Low-Q^2 constraints on the empirical parametrizations of the N^* transition amplitudes

Gilberto Ramalho

gilberto.ramalho2013@gmail.com

Laboratório de Física Téorica e Computacional,
Universidade Cruzeiro do Sul/Universidade Cidade de São Paulo,
São Paulo, SP, Brazil

GR, PLB **759**, 126 (2016); PRD **93**, 113012 (2016); PRD **94**, 114001 (2016);
EPJA **54**, 75 (2018); EPJA **55**, 32 (2019); G. Eichmann and GR, PRD **98**, 093007 (2018)

PWA11/ATHOS6
Centro Brasileiro de Pesquisas Físicas, Brazil
September 5, 2019

$\gamma^* N \to N^*$ transition form factors/helicity amplitudes

- The microscopic structure of the $\gamma^* N \to N^*$ current $\Rightarrow$ correlations between helicity amplitudes/form factors at **pseudothreshold (PT)**
  PT: N and N^* are **both** at rest: $q = (\omega, \mathbf{0})$, $\omega \to M_R - M_N$
  photon 3-momentum: $|\mathbf{q}| = 0$ and $Q^2 = -(M_R - M_N)^2 < 0$
- Correlations apear when we express helicity amplitudes/form factors in terms of **elementary form factors** $\quad P = \frac{1}{2}(P_R + P_N)$

$$\Gamma^\mu = F_1 \left(\gamma^\mu - \frac{\not q q^\mu}{q^2} \right) \mathbf{1}_P + F_2 \frac{i \sigma^{\mu\nu} q_\nu}{M_R + M_N} \mathbf{1}_P$$

$$\Gamma^{\alpha\mu} = G_1 \left(q^\alpha \gamma^\mu - \not q g^{\alpha\mu} \right) \mathbf{1}_P + G_2 \left(q^\alpha P^\mu - P \cdot q g^{\alpha\mu} \right) \mathbf{1}_P + G_3 \left(q^\alpha q^\mu - q^2 g^{\alpha\mu} \right) \mathbf{1}_P$$

$\mathbb{1}_P$: parity operators ($\mathbb{1}$ or γ_5)

- The correlations **cannot** be ignored at low-Q^2 in the empirical param.
  Most popular effect: Siegert's theorem: $S_{1/2} \propto E \frac{|\mathbf{q}|}{\omega}$
  Scalar amplitude $\propto$ (electric amplitude) $|\mathbf{q}|$
  $\Delta(1232)$: AJ Buchmann et al, PRC 58, 2478 (1998); Drechsel et al, EJPA 34, 69 (2007)

Helicity amplitudes ($\frac{1}{2} \to \frac{1}{2}, \frac{3}{2}$) – N^* rest frame

$N^*(S_z')$, $N(S_z)$, photon ($\epsilon_{+,0}$) $\quad K = \frac{M_R^2 - M_N^2}{2M_R}, \quad \alpha = \frac{e^2}{4\pi}$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{K}} \left\langle N^*, S_z' = +\tfrac{3}{2} \right| \epsilon_+ \cdot J \left| N, S_z = +\tfrac{1}{2} \right\rangle$$

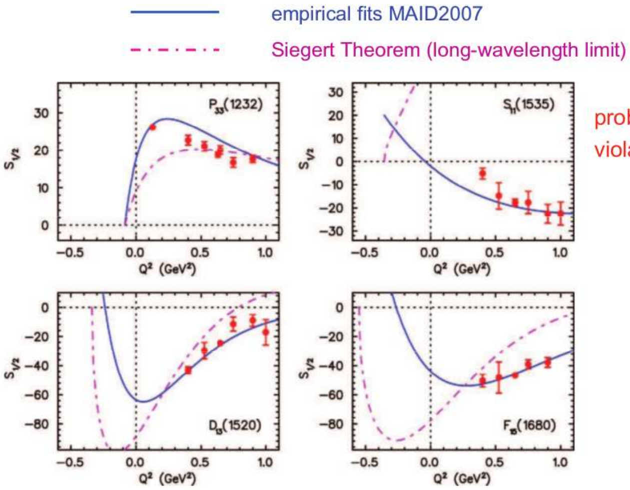
$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \left\langle N^*, S_z' = +\tfrac{1}{2} \right| \epsilon_+ \cdot J \left| N, S_z = -\tfrac{1}{2} \right\rangle$$

$$S_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \left\langle N^*, S_z' = +\tfrac{1}{2} \right| \epsilon_0 \cdot J \left| N, S_z = +\tfrac{1}{2} \right\rangle \frac{|\mathbf{q}|}{Q},$$

Constraints: consequence of:

- Gauge-invariant structure of the current J^μ
- N^* rest frame kinematics: $|N\rangle$, $|N^*\rangle$

transition form factors at low Q^2



problem with S11

violation of LWL ?

Lothar Tiator: Nucleon Resonances: From Photoproduction to High Photon Virtualities, Trento, Italy, October 2015 – Shape of amplitudes near PT L. Tiator, FBS 57, 1087 (2016)

Structure of the transition amplitudes

Notation: $Q_\pm^2 = (M_R \pm M_N)^2 + Q^2, \quad |\mathbf{q}| = \frac{\sqrt{Q_+^2 Q_-^2}}{2M_R}$ (N^* rest frame)

$N(1535)\frac{1}{2}^-$	$N(1520)\frac{3}{2}^-$	$\Delta(1232)\frac{3}{2}^+$
$A_{1/2} = 2b\tilde{F}_1$	$E_{2-} = \mathcal{O}(1)$	$E_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$
$S_{1/2} = \sqrt{2}b\frac{\lvert\mathbf{q}\rvert}{M_R - M_N}\tilde{F}_1$	$S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert)$	$S_{1/2} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$
	$M_{2-} = \mathcal{O}(\lvert\mathbf{q}\rvert^2)$	$M_{1+} = \mathcal{O}(\lvert\mathbf{q}\rvert)$
$\tilde{F}_1 = \mathcal{O}(1)$	$\frac{1}{2}E_{2-} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert}$	$\frac{E_{1+}}{\lvert\mathbf{q}\rvert} = \lambda_R \frac{S_{1/2}}{\lvert\mathbf{q}\rvert^2}$
	$A_{1/2} = \frac{1}{\sqrt{3}}A_{3/2}$	
	$\frac{1}{2}G_E = \frac{M_R - M_N}{2M_R}G_C$	$G_E = \frac{M_R - M_N}{2M_R}G_C$

GR, PLB 759, 126 (2016); PRD 93, 113012 (2016)

JD Bjorken and JD Walecka, Ann. Phys. 38, 35 (1966); D Drechsel, SS Kamalov and L Tiator, EPJA 34, 69 (2007)

$\gamma^* N \to N^*$ transition – Plan of the talk

How to ensure the correct form for the helicity amplitudes?

- PART 1
  Use relations between amplitudes and form factors
  (take into account implicitly the pseudothreshold conditions)
- PART 2
  Re-define empirical parametrization of the amplitudes at low-Q^2
  ($Q^2 < 0.5$ GeV^2) in order to **satisfay** the pseudothreshold conditions

PART 1

Use relation Amplitudes $\Longleftrightarrow$ Form Factors

- Parametrize the amplitudes according with the pseudothreshold constraints (correct the $|\mathbf{q}|$-dependence)
- Parametrize directly the *elementary* Form Factors

- From the analysis of the transition current – singularity-free FF
  Devenish et al, PRD 14, 3063 (1976)

$$J^\mu = F_1 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M_R + M_N}$$

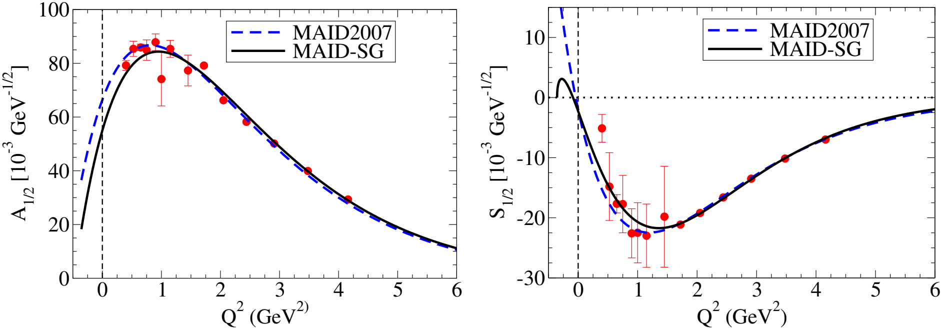
- Amplitudes at N^* rest frame $b = e\sqrt{\frac{Q_+^2}{8M(M_R - M_N)}}$
  $\tilde{F}_1 = F_1 + \eta F_2$, $\eta = \frac{M_R - M_N}{M_R + M_N}$

$$A_{1/2}(Q^2) = 2b\tilde{F}_1(Q^2),$$

$$S_{1/2}(Q^2) = -\sqrt{2} b (M_R - M_N) \frac{|\mathbf{q}|}{Q^2} \times \left[\tilde{F}_1(Q^2) - \frac{4M_R^2 |\mathbf{q}|^2}{(M_R^2 - M_N^2) Q_+^2} F_2(Q^2) \right],$$

- **PT**: $A_{1/2} = 2b\tilde{F}_1$, $S_{1/2} = -\sqrt{2}(M_R - M_N) \frac{|\mathbf{q}|}{Q^2} \tilde{F}_1$, $\Rightarrow$ $A_{1/2} = \lambda \frac{S_{1/2}}{|\mathbf{q}|}$

$\gamma^* N \to N(1535)\frac{1}{2}^-$ – MAID-SG



Improved MAID-type parametrizations ($S_{1/2} = \mathcal{O}(|\mathbf{q}|)$); ST $\to$ s_0'

MAID data

$$A_{1/2} = a_0 \left(1 + a_1 Q^2\right) e^{-a_4 Q^2}, \quad S_{1/2} = \frac{2M_R |\mathbf{q}|}{Q_+^2} s_0' \left(1 + s_1 Q^2 + s_2 Q^4\right) e^{-s_4 Q^2}$$

ST satisfied using a turnig point GR, PLB 759, 126 (2016)

$\gamma^* N \to \Delta(1232)$ $\quad J^P = \frac{3}{2}^+$ $\quad \lambda = \sqrt{2}(M_R - M_N)$

Devenish et al, PRD 14, 3063 (1976); Jones and Scadron, Ann. Phy. 81, 1 (1973)

G_i $(i = 1, 2, 3)$ free of singularities at PT

$$G_M = Z_R \left[(M_R - M_N) G_5 + 4 M_N G_1 + \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{2M_R} - G_3 \right) \right],$$

$$G_E = Z_R \left[(M_R - M_N) G_5 - \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{2M_R} + G_3 \right) \right],$$

$$G_C = Z_R \left[2 M_R G_5 + \frac{4 M_R^2 |\mathbf{q}|^2}{Q_+^2} \left(\frac{1}{2} G_2 - G_3 \right) \right],$$

$$G_5 = G_1 + \frac{1}{2}(M_R + M_N) G_2 + (M_R - M_N) G_3, \qquad Z_R = \frac{2M_N}{3(M_R + M_N)}$$

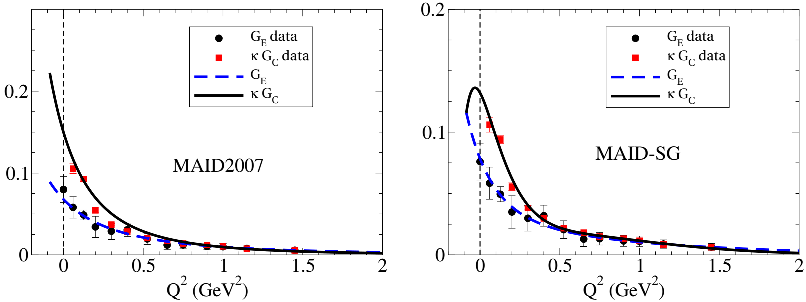
- Limit PT: $G_M = \mathcal{O}(1)$

  $G_E = \frac{M_R - M}{2M_R} G_C$ [Jones and Scadron (1973)]

- Amplitudes: $G_E = F_+ E_{1+}$, $G_C = \frac{2M_R}{|\mathbf{q}|}\sqrt{2} F_+ S_{1/2}$, $F_+ \propto 1/|\mathbf{q}|$

  $\frac{E_{1+}}{|\mathbf{q}|} = \lambda \frac{S_{1/2}}{|\mathbf{q}|^2}$ GR, PRD 93, 113012 (2016)

$\gamma^* N \to \Delta(1232) \quad J^P = \frac{3}{2}^+$ GR, PRD 93, 113012 (2016)



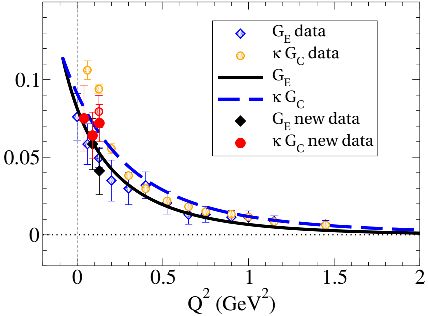
$$G_E \neq \overbrace{\frac{M_R - M_N}{2M_R}}^{\kappa} G_C \qquad\qquad G_E = \frac{M_R - M_N}{2M_R} G_C$$

$$E_{1+} = \lambda \frac{S_{1/2}}{|\mathbf{q}|} \to 0 \qquad\qquad \frac{E_{1+}}{|\mathbf{q}|} = \lambda \frac{S_{1/2}}{|\mathbf{q}|^2}$$

MAID-SG: Phenomenological parametrization – based on old data

$\gamma^* N \to \Delta(1232)$ $J^P = \frac{3}{2}^+$ – recent developments

- Low-Q^2 data corrected in recent analysis:
  A Blomberg, PLB 760, 267 (2016) – JLab/Hall A
- Data explained by model with combination of valence quark $\oplus$ pion cloud contributions (Large N_c) GR, EPJA 54, 75 (2018)



More details:

GR, PRD 94, 114001 (2016); EPJA 55, 32 (2019); GR and MT Peña, PRD 80, 013008 (2009)

$\gamma^* N \to N(1520) \quad J^P = \frac{3}{2}^- \quad \lambda = \sqrt{2}(M_R - M_N)$

Devenish et al, PRD 14, 3063 (1976) – G_i $(i = 1, 2, 3)$ free of singularities at PT

$$G_M = -Z_R \frac{4M_R|\mathbf{q}|^2}{Q_+^2} G_1,$$

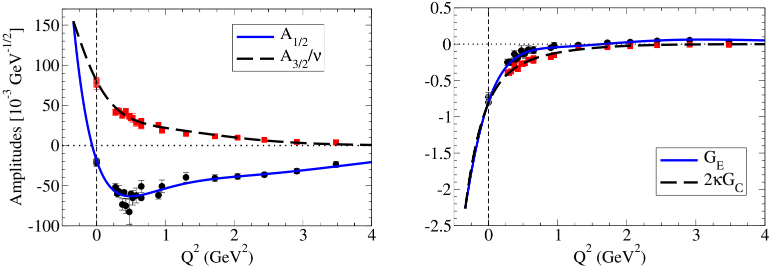
$$G_E = -Z_R \left[4(M_R - M_N) G_5 - \frac{4M_R^2|\mathbf{q}|^2}{Q_+^2} \left(\frac{G_1}{M_R} + 4G_3 \right) \right],$$

$$G_C = -Z_R \left[4M_R G_5 + \frac{4M_R^2|\mathbf{q}|^2}{Q_+^2} (G_2 - 2G_3) \right], \qquad \neq \text{ sign conventions}$$

$$G_5 = G_1 + \frac{1}{2}(M_R - M_N) G_2 + (M_R + M_N) G_3, \qquad Z_R = \frac{1}{\sqrt{6}} \frac{M}{M_R - M_M}$$

- Limit PT: $G_M = \mathcal{O}(|\mathbf{q}|^2)$
  $G_E = \frac{M_R - M_N}{M_R} G_C$
- Amplitudes: $G_E = F_- E_{2-}$, $G_C = \frac{2M_R}{|\mathbf{q}|}\sqrt{2} F_- S_{1/2}$, $F_- \propto 1/\sqrt{Q_+^2}$
  $\frac{1}{2} E_{2-} = \lambda \frac{S_{1/2}}{|\mathbf{q}|}$ $\qquad M_{2-} \propto \left(A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2} \right) \propto |\mathbf{q}|^2$

$\gamma^* N \to N(1520) \quad J^P = \frac{3}{2}^- \quad$ Jlab-SG $\kappa = \frac{M_R - M_N}{2M_R}$



Jlab-SG parametrization (fit to Jlab data): $D = K/\sqrt{Q_+^2}$; ST $\to$ b_0, c_0

$$A_{1/2} = D a_0 (1 + a_1 Q^2 + a_2 Q^4 + a_3 Q^6) e^{-a_4 Q^2},$$

$$A_{3/2} = D b_0 (1 + b_1 Q^2 + b_2 Q^4 + b_3 Q^6) e^{-b_4 Q^2},$$

$$S_{1/2} = \frac{|\mathbf{q}|}{K} c_0 (1 + c_1 Q^2 + c_2 Q^4 + c_3 Q^6) e^{-c_4 Q^2},$$

Data: https://userweb.jlab.org/~mokeev/resonance_electrocouplings/

Direct parametrization of form factores (CS)

Nucleon Compton scattering

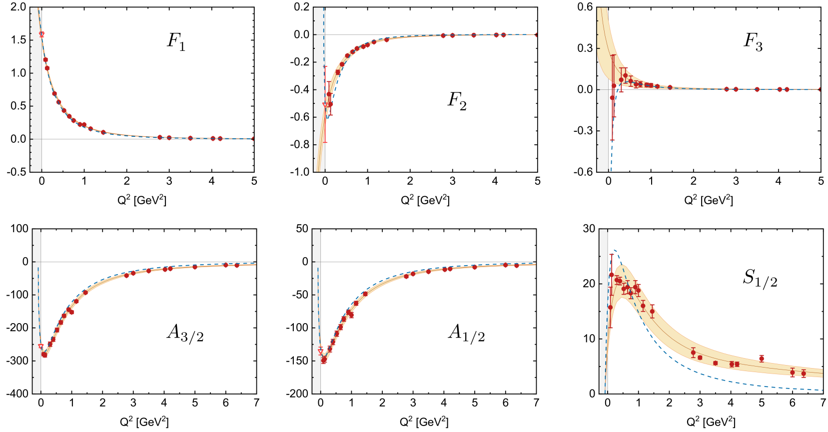
G Eichmann and GR, PRD 98, 093007 (2018)

- Study of the impact of resonances in the Nucleon CS
- Use appropriated form factors:

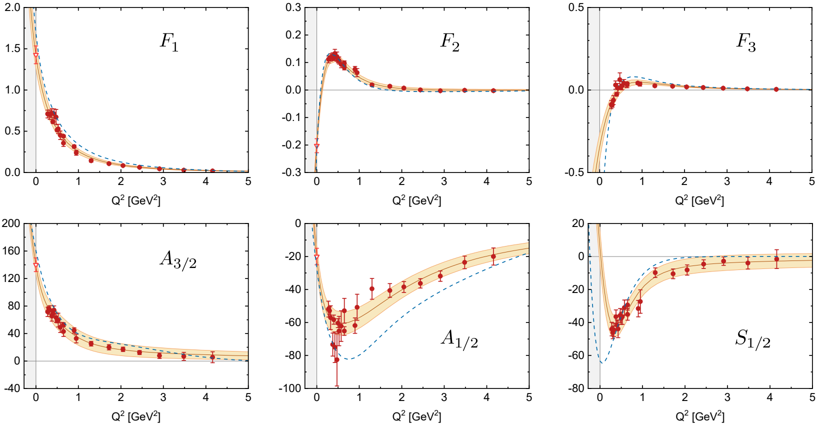
$$
\begin{aligned}
\Gamma^{\mu} &= \frac{F_1}{M_N^2}\left(\not{q}\, q^{\mu} - q^2\gamma^{\mu}\right)\mathbb{1}_P + \frac{F_2}{M_N} i\sigma^{\mu\nu} q_{\nu}\mathbb{1}_P \\
\Gamma^{\alpha\mu} &= \frac{F_1}{M_N^2}\gamma_5 \epsilon^{\alpha\mu\sigma\rho} p'_{\sigma} q_{\rho}\mathbb{1}_P + \frac{F_2}{M_N^2}(p'\cdot q\; g^{\alpha\mu} - q^{\alpha} p'^{\mu})\mathbb{1}_P \\
&\quad + \frac{F_3}{M_N^3}(\not{p}' g^{\alpha\sigma} - \gamma^{\alpha} p'^{\sigma})(q^2 g_{\sigma}{}^{\mu} - q_{\sigma} q^{\mu})\mathbb{1}_P
\end{aligned}
$$

- No poles; no zeros at $Q^2 = 0$; monotonous form (in general)
- **Pseudothreshold conditions automatically ensured**

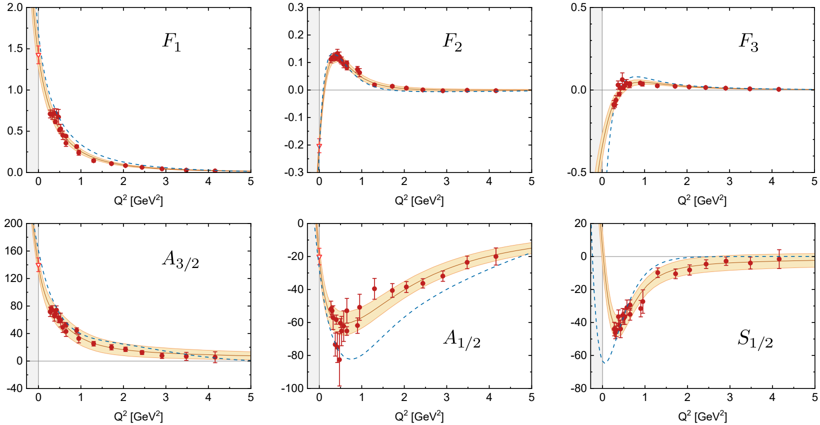
Direct parametrization of form factores – $\Delta(1232)$



Direct parametrization of form factores – $N(1520)$



Direct parametrization of form factores – $N(1520)$



Pseudothreshold conditions automatically ensured

PART 2

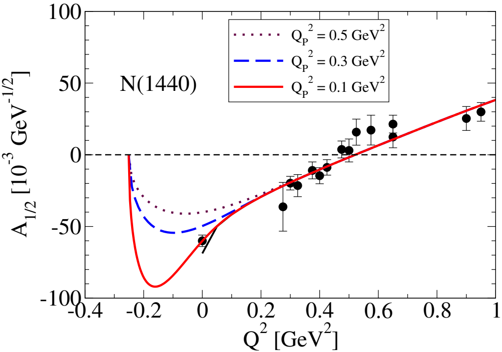
Re-define empirical parametrizations of the amplitudes A at low-Q^2

Basic idea:

- Select a set of empirical parametrizations of the amplitudes
  Example: JLab parametrization
  https://userweb.jlab.org/~isupov/couplings/
  (valid for several N^*, analytic functions)
- Extrapolate the amplitude below a threshold Q^2_P, to the PT, using
  - pseudothreshold conditions
  - smooth transition to the $-(M_R - M_N)^2 \leq Q^2 \leq Q^2_P$

arXiv:1909.00013 [hep-ph]

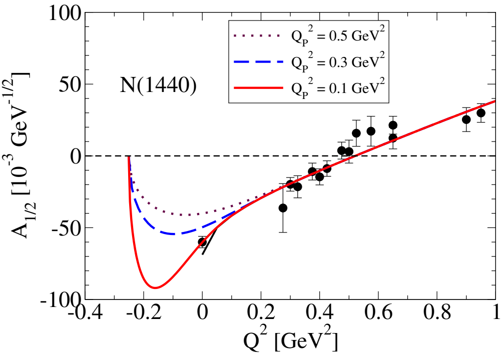
Analytic continuation to the timelike region $Q^2 < Q_P^2$



- Use different values of Q_P^2: $Q_P^2 = 0.1$, 0.3 and 0.5 GeV^2 to study deppendence on the pseudothreshold conditions
- Study sensitivity of analytic continuations to the low-Q^2 data

Data: https://userweb.jlab.org/~mokeev/resonance_electrocouplings/

Analytic continuation to the timelike region $Q^2 < Q_P^2$



- Use different values of Q_P^2: $Q_P^2 = 0.1$, 0.3 and 0.5 GeV^2 to study deppendence on the pseudothreshold conditions
- Study sensitivity of analytic continuations to the low-Q^2 data

Data: CLAS, PDG, MAMI, MIT-Bates, JLab-Hall A

Analytic continuation to the timelike region $Q^2 < Q_P^2$

$Q^2 \geq Q_P^2$ $\qquad A^{(k)} \equiv \frac{d^k A}{dQ^{2k}}(Q_P^2)$ $\qquad$ (input)

$$A(Q^2) = A^{(0)} + A^{(1)}(Q^2 - Q_P^2) + \frac{A^{(2)}}{2!}(Q^2 - Q_P^2)^2 + \frac{A^{(3)}}{3!}(Q^2 - Q_P^2)^3 + ...,$$

$-(M_R - M_N)^2 \leq Q^2 \leq Q_P^2$

$$A = |\mathbf{q}|^n(\alpha_0 + \alpha_1|\mathbf{q}|^2 + \alpha_2|\mathbf{q}|^4 + \alpha_3|\mathbf{q}|^6),$$

$n = 0, 1, 2$; α_l determined by $\qquad$ (α_l: output)

- pseudothreshold conditions
- continuity of A, A', A'', ... when $Q^2 = Q_P^2$

arXiv:1909.00013 [hep-ph]

Analytic continuation to the timelike region $Q^2 < Q_P^2$

$\frac{1}{2}^+$	$A_{1/2} \propto |\mathbf{q}|$,	$S_{1/2} \propto |\mathbf{q}|^2$	
$\frac{1}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto |\mathbf{q}|$	$S_{1/2} \propto A_{1/2}|\mathbf{q}|$
$\frac{3}{2}^+$	$A_{1/2} \propto |\mathbf{q}|$,	$S_{1/2} \propto |\mathbf{q}|^2$	$S_{1/2} \propto (A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2})|\mathbf{q}|$
	$A_{3/2} \propto |\mathbf{q}|$,		
$\frac{3}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto |\mathbf{q}|$	$S_{1/2} \propto (A_{1/2} + \sqrt{3}A_{3/2})|\mathbf{q}|$
	$A_{3/2} \propto 1$,		$(A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}) \propto |\mathbf{q}|^2$

Redefine variable: $\tilde{q} = \frac{|\mathbf{q}|}{2M_R}$; all coefficient with dimension GeV$^{-1/2}$

$\frac{1}{2}^+$	$A_{1/2} = (a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6)\tilde{q}$		$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}^2$
$\frac{1}{2}^-$	$A_{1/2} = a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6$		$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}$
$\frac{3}{2}^+$	$A_{1/2} = (a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6)\tilde{q}$	$A_{3/2} = (b_0 + b_1\tilde{q}^2 + b_2\tilde{q}^4 + b_3\tilde{q}^6)\tilde{q}$	$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}^2$
$\frac{3}{2}^-$	$A_{1/2} = a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6$	$A_{3/2} = b_0 + b_1\tilde{q}^2 + b_2\tilde{q}^4 + b_3\tilde{q}^6$	$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}$

$A_{1/2}(0)$, $A_{3/2}(0)$ known; $S_{1/2}(0)$ unknown

Analytic continuation to the timelike region $Q^2 < Q^2_P$

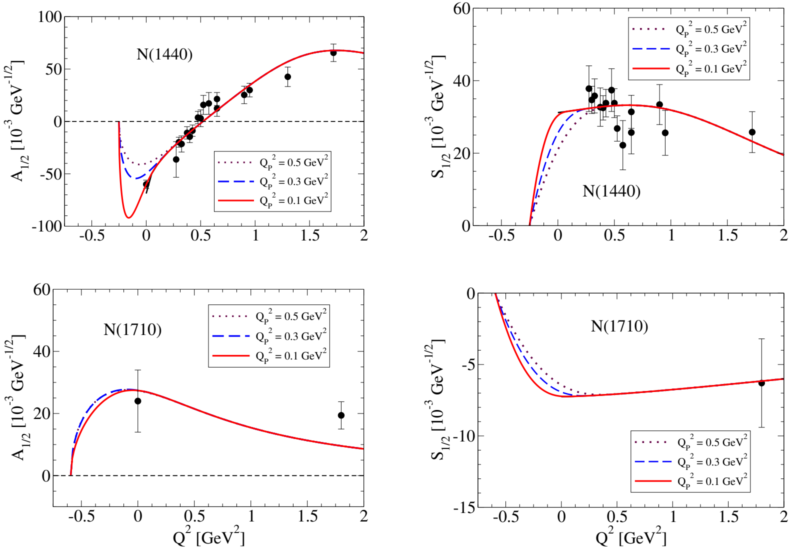
$\frac{1}{2}^+$	$A_{1/2} \propto |\mathbf{q}|$,	$S_{1/2} \propto |\mathbf{q}|^2$	
$\frac{1}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto |\mathbf{q}|$	$S_{1/2} \propto A_{1/2}|\mathbf{q}|$
$\frac{3}{2}^+$	$A_{1/2} \propto |\mathbf{q}|$,	$S_{1/2} \propto |\mathbf{q}|^2$	$S_{1/2} \propto (A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2})|\mathbf{q}|$
	$A_{3/2} \propto |\mathbf{q}|$,		
$\frac{3}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto |\mathbf{q}|$	$S_{1/2} \propto (A_{1/2} + \sqrt{3} A_{3/2})|\mathbf{q}|$
	$A_{3/2} \propto 1$,		$(A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}) \propto |\mathbf{q}|^2$

Redefine variable: $\tilde{q} = \frac{|\mathbf{q}|}{2M_R}$; all coefficient with dimension GeV$^{-1/2}$

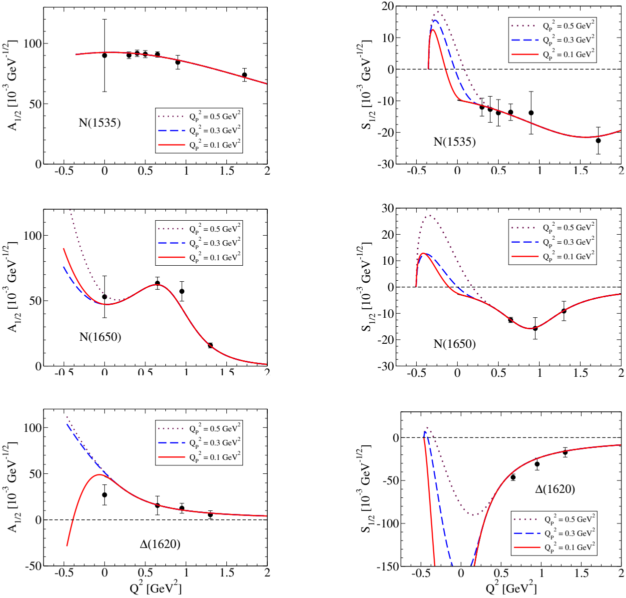
$\frac{1}{2}^+$	$A_{1/2} = (a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6)\tilde{q}$		$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}^2$
$\frac{1}{2}^-$	$A_{1/2} = a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6$		$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}$
$\frac{3}{2}^+$	$A_{1/2} = (a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6)\tilde{q}$	$A_{3/2} = (b_0 + b_1\tilde{q}^2 + b_2\tilde{q}^4 + b_3\tilde{q}^6)\tilde{q}$	$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}^2$
$\frac{3}{2}^-$	$A_{1/2} = a_0 + a_1\tilde{q}^2 + a_2\tilde{q}^4 + a_3\tilde{q}^6$	$A_{3/2} = b_0 + b_1\tilde{q}^2 + b_2\tilde{q}^4 + b_3\tilde{q}^6$	$S_{1/2} = (c_0 + c_1\tilde{q}^2 + c_2\tilde{q}^4 + c_3\tilde{q}^6)\tilde{q}$

$A_{1/2}$, $A_{3/2}$ determined by $A^{(3)}_{1/2,3/2}(0)$; $S_{1/2}$ dependent of $A_{1/2}$, $A_{3/2}$

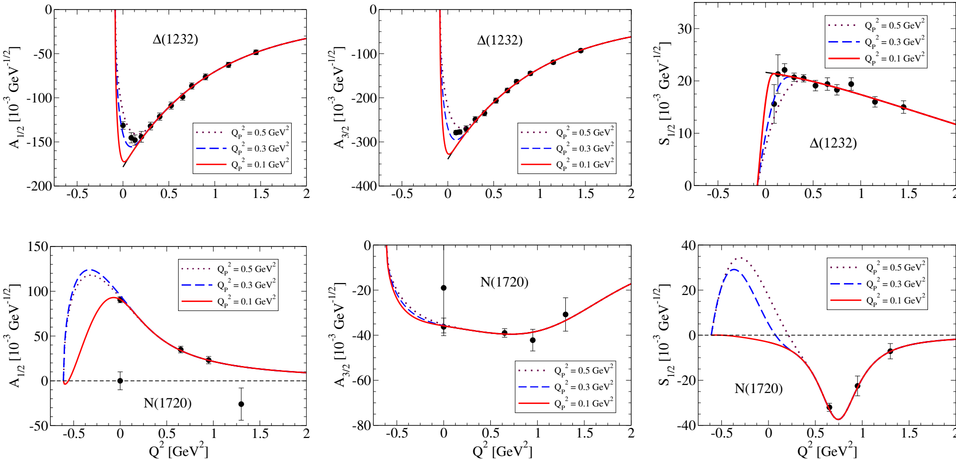
Analytic continuation: $\frac{1}{2}^+$



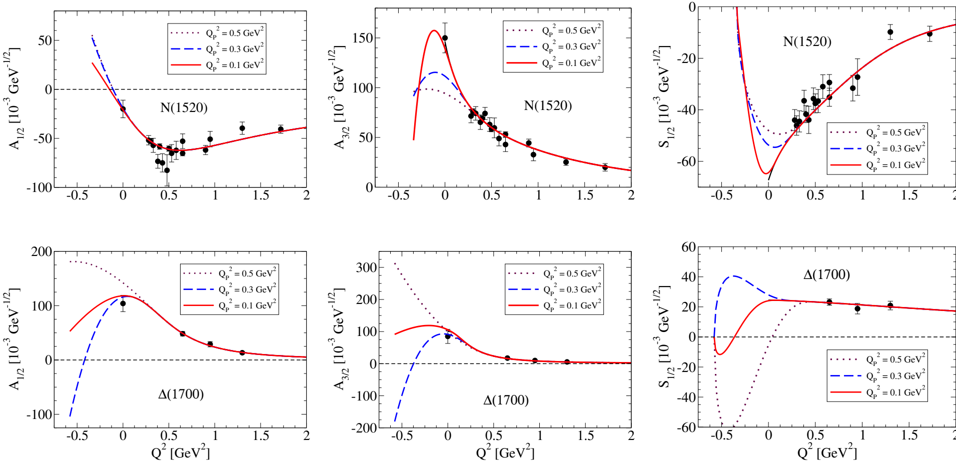
Analytic continuation: $\frac{1}{2}^-$



Analytic continuation: $\frac{3}{2}^+$



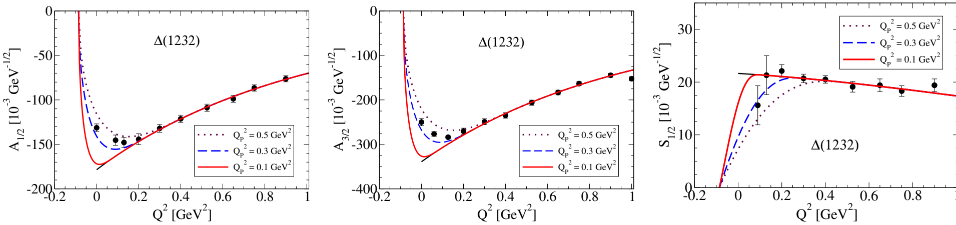
Analytic continuation: $\frac{3}{2}^-$



Analytic continuation: First summary

- Interesting cases:
  $N(1440)$, $N(1520)$, $\Delta(1232)$, $N(1535)$
- Other cases:
  The lack of data (at low Q^2 and in number):
  prevents definitive conclusions

Analytic continuation: $\Delta(1232)\frac{3}{2}^+$

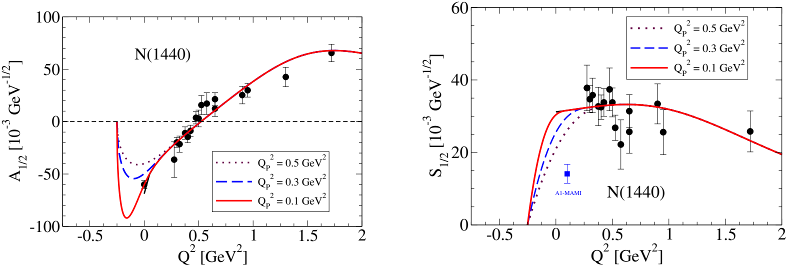


- Evidence of impact of pseudothreshold conditions (turning points)

$$A_{1/2} \simeq \frac{1}{\sqrt{3}} A_{3/2} \simeq -0.0909 \left(3.75 - 10.78 \frac{|\mathbf{q}|^2}{M_N^2} \right) |\mathbf{q}|$$

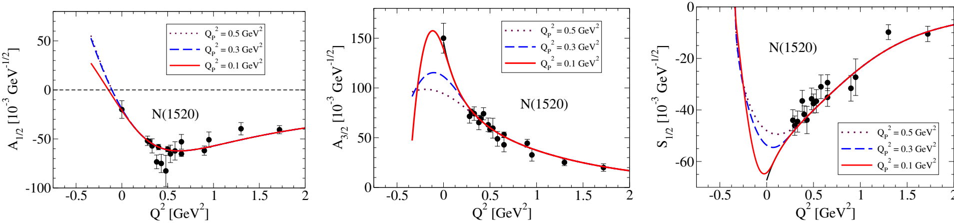
- First evidence of pseudothreshold limit
  (PT $Q^2 = -0.09$ GeV^2 close to $Q^2 = 0$)
- Best parametrization $Q_P^2 = 0.3$ GeV^2

Analytic continuation: $N(1440)\frac{1}{2}^+$



- No special constraints $A_{1/2} \propto |\mathbf{q}|$, $S_{1/2} \propto |\mathbf{q}|^2$
- Parametrizations sensitive to low-Q^2 data

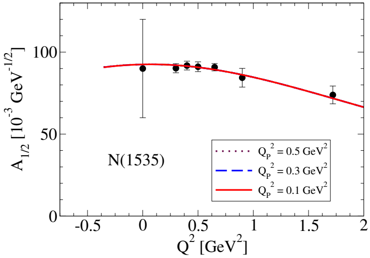
Analytic continuation: $N(1520)\frac{3}{2}^-$

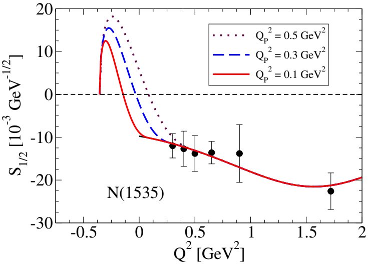


- Some evidence of pseudothreshold constraints
- Parametrizations differ from low-Q^2 data: $Q_P^2 = 0.5$ GeV^2, $Q_P^2 = 0.3$ GeV^2
- Best parametrization $Q_P^2 = 0.1$ GeV^2

Analytic continuation: $N(1535)\frac{1}{2}^-$ (1)

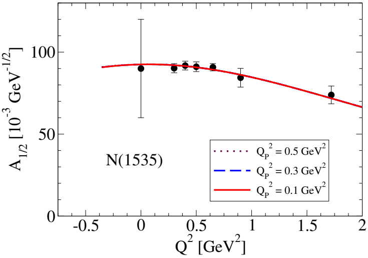
$A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$

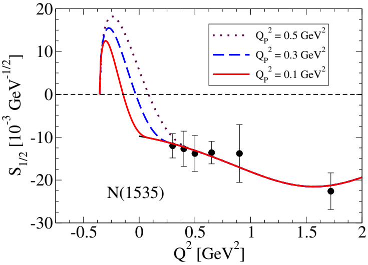




- Large uncertainty in $A_{1/2}(0)$ (PDG); $\neq$ results from $\neq$ groups
- Large uncertainty in the extension of $S_{1/2}$

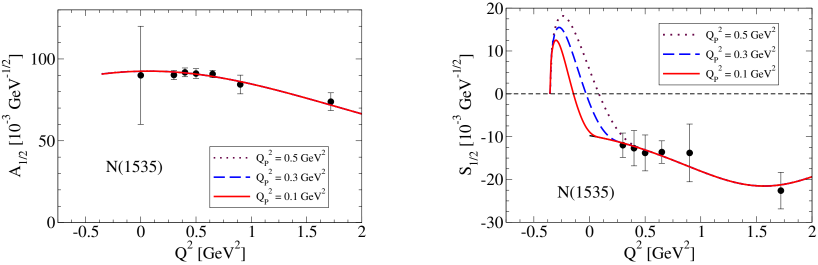
Analytic continuation: $N(1535)\frac{1}{2}^-$ (1) $\quad A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$





- Large uncertainty in $A_{1/2}(0)$ (PDG); $\neq$ results from $\neq$ groups
- Large uncertainty in the extension of $S_{1/2}$
- … we assumed that $A_{1/2}$ is smooth at low-Q^2:
  $\frac{d^3 A_{1/2}}{dQ^6}$ small $\Rightarrow$ $\frac{d^3 S_{1/2}}{dQ^6}$ large

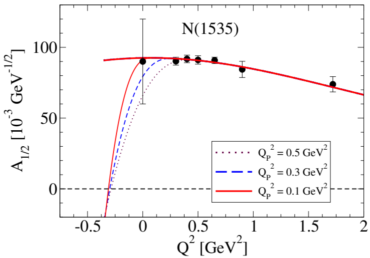
Analytic continuation: $N(1535)\frac{1}{2}^-$ (2) $A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$

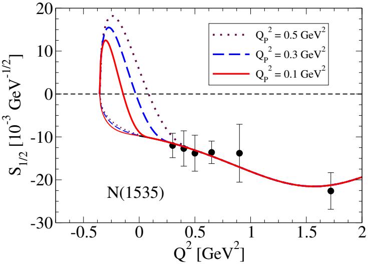


- … what happens is we require that $S_{1/2}$ is smooth at low-Q^2 ?
  $\frac{d^3 S_{1/2}}{dQ^6}$ determined by continuity (small)

Analytic continuation: $N(1535)\frac{1}{2}^-$ (2)

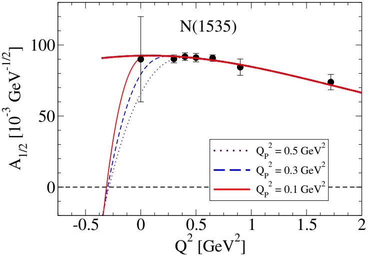
$A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$

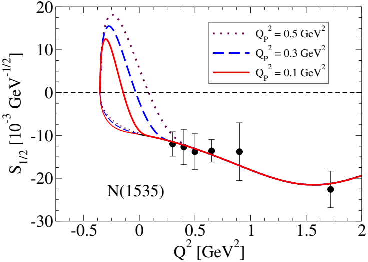




- ... what happens is we require that $S_{1/2}$ is smooth at low-Q^2 ? $\frac{d^3 S_{1/2}}{dQ^6}$ determined by continuity (small)
- New solutions (thin lines); Good results with $Q_P^2 = 0.3$, 0.5 GeV^2

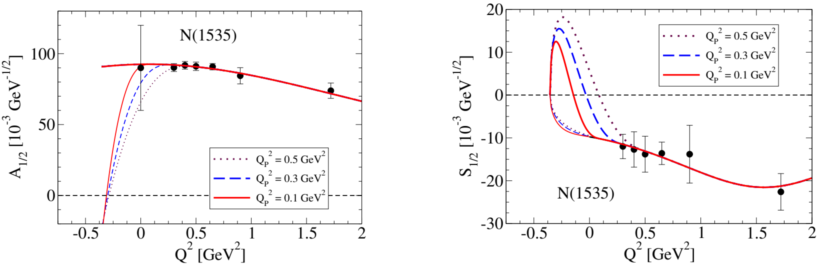
Analytic continuation: $N(1535)\frac{1}{2}^-$ (2) $\quad A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$





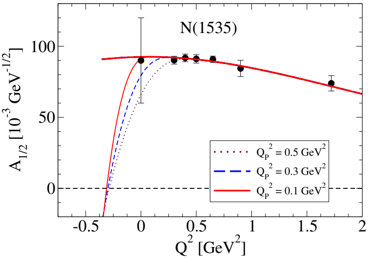
- ... what happens is we require that $S_{1/2}$ is smooth at low-Q^2 ?
  $\frac{d^3 S_{1/2}}{dQ^6}$ determined by continuity (small)
- New solutions (thin lines); Good results with $Q_P^2 = 0.3$, 0.5 GeV^2
- Smoother result for $S_{1/2}$; strong variation for $A_{1/2}$ (large $\frac{d^3 A_{1/2}}{dQ^6}$)
  Consistent with error of $A_{1/2}(0)$

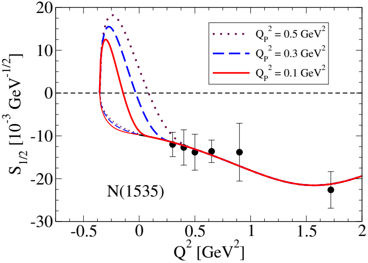
Analytic continuation: $N(1535)\frac{1}{2}^-$ (2') $A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$



- ... what happens is we require that $S_{1/2}$ is smooth at low-Q^2 ?
  $\frac{d^3 S_{1/2}}{dQ^6}$ determined by continuity (small)
- Smoother result for $S_{1/2}$; strong variation for $A_{1/2}$ (large $\frac{d^3 A_{1/2}}{dQ^6}$)
  Consistent with error of $A_{1/2}(0)$

Analytic continuation: $N(1535)\frac{1}{2}^-$ (2') $\quad A_{1/2} = \lambda_R \frac{S_{1/2}}{|\mathbf{q}|}$





- … what happens is we require that $S_{1/2}$ is smooth at low-Q^2 ?
  $\frac{d^3 S_{1/2}}{dQ^6}$ determined by continuity (small)
- Smoother result for $S_{1/2}$; strong variation for $A_{1/2}$ (large $\frac{d^3 A_{1/2}}{dQ^6}$)
  Consistent with error of $A_{1/2}(0)$
- **Only new data in the range Q^2 =0–0.3 GeV^2**
  can decide which extension is the best

Outlook and Conclusions

- The microscopic structure of the $\gamma^* N \to N^*$ current implies correlations between the amplitudes and constraints in the amplitudes
- Those constraints **cannot** be ignored in the empirical parametrizations of the amplitudes **at low-**Q^2 ($Q^2 < 0.3$ GeV^2): $\Delta(1232)$, $N(1535)$
- Best way to take into account those effects:
  use *proper* **form factors** instead of **helicity amplitudes**
  – **automatic verification** of the pseudothreshold constraints
- **If you do not want to bother using form factors:**
  Use our analytic extensions to derive the shape of the amplitudes near $Q^2 = 0$ & the transition to the timelike region
- **Accurate data** in the region $Q^2 =$0–0.3 GeV^2 is fundamental to determine the shape of some N^* amplitudes
  More critical case $N(1535)$; Very important for **Compton Scattering**

Thank you

Selected bibliography (1)

- **Low-Q^2 empirical parametrizations of the N^* helicity amplitudes**, G. Ramalho, arXiv:1909.00013 [hep-ph].
- **Improved empirical parametrizations of the $\gamma^* N \to N(1535)$ transition amplitudes and the Siegert's theorem**, G. Ramalho, Phys. Lett. B **759**, 126 (2016) [arXiv:1602.03444 [hep-ph]].
- **Improved empirical parametrizations of the $\gamma^* N \to \Delta(1232)$ and $\gamma^* N \to N(1520)$ helicity amplitudes and the Siegert's theorem**, G. Ramalho, Phys. Rev. D **93**, 113012 (2016) [arXiv:1602.03832 [hep-ph]].
- **Parametrizations of the $\gamma^* N \to \Delta(1232)$ quadrupole form factors and Siegert's theorem**, G. Ramalho, Phys. Rev. D **94**, 114001 (2016) [arXiv:1606.03042 [hep-ph]].

Selected bibliography (2)

- **New low-Q^2 measurements of the $\gamma^* N \to \Delta(1232)$ Coulomb quadrupole form factor, pion cloud parametrizations and Siegert's theorem**, G. Ramalho, Eur. Phys. J. A **54**, 75 (2018) [arXiv:1709.07412 [hep-ph]].
- **Nucleon resonances in Compton scattering**, G. Eichmann and G. Ramalho, Phys. Rev. D **98**, 093007 (2018) [arXiv:1806.04579 [hep-ph]].
- **Combined parametrization of G_{En} and $\gamma^* N \to \Delta(1232)$ quadrupole form factors**, G. Ramalho, Eur. Phys. J. A **55**, 32 (2019) [arXiv:1710.10527 [hep-ph]].