The Effect of Fluctuating Fuzzy Axion Haloes on Stellar Dynamics

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Subject matter:

For typical galactic speeds FDM has

De Broglie wavelength $\frac{h}{m\,v}$ $\sim$ 100 pc or more $\rightarrow$ $m \sim 10^{-22}\ \mathrm{eV}$

Outline:

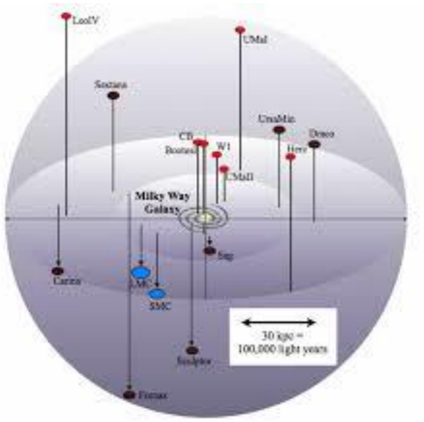
- Why Ultra light axions? (from a galactic perspective)?
- Origin and characterization of FDM fluctuations
- Effect on stellar dynamics and associated constraints

Galactic Scale Problems with CDM

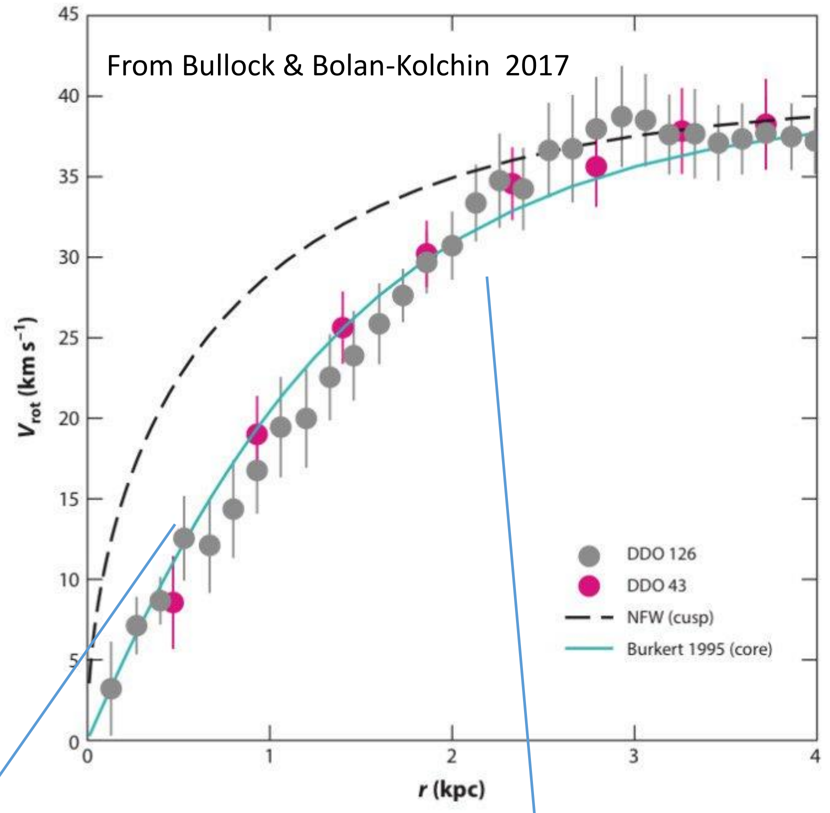
CDM compensates for mass deficit in outer parts

BUT contributes too much mass to central parts

****Probably related problems: Excess of small haloes and wrong dynamics**

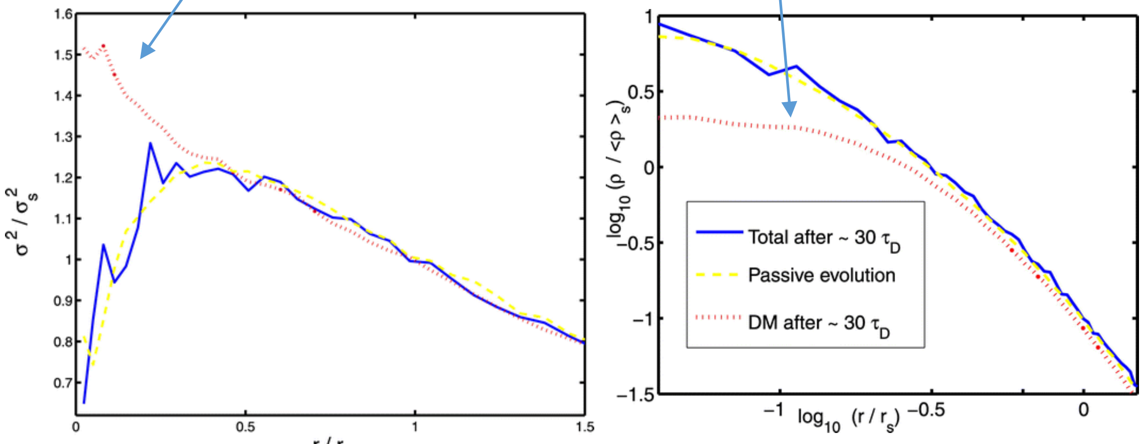


Simulation M.Y. size halo .vs. Dwarf galx. pop.



- **Need smaller density**

  + more random motion in centre of halo:



Heating of central cusp by DF; El-Zant (2008)

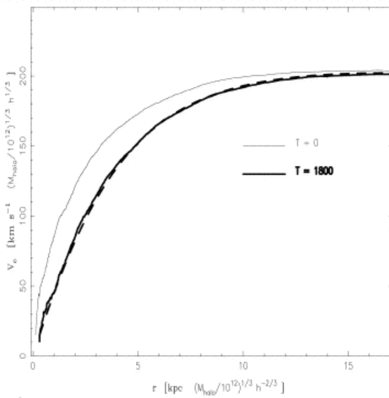
Some Proposed solutions (heating the CDM)

Pump energy → decrease DM density:

** Warm DM (smaller mass) → preheat!

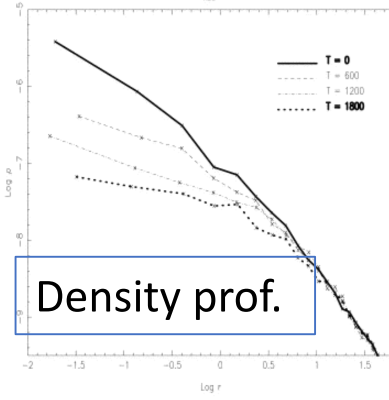
** **Self interacting DM** → Conduction

Rotation Curve



** **Baryonic solutions:** baryons give off energy to DM →

(e.g., El-Zant et. al 2001;2004; Pontzen & Governato 2014; El-Zant et. al. 2016)



** **Quantum) fluctuations** **(FDM)**

e.g., Hu et al. (2000), Peebles (2000), Hui et. al. (2017)

Ultralight Axion "Fuzzy DM"

Tiny Mass ~ →Astrophysical de Broglie wavelength

$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.92\,\mathrm{kpc}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\frac{10\,\mathrm{km\ s}^{-1}}{v}\right)$$

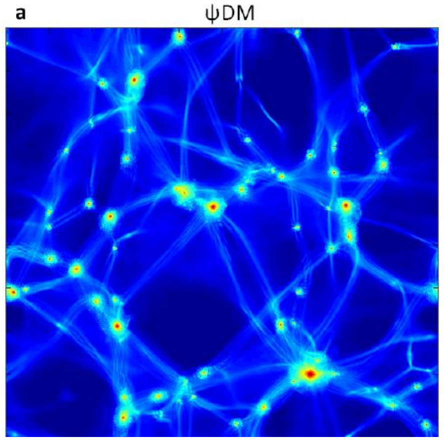
- Large number of particles in same state and non-relativistic **on galactic scales**

→ Schrodinger-Poisson system

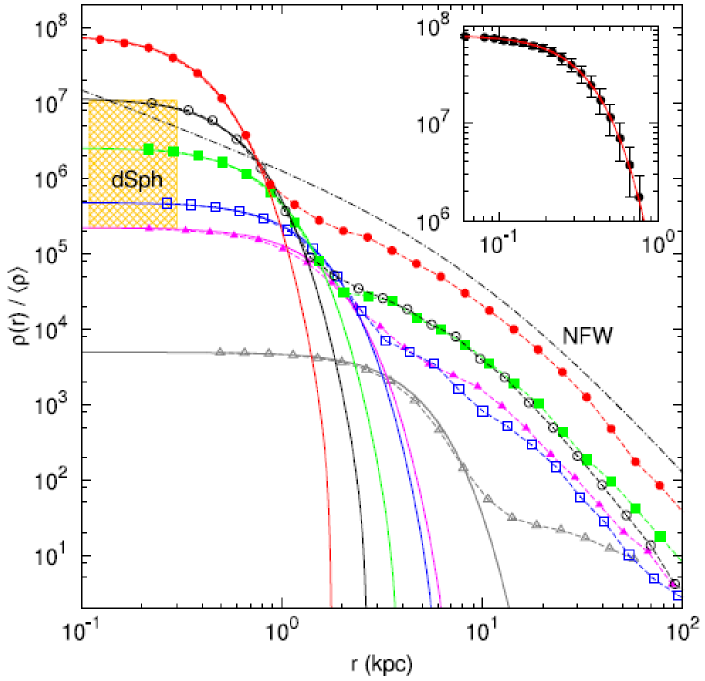
$$i\hbar\frac{d\Psi}{dt} = -\frac{\hbar^2}{2m_\chi}\nabla^2\Psi + m_\chi V\Psi,$$

$$\nabla^2 V = 4\pi G m_\chi |\Psi|^2.$$

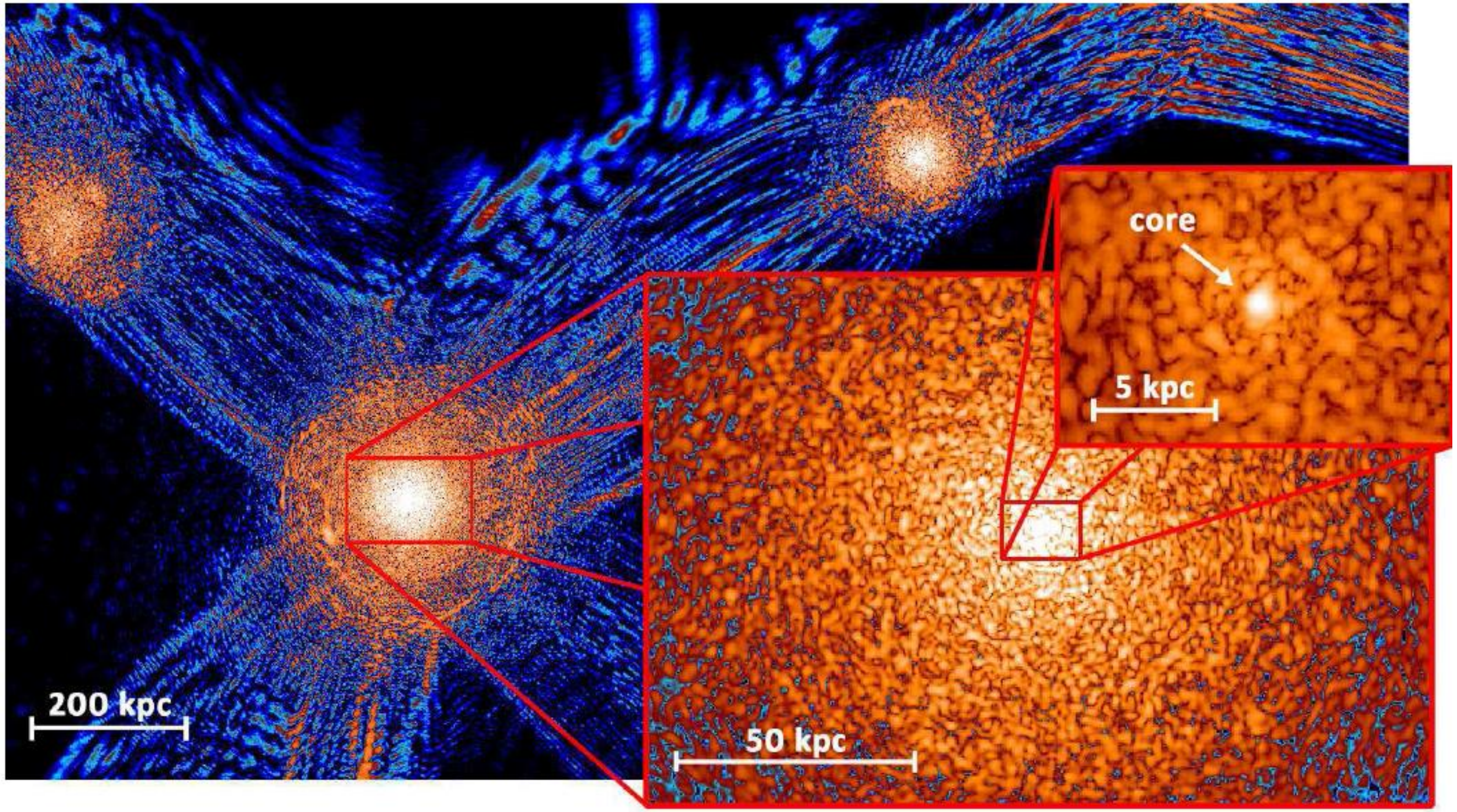
Structure Formation and fluctuations with fuzzy DM (Schive et. al. 2014)



~as CDM on large scales



~Constant density cores



Few smaller halos (instead interference pattern and fluctuations!)

Axion Fluctuations as Random Gaussian Field

Expand *fluctuations* in modes ρ_k moving at phase velocity v such that $\mathbf{k}.\mathbf{v} = \omega$

This is the case if

$$\phi_{\mathbf{k}}(t) = \phi_{\mathbf{k}}(0) e^{-i\mathbf{k}.\mathbf{v}t} \quad \text{and} \quad \psi(\mathbf{r}, t) = \int \phi_{\mathbf{k}}(t) e^{i\mathbf{k}.\mathbf{r}} \, d\mathbf{k}$$

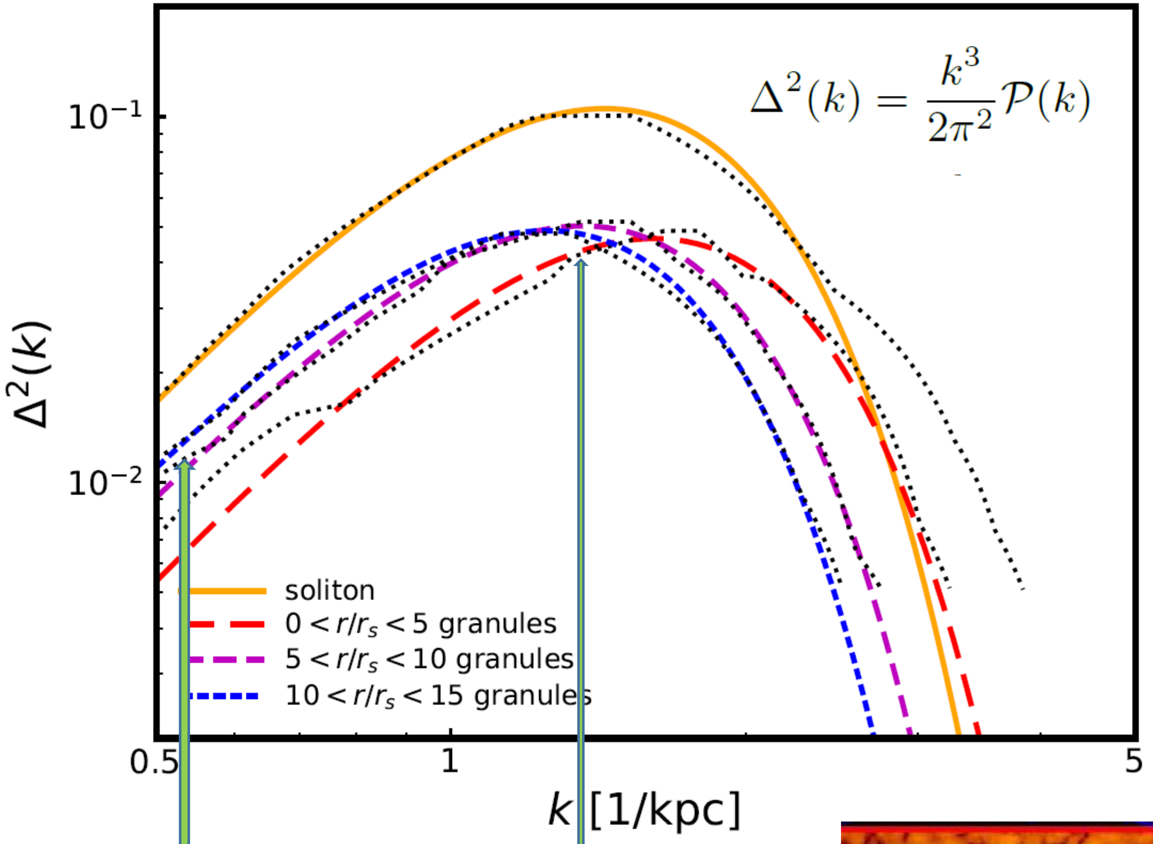
Wave function power spectrum → k-space density → $\langle \phi_{\mathbf{k}} \phi^{\star}_{\mathbf{k}'} \rangle = f_{\mathbf{k}}(\mathbf{k}) \delta_{\mathrm{D}}(\mathbf{k} - \mathbf{k}')$

Power spectrum of density fluctuations interference pattern →

$$\mathcal{P}(\mathbf{k}, t) = \frac{(2\pi)^3}{\rho_0^2} \times \iint f_{\mathbf{k}}(\mathbf{k}_1) f_{\mathbf{k}}(\mathbf{k}_2) e^{-i[\omega(\mathbf{k}_1) - \omega(\mathbf{k}_2)]t} \delta_{\mathrm{D}}(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2) d\mathbf{k}_1 d\mathbf{k}_2$$

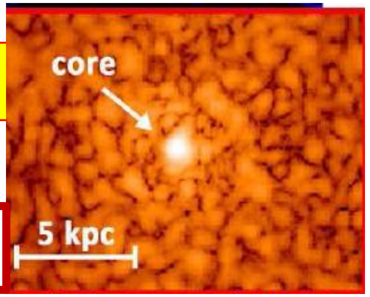
Power Spectrum of Density Fluctuations

Interpretation and Comparison with simulations (of Chan et. al. 2018)



White noise → **~ Randomly scattered masses ~ $m_{\rm eff}$**

Effective fluctuation scale



Dispersion relations $\omega = \frac{\hbar k^2}{2m}$

→ **Group velys of de Broglie wave packets**

→ **Correspondence of wavenumber and FDM vely distn function**

$$f_{\mathbf{k}}(\mathbf{k})d\mathbf{k} = f(\mathbf{v})d\mathbf{v}$$

Maxwellian velys

$$f(v) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} e^{-\frac{v^2}{2\sigma^2}}$$

→ Power spectrum

$$\mathcal{P}(\mathbf{k}, 0) = \left(\frac{2\sqrt{\pi}}{m_\hbar \sigma}\right)^3 e^{-\frac{k^2}{\sigma^2 m_\hbar^2}}$$

$m_\hbar = 2m/\hbar$

From Density to Force fluctuations

- Use Poisson equation

$$\nabla^2\Phi = 4\pi G\rho_0\delta$$

- Homogeneous process →

$$\phi_{\mathbf{k}} = -4\pi G\rho_0\delta_{\mathbf{k}}k^{-2}$$

- Force fluctuation power →

$$\mathcal{P}_F(k) = Vk^2\langle|\phi_k|^2\rangle$$

Fourier Transform → Force Correlation Function

$$\langle \mathbf{F}(0,0).\mathbf{F}(r,t)\rangle = \frac{1}{(2\pi)^3}\int \mathcal{P}_F(k,t)e^{i\mathbf{k}.\mathbf{r}}d\mathbf{k}$$

Stochastic equation → Random velocity from fluctuations

$$d\mathbf{v}/dt = \mathbf{F} \longrightarrow \langle(\Delta v_p)^2\rangle = 2\int_0^T (T-t)\langle \mathbf{F}(0).\mathbf{F}(t)\rangle dt$$

Maxwellian →

$$\langle(\Delta v_p)^2\rangle = T\frac{8\pi G^2\rho_0 m_{\rm eff}\ln\Lambda}{v_p}{\rm erf}(X_{\rm eff}) \quad m_{\rm eff} = \frac{8\pi^{3/2}\rho_0}{m_\hbar^3\sigma^3}$$

Observable Effect: Galactic Disk Velocity Dispersion

-- **Decompose energy input to disk via fluctuations into vertical and radial components**

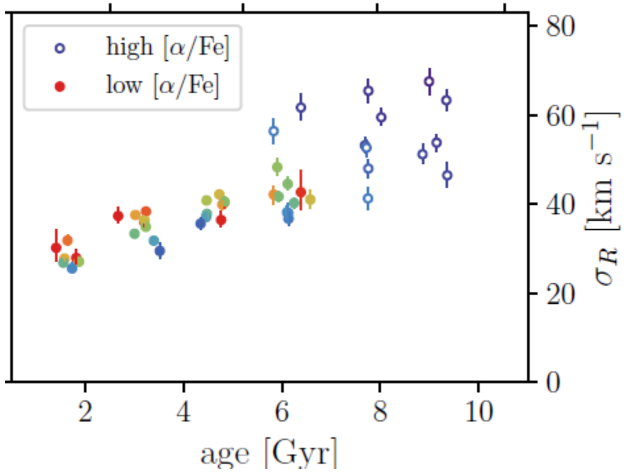
-- **Assume Virial equilibrium**

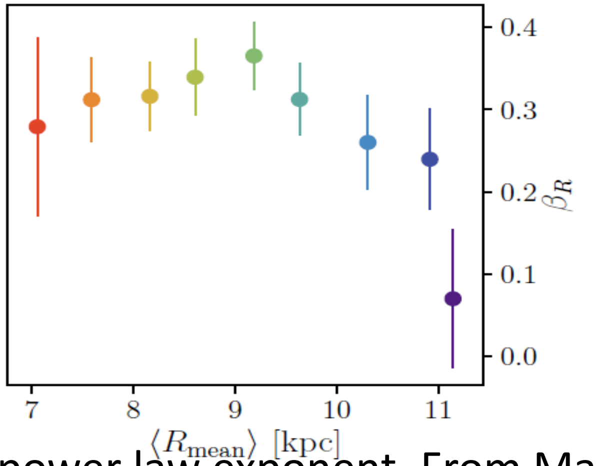
→ **Prediction: radial velocity dispersion of disk stars increases as**

$$\sigma_R = 4.5\ \mathrm{km/s}\left(\frac{10^{-22}\mathrm{eV}}{m}\right)^{3/2}\left(\frac{8\mathrm{kpc}}{r}\right)^{2}\left(\frac{T}{10\mathrm{Gyr}}\right)^{1/2}\ln\Lambda^{1/2}$$

Observed dispersion does increase BUT as $\sigma_R \sim t^{1/3}$ → **Axion fluctuation contribution**

$$\sigma_R \sim 3\mathrm{km/s}$$



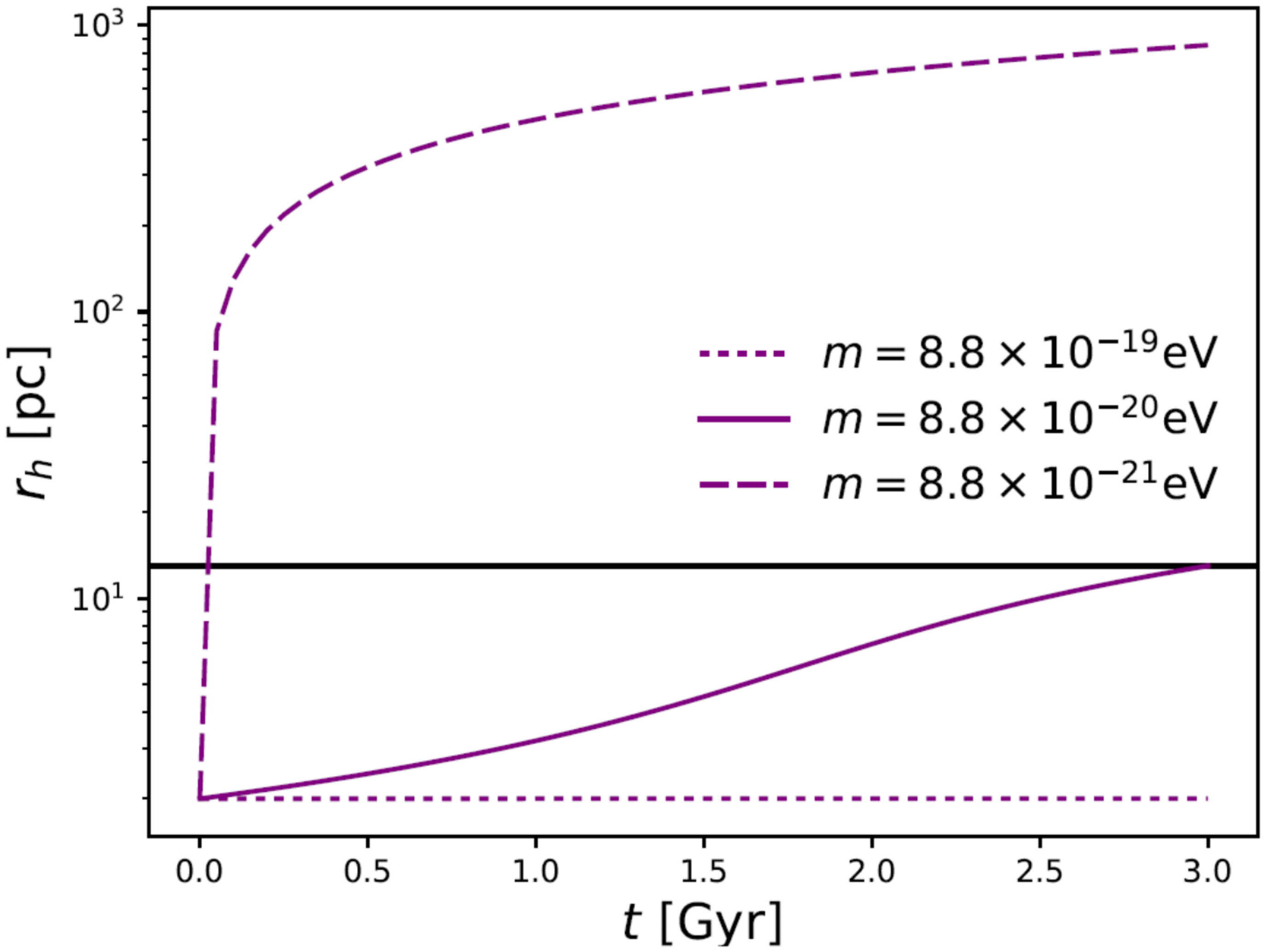


→ **Limit on axion mass**

$$m \gtrsim 2\times 10^{-22}\mathrm{eV}$$

Observed radial dispersion increase and power law exponent. From Mackareth et. al. (2019)

Expansion of the Central Cluster of Dwarf Eridanus II



Cluster expansion in context of El-Zant et. al. (2019) model

Basic idea (Marsh & Niemeyer 2019):

-- Fluctuations cause central cluster to expand

-- If axion mass too small → cluster too large

→ quite severe constraints on axion mass

BUT does cluster expand or gets displaced?

$$\frac{dr_h}{dt} = \frac{D}{G}\left(\frac{\alpha M_\star}{r_h^2} + 2\beta\rho_0 r_h\right)^{-1}$$

$$D\left[(\Delta v)^2\right] = \frac{4\sqrt{2}\pi G^2 \rho_0 m_{\rm eff}}{\sigma_{\rm eff}} \ln\Lambda \left[\frac{\mathrm{erf}(X_{\rm eff})}{X_{\rm eff}}\right]$$

Conclusions and Prospects

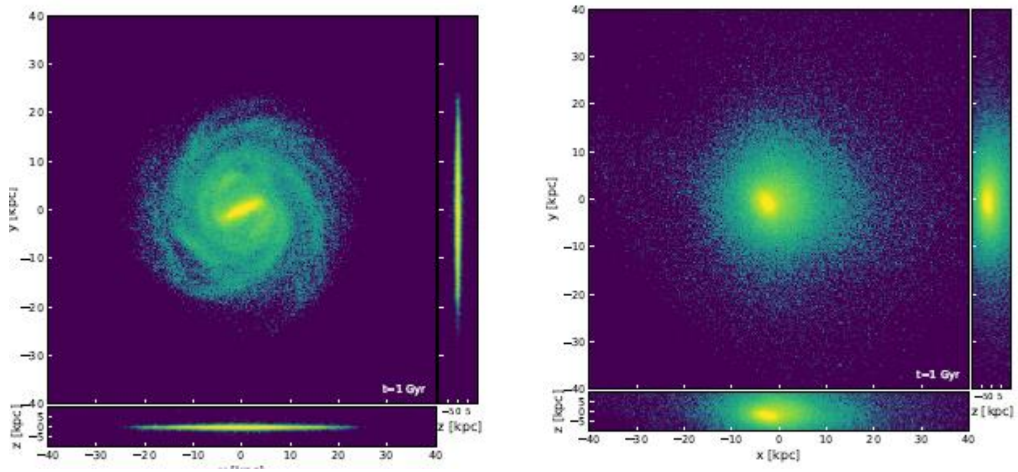
- **Galactic scale problems are part of a parcel that threatens CDM**
- **Core in CDM haloes can be produced by stochastic gas fluctuation**
- **Fluctuations from uncertainty principle can play roughly similar role**
- **But are fluctuations needed to solve core-cusp problem etc, too large?**

Ongoing and prospective work:

- **Simulations of disks with added noise**

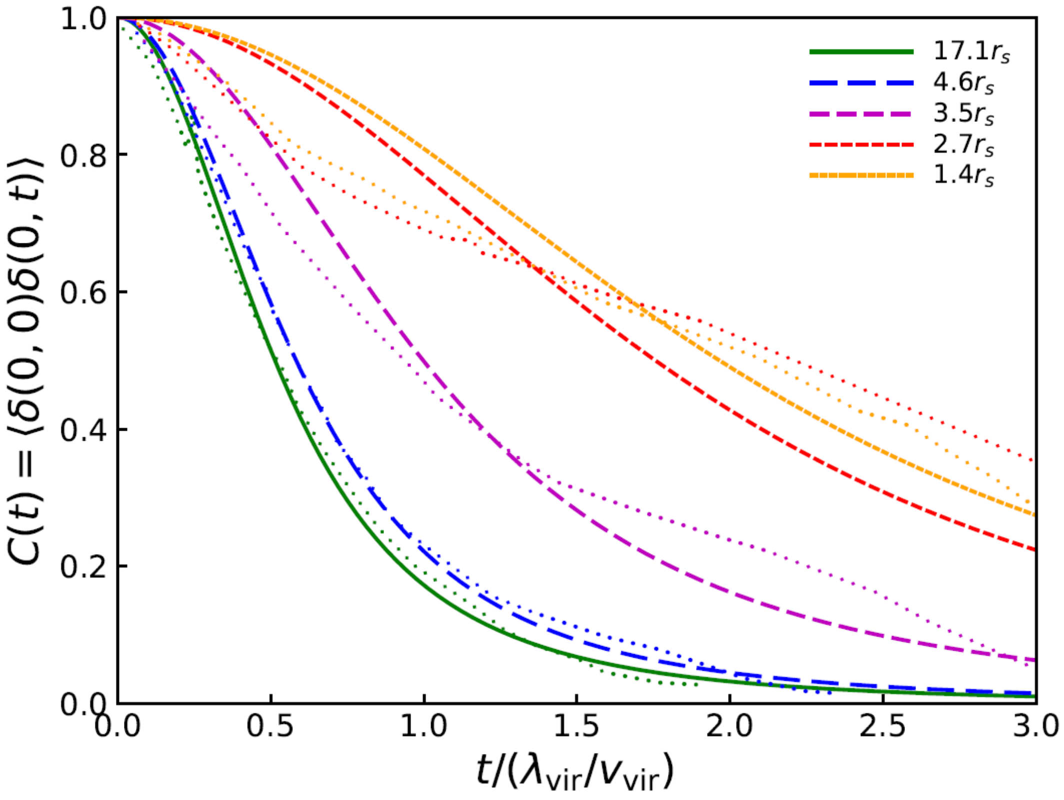
(expectation: self consistent response amplifies effect of fluctuations)

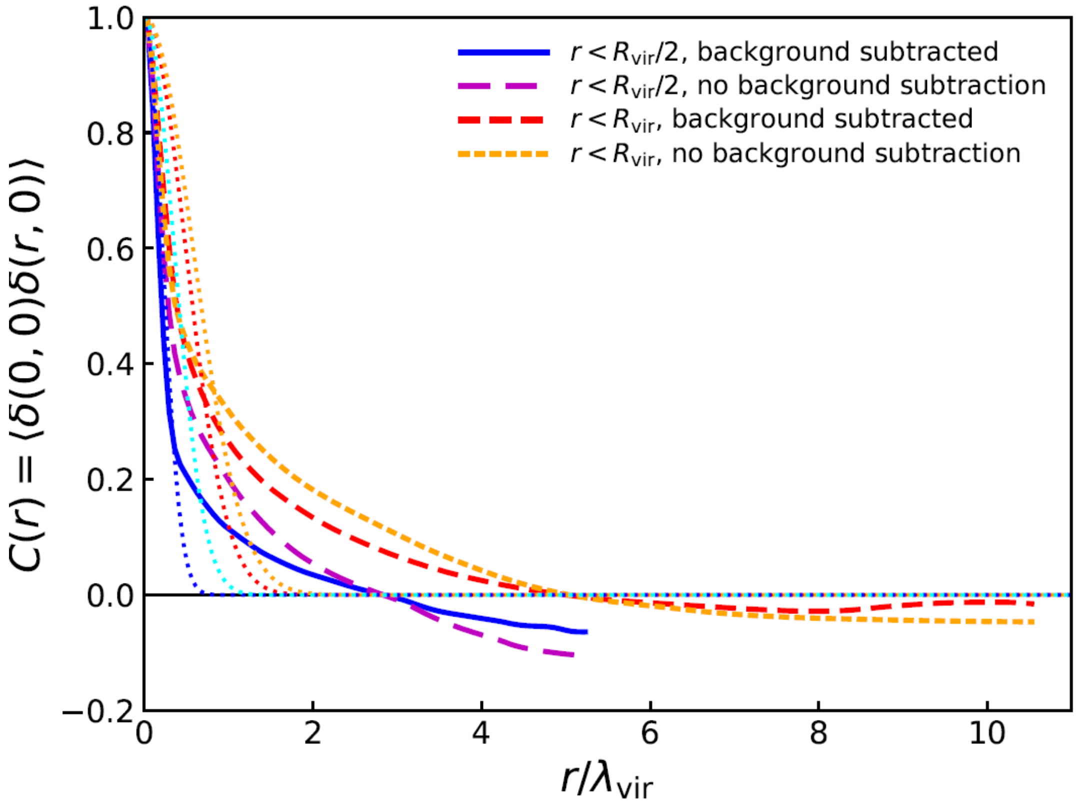
- **Effect on central BH and tidal stream**
- **FDM self interaction…**



Disk heating simulations (preliminary results)

Space and Time Correlations



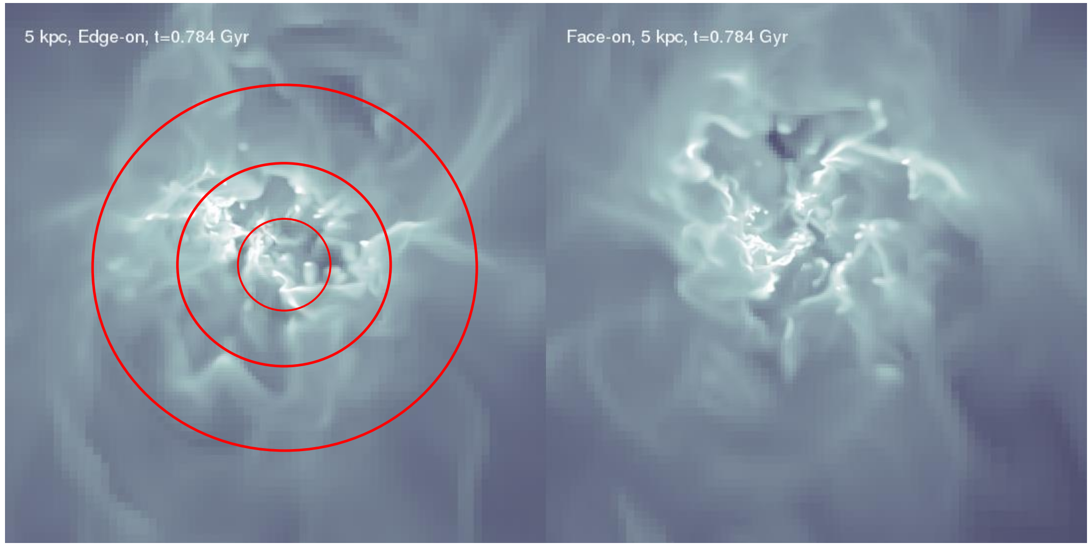


$$\langle\delta(0,0)\delta(r,t)\rangle = \frac{1}{(1+\sigma^2 t^2/\lambda_\sigma^2)^{3/2}} e^{-\frac{r^2/\lambda_\sigma^2}{1+\sigma^2 t^2/\lambda_\sigma^2}}$$

Characterising turbulent density fluctuations:

Within volume V fluctuations describe a **stationary Gaussian process**

Courtesy J. Read

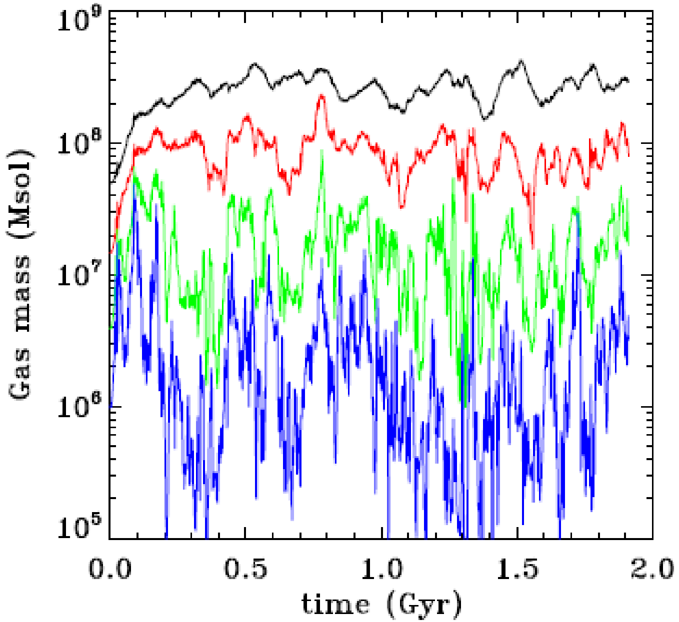


$$\delta(\mathbf{r}) = \frac{V}{(2\pi)^3}\int \delta_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}} d\mathbf{k}$$

$$\left(\delta\ (r) = \frac{\rho\ (r) - \langle\rho\rangle}{\rho}\right)$$

Power spectrum

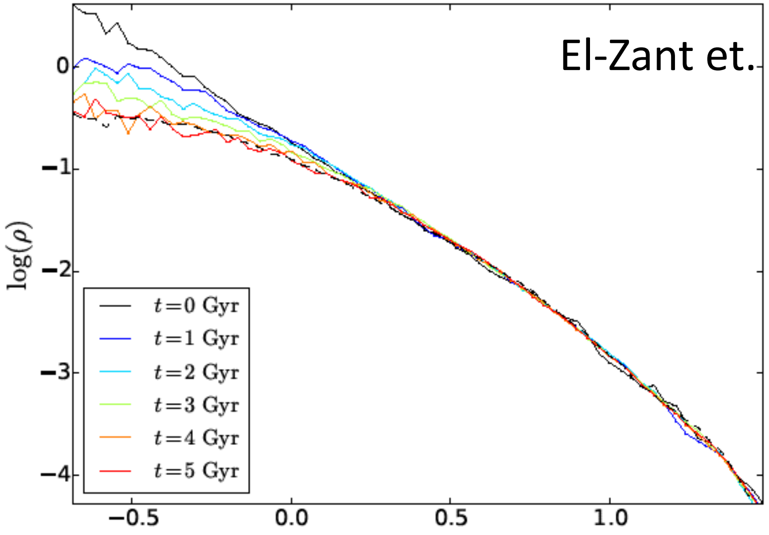
$$\mathcal{P}(\mathbf{k}) = V\langle|\delta_{\mathbf{k}}|^2\rangle$$



→ Density fluctuations

→ Flattening of cusp

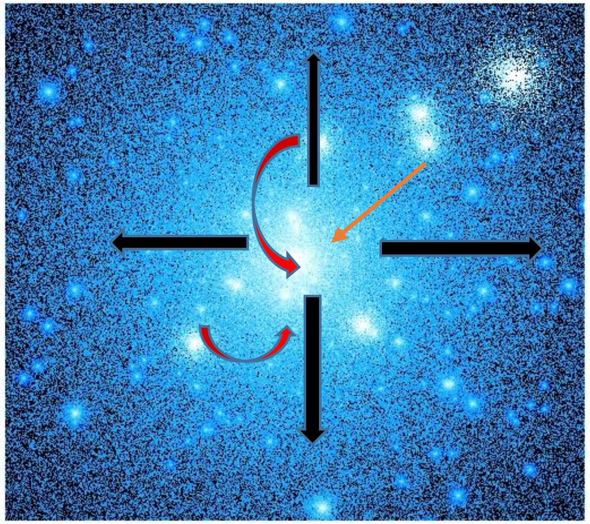
El-Zant et. al. 2016)



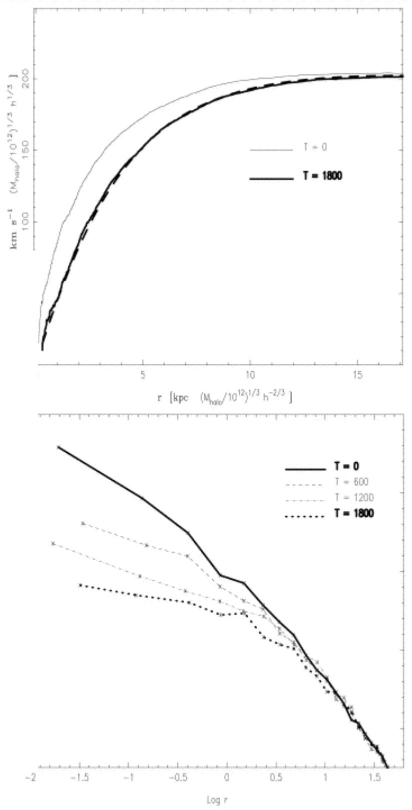
Fluctuating Baryons

Baryonic clumps couple to CDM via dynamical friction

→ Lose energy → Heat CDM



El-Zant et. al. (2006)



Questions:

Can clumps survive? Have enough energy?

→ **Alternative** (e.g., Ponzen & Governato 2014, Nat. 506, 171):

Clumps are not monolithic

→ They are density fluctuations

→ Energetically driven by supernovae/AGN