coupled-channel scattering from lattice QCD

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Science Foundation Ireland
For what's next
THE ROYAL SOCIETY

two-to-two scattering of (coupled) pseudoscalars

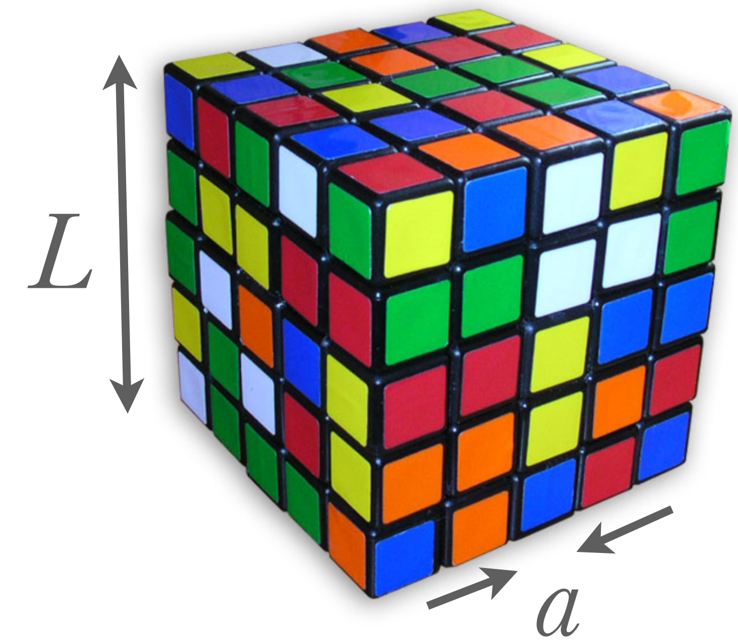
- we have extracted 3x3 scattering matrices (can do more)
- straightforward to extract the first few partial waves

typically heavier-than-physical pion masses

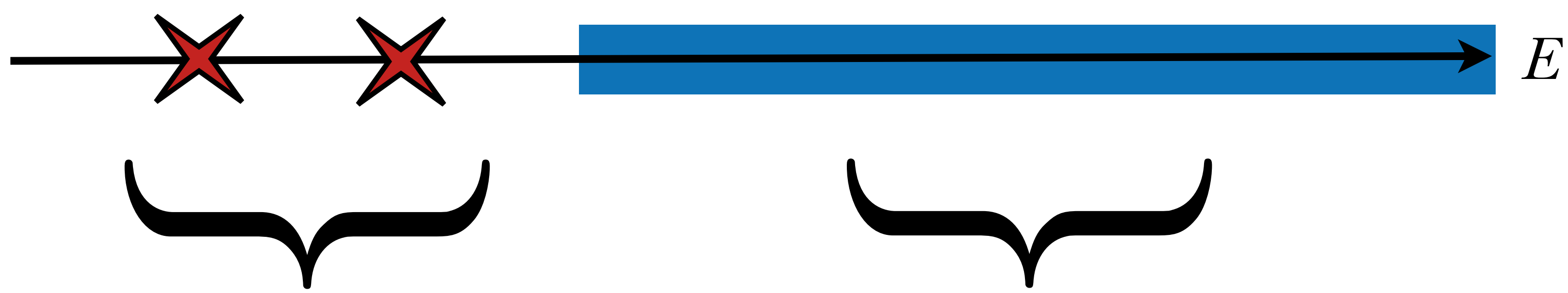
- choice to avoid many-body effects

three-body formalism is rapidly maturing

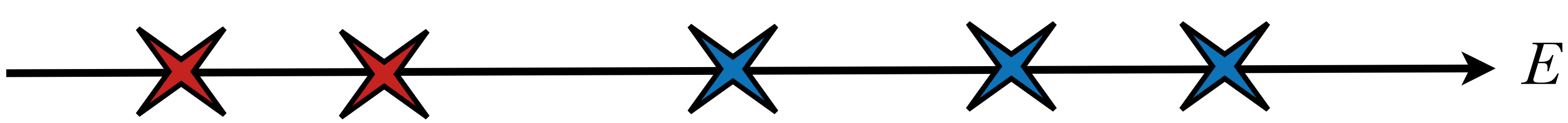
- to allow for lighter pions, higher resonances



Infinite volume

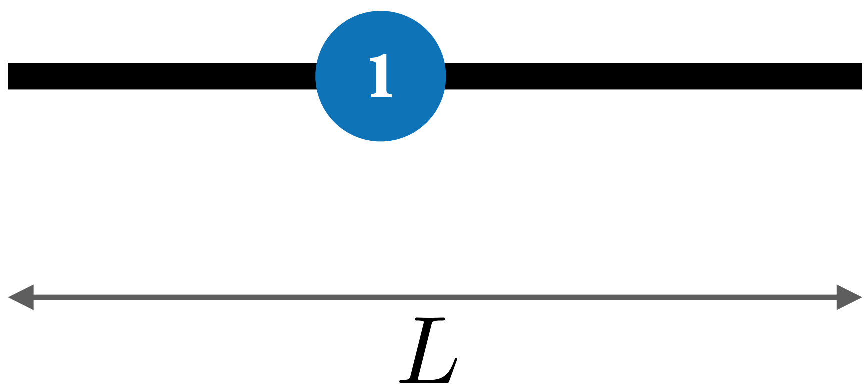


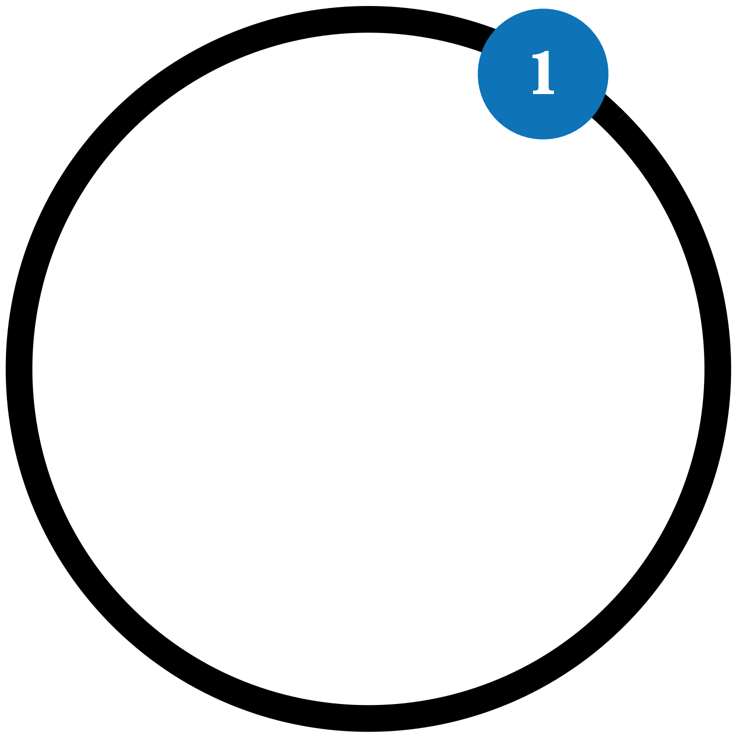
Finite volume



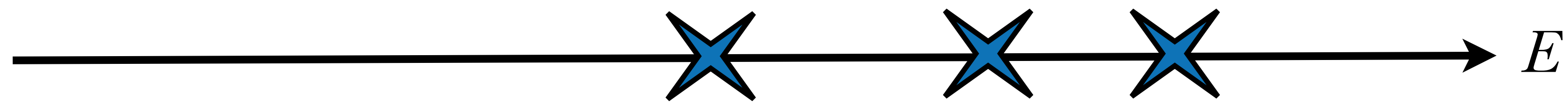
Momentum is quantised - no continuum $\vec{p} = \frac{2\pi}{L}\vec{n}$

1-dimensional QM, periodic BC, single particle:

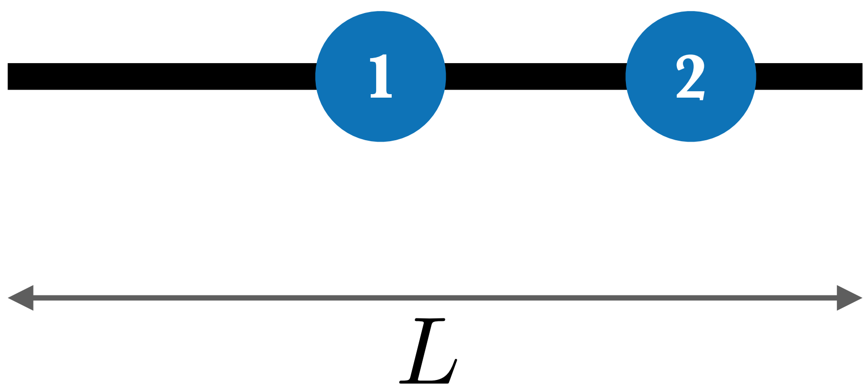


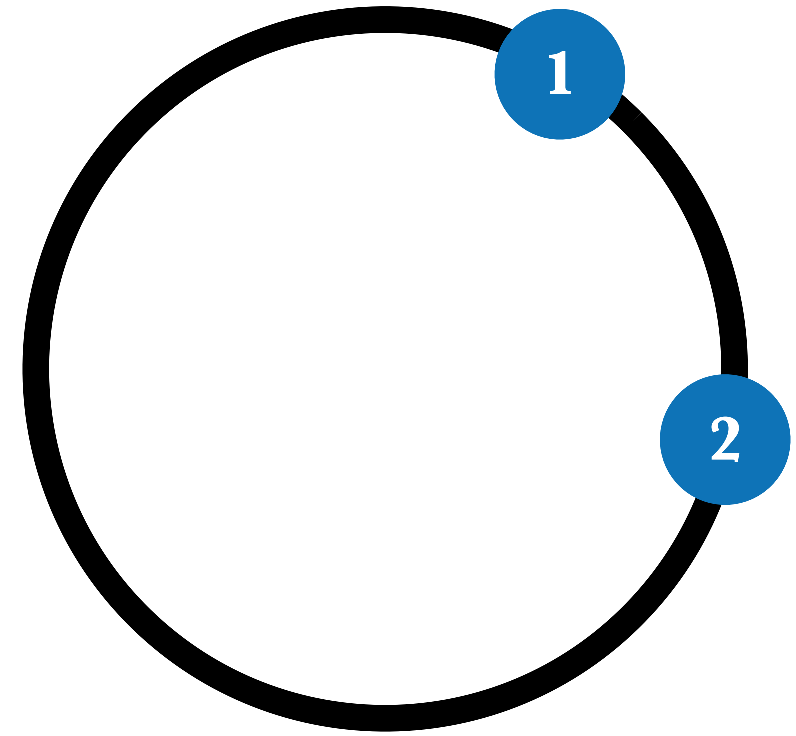


momentum is quantised: $p = \frac{2\pi n}{L}$



1-dimensional QM, periodic BC, two particles, no interactions

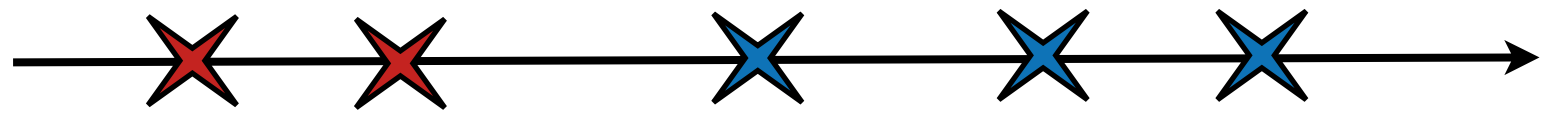




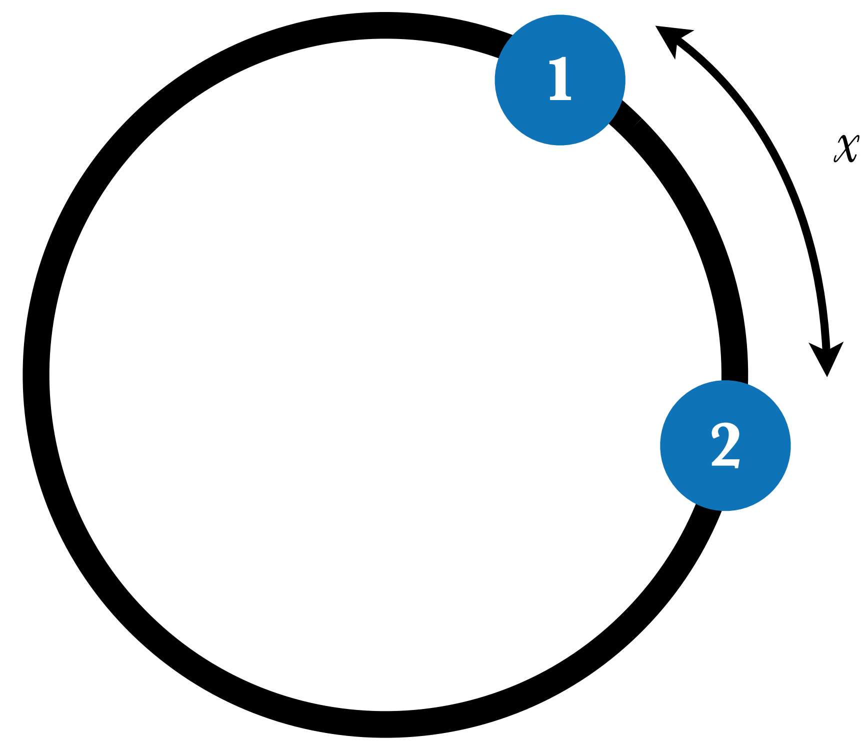
momentum is quantised: $p_i = \frac{2\pi n_i}{L}$

two particle energies are discrete:

$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$



1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$



$$\psi(0) = \psi(L), \quad \frac{\partial\psi}{\partial x}\Big|_{x=0} = \frac{\partial\psi}{\partial x}\Big|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

Phase shifts via Lüscher's method: $$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering *t*-matrix

known finite-volume functions

Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

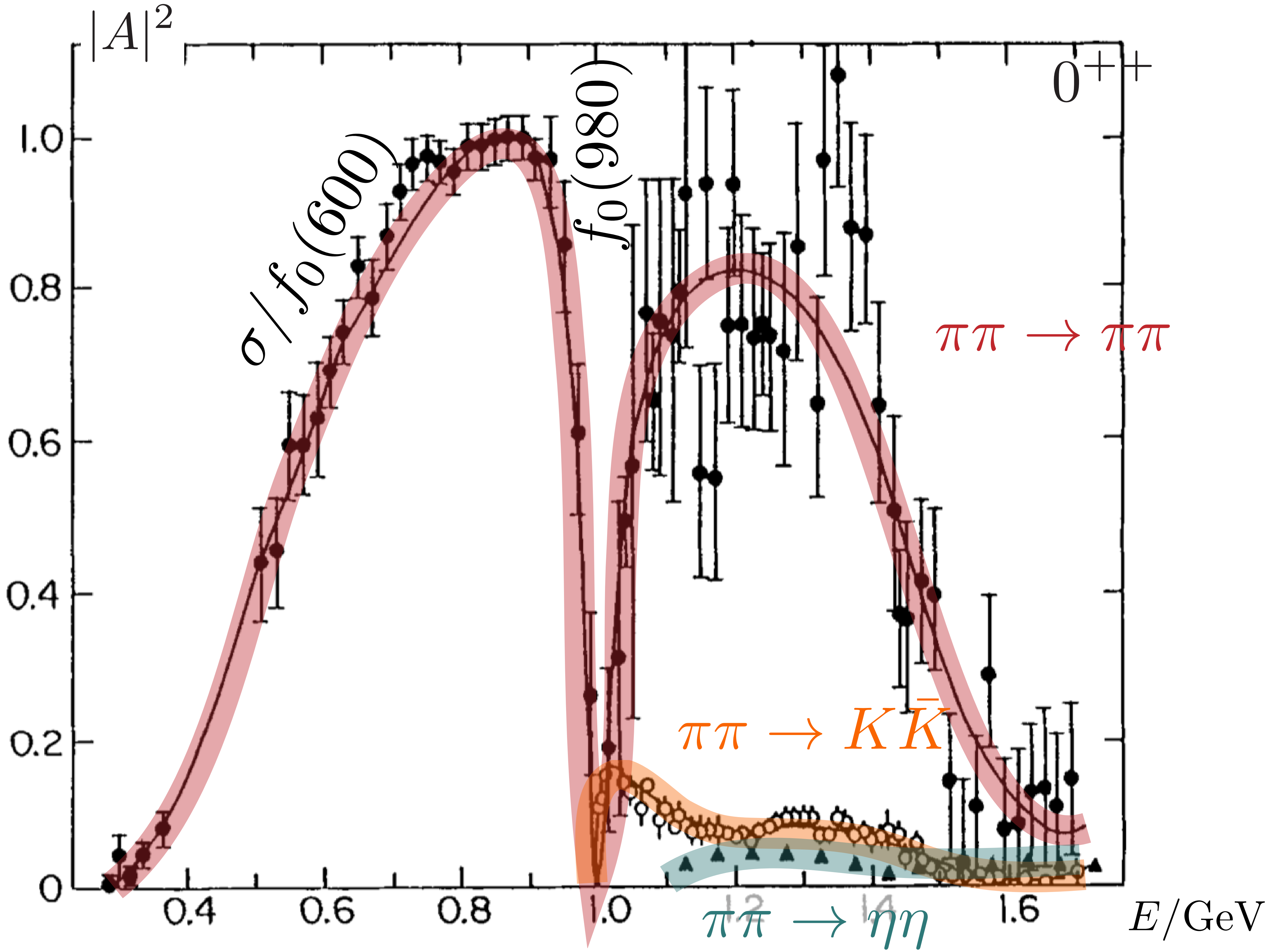
Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

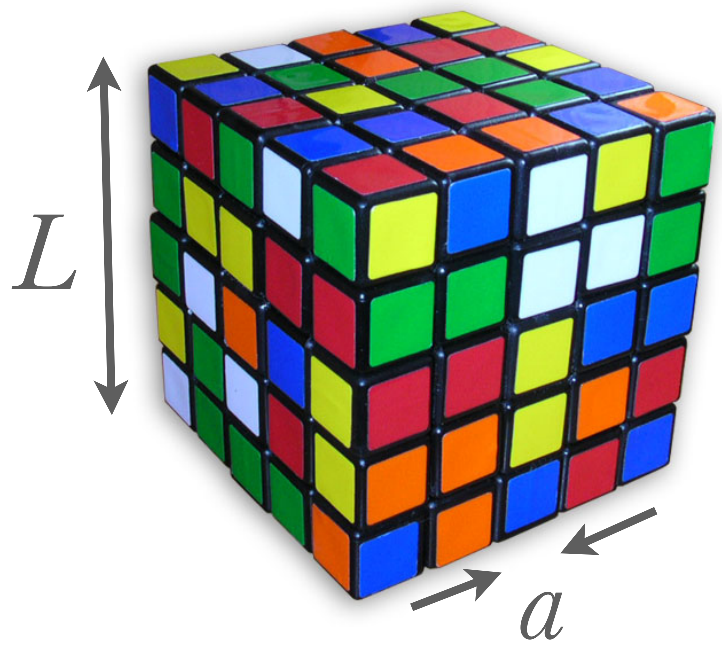
Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

developments towards a general 3-body quantization condition are being made

CERN-Munich, ANL, BNL





3 volumes
L=16, 20, 24
anisotropic (3.5 finer spacing in time)
Wilson-Clover

m_π=391MeV

m_K=549MeV

$$\begin{bmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{bmatrix}$$

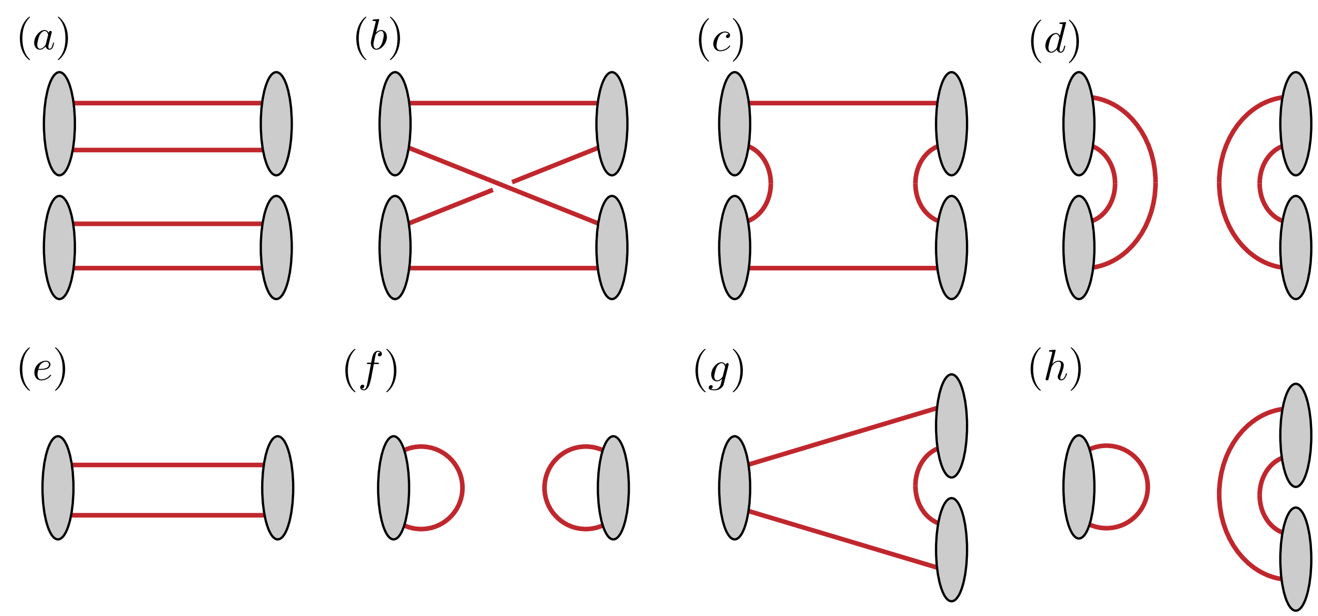
operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D}\ \psi$ local qq-like constructions

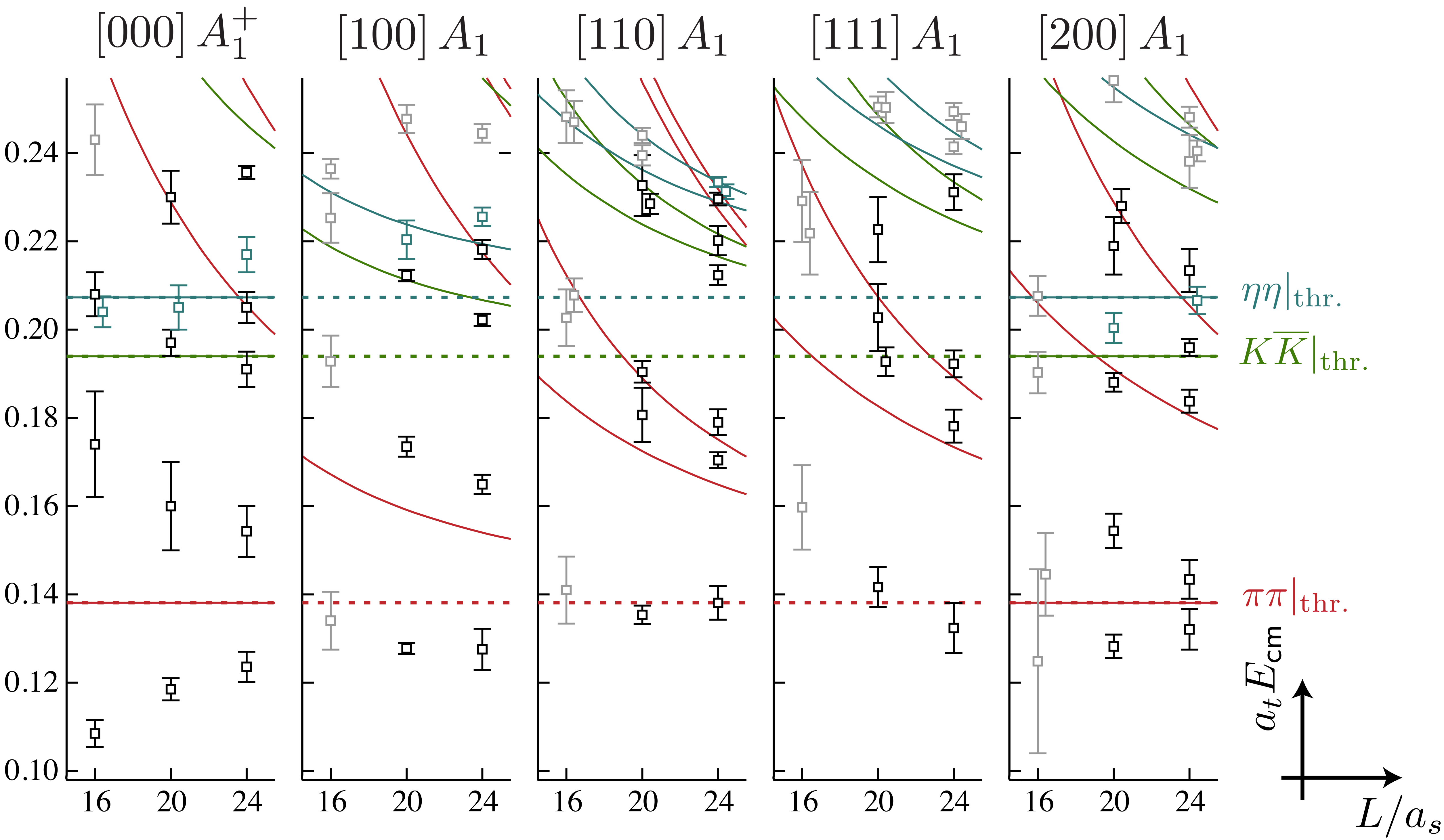
$$\sum_{\vec{p}_1+\vec{p}_2 \in \vec{p}} C(\vec{p}_1, \vec{p}_2; \vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$ two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many wick contractions, eg just pi-pi & qq operators:



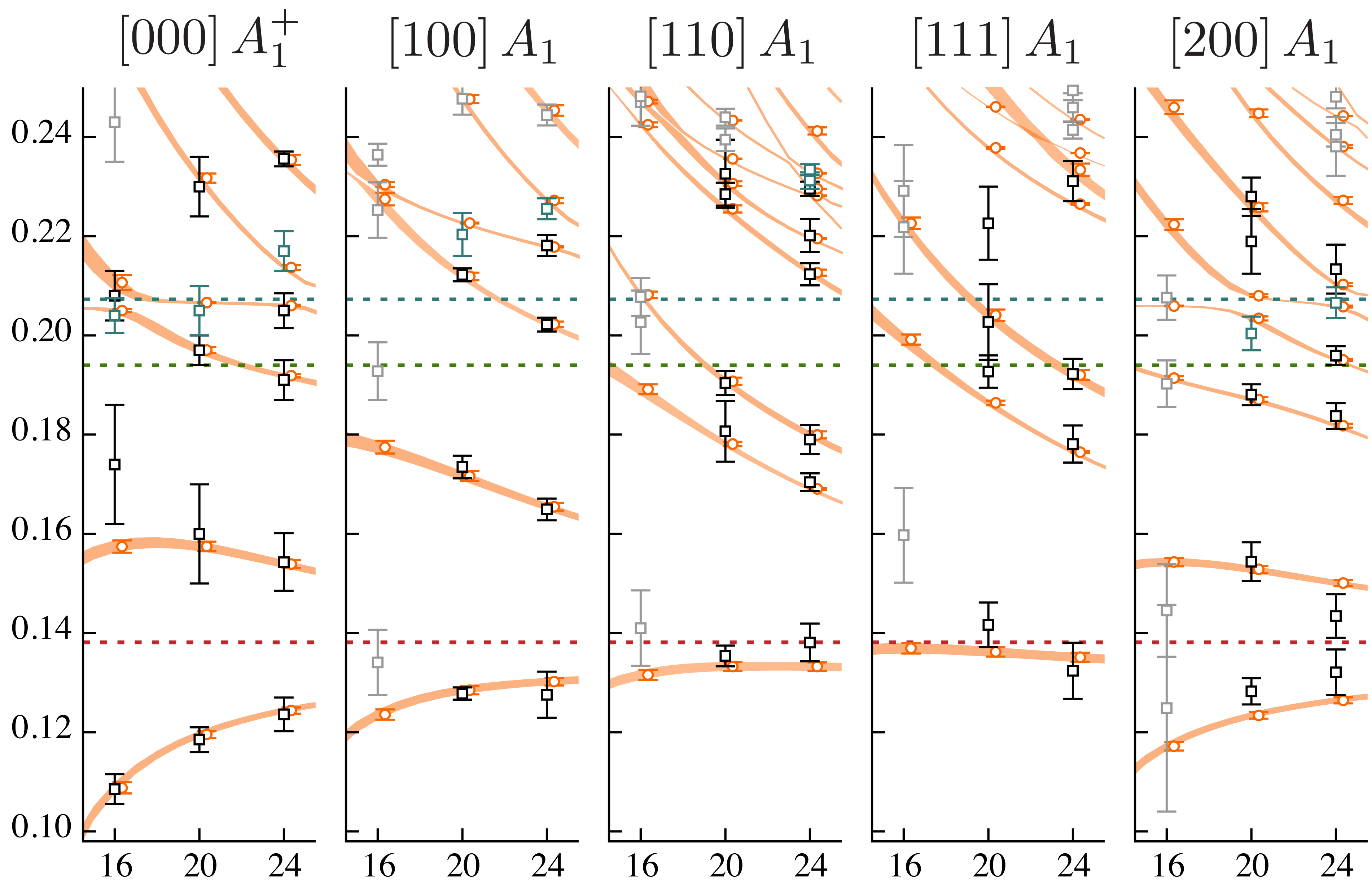
- we compute a large correlation matrix
- then use GEVP to extract energies



~local $q\bar{q}$ & 2-hadron operators
conservatively **57 energy levels**
dominated by S-wave interactions

An example S-wave spectrum fit

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$$



$$\chi^2/N_{\text{dof}} = \frac{44.0}{57-8} = 0.90$$

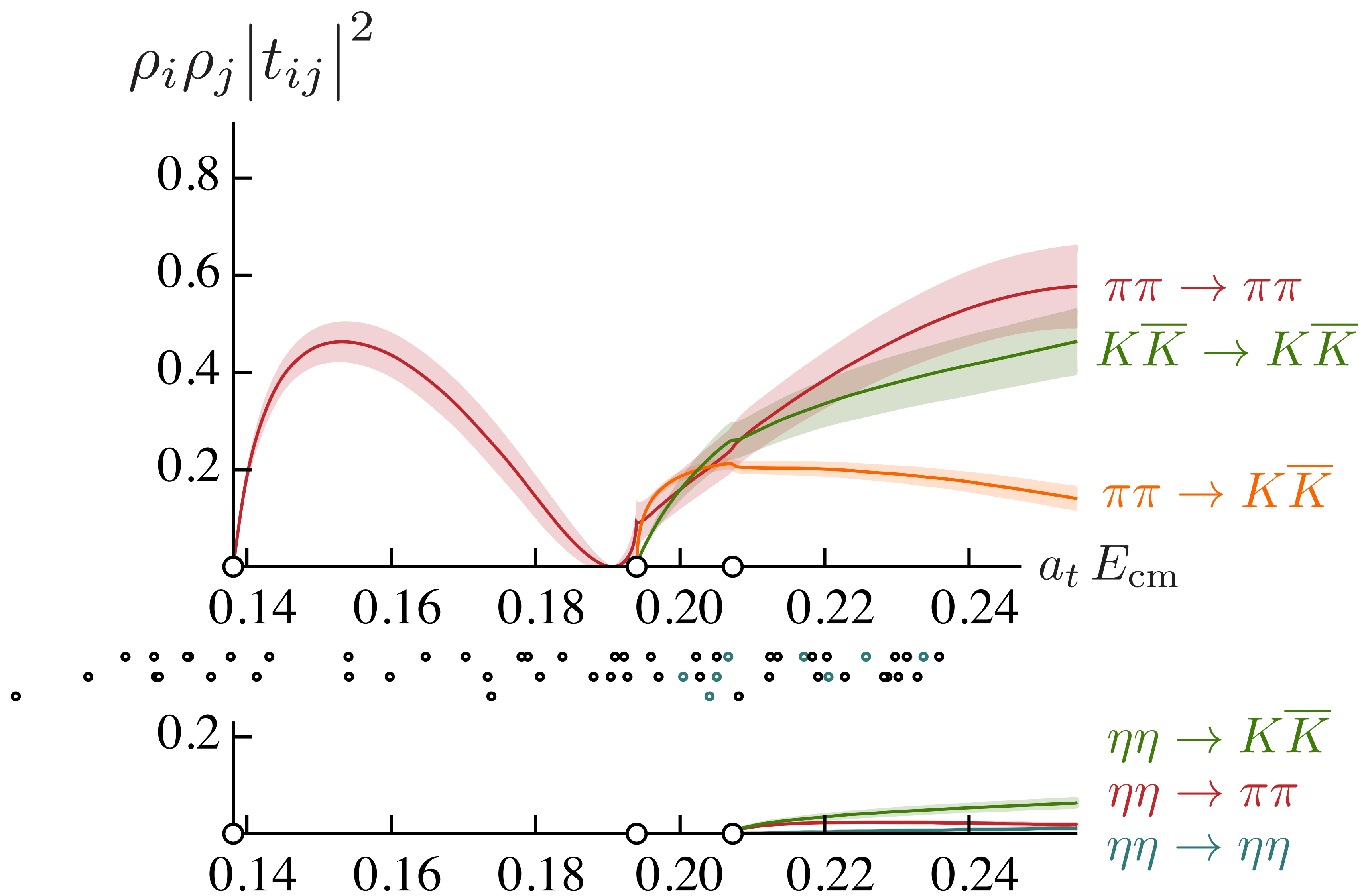
An example S-wave spectrum fit

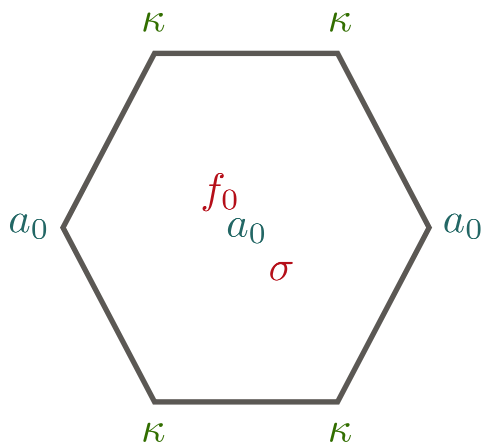
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\text{dof}} = \frac{44.0}{57-8} = 0.90$$

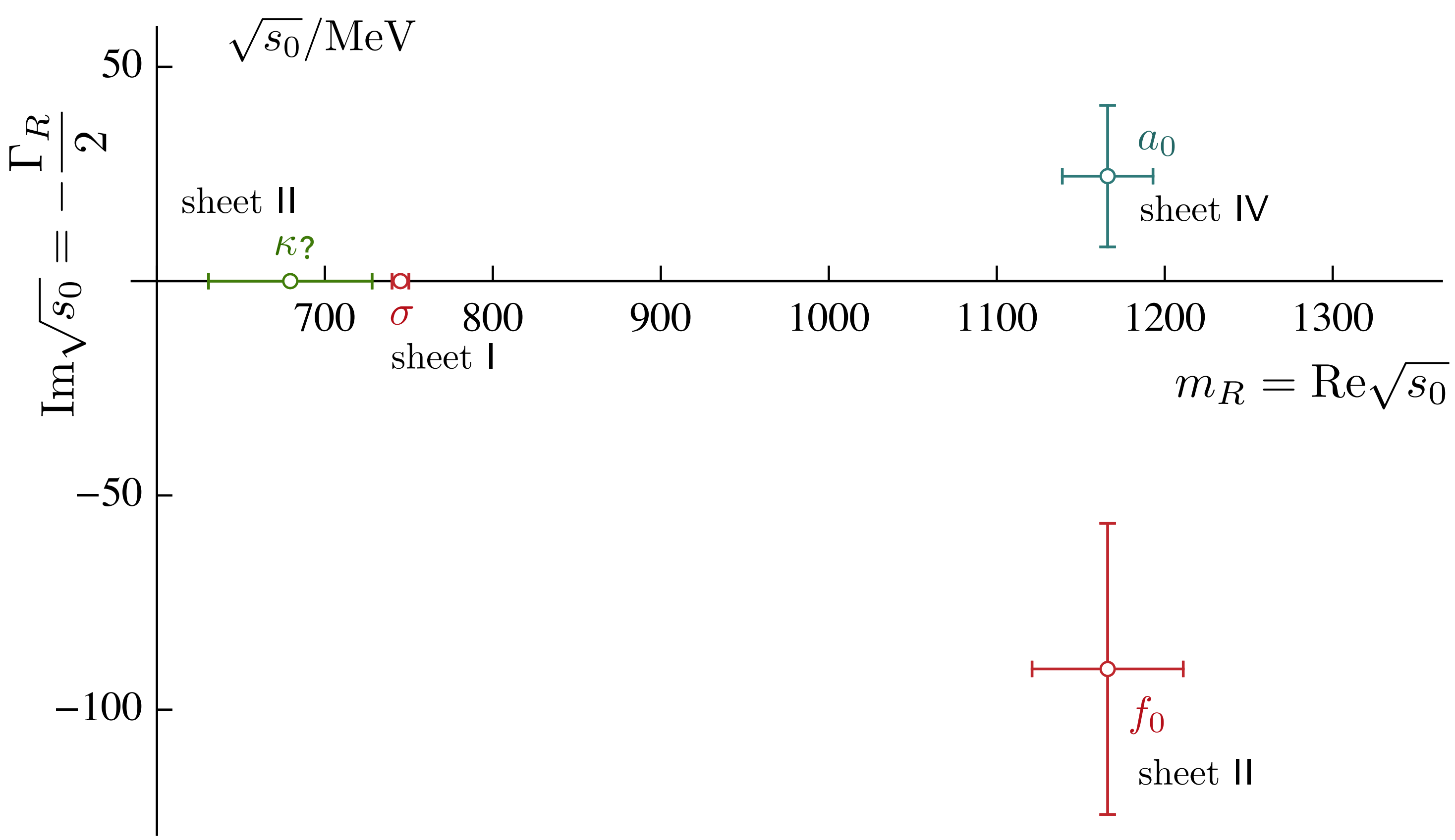
57 energy levels

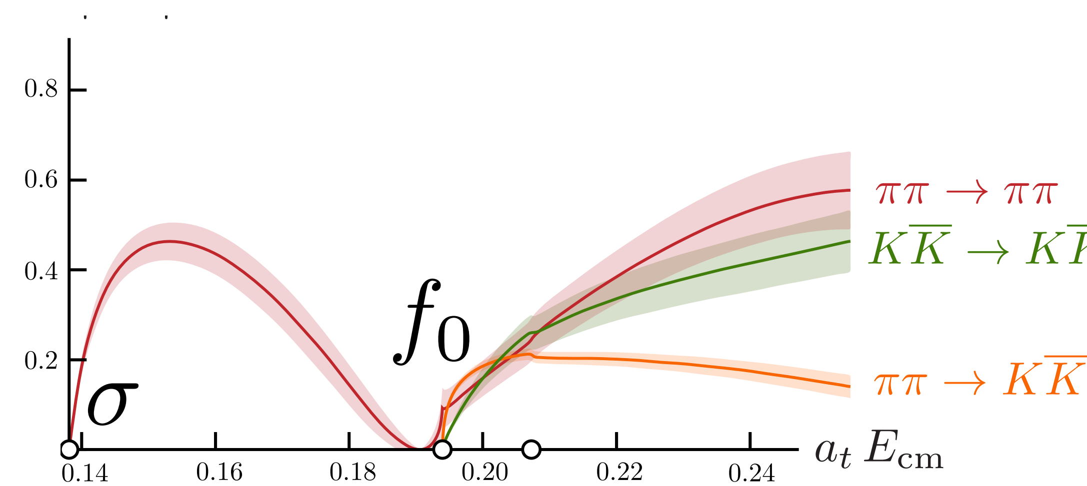


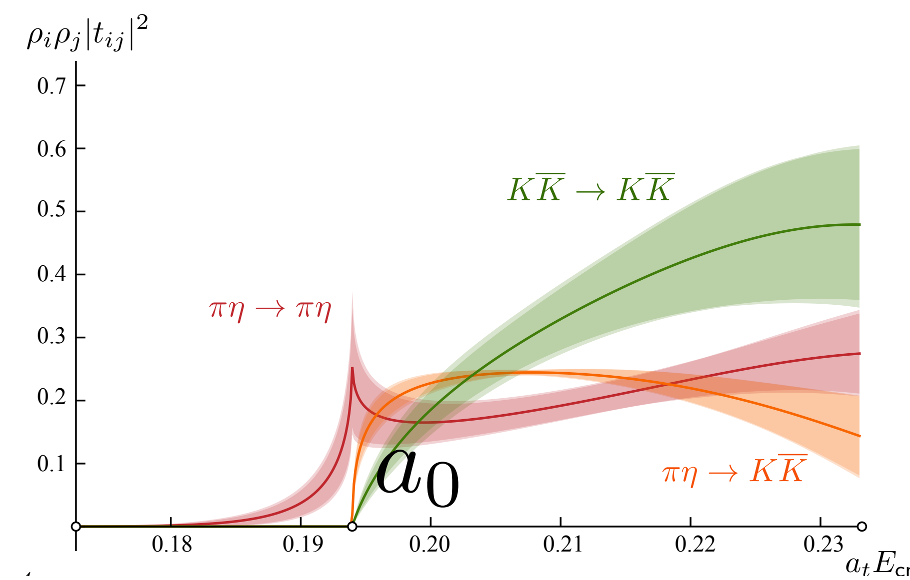


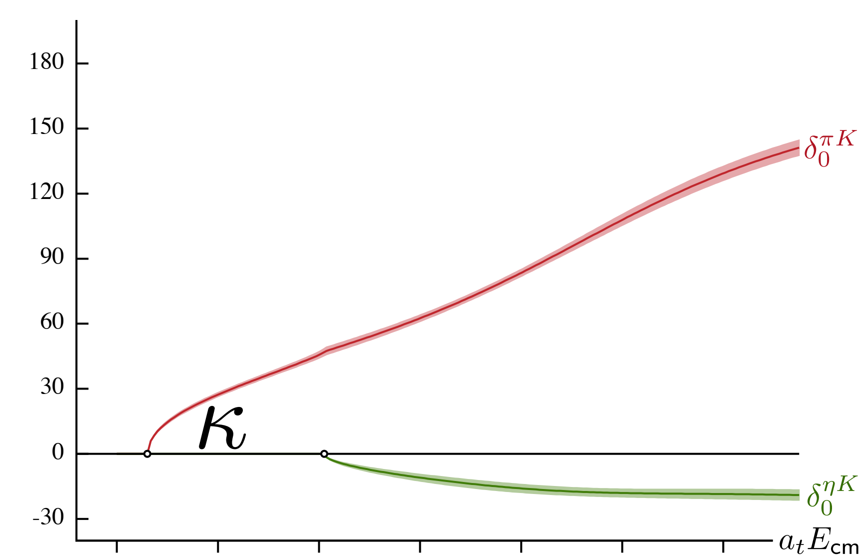
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

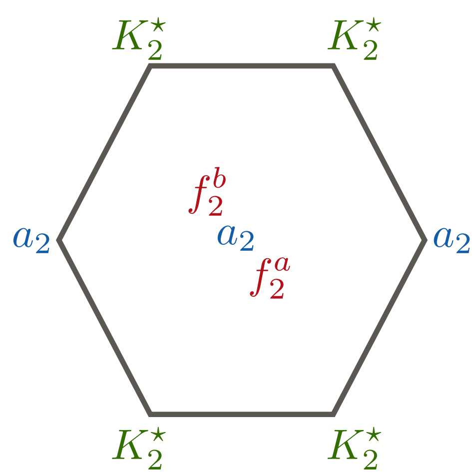
$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$





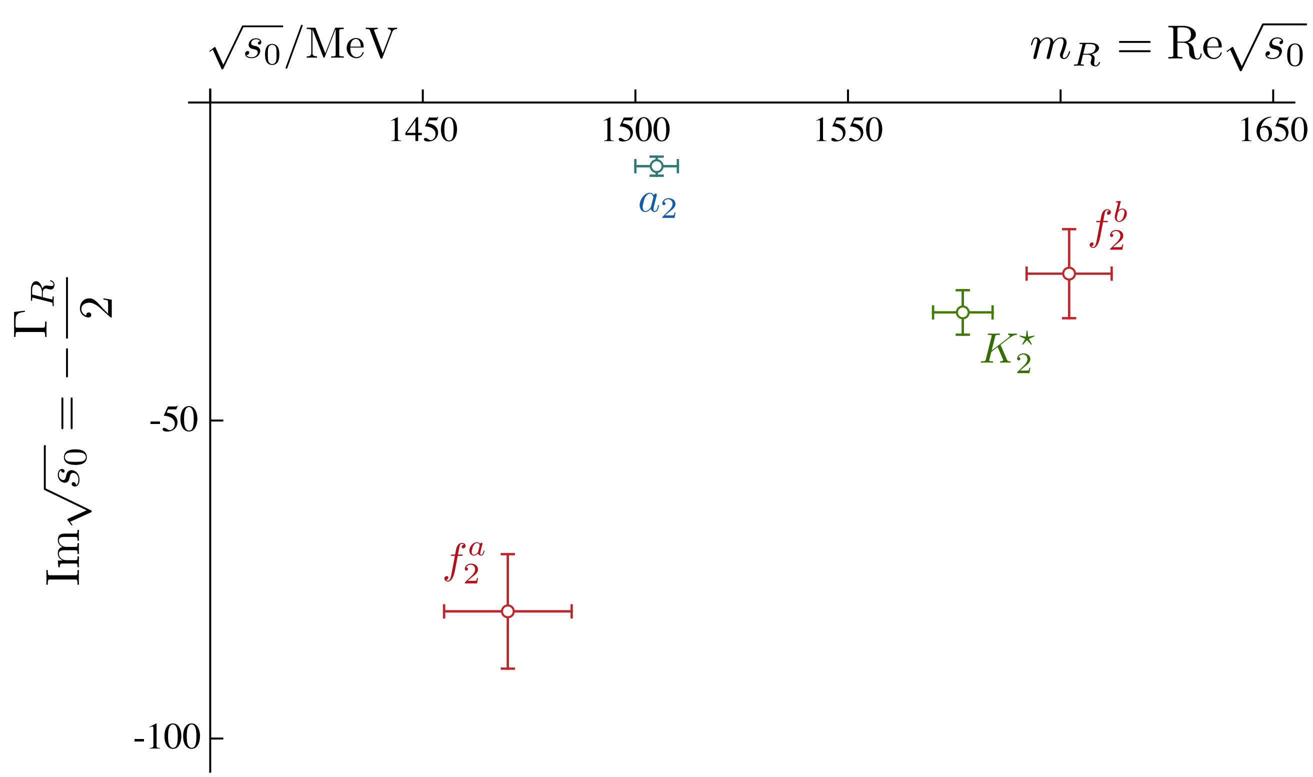


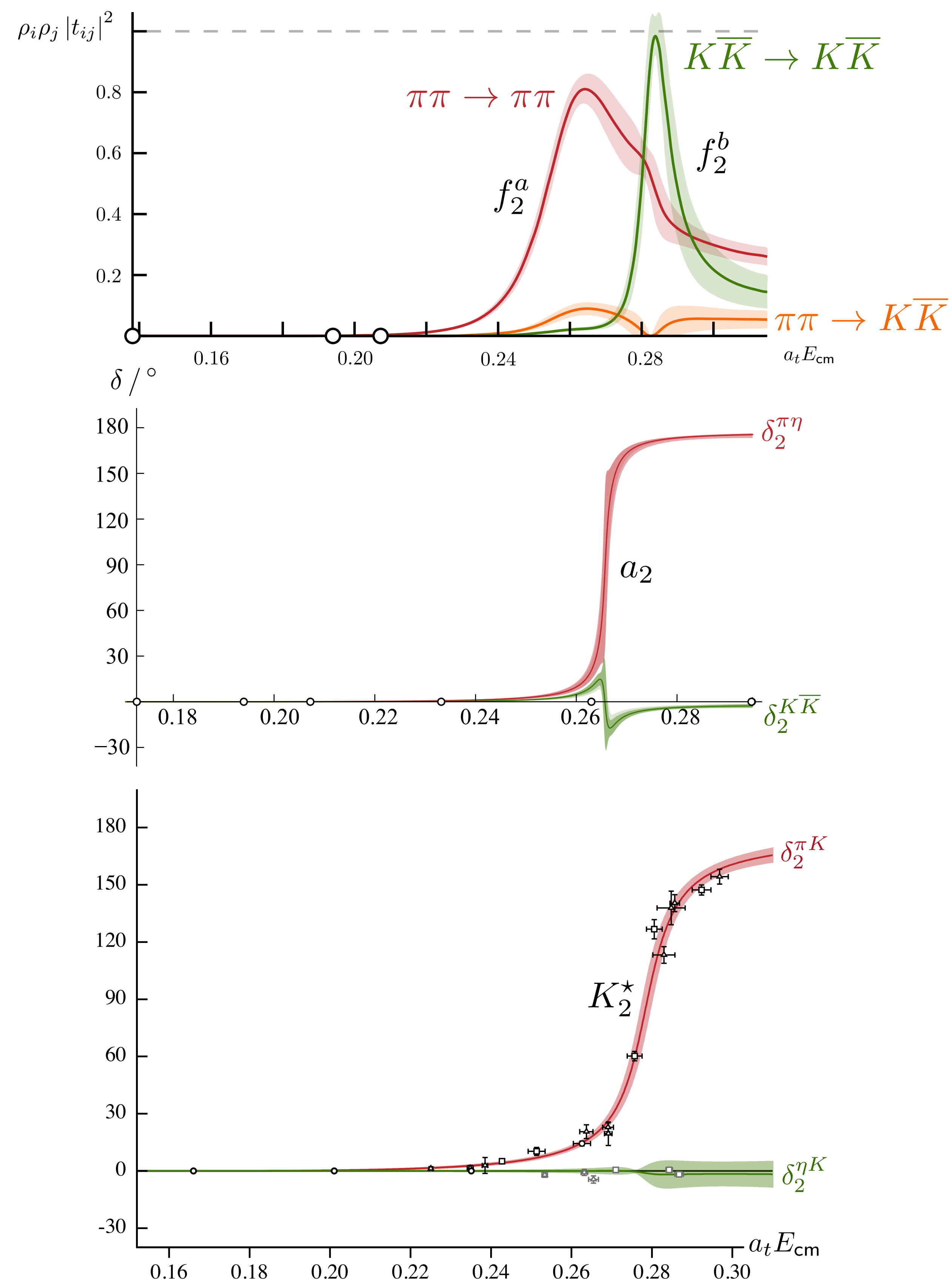




$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$





based on: **arXiv:1904.04136**
Antoni Woss, Christopher Thomas, Jo Dudek,
Robert Edwards, David Wilson -

'natural' spin-parities: $J^P = 0^+, 1^-, 2^+, \ldots$

e.g.: $\sigma(600),\ \rho(770),\ f_2(1270)$

seen in pseudoscalar-pseudoscalar scattering

'unnatural' spin-parities: $J^P = 0^-, 1^+, 2^-, \ldots$

e.g.: $\pi, a_1(1260),\ a_2(1320)$
$b_1(1235),$

needs something more than pseudoscalar-pseudoscalar
e.g.: pseudoscalar-vector scattering or three-body scattering

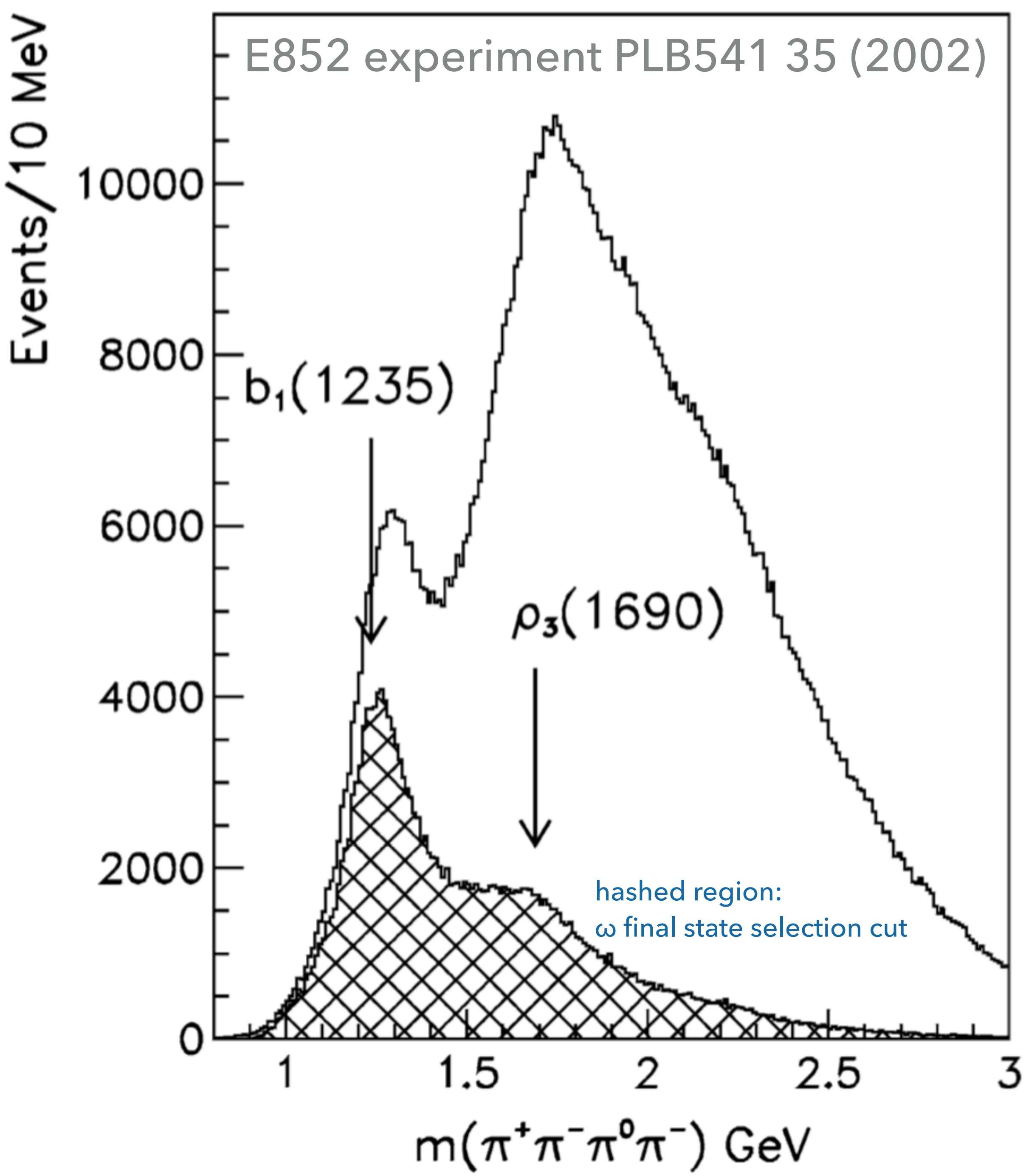
experimentally the $b_1(1235)$ is the lightest axial vector resonance

important step on the road to understanding highly excited states
- e.g.: hybrids

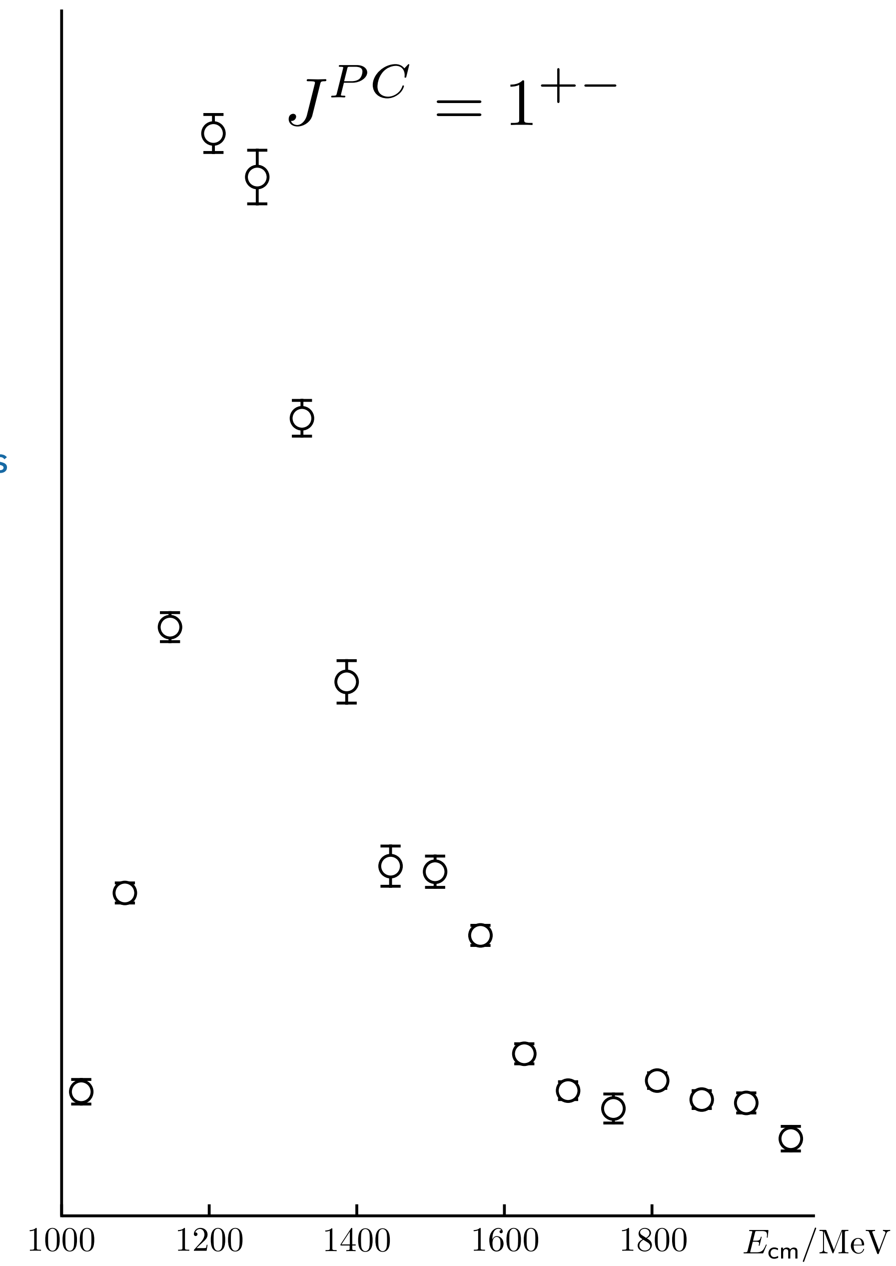
In the quark model:

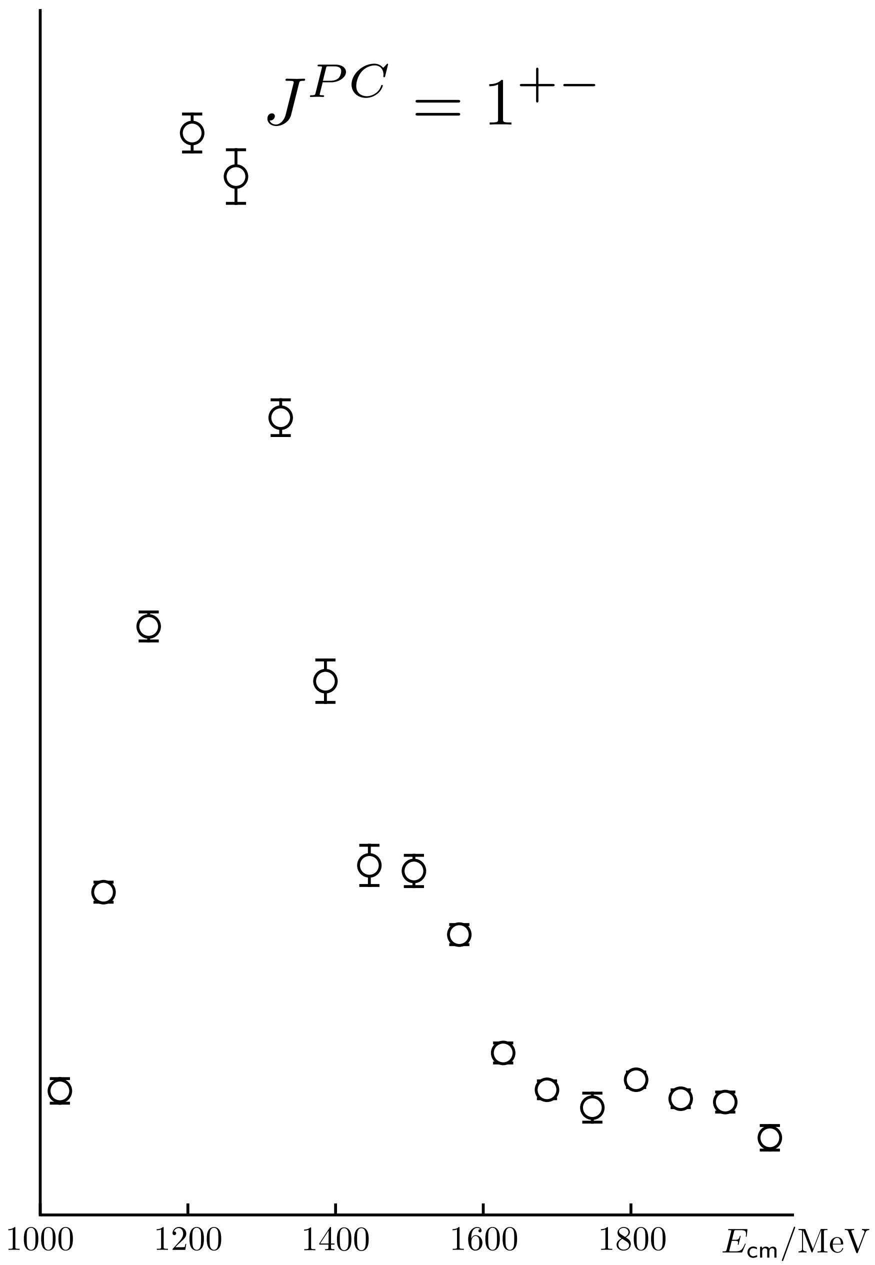
b_1 $\quad q\bar{q}\,(^1P_1) \quad J^{PC} = 1^{+-}$ $\quad \to \pi\omega \to \pi(\pi\pi\pi)$

$\to \ldots$



partial wave analysis →

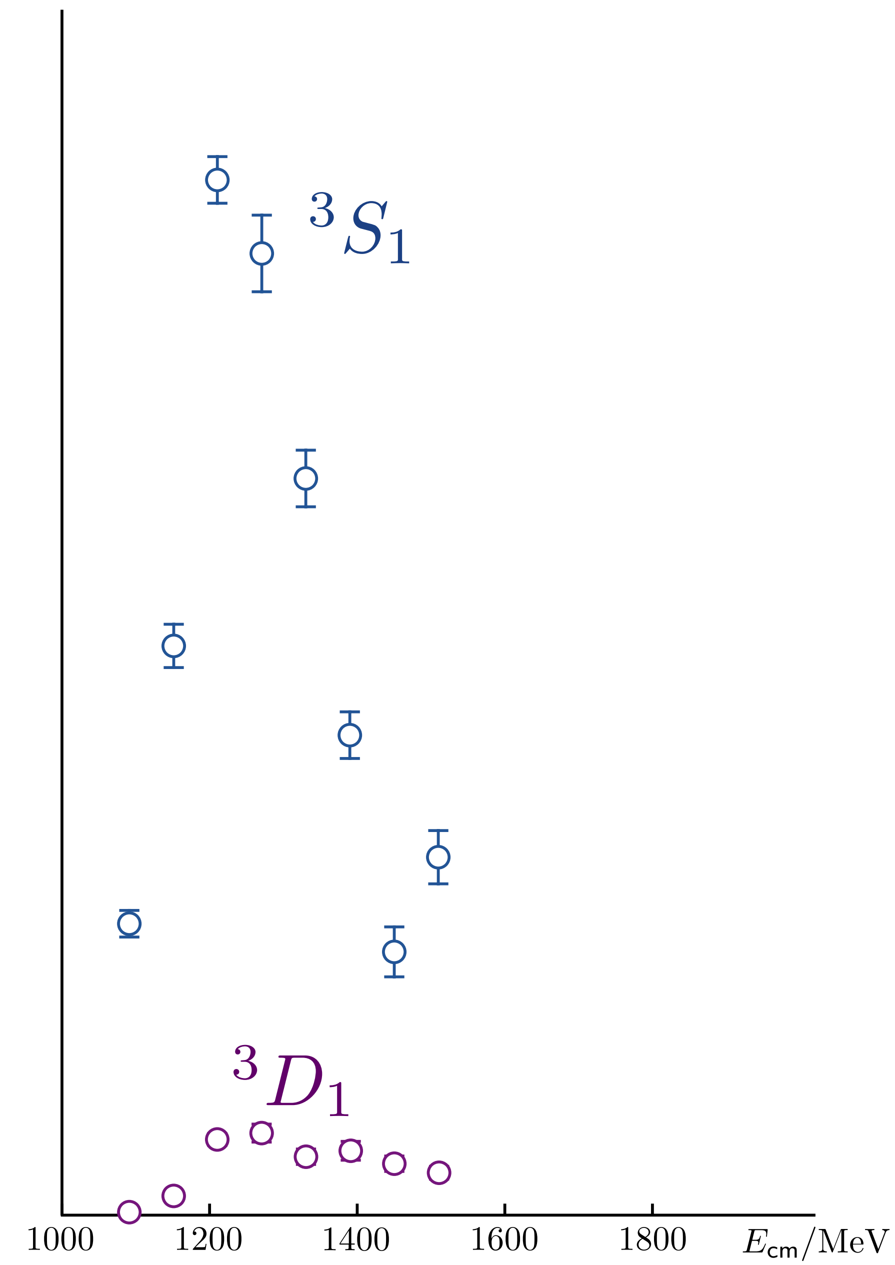




$b_1 \to \pi\omega \quad {}^3S_1$

$\to \pi\omega \quad {}^3D_1$

$\to \dots$



E852 experiment PLB541 35 (2002)

In the quark model:

b$_1$ $q\bar{q}\left({}^1P_1\right)$ $J^{PC}=1^{+-}$ $\rightarrow \pi\omega \rightarrow \pi(\pi\pi\pi)$

$\rightarrow \ldots$

cf: a$_1$ $q\bar{q}\left({}^3P_1\right)$ $J^{PC}=1^{++}$ $\rightarrow \pi\pi\pi$

$\rightarrow \ldots$

working at a heavier than physical pion mass:

$$m_\pi = 391 \text{ MeV}$$
$$m_\omega = 881 \text{ MeV}$$

$m_\omega < 3m_\pi$ $\rightarrow$ ω is stable

earlier studies:
Lang et al JHEP 04 162 (2014)
Michael & McNeile PRD 73 074506

3 volumes: 2 - 3 fm

$L/a_s = 16, 20, 24 \quad T/a_t = 128 \quad m_\pi L \sim 4-6$

anisotropic action: $\xi = a_s/a_t \sim 3.5$

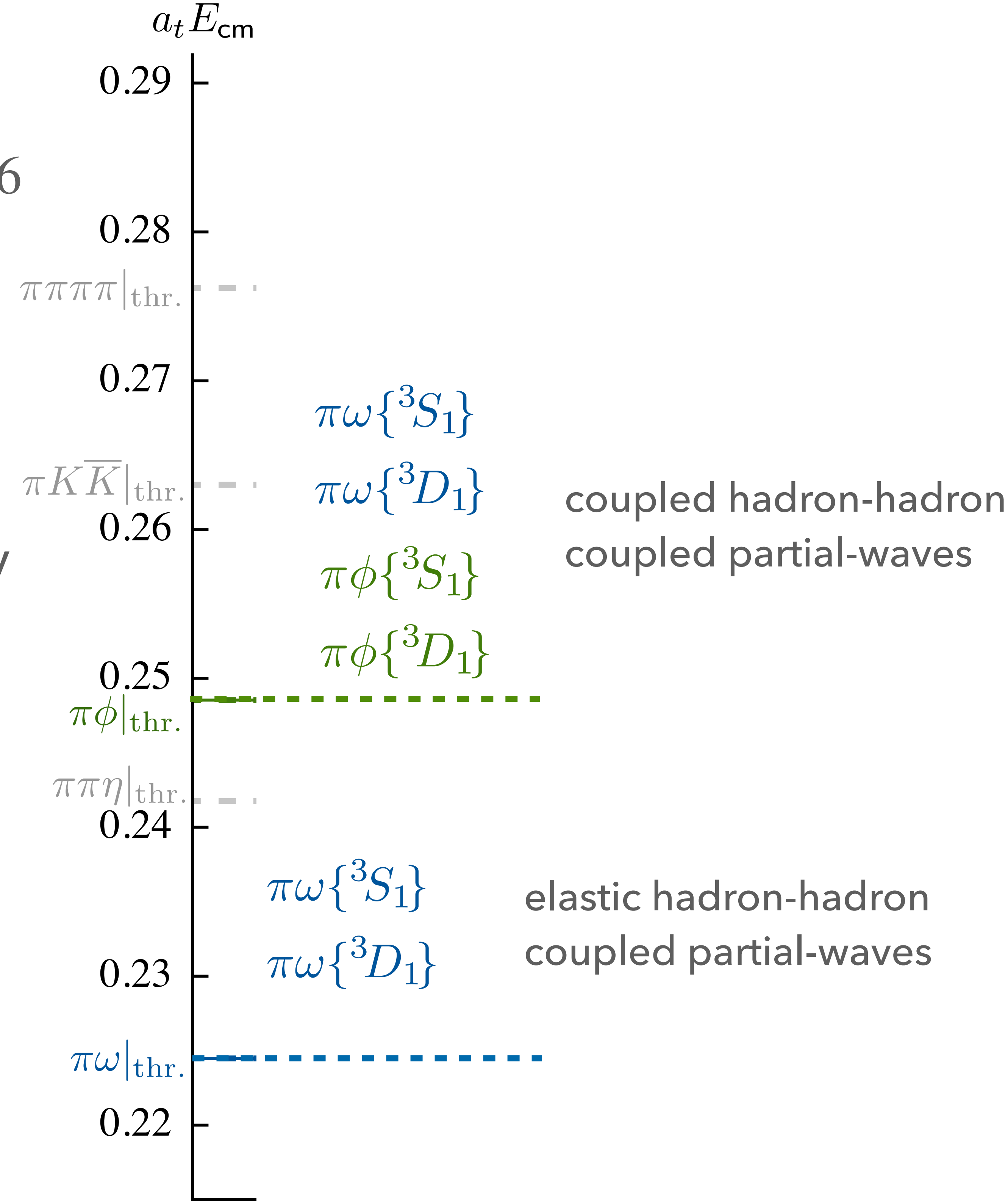
Symanzik-improved Wilson-Clover fermions

Distillation (Peardon *et al* 2009) to efficiently handle the many wick contractions

heavier-than-physical light quark masses

$m_\pi \sim 391$ MeV

used in many calculations to-date



optimised operator formed from the eigenvector from the variational method, e.g.:

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

local qq-like constructions

$$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$$

two-hadron constructions

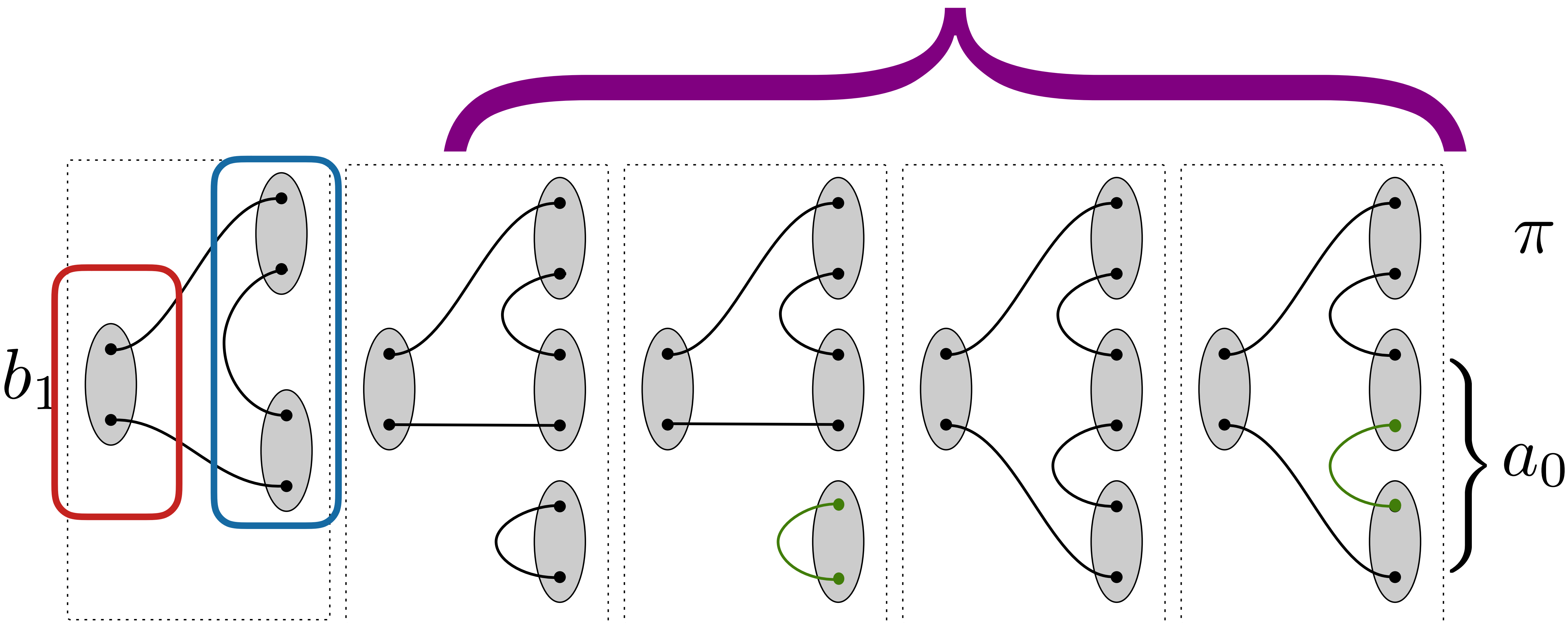
$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\omega(\vec{p}_2)$$

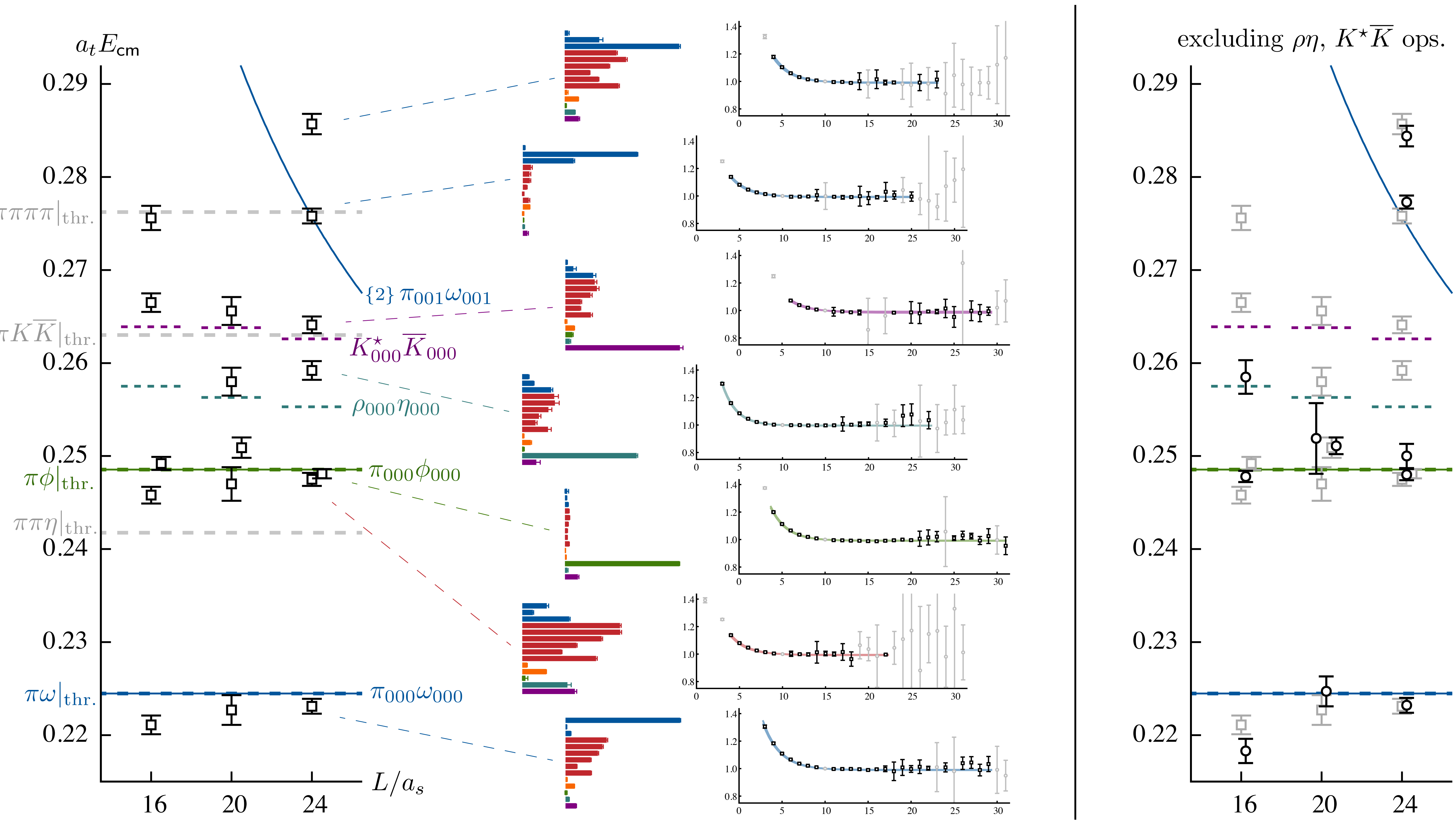
three-hadron constructions

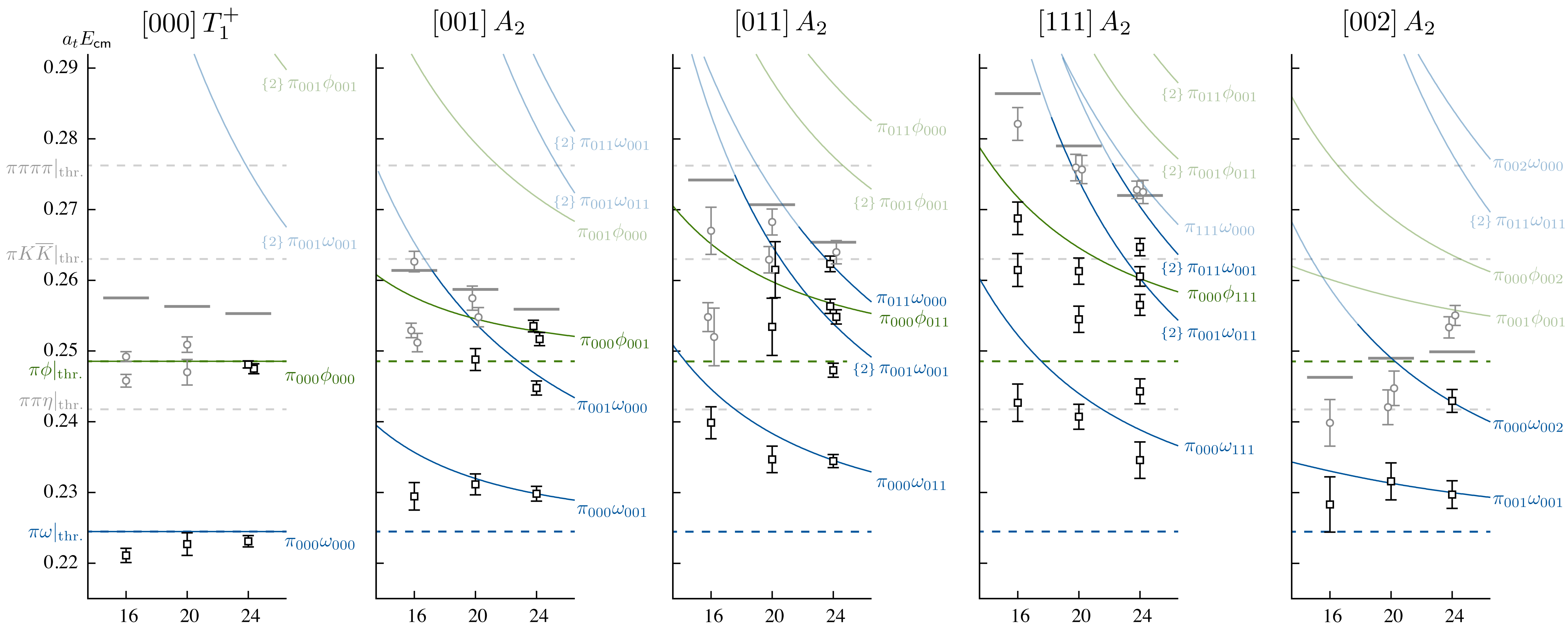
$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2\ \vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_{a_0}(\vec{p}_2)$$

these "two-hadron" parts constructed from a_0, ρ, K* variational analysis

contains both meson-meson and local qq-like constructions




$a_t E_{\text{cm}}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\pi\pi\pi\pi|_{\text{thr.}}$
$\pi K\overline{K}|_{\text{thr.}}$
$\pi\phi|_{\text{thr.}}$
$\pi\pi\eta|_{\text{thr.}}$
$\pi\omega|_{\text{thr.}}$
$\{2\}\ \pi_{001}\omega_{001}$
$K^\star_{000}\overline{K}_{000}$
$\rho_{000}\eta_{000}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
16
20
24
L/a_s
excluding $\rho\eta$, $K^\star\overline{K}$ ops.


[000] T_1^+
[001] A_2
[011] A_2
[111] A_2
[002] A_2
$a_t E_{\mathsf{cm}}$
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$\pi\pi\pi\pi|_{\text{thr.}}$
$\pi K\overline{K}|_{\text{thr.}}$
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16
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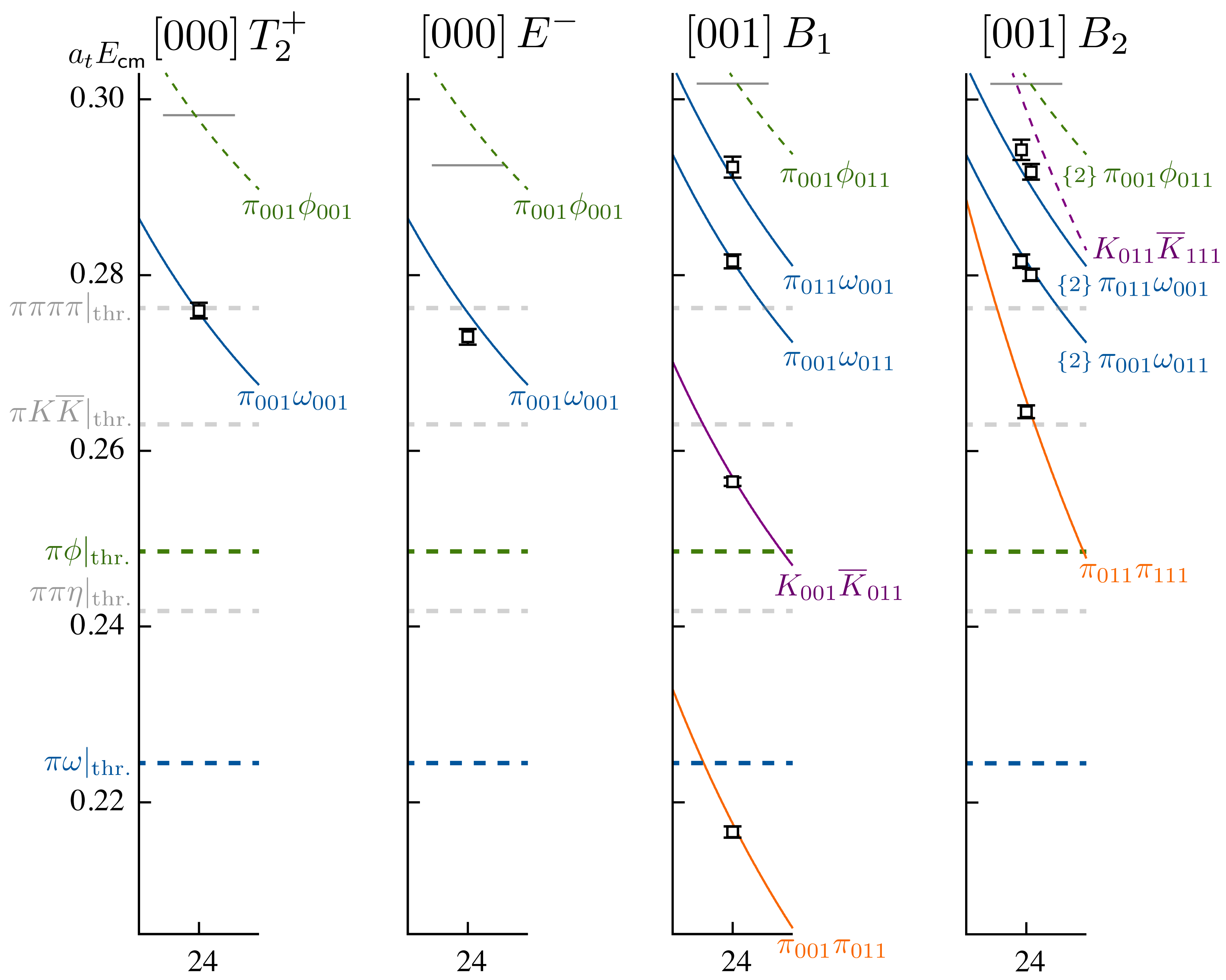

$a_t E_{\mathsf{cm}}$
$[000]\ T_2^+$
$[000]\ E^-$
$[001]\ B_1$
$[001]\ B_2$
0.30
0.28
0.26
0.24
0.22
$\pi\pi\pi\pi|_{\rm thr.}$
$\pi K\overline{K}|_{\rm thr.}$
$\pi\phi|_{\rm thr.}$
$\pi\pi\eta|_{\rm thr.}$
$\pi\omega|_{\rm thr.}$
$\pi_{001}\phi_{001}$
$\pi_{001}\omega_{001}$
$\pi_{001}\phi_{011}$
$\pi_{011}\omega_{001}$
$\pi_{001}\omega_{011}$
$K_{001}\overline{K}_{011}$
$\pi_{001}\pi_{011}$
$\{2\}\ \pi_{001}\phi_{011}$
$K_{011}\overline{K}_{111}$
$\{2\}\ \pi_{011}\omega_{001}$
$\{2\}\ \pi_{001}\omega_{011}$
$\pi_{011}\pi_{111}$
24
24
24
24

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

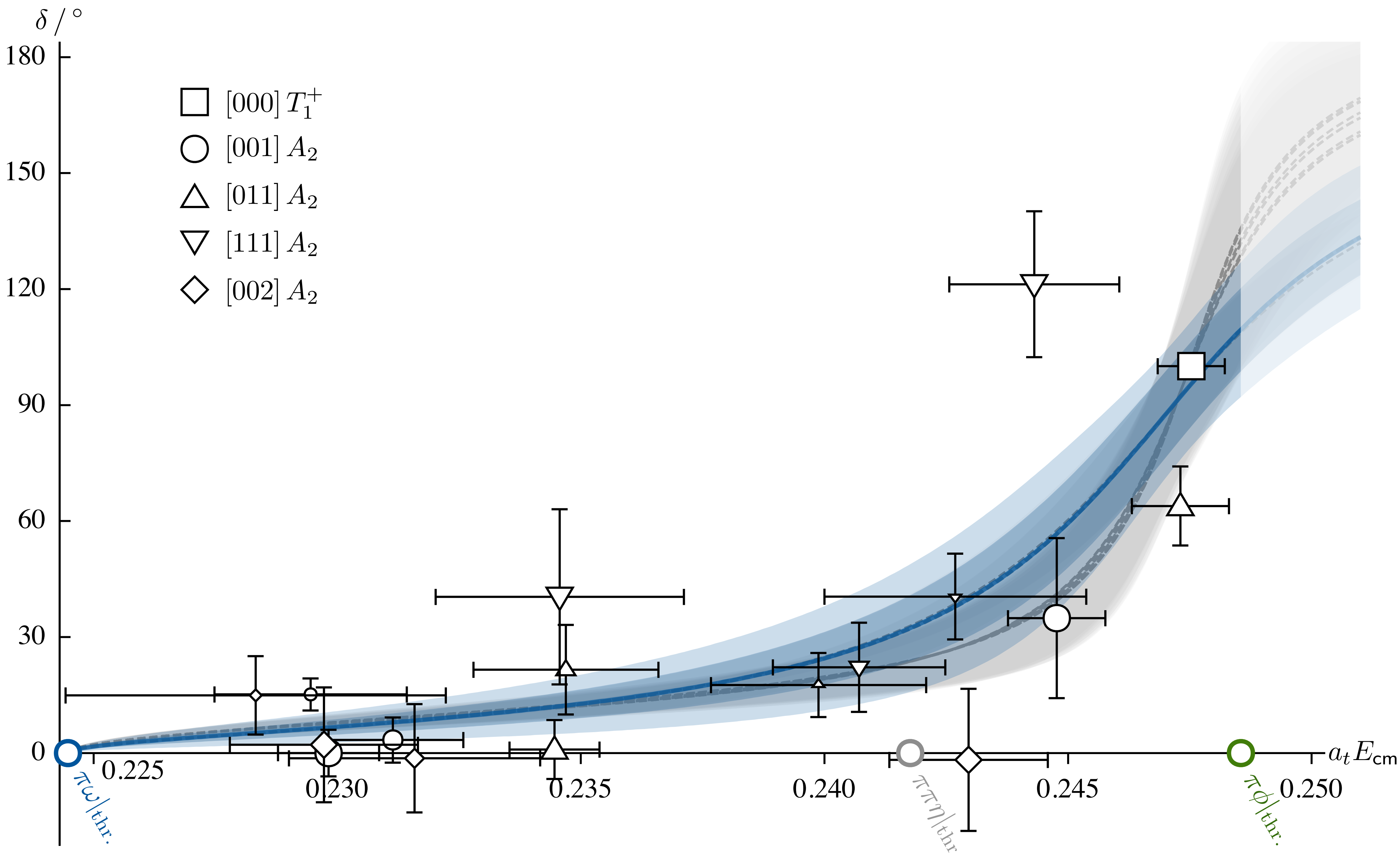
one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) \\ t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) \end{pmatrix}$$

K-matrix approach: $\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$

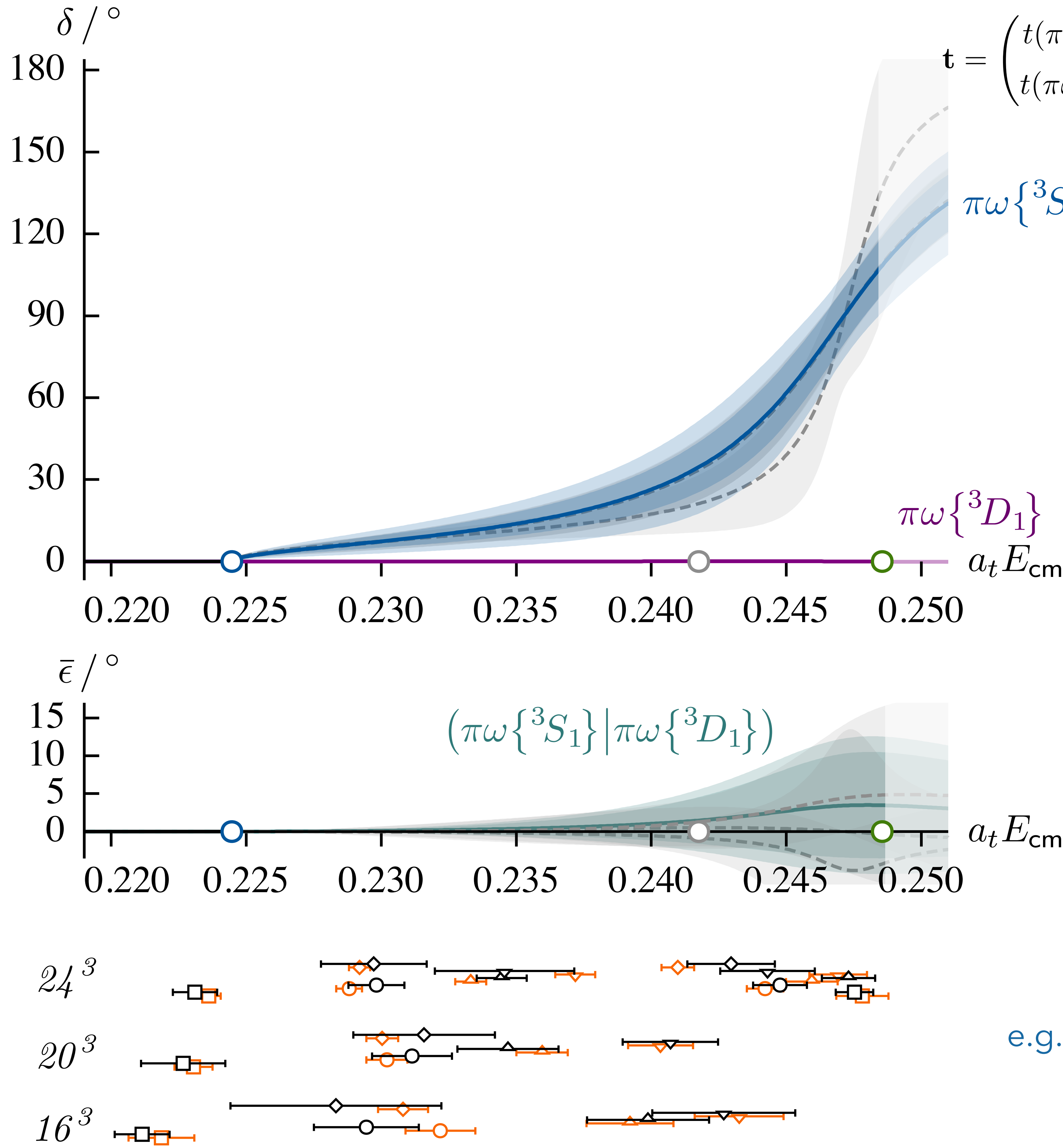
Chew-Mandelstam phase space: $\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$ use a dispersion relation to generate a real part from iρ

$$t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\})$$



e.g.: single-channel K-matrix with one pole: $\chi^2/N_{\mathrm{dof}} = \dfrac{15.1}{20-2} = 0.84$

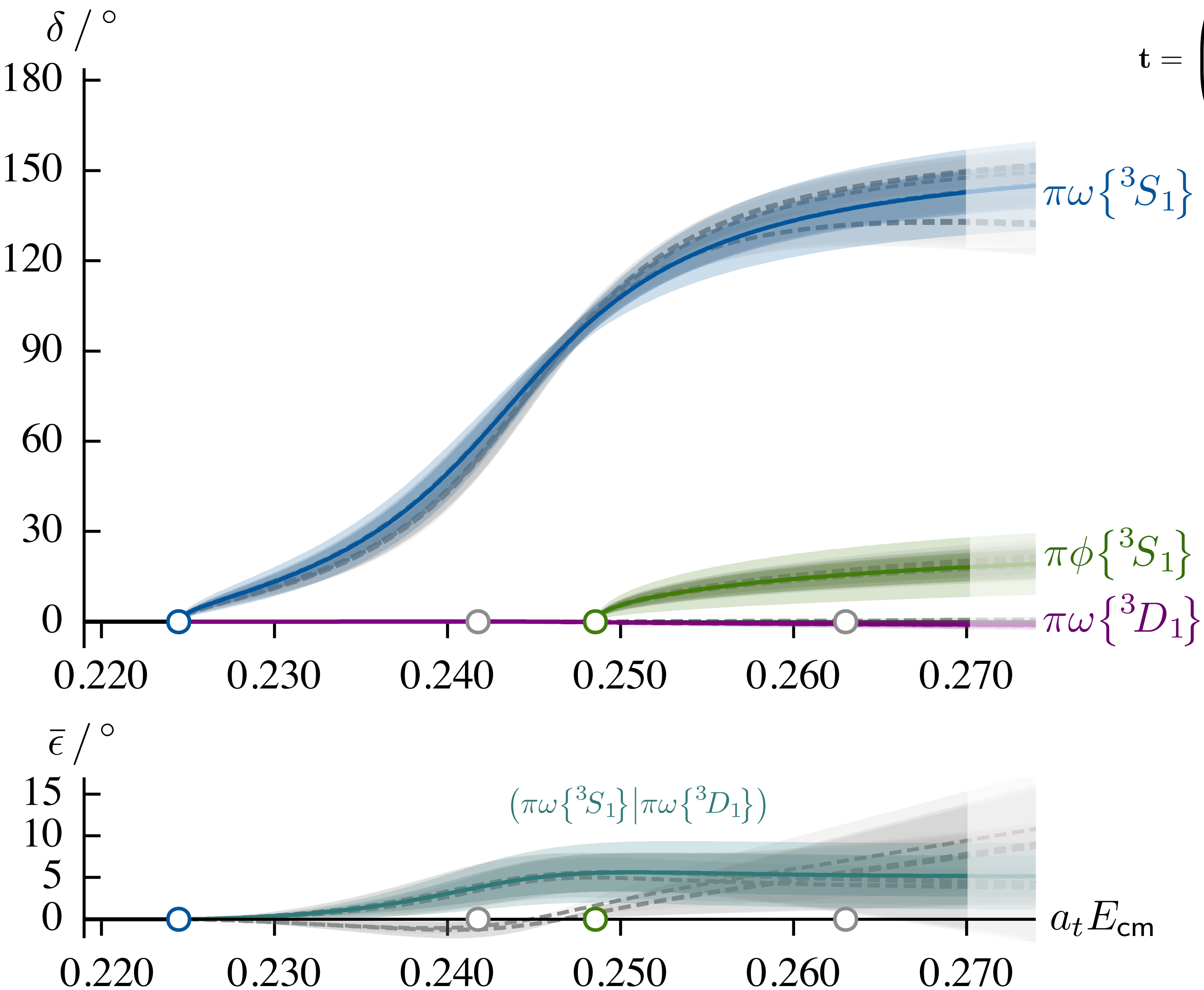
$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) \\ t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}), \end{pmatrix}$$

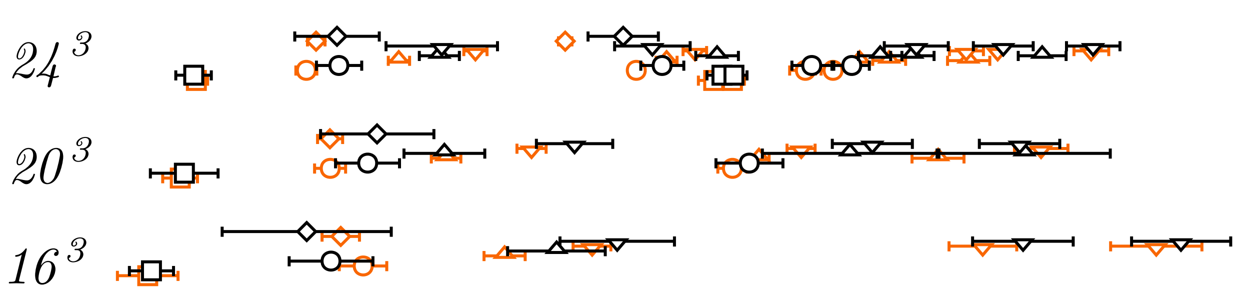


e.g.: two-channel K-matrix with one pole:

$$\chi^2/N_{\text{dof}} = \frac{14.9}{20-3} = 0.87$$

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$





e.g.: three-channel K-matrix with a pole and constants:

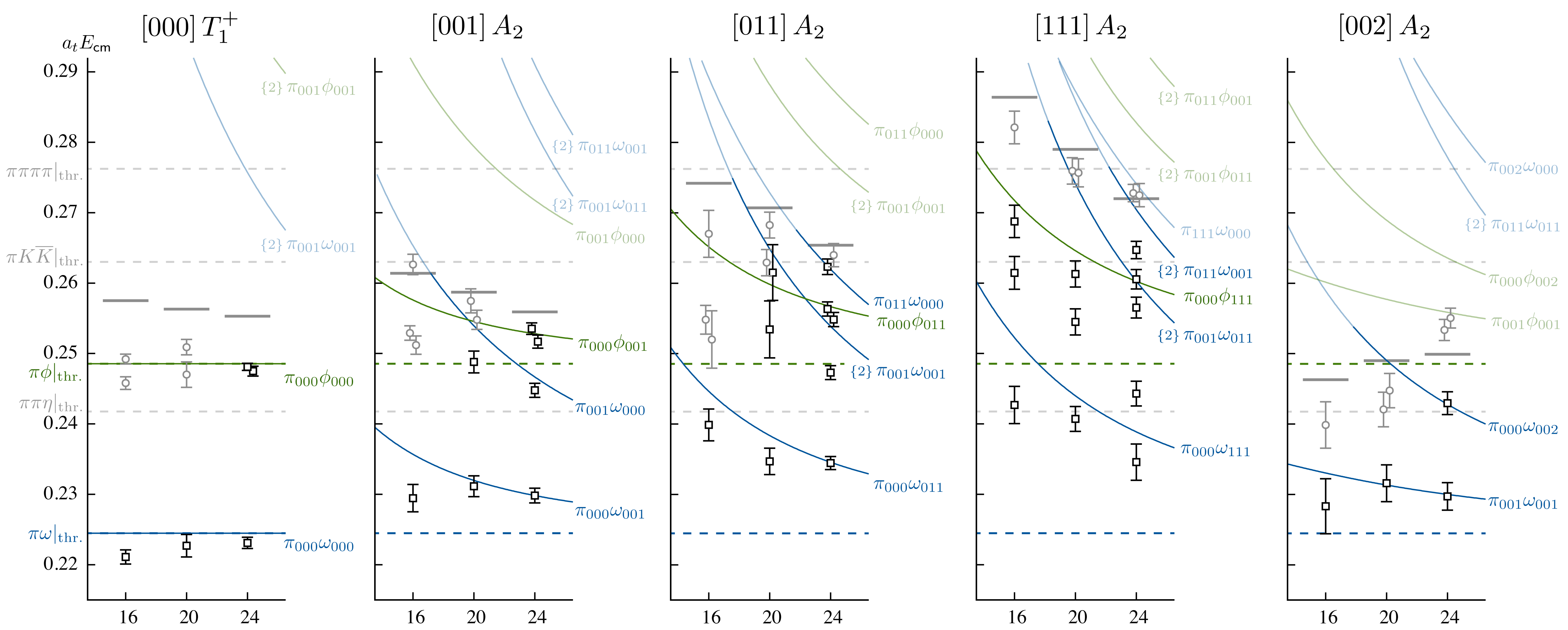
$$[t^{-1}]_{\ell Ja,\,\ell' Jb} = (2k^{(a)})^{-\ell}\,[K^{-1}]_{\ell Ja,\,\ell' Jb}\,(2k^{(a)})^{-\ell} + \delta_{\ell'\ell} I_{ab}$$

$$K_{\ell Ja,\,\ell' Jb} = \frac{g_{\ell Ja}\, g_{\ell' Jb}}{m^2 - s} + \gamma_{\ell Ja,\,\ell' Jb}$$

$$m = (0.2465 \pm 0.0007 \pm 0.0001) \cdot a_t^{-1}$$
$$g_{\pi\omega\{^3S_1\}} = (0.106 \pm 0.007 \pm 0.007) \cdot a_t^{-1}$$
$$g_{\pi\omega\{^3D_1\}} = (1.08 \pm 0.47 \pm 0.28) \cdot a_t$$
$$\gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} = -0.35 \pm 0.19 \pm 0.18$$
$$\gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} = 0.90 \pm 0.24 \pm 0.27$$

1	−0.05	0.05	−0.01	−0.23
	1	0.70	−0.54	−0.06
		1	−0.39	−0.06
			1	0.22
				1

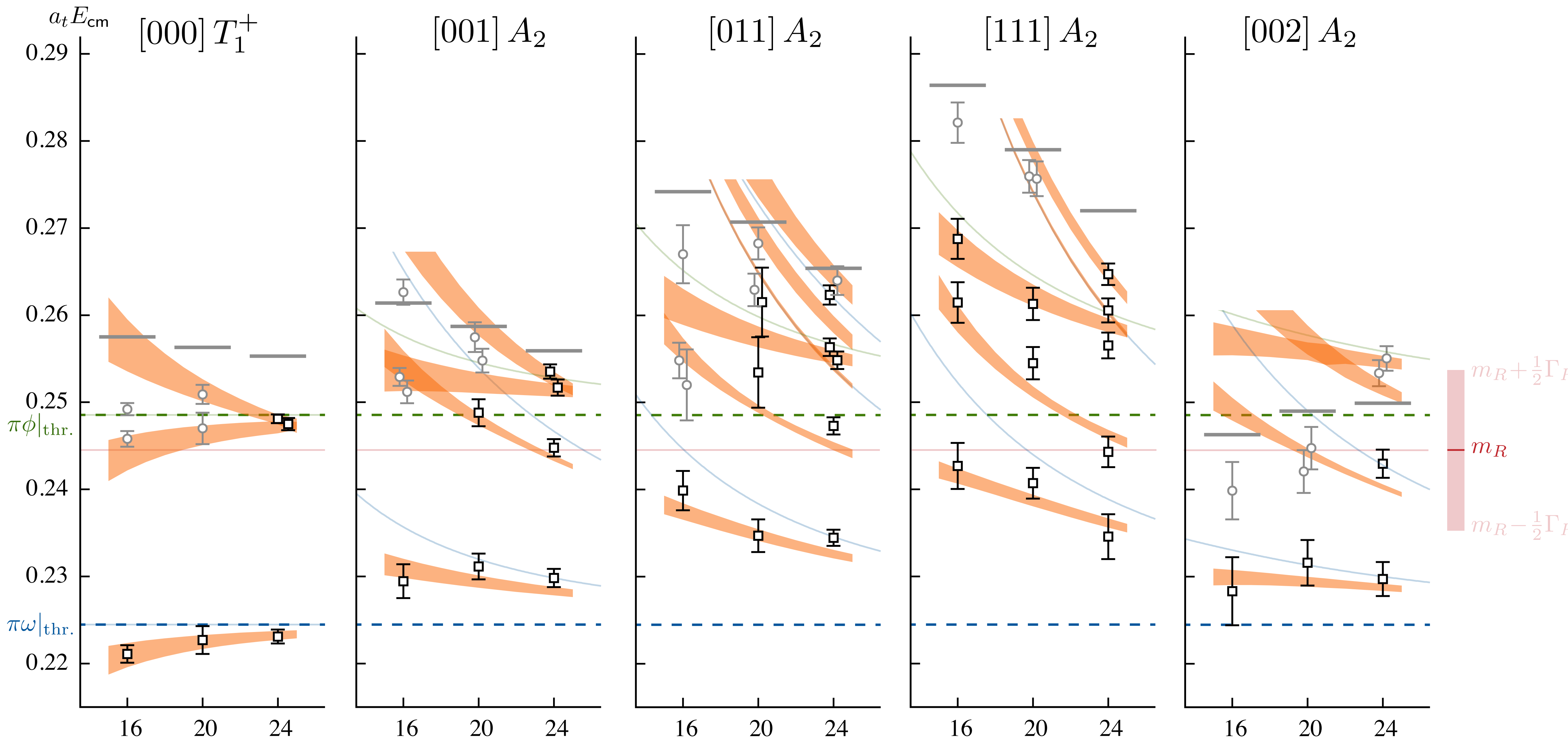
$$\chi^2/N_{\mathrm{dof}} = \frac{36.8}{36-5} = 1.19.$$



[000] T_1^+
[001] A_2
[011] A_2
[111] A_2
[002] A_2
$a_t E_{cm}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\pi\pi\pi\pi|_{thr.}$
$\pi K\overline{K}|_{thr.}$
$\pi\phi|_{thr.}$
$\pi\pi\eta|_{thr.}$
$\pi\omega|_{thr.}$
16
20
24
{2} $\pi_{001}\phi_{001}$
{2} $\pi_{001}\omega_{001}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
{2} $\pi_{011}\omega_{001}$
{2} $\pi_{001}\omega_{011}$
$\pi_{001}\phi_{000}$
$\pi_{000}\phi_{001}$
$\pi_{001}\omega_{000}$
$\pi_{000}\omega_{001}$
$\pi_{011}\phi_{000}$
{2} $\pi_{001}\phi_{001}$
$\pi_{011}\omega_{000}$
$\pi_{000}\phi_{011}$
{2} $\pi_{001}\omega_{001}$
$\pi_{000}\omega_{011}$
{2} $\pi_{011}\phi_{001}$
{2} $\pi_{001}\phi_{011}$
$\pi_{111}\omega_{000}$
{2} $\pi_{011}\omega_{001}$
$\pi_{000}\phi_{111}$
{2} $\pi_{001}\omega_{011}$
$\pi_{000}\omega_{111}$
$\pi_{002}\omega_{000}$
{2} $\pi_{011}\omega_{011}$
$\pi_{000}\phi_{002}$
$\pi_{001}\phi_{001}$
$\pi_{000}\omega_{002}$
$\pi_{001}\omega_{001}$

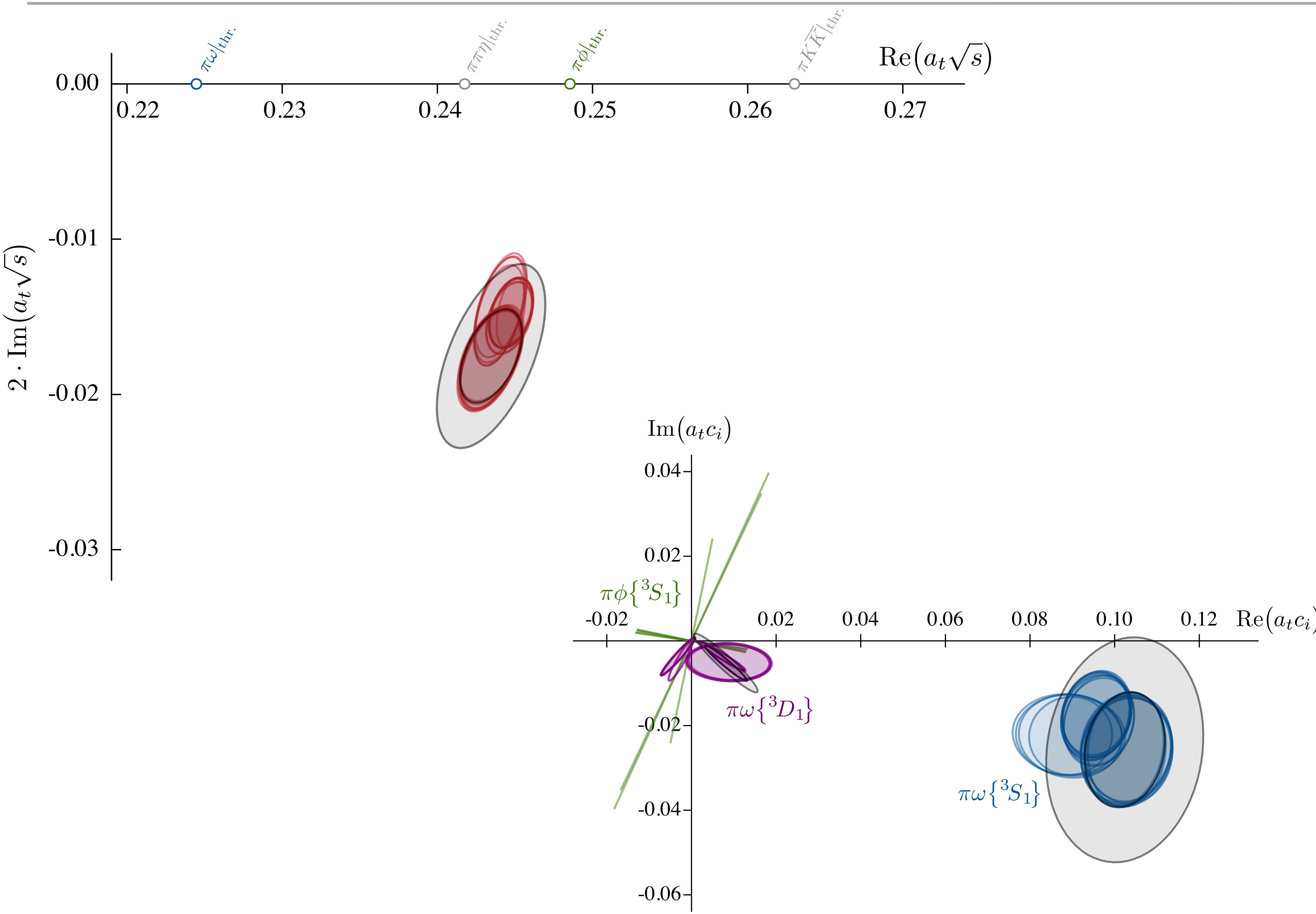
$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$

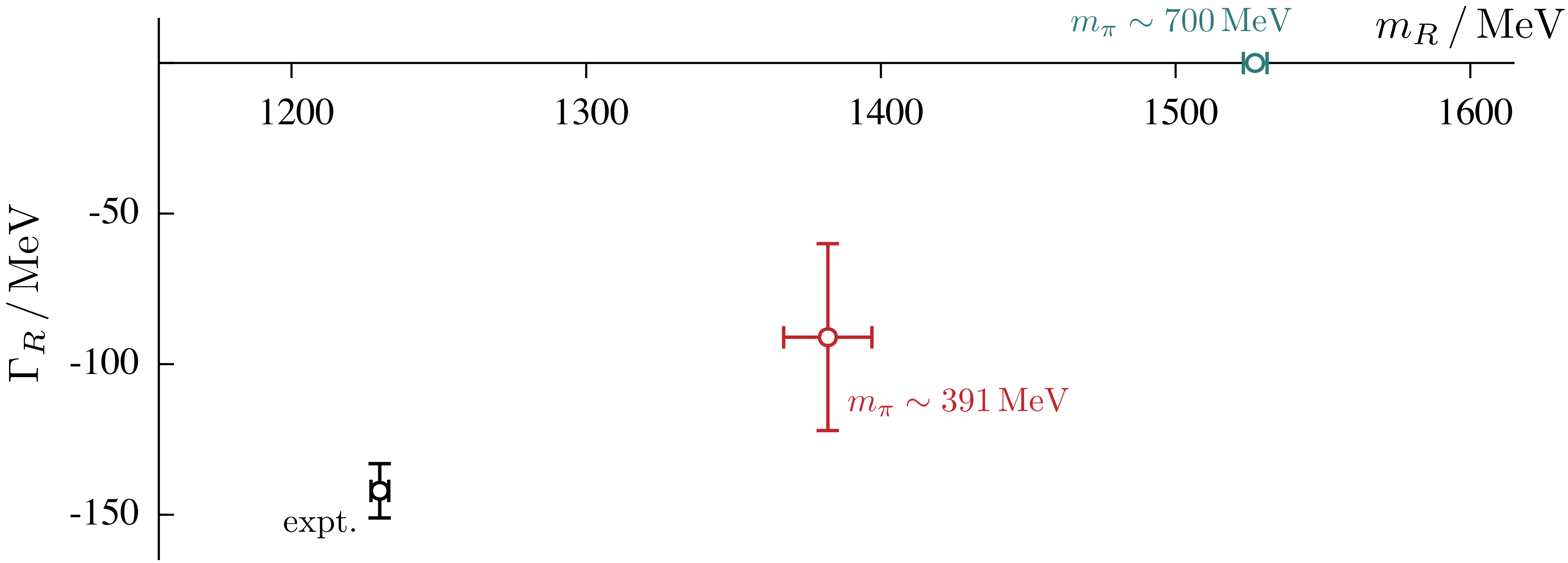


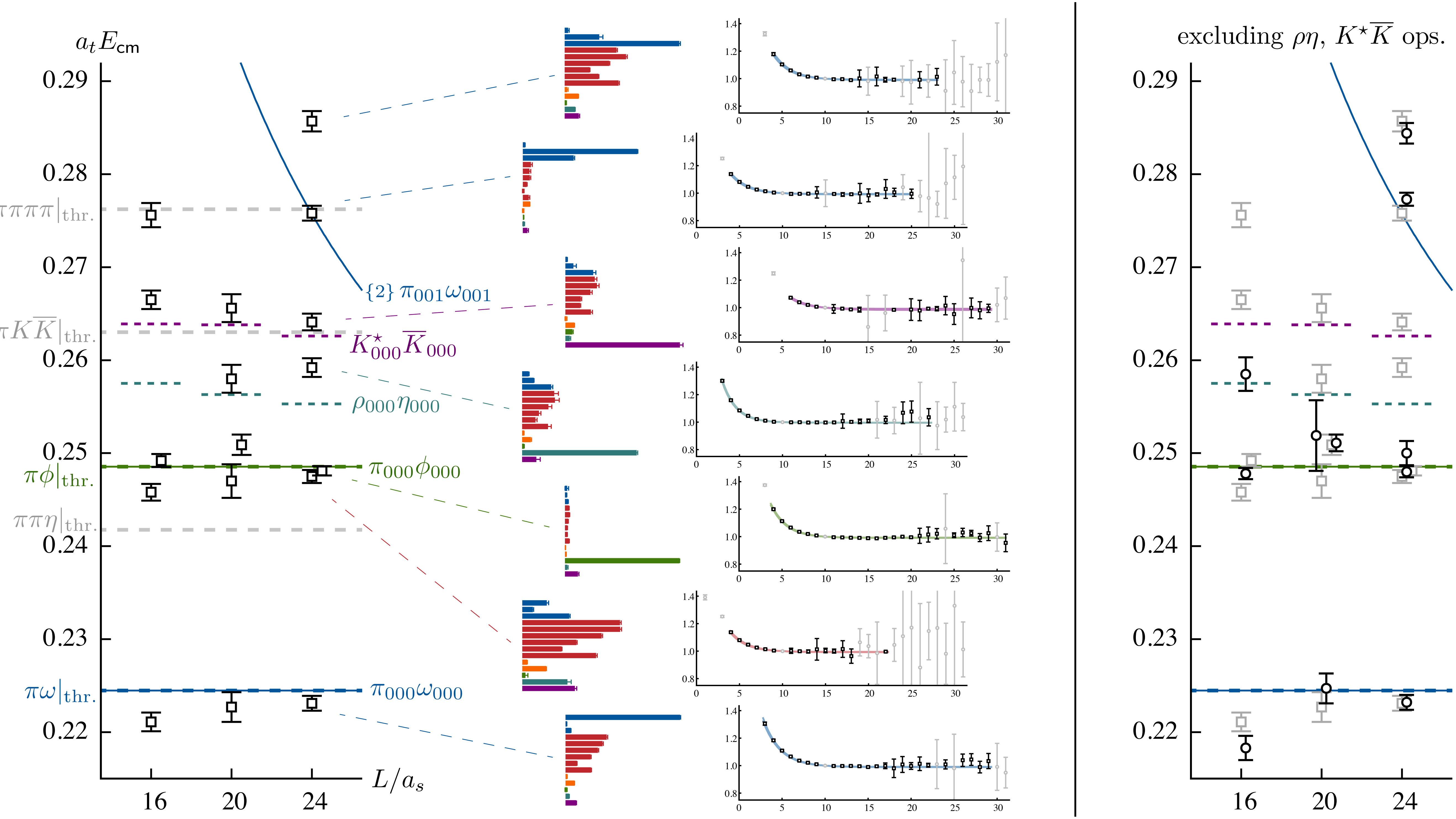
$$\begin{aligned}
m &= (0.2465 \pm 0.0007 \pm 0.0001)\cdot a_t^{-1} \\
g_{\pi\omega\{^3S_1\}} &= (0.106 \pm 0.007 \pm 0.007)\cdot a_t^{-1} \\
g_{\pi\omega\{^3D_1\}} &= (1.08 \pm 0.47 \pm 0.28)\cdot a_t \\
\gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} &= -0.35 \pm 0.19 \pm 0.18 \\
\gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} &= 0.90 \pm 0.24 \pm 0.27
\end{aligned}
\qquad
\begin{bmatrix}
1 & -0.05 & 0.05 & -0.01 & -0.23 \\
 & 1 & 0.70 & -0.54 & -0.06 \\
 & & 1 & -0.39 & -0.06 \\
 & & & 1 & 0.22 \\
 & & & & 1
\end{bmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{36.8}{36-5} = 1.19.$$



$\pi\omega|_{\text{thr.}}$
$\pi\pi\eta|_{\text{thr.}}$
$\pi\phi|_{\text{thr.}}$
$\pi K\bar{K}|_{\text{thr.}}$
$\mathrm{Re}(a_t\sqrt{s})$
0.00
-0.01
-0.02
-0.03
0.22
0.23
0.24
0.25
0.26
0.27
$2\cdot\mathrm{Im}(a_t\sqrt{s})$
$\mathrm{Im}(a_t c_i)$
0.04
0.02
-0.02
-0.04
-0.06
-0.02
0.02
0.04
0.06
0.08
0.10
0.12
$\mathrm{Re}(a_t c_i)$
$\pi\phi\{^3S_1\}$
$\pi\omega\{^3D_1\}$
$\pi\omega\{^3S_1\}$

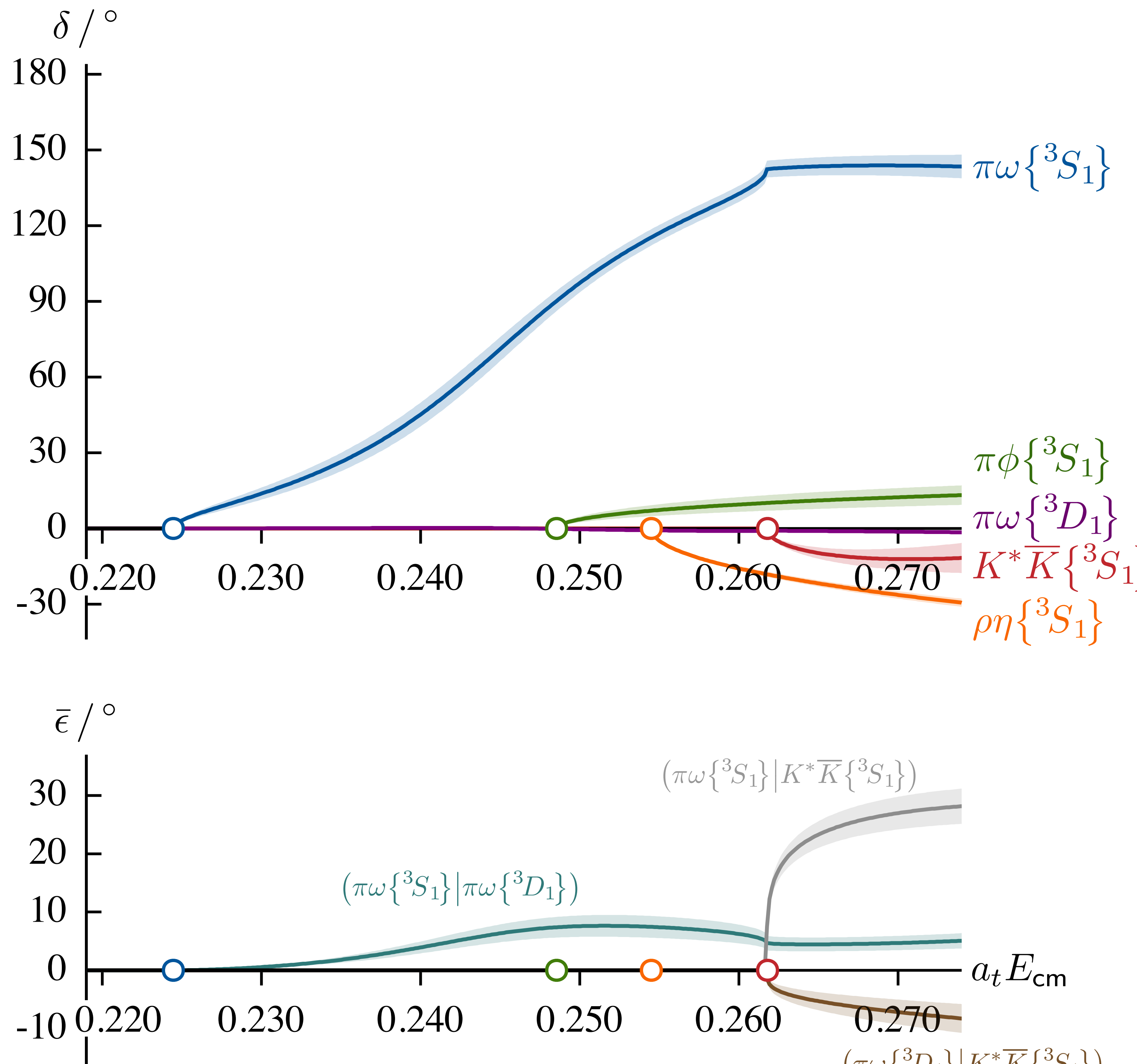

$m_\pi \sim 700\,\mathrm{MeV}$
m_R / MeV
1200
1300
1400
1500
1600
Γ_R / MeV
-50
-100
-150
$m_\pi \sim 391\,\mathrm{MeV}$
expt.


$a_t E_{\mathsf{cm}}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\pi\pi\pi\pi|_{\mathrm{thr.}}$
$\pi K\overline{K}|_{\mathrm{thr.}}$
$\pi\phi|_{\mathrm{thr.}}$
$\pi\pi\eta|_{\mathrm{thr.}}$
$\pi\omega|_{\mathrm{thr.}}$
$\{2\}\ \pi_{001}\omega_{001}$
$K^\star_{000}\overline{K}_{000}$
$\rho_{000}\eta_{000}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
L/a_s
16
20
24
excluding $\rho\eta$, $K^\star\overline{K}$ ops.

treating ρ, K* as stable states

useful to check that the πω, πφ amplitudes are insensitive to the effects of these nearby channels

resonance pole position is robust



$$\chi^2/N_{\rm dof} = \frac{46.6}{48-9} = 1.19$$

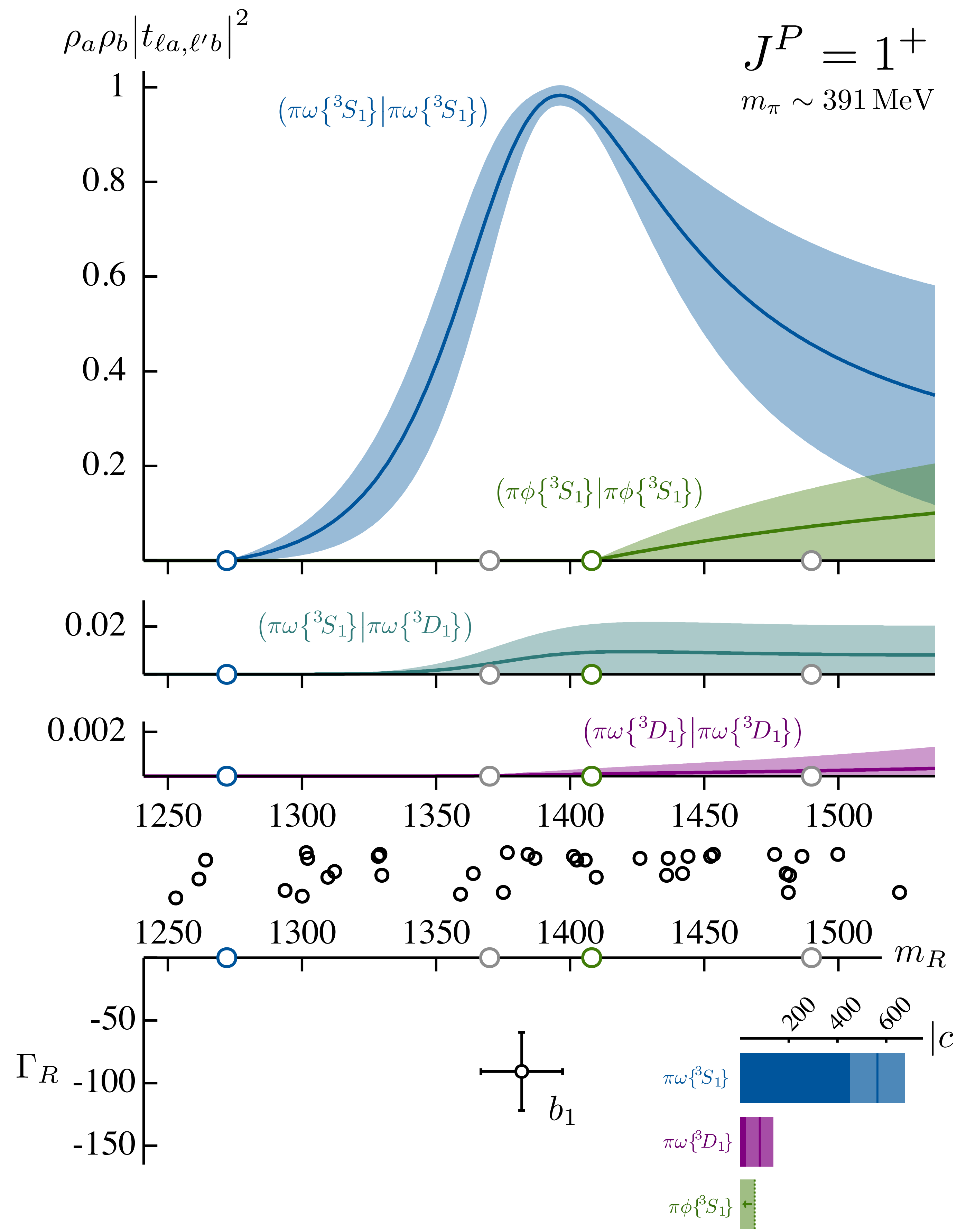
we have computed the b_1 resonance in dynamically coupled partial waves at $m_\pi = 391$ MeV

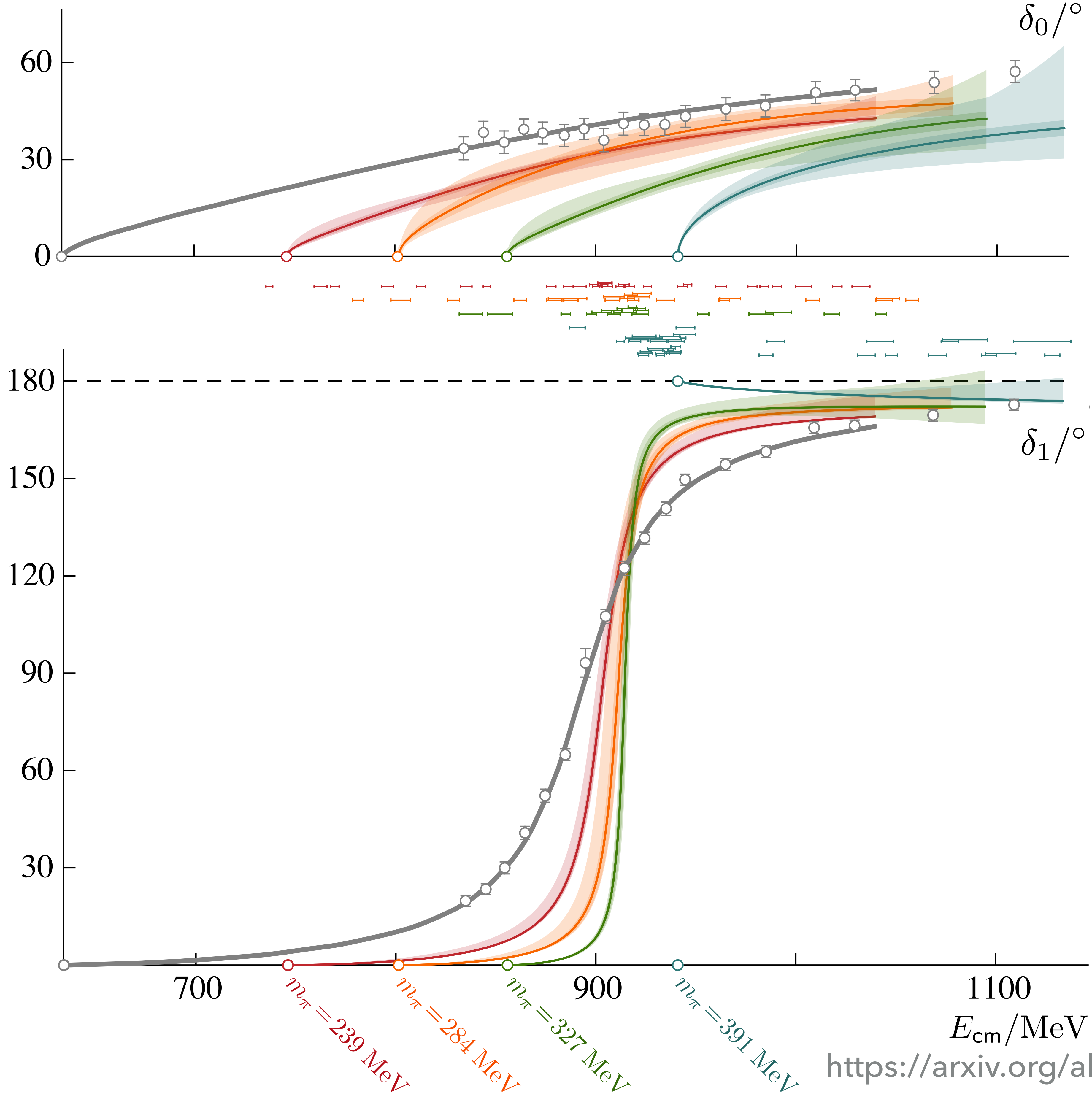
it has all the hallmarks of the experimental b_1 - a broad resonance pole with a dominant ${}^3S_1$ and subleading ${}^3D_1$ components

3-body operators were used in extracting the spectrum

3 and 5 coupled-channel systems were obtained

a consistent pole position was found





https://arxiv.org/abs/1904.03188

accepted in PRL

scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$

P-wave: