PWA/ATHOS
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Three body dynamics and resonance formation

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Collaborators on the topic (over the years)

- A. Martínez Torres (USP, São Paulo)
- E. Oset (IFIC, Valencia)
- D. Jido (YITP, Kyoto)
- A. Hosaka (RCNP, Osaka)
- Y. Kanada-En'yo (Kyoto univ.)
- L. Geng (Beihang univ, Beijing)
- D. Gamermann(URGS, Rio grande do Sul)
- F. Navarra (USP, São Paulo)
- M. Nielsen (USP, São Paulo)

Our motivation to study three-body amplitudes

- What our motivation is not:
  - binding energy calculations of light nuclei/hypernuclei
- What we are interested in:
  - Three-meson or two meson-one baryon systems
  - study coupled channel dynamics and look for hadrons arising from three-body dynamics (exotic hadrons)

Exotic hadrons

- Various possible configurations of (``valence”) quarks and gluons are allowed within QCD
  - tetraquark/pentaquark
  - molecule-like hadrons
  - glueballs
- Large census exist on a two-body molecule-like nature for several hadrons:
  - $\Lambda(1405)$
  - $D_s(2317)$, $X(3872)$

Exotic hadrons

- Under debate:
  - $P_c(4312)$, $P_c(4440)$, $P_c(4457)$
  - $Z(3900)$
  - $D_{s1}(2460)$
- Formalism: low-energy (s-wave) interactions, close to threshold.
- Effective field treatment.
- Our focus: if similar ``binding” occurs in three-hadron systems

Exotic hadrons

- Three-hadron ``bound state” :
  - $\bar{K}NN$, $\bar{K}\bar{K}N$ (former case, discussions on deeply bound state, see review A. Gal, E.V. Hungerford, D.J. Millener, Rev.Mod.Phys. 88 (2016), 035004)
  - DNN
  - $\phi K\bar{K}$ [$\phi(2150)$]

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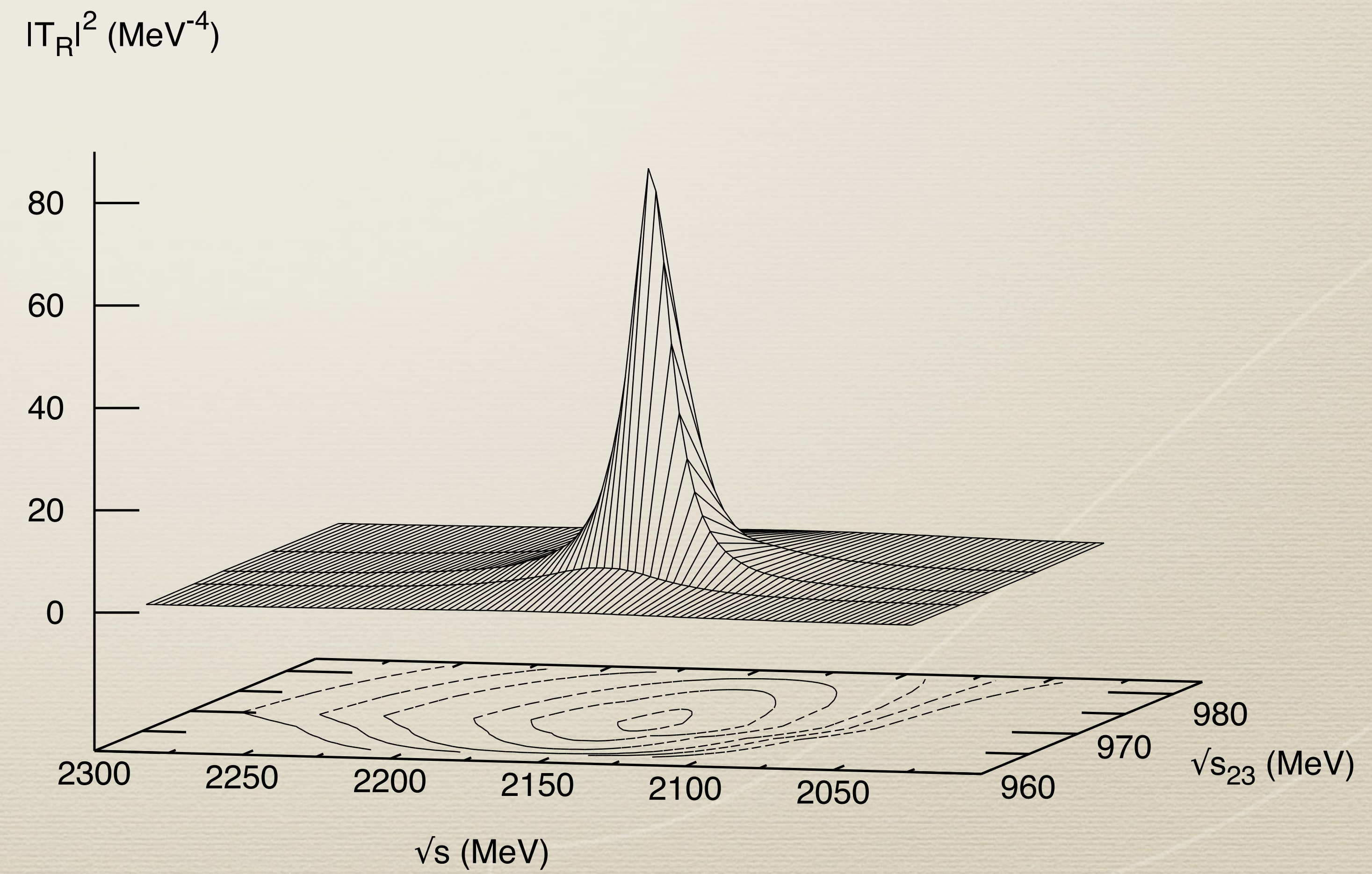
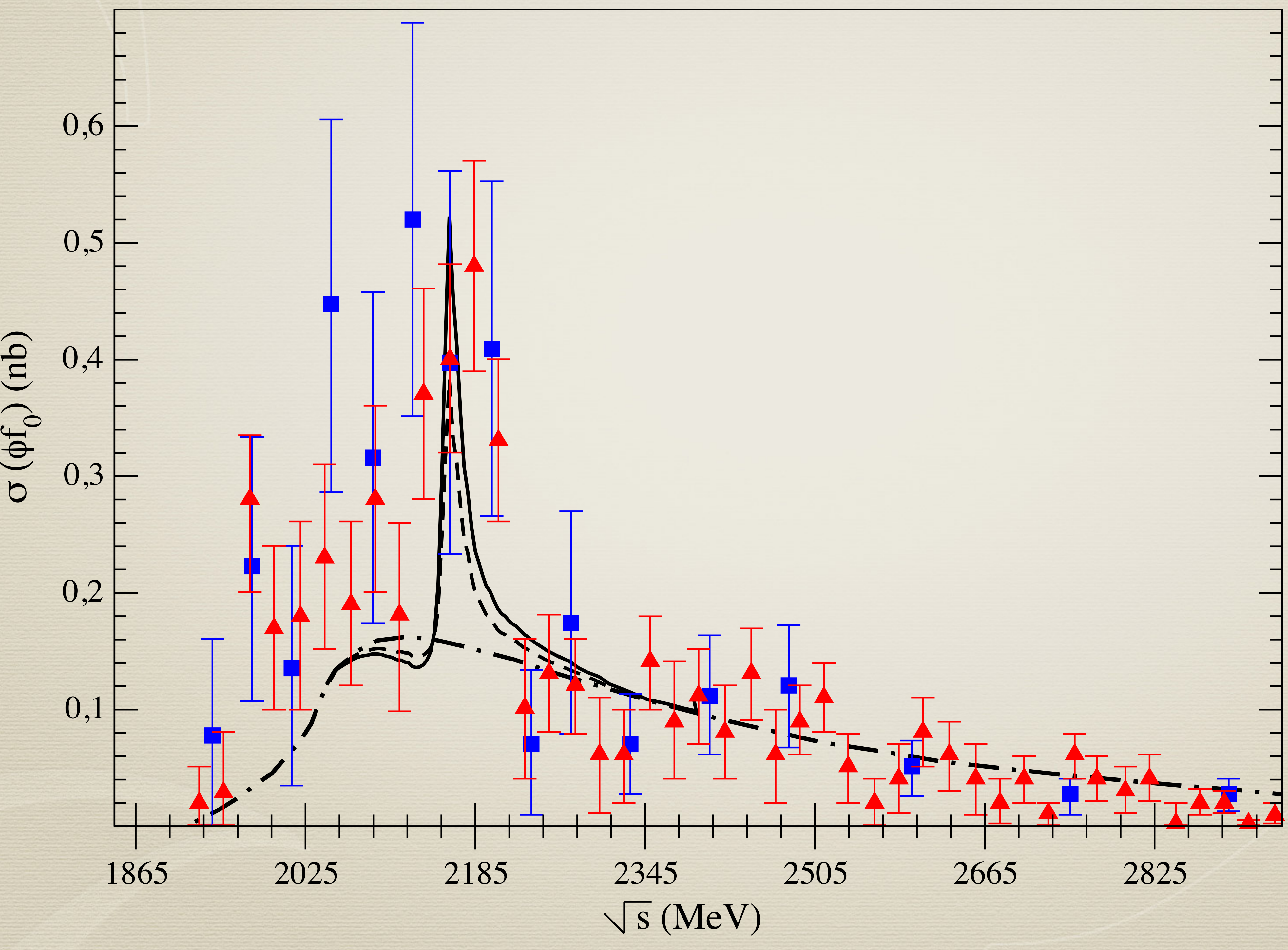


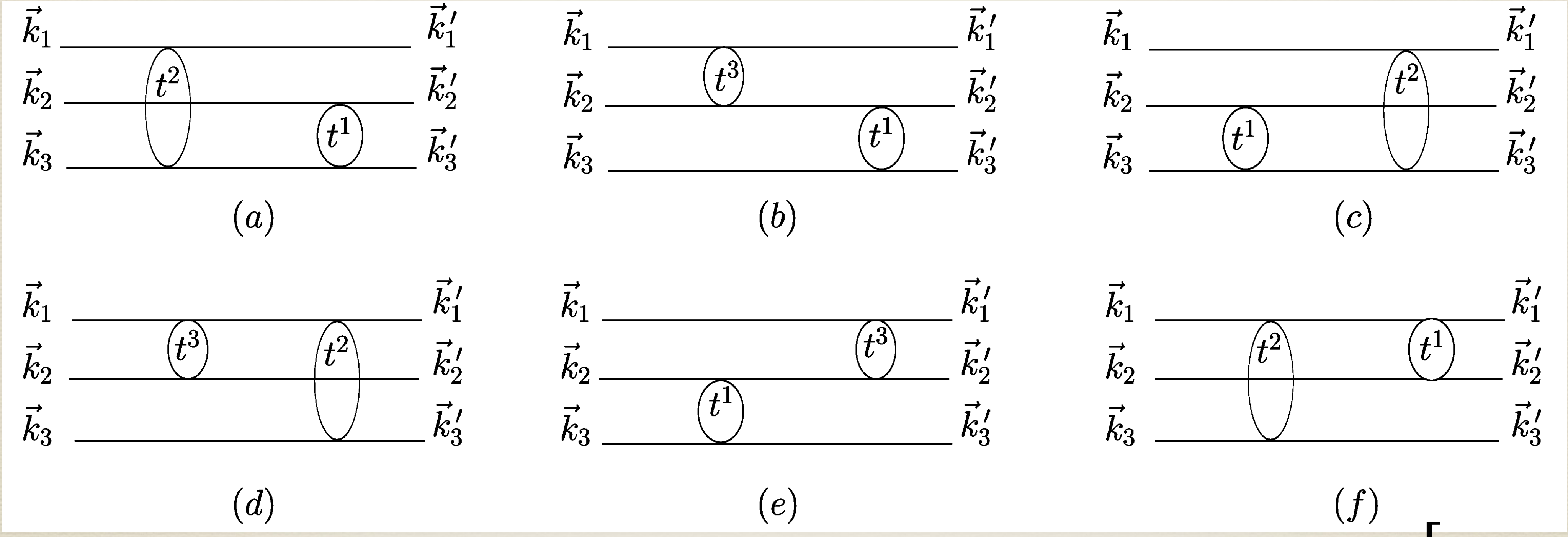
Exotic hadrons



Formalism

- Three-body scattering equations: Faddeev equations

Lowest order diagrams



$$T_1 = t_1 + t_1 G \left[T_2 + T_3 \right]$$

$$T_2 = t_2 + t_2 G \left[T_1 + T_3 \right]$$

$$T_3 = t_3 + t_3 G \left[T_1 + T_2 \right]$$

Formalism

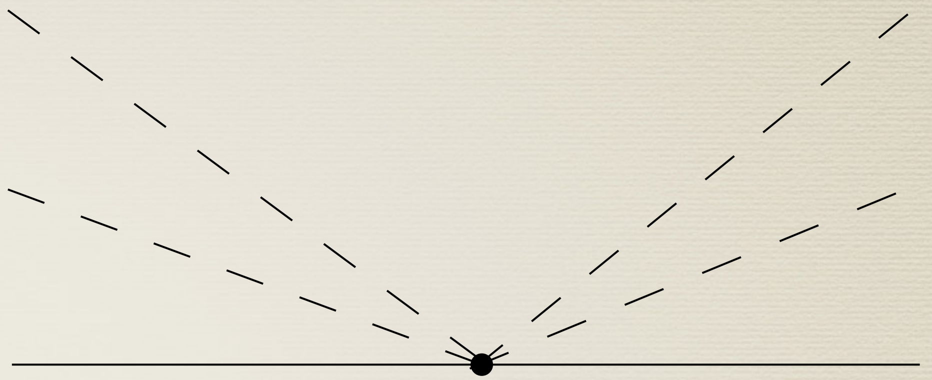
- Three-body scattering equations: Faddeev equations
- Various methods of solving these equations exist.
  - particle-dimer scattering
  - separable formulation

$$T_1 = t_1 + t_1 G \left[T_2 + T_3\right]$$
$$T_2 = t_2 + t_2 G \left[T_1 + T_3\right]$$
$$T_3 = t_3 + t_3 G \left[T_1 + T_2\right]$$

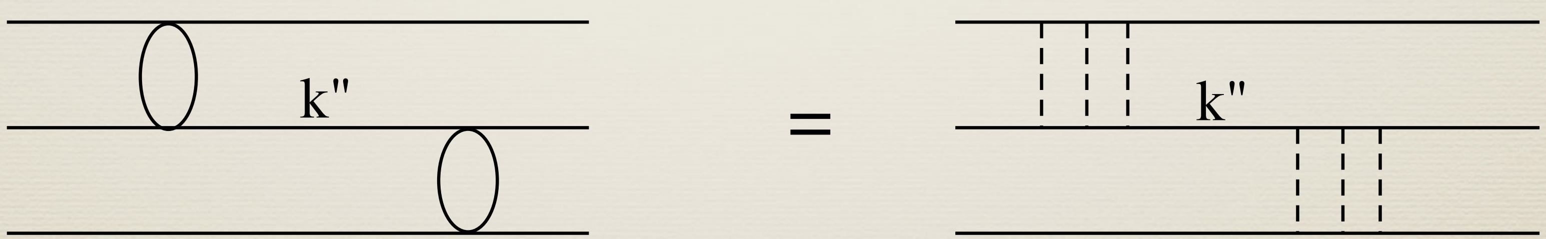
- Keeping in mind our interest in low energy scattering:
  - we calculate t_i by solving Bethe-Salpeter equation obtaining kernel from the lowest Lagrangian

Formalism

- More diagrams can contribute
- Three-Body contact terms

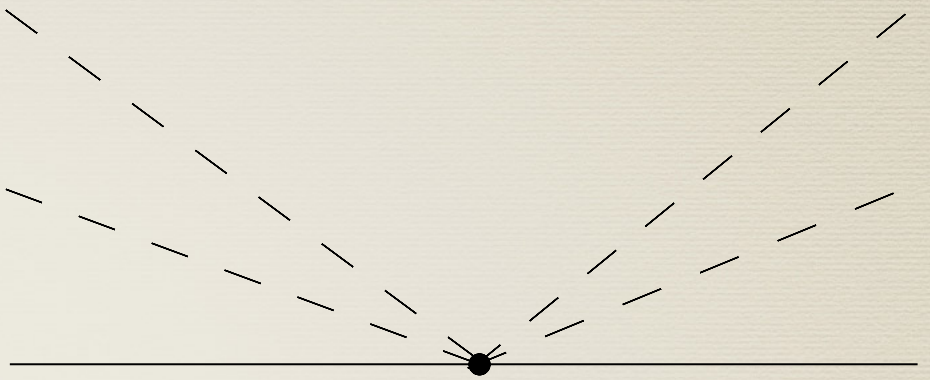


- We find more contact interactions



Formalism

- More diagrams can contribute
- Three-Body contact terms



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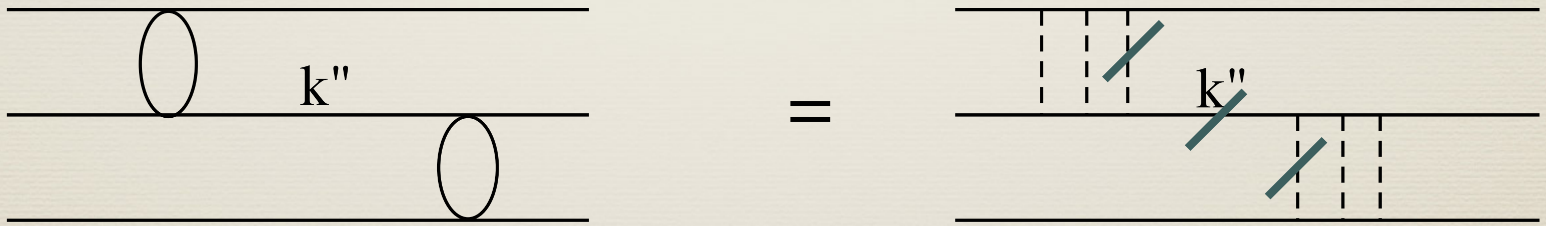
off-shell part



Formalism

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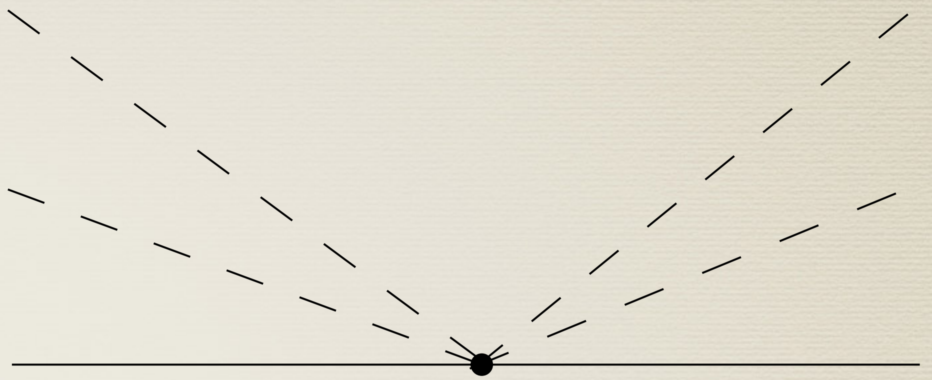
- Three-Body contact terms



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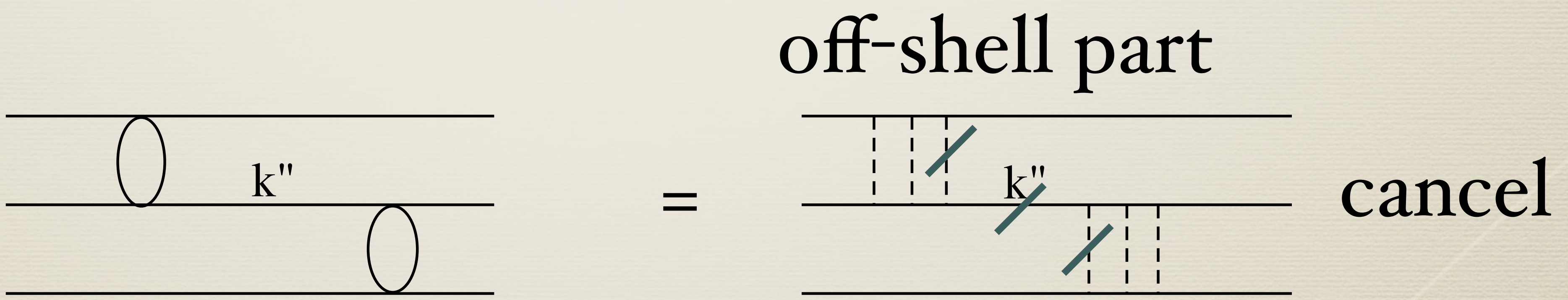

Formalism

- More diagrams can contribute
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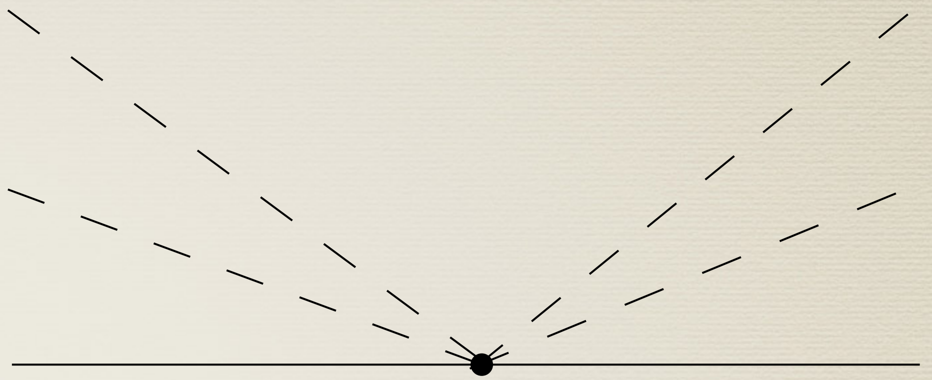

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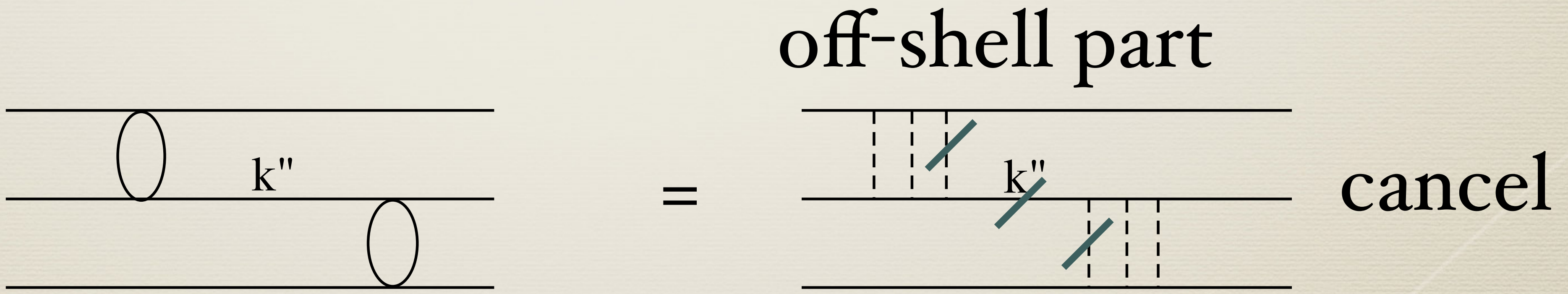

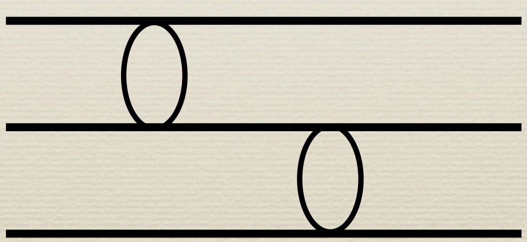

Formalism

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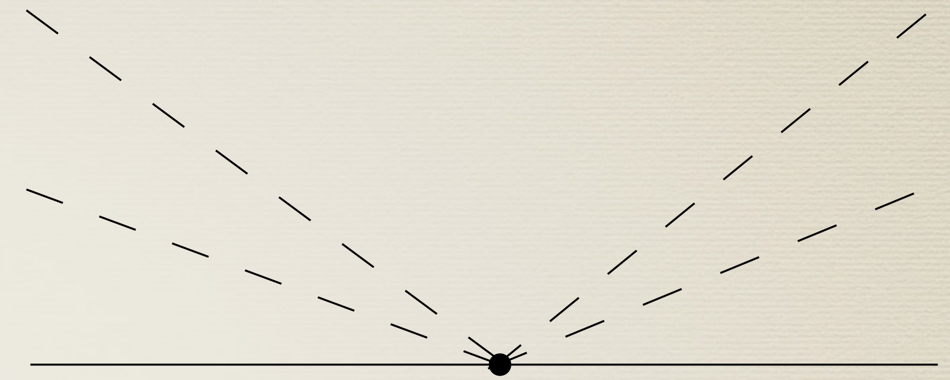
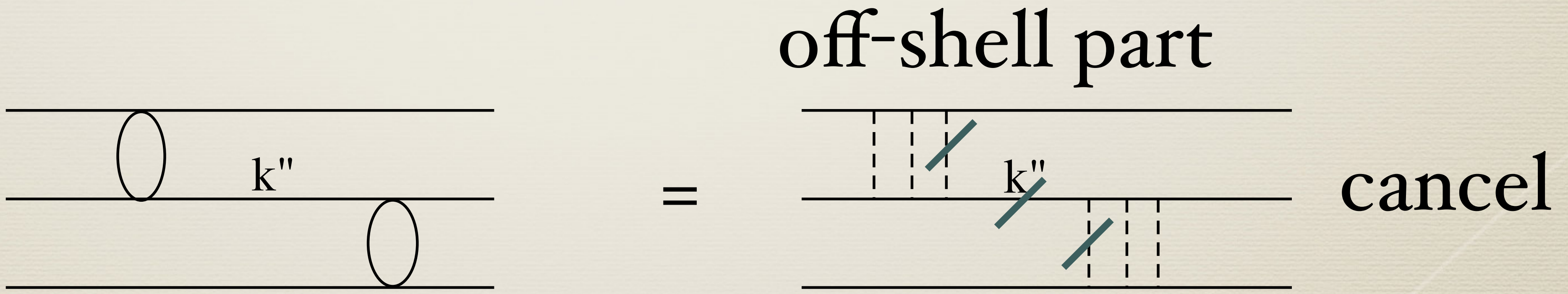
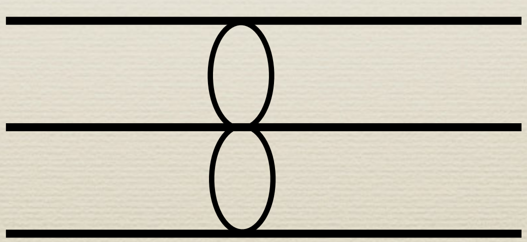
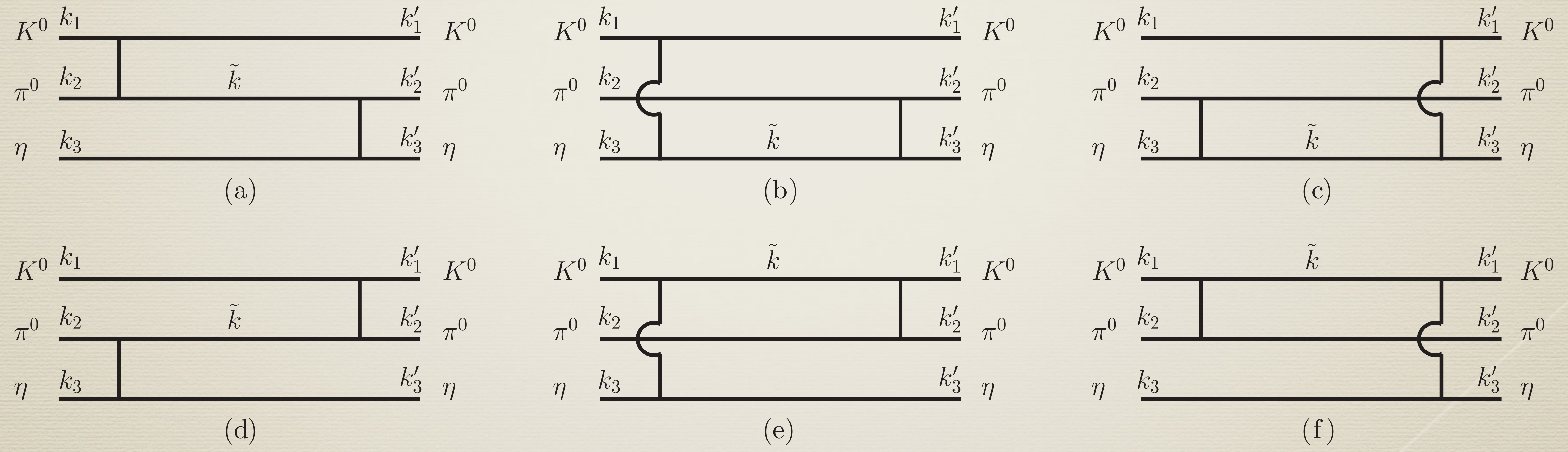






Formalism

- Example of cancellation:



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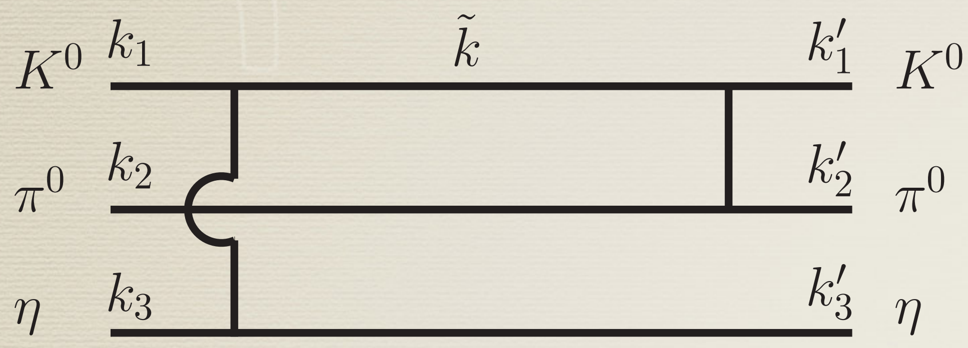

$$V_{K^0\pi^0 \to K^0\pi^0} = \frac{1}{12f^2}\left[-3t + \sum_i \left(k_i^2 - m_i^2\right)\right]$$

$$V_{K^0\pi^0 \to K^0\eta} = -\frac{1}{12\sqrt{3}f^2}\left[-9t + 8m_K^2 + m_\pi^2 + 3m_\eta^2 + 3\sum_i \left(k_i^2 - m_i^2\right)\right],$$

$$V_{K^0\eta \to K^0\eta} = \frac{1}{12f^2}\left[-9t + 6m_\eta^2 + 2m_\pi^2 + 3\sum_i \left(k_i^2 - m_i^2\right)\right]$$

$$T = \frac{1}{144f^4}\left[-9(k_3 - k_3')^2 + 6m_\eta^2 + 2m_\pi^2 + 3(\tilde{k}^2 - m_K^2)\right]\frac{1}{\tilde{k}^2 - m_k^2}\left[-3(k_2 - k_2')^2 + (\tilde{k}^2 - m_k^2)\right]$$

Formalism

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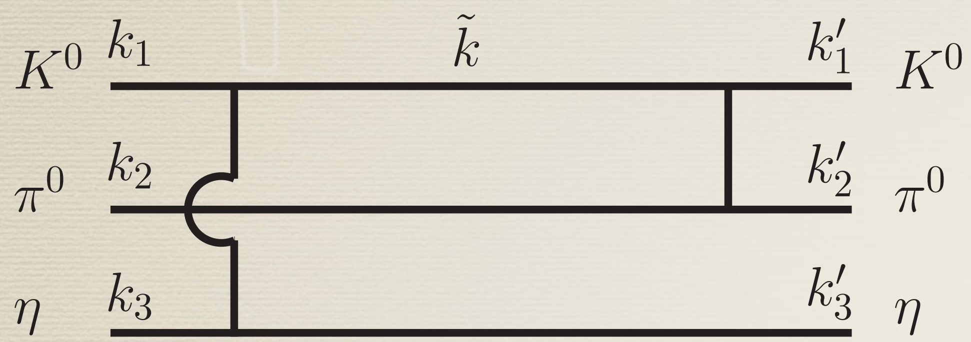

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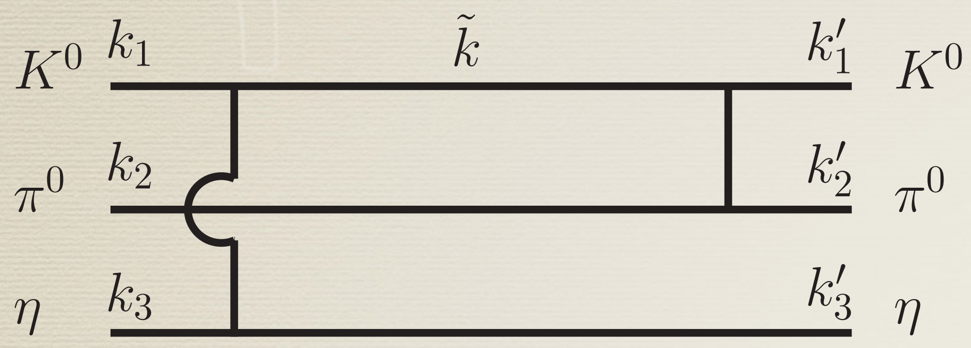
Formalism

- Example of cancellation:



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Formalism

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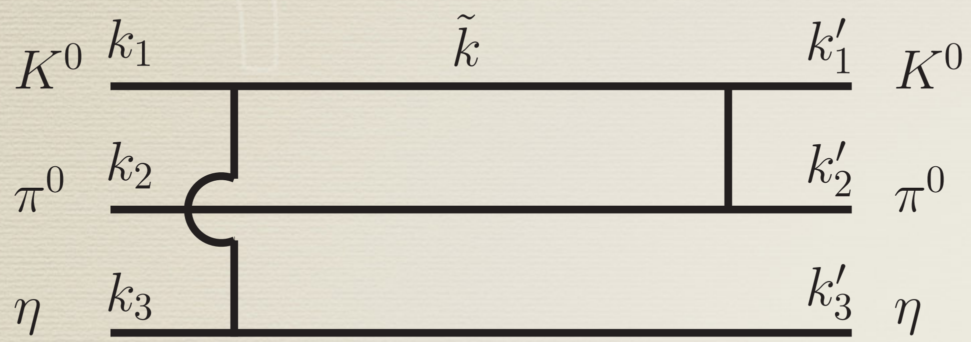

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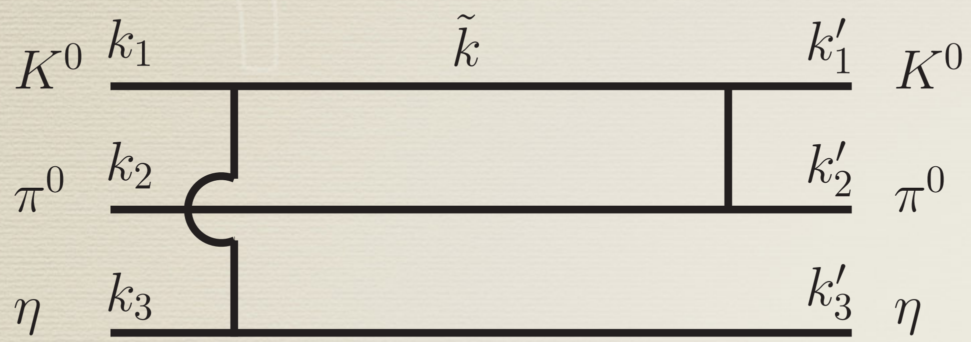

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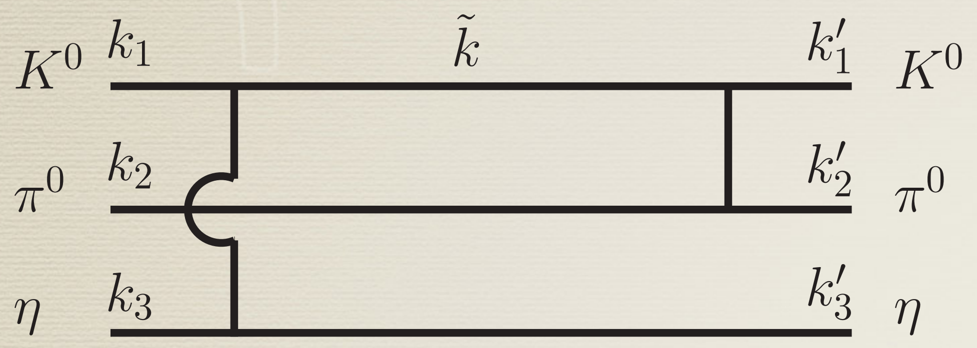

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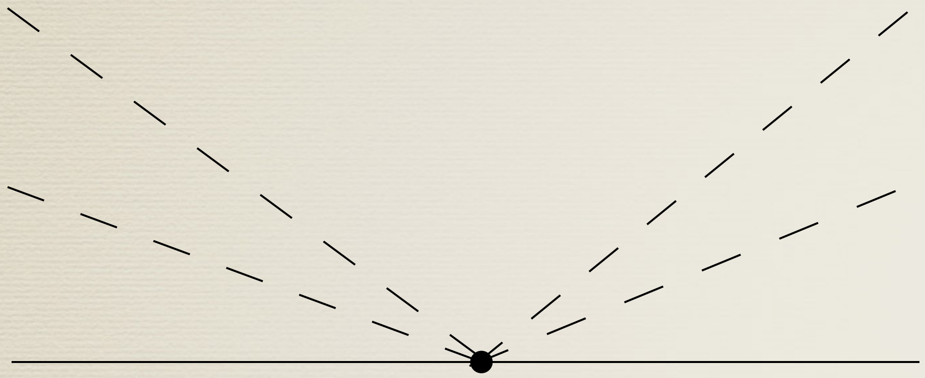
$$T_{\rm on} = -\frac{1}{48f^4}\left[-9\Delta {k_3}^2 + 6m_\eta^2 + 2m_\pi^2\right]\frac{\Delta {k_2}^2}{\Delta {k_2}^2 - 2k'_1\Delta k_2},$$

$$T_{\rm off} = \frac{1}{144f^4}\left[-9\Delta {k_3}^2 - 6\Delta {k_2}^2 - 6k'_1\Delta k_2 + 6m_\eta^2 + 2m_\pi^2\right].$$

Formalism

- Example of cancellation:



$$t_3 \;=\; \frac{1}{6f^4}\Delta {k_1}^2 - \frac{1}{90f^4}\big(16m_K^2 + 3m_\eta^2 + m_\pi^2\big).$$

$$\begin{aligned}\mathcal{L}_{6P} = \frac{1}{360f^4}\langle &-9\partial_\mu\Phi\Phi\partial^\mu\Phi\Phi^3 + 11\partial_\mu\Phi\Phi^2\partial^\mu\Phi\Phi^2 - 4\partial_\mu\Phi\Phi^3\partial^\mu\Phi\Phi + 2\partial_\mu\Phi\Phi^4\partial^\mu\Phi - 4\Phi\partial_\mu\Phi\Phi^3\partial^\mu\Phi \\ &+ 11\Phi^2\partial_\mu\Phi\Phi^2\partial^\mu\Phi - 9\Phi^3\partial_\mu\Phi\Phi\partial^\mu\Phi + 6\partial_\mu\Phi\partial^\mu\Phi\Phi^4 + 6\Phi^4\partial_\mu\Phi\partial^\mu\Phi - 15\Phi\partial_\mu\Phi\partial^\mu\Phi\Phi^3 + 5\Phi\partial_\mu\Phi\Phi\partial^\mu\Phi\Phi^2 \\ &- 10\Phi\partial_\mu\Phi\Phi^2\partial^\mu\Phi\Phi + 5\Phi^2\partial_\mu\Phi\Phi\partial^\mu\Phi\Phi - 15\Phi^3\partial_\mu\Phi\partial^\mu\Phi\Phi + 20\Phi^2\partial_\mu\Phi\partial^\mu\Phi\Phi^2 - 2M\Phi^6\rangle.\end{aligned}$$

Formalism

- SUM of all contact like diagrams

$$\sum_{i=g}^{h} T_{\rm off}^{(i)} + \sum_{i=b}^{c} t_{3\,{\rm off}}^{(i)} = \frac{1}{24f^4}\left[-10m_\eta^2 - 10m_\pi^2 + 2m_K^2 + 5k_3k_2' + 5k_2k_3' - 5k_2k_3 - 5k_2'k_3' + \Delta k_1(\Delta k_2 + \Delta k_3)\right]$$
$$+ \frac{1}{36f^4}(13m_K^2 + 2m_\pi^2 + 6m_\eta^2) = \frac{1}{24f^4}\left[-10m_\eta^2 - 10m_\pi^2 + 2m_K^2 - 5\Delta k_2\Delta k_3 - \Delta k_1{}^2\right]$$
$$+ \frac{1}{36f^4}(13m_K^2 + 2m_\pi^2 + 6m_\eta^2),$$

$$\sum_{i=a}^{f} T_{\rm off}^{(i)} + t_3^{(a)} = \frac{1}{24f^4}(\Delta k_1{}^2 + 5\Delta k_2\Delta k_3) - \frac{1}{180f^4}(32m_K^2 - 9m_\eta^2 + 37m_\pi^2)$$

<5% of the on shell contribution in a realistic case

$$\sum_{i=a}^{h} T_{\rm off}^{(i)} + \sum_{i=a}^{c} t_{3\,{\rm off}}^{(i)} = -\frac{m_\pi^2}{2f^4},$$

zero in the chiral limit!!

Formalism

$$T_R^{12} = t^1 g^{12} t^2 + t^1 \left[G^{121} T_R^{21} + G^{123} T_R^{23} \right]$$
$$T_R^{13} = t^1 g^{13} t^3 + t^1 \left[G^{131} T_R^{31} + G^{132} T_R^{32} \right]$$
$$T_R^{21} = t^2 g^{21} t^1 + t^2 \left[G^{212} T_R^{12} + G^{213} T_R^{13} \right]$$
$$T_R^{23} = t^2 g^{23} t^3 + t^2 \left[G^{231} T_R^{31} + G^{232} T_R^{32} \right]$$
$$T_R^{31} = t^3 g^{31} t^1 + t^3 \left[G^{312} T_R^{12} + G^{313} T_R^{13} \right]$$
$$T_R^{32} = t^3 g^{32} t^2 + t^3 \left[G^{321} T_R^{21} + G^{323} T_R^{23} \right]$$

Six coupled matrix equations

- Solve the Faddeev equations (connected diagrams) using the on shell part of the t-matrices.
- Solve Bethe-Salpeter equation in on-shell factorization approach to obtain (coupled channel formalism)

Formalism

$$T_R^{12} = t^1 g^{12} t^2 + t^1 \left[G^{121}\, T_R^{21} + G^{123}\, T_R^{23} \right]$$

$$T_R^{13} = t^1 g^{13} t^3 + t^1 \left[G^{131}\, T_R^{31} + G^{132}\, T_R^{32} \right]$$

$$T_R^{21} = t^2 g^{21} t^1 + t^2 \left[G^{212}\, T_R^{12} + G^{213}\, T_R^{13} \right]$$

$$T_R^{23} = t^2 g^{23} t^3 + t^2 \left[G^{231}\, T_R^{31} + G^{232}\, T_R^{32} \right]$$

$$T_R^{31} = t^3 g^{31} t^1 + t^3 \left[G^{312}\, T_R^{12} + G^{313}\, T_R^{13} \right]$$

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$$g^{ij}(\vec{k_i}', \vec{k_j}) = \left(\prod_{r=1}^{D} \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(\vec{k_i}') - E_l(\vec{k_i}' + \vec{k_j}) - E_j(\vec{k_j})},$$

$$l \neq i, \ l \neq j = 1, 2, 3,$$

$$G^{i\,j\,k} = \int \frac{d^3 k''}{(2\pi)^3} \frac{N_l}{2E_l} \frac{N_m}{2E_m} \frac{F^{i\,j\,k}(\sqrt{s}, \vec{k}'')}{\sqrt{s_{lm}} - E_l(\vec{k}'') - E_m(\vec{k}'') + i\epsilon}$$

Formalism

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$$T_R^{13} = t^1 g^{13} t^3 + t^1 \left[G^{131} T_R^{31} + G^{132} T_R^{32} \right]$$

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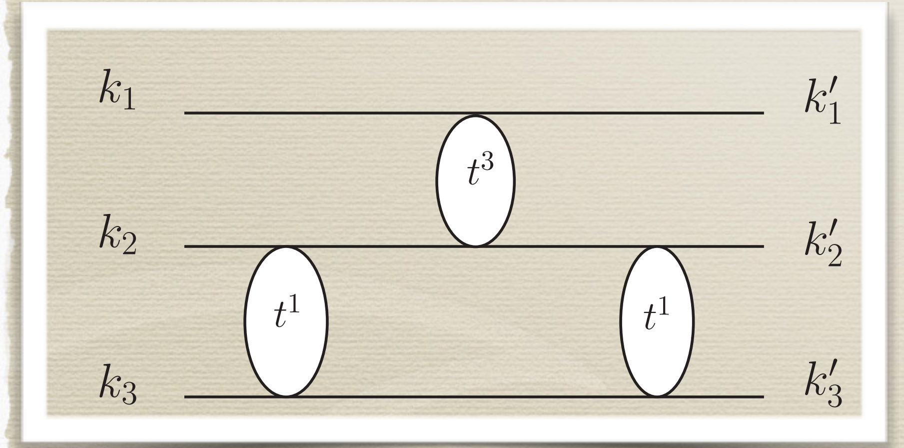


$$G^{i\,j\,k} = \int \frac{d^3k''}{(2\pi)^3} \frac{N_l}{2E_l} \frac{N_m}{2E_m} \frac{F^{i\,j\,k}(\sqrt{s}, \vec{k}'')}{\sqrt{s_{lm}} - E_l(\vec{k}'') - E_m(\vec{k}'') + i\epsilon}$$

$$F^{i\,j\,k} = t^j(\sqrt{s_{\text{int}}}(\vec{k}'')) \left(\frac{g^{jk}|_{\text{off-shell}}}{g^{jk}|_{\text{on-shell}}} \right) \left[t^j(\sqrt{s_{\text{int}}}(\vec{k}_{j'})) \right]^{-1}$$

Formalism

- The final three-body t-matrix needs to be projected on an isospin base
- Defined in terms of total isospin and isospin of a two-body subsystem
- Example: $\pi\pi N$-$\pi K\Lambda$-$\pi K\Sigma$-$\pi\eta N$ coupled channels

$$
\begin{aligned}
|\pi^0\,\pi^0\,n\rangle &= |1,0\rangle \otimes |1,0\rangle \otimes |1/2,-1/2\rangle \\
&= \left\{\sqrt{\frac{2}{3}}|I_{\pi\pi}=2, I^z_{\pi\pi}=0\rangle - \sqrt{\frac{1}{3}}|I_{\pi\pi}=0, I^z_{\pi\pi}=0\rangle\right\} \\
&\quad \otimes|1/2,-1/2\rangle \\
&= \sqrt{\frac{2}{5}}|I=5/2, I_{\pi\pi}=2\rangle + \frac{2}{\sqrt{15}}|I=3/2, I_{\pi\pi}=2\rangle \\
&\quad -\sqrt{\frac{1}{3}}|I=1/2, I_{\pi\pi}=0\rangle.
\end{aligned}
$$

$$
|\pi^0\,\pi^0\,n\rangle = \sqrt{\frac{2}{5}}|5/2,2\rangle + \frac{2}{\sqrt{15}}|3/2,2\rangle - \sqrt{\frac{1}{3}}|1/2,0\rangle
$$

Formalism

$$
\begin{aligned}
|\pi^+ \pi^- n\rangle &= -\sqrt{\frac{1}{10}}|5/2,2\rangle - \sqrt{\frac{1}{15}}|3/2,2\rangle \\
&\quad -\sqrt{\frac{1}{3}}|3/2,1\rangle - \sqrt{\frac{1}{6}}|1/2,1\rangle - \sqrt{\frac{1}{3}}|1/2,0\rangle, \\
|\pi^- \pi^+ n\rangle &= -\sqrt{\frac{1}{10}}|5/2,2\rangle - \sqrt{\frac{1}{15}}|3/2,2\rangle \\
&\quad +\sqrt{\frac{1}{3}}|3/2,1\rangle + \sqrt{\frac{1}{6}}|1/2,1\rangle - \sqrt{\frac{1}{3}}|1/2,0\rangle, \\
|\pi^- \pi^0 p\rangle &= \sqrt{\frac{1}{5}}|5/2,2\rangle - \sqrt{\frac{3}{10}}|3/2,2\rangle \\
&\quad -\sqrt{\frac{1}{6}}|3/2,1\rangle + \sqrt{\frac{1}{3}}|1/2,1\rangle, \\
|\pi^0 \pi^- p\rangle &= \sqrt{\frac{1}{5}}|5/2,2\rangle - \sqrt{\frac{3}{10}}|3/2,2\rangle \\
&\quad +\sqrt{\frac{1}{6}}|3/2,1\rangle - \sqrt{\frac{1}{3}}|1/2,1\rangle.
\end{aligned}
$$

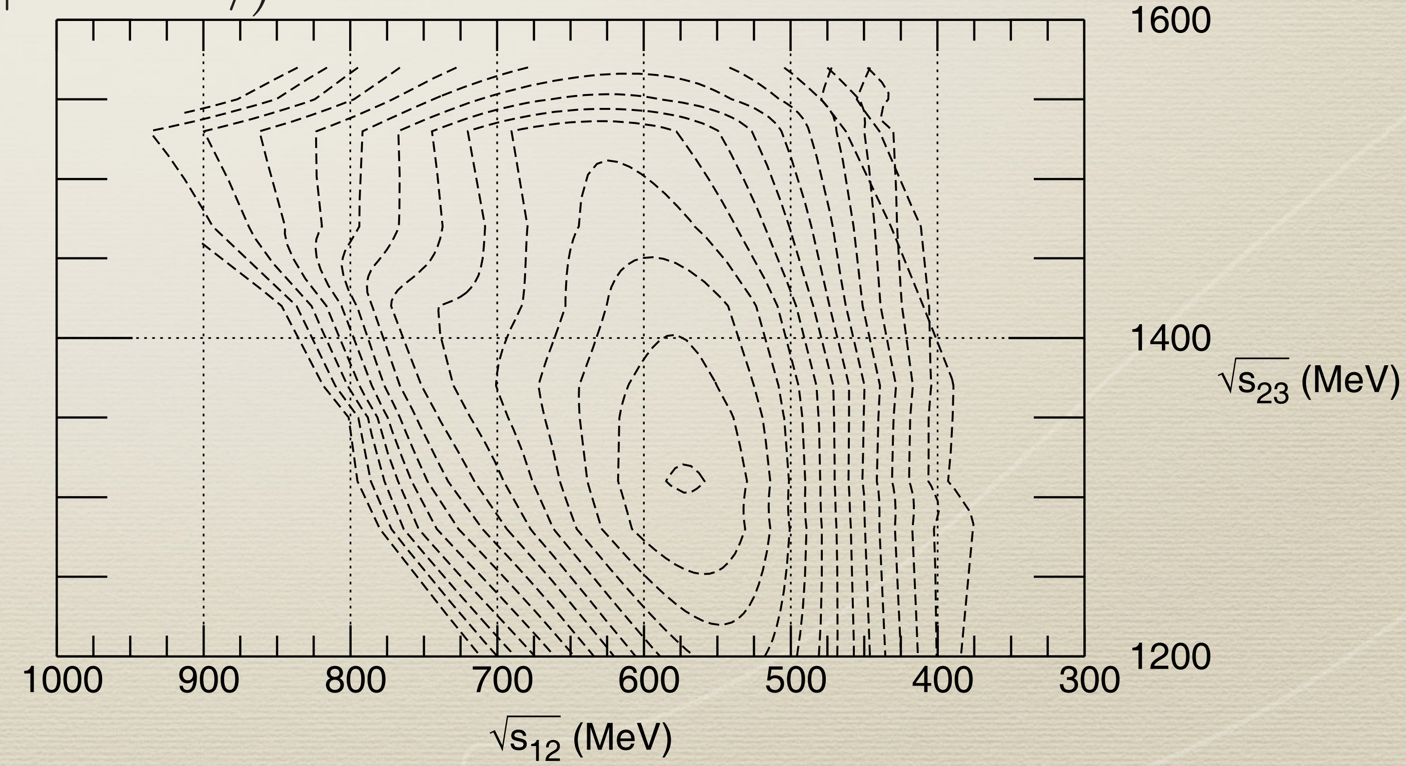
Formalism

$$|5/2,2\rangle = \sqrt{\frac{1}{5}}\Big(\sqrt{2}|\pi^0\,\pi^0\,n\rangle + |\pi^0\,\pi^-\,p\rangle + |\pi^-\,\pi^0\,p\rangle - \sqrt{\frac{1}{2}}|\pi^+\,\pi^-\,n\rangle - \sqrt{\frac{1}{2}}|\pi^-\,\pi^+\,n\rangle\Big),$$

$$|3/2,2\rangle = \sqrt{\frac{1}{15}}\Big(2|\pi^0\,\pi^0\,n\rangle - \frac{3}{\sqrt{2}}|\pi^0\,\pi^-\,p\rangle - \frac{3}{\sqrt{2}}|\pi^-\,\pi^0\,p\rangle - |\pi^+\,\pi^-\,n\rangle - |\pi^-\,\pi^+\,n\rangle\Big),$$

$$|1/2,0\rangle = -\sqrt{\frac{1}{3}}(|\pi^0\,\pi^0\,n\rangle + |\pi^+\,\pi^-\,n\rangle + |\pi^-\,\pi^+\,n\rangle).$$

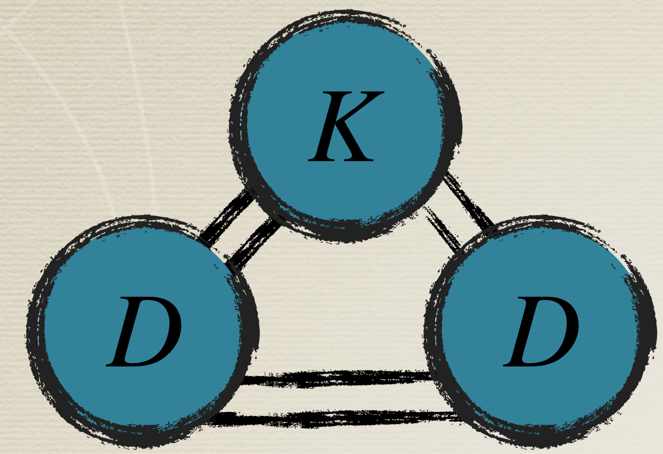
$N^*(1710) \sim \sigma(600)N$



Recent applications

- In recent times, focus has widened to charm, bottom hadrons.
- Lots of attention being paid to explicit charm, double charm, etc. systems (T_{cc}, Ξ_{cc}^{+}, Ξ_{cc}^{++}, Ω_{cc}^{+}, $\Xi_{cc}D$, $\Xi_{cc}D^*$ $\Xi_{cc}\Lambda_c$, $\Xi_{cc}\Sigma_c$, $BD\bar{D}$, BDD, BBB^*)
- With data available in 3-5 GeV, the non-charm, non-bottom physics can also be explored
- Example: Kaon physics, last kaon listed K(3100), +25 years ago.
- There is data on processes like $B \to J/\psi K\pi\pi$

Recent applications

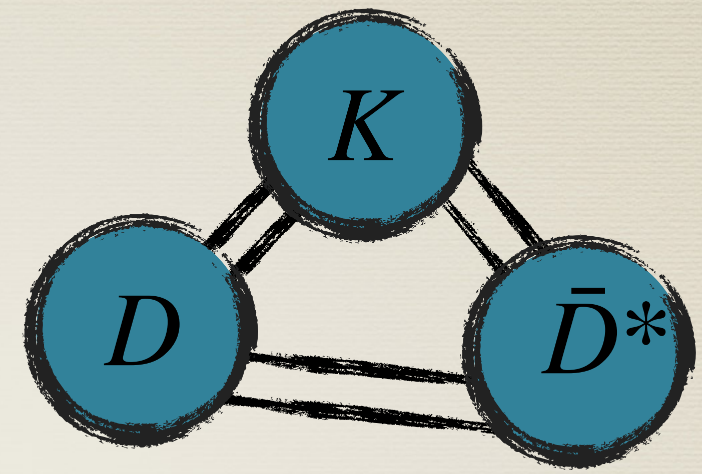


$D - D^*_{s0}(2317)$

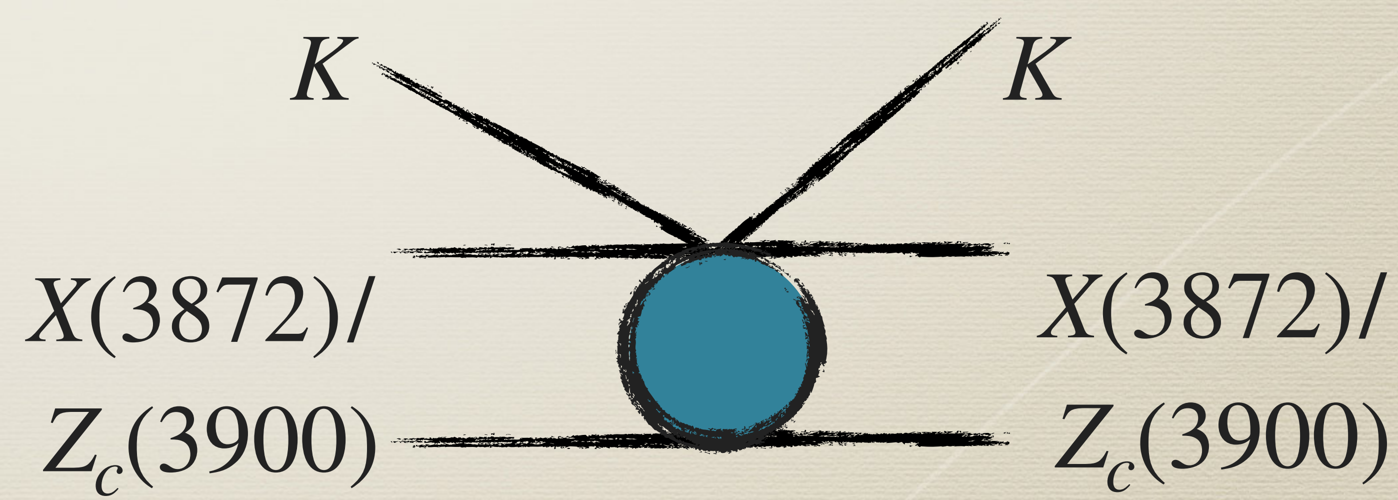
$DDK,\ DD_s\eta,\ DD_s\pi$

$DK,\ D_s\eta,\ D_s\pi \rightarrow D^*_{s0}(2317)$

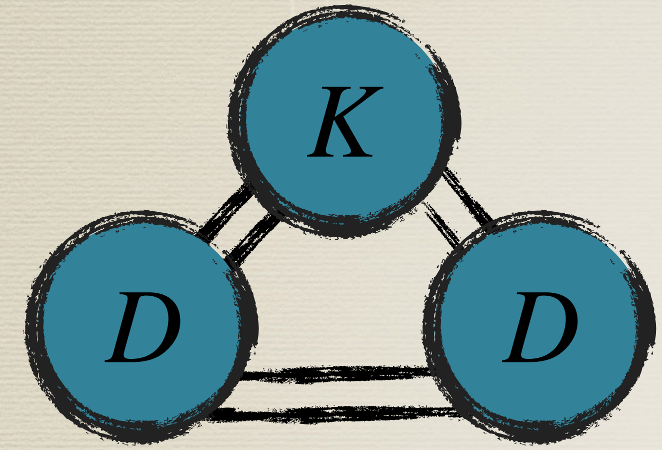
DD, DD_s → Vector meson exchange *t*, *u* channels



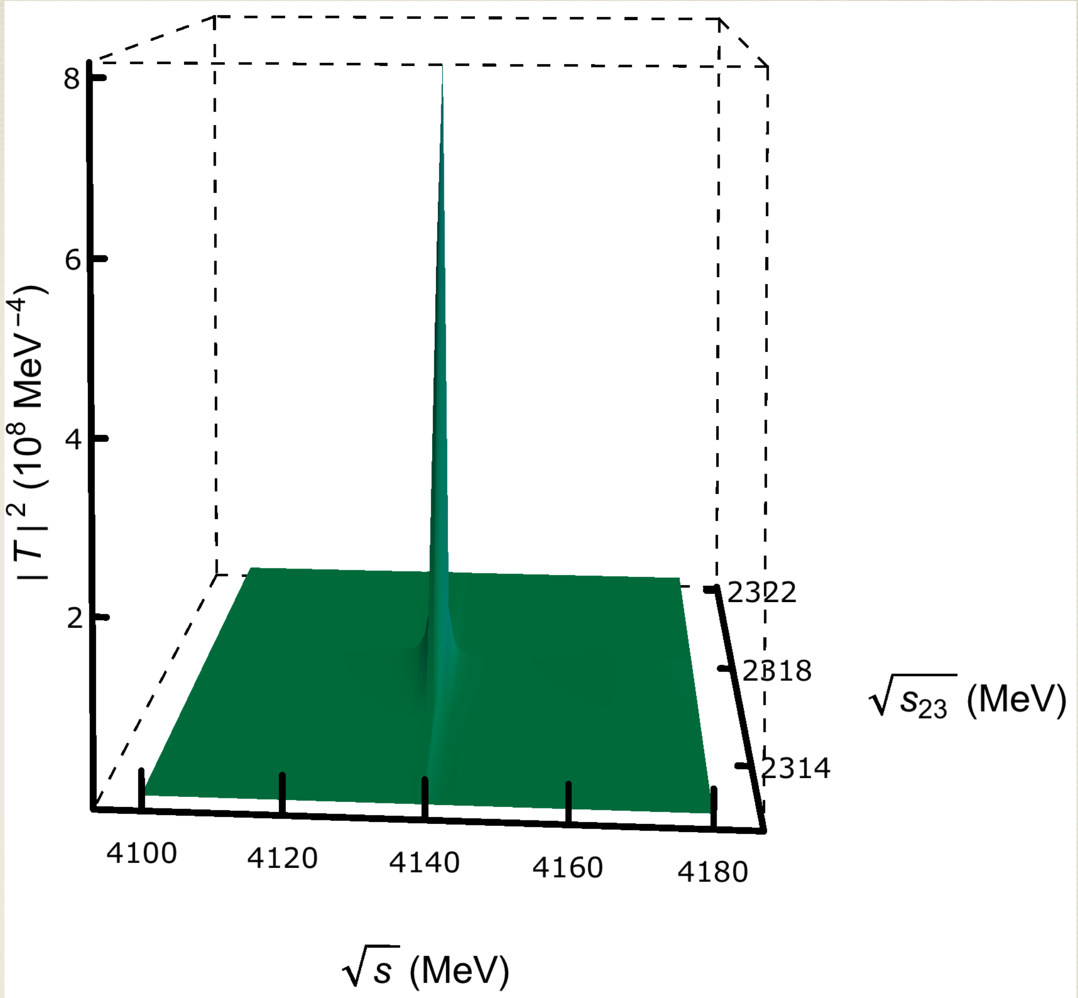
$DD^*K \rightarrow D\bar{D}^*K$



Results: Recent applications



Phys. Rev. D99, 076017 (2019)



I=1/2, 4140 MeV

Results: Recent applications

Effective $D - D_{s0}^*(2317)$

Size estimation:

$$\langle \vec{x} | \psi \rangle = \alpha \sqrt{\frac{2}{\pi}} \frac{1}{r} \mathrm{Im} \left[\int_0^\Lambda dp\, p \frac{e^{ipr}}{M_R - M_D - M_{D_{s0}^*} - \frac{p^2}{2\mu}} \right]$$

$$-\left[\left. \frac{dG}{ds} \right|_{s=M_R^2} \right]^{-1} = 64\pi^3 \mu B^2 \alpha^2$$

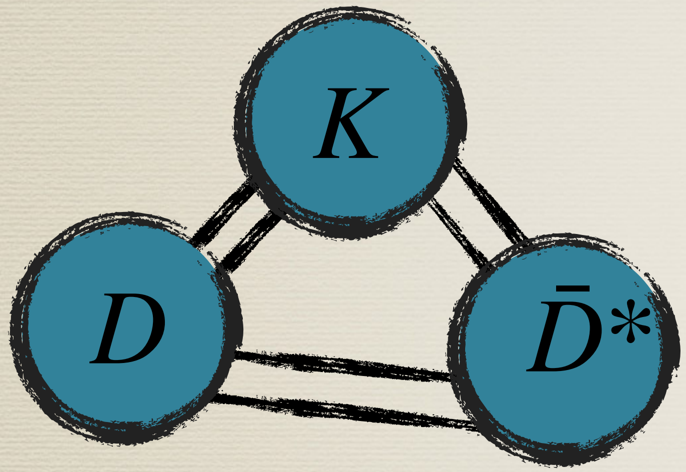
$$\sqrt{\langle r^2 \rangle} \sim 1.0\text{–}1.4 \text{ fm}$$

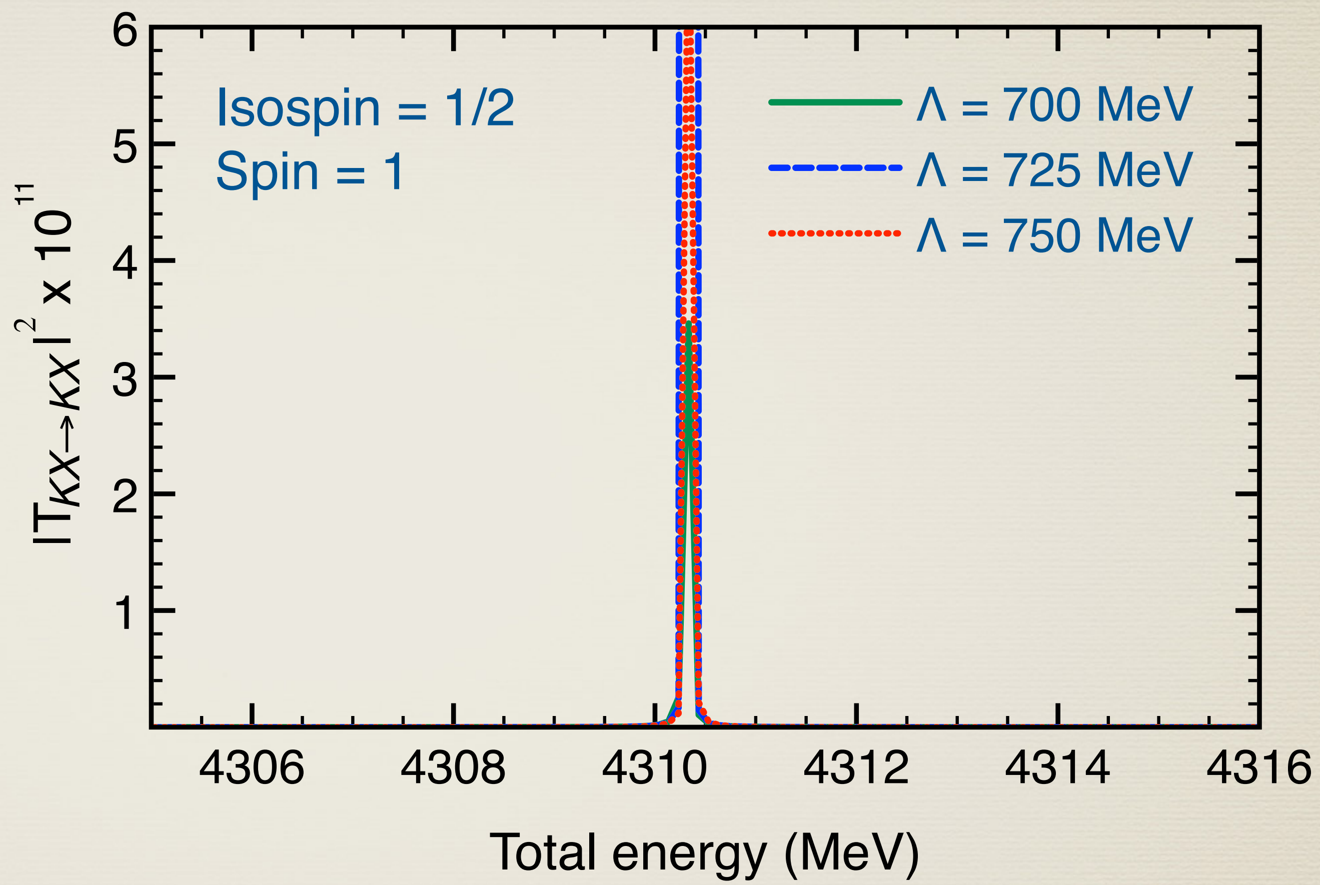
Sanchez, Geng, Lu, Hyodo, Valderrama PRD98, 054001 (2018): 1.0-1.6 fm

Wu, Liu, Geng, Hiyama, Valderrama, Gaussian Expansio
Method, arxiv: 1906.11995 [hep-ph].

Gamermann, Nieves, Oset, Arriola, PRD81, 014029 (2010)

Results: Recent applications



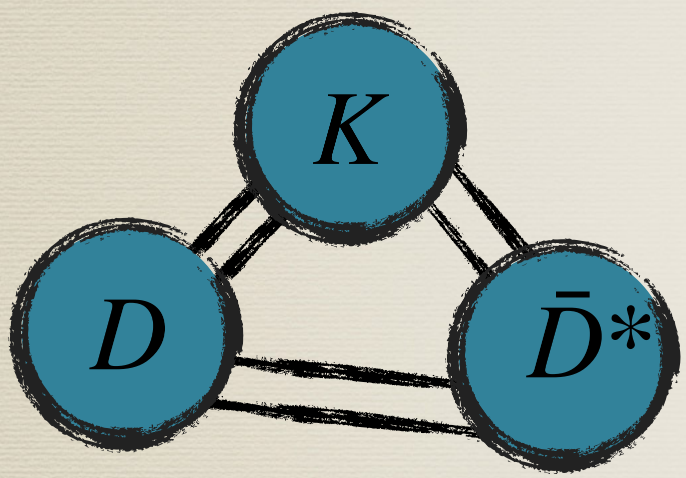


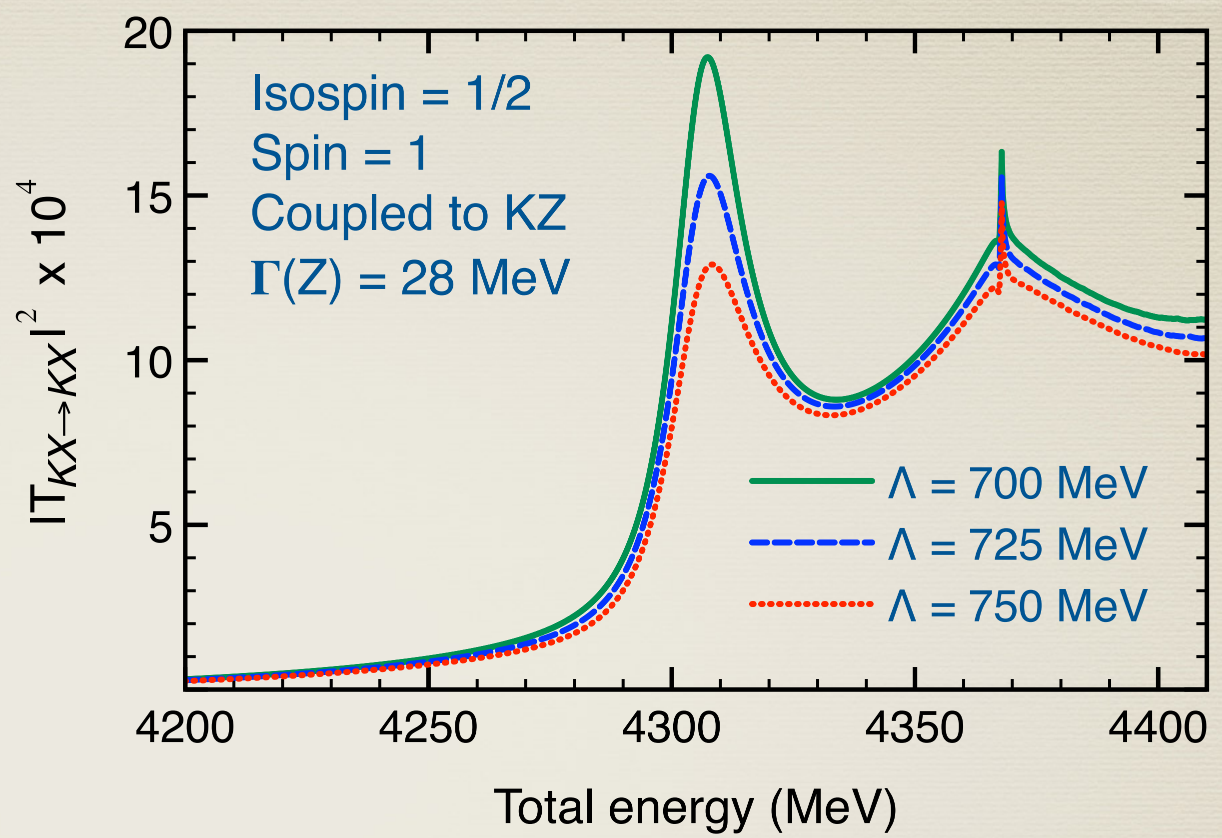
I=1/2, 4307 MeV,

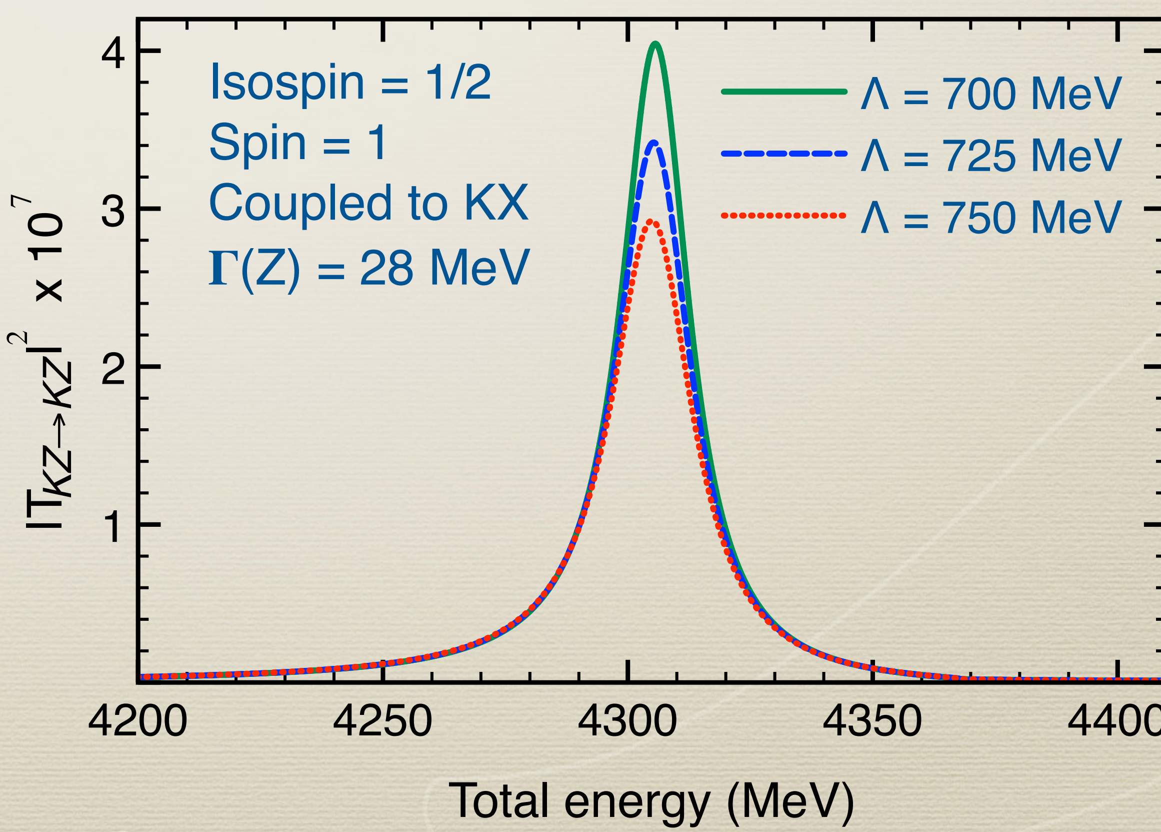
$\Gamma \sim 18$ MeV

Phys. Lett. B785, 112-117(2018)

Results: Recent applications







I=1/2, 4307 MeV,

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Phys. Lett. B785, 112-117(2018)

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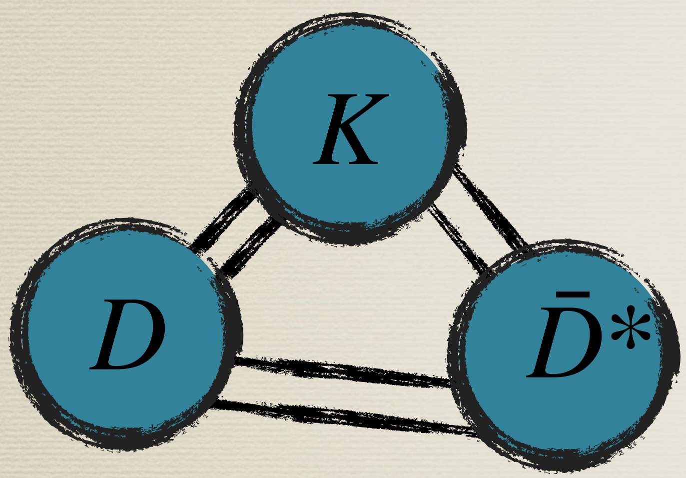

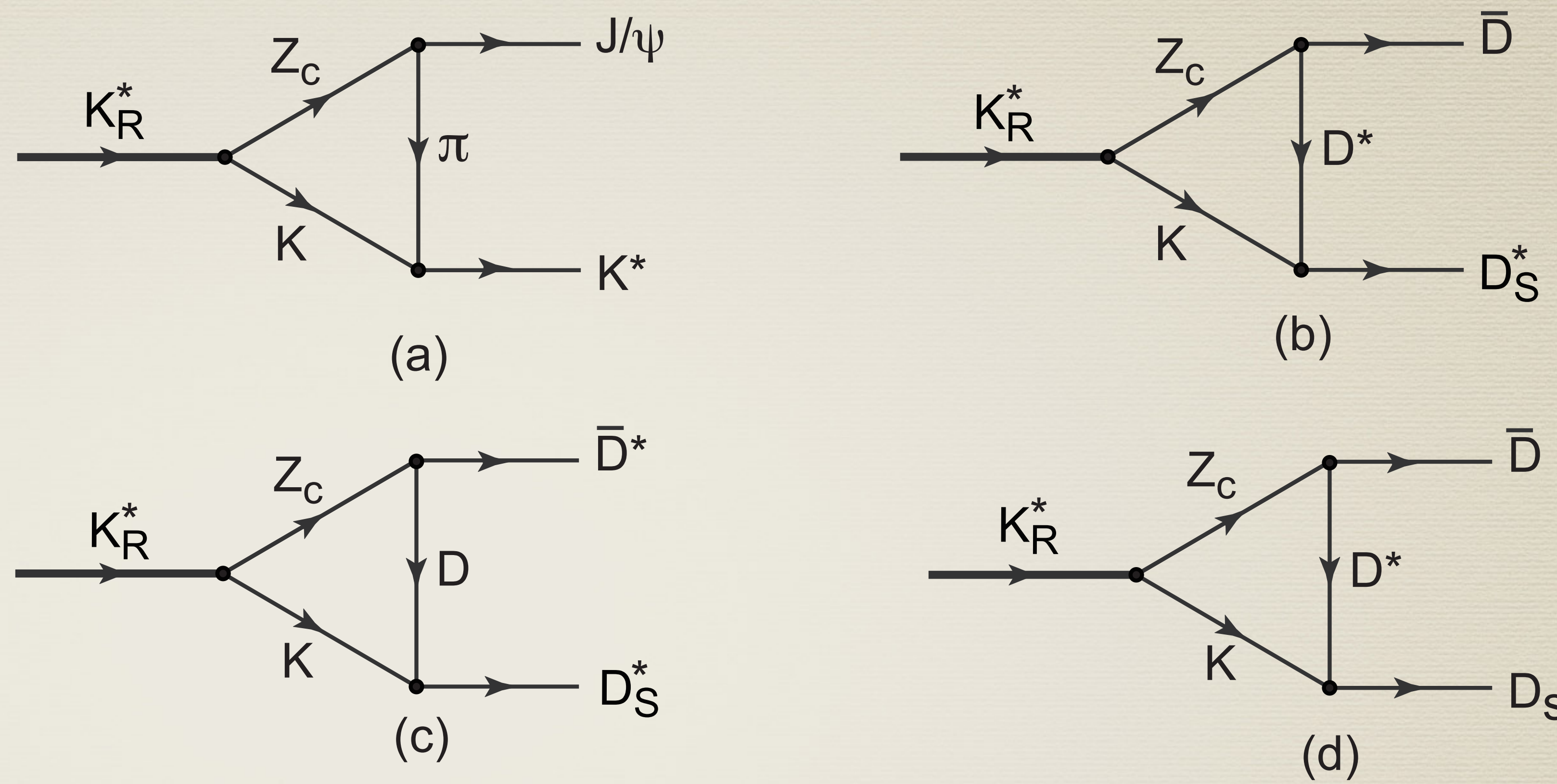

I=1/2, 4307 MeV,

$\Gamma \sim 18$ MeV

$\Gamma_a \sim 7$ MeV, $\Gamma_b \sim \Gamma_c \sim 0.5$ MeV,

$\Gamma_d \sim 1$ MeV

Phys. Lett. B785, 112-117(2018)

JHEP 1905, 103 (2019)

Outlook/work in progress: