# Toward a minimum spectrum of excited baryons


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PRD99, 016001 (2019)

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# *Resonance determination: issues one faces*

Basic issues in baryon resonance determination from data:

- Resonance extraction from experimental data is non-trivial since the separation of the signal from background is model-dependent.

- Light resonances may be broad and potentially overlapping resonances are difficult to distinguish from the background.

- Many experimental data, especially, in hadronic reactions suffer from large uncertainties. Some of them suffer from under- or over-estimation of the systematical uncertainties. Furthermore, many data sets are often inconsistent with each other, even for recent high-precision photoproduction data (e.g., differential cross sections in $\eta$, $\eta'$ and $\omega$ photoproduction from the CLAS and CBELSA/TAPS collaborations [V. Crede, *et al*., PRC80'2009; A. Wilson *et al*., PLB479'2015] ).

# *Resonance determination: standard procedures*

Resonances are determined from experimental data by fitting the data using some model. Except for those models where the resonances are generated dynamically (e.g., U$\chi$PT), one usually considers the resonances:

a) From some quark (or related) theoretical models
b) Known from PDG (1-, 2-, 3-, 4-star resonances)
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Problem: Many potentially relevant resonances.

# *Resonance determination: standard procedures*

S=-1 hyperon resonances listed in PDG (35 resonances):

| $\Lambda$ states | | | | $\Sigma$ states | | | |
|---|---|---|---|---|---|---|---|
| State | $m_R$ (MeV) | $\Gamma_R$ (MeV) | Rating | State | $m_R$ (MeV) | $\Gamma_R$ (MeV) | Rating |
| $\Lambda(1116)\ 1/2^+$ | 1115.7 | | **** | $\Sigma(1193)\ 1/2^+$ | 1193 | | **** |
| $\Lambda(1405)\ 1/2^-$ | 1406 | 50 | **** | $\Sigma(1385)\ 3/2^+$ | 1385 | 37 | **** |
| $\Lambda(1520)\ 3/2^-$ | 1520 | 16 | **** | | | | |
| $\Lambda(1600)\ 1/2^+$ | 1600 | 150 | *** | $\Sigma(1660)\ 1/2^+$ | 1660 | 100 | *** |
| $\Lambda(1670)\ 1/2^-$ | 1670 | 35 | **** | $\Sigma(1670)\ 3/2^-$ | 1670 | 60 | **** |
| $\Lambda(1690)\ 3/2^-$ | 1690 | 60 | **** | $\Sigma(1750)\ 1/2^-$ | 1750 | 90 | *** |
| $\Lambda(1800)\ 1/2^-$ | 1800 | 300 | *** | $\Sigma(1775)\ 5/2^-$ | 1775 | 120 | **** |
| $\Lambda(1810)\ 1/2^+$ | 1810 | 150 | *** | $\Sigma(1840)\ 3/2^+$ | 1840 | 100 | * |
| $\Lambda(1820)\ 5/2^+$ | 1820 | 80 | **** | $\Sigma(1880)\ 1/2^+$ | 1880 | 194 | ** |
| $\Lambda(1830)\ 5/2^-$ | 1830 | 95 | **** | $\Sigma(1900)\ 1/2^-$ | 1900 | 191 | * |
| $\Lambda(1890)\ 3/2^+$ | 1890 | 100 | **** | $\Sigma(1915)\ 5/2^+$ | 1915 | 120 | **** |
| $\Lambda(2000)\ ?^?$ | 2000 | 167 | * | $\Sigma(1940)\ 3/2^+$ | 1941 | 400 | * |
| $\Lambda(2020)\ 7/2^+$ | 2020 | 195 | * | $\Sigma(1940)\ 3/2^-$ | 1940 | 220 | *** |
| $\Lambda(2100)\ 7/2^-$ | 2100 | 200 | **** | $\Sigma(2000)\ 1/2^-$ | 2000 | 273 | * |
| $\Lambda(2110)\ 5/2^+$ | 2110 | 200 | *** | $\Sigma(2030)\ 7/2^+$ | 2030 | 180 | **** |
| $\Lambda(2325)\ 3/2^-$ | 2325 | 169 | * | $\Sigma(2070)\ 5/2^+$ | 2070 | 220 | * |
| $\Lambda(2350)\ 9/2^+$ | 2350 | 150 | *** | $\Sigma(2080)\ 3/2^+$ | 2080 | 177 | ** |
| | | | | $\Sigma(2100)\ 7/2^-$ | 2100 | 103 | * |
| | | | | $\Sigma(2250)\ ?^?$ | 2265 | 100 | *** |

# *Resonance determination: standard procedures*

Resonances are determined from experimental data by fitting the data using some model. Except for those models where the resonances are generated dynamically (e.g., U$\chi$PT), one usually considers the resonances:

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It is impractical to consider all of them.
Which ones to consider is subjective.
Problem of overfitting.

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In particular, if a flexible background with resonance terms on top of it is provided, the task consists of minimizing the number of resonances and only accepting them as physically significant if the background cannot provide a satisfactory description.

# *Resonance determination: LASSO+ITC*

A blindfold determination of the resonances based on the Least Absolute Shrinkage and Selection Operator (LASSO) in combination with Information Theory Criteria (ITC)

R. Tibshirani, J. R. Stat. Soc. B **58**, 267 (1996);
T. Hasti *et al., The Elements of Statistical Learning: Data Mining, Inference, and Prediction* (Springer-Verlag, N.Y., 2009);
G. James, et al., A*n Introduction to Statistical Learning* (Springer-Verlag, N. Y., 2013).

LASSO : minimize

$$\chi_T^2 = \chi^2 + \lambda^2 \sum_R |f_R| \;\; ; \quad \lambda = \text{penalty parameter}$$

$f_R$ = penalty function (resonances in our case)

as a function of $\lambda$.

ITC : selection of optimal value of $\lambda$ based on Information Theory Criteria:

Akaike Information Criterion (AIC)
Bayesian Information Criterion (BIC)
& their variatiants

H. Akaike, IEEE Trans. Autom. Control **19**, 716 (1974);
K. Burnham and D. Anderson, *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach* (Springer-Verlag, New York, 2003);
G. Schwarz, Ann. Stat. **6**, 461 (1978);
J. E. Cavanaugh, Statist. Probab. Lett. **33**, 201 (1997).

AIC/BIC attempt to resolve the overfitting problem by introducing a penalty term for the number of parameters in the model.

# *Resonance determination: LASSO in hadron physics*

Applied in partial-wave analysis in mesonic systems
[B. Guegan *et al*., JINST **10**, P09002 (2015)]
See also [M. Williams, JINST 12, P09034 (2017)]
Stefan Wallner' talk this morning.

Bayesian inference to determine the baryon resonance spectrum (Ghent group)
[L. De Cruz et al., PRC86' 015212 (2012); PRL108' 182002 (2012)]
See also [J. Nys et al., PLB759' 260 (2016)]

LASSO +different criteria to determine the multipole content in pion photoproduction
[J. Landay *et al*., PRC95'17]

This work : LASSO + BIC to determine the minimum baryon resonance content.
[J. Landay et al., PRD99' 016001(2019)] (detailed study of LASSO+ITC)

# *Resonance determination: expt. data prunning*

## Self-consistent 3σ criterion:

[Perez, Amaro, Arriola, PRC88'13, 89'14]

For a set of $n$ measurements with Gaussian distribution, the quantity $z = \chi^2 / n$ will satisfy the normalized probability distribution:

$$P_n(z) = \frac{n(nz/2)^{-1/2}}{2\Gamma(n/2)} e^{-nz/2}$$

**3σ criterion :** a dataset is inconsistent with the rest of the database if $z$ has a probability smaller than 0.27%. Then, for every $n$, the allowed $z$ is given by $z_{min} < z < z_{max}$, where

$$\int_{z_{max}}^{\infty} P_n(z)\, dz = \frac{\Gamma(n/2, nz_{min}/2)}{\Gamma(n/2)} = 0.0027$$

$$\int_{0}^{z_{min}} P_n(z)\, dz = 1 - \frac{\Gamma(n/2, nz_{min}/2)}{\Gamma(n/2)} = 0.0027$$

(absent for a set with $n$=1)

**self-consistency :** fit the entire unpruned data → apply 3σ criterion & prune the data → fit the pruned data anew & apply 3σ criterion to the entire unpruned data to get a new pruned data → repeat the process.

# *Resonance determination:* $\chi^2$ *merit function*

Theoretical description of a given experimental dataset with *n* data points is achieved by fitting the model parameters through a minimization procedure of the $\chi^2$ merit function :

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i^{exp}/Z - O_i^{the})^2}{(\delta O_i/Z)^2} + \frac{(1 - 1/Z)^2}{(\delta_{sys}/Z)^2}$$

where

$O_i^{exp}$ ($O_i^{the}$) = experimental (theoretical) value of the observable

$\delta O_i$ = statistical uncertainty

$\delta_{sys}$ = systematic uncertainty of the experiment

$Z$ = scalling factor:

no $\delta_{sys}$ → absolute dataset ($\delta_{sys} = 0$) → $Z = 1$

known common $\delta_{sys}$ → normalized dataset

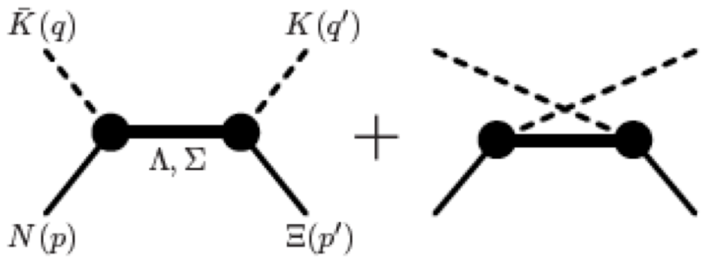
arbitrarily large $\delta_{sys}$ → floated dataset

minimize $\chi^2$ w.r.t. $Z$ :

$$Z = \left[\sum_{i=1}^{n} \frac{O_i^{exp} O_i^{the}}{\delta O_i^2} + \frac{1}{\delta_{sys}^2}\right] \Big/ \left[\sum_{i=1}^{n} \frac{O_i^{\exp 2}}{\delta O_i^2} + \frac{1}{\delta_{sys}^2}\right]$$

# *Illustration of the approach:* $\bar{K}N \to K\Xi$

1) Basic reaction mechanism:

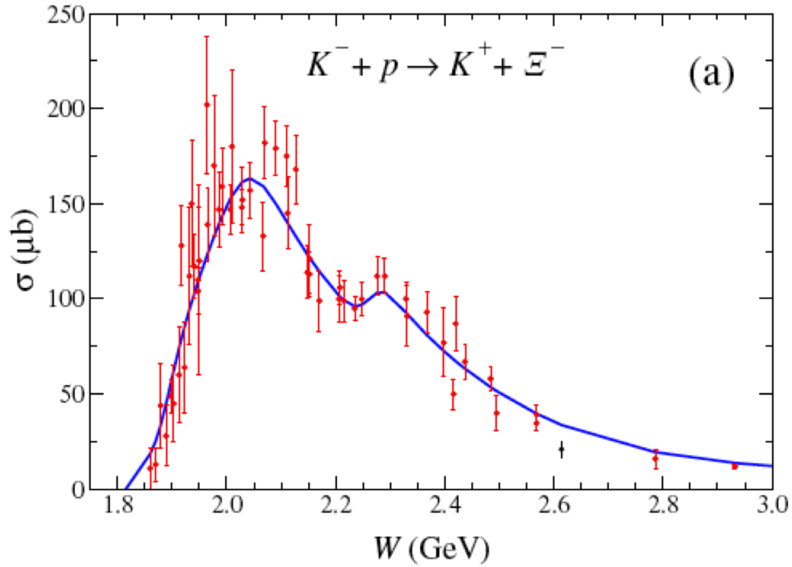


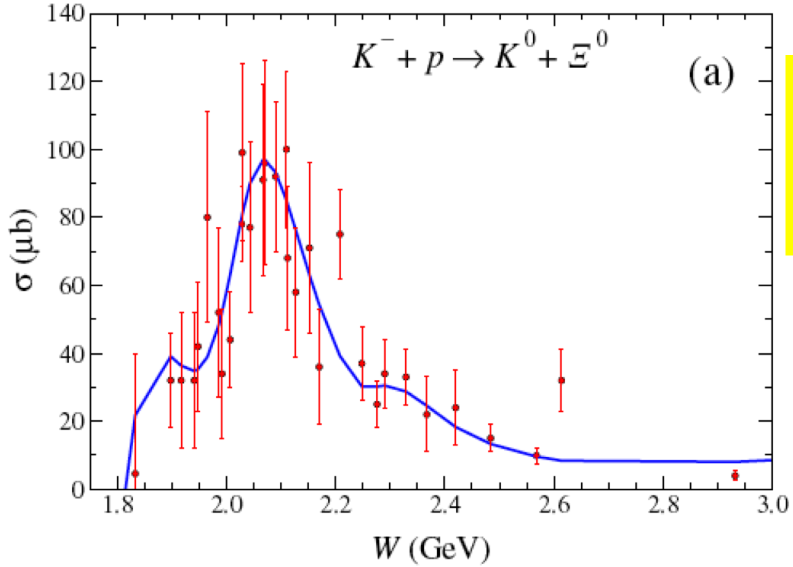

hyperon exchanges

no t-channel exchange
(absence of S=2 exotic mesons)

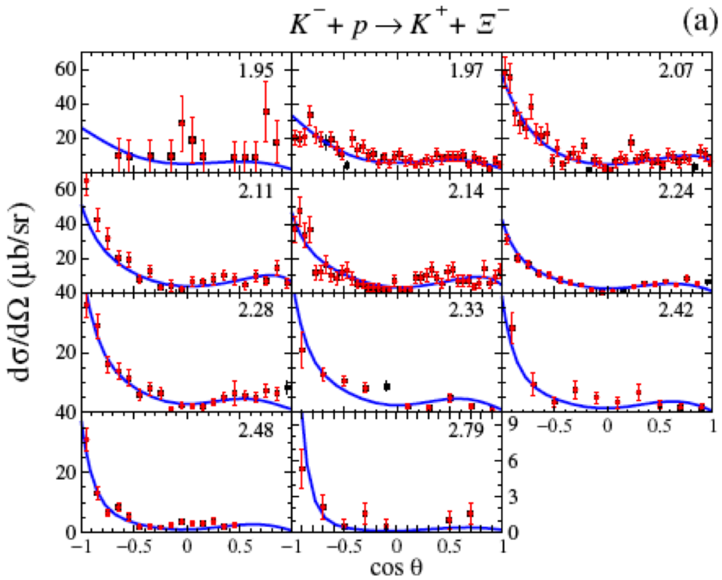
2) existing data are of poor quality: does LASSO+ITC work?
for high-quality data (pion photoproduction): works [J. Landay *et al*., PRC95'17]

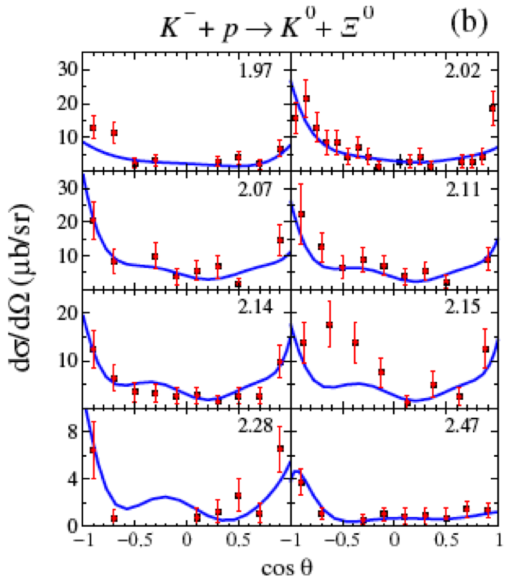
# $\bar{K}N \rightarrow K\Xi$ : experimental data prunning

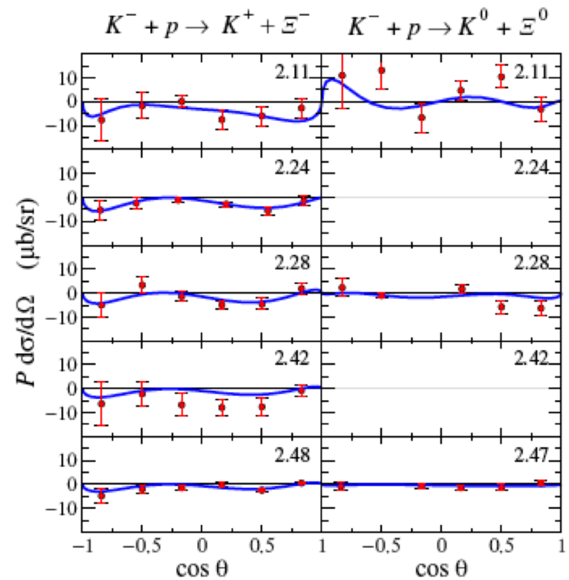



448 data points.
10 out of the allowed range of $z$ .



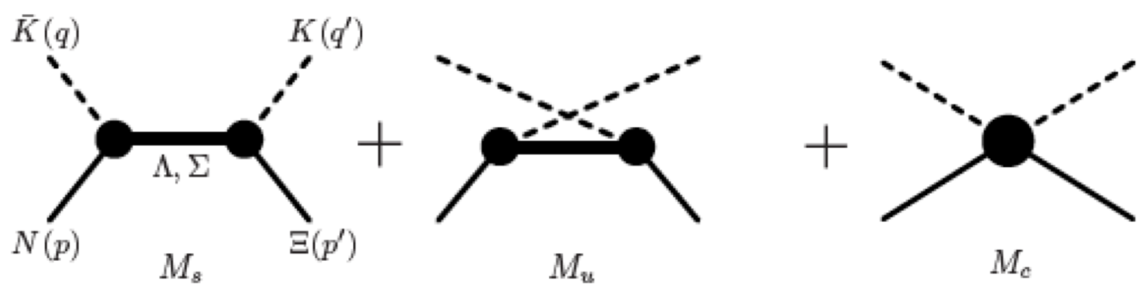



# $\bar{K}N \to K\Xi$ : *model*

*[Jackson, Oh, Haberzettl, K.N., PRC91(2015)065208]*

$$T = V + VG_0T = T^P + T^{NP}$$

$$\begin{cases} T^P = \sum_r |F_r\rangle S_r \langle F_r| \;\rightarrow\; M_s \\ T^{NP} = V^{NP} + \underbrace{V^{NP}G_0T^{NP}}_{} \;\rightarrow\; M_u + M_c \end{cases}$$

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$$M_c \begin{cases} M^T_{c++} = M^T_{c--} = \sum_L a_{LT} \left(\dfrac{p'}{\Lambda_S}\right)^L \exp\left[-\alpha^{LT}\dfrac{p'^2}{\Lambda_S^2}\right] P_L(\theta) \\ M^T_{c+-} = -M^T_{c-+} = \sum_L b_{LT} \left(\dfrac{p'}{\Lambda_S}\right)^L \exp\left[-\alpha^{LT}\dfrac{p'^2}{\Lambda_S^2}\right] P^1_L(\theta) \end{cases}$$

$a_{LT}$, $b_{LT}$, $\alpha^{LT}$ = fit parameters

$\Lambda_S \sim 1$ GeV
(scale parameter)

# $\bar{K}N \to K\Xi$ : *hyperon resonances considered*

$\Lambda(1116)$ $\Sigma(1193)$
$\Lambda(1405)$ $\Sigma(1385)$
$\Lambda(1520)$

the model parameters can be fixed from the relevant decay rates(PDG) and/or quark models and SU(3) symmetry considerations.

All 21 above-threshold S=-1 hyperon resonances listed in PDG:

| Λ states | | | | Σ states | | | |
|---|---|---|---|---|---|---|---|
| State | $m_R$ (MeV) | $\Gamma_R$ (MeV) | Rating | State | $m_R$ (MeV) | $\Gamma_R$ (MeV) | Rating |
| $\Lambda(1810)\ 1/2^+$ | 1810 | 150 | *** | $\Sigma(1840)\ 3/2^+$ | 1840 | 100 | * |
| $\Lambda(1820)\ 5/2^+$ | 1820 | 80 | **** | $\Sigma(1880)\ 1/2^+$ | 1880 | 194 | ** |
| $\Lambda(1830)\ 5/2^-$ | 1830 | 95 | **** | $\Sigma(1900)\ 1/2^-$ | 1900 | 191 | * |
| $\Lambda(1890)\ 3/2^+$ | 1890 | 100 | **** | $\Sigma(1915)\ 5/2^+$ | 1915 | 120 | **** |
| $\Lambda(2000)\ ?^?$ | 2000 | 167 | * | $\Sigma(1940)\ 3/2^+$ | 1941 | 400 | * |
| $\Lambda(2020)\ 7/2^+$ | 2020 | 195 | * | $\Sigma(1940)\ 3/2^-$ | 1940 | 220 | *** |
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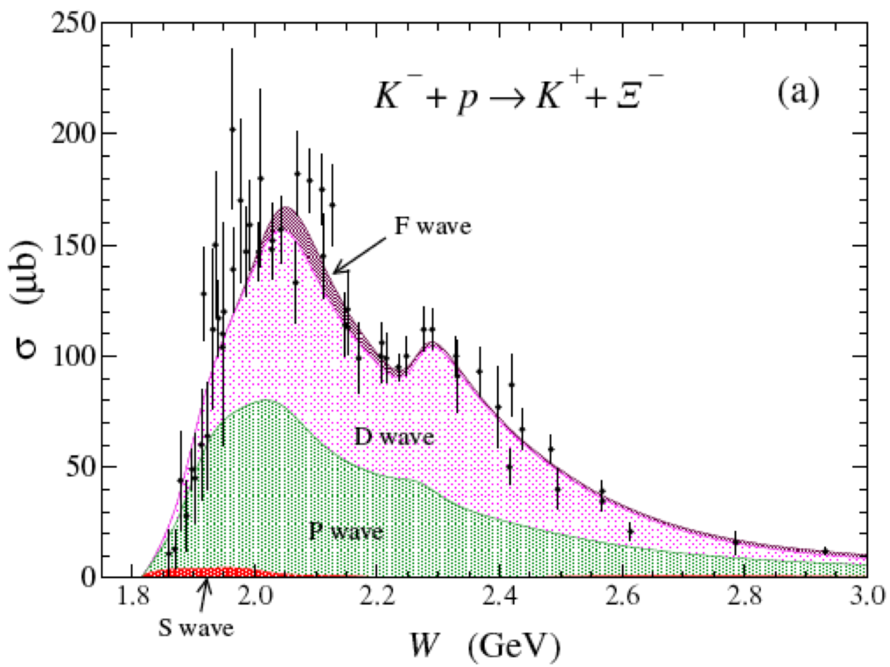
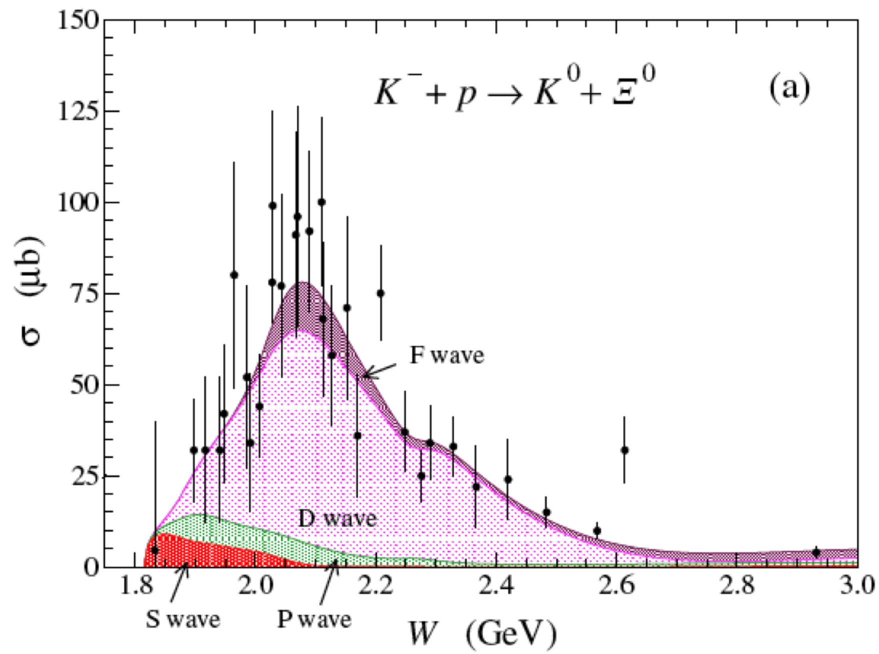
| | | | | $\Sigma(2080)\ 3/2^+$ | 2080 | 177 | ** |
| | | | | $\Sigma(2100)\ 7/2^-$ | 2100 | 103 | * |
| | | | | $\Sigma(2250)\ ?^?$ | 2265 | 100 | *** |

except $\Lambda(2350)9/2^+$ (results saturates after L > 3)

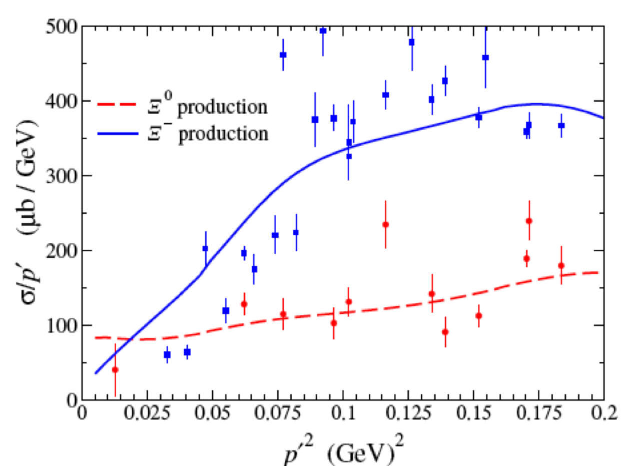
# $\bar{K}N \to K\Xi$ : *phenom. model for data prunning*




For hard processes ($p'^{2L}$):

$$\frac{\sigma}{p'} = c_0 + c_1 p'^2 + c_2 p'^4 + \dots ,$$



# $KN \to K\Xi$ : *LASSO + BIC*

LASSO : $\chi_T^2 = \chi^2 + \lambda^2 \sum_R |f_R|$ ; $\lambda$ = penalty parameter

$f_R$ = penalty function (res. parameter)

as a function of $\lambda$.

$$f_R(J\pm) = M_s(\text{on}-\text{shell res.}) \propto \begin{cases} g_{RJ^\pm} \sqrt{\dfrac{(\varepsilon_N \mp m_R)(\varepsilon_\Xi \mp m_R)}{(\varepsilon_N + m_R)(\varepsilon_\Xi + m_R)}} \dfrac{N_{RJ}}{\Gamma_R}, & \text{if } J = \frac{1}{2}, \frac{5}{2} \\ g_{RJ^\pm} \sqrt{\dfrac{(\varepsilon_N \pm m_R)(\varepsilon_\Xi \pm m_R)}{(\varepsilon_N + m_R)(\varepsilon_\Xi + m_R)}} \dfrac{N_{RJ}}{\Gamma_R}, & \text{if } J = \frac{3}{2}, \frac{7}{2} \end{cases}$$

$N_{RJ}$ = normalization constant

BIC : select the optimal value of $\lambda$ by minimizing :

$$\text{BIC} = k \, ln(N) - 2 \, ln(L)$$

$k$ = # of fit parameters

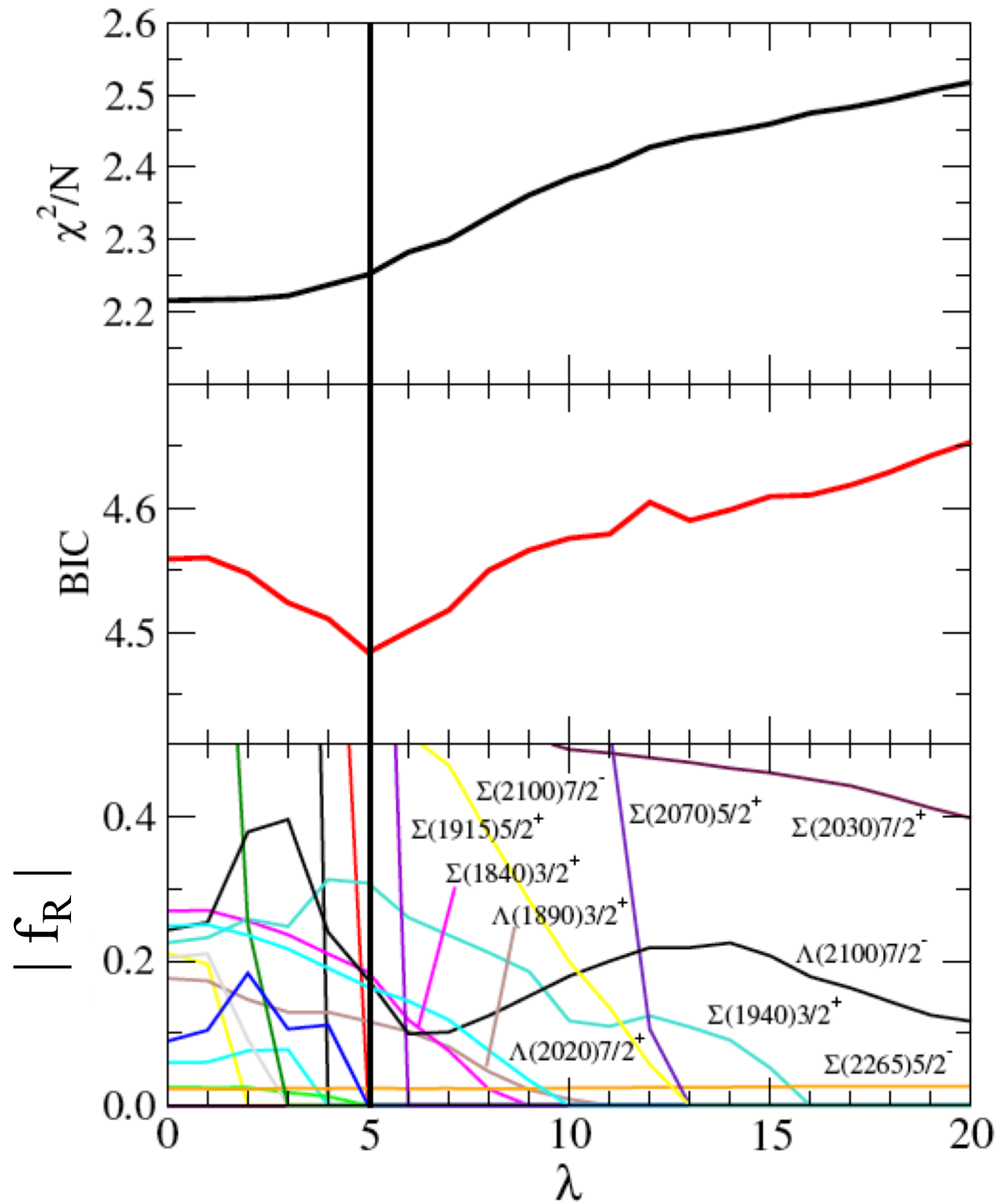
$N$ = # of data points

$L$ = likelihood

$= -\chi^2/2 + \ldots$ (for normal distribution)

$(\text{BIC})_{\text{min}} \to$ lowest test error

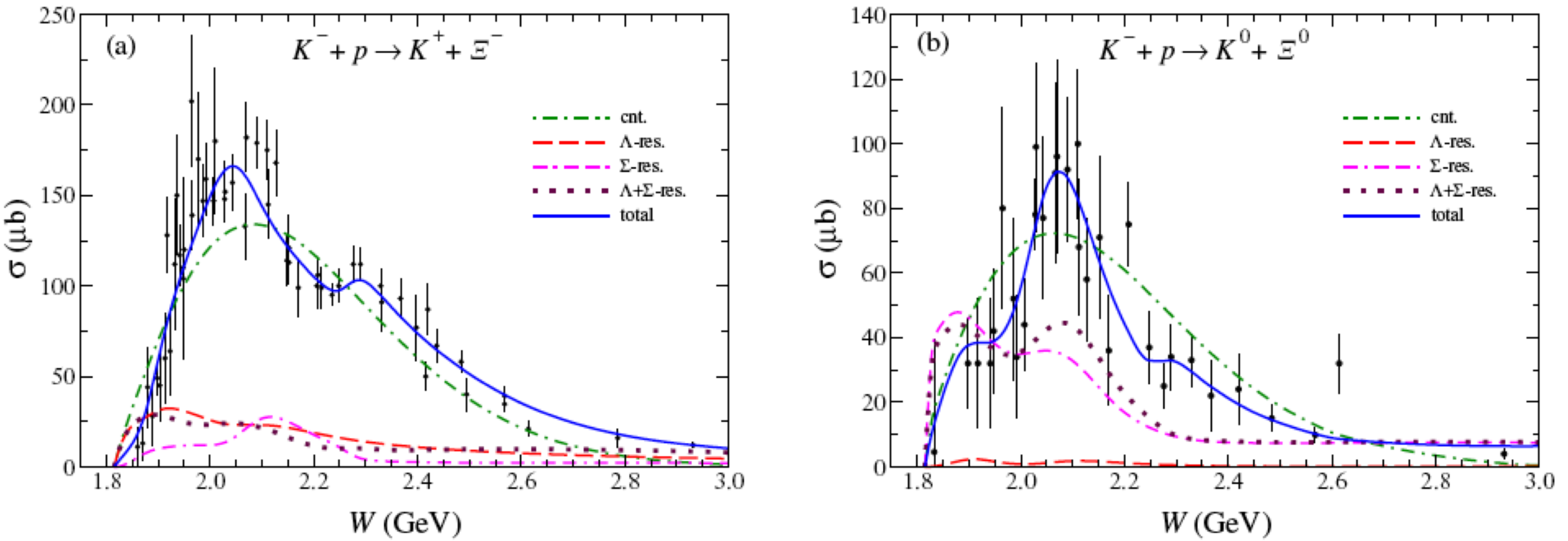
# $\bar{K}N \to K\Xi$ : *LASSO + BIC results*

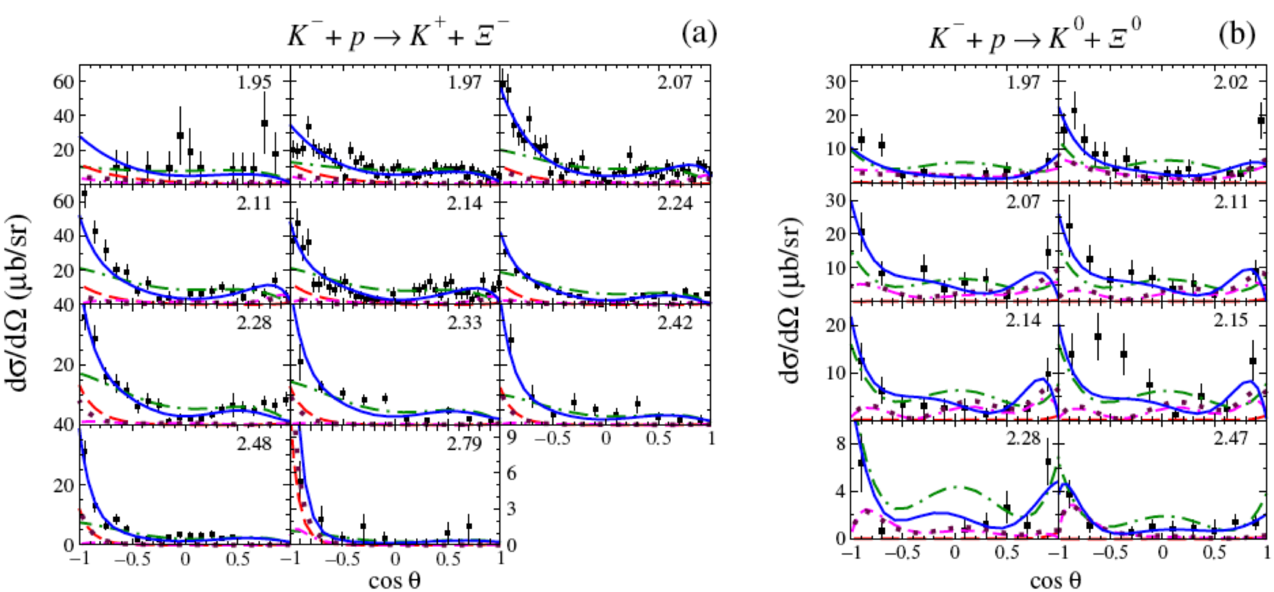


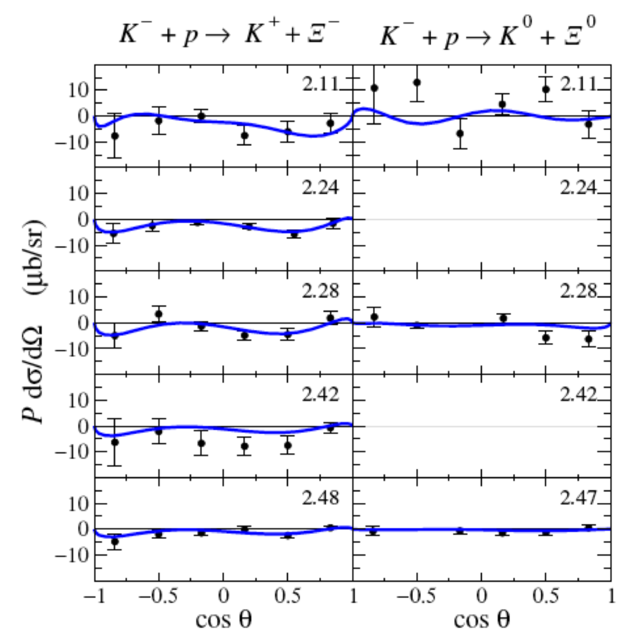

<u>10 res. out of 21:</u>

$\Sigma(2030)7/2^+$
$\Sigma(1940)3/2^+$
$\Sigma(2100)7/2^-$
$\Lambda(2020)7/2^+$
$\Sigma(1840)3/2^+$
$\Lambda(1890)3/2^+$
$\Sigma(2265)5/2^-$
$\Sigma(2070)5/2^+$
$\Sigma(1915)5/2^+$
$\Lambda(2100)7/2^-$

# $\bar{K}N \to K\Xi$ : *LASSO + BIC results*



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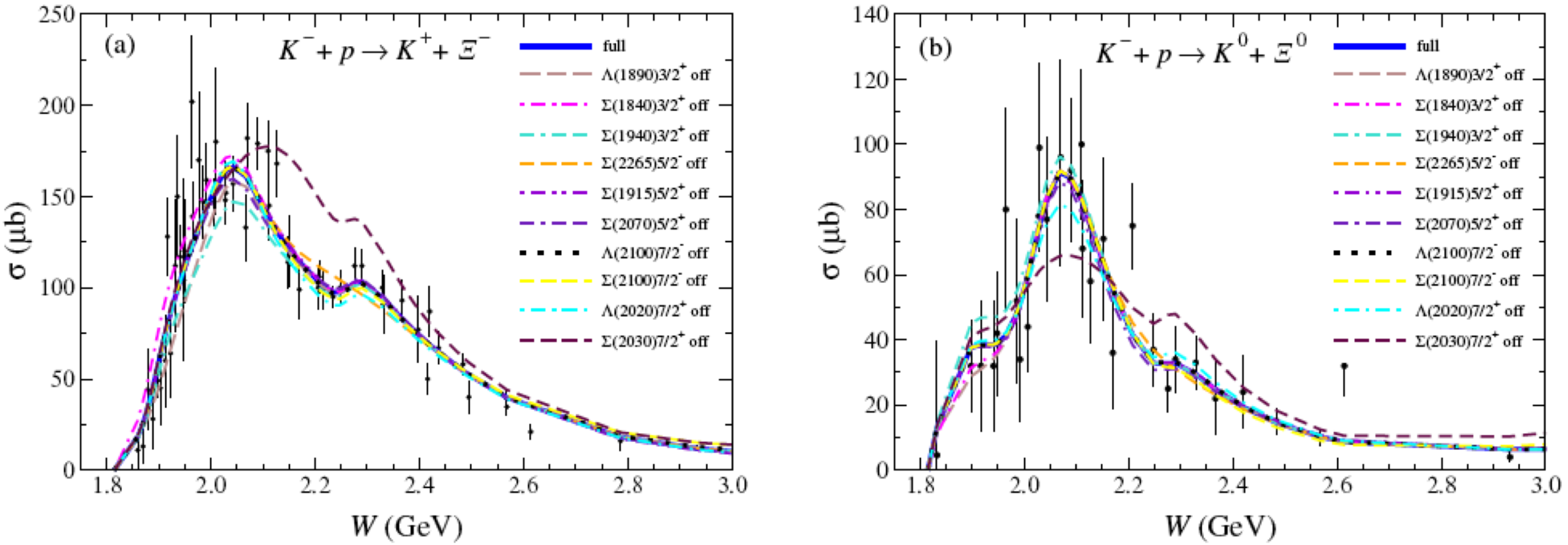
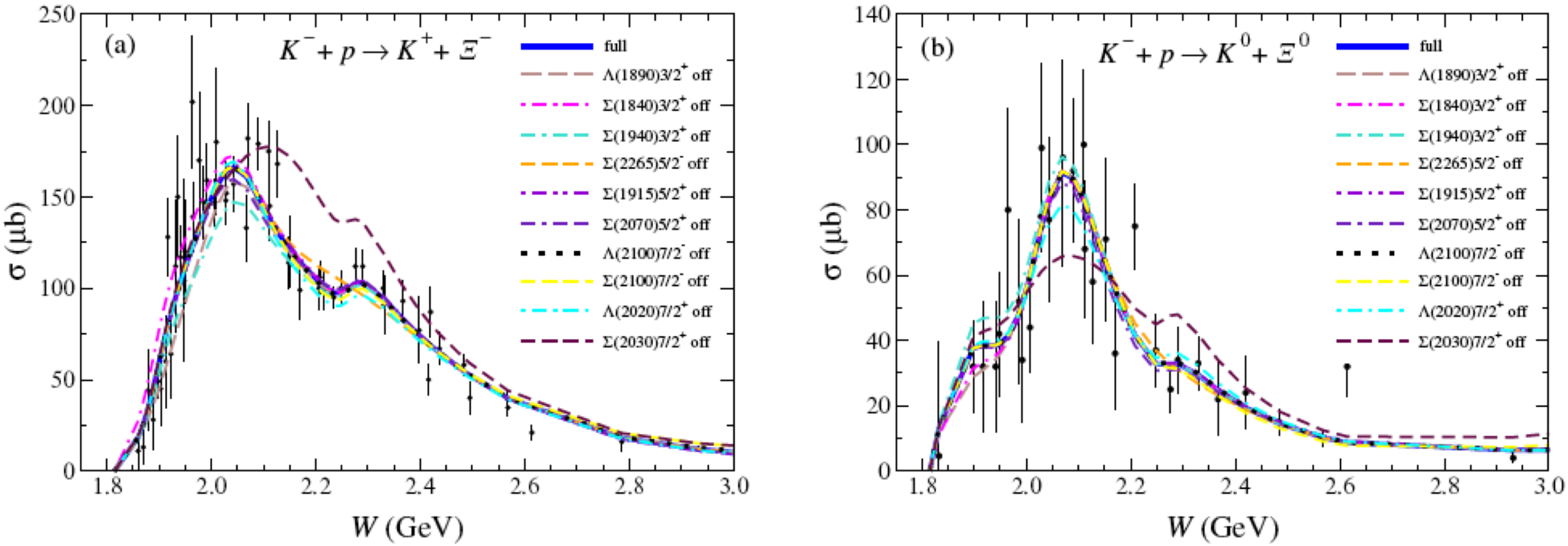



| resonance switched off | rating | $\chi^2/N$ | $\delta\chi^2(\%)$ |
|---|---|---|---|
| none (full result) | - | 2.25 | - |
| $\Sigma(2030)7/2^+$ | **** | 5.59 | 59.76 |
| $\Sigma(1940)3/2^+$ | * | 2.49 | 9.60 |
| $\Sigma(2100)7/2^-$ | * | 2.46 | 8.36 |
| $\Lambda(2020)7/2^+$ | * | 2.41 | 6.63 |
| $\Sigma(1840)3/2^+$ | * | 2.41 | 6.52 |
| $\Lambda(1890)3/2^+$ | **** | 2.40 | 6.18 |
| $\Sigma(2265)5/2^-$ | *** | 2.35 | 4.37 |
| $\Sigma(2070)5/2^+$ | * | 2.33 | 3.36 |
| $\Sigma(1915)5/2^+$ | **** | 2.29 | 1.69 |
| $\Lambda(2100)7/2^-$ | **** | 2.26 | 0.48 |

# $\bar{K}N \to K\Xi$ : *LASSO + BIC results*

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| $\Lambda(2100)7/2^-$ | **** | 2.26 | 0.48 |

Jackson et al., PRC91 ’2015 :

(considering only 3- & 4-star resonances)

$\Sigma(2030)7/2^+$
$\Sigma(2265)5/2^-$
$\Lambda(1890)3/2^+$

(consistent with LASSO+BIC)

# *Summary & Outlook :*

A blindfold determination of hyperon resonances based on the Least Absolute Shrinkage Selection Operator (LASSO) method together with the Bayesian Information Criterion (BIC) has been applied to $\bar{K}N \rightarrow K\Xi$ to provide a model that describes the existing data with a minimum number of hyperon resonances.

The approach works well even for relatively poor quality data. In particular, it has identified 10 resonances (5 of them 1-star rated) to be significant out of all 21 above-threshold resonances listed in PDG.

We expect the LASSO+ITC method to become a standard tool in the determination of baryon spectrum.

*The End*

# *BIC :*

$$BIC = k_{eff} \ln(N) - 2\ln(L)$$

$L$= Likelihood

$N$ = # of data points

$$\bar{\hat{y}}_i = \sum_{j=1}^{m} \hat{y}_{i,j}/m.$$

$$k_{\text{eff}} = \sum_{i=1}^{n} \text{COV}(\hat{y}_i, y_i)$$

$$\text{COV}(\hat{y}_i, y_i) = \sum_{j=1}^{m} \frac{(\hat{y}_{i,j} - \bar{\hat{y}}_i)(y_{i,j} - \bar{y}_i)}{m-1}$$

d.o.f = $N$ - $k_{eff}$