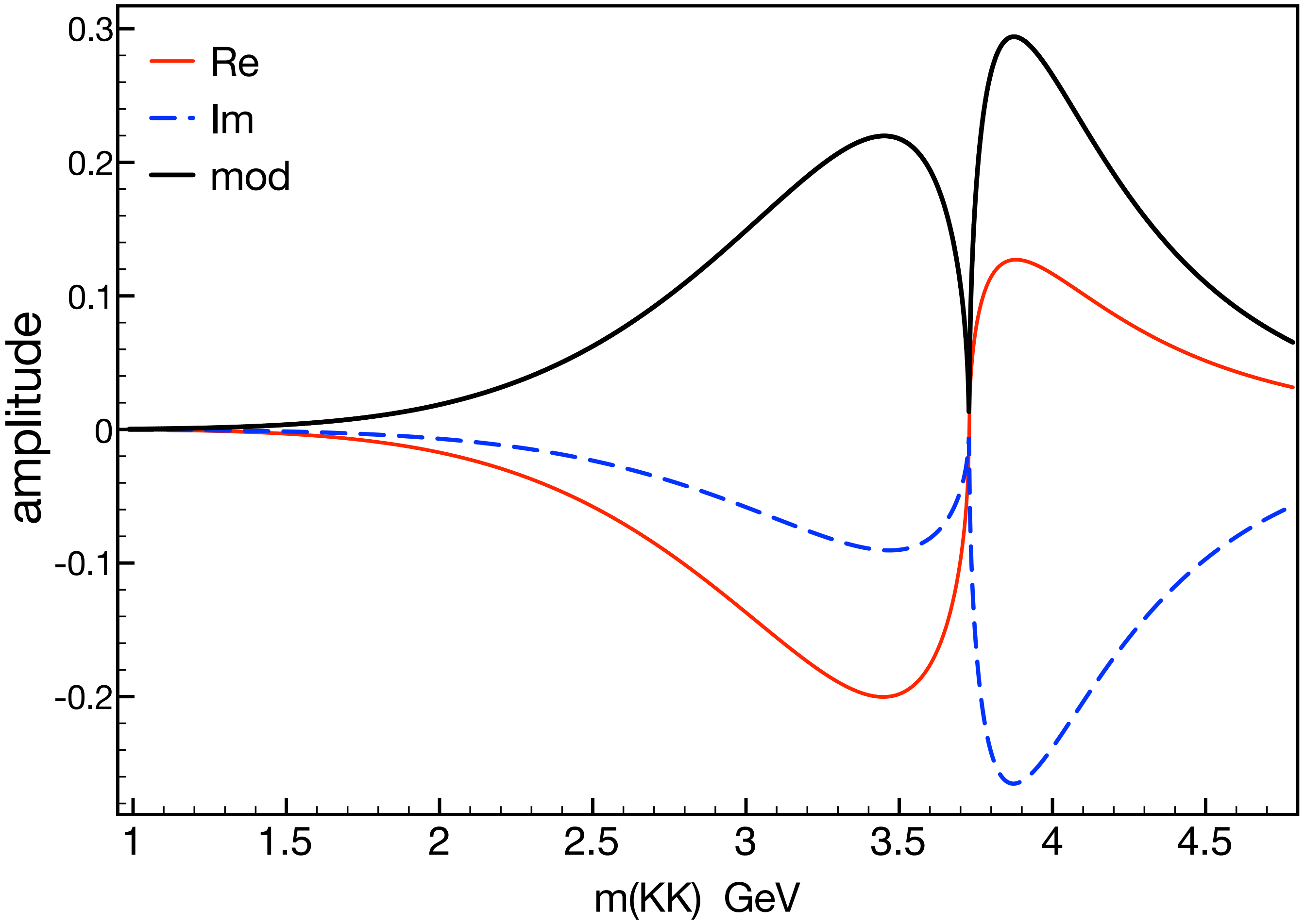
→ parameters fix by data!



$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^\alpha}{s^\alpha_{th\,D\bar{D}}}\,\frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}},\ s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi}\left(\frac{m_0}{s-m_0}\right)^{\beta}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}},\ s \geq s_{th\,D\bar{D}}$$

→ parameters fix by data!

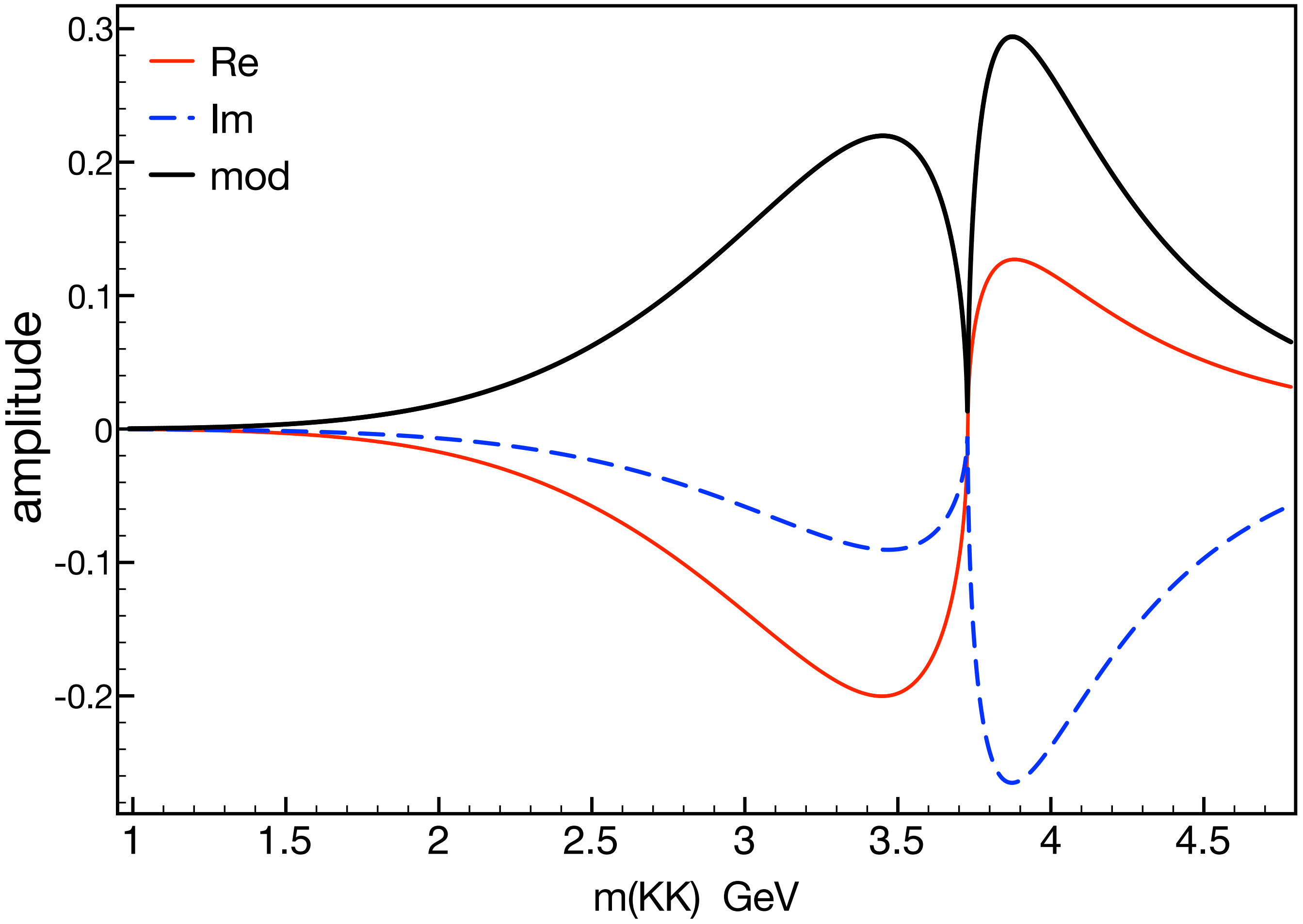


→ zero at threshold

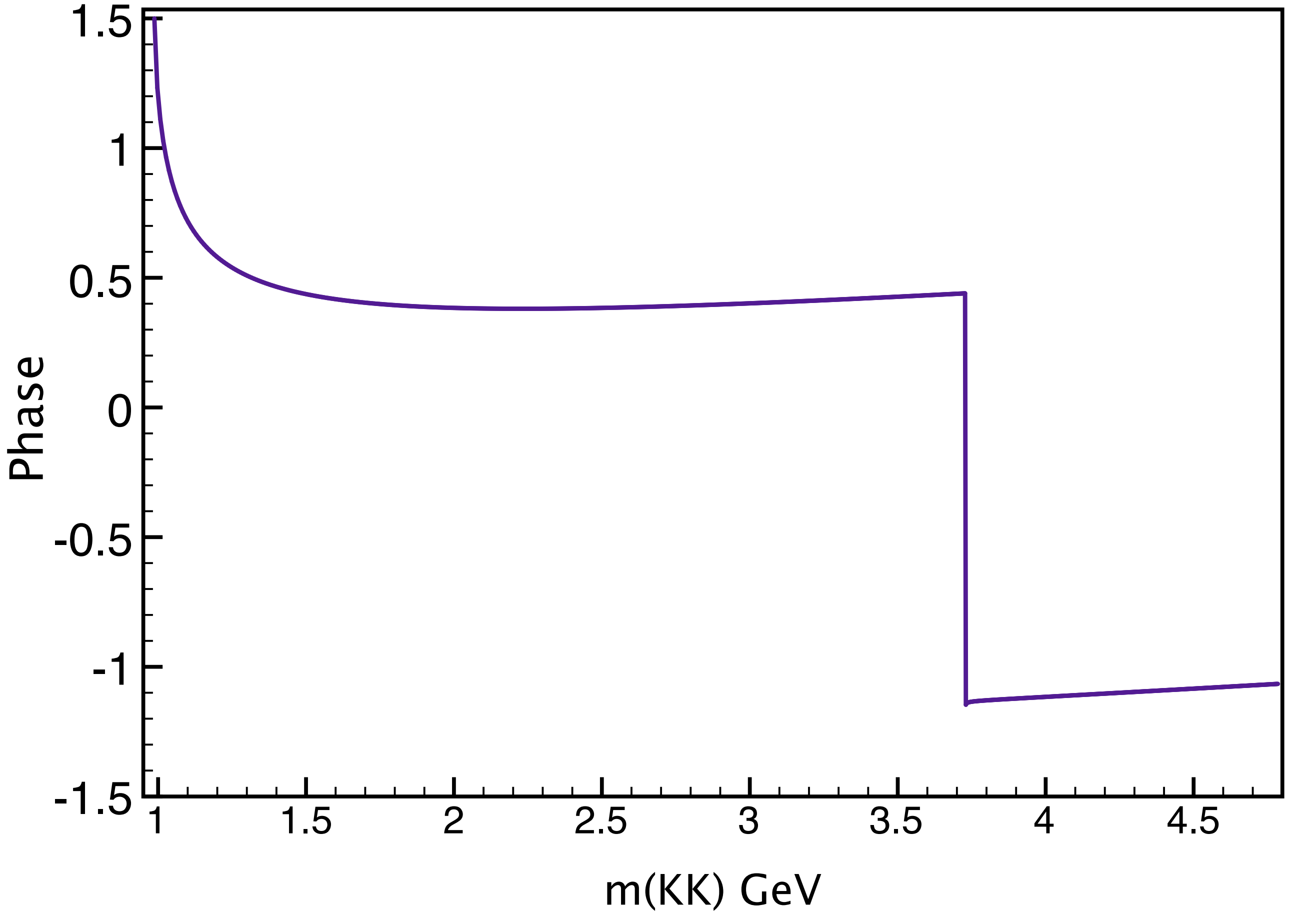
$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^{\alpha}}{s^{\alpha}_{th\,D\bar{D}}}\,\frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}},\ s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi}\left(\frac{m_0}{s-m_0}\right)^{\beta}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}},s \geq s_{th\,D\bar{D}}$$

→ parameters fix by data!



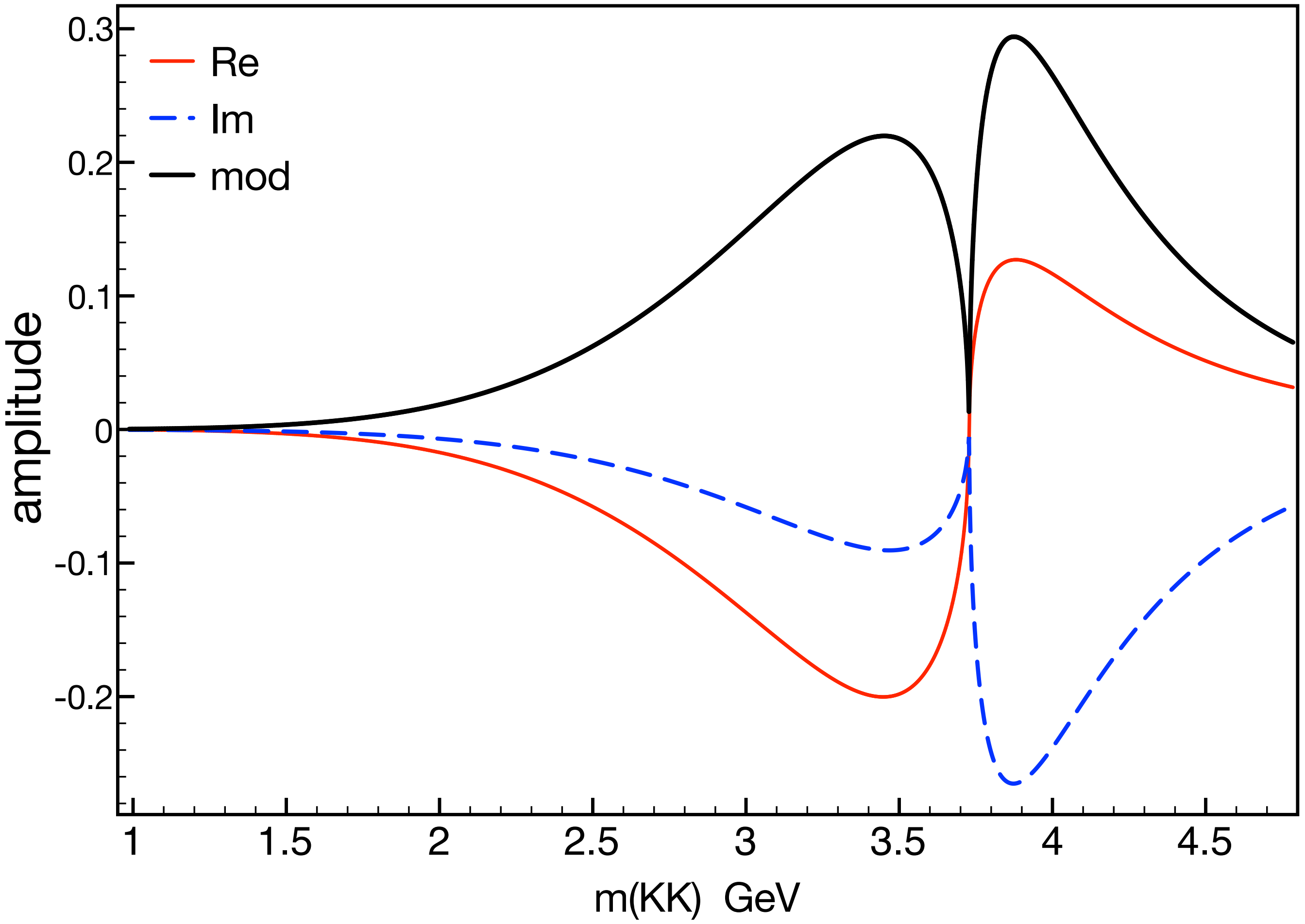
→ zero at threshold



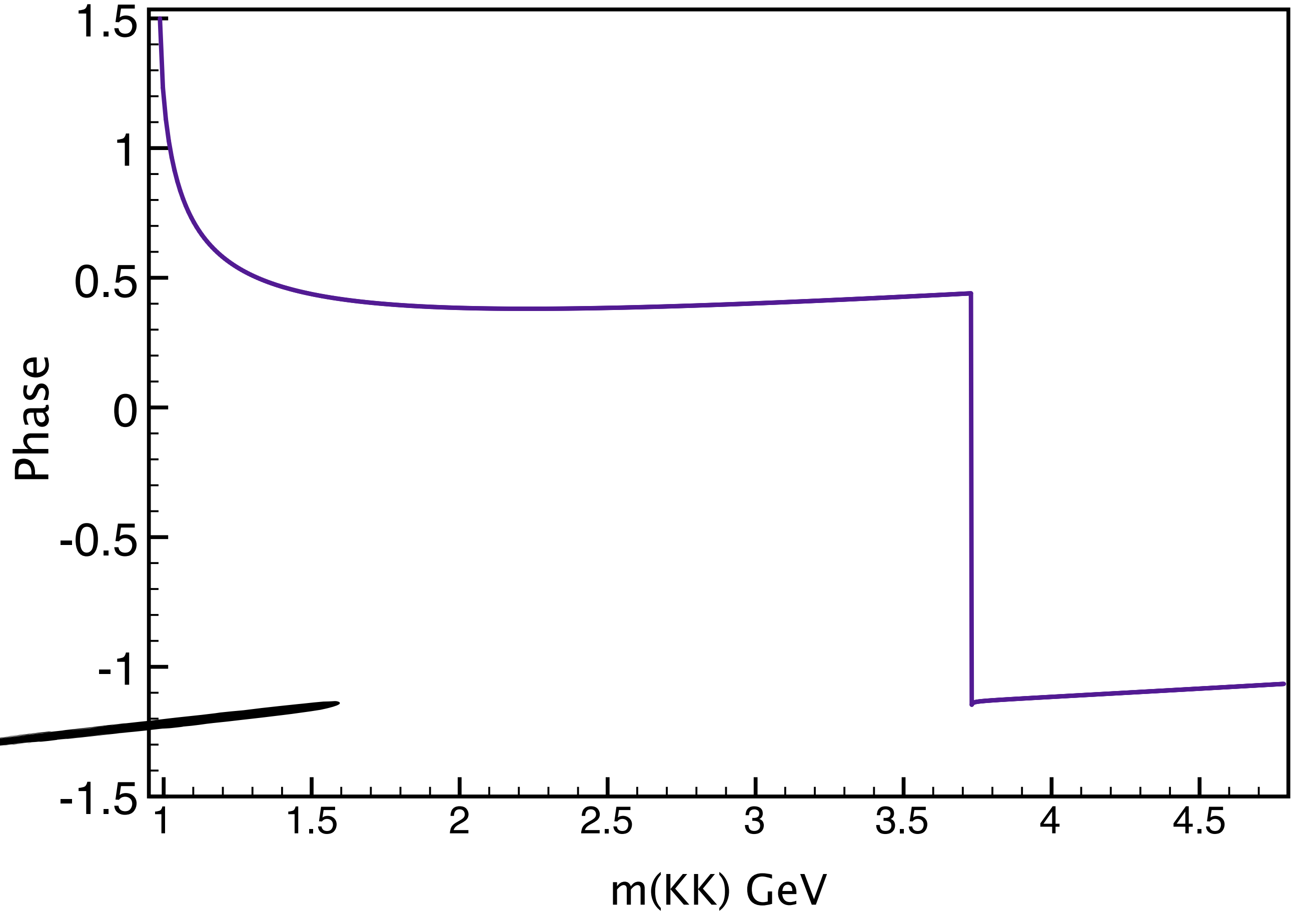
$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^{\alpha}}{s^{\alpha}_{th\,D\bar{D}}} \frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right) \left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}}, \; s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}} \left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi} \left(\frac{m_0}{s-m_0}\right)^{\beta} \left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right) \left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}}, s \geq s_{th\,D\bar{D}}$$

→ parameters fix by data!



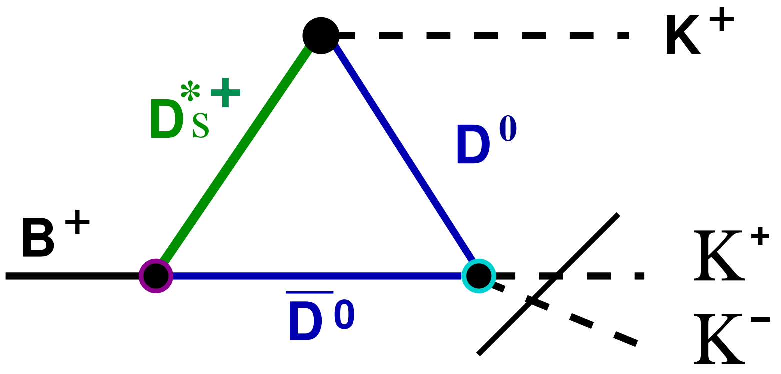
zero at threshold

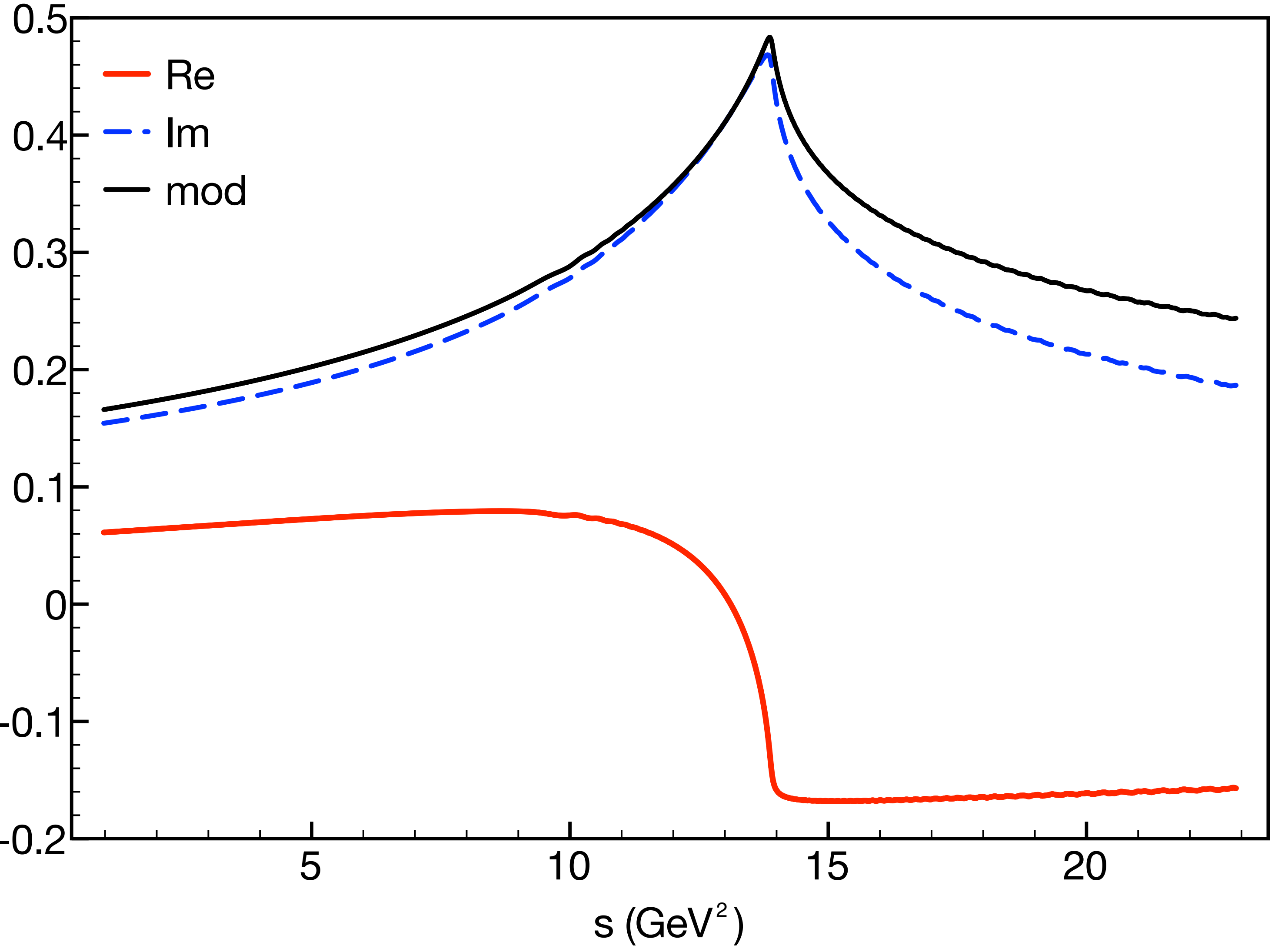


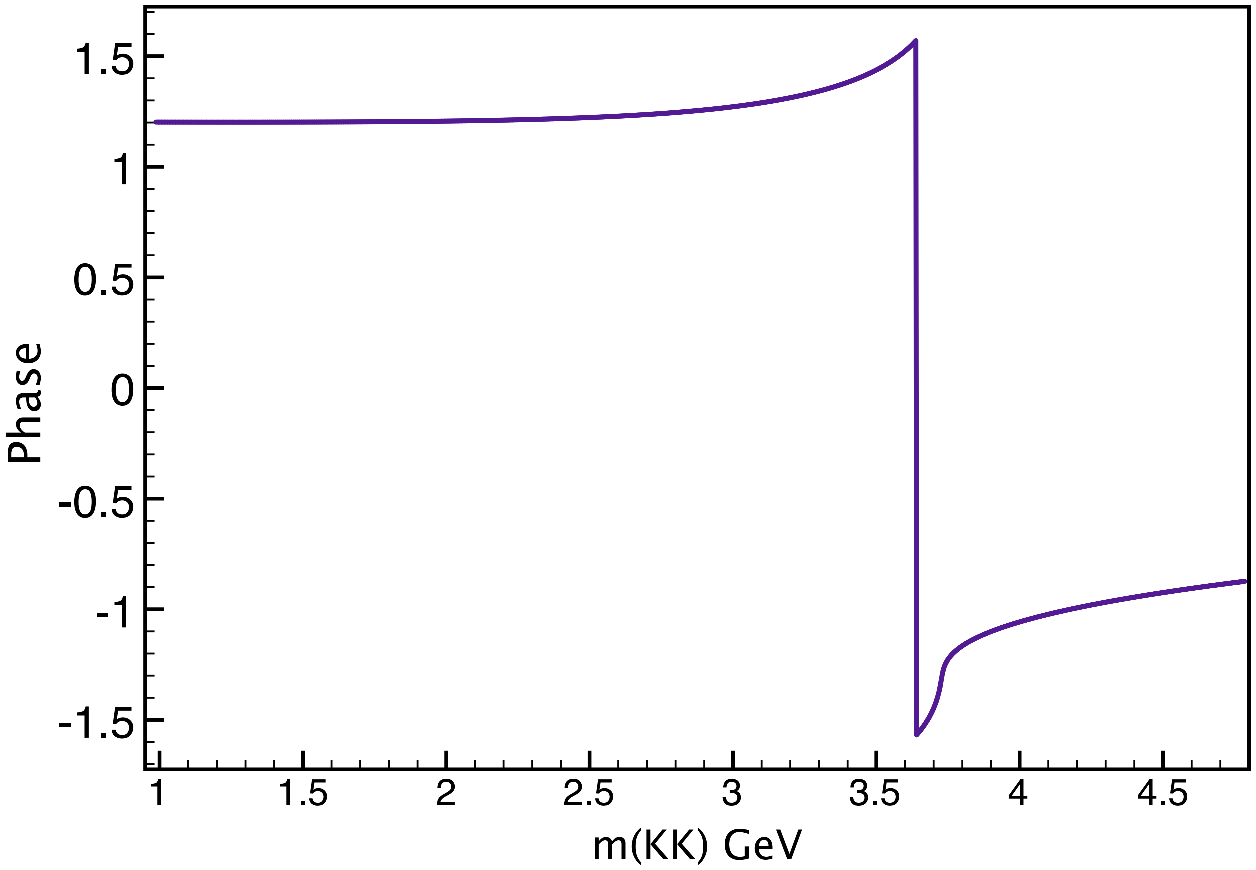
discontinuity at threshold

hadronic loop

- $$Loop = i \int \frac{d^4\ell}{(2\pi)^4} \, \frac{\Delta_{D^0} + 2\,\Delta_{\bar{D}^0} - \, 2\,s_{23} + 3\,M_K^2 + M_B^2 - l^2}{\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_{D*}\,[l^2 - m_{B^*}]}$$

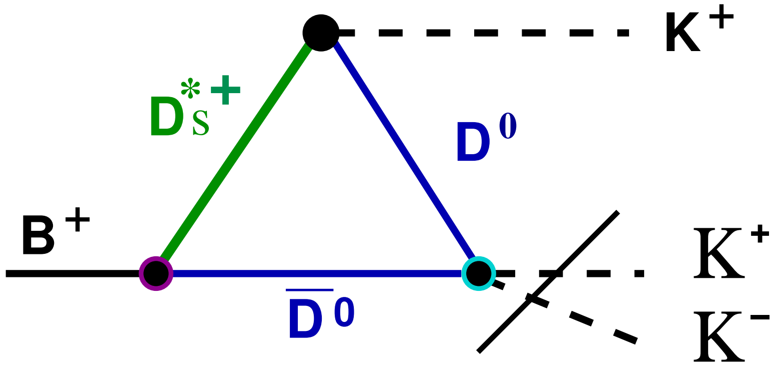


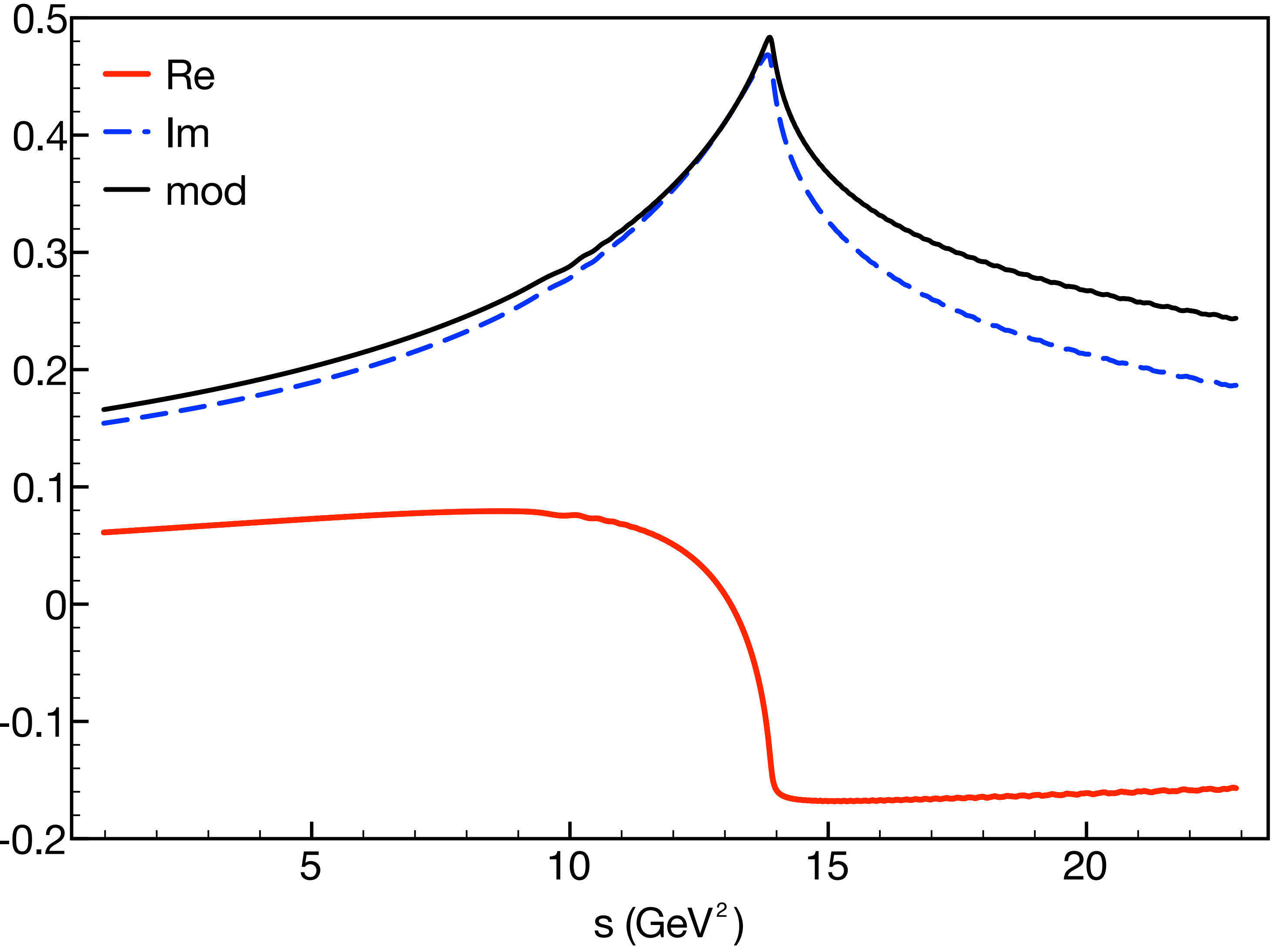


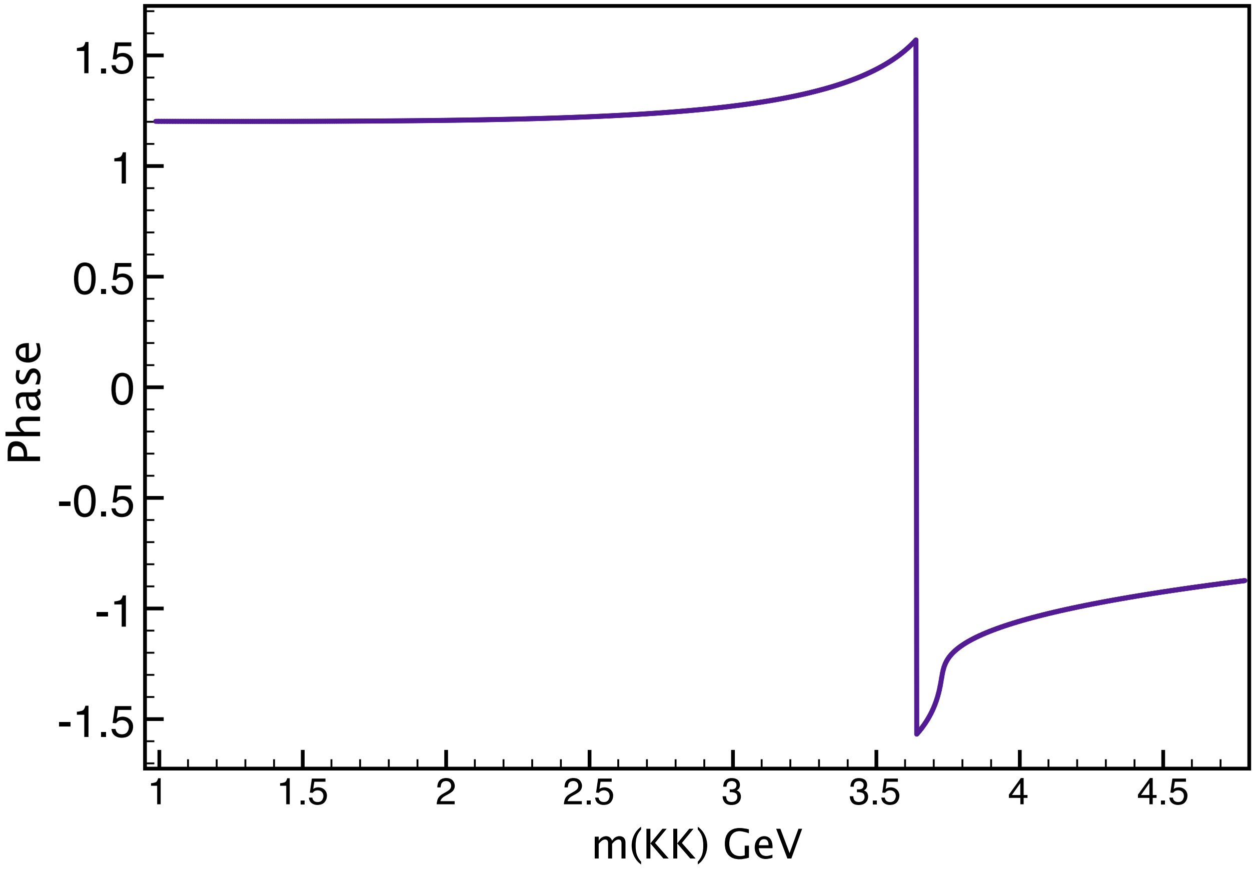


hadronic loop

- $$Loop = i \int \frac{d^4\ell}{(2\pi)^4} \, \frac{\Delta_{D^0} + 2\,\Delta_{\bar{D}^0} - \, 2\,s_{23} + 3\,M_K^2 + M_B^2 - l^2}{\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_{D*}\,[l^2 - m_{B^*}]}$$



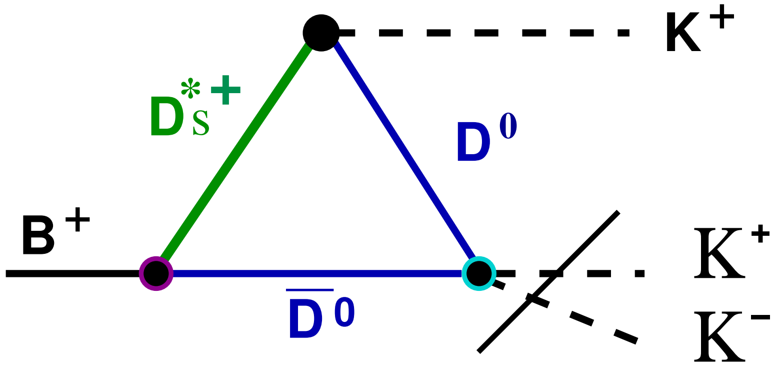


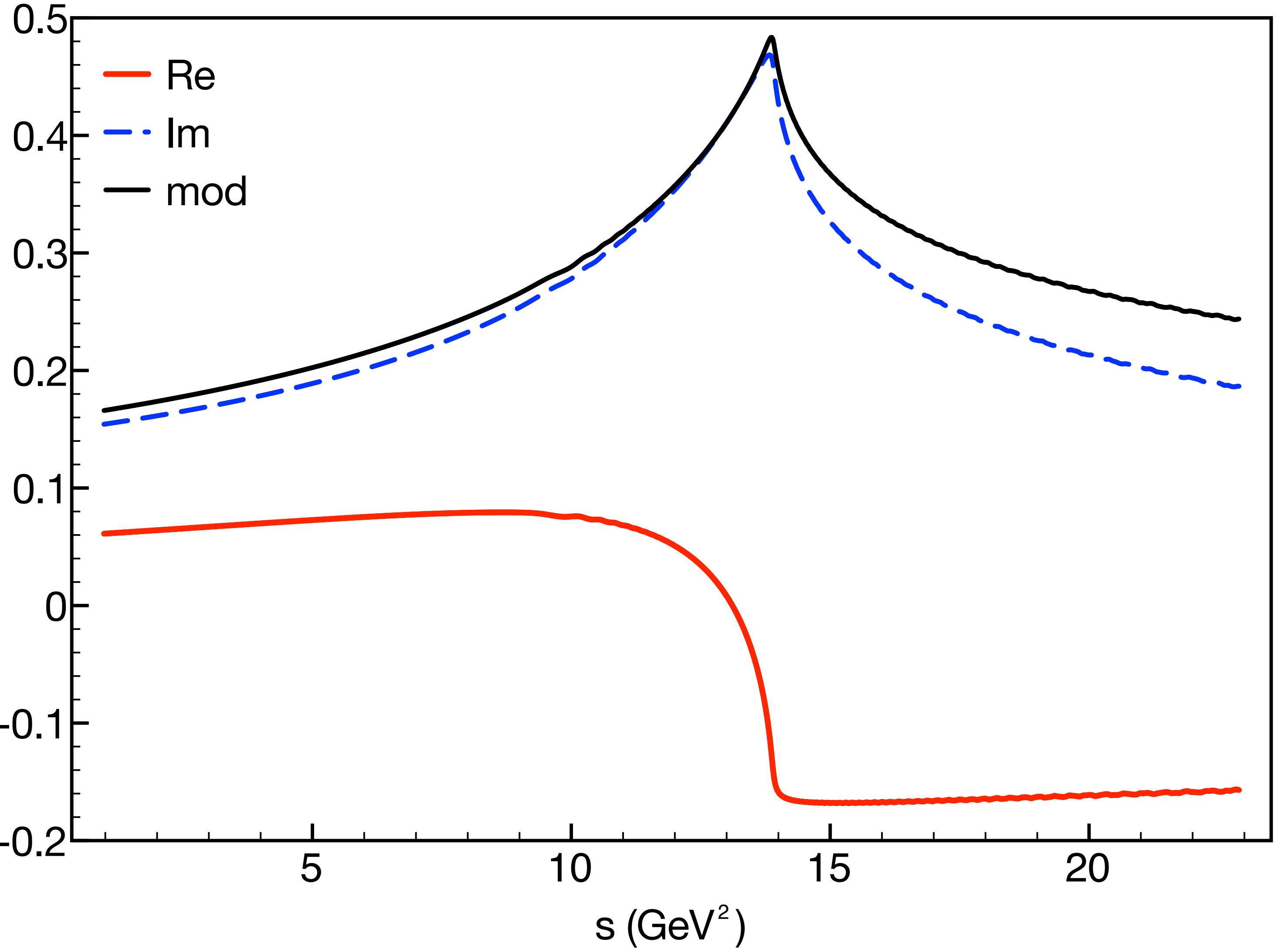


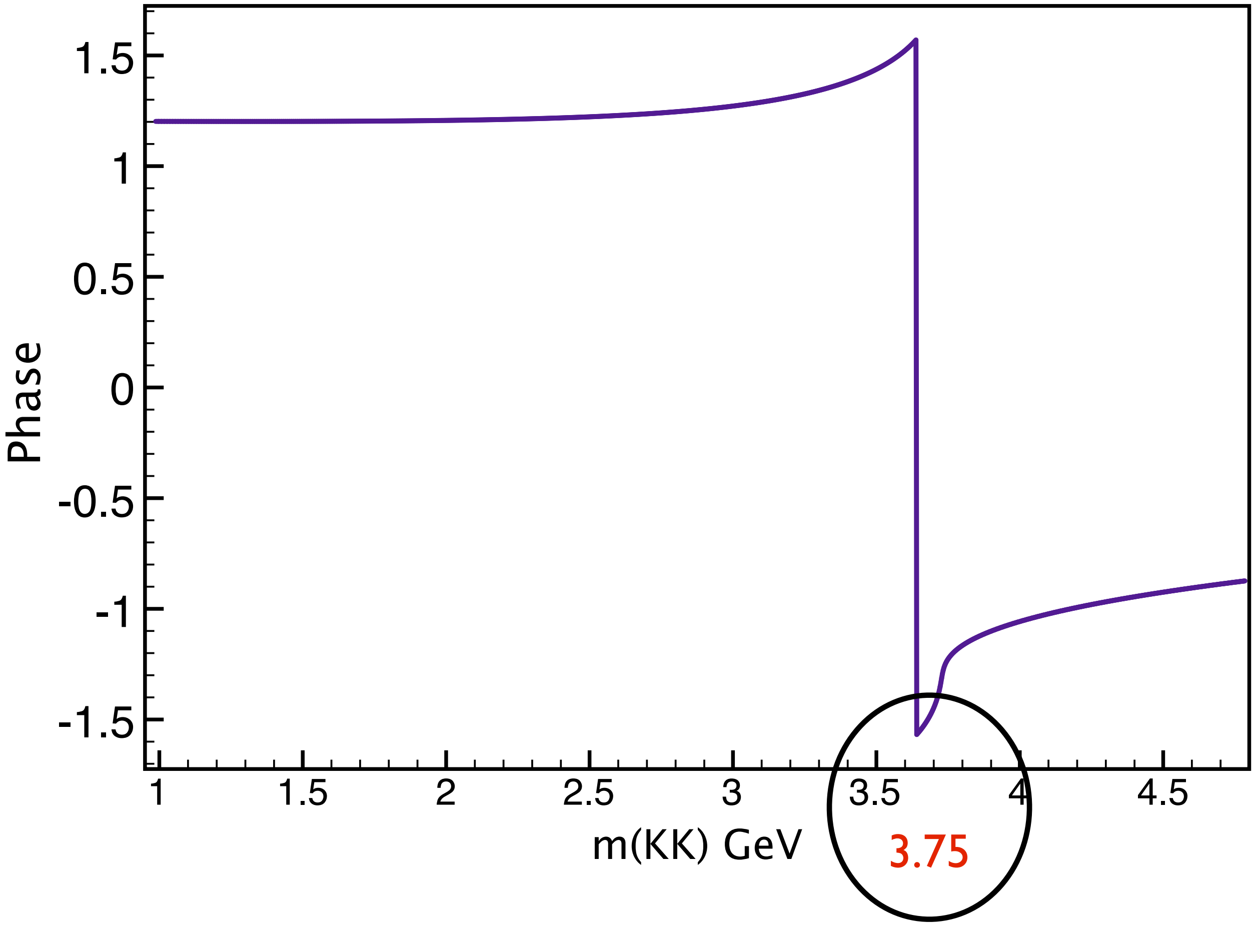
discontinuity at threshold

hadronic loop

- $$Loop = i \int \frac{d^4\ell}{(2\pi)^4} \, \frac{\Delta_{D^0} + 2\,\Delta_{\bar{D}^0} - \,2\,s_{23} + 3\,M_K^2 + M_B^2 - l^2}{\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_{D*}\,[l^2 - m_{B^*}]}$$





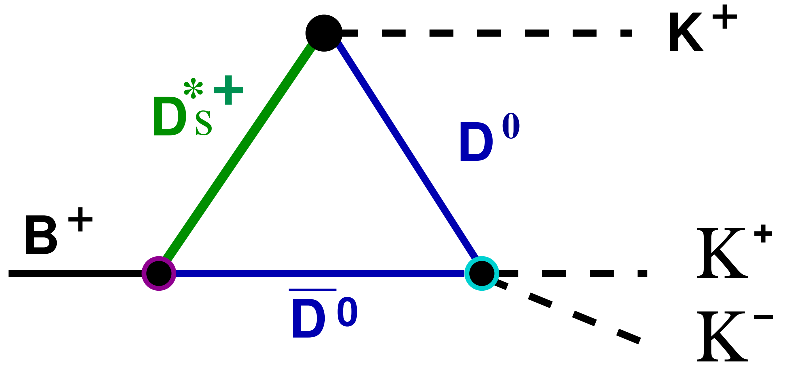


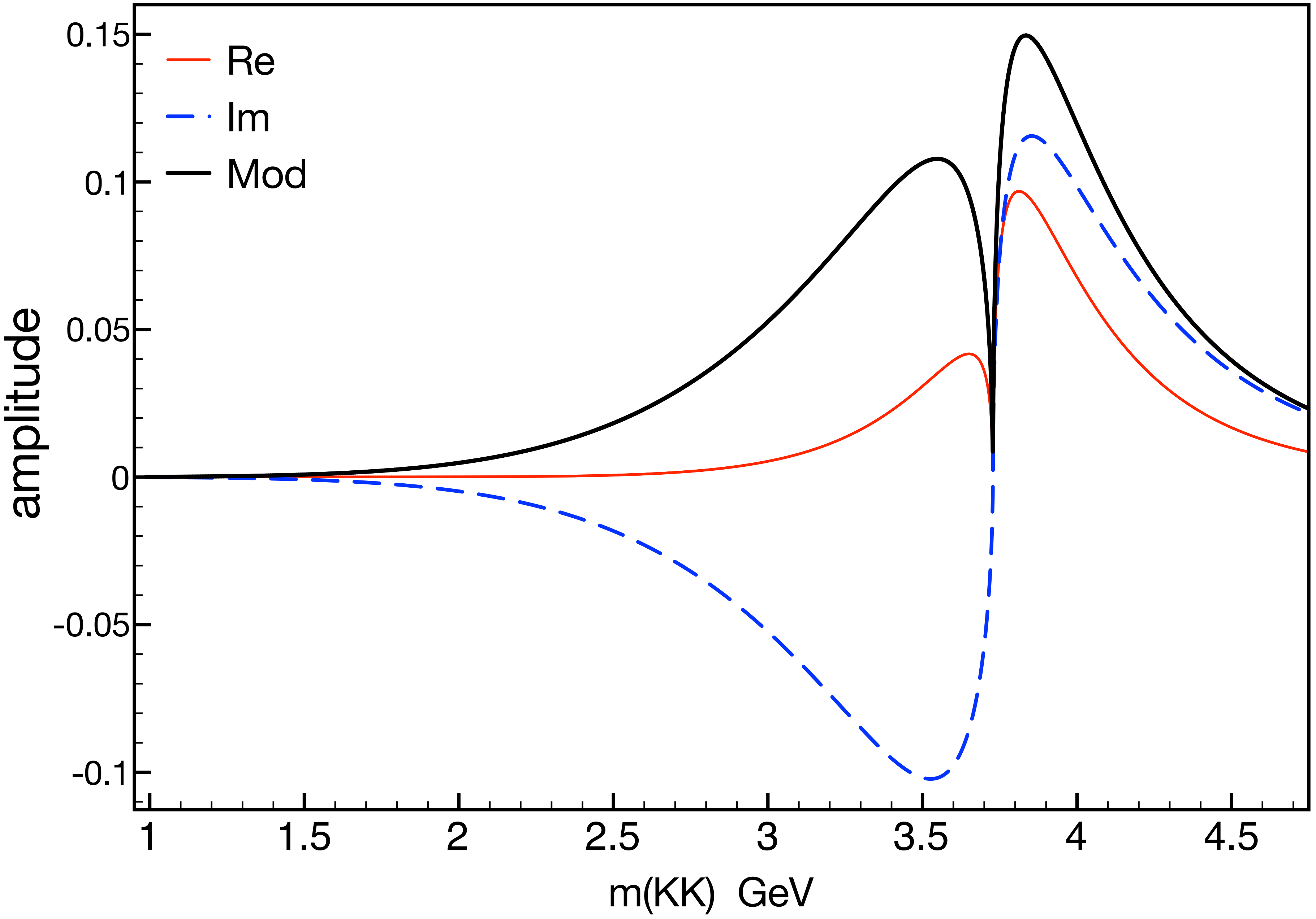
discontinuity at threshold

→ change sign at threshold

Final Amplitude

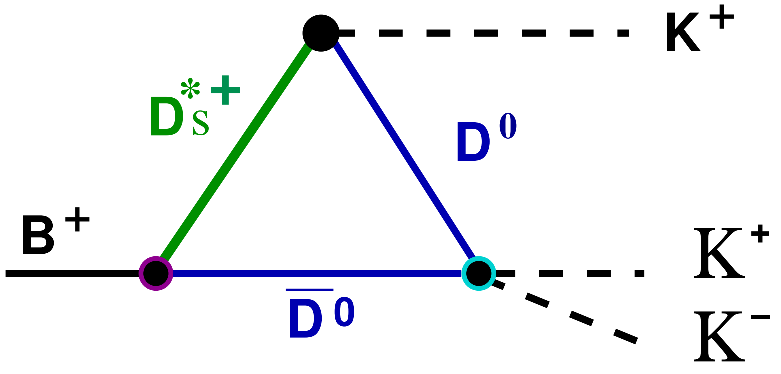
- $$A = iC \;\; m_a^2 \int \frac{d^4\ell}{(2\pi)^4} \; \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23})\,[-2\,p_3' \;\cdot (p_2' - p_1)]}{\Delta_{D^{+*}}\, \Delta_{D^0}\; \Delta_{\bar{D}^0}\; \Delta_a} \;,$$

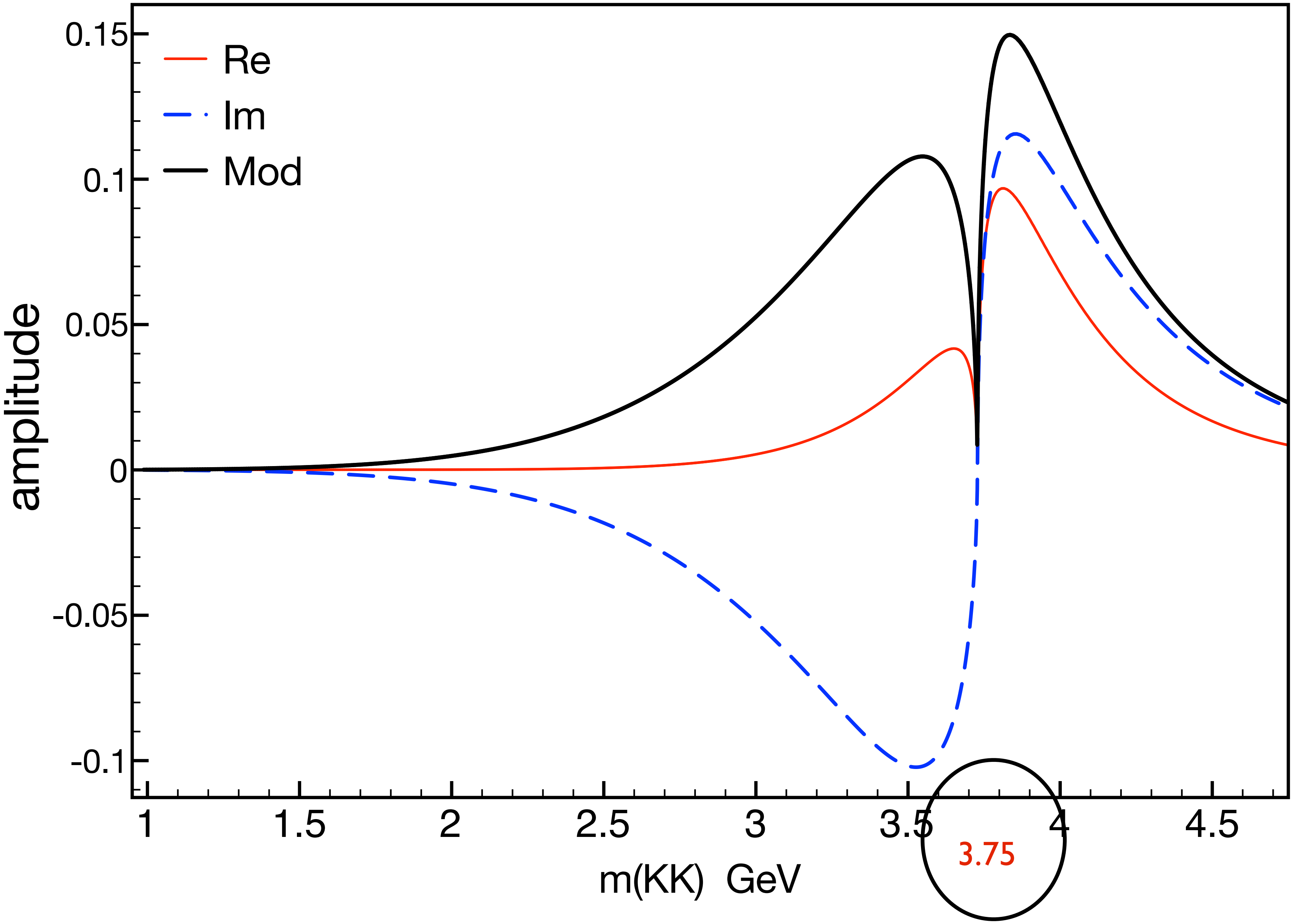




Final Amplitude

- $$A = iC\ m_a^2 \int \frac{d^4\ell}{(2\pi)^4}\ \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23})\,[-2\,p_3' \cdot (p_2' - p_1)]}{\Delta_{D^{+*}}\,\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_a}\ ,$$

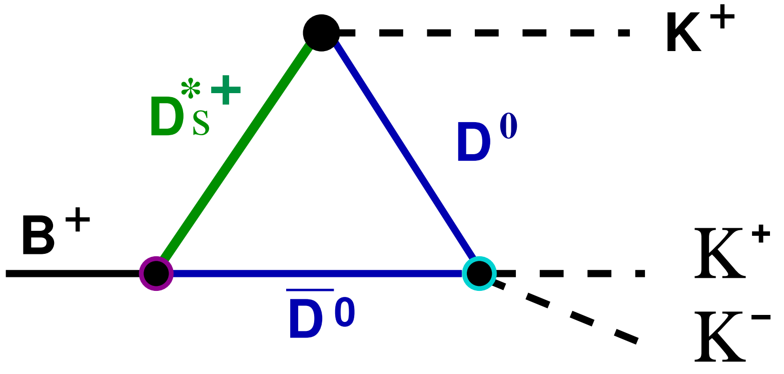


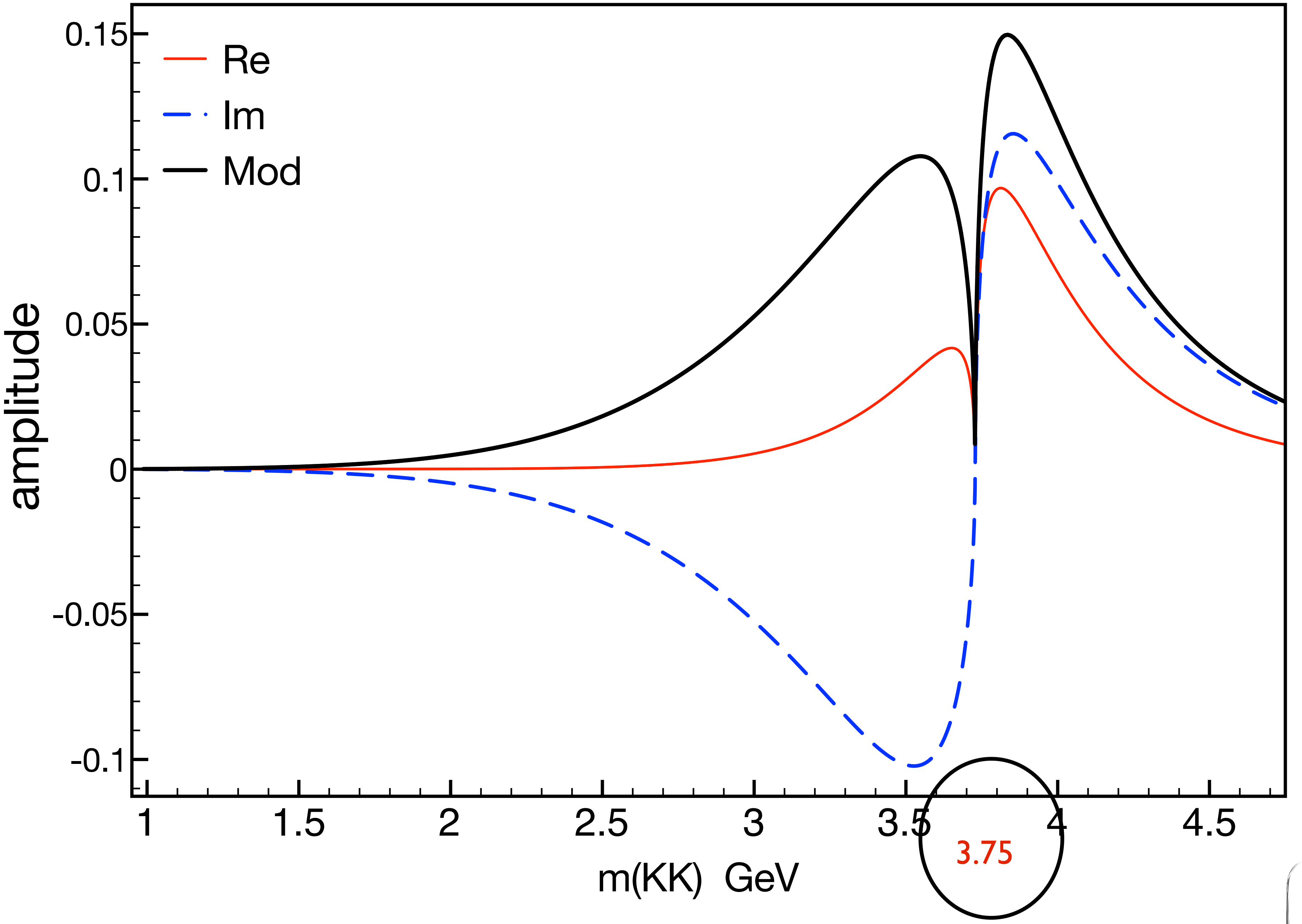


→ zero in between two bumps

Final Amplitude

- $$A = iC \; m_a^2 \int \frac{d^4\ell}{(2\pi)^4} \, \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23}) \, [-2\, p'_3 \; \cdot (p'_2 - p_1)]}{\Delta_{D^{+*}} \Delta_{D^0} \, \Delta_{\bar{D}^0} \, \Delta_a} \; ,$$





→ zero in between two bumps

rescattering $D^0 \bar{D}^0 \to K^+ K^-$ play a major role