

The Odderon and elastic pp scattering

Alan Martin (IPPP, Durham)
with Valery Khoze & Misha Ryskin

Forward Physics and Diffraction
at the LHC
UCD, Dublin, June 10-13th, 2019

Odderon

Very nice review by Carlo Ewerz

The Odderon in QCD
hep-ph/0306137 (2003)

Properties of odd-signature high-energy amp studied in early 70's

Odderon first promoted in 1973 (Lukaszuk, Nicolescu) by Regge exchange for high-energy cross sections; for example **pp and p \bar{p}**

$$A_{\pm} = A(pp) \pm A(p\bar{p})$$

simple poles $\alpha_{p,O}(0) \sim 1$

$$A_{+}(pp) = A_{+}(p\bar{p}) \quad C = +1$$

Pomeron --- dominately imag

$$A_{-}(pp) = - A_{-}(p\bar{p}) \quad C = -1$$

Odderon --- dominately real

Maximal Odderon (MO)

$$\text{Im}A_{+} \leq c \ln^2 s \quad (\text{Froissart})$$

$$\text{Re}A_{-} \leq c' \ln^2 s \quad (\text{MO analogy})$$

allowed by asymptotic theorems

1. Pomeranchuk theorem $\Delta\sigma \equiv \sigma(\bar{p}p) - \sigma(pp) \sim \text{Im}A_{-} \rightarrow 0 \quad \text{as } s \rightarrow \infty$

2. Generalized Pomeranchuk th: $\frac{\sigma(\bar{p}p)}{\sigma(pp)} \rightarrow 1 \quad \text{as } s \rightarrow \infty$

1. Pommeranchuk theorem $\Delta\sigma \equiv \sigma(\bar{p}p) - \sigma(pp) \sim \text{Im}A_- \rightarrow 0$ as $s \rightarrow \infty$

2. Generalized Pommeranchuk th: $\frac{\sigma(\bar{p}p)}{\sigma(pp)} \rightarrow 1$ as $s \rightarrow \infty$

1 and 2 are not equivalent

$$\sigma(\bar{p}p) = A \ln^2 s + B \ln s + C$$

$$\sigma(pp) = A \ln^2 s + B' \ln s + C'$$

if $B \neq B'$ then satisfy 2, but not 1

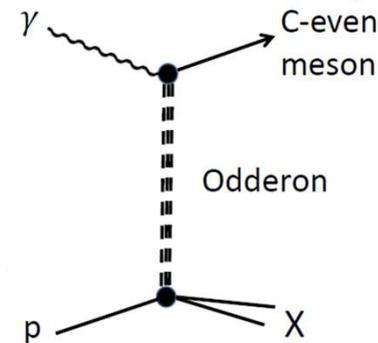
In general $\Delta\sigma \leq c \ln s$

There was a hint of the Odderon from non-zero $d\Delta\sigma/dt$ in dip region at 53 GeV, but at this energy it could be due to the Pomeron- ω cut

Then in 1980 the Odderon is found to be a **firm prediction of QCD**

But **no evidence** of Odderon exchange from HERA data for exclusive photoprod. of C-even mesons $\gamma p \rightarrow \pi^0 p, \eta p, f_2 p \dots$ (Nachtmann et al)

Here we will discuss this and the possibility of signals from the LHC.



First, explain why **Maximal Odderon violates unitarity** \rightarrow

Why the Maximal Odderon violates unitarity

Khoze, Martin, Ryskin
arXiv: 1801.07065

1. Unitarity

$$SS^\dagger = I \quad (\text{let } S = I + iA) \quad \rightarrow \quad \underline{i(A^\dagger - A) = A^\dagger A}$$

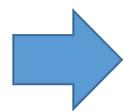
Unitarity equation

$$\underline{2 \operatorname{Im} A_{\text{el}}(b)} = \sum_n |A_{i \rightarrow n}(b)|^2 = \underline{|A_{\text{el}}(b)|^2 + G_{\text{inel}}(b)}$$

where $G_{\text{inel}}(b) = \sum_{n \neq i} |A_{i \rightarrow n}(b)|^2 < 1 =$ probability of inelastic scatt.

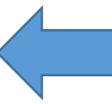
Solution of unitarity eq.

$$A(b) \equiv \underline{A_{\text{el}}(b)} = i(1 - e^{-\Omega(b)/2}) \quad \text{with } \operatorname{Re}\Omega(b) \geq 0$$



No solution of unitarity eq. if $G_{\text{inel}}(b) > 1.$

Let us calculate $G_{\text{inel}}(b)$



exp(2iδ_l) in terms of
partial waves $l = bv/s/2$

2. Finkelstein-Kajantie problem: $\sigma(\text{diff}^{\text{ve}}) > \sigma(\text{total})$ due to $\int_0^{\ln s} dy \dots \sim \ln s$

Simple example: Central Exclusive Prod. $pp \rightarrow p+X+p$

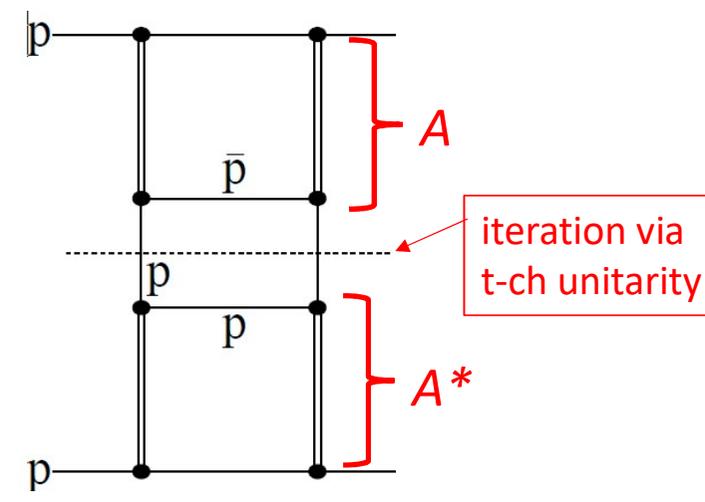
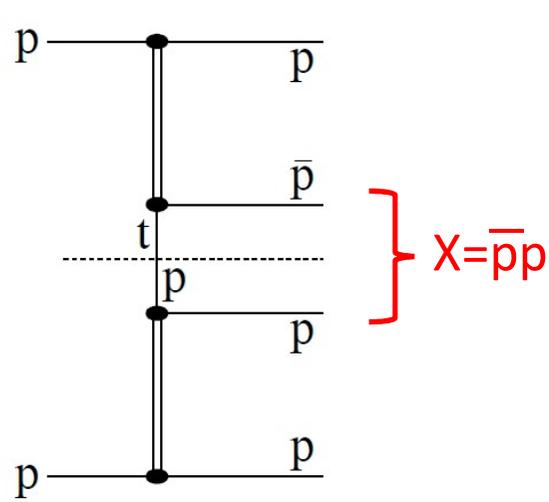
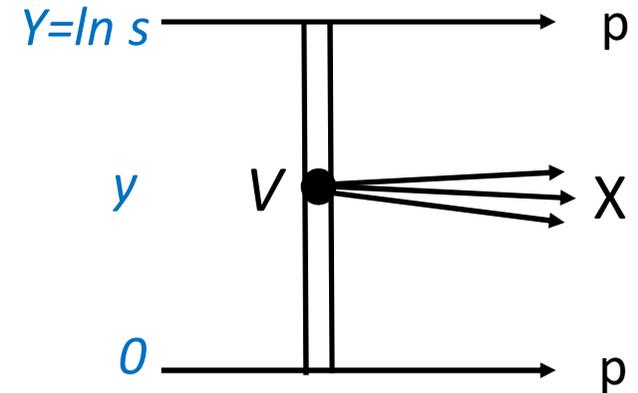
In the Froissart limit $\sigma_{\text{CEP}} \sim \ln^5 s$

so $\sigma_{\text{CEP}} > \sigma_{\text{tot}} \sim \ln^2 s$

Could the explanation be that vertex $V = 0$? **No**

Can show, for example, that the $p\bar{p}$ component of X generated by t-channel unitarity has $V \neq 0$, and cannot be compensated due to the singularity/pole at $t=m_p^2$.

So starting from A_{el} we see t-ch unitarity gives a component of $G_{\text{inel}}(b)$ increasing faster than $\int_0^{\ln s} dy \dots \sim \ln s$



Figs: amplitude (left) and cross section (right) of $\bar{p}p$ Central Exclusive Prod. generated by t-ch unitarity

3. Solution to the Finkelstein-Kajantie problem

Complete CEP **must include** rescattering S_{el} (that is the **survival probability** $S^2 = |S_{el}|^2$ of the rapidity gaps)

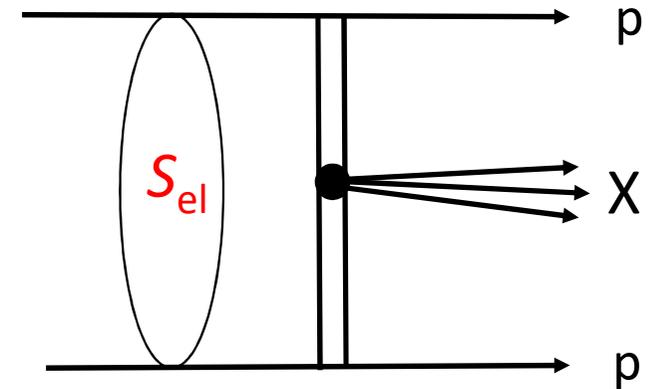
$$A_{CEP}(b) = S_{el}(b) A_{bare}(b)$$

where $|S_{el}(b)|^2 = |1 + iA_{el}(b)|^2 = e^{-\text{Re}\Omega(b)}$

Black disc asymptotics: $\text{Re}\Omega \rightarrow \infty$, $A_{el}(b) \rightarrow i$, $S^2(b) \rightarrow 0$ for $b < R$

If σ_{tot} increases, Black disc is the only known solution to the FK problem

To repeat, if at least one component of $G_{inel}(b)$ increases (due to $\int dy \sim \ln s$) then unitarity is violated as $s \rightarrow \infty$. The only way to restore unitarity is to have $S(b) \rightarrow 0$



4. Maximal Odderon contradicts unitarity as $s \rightarrow \infty$

Maximal Odderon

Asymptotically MO means $\text{Re}A/\text{Im}A \rightarrow \text{constant} \neq 0$

In this case $S^2(b) = |1 + iA(b)|^2 \geq |\text{Re}A(b)|^2 \neq 0$

so there is no possibility to compensate the growth of σ_{CEP} .

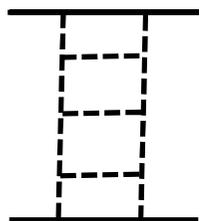
The Odderon exists in QCD

Need the existence of symmetric tensor d_{abc} of non-Abelian $SU(3)_{\text{col}}$ to form colourless ggg exchange with $C=-1$

Pomeron (gg)

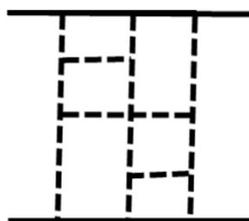
Odderon (ggg)

BFKL eq.



resum
 $\alpha_p(0) > 1$

BKP eq.



resum
 $\alpha_o(0) \approx 1$

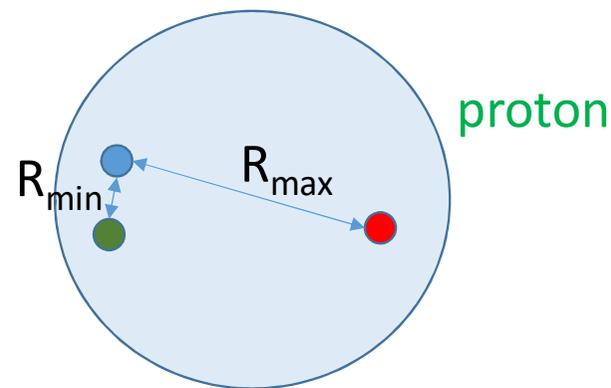
Bartels; Kwiecinski, Praszalowicz 1980

Janik-Wosiek solution
Bartels-Lipatov-Vacca solution,
2000

Theoretically the **Odderon** exists (pQCD), but the amplitude is small in comparison with the **Pomeron**

$$A_{\text{Odd}} \sim \alpha_s^3 R_{\text{min}}^2$$

$$A_{\text{Pom}} \sim \alpha_s^2 R_{\text{max}}^2$$



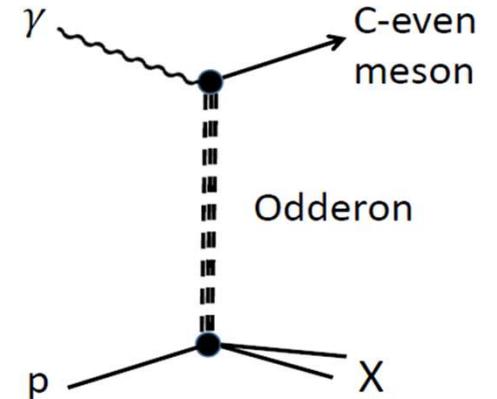
Ways to observe the Odderon

(1) To measure $d\Delta\sigma/dt$ in the dip region
 a difference in pp and $pp(\bar{p})$ was seen at energy 53 GeV (ISR),
 but cannot disentangle from b' ground due to the Pomeron- ω cut

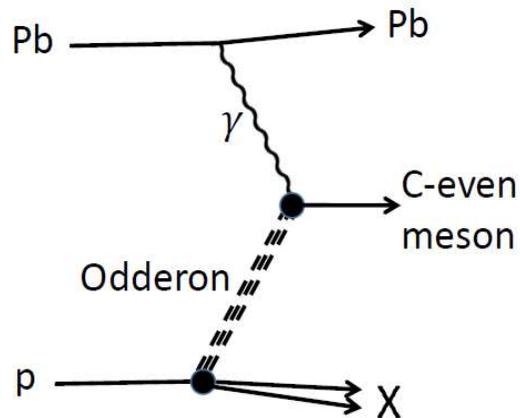
(2) $\text{Re } A/\text{Im } A$ at $t=0$ in pp elastic scattering at the LHC



(3) photoproduction of **C-even** mesons: π^0, f_2, \dots (HERA)
 but dominated by,
 γ exchange b' gd



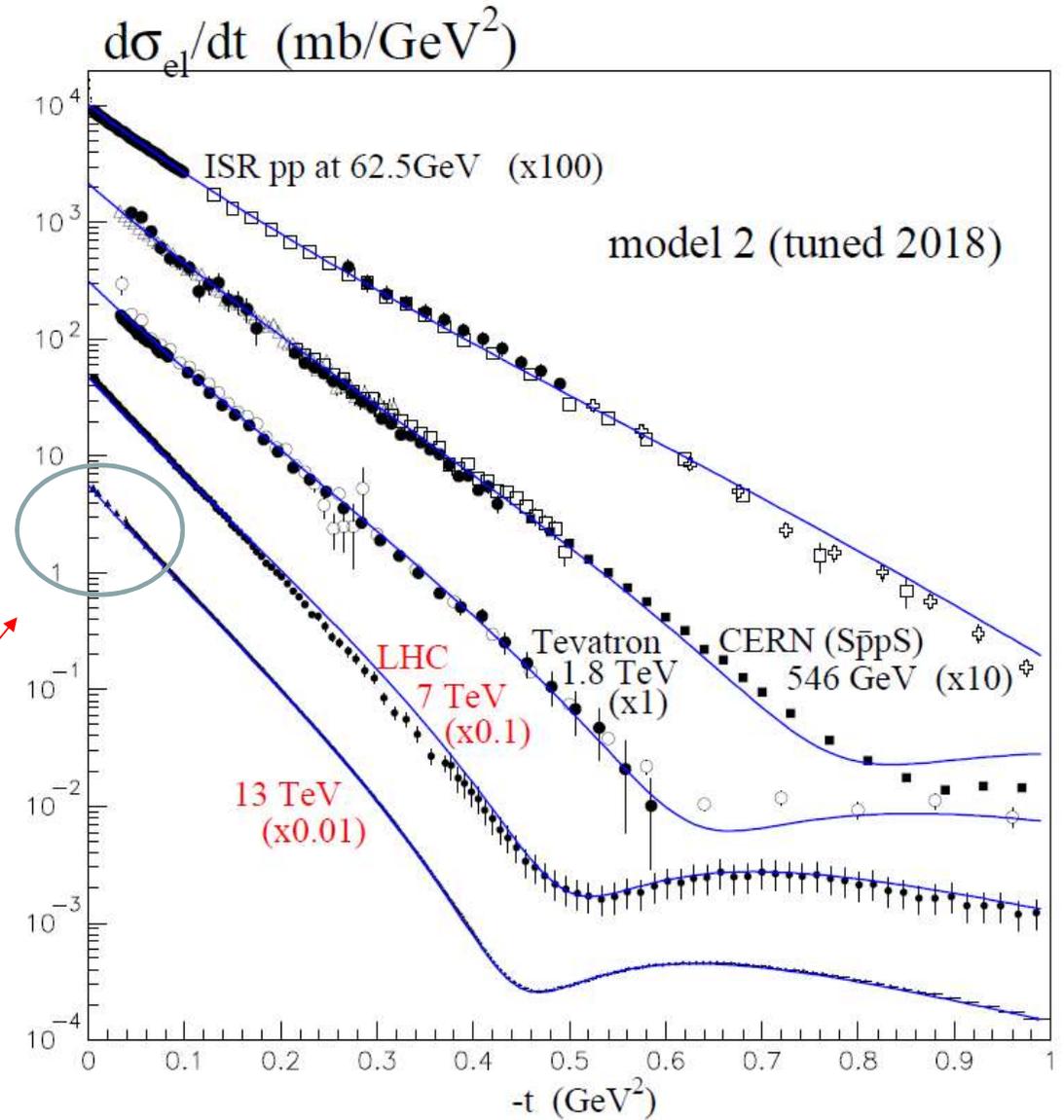
(4) ultraperipheral
 prodⁿ in **p-Pb**
 collisions (LHC)
 Z^2 in γ flux



2-channel eikonal global fit
to describe all high-energy
 $d\sigma_{el}/dt$, σ_{tot} , σ_{lowM}^{diff} pp data
(KMR 1806.05970)

11 parameters in total
4 for Pom: σ_0 , $\alpha_P(0)$, α'_P , γ
7 for two GW eigenstates

Is Odderon-exchange seen in
 $\text{Re}A/\text{Im}A$ at $t=0$
of 13 TeV TOTEM data ??



pp elastic scattering at 13 TeV

In the analysis of their data TOTEM include the norm $n=1\pm 0.055$ just in overall error deterⁿ

KMR obtain lower $\sigma(\text{tot})$ as they fit to data over range of energies

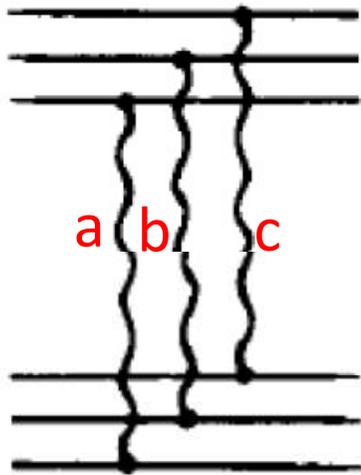
C-S perform Bayesian analysis of first 79 points of 13 TeV data ($0.0008 < -t < 0.07$) including norm n with $\pm 5.5\%$ error

	$\sigma(\text{tot})$ mb	$\rho = \text{Re}A/\text{Im}A$
TOTEM	110.6 ± 3.4	0.10 ± 0.01
KMR	104.2	0.109
Cudell-Selyugin 1901.05863	$107.5 \pm 1.5 - 2$	0.096 ± 0.006

from form factor uncertainty

Estimate of Odderon contribⁿ

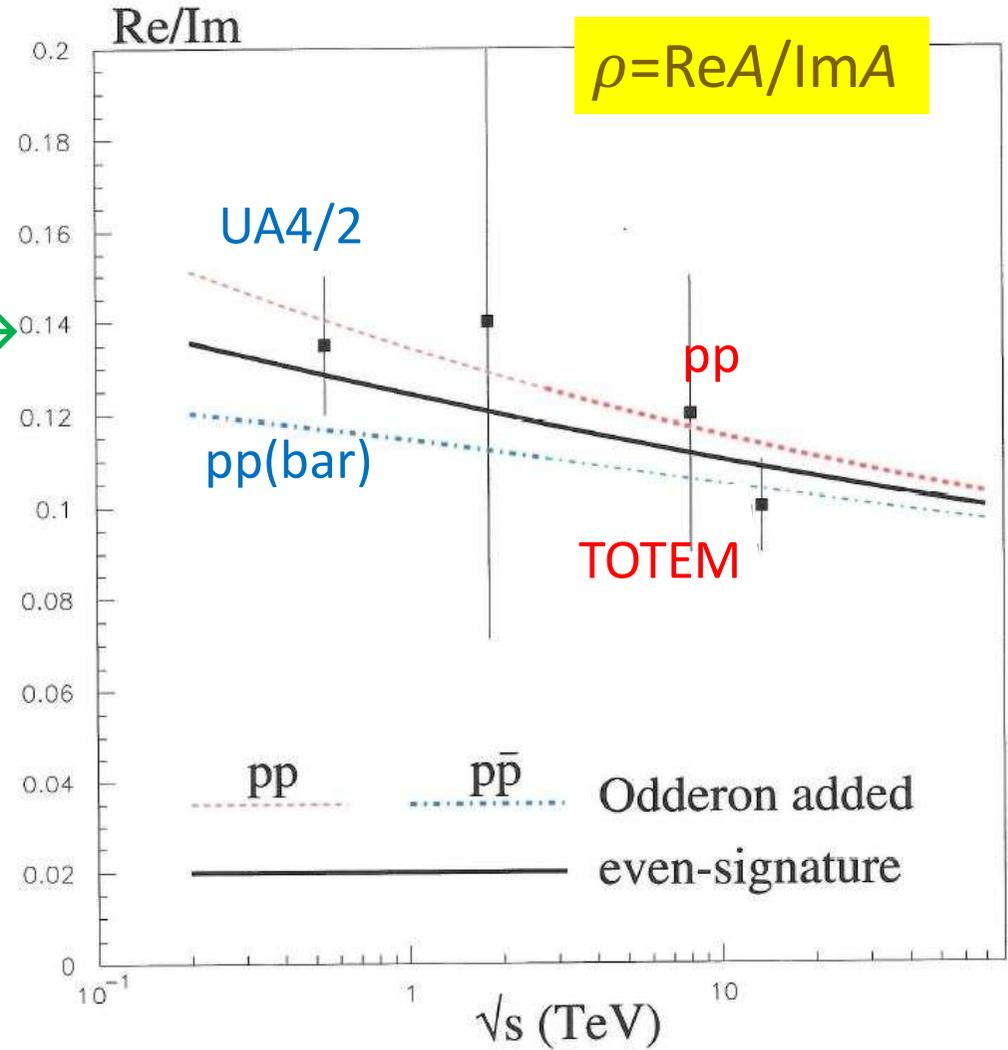
QCD lowest α_s order Ryskin '87
 (Fukugita, Kwiecinski '79;
 Kwiecinski, Motyka.. '96 (η_c at HERA))



enhanced x2 →

x d_{abc}

KMR: 1806.05970



Must include full Ω in amplitude

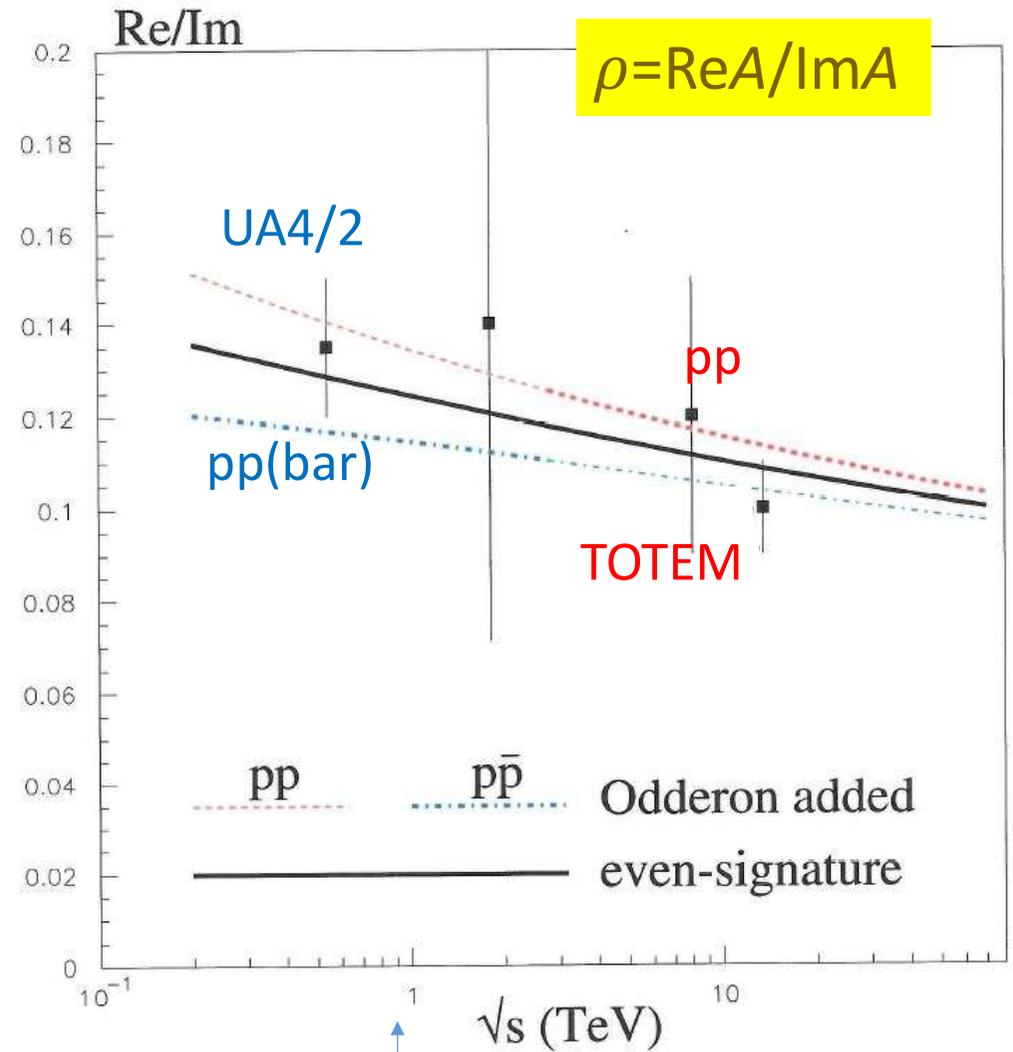
$$A(b) = i \left(1 - e^{-\Omega(b)/2} \right)$$

with $\Omega = \Omega_{\text{even}} + \Omega_{\text{odd}}$

Automatically accounts for absorptive effect caused by elastic rescattering

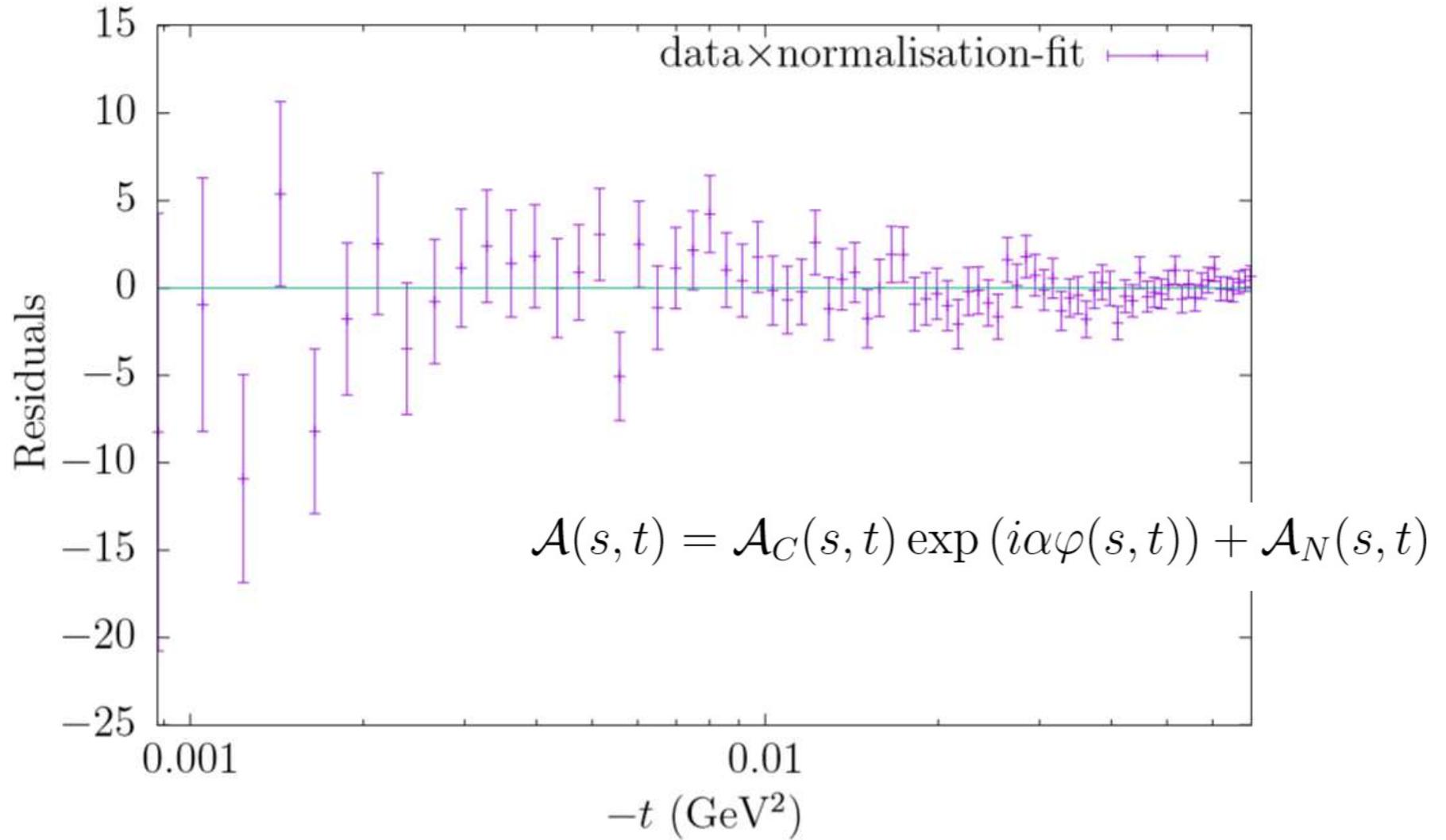
No evidence for the Odderon from $\text{Re}A/\text{Im}A$ at $t=0$
C-S come to same conclusion as KMR from just fitting to 13 TeV data

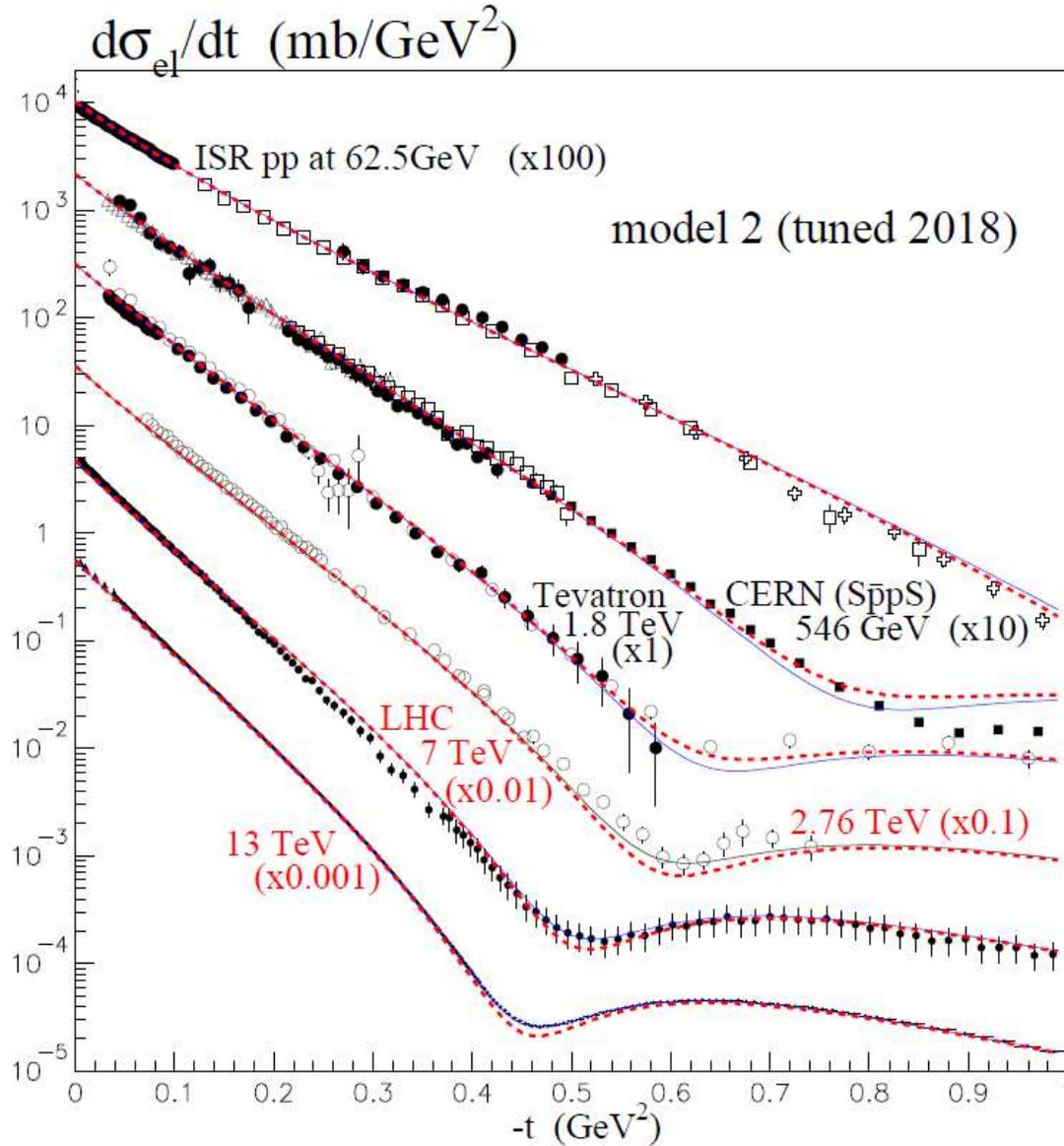
KMR: 1806.05970



TOTEM measurement 0.9 TeV
could be informative?

Cudell, Selyugin Data/Fit for 13 TeV pp elastic scatt.





Previous fit, but now red-dotted curves show the effect of the Odderon fixed to agree with $\rho = \text{Re}A/\text{Im}A$

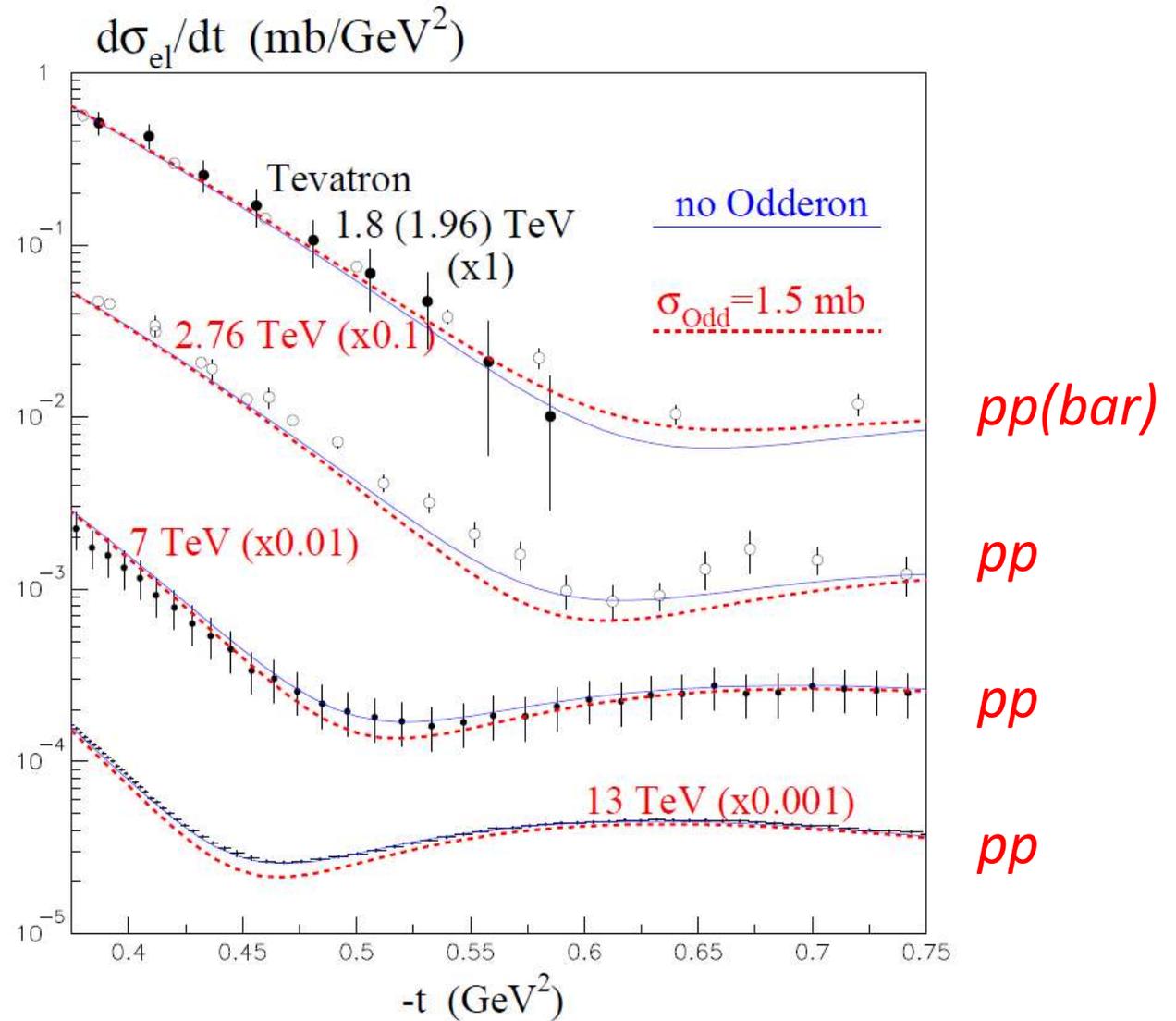
Actually in approx. LO QCD calculation Odderon would be expected to decrease $pp(\bar{p})$, increase pp (opposite to effect shown in this plot) Main effect in dip region, but v.small

← New TOTEM data at 2.76 TeV

Dip region

No conclusive evidence for a larger Odderon

Approx QCD sign would have pp shift above no Odderon curve



Odderon signals

- **pp scatt** Odderon exch. is a small correction to even-signature term $(g_{p0})^2$

- **photoproduction of C even mesons**

$\pi^0, f_2, \eta \dots$

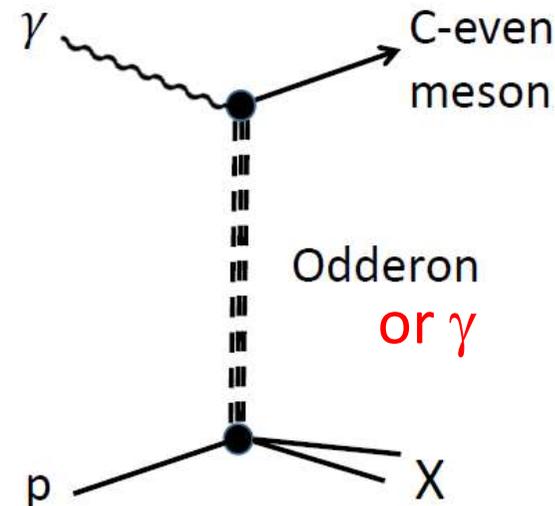
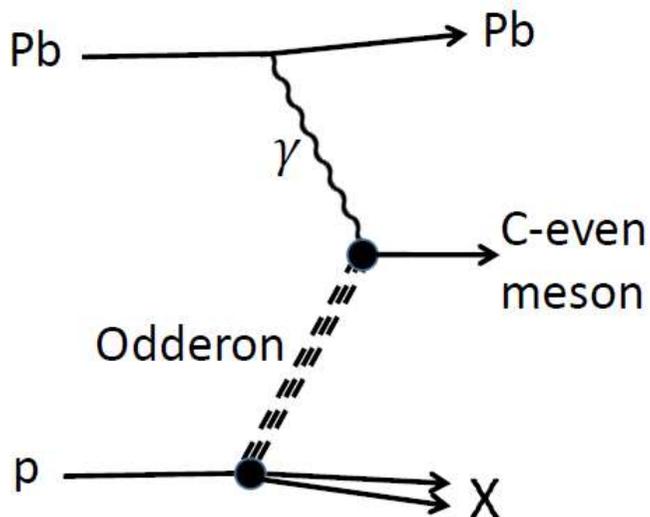
No evidence in HERA data

upper limits $\sigma(\pi^0)=39\text{nb}$, $\sigma(f_2)=16\text{nb}$

Need to suppress back^{gd} due to γ exchange

- **ultraperipheral production in p-Pb collisions**

Z^2 in photon flux



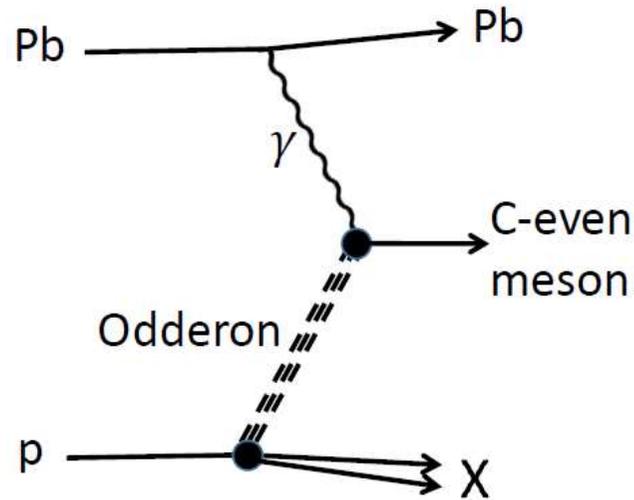
$(g_{p0})^2$

g_{p0}

Odderon signal in p-Pb collisions?

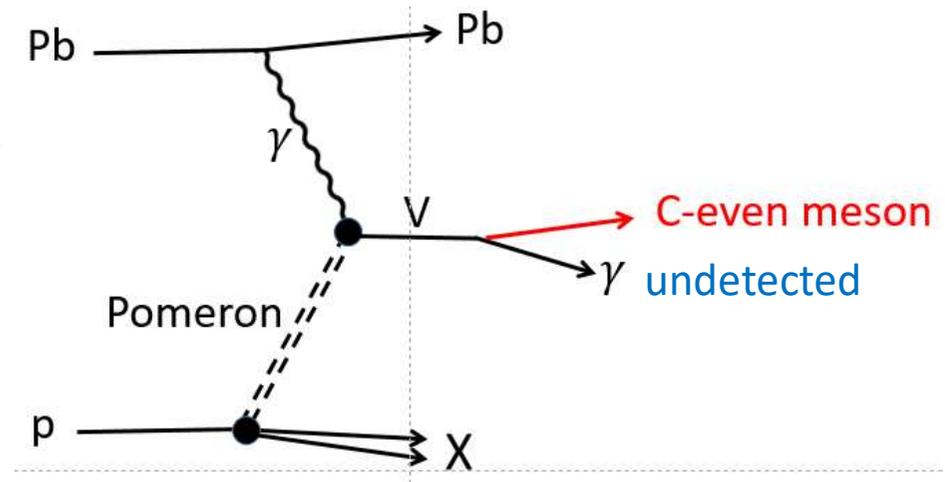
$d\sigma/dy_M _{y_M=0}$	Expected upper limits [μb]
π^0	7.4
η	3.4
$f_2(1270)$	3.0

Healthy signal,
but backgrounds
are due to



production of C-even meson by

1. $\gamma\gamma$ fusion
2. Pomeron-Pomeron fusion
3. Via vector meson
 $V \rightarrow$ C-even meson + undetected γ



π^0

$\sigma(\pi^0)$ from $\gamma\gamma$ fusion is well known. Estimating the cross section due to Odderon exchange, allowing for the colour factors etc. and integrating over $0.04 < |t| < 1 \text{ GeV}^2$ we find

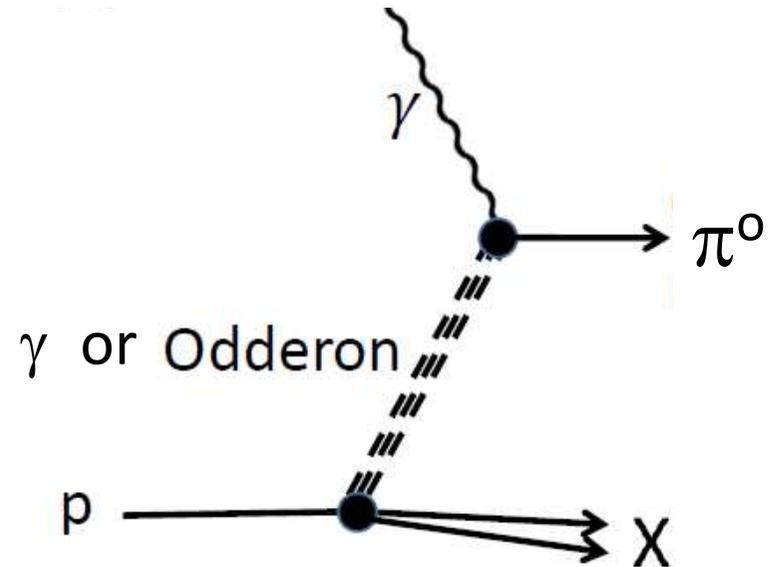
$$\sigma_{\text{Odd}}(\gamma p \rightarrow \pi^0 + X) \sim 5(1) \text{ nb}$$

for the cutoff $\mu = 0.3(0.5) \text{ GeV}$. The t cut adequately suppresses the $\gamma\gamma$ fusion background.

Pomeron-Pomeron background entirely absent by SU(3) flavour

However the reducible background from radiative ω decay is very large

$$\omega \rightarrow \pi^0 + \gamma \text{ (undetected)}$$

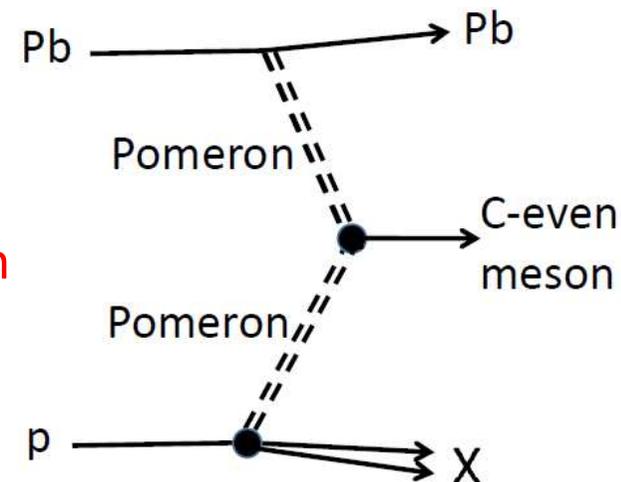


f_2

There is a very low background due to radiative V decay. However the problem here is the **v.large Pomeron-Pomeron** background. The signal-to-bkgd may be suppressed by observing central (semi)exclusive production (CEP*) of C-even mesons in which the proton may break up but the **Pb-ion remains intact**. For such events we expect a larger possibility of break-up for Odderon exchange --- exptally challenging.

In any nucleon-proton interaction creating the C-even meson there is a large probability of inelastic nucleon-proton interactions which will populate the rapidity gaps. Only in very **peripheral** ion-proton collisions is there a chance to observe a CEP* event.

Can show the A dependence of CEP* events scales as $A^{1/3}$. Recall the photoprodⁿ cross section (the signal) scales as Z^2 , so the expected $A^{1/3}$ back^{gd} scaling is much milder.



η Pom-Pom background is small as η has small SU(3) singlet compt. However again the reducible backgrounds coming from $\phi \rightarrow \eta\gamma$ and $\eta' \rightarrow \eta\pi^0\pi^0$ are rather large

η_c In principle, viable channel but has a much smaller production rate.

C-even meson (M)	Odderon Signal		Backgrounds		
	Upper Limit	QCD Prediction	$\gamma\gamma$	Pomeron-Pomeron	$V \rightarrow M + \gamma$
π^0	7.4	0.1 - 1	0.044	–	<u>30</u>
$f_2(1270)$	3	0.05 - 0.5	0.020	<u>3 - 4.5</u>	0.02
$\eta(548)$	3.4	0.05 - 0.5	0.042	negligible	<u>3</u> $\phi \rightarrow \eta\gamma$
η_c	–	<u>$(0.1 - 0.5) \cdot 10^{-3}$</u>	0.0025	$\sim 10^{-5}$	0.012

0.05 included

μb

signal and background for $d\sigma(\text{Pb p} \rightarrow \text{Pb} + M + X)/dY$ at $Y=0$

$d\sigma/dY_M$ at $Y_M = 0$ in μb

C-even meson (M)	Odderon Signal		Backgrounds		
	Upper Limit	QCD Prediction	$\gamma\gamma$	Pomeron-Pomeron	$V \rightarrow M + \gamma$
π^0	7.4	0.1 - 1	0.044	—	<u>30</u> ($\omega \rightarrow \pi^0\gamma$)
$f_2(1270)$	3	0.05 - 0.5	0.020	<u>3 - 4.5</u>	0.02 ($J/\psi \rightarrow f_2\gamma$)
$\eta(548)$	3.4	0.05 - 0.5	0.042	negligible	<u>3</u> ($\phi \rightarrow \eta\gamma$)
η_c	—	<u>$(0.1 - 0.5) \cdot 10^{-3}$</u>	0.0025	$\sim 10^{-5}$	0.012 ($J/\psi \rightarrow \eta_c\gamma$)

γ unobserved

$\eta_c \times 0.05$ for observable BR included

$p p \rightarrow p + M + X$ Pom – Pom background overwhelming
 $\text{Pb Pb} \rightarrow \text{Pb} + M + \text{Pb}$ $\gamma\gamma$ background overwhelming

Ronan McNulty: Pb-Pb data could check model for Pom-Pom bk^{gd} for f_2 ; $\text{BR}(f_2 \rightarrow \gamma\gamma) \sim 10^{-5}$

Conclusion

Theoretically the **Odderon** exists (pQCD), but the amplitude is small in comparison with that for Pomeron exchange.

So at present there is no experimental evidence for its existence. The Odderon remains elusive, but with experimental ingenuity and precision it stands a chance of being cornered.

“Soft” and “Hard” Pomerons

A vacuum-exchange object drives soft HE interactions. Not a simple pole, but an enigmatic non-local object. Rising σ_{tot} means multi-Pom diags (with Regge cuts) are necessary to restore unitarity. σ_{tot} , $d\sigma_{\text{el}}/dt$, $\sigma_{\text{lowM}}^{\text{diff}}$, described, by an effective pole
 $\alpha_{\text{P}}^{\text{eff}} = 1.13 + 0.05t$

Sum of ladders of Reggeized gluons with, in LLx BFKL, a singularity which is a cut and not a pole. When HO are included the intercept of the BFKL/hard Pomeron is
 $\alpha_{\text{P}}^{\text{bare}}(0) \sim 1.3$
 $\Delta = \alpha_{\text{P}}(0) - 1 \sim 0.3$

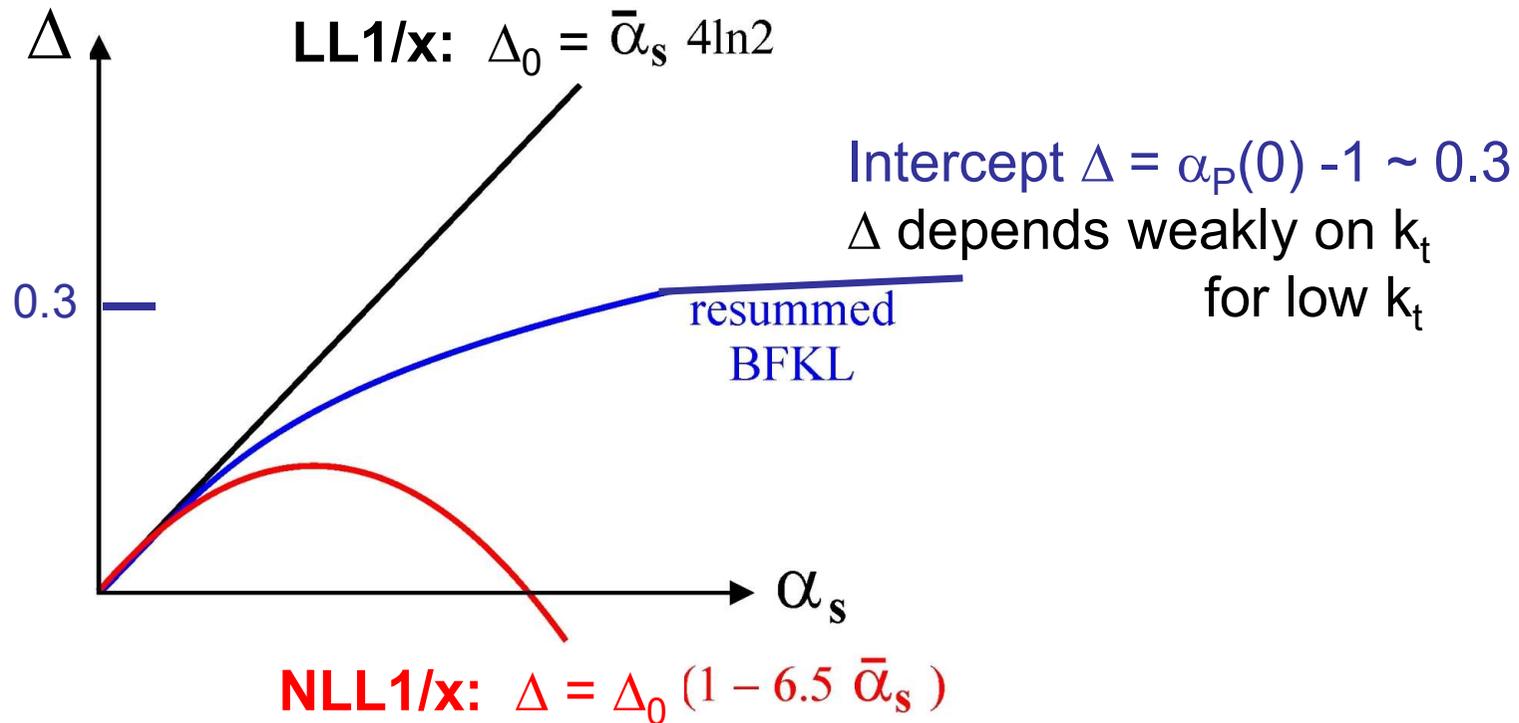
$$\alpha_{\text{P}}^{\text{eff}} \sim 1.13 + 0.05 t$$

Accounting for absorptive
(multi-Pomeron) effects

$$\alpha_{\text{P}}^{\text{bare}} \sim 1.3 + 0 t$$

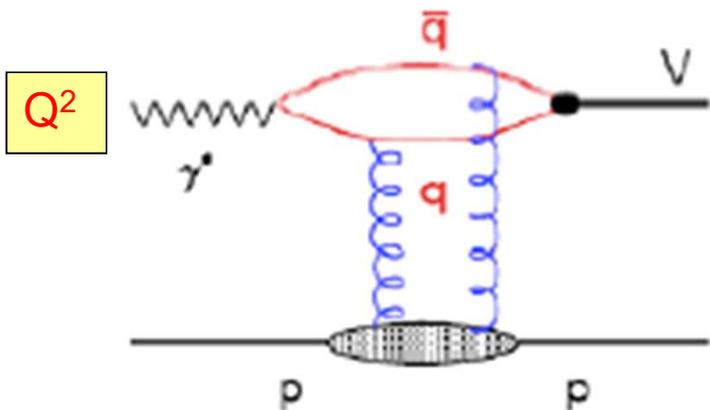
BFKL stabilized

$$\Delta = \alpha_P(0) - 1$$

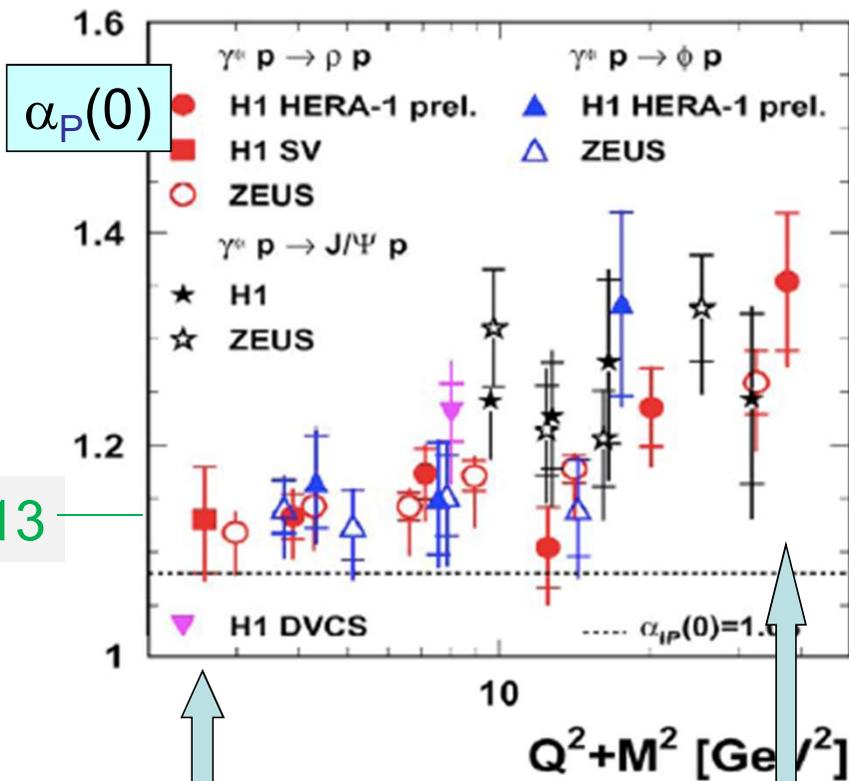


Small-size “BFKL” Pomeron is natural object to continue from “hard” to “soft” domain

Vector meson prodⁿ at HERA
 ~ bare QCD Pom. at high Q^2
 ~ no absorption



hard energy dependences



$\alpha_P(0)$

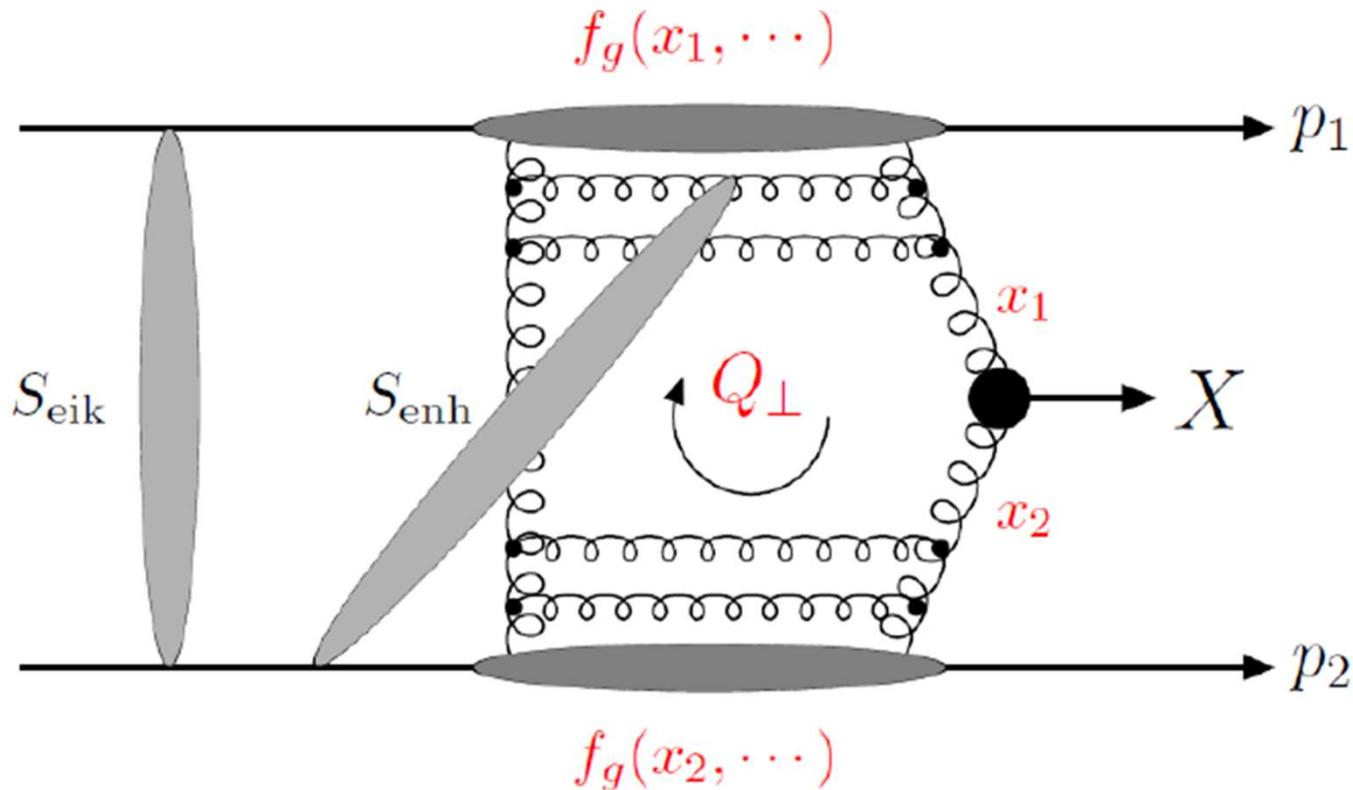
1.13

$\alpha_P(0) \sim 1.1$
 after absorption

$\alpha_P^{\text{bare}}(0) \sim 1.3$

Large number of intermediate partons

In general “enhanced” screening is small for large M_x due to strong k_t ordering of intermediate partons



QCD bremsstrahlung --- Sudakov suppression

$$pp \rightarrow p + H + p$$

Survival prob. of rap. gaps

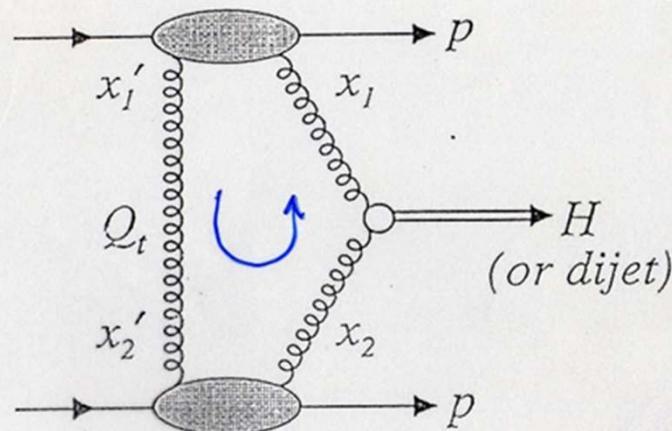
$$W = S^2 T^2$$

price for no soft rescatt.

no g radⁿ in $gg \rightarrow H$

Not a simple multiplication. Need to embed the Sudakov factor in the pQCD integral over Q_t to ensure infrared convergence

$$f(x_1, x_1', Q_t, M_H)$$



$$\Lambda_{\text{QCD}}^2 \ll Q_t^2 \ll M_H^2 \rightarrow \text{pQCD}$$

$$\left(x' \sim \frac{Q_t}{\sqrt{s}}\right) \ll \left(x \sim \frac{M_H}{\sqrt{s}}\right) \ll 1$$

need uninteg. skewed gluons
 $f(x_i, x_i', \dots)$

Sudakov factor $T(Q_t, \mu) \sim \exp(-\alpha_s \ln^2(Q_t^2/M_H^2))$ ensures **no** gluon emission from the fusing gluon as it evolves from Q_t to hard scale μ . It ensures infrared convergence of Q_t integral

$$\left(x' \sim \frac{Q_t}{\sqrt{s}}\right) \ll \left(x \sim \frac{M_H}{\sqrt{s}}\right) \ll 1$$

need uninteg. skewed gluons

$$\mathcal{M} = \frac{A}{M_H^2} \int \vec{Q}_{1t} \cdot \vec{Q}_{2t} \frac{d^2 Q_t}{Q_t^6} f(x_1, x'_1, Q_t^2, \frac{M_H^2}{4}) f(x_2, x'_2, Q_t^2, \frac{M_H^2}{4})$$

where $f(x, x', Q_t^2, \mu^2) \approx R \frac{\partial}{\partial \ln Q_t^2} \left[\sqrt{T(Q_t, \mu)} x g(x, Q_t^2) \right]$

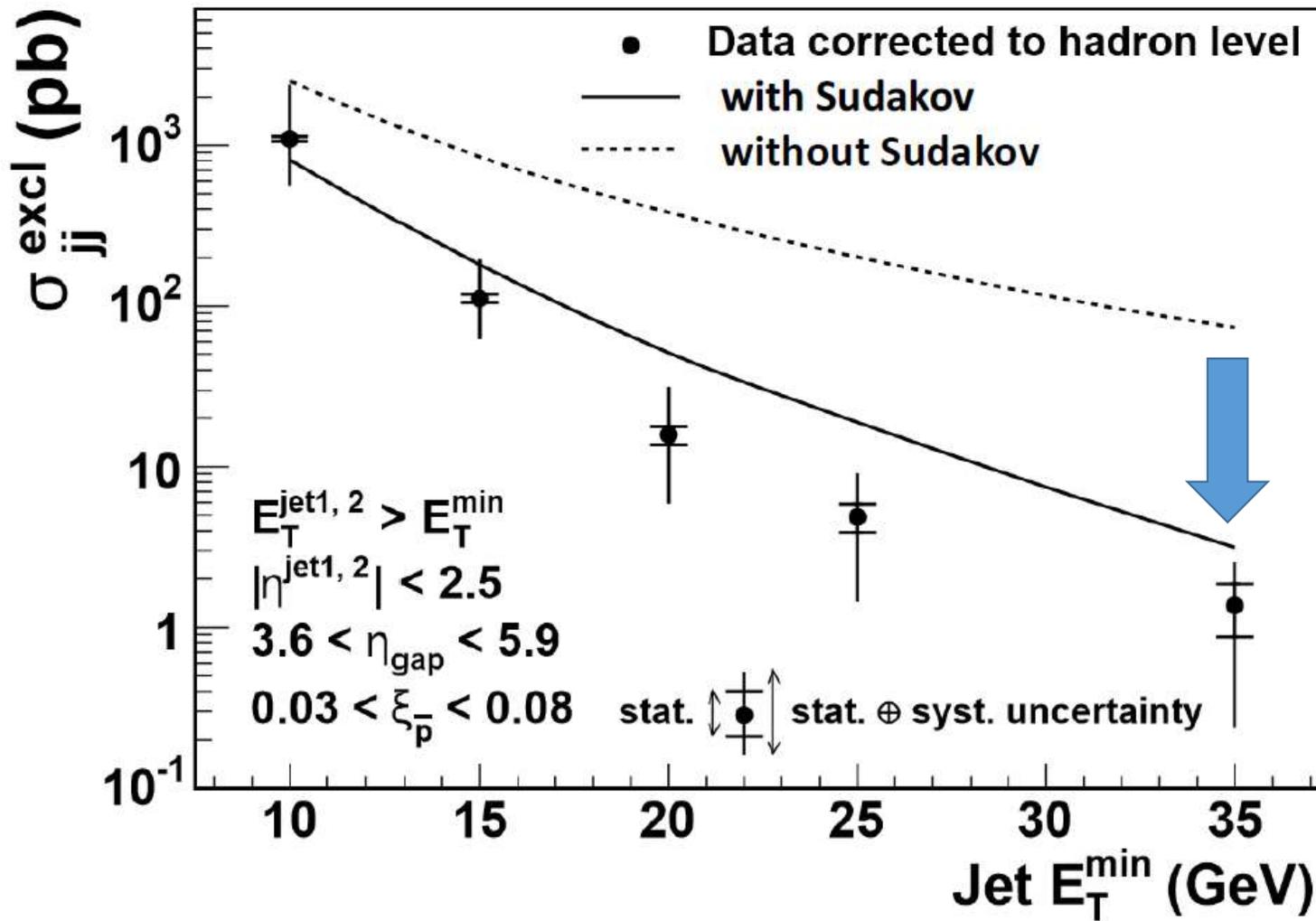
R is calculable skewed effect
(R=1.2 at LHC)

$$T(Q_t, \mu) = \exp\left(-\int_{Q_t^2}^{\mu^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_s}{2\pi} \int_0^{1-k_t/\mu} dz z P_{gg} \dots\right)$$

strongly suppresses Q_t infrared region

no emission when $(\lambda \sim 1/k_t) > (d \sim 1/Q_t)$
i.e. only emission with $k_t > Q_t$

Exclusive dijet production



Factor 25
Sudakov
suppression

Even better descripⁿ
data \rightarrow higher E_T