PWA11/ATHOS6, 2–6 September 2019, Rio de Janeiro

Theory of two-pion photo- and electroproduction off the nucleon

PRD**99**,053001(2019); see also arXiv:1811.01475v1[nucl-th]

Helmut Haberzettl

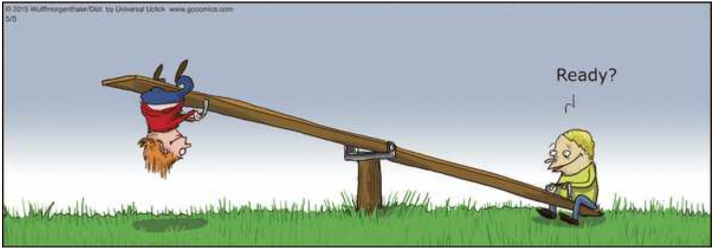
Institute for Nuclear Studies and Department of Physics

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

Collaborators: Kanzo Nakayama (UGA), Yongseok Oh (KNU)

Supported by DOE Award DE-SC0016582


Ready?

Motivation

- Two-pion production $\gamma N \to \pi\pi N$ is being measured now
- No comprehensive formulation of two-pion photoproduction processes exists that is at the same level of rigor as single-pion production
- Also interested in strangeness production in processes like $\gamma N \to KK\Xi$

Goal

- Provide microscopically consistent formulation of photo- and electroproduction processes of two pions off the nucleon
- Full implementation of local gauge invariance (Generalized Ward-Takahashi identities)

Procedure

- Use field theory based on hadronic Lagrangians
- Employ LSZ-type mechanisms to couple electromagnetic field to fully dressed hadronic propagators and vertices

Outline

- A word about Gauge Invariance
- Recap single-pion production formalism
- Two-pion production
- An application
- Summary

NB:

(1) Pions and nucleons are placeholders for arbitrary mesons and baryons.

(2) Formulation is valid for real *and* virtual photons.

(3) In all diagrams, time runs from right to left to conform with the usual quantum-mechanical evaluation of matrix elements, $\langle \mathit{final} | \text{operator} | \mathit{initial} \rangle$

A word about Gauge Invariance

Global gauge invariance: Physics invariant under global phase transformation of field

$$\phi \to \phi \, \mathrm{e}^{-i\lambda} \qquad \lambda\text{: real, constant}$$

$$\boxed{k_\mu J^\mu = 0}$$

implies a conserved charge

Local gauge invariance: Physics invariant under local phase transformation of field

$$\phi \to \phi \, \mathrm{e}^{-i\Lambda(x)} \qquad \Lambda\text{: real function of } x$$

implies a conserved charge

In addition, it implies the very existence of the electromagnetic field A^μ, i.e., it provides Maxwell's equations:

$$\partial_\nu F^{\mu\nu} = -J^\mu \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Violation of *local* gauge invariance tampers with consistent implementation of the electromagnetic field!

$\Longrightarrow$

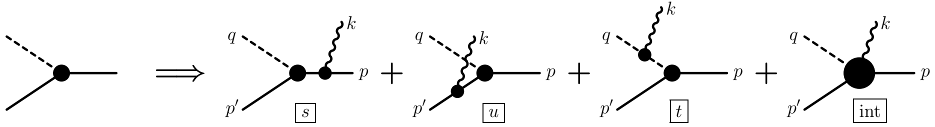
Always insist that currents satisfy (generalized) Ward-Takahashi identities!

First, something simple. . .

Single-pion Production

Construct Single-pion Production Current

Attach photon to πNN vertex:

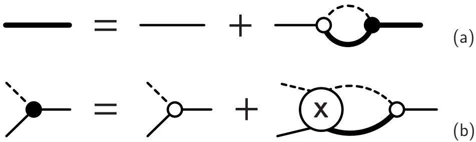


- Simple at tree level
- Quite complicated for fully *dressed* problem

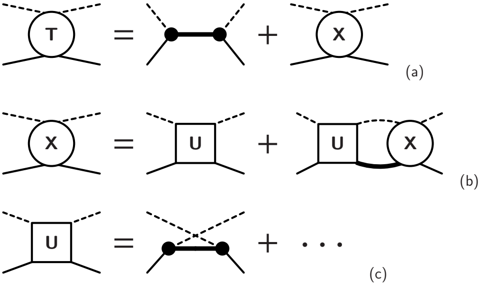
Need:

- dressed propagators
- dressed vertices
- dressed single-particle currents
- dressed interaction current

Dressed propagator and vertex:



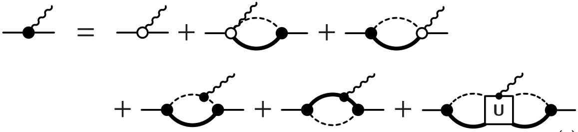
Dressed $\pi N \to \pi N$ T-matrix:



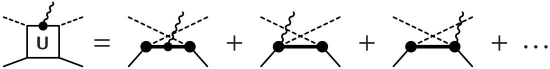
- Tower of *nonlinear* Dyson-Schwinger-type equations

Gathering ingredients. . . (electromagnetic)

Dressed nucleon current:

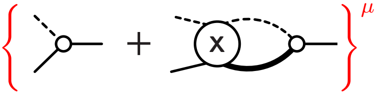


(a)



(b)

Dressed interaction current:

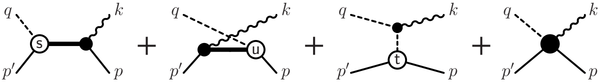
$M^{\mu}_{\text{int}} = -$ 

contains πN final-state interaction

$\left\{\cdots\right\}^{\mu}$ denotes gauge derivative; see PRC**56**, 2041 (1997)

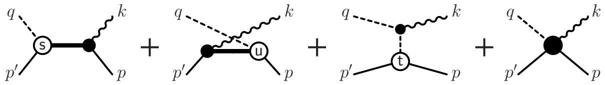
Fully Dressed Production Current

$M^\mu =$



M_s^μ + M_u^μ + M_t^μ + M_{int}^μ

Fully Dressed Production Current



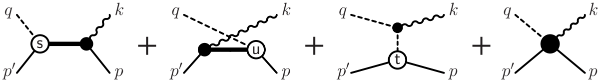
$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$

Aside: Considering *fully dressed* hadron propagators and consistently constructed *fully dressed* particle currents, one finds huge cancellations of dressing effects where *only* the πNN vertices and the interaction current M_{int}^μ retains any effects of dressing.

HH, PRD**99**,016022(2019)

Gauge Invariance: Generalized WTI

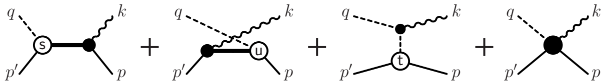
$$M^\mu =$$



$$M_s^\mu \qquad M_u^\mu \qquad M_t^\mu \qquad M_{\text{int}}^\mu$$

Gauge Invariance: Generalized WTI

$M^\mu =$



$M_s^\mu \qquad M_u^\mu \qquad M_t^\mu \qquad M_{\text{int}}^\mu$

Generalized WTI for the full current M^μ:

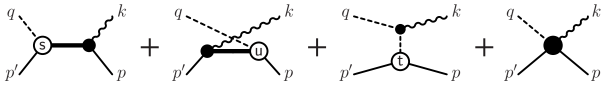
$$k_\mu M^\mu = -\underbrace{F_s S(p+k) Q_i S^{-1}(p)}_{s-\text{channel}} + \underbrace{S^{-1}(p') Q_f S(p'-k) F_u}_{u-\text{channel}} + \underbrace{\Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t}_{t-\text{channel}}$$

Off-shell constraint!

Hadrons on-shell: $k_\mu M^\mu = 0$

Gauge Invariance: Generalized WTI

$M^\mu =$



$M_s^\mu \quad M_u^\mu \quad M_t^\mu \quad M_{\text{int}}^\mu$

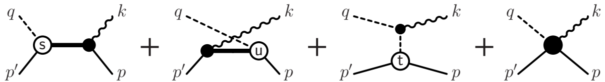
■ **Generalized WTI for the full current M^μ:**

$$k_\mu M^\mu = -\underbrace{F_s S(p+k) Q_i S^{-1}(p)}_{s-\text{channel}} + \underbrace{S^{-1}(p') Q_f S(p'-k) F_u}_{u-\text{channel}} + \underbrace{\Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t}_{t-\text{channel}}$$

Approximations destroy local gauge invariance!

Gauge Invariance: Generalized WTI

$M^\mu =$



$M_s^\mu \qquad M_u^\mu \qquad M_t^\mu \qquad M_{\text{int}}^\mu$

- **Generalized WTI for the full current M^μ:**

$$k_\mu M^\mu = \underbrace{-F_s S(p+k) Q_i S^{-1}(p)}_{s-\text{channel}} + \underbrace{S^{-1}(p') Q_f S(p'-k) F_u}_{u-\text{channel}} + \underbrace{\Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t}_{t-\text{channel}}$$

To the rescue:

- **Equivalent Generalized WTI for the interaction current M_{int}^μ:**

$$k_\mu M_{\text{int}}^\mu = -F_s \hat{Q}_i + \hat{Q}_f F_u + \hat{Q}_\pi F_t$$

Gauge Invariance: Generalized WTI

- **Equivalent Generalized WTI for the interaction current M^{μ}_{int}:**

$$k_\mu M^\mu_{\mathsf{int}} = -F_s \hat{Q}_i + \hat{Q}_f F_u + \hat{Q}_\pi F_t$$

Task: Determine approximations of M^μ_{int} that preserve this WTI.

Gauge Invariance: Generalized WTI

- **Equivalent Generalized WTI for the interaction current M^μ_{int}:**

$$k_\mu M^\mu_{\text{int}} = -F_s \hat{Q}_i + \hat{Q}_f F_u + \hat{Q}_\pi F_t$$

Task: Determine approximations of M^μ_{int} that preserve this WTI.

Can be done straightforwardly at various levels of sophistication. ✓

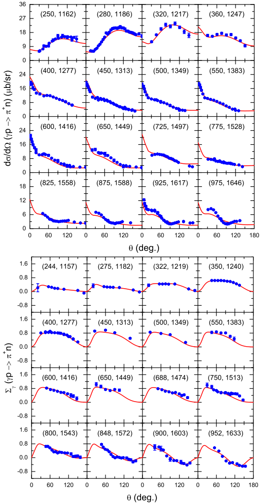
For details, see: PRC**56**,2041(1997); PRC**62**,034605(2000); PRC**74**,045202(2006); PRC**83**,065502(2011); PRC**92**,055503(2015)

Does it work? — Yes!

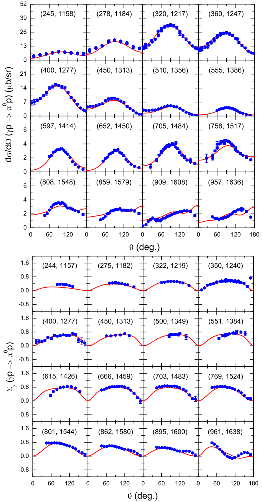
Results for $\gamma N \to \pi N$

$\frac{d\sigma}{d\Omega}$

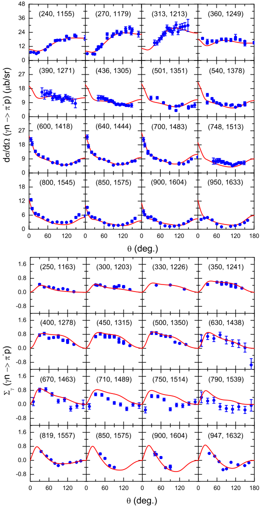
Σ



$\gamma p \to \pi^+ n$



$\gamma p \to \pi^0 p$



$\gamma n \to \pi^- p$

F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama, PRC**85**, 054003 (2012)

Now, the real thing...

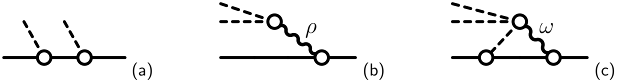
Two-pion Production

Now, the real thing. . .

Two-pion Production

Needed for LSZ formalism: Hadronic Green's function for $N \to \pi\pi N$

Basic Hadronic Two-pion Production Processes



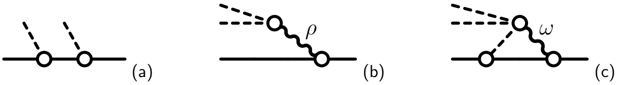
etc.

(a) sequential production off nucleon

(b) production off intermediate vector meson

(c) production off intermediate three- or more-pion vertex

Basic Hadronic Two-pion Production Processes



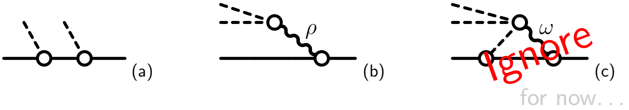
etc.

(a) sequential production off nucleon
(b) production off intermediate vector meson
(c) production off intermediate three- or more-pion vertex

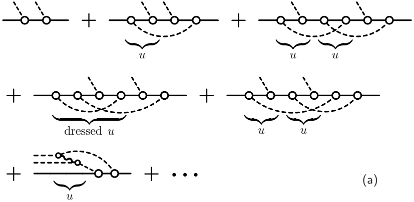
Procedure

(1) Iterate bare hadronic processes and sum up to obtain dressed mechanisms
(2) Attach photon — employ (gauge-invariant!!) single-pion amplitudes

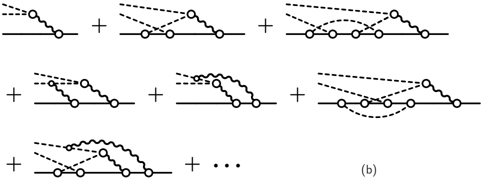
Iterated Hadronic Two-pion Production Processes



etc.

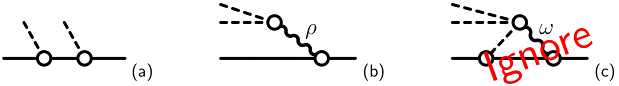


(a)



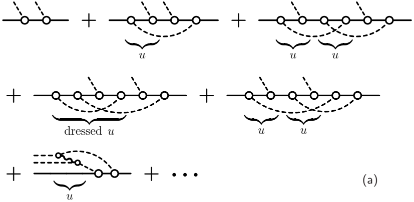
(b)

Iterated Hadronic Two-pion Production Processes

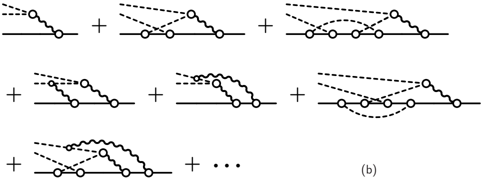


for now. . .

etc.

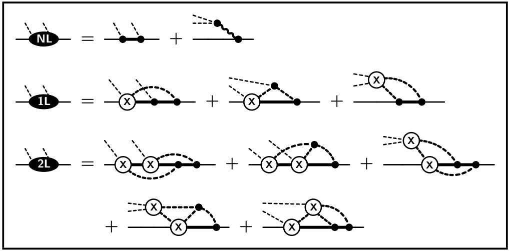


(a)



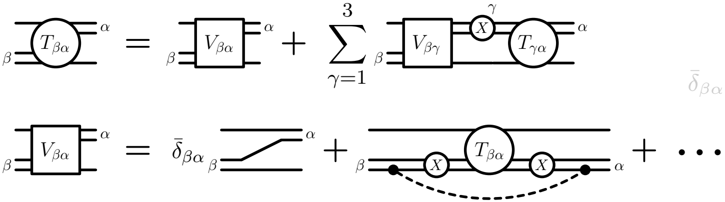
(b)

Lowest orders of
3-body multiple scattering series



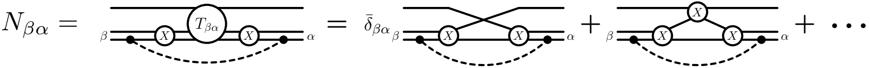
Faddeev-type Alt-Grassberger-Sandhas Equations

NP **B2**, 167 (1967)



$\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$

Non-linear contributions:



Cluster indices:

$\alpha, \beta, \gamma :$ "1" $= (\pi_1 N, \pi_2)$
"2" $= (\pi_2 N, \pi_1)$
"3" $= (\pi_1 \pi_2, N)$

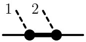
$$T_{\beta\alpha} = V_{\beta\alpha} + \sum_{\gamma=1}^{3} V_{\beta\gamma} G_0 X_\gamma G_0 T_{\gamma\alpha}$$

Matrix LS structure: $\mathrm{T} = \mathrm{V} + \mathrm{V} G_0 \mathrm{T}$

Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha + \sum_\gamma \left(\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0 \right) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

AGS amplitudes $T_{\beta\alpha}$ subsume all explicit three-body effects, with contributions of infinitely many mesons provided by their non-linear driving terms $N_{\beta\alpha}$

where $f_1 =$ 

$f_2 =$ 

$f_3 =$ 

Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha$$

$$+ \sum_\gamma (\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

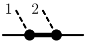
Loop Expansion. . .

$$= f_\beta$$ $\Leftarrow$ no loop

$$+ \sum_{\gamma,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} X_\gamma G_0 f_\alpha$$ $\Leftarrow$ one loop

$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha$$ $\Leftarrow$ two loops

$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$ appear only at 2-loop level — *Relief!*

where $f_1 =$ 

$f_2 =$ 

$f_3 =$ 

Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha + \sum_\gamma \left(\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0\right) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

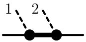
$$= f_\beta \qquad \Leftarrow \text{ no loop}$$

$$+ \sum_{\gamma,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} X_\gamma G_0 f_\alpha \qquad \Leftarrow \text{ one loop}$$

$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha \qquad \Leftarrow \text{ two loops}$$

$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$

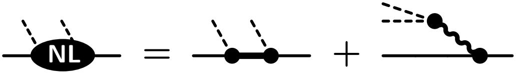
attach photon order by order

where $f_1 =$ 

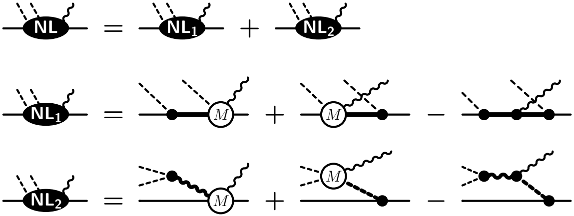
$f_2 =$ 

$f_3 =$ 

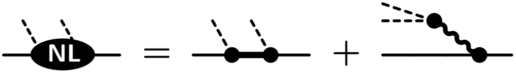
Attach Photon — No-loop Graphs



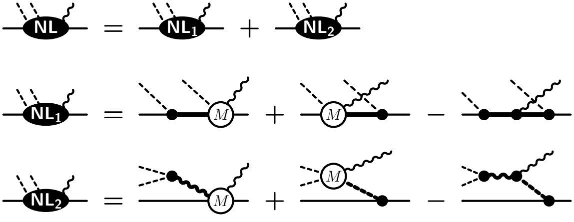
$\Downarrow$ attach photon



Attach Photon — No-loop Graphs



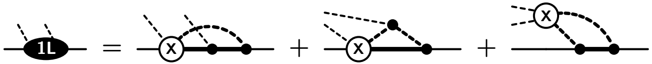
⇓ attach photon



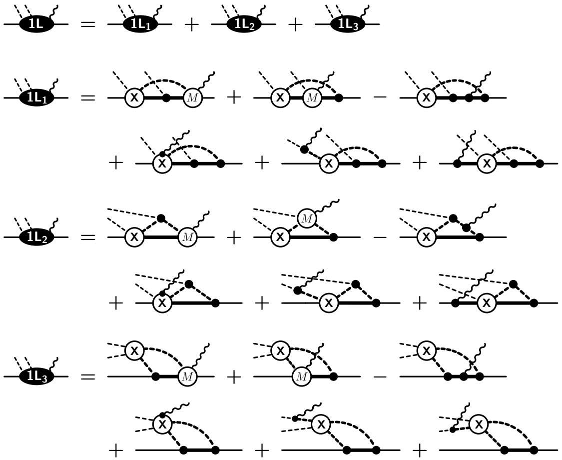
separately gauge invariant!

Local gauge invariance follows as a matter of course since all ingredients satisfy their respective off-shell WTIs

Attach Photon — One-loop Graphs



⇓ attach photon



separately gauge invariant!

10 graphs for each group

In general, for n loops, there are $7+3n$ graphs in each group

Attaching Photon at All Orders

$$M^{\mu}_{\pi\pi} = \sum_{i=0}^{\infty} M^{\mu}_{\pi\pi}[i]$$

$$= \underbrace{M^{\mu}_{\pi\pi}[0]}_{\text{no loop}} + \underbrace{M^{\mu}_{\pi\pi}[1]}_{\text{one loop}} + \underbrace{M^{\mu}_{\pi\pi}[2] + \cdots}_{\text{higher orders}}$$

Attaching Photon at All Orders

$$
\begin{aligned}
M^{\mu}_{\pi\pi} &= \sum_{i=0}^{\infty} M^{\mu}_{\pi\pi}[i] \\
&= \underbrace{M^{\mu}_{\pi\pi}[0]}_{\text{no loop}} + \underbrace{M^{\mu}_{\pi\pi}[1]}_{\text{one loop}} + \underbrace{M^{\mu}_{\pi\pi}[2] + \cdots}_{\text{higher orders}}
\end{aligned}
$$

not possible in practice

Attaching Photon at All Orders

$$M^{\mu}_{\pi\pi} = \sum_{i=0}^{\infty} M^{\mu}_{\pi\pi}[i] = \underbrace{M^{\mu}_{\pi\pi}[0]}_{\text{no loop}} + \underbrace{M^{\mu}_{\pi\pi}[1]}_{\text{one loop}} + \underbrace{M^{\mu}_{\pi\pi}[2] + \cdots}_{\text{higher orders}}$$

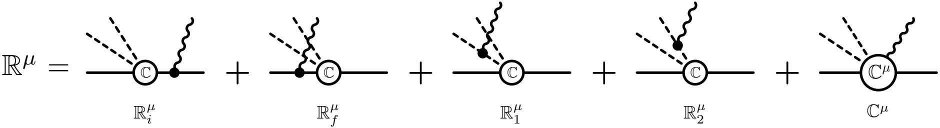
Truncated Expansion

$$M^{\mu}_{\pi\pi} \approx \sum_{i=0}^{N} M^{\mu}_{\pi\pi}[i] + \mathbb{R}^{\mu}[N]$$

Remainder term $\mathbb{R}^{\boldsymbol{\mu}}[\boldsymbol{N}]$ needed to mock up higher orders

Remainder Term

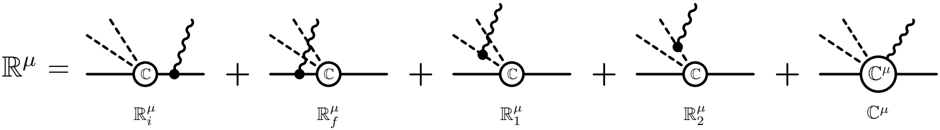
Ansatz:



Needed: Hadronic contact term $\mathbb{C}$ and contact current $\mathbb{C}^\mu$

Remainder Term

Ansatz:



Simple parametrization:

$$\mathbb{C} = \left(a_1 + a_2 \frac{\not{p}}{m} + a_3 \frac{\not{p}'}{m'} + a_4 \frac{\not{p}'\not{p}}{m'm} + b_1 \frac{\not{q}}{m_\pi} + b_2 \frac{\not{q}\not{p}}{m_\pi m} + b_3 \frac{\not{p}'\not{q}}{m' m_\pi} + b_4 \frac{\not{p}'\not{q}\not{p}}{m' m_\pi m} \right) \boldsymbol{F}$$

$$\begin{aligned} \mathbb{C}^\mu = & -e_i \boldsymbol{F_i} \left[\left(a_2 + a_4\right) + \left(b_2 + b_4\right) \frac{\not{q}}{m_\pi} \right] \frac{\gamma^\mu}{m} - e_f \boldsymbol{F_f} \frac{\gamma^\mu}{m'} \left[\left(a_3 + a_4\right) + \left(b_3 + b_4\right) \frac{\not{q}}{m_\pi} \right] \\ & - \left(e_1 \boldsymbol{F_1} - e_2 \boldsymbol{F_2}\right) \left(b_1 + b_2 + b_3 + b_4\right) \frac{\gamma^\mu}{m_\pi} \\ & + \left[\left(a_1 + a_2 + a_3 + a_4\right) + \frac{\not{q}}{m_\pi} \left(b_1 + b_2 + b_3 + b_4\right) \right] \boldsymbol{C^\mu_F} \end{aligned}$$

$\boldsymbol{F_i}$, $\boldsymbol{F_f}$, $\boldsymbol{F_1}$, $\boldsymbol{F_2}$: various kinematical situations of form factor $\boldsymbol{F}$
$\boldsymbol{C^\mu_F}$: nonsingular contact current determined from form factor $\boldsymbol{F}$

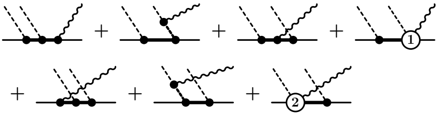
Read paper!

Application to $\gamma p \to K^+ K^+ \Xi^-$ at the No-loop Level

$$M^\mu_{KK} = \mathbb{M}^\mu_Y + \mathbb{R}^\mu \ ,$$

where

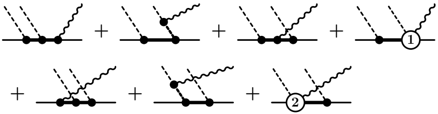
$$\mathbb{M}^\mu_Y =$$

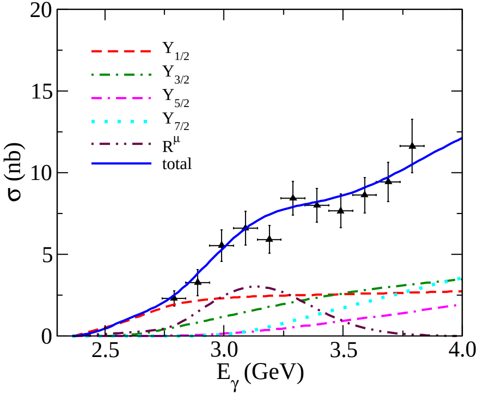


intermediate hyperons Λ, Σ

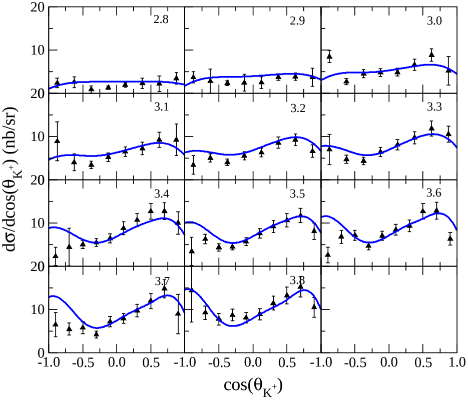
Application to $\gamma p \to K^+K^+\Xi^-$ at the No-loop Level

$$M^\mu_{KK} = \mathbb{M}^\mu_Y + \mathbb{R}^\mu \,, \qquad \text{where} \qquad \mathbb{M}^\mu_Y =$$





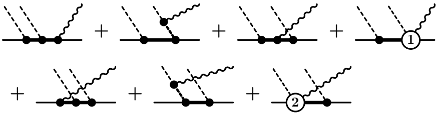
Total cross section

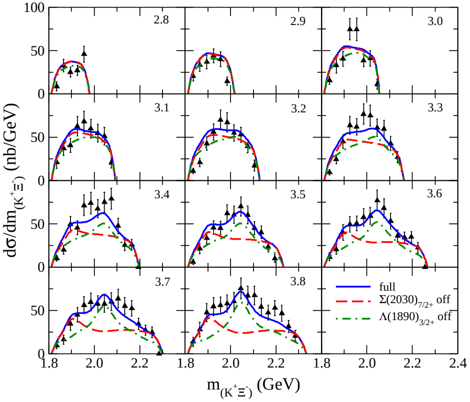


Differential cross section

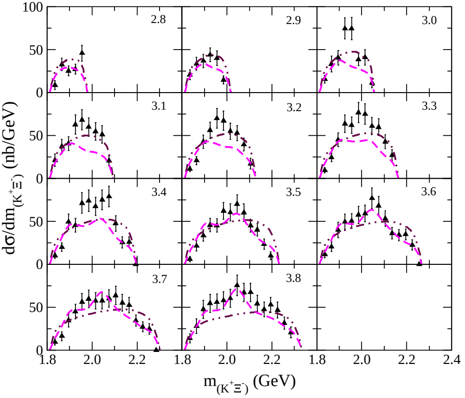
Application to $\gamma p \to K^+ K^+ \Xi^-$ at the No-loop Level

$$M^\mu_{KK} = \mathbb{M}^\mu_Y + \mathbb{R}^\mu \,, \qquad \text{where} \qquad \mathbb{M}^\mu_Y =$$



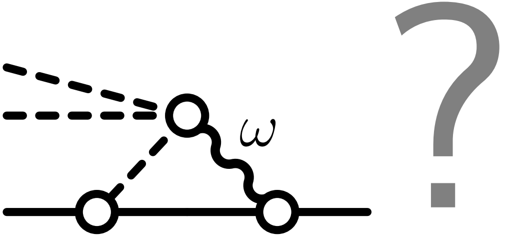


Invariant-mass distribution

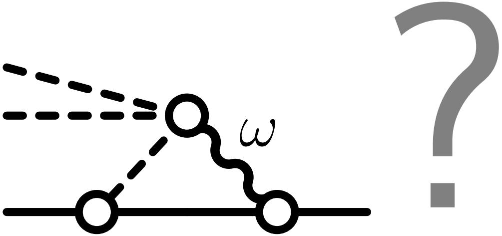


Effect of remainder $\mathbb{R}^\mu$ (magenta w/o $\mathbb{R}^\mu$)

What about...



What about. . .



Can be done. Complicated. — I'll skip this. . . See PRD**99**,053001(2019)

Summary

- Theory presented provides a complete description of the $\pi\pi$ production process based on field theory. (*This is not a model!* — In principle, the formalism could be implemented to an arbitrary degree of sophistication for any given set of interaction Lagrangians.)
- Consistent expansion of the two-pion production current in terms of the $\pi\pi N$ Faddeev ordering structure.
- Full implementation of *local* gauge invariance order by order in terms of *Generalized Ward–Takahashi Identities* at all levels of the reaction dynamics.
  $\Longrightarrow$ Essential for the microscopic consistency of all reaction mechanisms.
- Valid for hadronic two- and three-point functions dressed by arbitrary internal mechanisms — even nonlinear ones.
- Resulting $\pi\pi$ production current written in closed form accounting for full three-body dynamics.
- Extension beyond one-photon approximation straightforward.
- Translation to other two-meson production processes straightforward.
- Calculations are in progress ($K\bar{K}N$, $KK\Xi$)

Thank you! Obrigado!

PWA11/ATHOS6, 2–6 September 2019, Rio de Janeiro