Update on longitudinal beam stability in HL-LHC

Ivan Karpov, CERN

Acknowledgements:

Theodoros Argyropoulos, Elena Shaposhnikova, Alexey Burov, Rama Calaga, James A. Mitchell, Sergey Antipov, Helga Timko

the 9th HL-LHC Collaboration Meeting Fermilab, 14-16 October 2019

Longitudinal single-bunch stability

Loss of Landau damping (LD) in longitudinal plane is important intensity limitation in LHC.

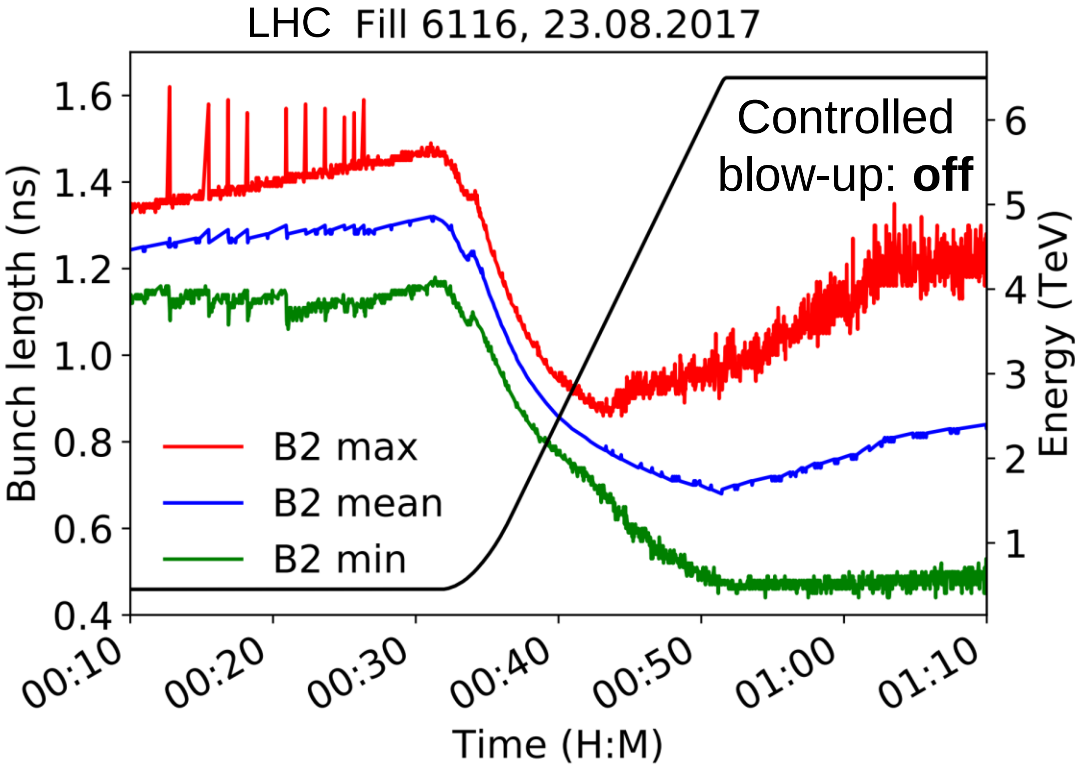
Based on Sacherer formalism the intensity threshold is *(F. Sacherer, 1971)*

rf voltage

bunch length*

$$N_b^{\mathrm{thr}} \propto \frac{V_{\mathrm{rf}}\tau^5}{(\mathrm{Im}Z/n)_{\mathrm{eff}}}$$

effective impedance



Bunch length shrinks during acceleration → Controlled blow-up must be applied to keep beam stable

*$\tau = \tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$ is scaled from full-width half-maximum (FWHM) bunch length

Longitudinal single-bunch stability

Loss of Landau damping (LD) in longitudinal plane is important intensity limitation in LHC.

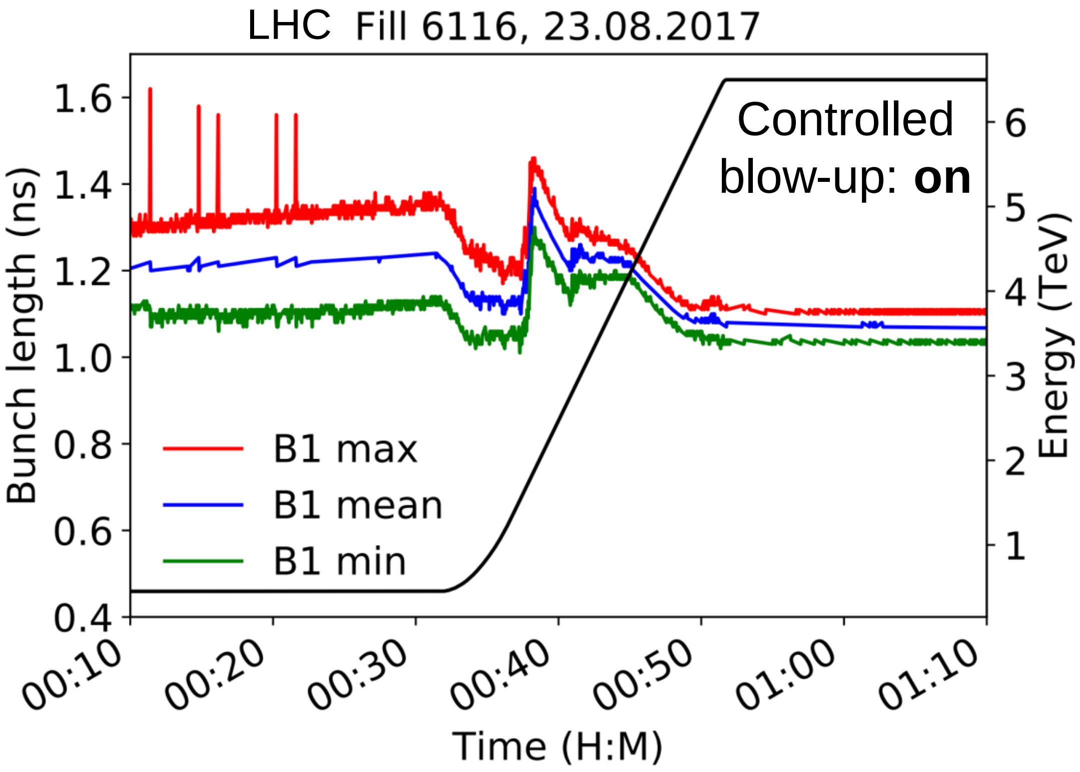
Based on Sacherer formalism the intensity threshold is *(F. Sacherer, 1971)*

rf voltage

bunch length*

$$N_b^{\mathrm{thr}} \propto \frac{V_{\mathrm{rf}}\tau^5}{(\mathrm{Im}Z/n)_{\mathrm{eff}}}$$

effective impedance

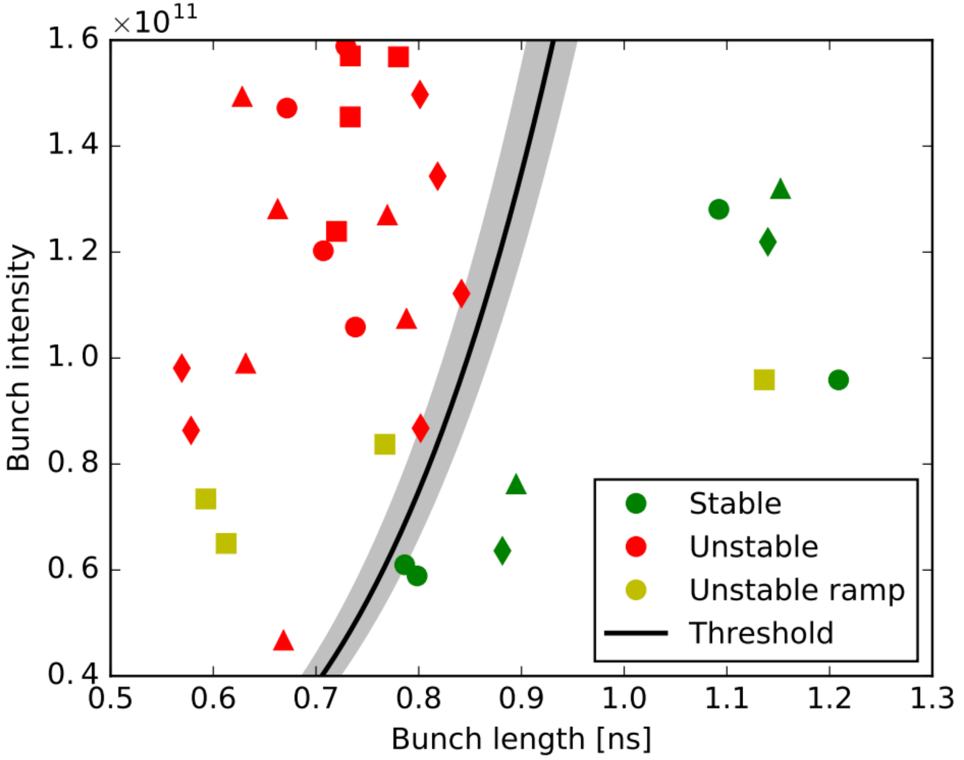


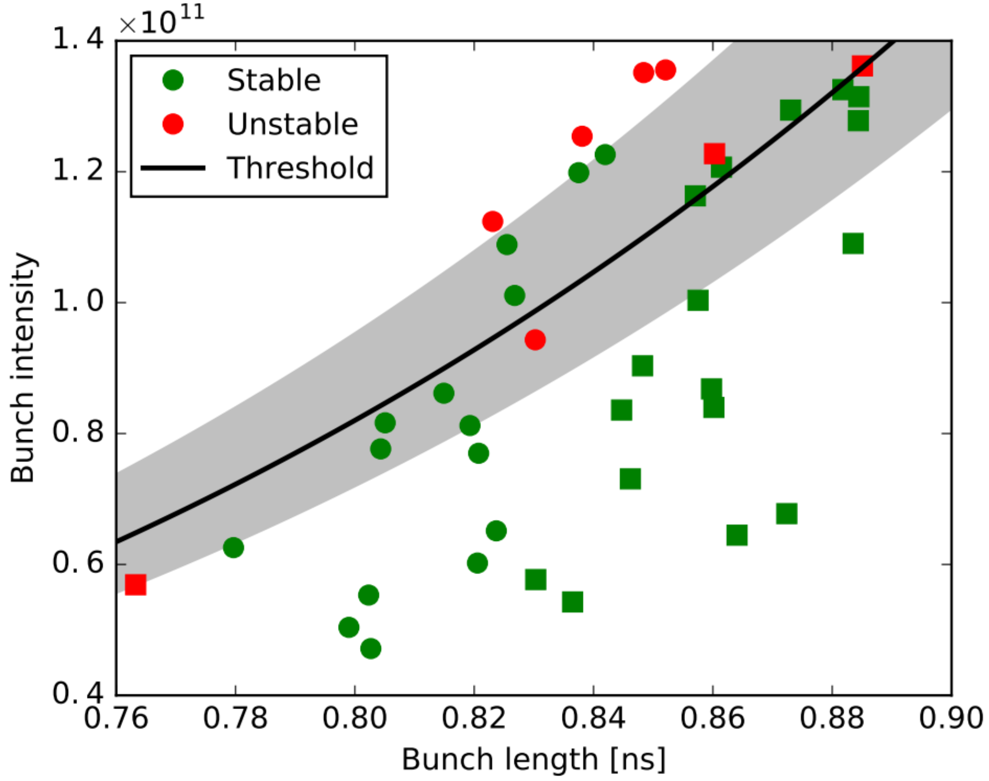
Bunch length shrinks during acceleration → Controlled blow-up must be applied to keep beam stable

*$\tau = \tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$ is scaled from full-width half-maximum (FWHM) bunch length

Loss of Landau damping in LHC: measurements

MDs in 2015, V_{rf} = 12 MV, beam energy 6.5 TeV

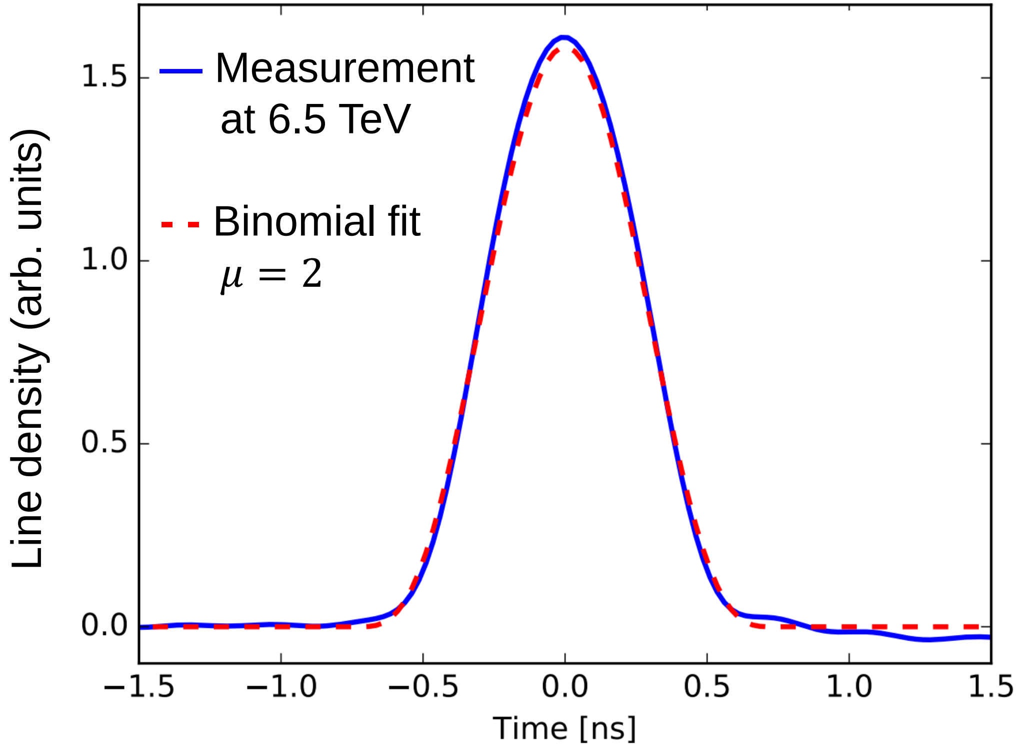


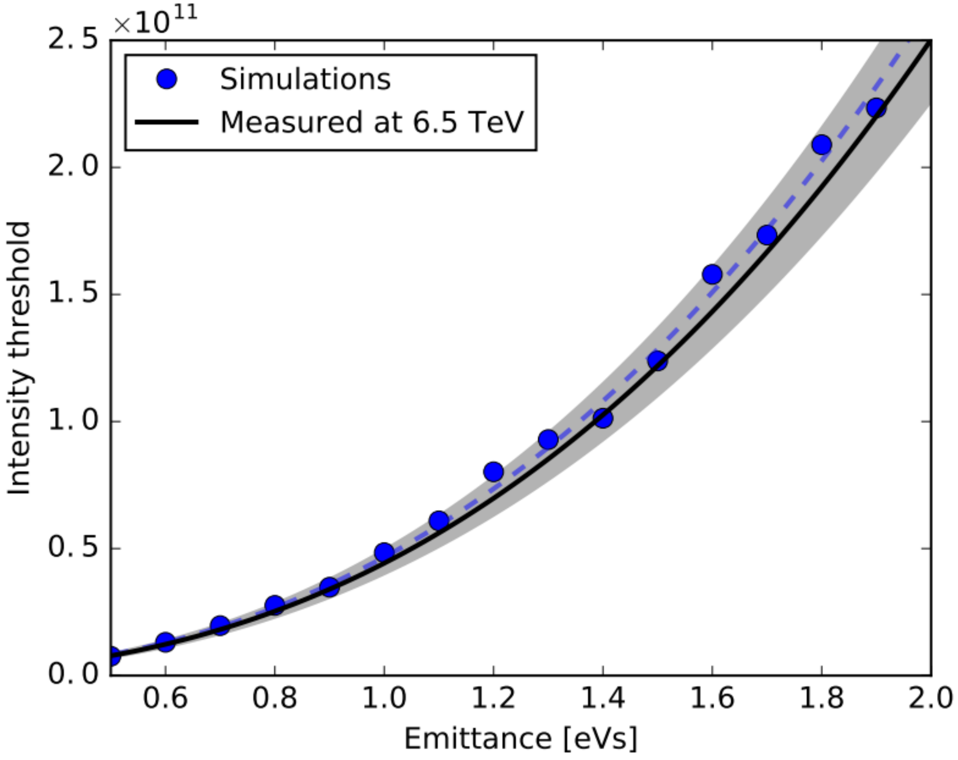


Measurements performed at different conditions and stability parameter was calculated for all bunches $\xi = V_{\mathrm{rf}}\tau^5/N_b$ *(PhD thesis J. E. Muller, 2016)*

→ The threshold $\xi_{\mathrm{th}} = 0.5 \times (\max \xi_{\mathrm{unst}} + \min \xi_{\mathrm{st}}) = \; (5.0 \pm 0.5) \times 10^{-5}\,(\mathrm{ns})^5\mathrm{V}$

Comparison of measurements and simulations

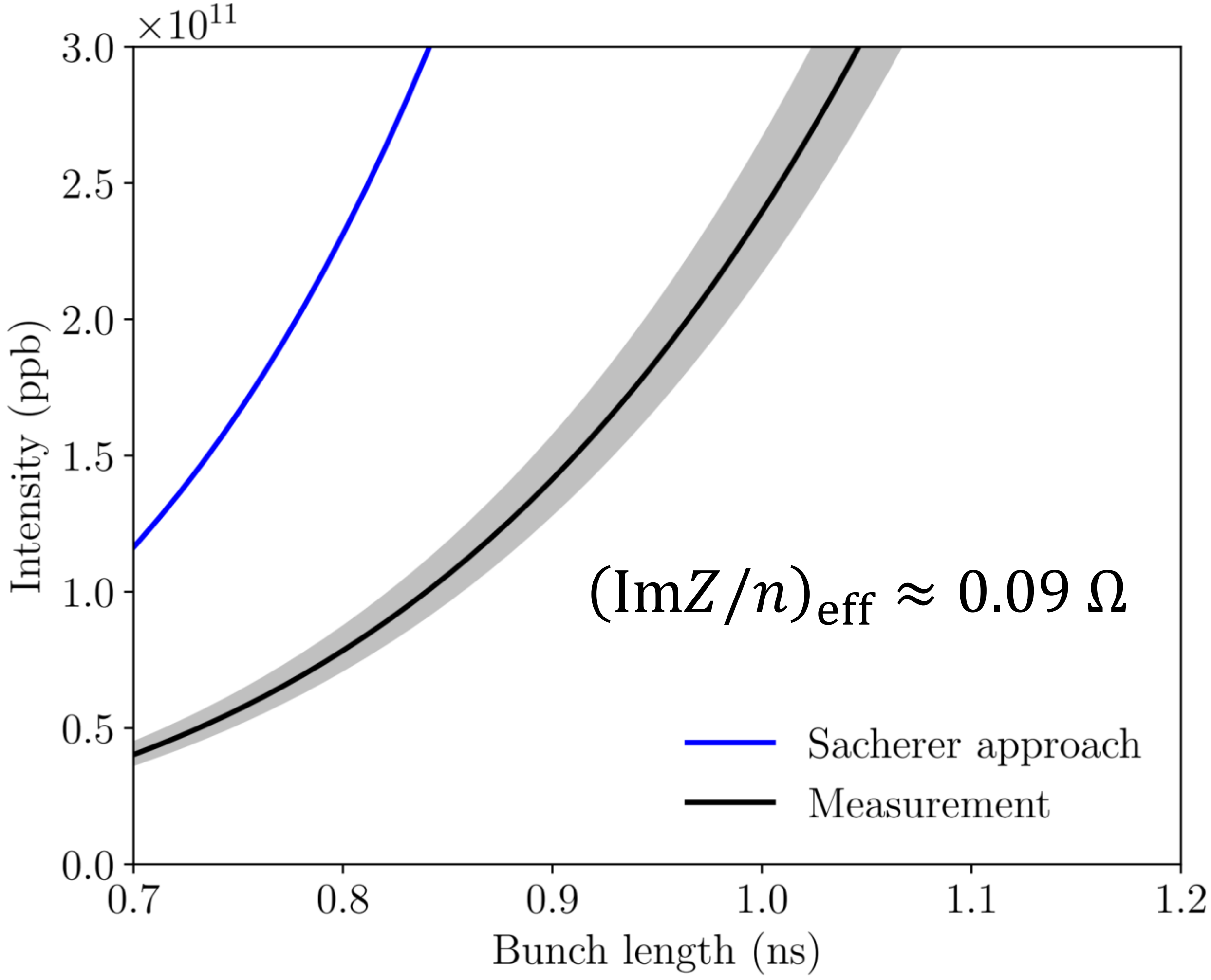




Simulation with full LHC longitudinal impedance model *(B. Salvant et al., HB2012)* & binomial bunch distribution $\lambda(t) = \lambda_0[1 - 4t^2/\tau^2]^{\mu+0.5}$

→ Very good agreement between measurements and BLonD particle tracking simulations (*PhD thesis J. E. Muller, 2016*).

Comparison of measurements and analytic calculations



→ The analytically calculated thresholds using Sacherer approach are 3 – 4 times higher.

→ We need a different approach to evaluate stability for future machines (for example FCC-hh).

Analytic approaches

Vlasov equation for a small perturbation of stationary distribution can be solved by using:

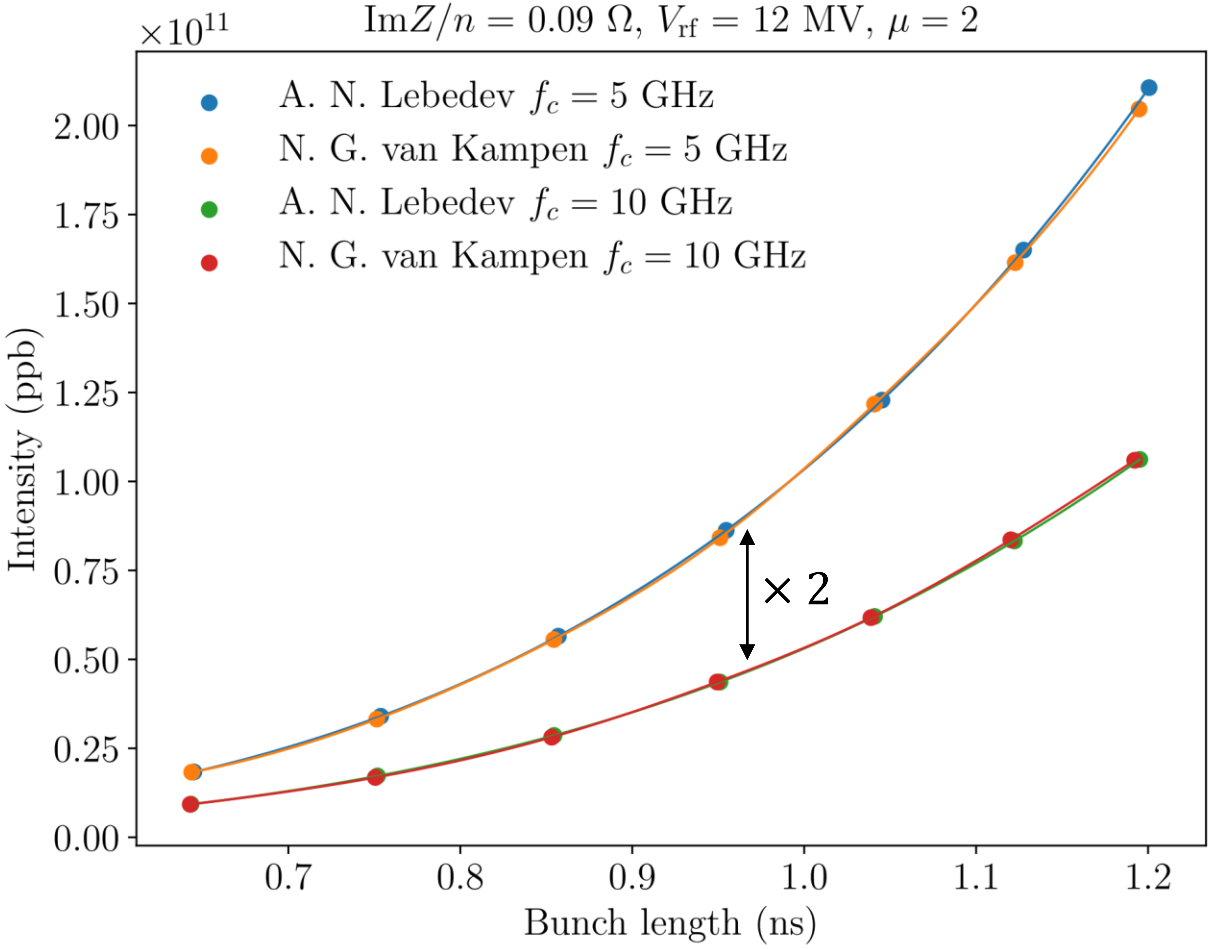
1. Orthogonal mode expansion *(K. Oide & K. Yokoya, 1990)*.
   Loss of LD is interpreted as emerged van Kampen coherent modes (semi-analytic code by *A. Burov, 2010*, recently translated in Python by *T. Argyropoulos, 2019*).

2. Another form of solution *(A. N. Lebedev, 1967)*.
   Matrix equation defines presence of undamped modes

Both approaches were incorporated in a new semi-analytic framework MELODY (Matrix Equations for LOngituDinal stabilitY evaluation, 2019)

Comparison of analytic approaches

LHC at 6.5 TeV, $\mathrm{Im}Z/n = 0.09\ \Omega$ = const

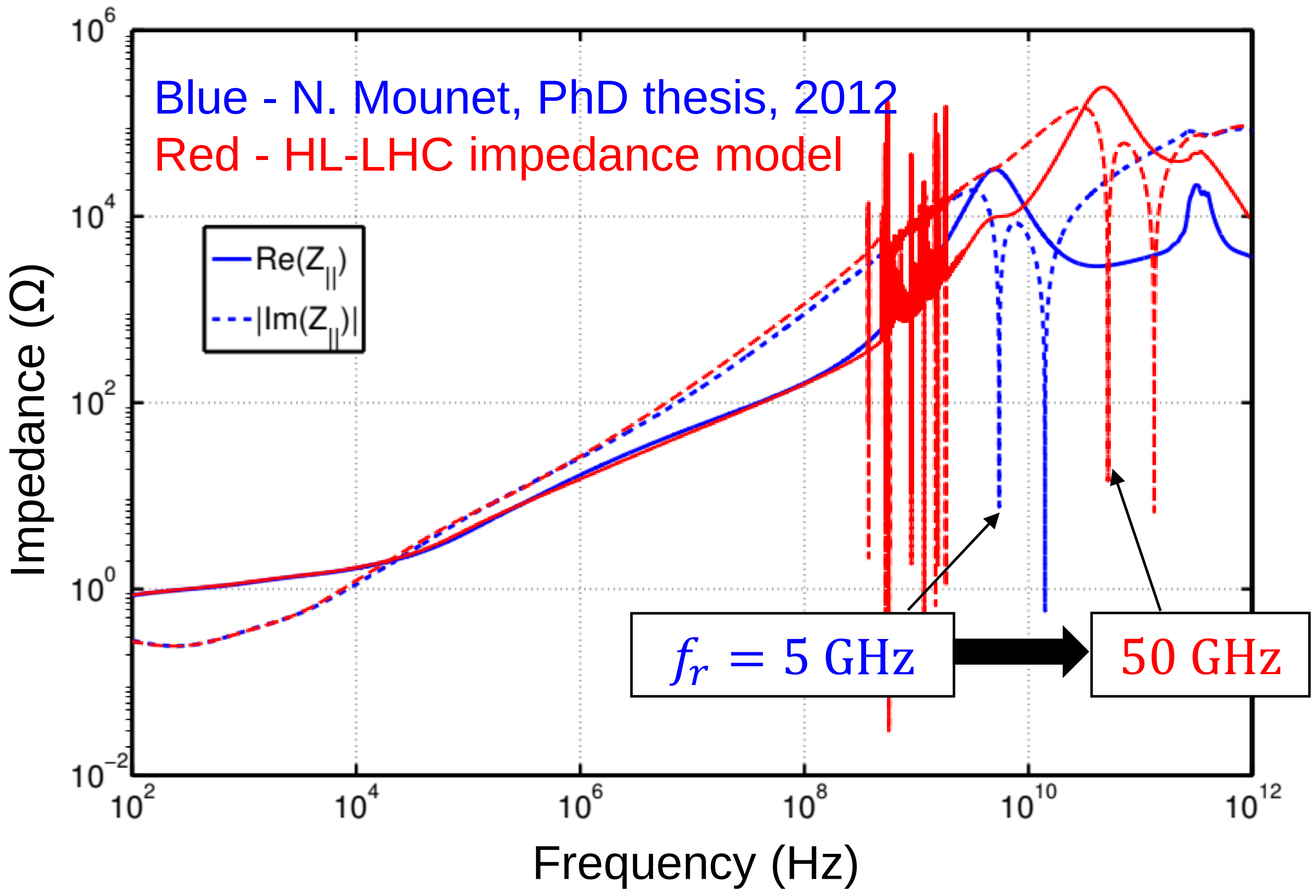


→ Very good agreement between both approaches

→ $\mathrm{Im}Z/n$ = const, often used for estimations turns out to be not physical (threshold depends on cut-off frequency f_c)

→ Realistic impedance model needs to be used

LHC/HL-LHC impedance model



Broad-band impedance model
(LHC Design Report, 2004)

$$R = Q\ \mathrm{Im}Z/n\,\frac{f_r}{f_0}$$

$Q = 1, f_r = 5$ GHz

$\mathrm{Im}Z/n = 0.07\ \Omega$ for injection optics

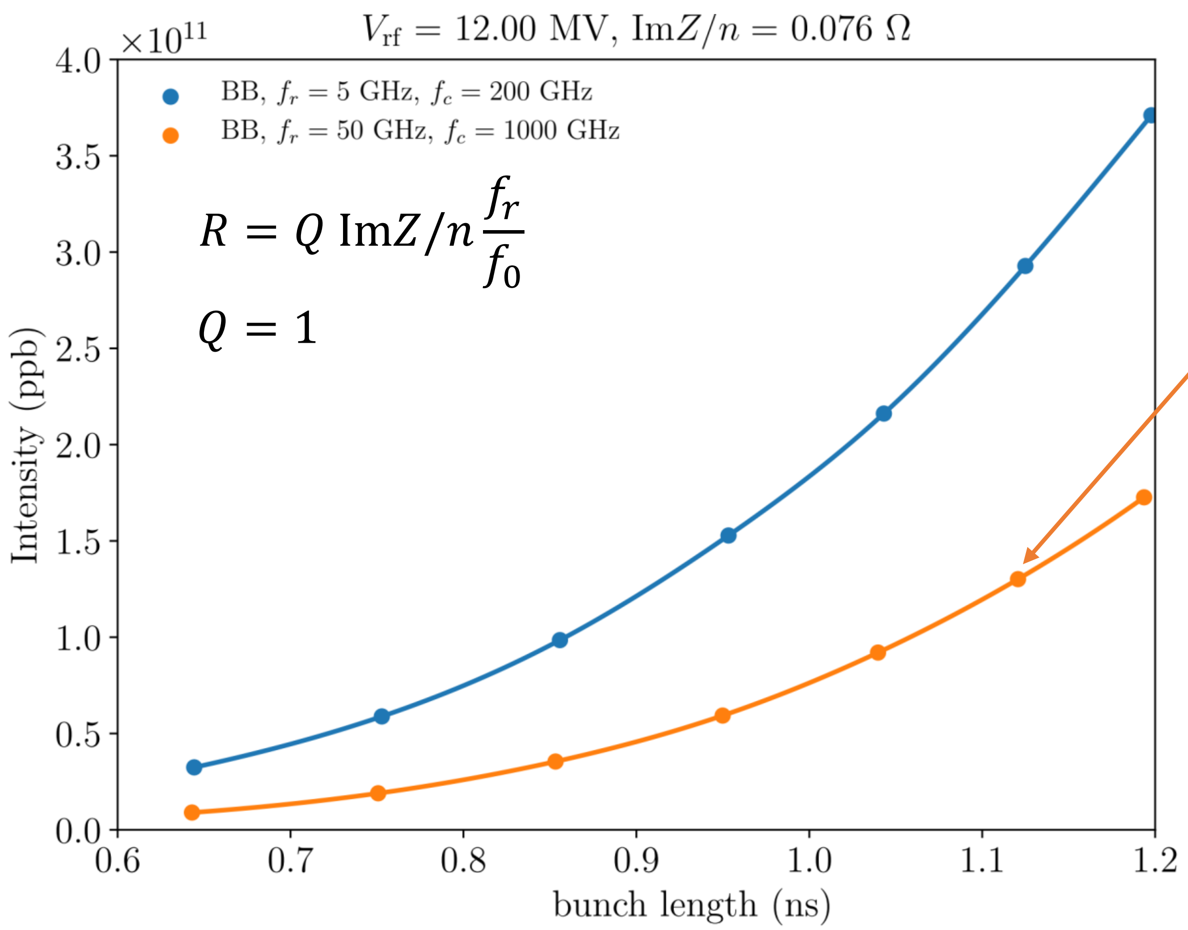
$\mathrm{Im}Z/n = 0.076\ \Omega$ for squeezed optics

→ The resonant frequency of the broadband model was changed from 5 GHz to 50 GHz (*D. Amorim, 2018*).

→ This change has a significant impact on longitudinal single-bunch stability

Stability threshold for broad-band impedance

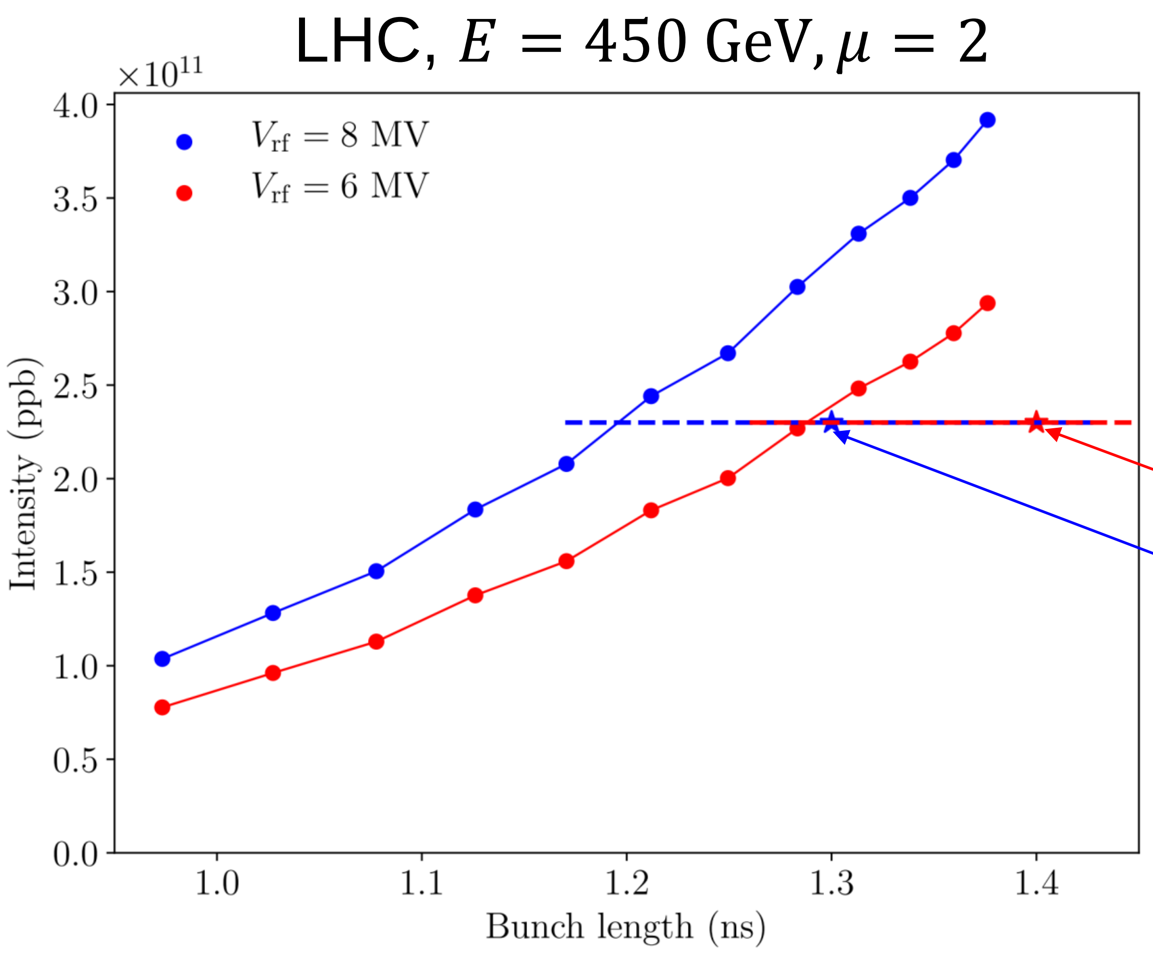
MELODY results



→ Changing resonant frequency from 5 GHz to 50 GHz results in reduction of the threshold by factor of 3.

→ The LHC/HL-LHC broad-band impedance model needs to be revised.

Single-bunch stability at 450 GeV



Results using MELODY for smoothed impedance (resistive wall + broad-band model at 5 GHz)

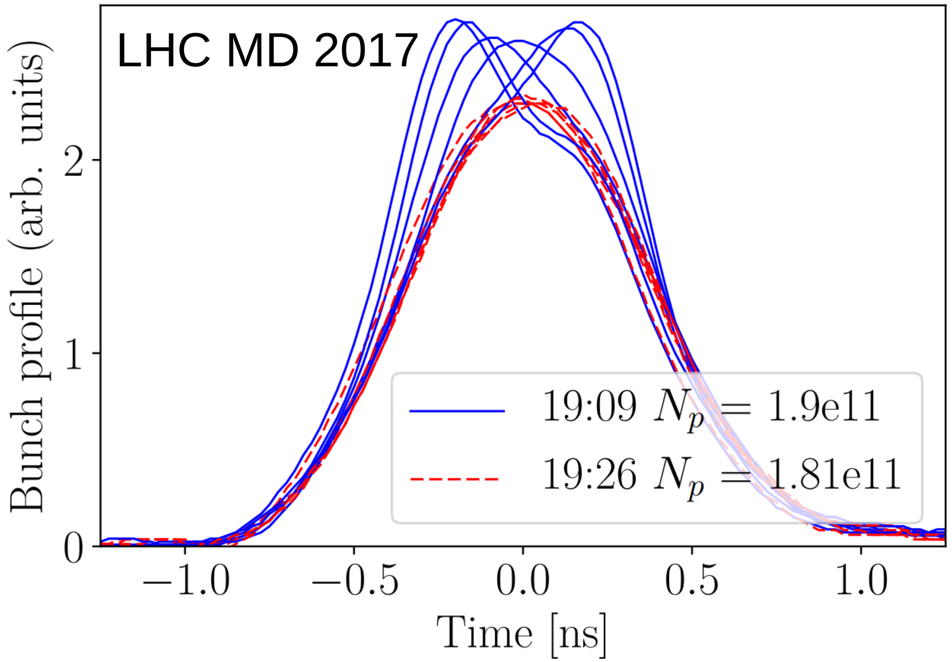
For LIU bunch from SPS (1.65 ns, 10MV@200MHz + 1.6 MV@800 MHz), bunch length in LHC (in absence of injection errors):

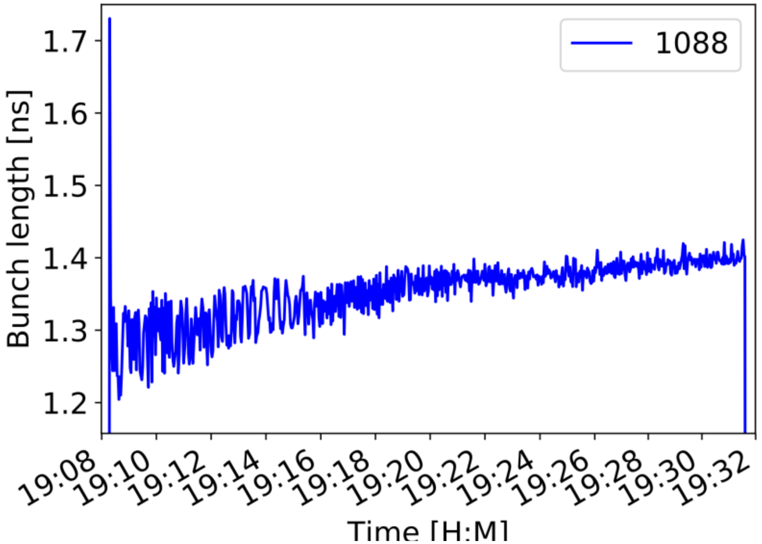
- 1.4 ns for 6 MV (LHC nominal 2017)
- 1.3 ns for 8 MV (HL-LHC design report)

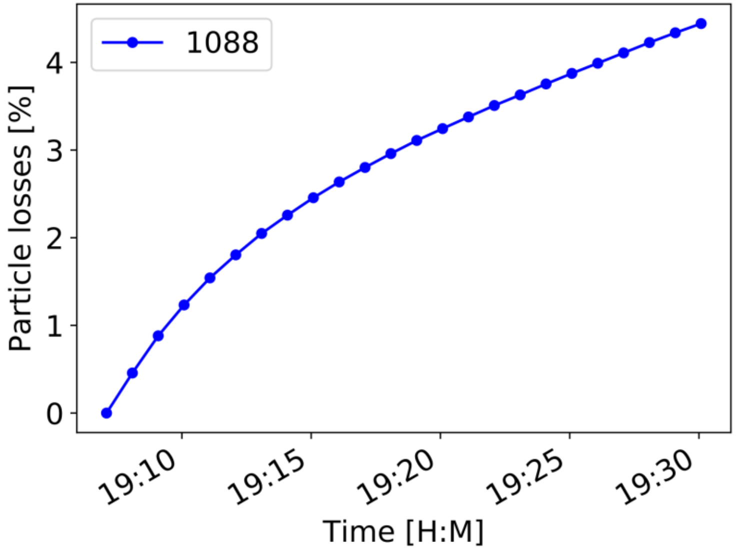
Two voltages V_{rf} provide similar single-bunch stability

There are constrains due to injection losses and rf power consumption *(see talk of H. Timko)*

Persistent oscillations after injection





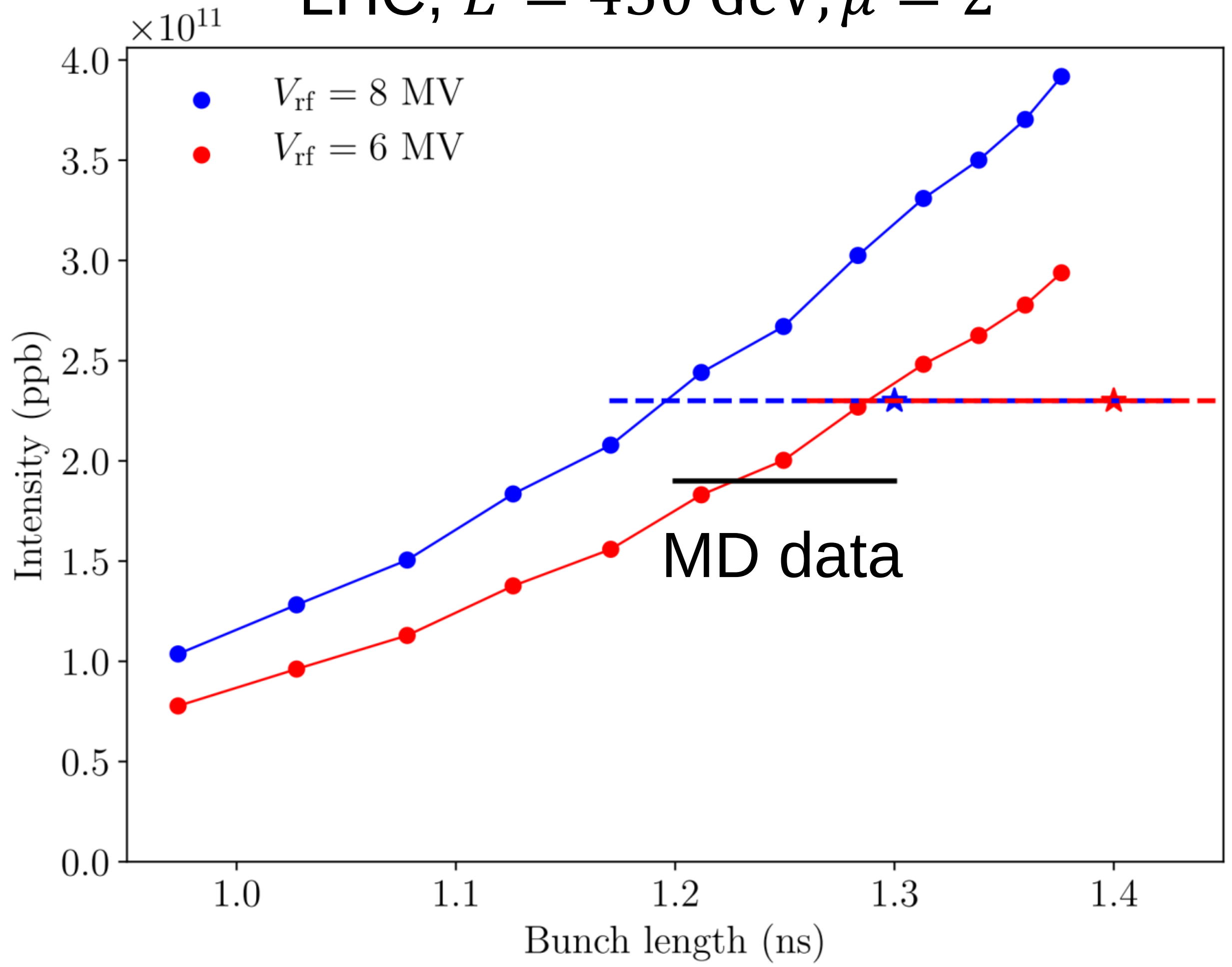


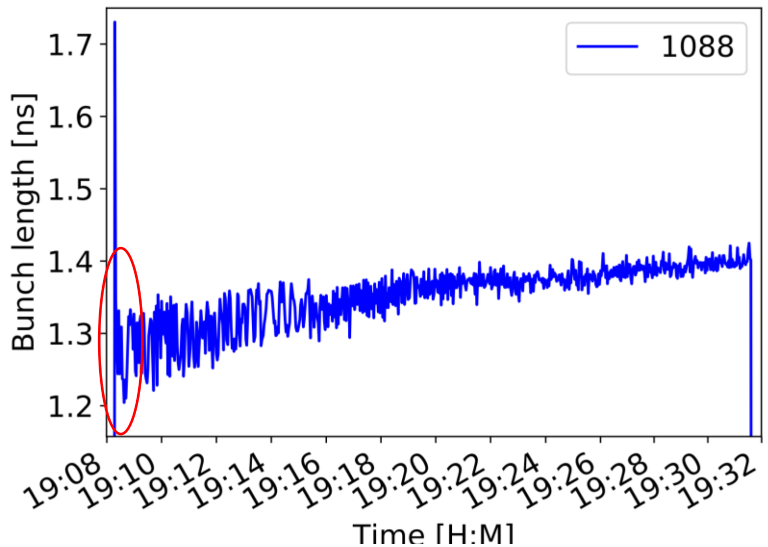
During 20 min oscillations lead to ~10 % bunch lengthening and ~5% particle loss *(H. Timko et al., HB2018)*

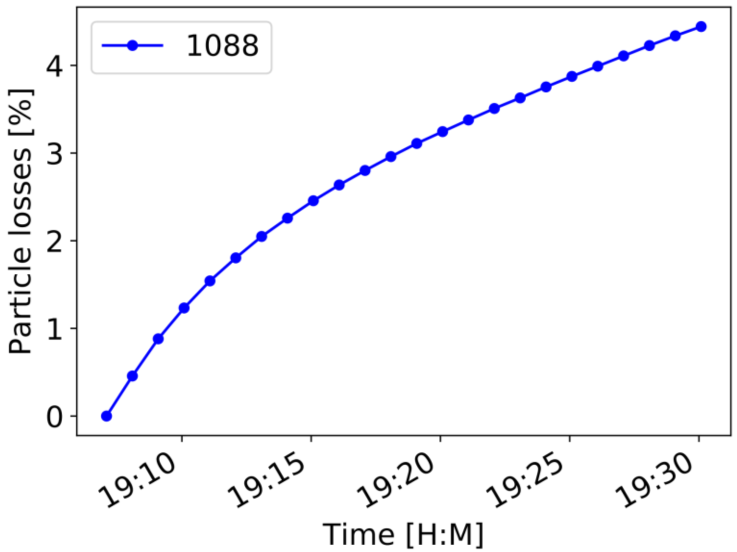
Similar oscillations were observed in Tevatron (*R. Moore, PAC2003*)

Persistent oscillations after injection

LHC, $E = 450$ GeV$, \mu = 2$

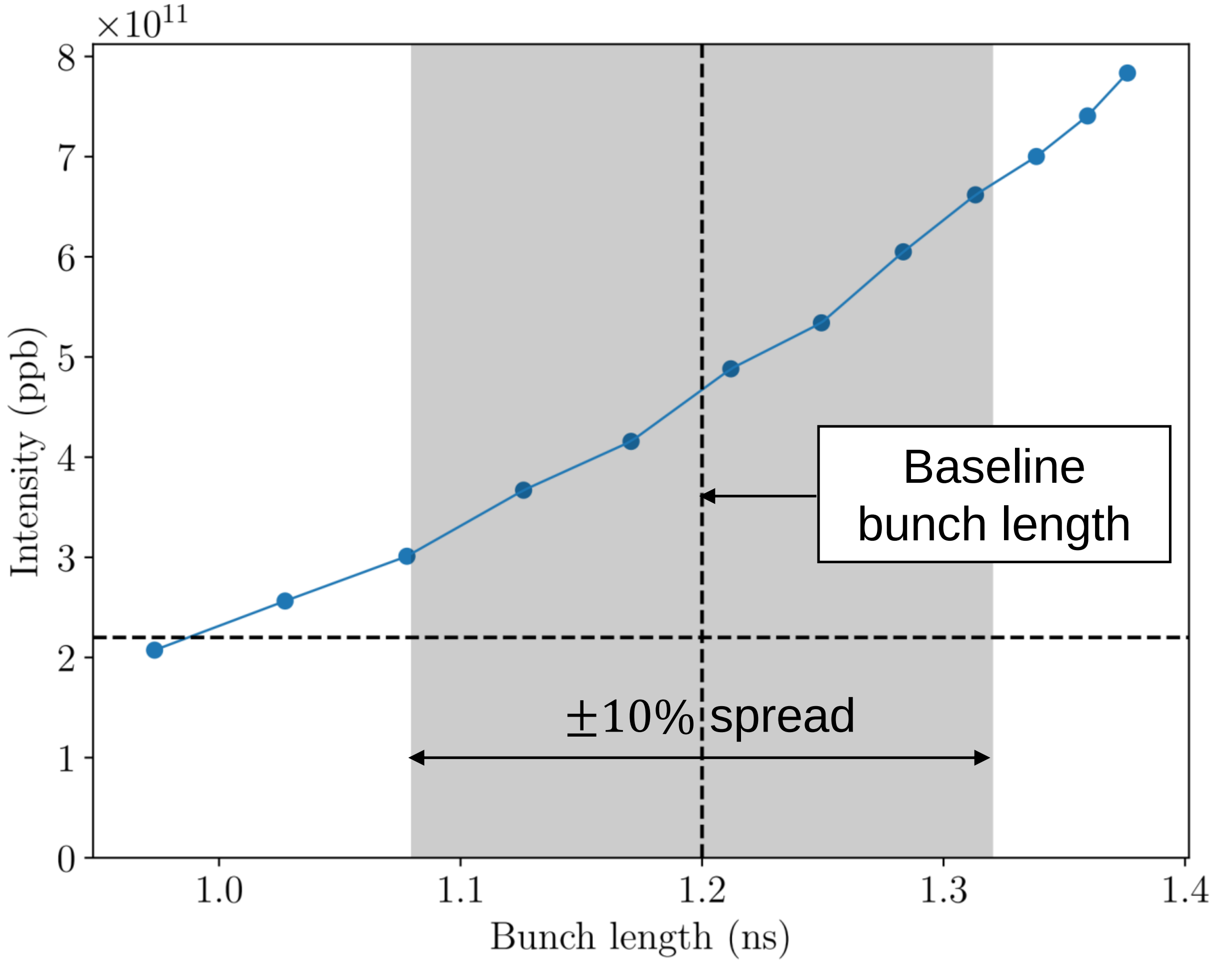






Single-bunch stability at 7 TeV

LHC, $V_{\mathrm{rf}} = 16\ \mathrm{MV}, E = 7\ \mathrm{TeV}, \mu = 2$



Results using MELODY for smoothed impedance (resistive wall + broad-band model at 5 GHz)

→ Sufficient stability for $\tau = 1.2$ ns with margin for $\pm 10\%$ bunch length spread

Next steps:
→ To repeat calculations with revised broad-band impedance model
→ To study effect of high-order-modes (HOM) on single-bunch stability

Stability of multi-bunch beam

Multi-bunch instabilities were not observed so far in LHC

HL-LHC: higher intensity & HOMs of crab cavities (CC)

For ≈ 3000 bunches macro-particle simulations are computationally expensive

$\rightarrow$ Analytical approaches are used to define requirements for HOM damping

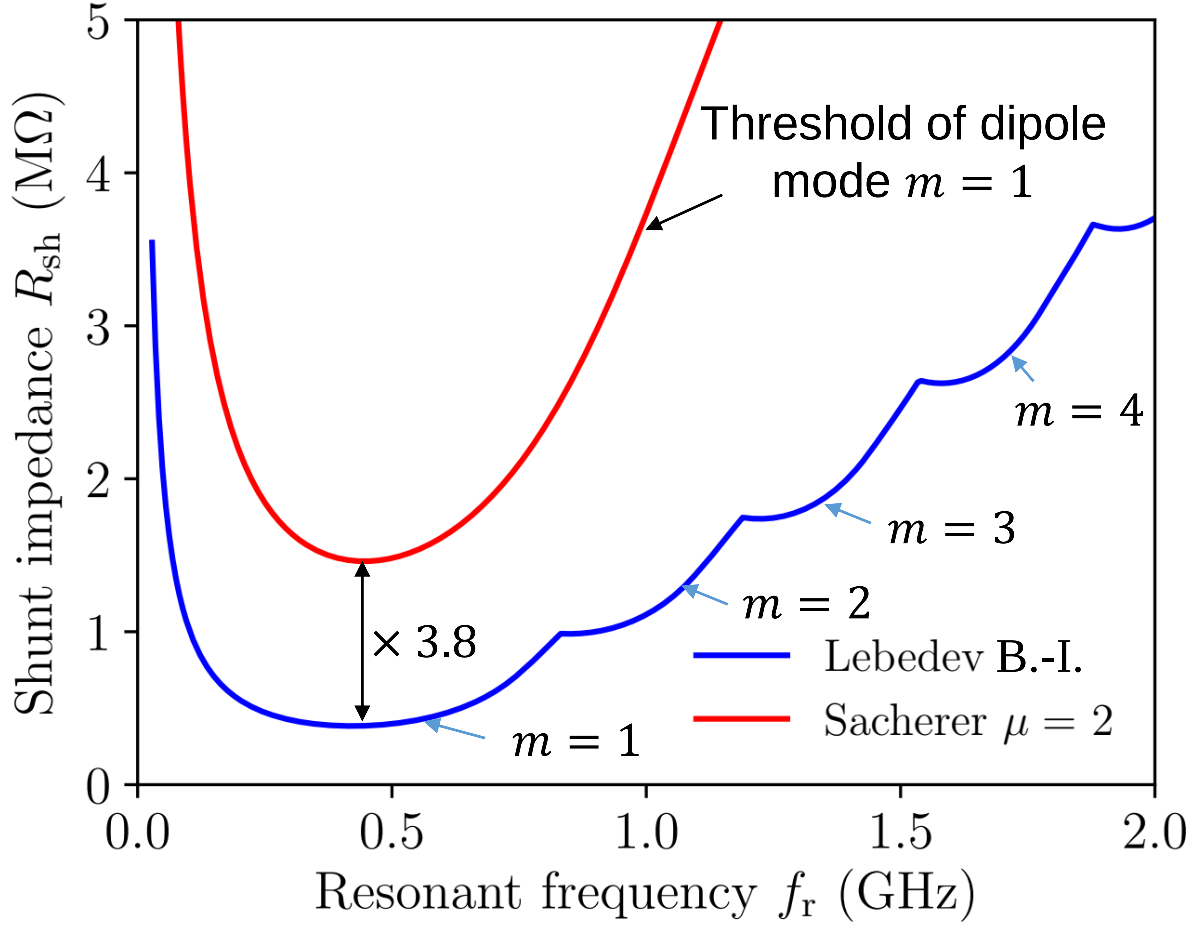
Analytical stability evaluation can be based on:

- Sacherer stability diagram *(F. Sacherer, 1971)*
- Lebedev equation (stability diagram by *V. Balbekov, S. Ivanov, 1987)*

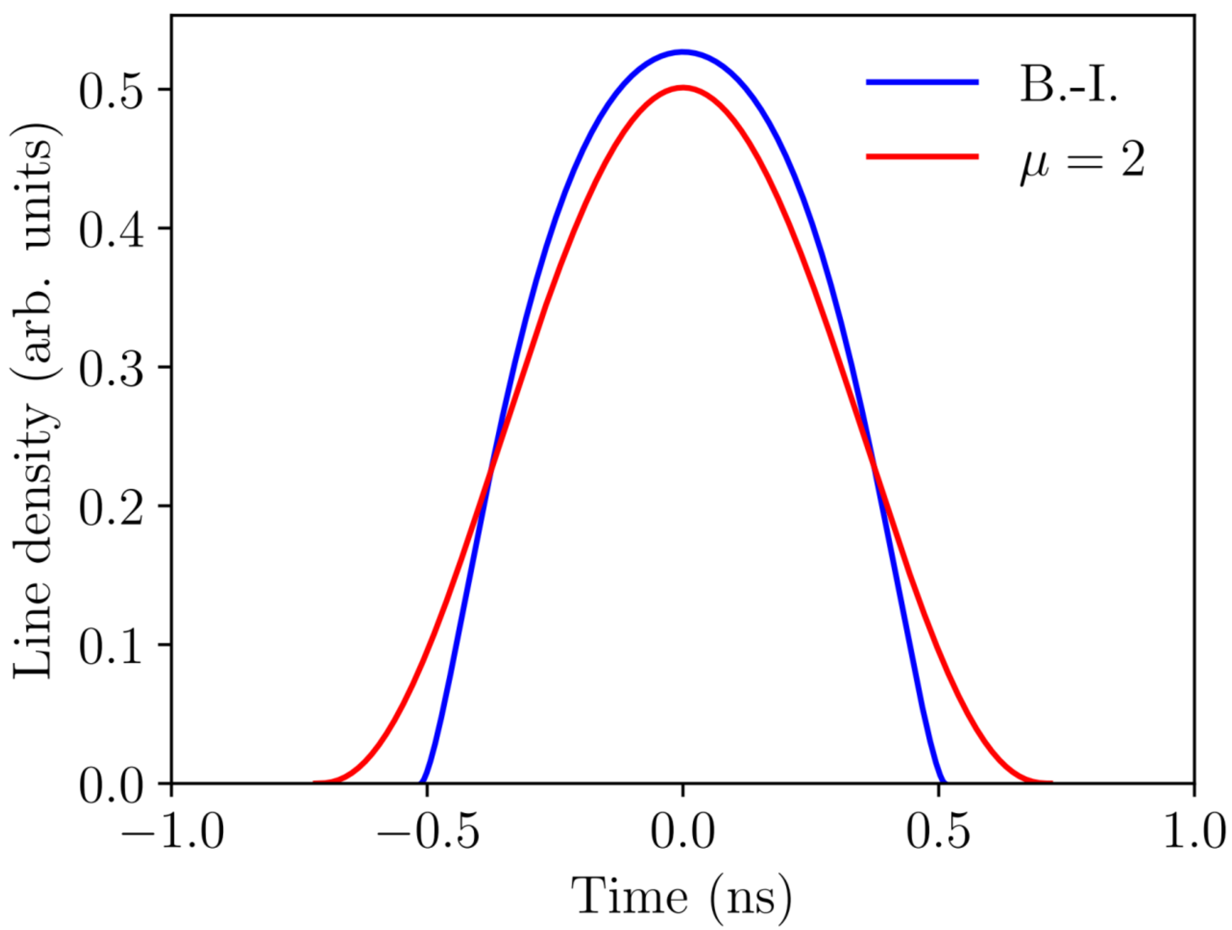
There was a significant discrepancy between the results of two approaches (*E. Shaposhnikova, LHC-CC'10* and *A. Burov, LHC-CC'11*)

Lebedev vs Sacherer approach

$V_{\mathrm{rf}} = 16\ \mathrm{MV}, \tau = 1.2\ \mathrm{ns}, E = 7\ \mathrm{TeV}$

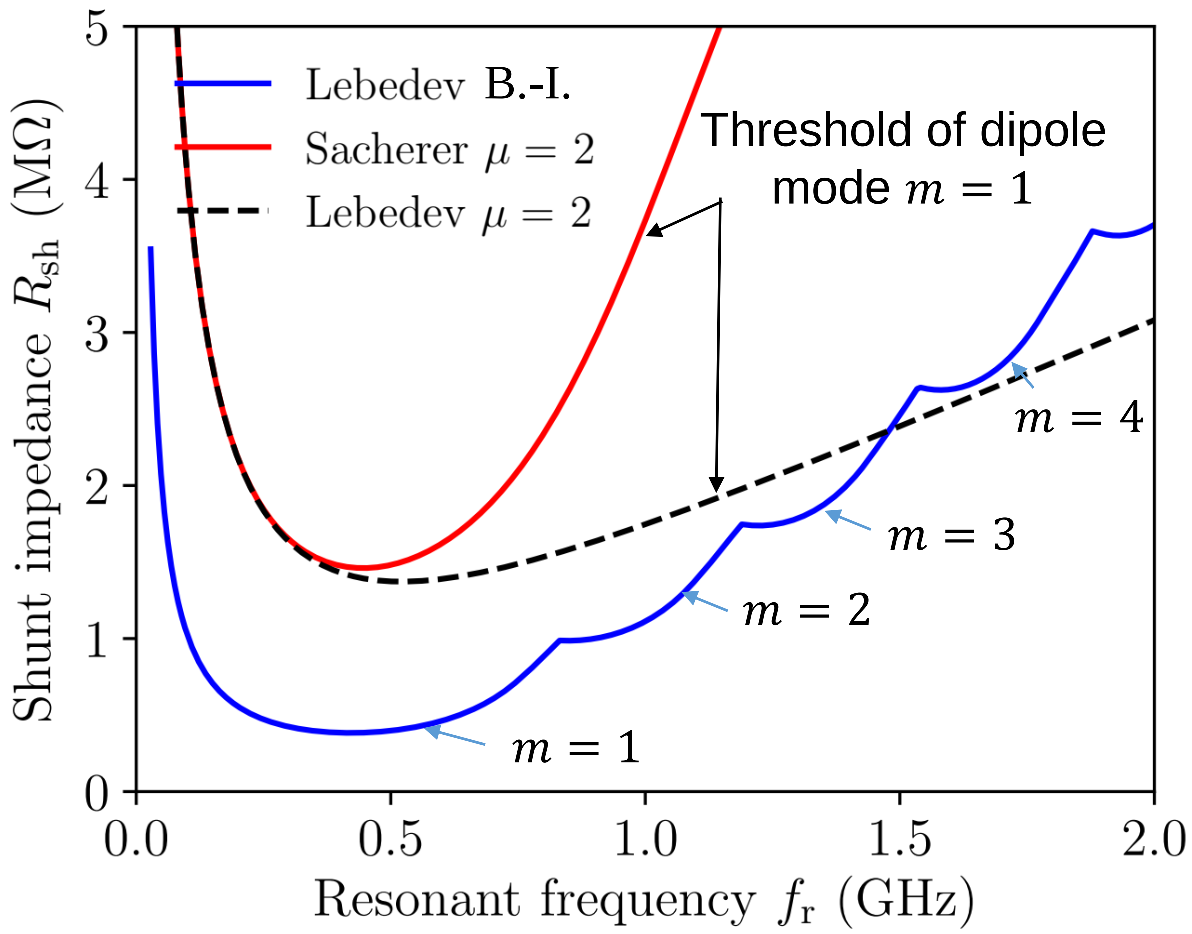


Binomial vs Balbekov-Ivanov distribution



Lebedev vs Sacherer approach

$V_{\text{rf}} = 16\ \text{MV}, \tau = 1.2\ \text{ns}, E = 7\ \text{TeV}$



→ Factor of 4 difference is due to different distribution function.

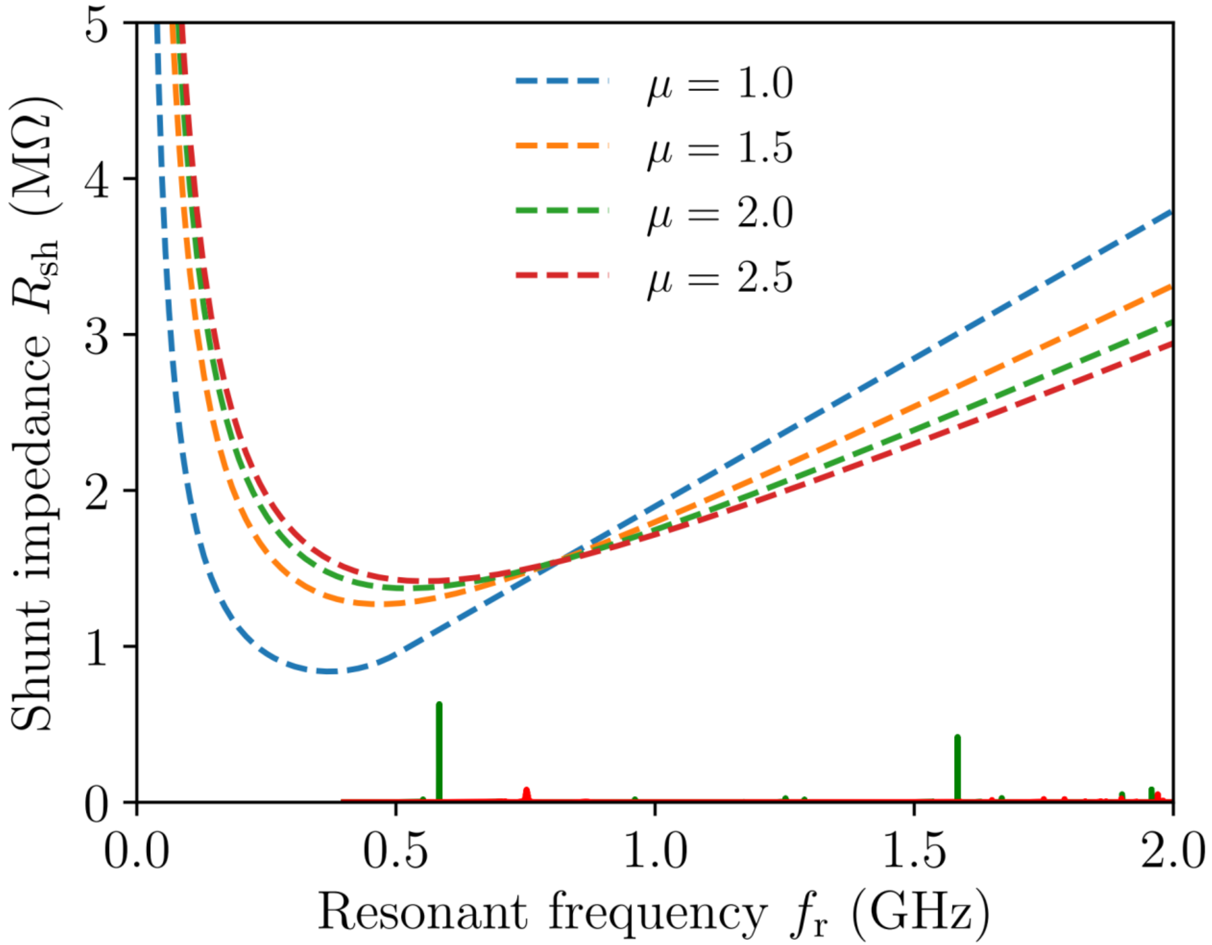
→ Stability diagram approach based on Lebedev equation was extended to binomial distribution.

→ For $\mu = 2$, the minimum thresholds are similar, but Sacherer approach underestimates threshold at higher frequencies

→ Sacherer approach can be obtained as a low frequency expansion of Lebedev equation (*E. Shaposhnikova et al., MCBI19*)

Results for HL-LHC flat top

$V_{\mathrm{rf}} = 16\ \mathrm{MV}, \tau = 1.2\ \mathrm{ns}, E = 7\ \mathrm{TeV}$

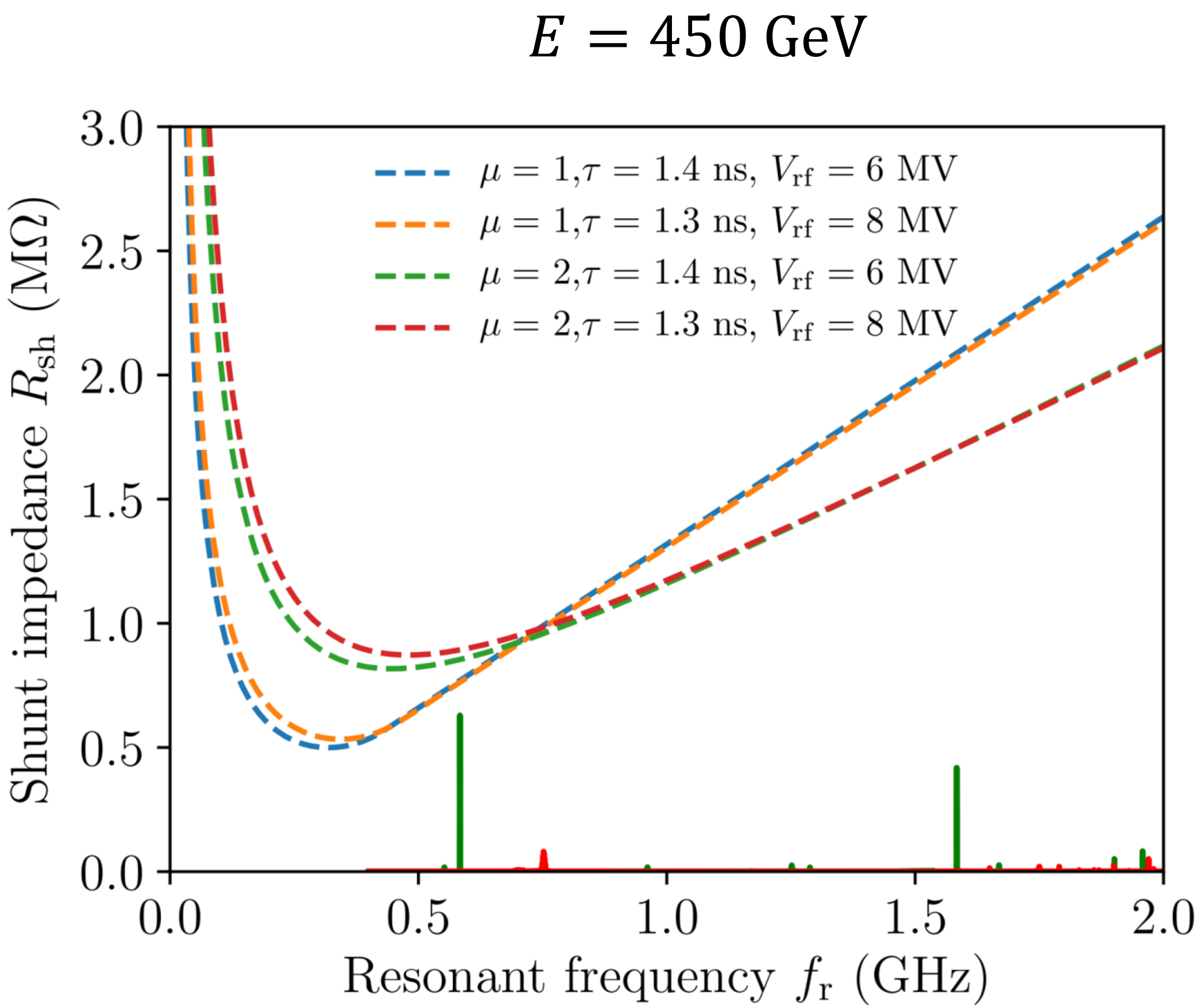


Crab cavity HOMs:
HL-LHC Double Quarter Wave (DQW) × 4
HL-LHC RF-Dipole (RFD) × 4

→ Thresholds for distributions with different μ and the same FWHM bunch length are similar (except $\mu = 1$)

→ Only one HOM is close to the stability limit for the worst-case scenario without frequency spread between CC.

Results for HL-LHC flat bottom



Crab cavity HOMs:
HL-LHC Double Quarter Wave (DQW) × 4
HL-LHC RF-Dipole (RFD) × 4

→ Thresholds are similar for 6 MV and 8 MV of rf voltage for the same bunch parameters at the SPS extraction.

→ Recommendation: further damping of the first high Q mode of DQW CC could be addressed for margin in machine operation.

Summary

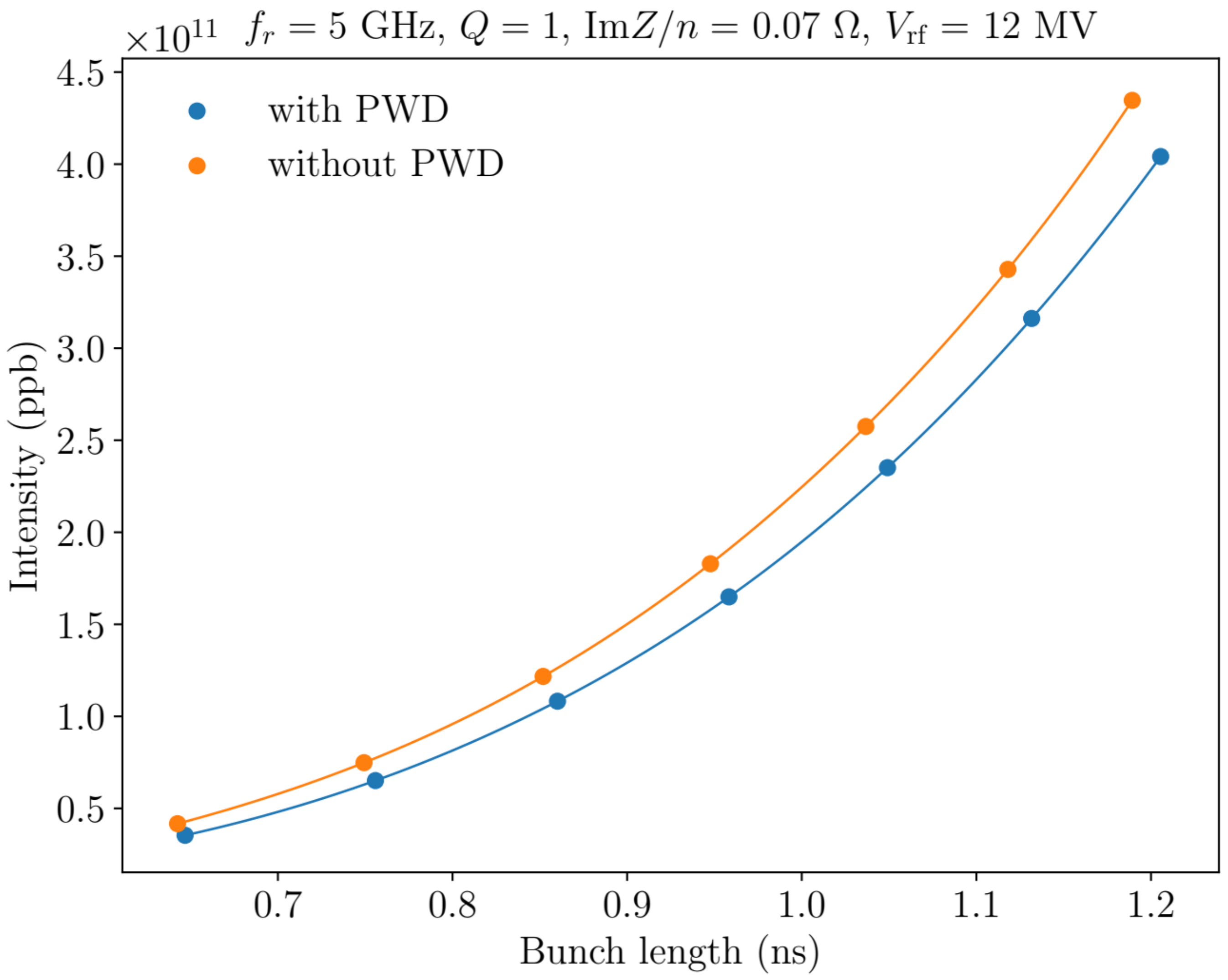
Single-bunch stability:
- Bunch parameters are affected by the loss of Landau damping
- Sacherer stability diagram in longitudinal plane should be used with caution. More complete formalisms (van-Kampen modes and Lebedev equation) are available for accurate semi-analytical threshold estimations.
- LHC/HL-LHC impedance model needs to be revised for longitudinal stability evaluation.

Multi-bunch stability:
- Thresholds of coupled-bunch instability depend on distribution function but are similar for the same FWHM bunch length (binominal distribution).
- To increase stability margin, a spread of HOM frequencies between crab cavities needs to be introduced and further damping of the first high Q mode of DQW CC is recommended.

Thank you for your attention!

Spare slides

Impact of potential well distortion



Difference is about 10 %

Landau damping for multi-bunch beam

For narrow band impedance with ω_r, only one resonant harmonic $k_r = \omega_r/\omega_0 = lM + n$ can be kept (M - number of equidistant bunches) in Lebedev' equation:

$$\frac{k}{Z_k} = -\frac{i I_0\, h M}{V \cos\phi_s} G_{kk}(\Omega)$$

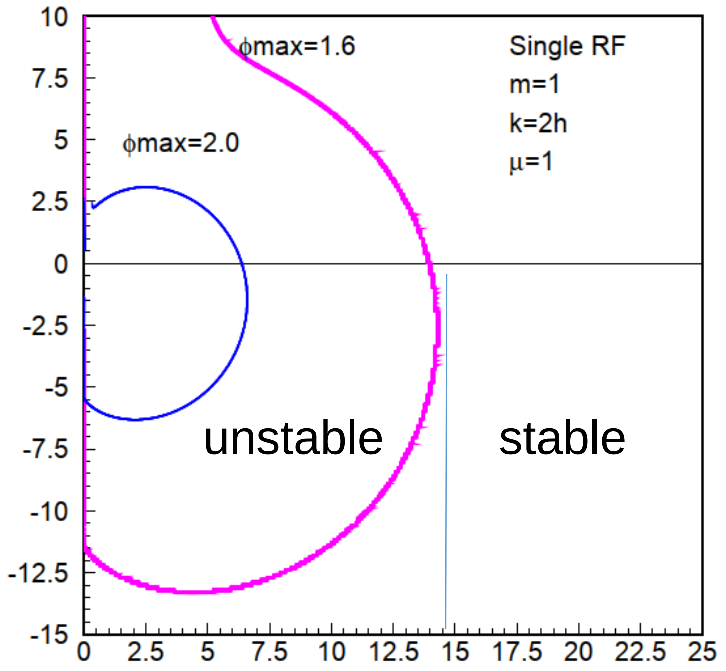
From stability diagram
(V. Balbekov, S. Ivanov, 1987):

$$\frac{k}{Z_k} > \frac{I_0\, h M}{V \cos\phi_s} \mathrm{Im} G_{kk}^{max}$$

with $$\mathrm{Im} G_{kk}(\Omega) = \frac{\pi}{A_N} \sum_{m=1}^{\infty} \frac{F'(\mathcal{E}_m) I_{mk}^2(\mathcal{E}_m)}{\omega_s'(\mathcal{E}_m)}$$

→ Beam is stable if vertical line $1/R_{sh}$ is inside stability region

Stability diagram for
$F(\mathcal{E}) = F_0(1 - \mathcal{E}/\mathcal{E}_{max})^{\mu}$



$$Z_k^{-1} = 1/R_{sh} + i\, Q(\omega/\omega_r - \omega_r/\omega)/R_{sh}$$

Multi-bunch threshold

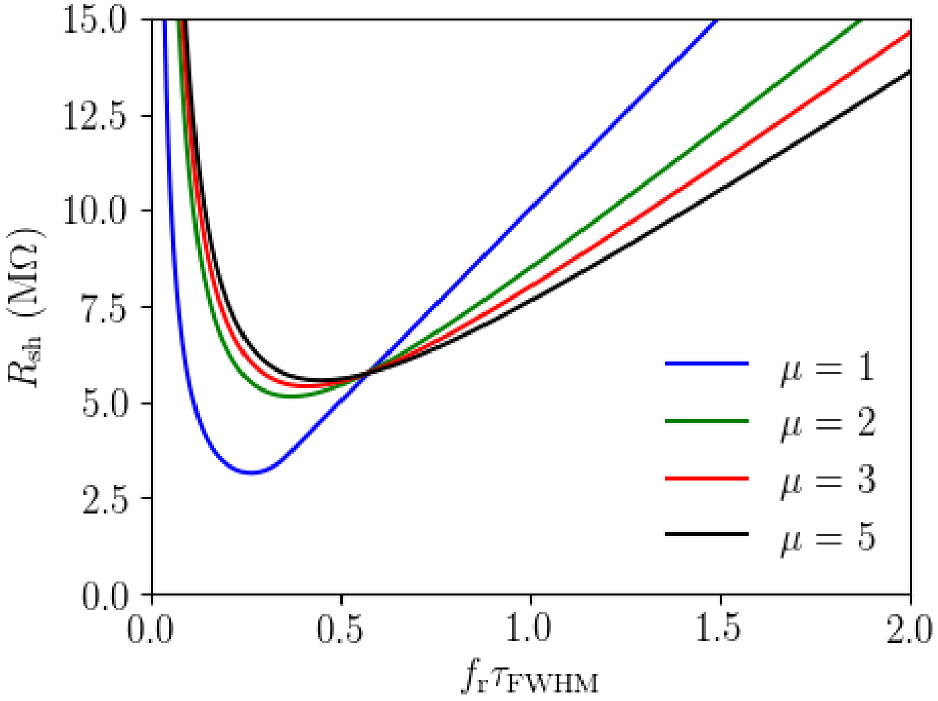
In single RF system the threshold (no acceleration) for binomial distribution

$$F(\mathcal{E}) = F_0(1 - \mathcal{E}/\mathcal{E}_{max})^{\mu}$$

$$R_{sh} < \frac{V\ (\pi f_{rf} \tau_b)^3}{32\, I_0}\, G_\mu(f_r \tau_b)$$ where

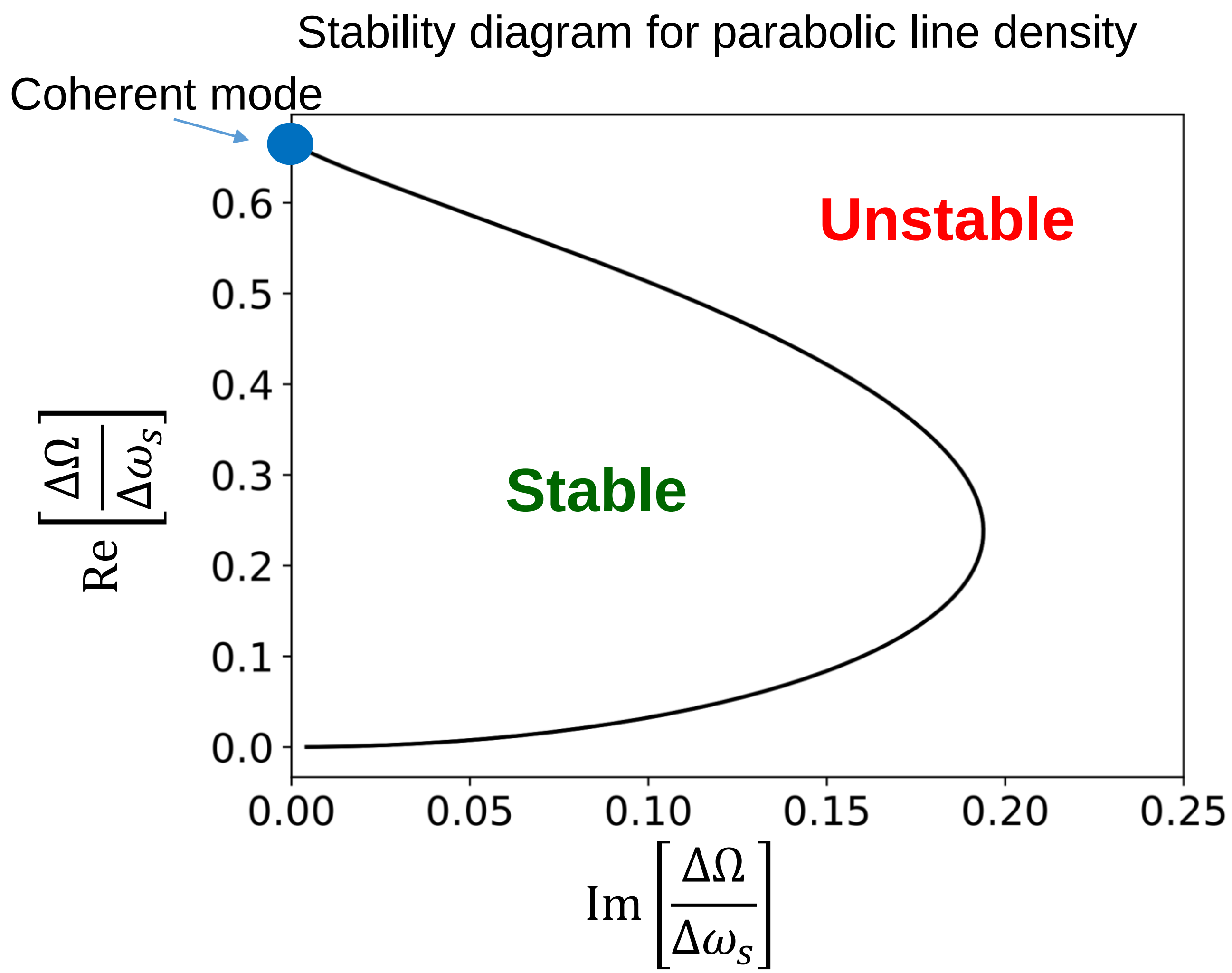
$$G_\mu(x) = \frac{x}{\mu(\mu+1)} \min_{y\in[0,1]} \left[\sum_{m=1}^{\infty} (1-y^2)^{\mu-1}\, J_m^2(\pi x y)\right]^{-1}$$

→ The FWHM bunch length is important



Threshold R_{sh} for coupled-bunch instabilities in FCC-hh at 50 TeV for nominal intensity N_b=10^{11}, V_{rf} = 38 MV, γ_t =99.3 *(I. K, E.Shaposhnikova, IPAC'19)*

Sacherer's formalism



Landau damping is lost if coherent mode shift $\Delta\Omega$ normalized by incoherent spread $\Delta\omega_s$ lies outside of stability diagram (F. Sacherer, 1971)

Simplified threshold $N_b = V_{\mathrm{rf}}\tau^5/\xi$

Stability parameter $\xi \propto (Z/n)_{\mathrm{eff}}$

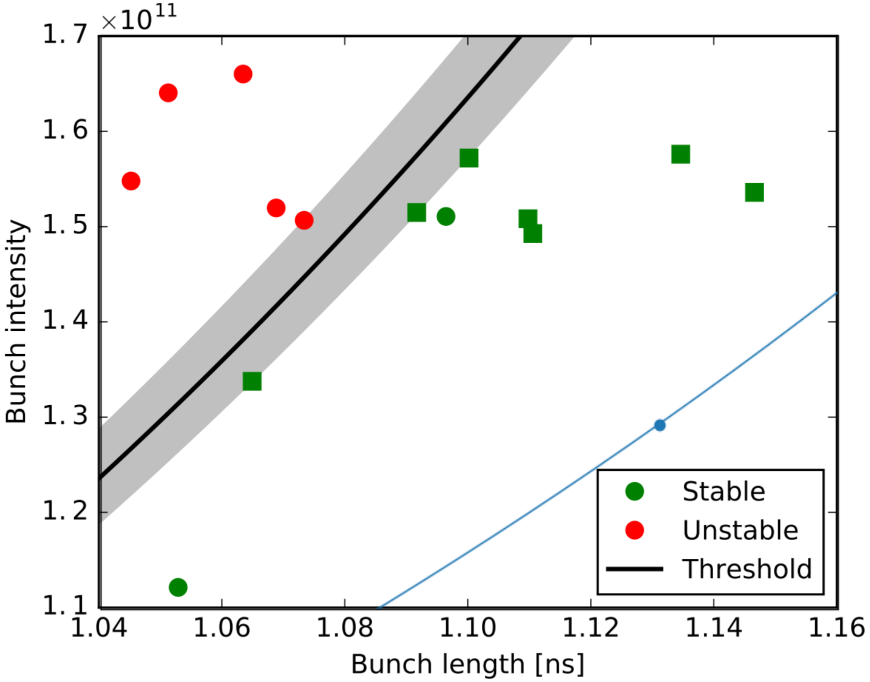
Effective impedance

$$(Z/n)_{\mathrm{eff}} = \omega_0 \frac{\sum_q \frac{Z^{\parallel}(\omega_q)}{\omega_q} \frac{J_{3/2}^2(\omega_q \tau)}{|\omega_q \tau|}}{\sum_q \frac{J_{3/2}^2(\omega_q \tau)}{|\omega_q \tau|}} \qquad \omega_q = q\omega_0 + \omega_s$$

→ For the case of the LHC impedance model $\xi \approx 1.4 \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$, ($(Z/n)_{\mathrm{eff}} \approx 0.09\ \Omega$)

Measurements of the loss of Landau damping threshold in LHC

MD 2011,
V_{rf} = 5 MV, E = 450 GeV



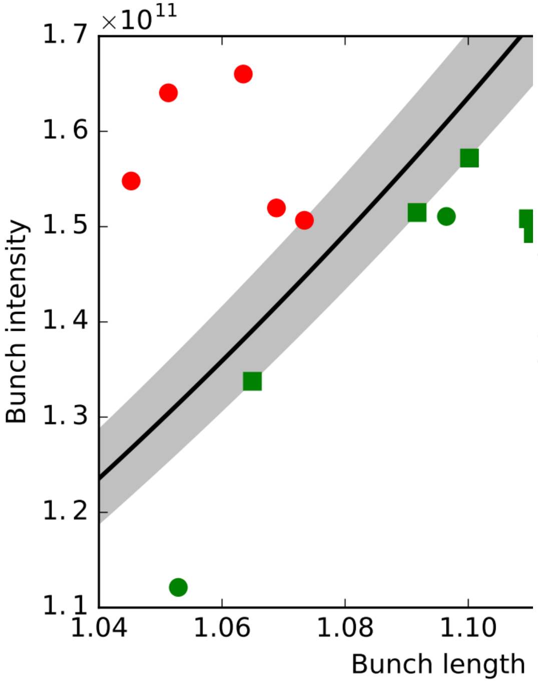
Measurement were performed at different conditions, but with all efforts only a limited parameter space was available during each of MDs (*PhD thesis J. E. Muller, 2016*). Threshold curves correspond to a fit $N_b = V\tau^5/\xi$.

→ As the result, $\xi \;=\; (5.0 \pm 0.5) \times 10^{-5}\;(\mathrm{ns})^5\mathrm{V}$ was obtained.

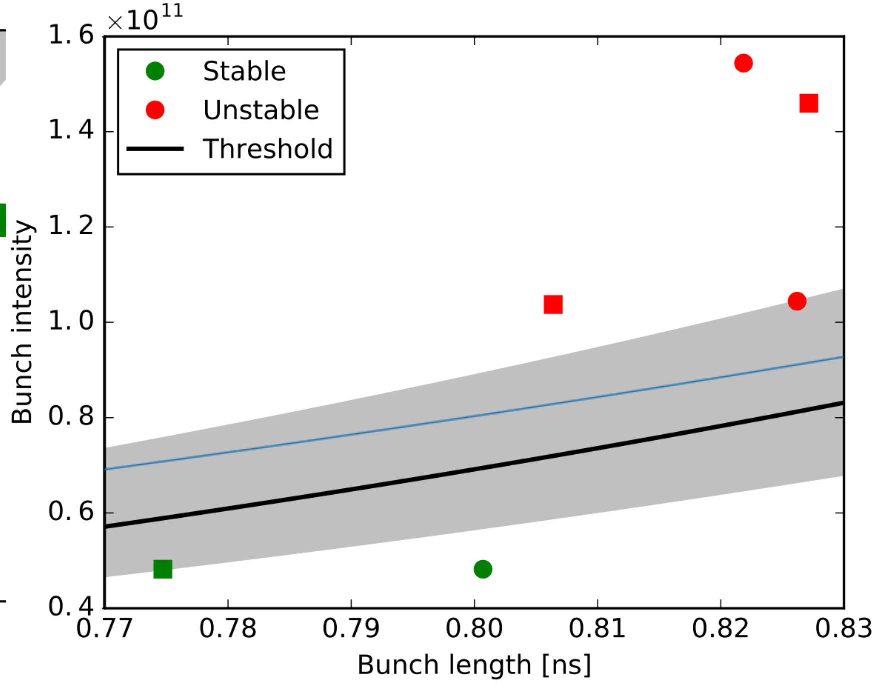
→ The thresholds predicted from Sacherer' stability diagrams are 3 – 4 times higher than measured thresholds.

Measurements of the loss of Landau damping threshold in LHC

MD 2011,
V_{rf} = 5 MV, E = 450 GeV



MD 2011,
V_{rf} = 12 MV, E = 4 TeV

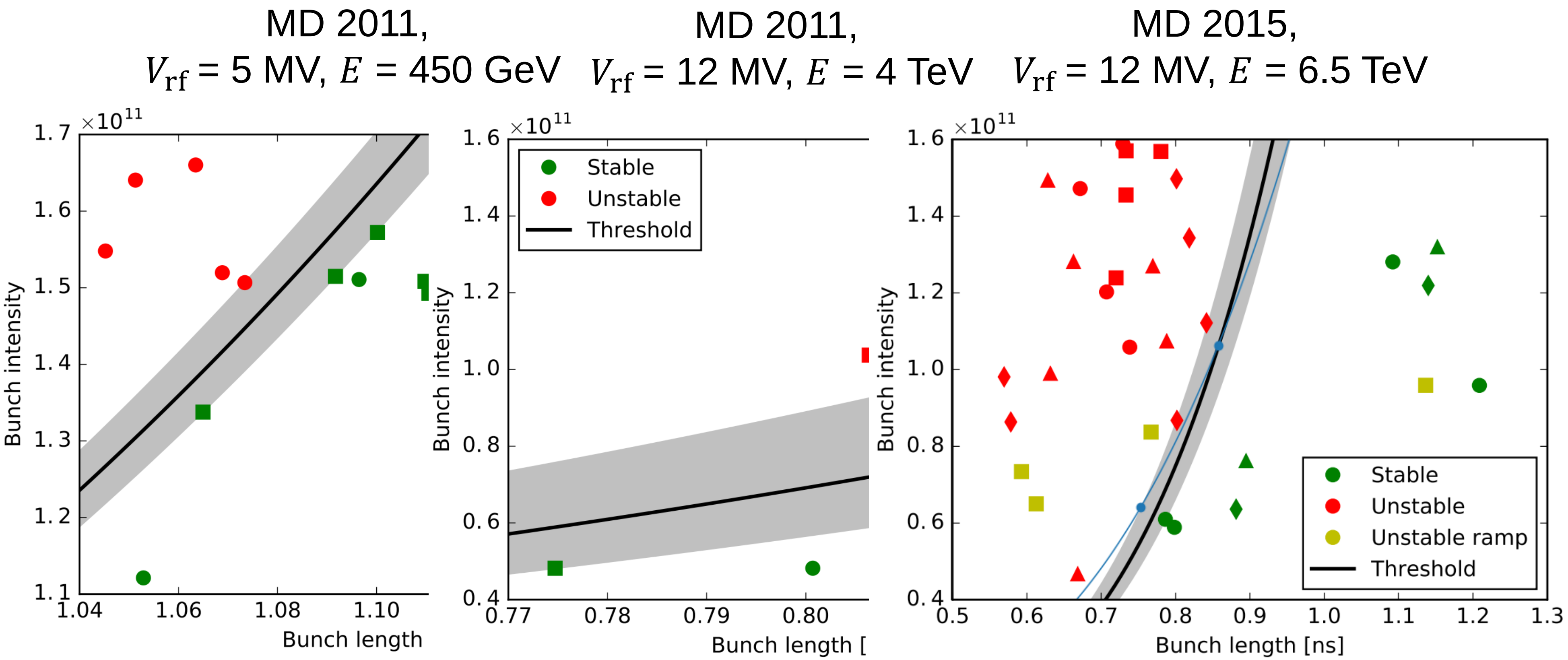


Measurement were performed at different conditions, but with all efforts only a limited parameter space was available during each of MDs (*PhD thesis J. E. Muller, 2016*). Threshold curves correspond to a fit $N_b = V\tau^5/\xi$.

→ As the result, $\xi \;=\; (5.0 \pm 0.5) \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$ was obtained.

→ The thresholds predicted from Sacherer' stability diagrams are 3 – 4 times higher than measured thresholds.

Measurements of the loss of Landau damping threshold in LHC



Measurement were performed at different conditions, but with all efforts only a limited parameter space was available during each of MDs (*PhD thesis J. E. Muller, 2016*). Threshold curves correspond to a fit $N_b = V\tau^5/\xi$.

→ As the result, $\xi = (5.0 \pm 0.5) \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$ was obtained.

→ The thresholds predicted from Sacherer' stability diagrams are 3 – 4 times higher than measured thresholds.

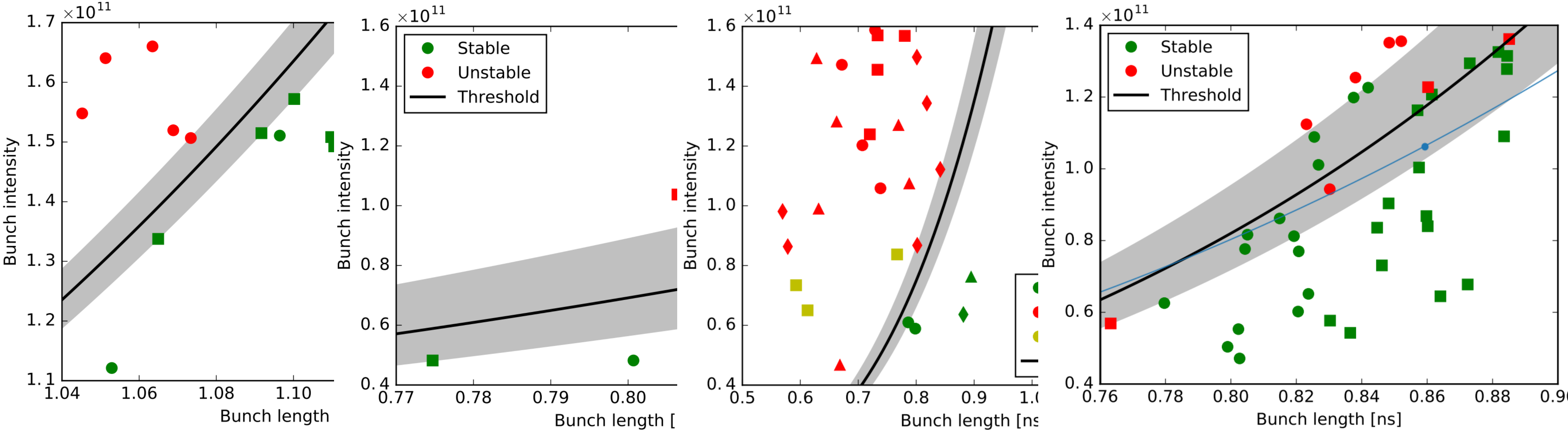
Measurements of the loss of Landau damping threshold in LHC

MD 2011, V_{rf} = 5 MV, E = 450 GeV

MD 2011, V_{rf} = 12 MV, E = 4 TeV

MD 2015, V_{rf} = 12 MV, E = 6.5 TeV

MD 2015, V_{rf} = 12 MV, E = 6.5 TeV



Measurement were performed at different conditions, but with all efforts only a limited parameter space was available during each of MDs (*PhD thesis J. E. Muller, 2016*). Threshold curves correspond to a fit $N_b = V\tau^5/\xi$.

→ As the result, $\xi = (5.0 \pm 0.5) \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$ was obtained.

→ The thresholds predicted from Sacherer' stability diagrams are 3 – 4 times higher than measured thresholds.

Comparisons with simulations

Simulation setup (*PhD thesis J. E. Muller, 2016*):

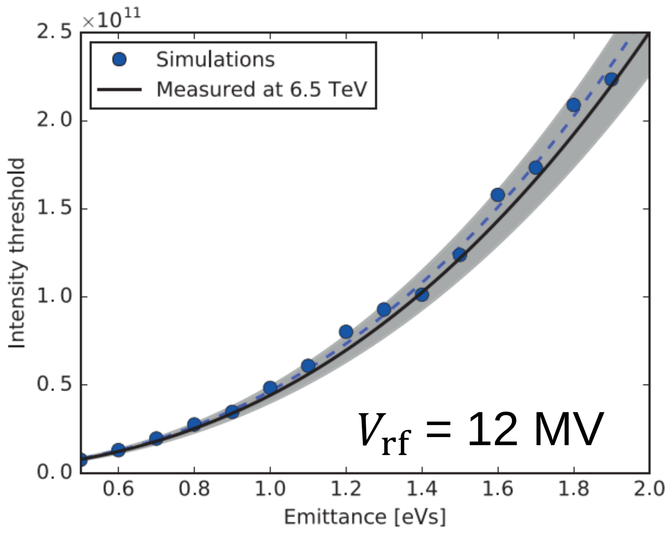
- Number of macro-particles 5×10^5
- 50 slices per bucket (f_c = 10 GHz) for induced voltage calculation using full impedance model
- Initially matched bunched is kicked by 1 degree

Stability criterion:
Bunch is stable if oscillation amplitude is reduced below 0.2 degrees

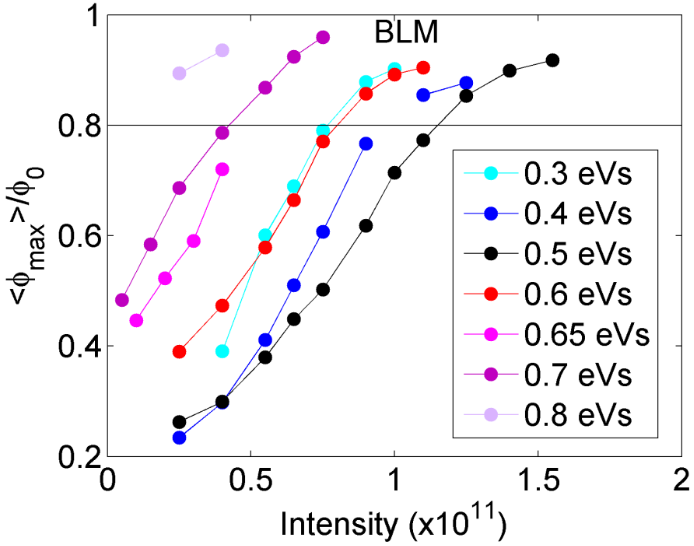
BLonD simulations using full LHC impedance model agree very well with measurements

However, the stability threshold can significantly vary depending on the chosen criteria based on the final oscillation amplitude (PhD thesis T. Argyropoulos, 2015)

→ Different method to find absolute threshold is needed



Simulations for double rf system**



Lebedev' approach

Matrix equation (A. N. Lebedev, 1967) in $(\mathcal{E}, \psi)$ variables with $\mathcal{E} = \dot{\phi}^2/(2\omega_{s0}^2) + U(\phi)/(V \cos\phi_s)$

$$\boxed{j_p(\Omega) = -\frac{iI_0 h}{V\cos\phi_s} \sum_{k=-\infty}^{\infty} G_{pk}(\Omega) \frac{Z_k(\Omega)}{k} j_k(\Omega)}$$

where $$G_{pk}(\Omega) = \frac{1}{A_N} \sum_{m=-\infty}^{\infty} \int_0^\infty F'(\mathcal{E}) \frac{I_{mp}(\mathcal{E})\, I_{mk}^*(\mathcal{E})}{\Omega/m - \omega_s(\mathcal{E})} d\mathcal{E}$$

where $$I_{mk}(\mathcal{E}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{k\phi(\mathcal{E},\psi)/h - im\psi}\, d\psi$$ and normalization $$A_N = \int_0^\infty \frac{F(\mathcal{E})}{\omega_s(\mathcal{E})} d\mathcal{E}$$

It allows to evaluate both single- and multi-bunch stability
Was extensively used to evaluate coupled-bunch instability thresholds due to narrow-band impedance *(V. Balbekov, S. Ivanov, 1987)* and for combination of narrow-band resonator and Im*Z/n* = const (*M. Blaskiewicz, 2009)*

→ For single-bunch case it was considered to be not numerically tractable

Landau damping: Van-Kampen modes

Method (A. Burov, 2010): find Van-Kampen modes solving Vlasov equation for perturbation *f(J,ψ,t)* of stationary distribution function *F(J)* expanded (Oide & Yokoya, 1990) as

$$f(J,\psi,t) = \mathrm{e}^{-i\Omega t} \sum \left[f_m(J) \cos m\psi + g_m(J) \sin m\psi \right]$$

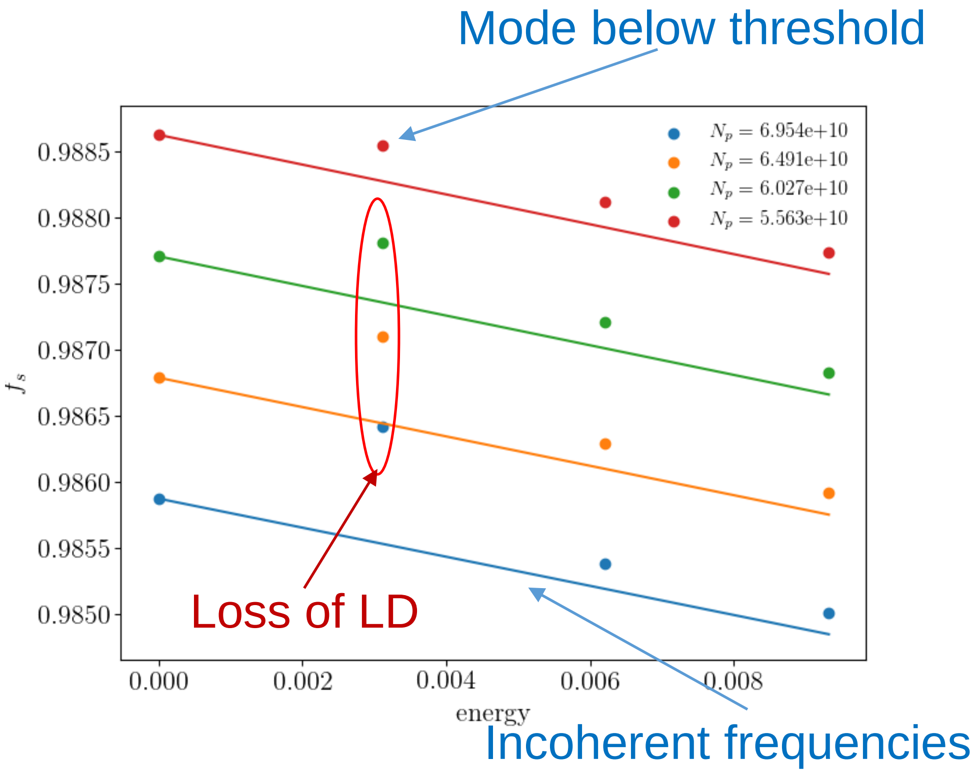
Without mode coupling the matrix equation (in action J) is

$$(\Omega^2 - m^2\omega_s^2) f_m(J) = m^2 \omega_s(J) F'(J) \int V_m(J,J') f_m(J') dJ'$$

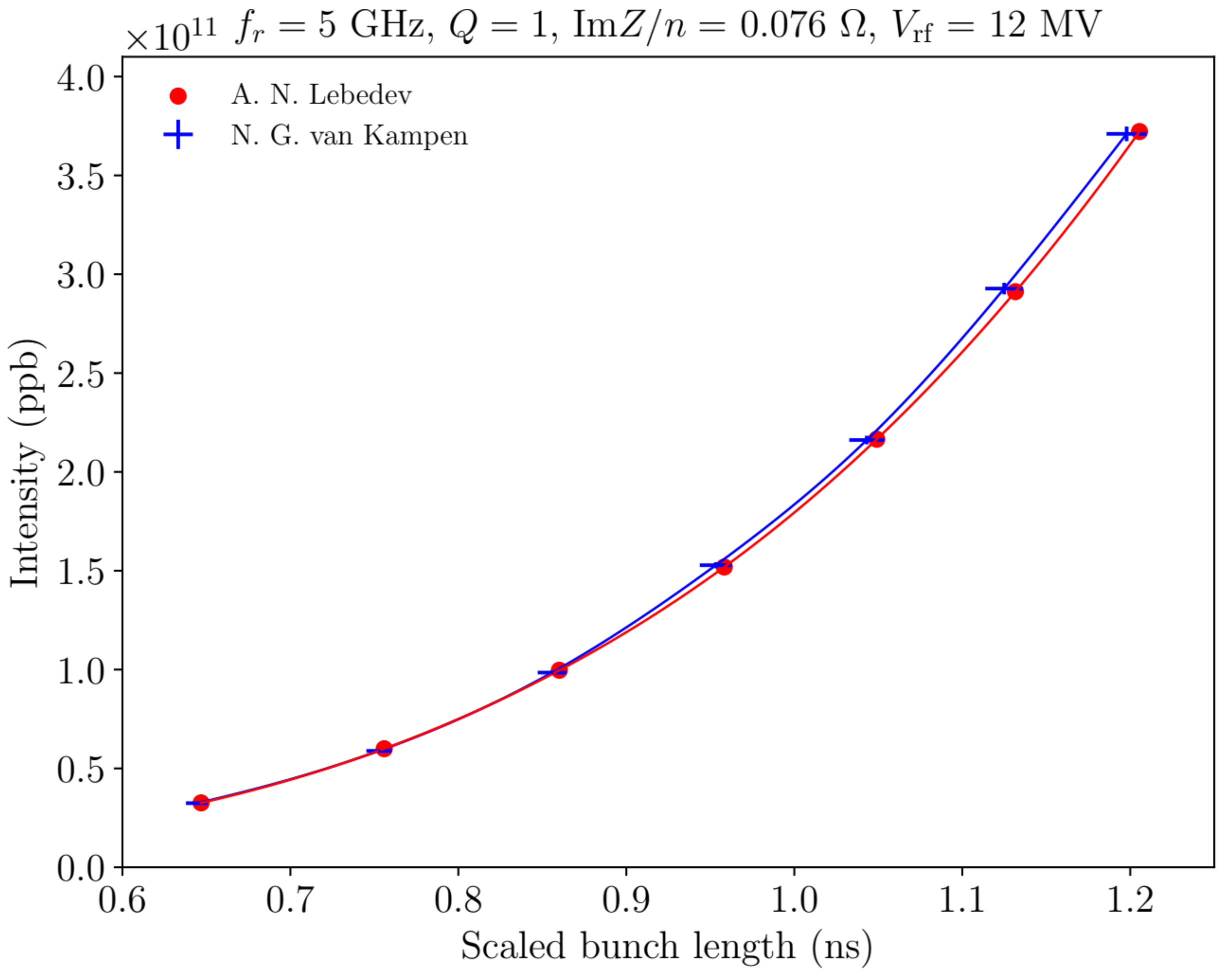
where $$V_m(J,J') = 2\,\mathrm{Im} \sum_k \frac{Z_k}{k} I_{mk}(J) I^*_{mk}(J')$$

and $$I_{mk}(J) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{k\phi(J,\psi)/h - im\psi}\, d\psi$$

Continuous spectrum - singular modes from incoherent band. Discrete modes - coherent solutions described by regular eigenfunctions → their existence outside incoherent band serve as criterion for loss of LD



Van-Kampen vs Lebedev approaches



A new numerical code was developed to solve Lebedev eigenvalue problem

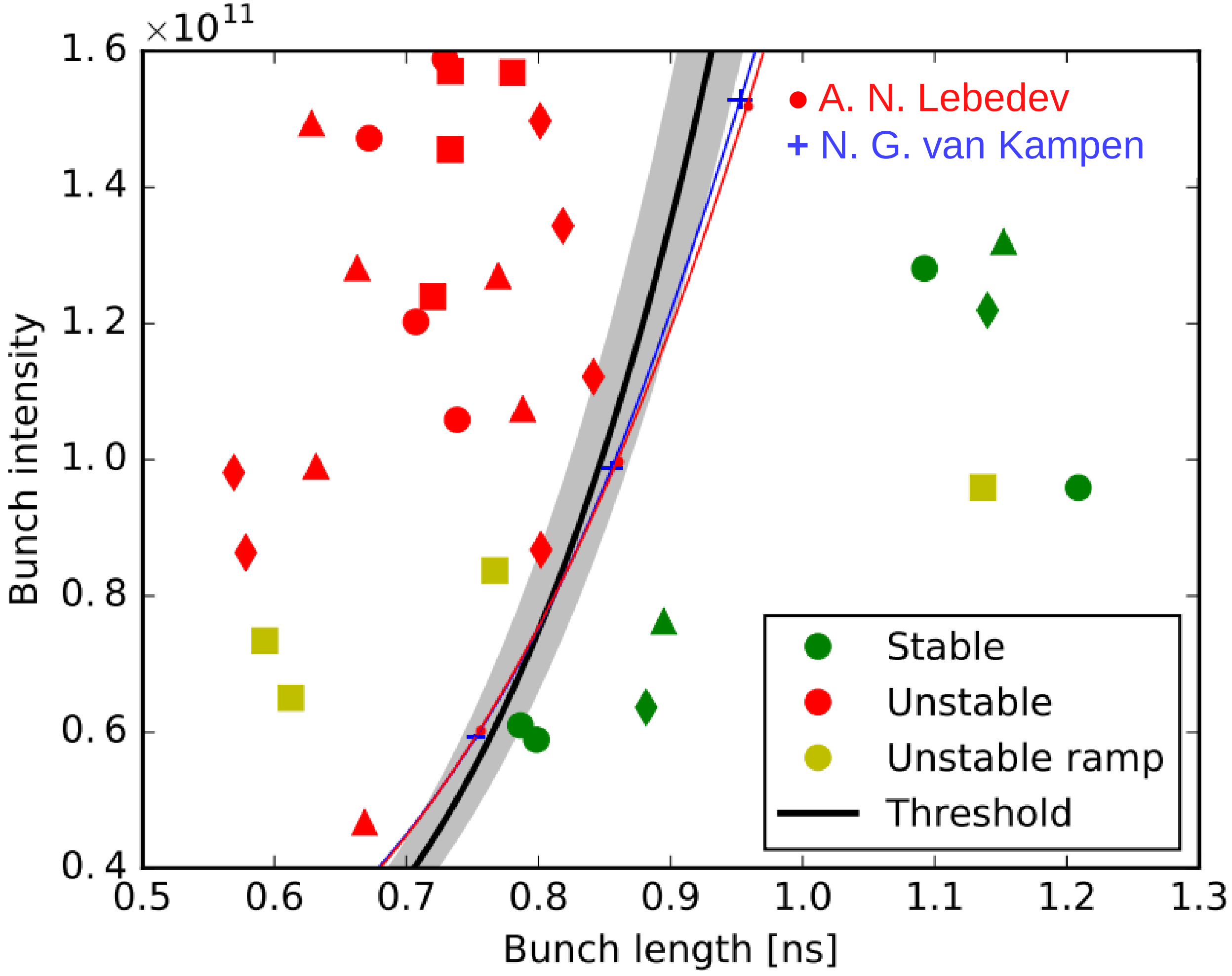
The thresholds are calculated using Lebedev and Van-Kampen and approaches for broadband impedance and following parameters:
$R = (\mathrm{Im}Z/n) f_r/f_0, f_r = 5\ \mathrm{GHz}, Q = 1,$
$f_c = 20\ \mathrm{GHz},\ \mathrm{Im}Z/n = 0.076, \mu = 2,$
$V_{\mathrm{rf}} = 12\ \mathrm{MV}$

Distribution function $F(\mathcal{E}) = F_0(1 - \mathcal{E}/\mathcal{E}_{max})^{\mu}$

→ Very good agreement between two approaches

Preliminary comparisons with measurements

Measurements at 6.5 TeV, $V_{\mathrm{rf}} = 12$ MV (*PhD thesis J. E. Muller, 2016*)



The thresholds are calculated using Lebedev and Van-Kampen and approaches for broadband impedance and following parameters:

$R = (\mathrm{Im}Z/n) f_r/f_0, f_r = 5\ \mathrm{GHz}, Q = 1,$

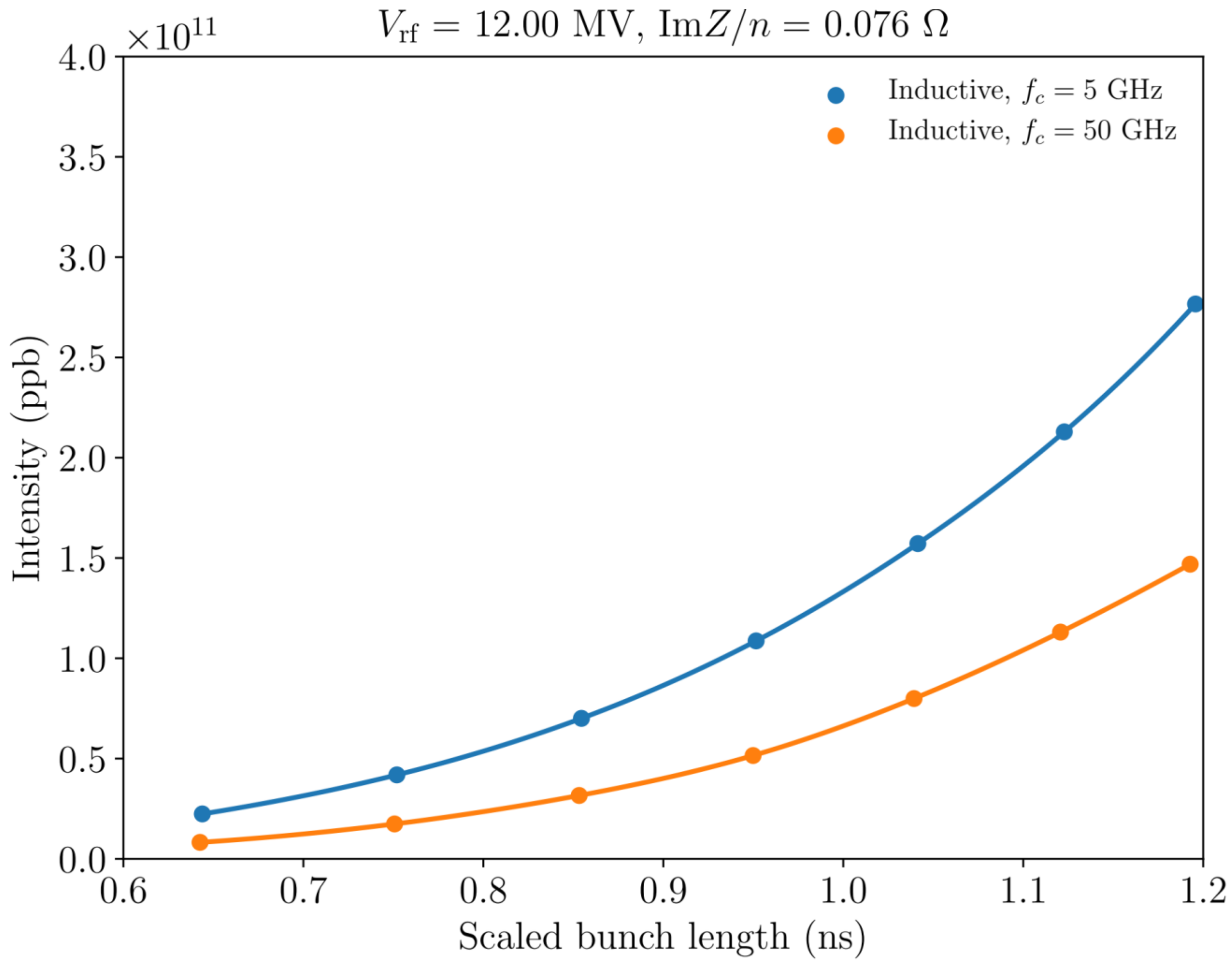
$f_c = 20\ \mathrm{GHz},\ \mathrm{Im}Z/n = 0.076, \mu = 2,$

$V_{\mathrm{rf}} = 12\ \mathrm{MV}$

→ Reasonable agreement between measurements and semi-analytic calculations.

→ Scaling law is different in comparison to simplified Sacherer's approach

Calculation results



Case of inductive impedance $\mathrm{Im}Z/n = \mathrm{const}$.

Threshold significantly depends on the cut-off frequency f_c

For the case of symmetric potential well

$$I_{mk}(\mathcal{J}) \approx i^m J_m\left(\frac{k}{h}\sqrt{2\mathcal{J}}\right)$$

So diagonal elements of the matrix diverge

$$V_m(\mathcal{J},\mathcal{J}) = 2\mathrm{Im}Z/n \sum_k J_m^2\left(\frac{k}{h}\sqrt{2\mathcal{J}}\right) \to \infty$$

$$V_m(\mathcal{J},\mathcal{J}) = 2\mathrm{Im}Z/n \sum_k J_m^2\left(\frac{k}{h}\sqrt{2\mathcal{J}}\right) \to \infty$$

→ Realistic impedance model needs to be used