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# Emittance blow-up due to burn-off

## BB and Luminosity meeting, INDICO 806869

G. Sterbini with valuable inputs from R. Bruce, I. Efthymiopoulous, J. Jowett, Y. Papaphilippou, G. Trad... In particular refer to the work in [1, 2, 3, 4].
https://www.overleaf.com/read/ycmrpyptwwjd

# Introduction

We will discuss about the impact of the burn-off to the LHC emittance blow-up.

Due to the inelastic collisions, the depletion of the bunch transverse core is faster than the one of the tail, resulting in emittance blow-up.

In the following we will use a semi-analytical perturbative approach based on [1, 2].

# The analytical approach (I)

The key point is to express the probability of collision, $P_1$, of the B1 particle with action $J_x$ and $J_y$. This was analytically done in [1] for **round Gaussian beams**:

$$P_1 = K \underbrace{e^{-\frac{J_x}{2\epsilon_{2x}}} I_0\left(-\frac{J_x}{2\epsilon_{2x}}\right)}_{f(J_x)\ factor} \times \underbrace{e^{-\frac{J_y}{2\epsilon_{2y}}} I_0\left(-\frac{J_y}{2\epsilon_{2y}}\right)}_{f(J_y)\ factor} \tag{1}$$

# The analytical approach (II)

with

$$K = \sigma \frac{N_2 \ R_{tot}}{2\pi \beta^*_{xy} \sqrt{\epsilon_{2x} \epsilon_{2y}}}$$

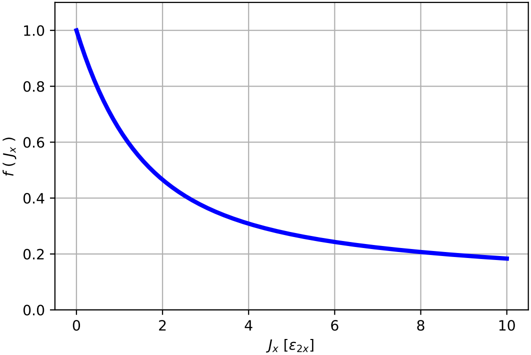
- $\epsilon_{2x,y}$ refers to the geometrical B2 emittance.
- $I_0$ is the modified Bessel function of first type with $\alpha = 0$ [5].
- $\sigma$ is the interaction cross section. For pp inelastic cross-section at LHC collision energy we consider $\sigma = 80$ mbarn.
- $N_2$ is the number of particle in the B2 bunch.

# The analytical approach (III)

- $R_{tot}$ is the total reduction factor due to crossing angle and/or hourglass effect.
- $\beta^*_{xy}$ is the $\beta$-function at the IP (**round optics**).

# Probability as function of $J_{x,y}$ (I)

The relative probability of $J_x$-amplitude collision is



# Probability as function of $J_{x,y}$ (II)

*If we launch a particle A of* $J_X = 0$ *against a Gaussian bunch of B2 the probability to have a collision is more than twice the one of a particle B of* $J_X = 2\epsilon_{2x}$.

This function gives not the probability in absolute terms (as Eq.1). It is NOT a probability density (**pdf**) function since does not describe the particle probability to get different $J_X$ (its integral is not finite).

# Normalize trace-space and pdf's (I)

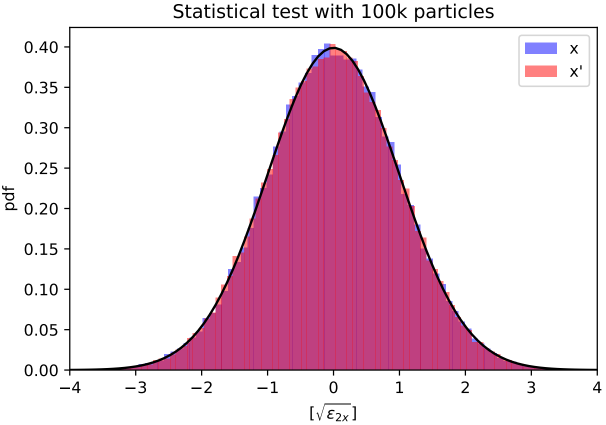
We will work in the in the **normalized** $xx'$ **trace-space**. For Gaussian beams we have, in units of $\sigma_x = \sigma_{x'} = \sqrt{\epsilon}$

- $x \sim N(0,1)$,
- $x' \sim N(0,1)$.

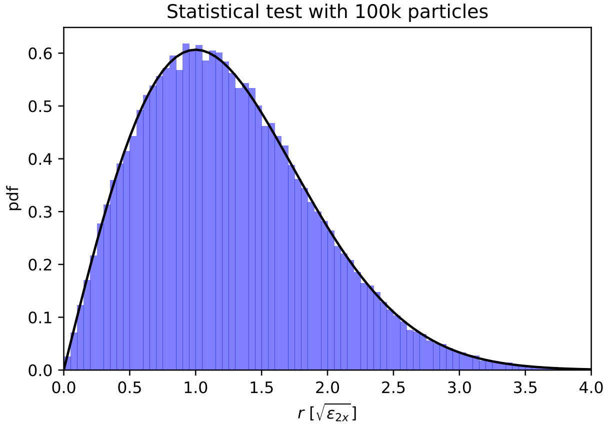
Therefore

- $r = \sqrt{x^2 + x'^2} \sim \chi_2 \rightarrow r \sim r\ e^{-\frac{r^2}{2}}$
- $2J_x = r^2 \sim \chi_2^2 \rightarrow J_x \sim e^{-J_x}$

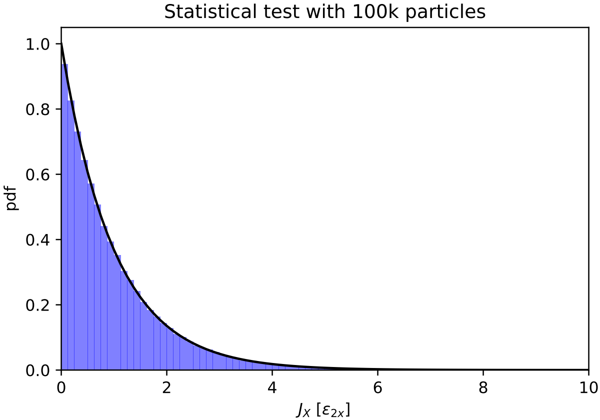
# Normalize trace-space and pdf's (II)



# Normalize trace-space and pdf's (III)



# Normalize trace-space and pdf's (IV)



# Evolution of the $J_x$ pdf (I)

The aim is to compute, in a perturbative approach, the evolution of the $J_x$ distribution, pdf($J_x$, $BO$), where $BO$ is the fraction of the **B**urnt-**O**ff intensity:

$$BO(t) = \frac{\sigma \int_{t_0}^{t} \mathcal{L}(\tau) d\tau}{N_1(t_0)} \tag{2}$$

In the 2018 LHC run, the average $BO$ at the End-of-Fill was $\approx$ 15% ($\approx$ 130 fb$^{-1}\times$ 80 mb / 240 fills / (1.1e11 ppb $\times$ 2556 bunches)).

# Evolution of the $J_x$ pdf (II)

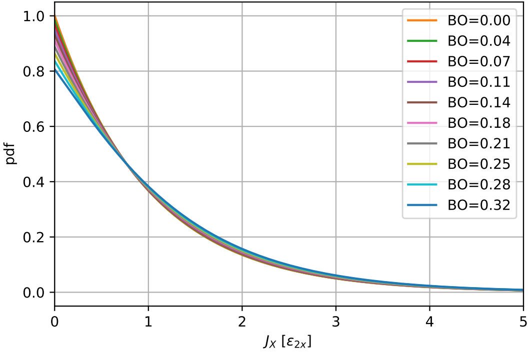
The pdf($J_x$, $BO$) can be approximate as

$$\frac{\overbrace{pdf(J_x,0)\left(1-k\ e^{-\frac{J_y}{2}} I_0\left(-\frac{J_y}{2}\right) pdf(J_x,0)\right)}^{g(J_x)}}{\int g(J_x) dJ_X}$$

and

$$BO = \int k\ e^{-\frac{J_y}{2}} I_0\left(-\frac{J_y}{2}\right) \underbrace{pdf(J_x,0)}_{e^{-J_X}} dJ_x$$

# Evolution of the $J_x$ pdf (III)



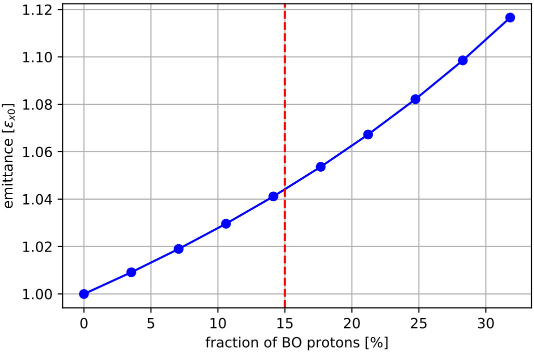

See the depletion of the low-$J_x$ particle.

# From action to rms emittance (I)

For matched beams we have

$$\boxed{\epsilon_{rms} = < J_x > = \int J_x pdf(J_x) dJ_x}$$

# From action to rms emittance (II)



# From action to rms emittance (III)

A 4% emittance BU is expected on the averaged LHC fill. This effect is larger for the good (longer) fills.

# From $pdf(J_x)$ to $x$-profiles

To do that we need to introduce an additional (very useful) tool: the **Abel transform**.

It is among the standard tools for tomography.

# The Abel transform (I)

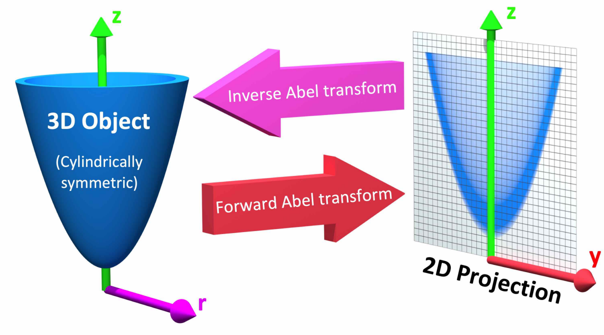



Figure: Courtesy of [7].

# The Abel transform (II)

The Abel Transform [6] give us the possibility to link the rotating function, $f(r)$, to the x-projected density of the beam profile $\rho(x)$.

$$\rho(x) = \mathcal{A}(f(r)) = 2\int_x^\infty \frac{r\ f(r)}{\sqrt{r^2 - x^2}} dr$$

*A matched distribution in the $xx'$ trace space is a rotation of a $f(r)$ and its x-projection is the beam profile.*

# Example

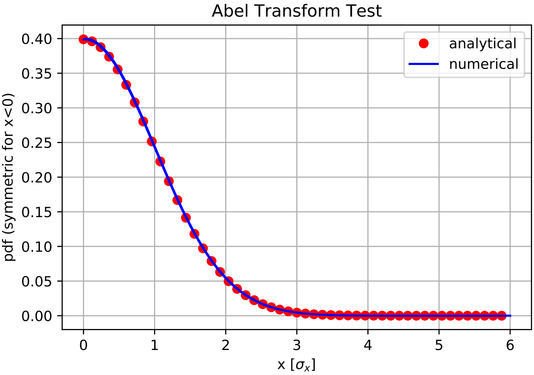
From [6], for

$$f(r) = \frac{1}{2\pi} e^{-\frac{r^2}{2}},$$

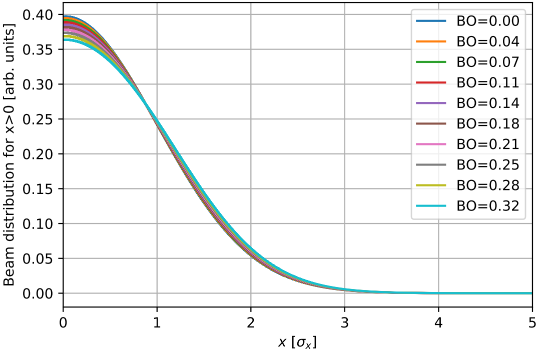
we have

$$\mathcal{A}(f(r)) = \rho(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

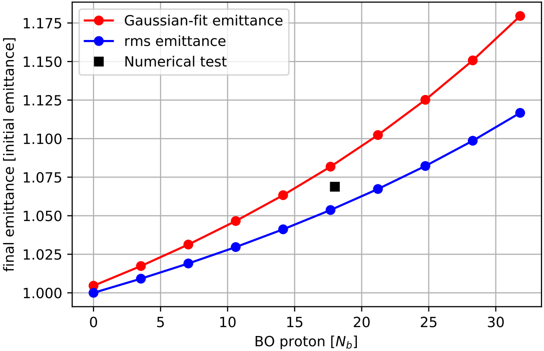
# Using pyAbel [7]



# The beam x-profiles



# The effect of the Gaussian fit




The numerical test is described in [8].

# Conclusion/Discussion

- We should consider to include this effect in Luminosity Model.
- How? Is the perturbative and tabular approach enough to start with?

# 2019 Abel Prize

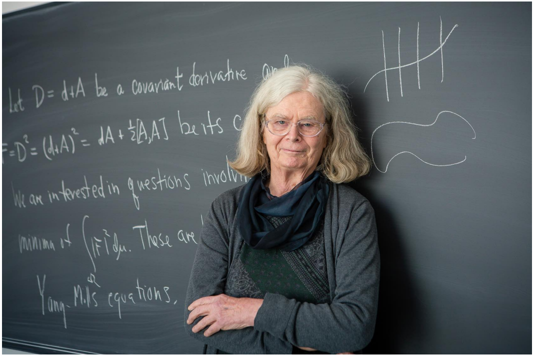



Figure: By winning the 2019 Abel Prize, U.S. mathematician Karen Uhlenbeck is first woman to ever receive the prestigious honor. Photo courtesy of Andrea Kane/Institute for Advanced Study.

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- https://github.com/PyAbel/PyAbel
- https://indico.cern.ch/event/790733

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