Central exclusive production at the LHC and the tensor-pomeron model

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3 From HERA to ultraperipheral Ap collisions
4 Tensor pomeron and vector odderon
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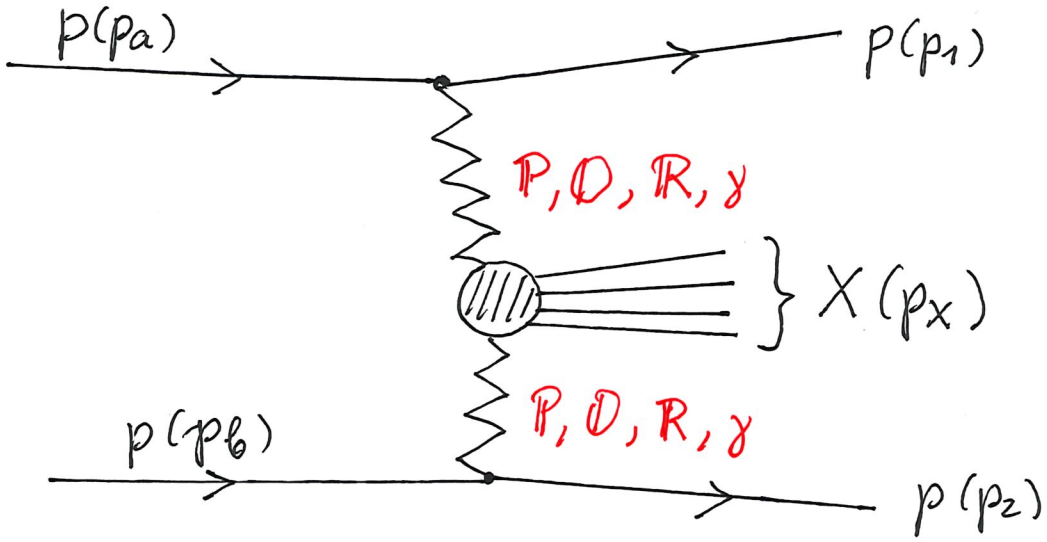
1 Introduction

At this workshop I shall discuss CEP, mainly in proton - proton collisions at high energies

$$p(p_a) + p(p_b) \longrightarrow p(p_1) + X(p_X) + p(p_2)$$



In the place of protons we can have nuclei A.

X may be a single particle, a group of particles, jets, etc.

We require large rapidity gaps between $p(p_1)$ and X, $p(p_2)$ and X.

Kinematic variables:

$s = (p_a + p_b)^2$ total c.m. energy squared,

$s_1 = (p_1 + p_X)^2$ inv. mass squared of proton p_1 and X,

$s_2 = (p_2 + p_X)^2$ inv. mass squared of proton p_2 and X,

$s_1 s_2 \cong s\, m_X^2$.

Exchange objects:

$\mathbb{P}$ $(C = +1)$ pomeron, should dominate for large s_1, s_2

$\mathbb{O}$ $(C = -1)$ odderon

$\mathbb{R}$: $\left.\begin{array}{l} f_{2R}\,,\ a_{2R}\ (C = +1) \\ \omega_R\,,\ \rho_R\ (C = -1) \end{array}\right\}$ reggeons

γ $(C = -1)$ photon

What can we hope to learn from CEP?

- (1) Properties and the couplings of the exchange objects $\mathbb{P}$, $\mathbb{O}$, $\mathbb{R}$, γ to the external protons and to the system X.

Here $\mathbb{P}$, $\mathbb{R}$, γ are well established. But what is the spin structure of the pomeron $\mathbb{P}$? I think that we could clarify this question. Does the "elusive odderon" exist? I hope to show you that CEP offers excellent possibilities to study odderon effects.

- (2) Properties of the system X
  - Search for and characterisation of resonances, e.g. glueballs.
  - X = jet(1) + jet(2) : "hard" diffraction,
    G. Ingelman, P. Schlein, PLB 152 (1985) 256
  - X = Higgs particle or Higgs particle + soft jets
    A. Schäfer, O.N., R. Schöpf, PLB 249 (1990) 331,
    A. Bialas, P.V. Landshoff, PLB 256 (1991) 540,
    ....
    Extensive work e.g. by the Durham group
  - X = e^+e^-, $\mu^+\mu^-$ etc. electroweak physics,
    non SM Higgs, ... physics beyond the SM.

In my talk I shall consider hadronic systems X of low to moderate mass m_X.

X a resonance : η, η', f_0, f_2, ---

X a group of hadrons : $\pi^+\pi^-$, K^+K^-, $p\bar{p}$, ----

Calculations of CEP for such systems X from first principles of QCD are still too difficult for theorists at present. We need models. We have developed one such model:

"A model for soft high-energy scattering: Tensor pomeron and vector odderon"

C. Ewerz, M. Maniatis, O.N., Ann. Phys. 342 (2014) 31

Applications of the tensor-pomeron model

$\gamma p \to \pi^+\pi^- p$ Bolz, Ewerz, Maniatis, O.N., Sauter, Schöning, JHEP 01 (2015) 151

$pp \to pp$, spin dependence: Ewerz, Lebiedowicz, O.N., Szczurek, PL B763 (2016) 382

CEP, X, Lebiedowicz, O.N., Szczurek

η, η', f_0 Ann. Phys. 344 (2014) 301

ρ_0 P.R. D91, 074023 (2015)

$\pi^+\pi^-, f_0, f_2$ P.R. D93, 054015 (2016)

$\pi^+\pi^- \pi^+\pi^-$ P.R. D94, 034017 (2016)

ρ^0 with proton diss. P.R. D95, 034036 (2017)

$p\bar{p}$ P.R. D97, 094027 (2018)

K^+K^- P.R. D98, 014001 (2018)

CEP X

$p\bar{p}$ in ultraperipheral heavy ion collisions — Klusek-Gawenda, Lebiedowicz, O.N., Szczurek, PRD 96 (2017) 094029

$K^+K^-K^+K^-$ via $\phi\phi$ — Lebiedowicz, O.N. Szczurek, arXiv: 1901.11490 (2019)

$f_2 \to \pi^+\pi^-$ — arXiv: 1901.07788 (2019)

$\phi \to K^+K^-, \mu^+\mu^-$ — in preparation

Photoproduction and low x DIS

Britzger, Ewerz, Glazov, O.N., Schmitt, arXiv: 1901.08524 (2019)

Many thanks go to all with whom I had the pleasure to collaborate on these projects.

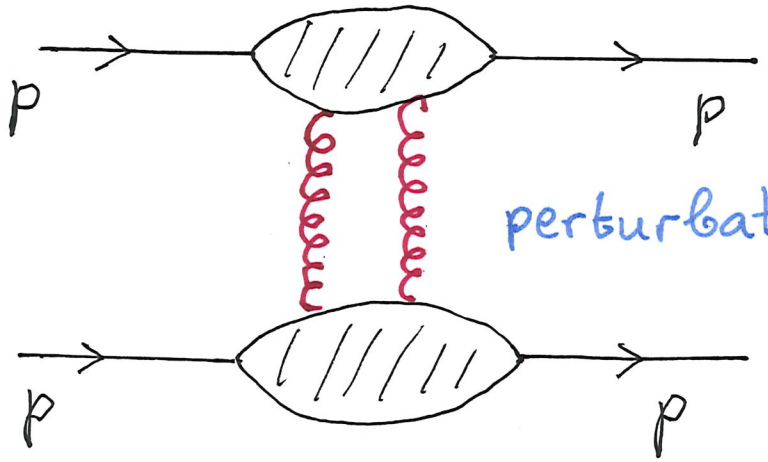
2 QCD, the pomeron and the odderon

Theoretical ideas on the pomeron in QCD were first put forward by

F. E. Low, PR D12 (1975) 163,

S. Nussinov, PRL 34 (1975) 1286.

They calculated the exchange of two gluons between colour singlet hadrons, e.g. protons.



perturbative gluons, regulated by a small mass

Studies of the pomeron in QCD perturbation theory with correct massless gluons have become a big industry starting from the famous works

E. A. Kuraev, L. N. Lipatov, V. S. Fadin, Sov. Phys. JETP 45 (1977) 199,

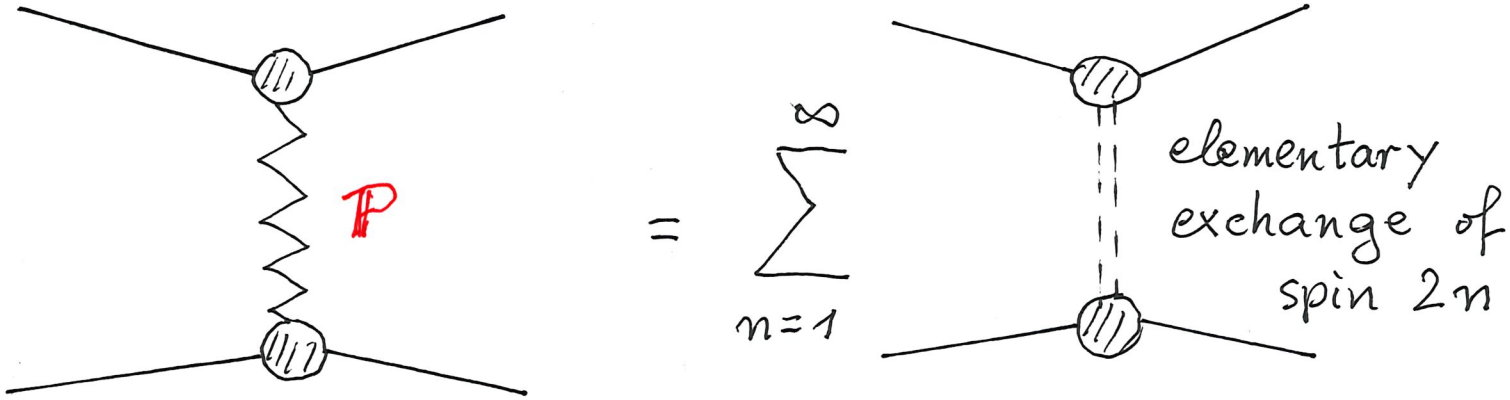
I. I. Balitsky, L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.

But there are obvious problems applying perturbation theory, pQCD, to soft hadronic processes.

Peter Landshoff and myself asked ourselves if we could understand the soft pomeron in terms of non perturbative QCD. We investigated a toy model in Z. Phys. C 35 (1987) 405. Then I tried to extend this approach to real QCD using functional techniques

O.N. Ann. Phys. 209 (1991) 436

In this approach the pomeron could be written as the coherent exchange of elementary exchanges of spin 2+4+6+....



Note that this series starts with spin 2 and that spin 0 is missing. This is a direct consequence of the vector nature of the basic quark-gluon coupling in QCD.

We also found an <u>odderon</u> with <u>vector character</u>.

In our present model we have incorporated exactly these structures for $\mathbb{P}$ and $\mathbb{O}$ giving it a good basis in QCD.

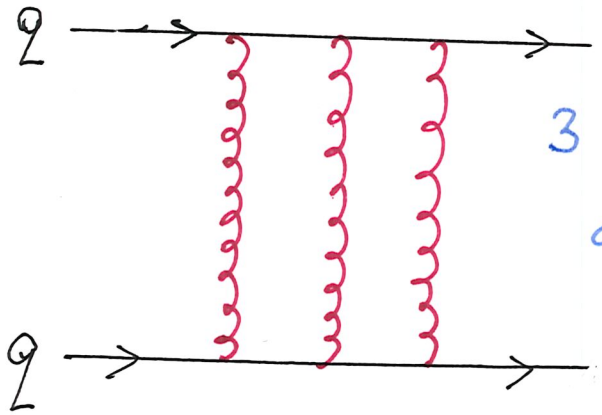
Investigations of the pomeron using the methods of AdS / CFT correspondence also prefer a tensor nature of the pomeron:

Domokos, Harvey, Mann, PR D 80 (2009) 126015

Iatrakis, Ramamurti, Shuryak, PR D 94 (2016) 045005

Let me remark that the "odderon question" relates to the existence of the <u>nonperturbative odderon</u>.

In perturbative QCD clearly odderon effects exist. The simplest realisation is 3-gluon exchange in a colour singlet state d_{abc}, e.g. in quark-quark scattering



3 gluons in colour d_{abc} state, $C = -1$

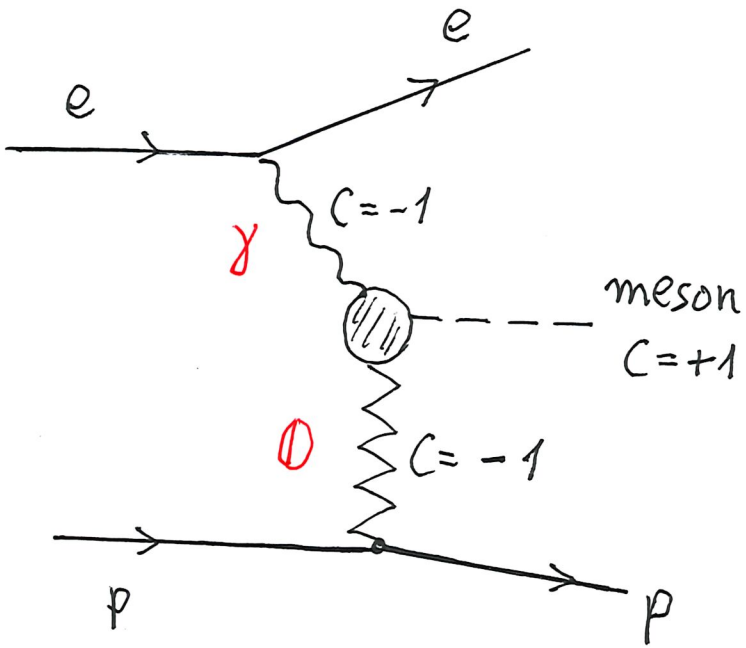
3 From HERA to ultraperipheral Ap collisions

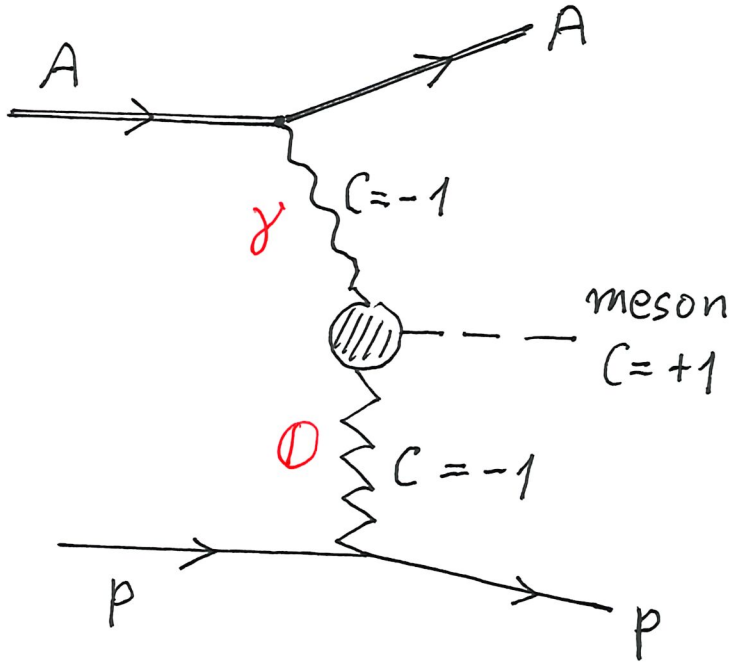
In 1990/91 there was a workshop at DESY: "Physics at HERA". HERA experiments started in 1992. At this workshop we suggested to look for the odderon in photo- and electroproduction of $C=+1$ mesons, e.g. η, η', η_c.

A. Schäfer, L. Mankiewicz, O.N., Proc. "Physics at HERA", 1991

The same idea was put forward, independently, by

V.V. Barakhosky, I.R. Zhitnitsky, A.N. Shelkovenko, P.L. B 267 (1991) 532





mesons with $C=+1$: π^0, η, η', η_c, f_2

The same reactions as at HERA can be studied in CEP in ultraperipheral Ap collisions at the LHC.

Results from HERA are directly relevant for the LHC.

We think that a rather promising reaction to look for odderon effects is $f_2(1270)$ photoproduction at HERA or in CEP in ultraper. Ap collisions at LHC.

Around 2010 I attended a seminar from André Schöning's group where nice results for ρ photoproduction were shown.

$$\gamma + p \longrightarrow \rho + p, \quad \rho \to \pi^+ \pi^-$$

$\mathbb{P}$ exchange reaction

After the seminar I asked André if he could also look at higher $\pi^+\pi^-$ invariant masses, in the f_2 region,

$$\gamma + p \longrightarrow f_2 + p, \quad f_2 \to \pi^+ \pi^-$$

$\mathbb{O}$ exchange reaction

There will be interference between these processes and as a consequence $\pi^+ \pi^-$ charge asymmetries.

Charge asymmetries as a signal for odderon exchange were first discussed by

Brodsky, Rathsman, Merino, P.L. B 461 (1999) 114

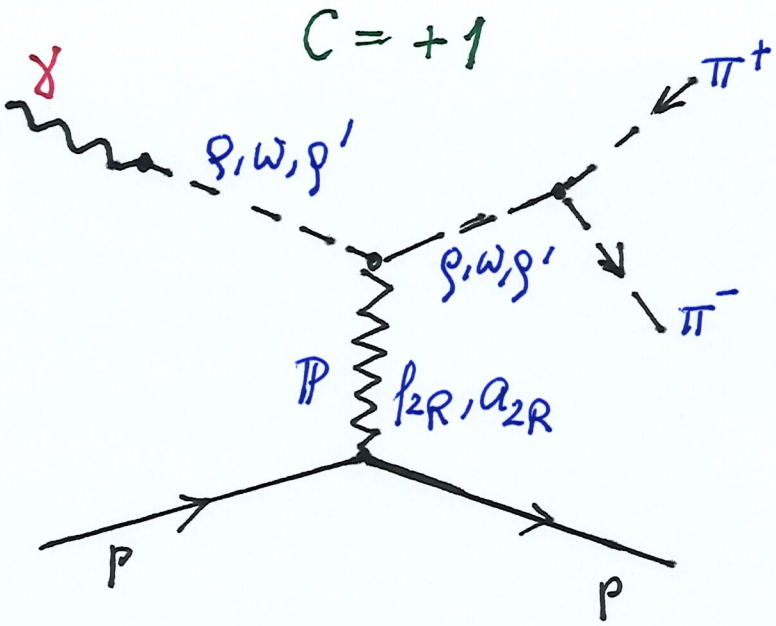
Ivanov, Nikolaev, Ginzburg, Proc. DIS 2001, hep-ph/0110181

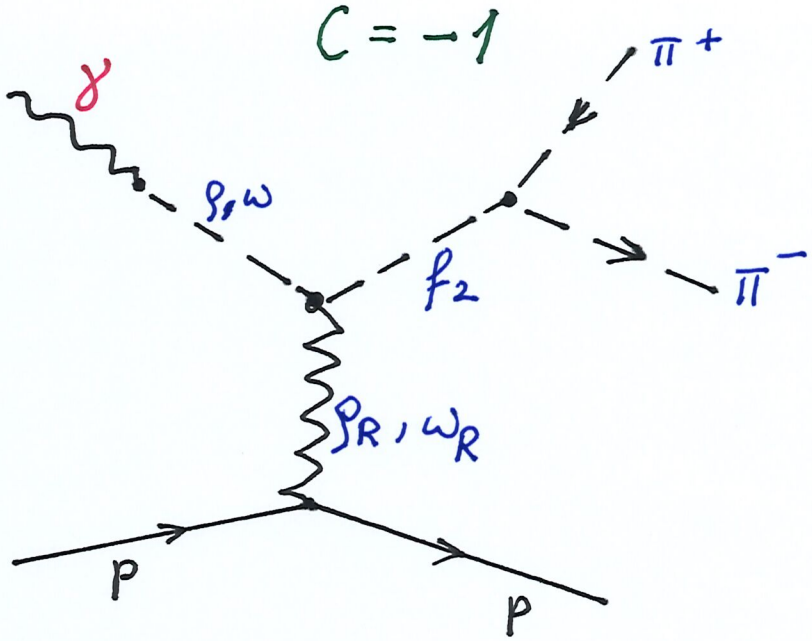
André's answer was that he in turn asked me 3 questions:

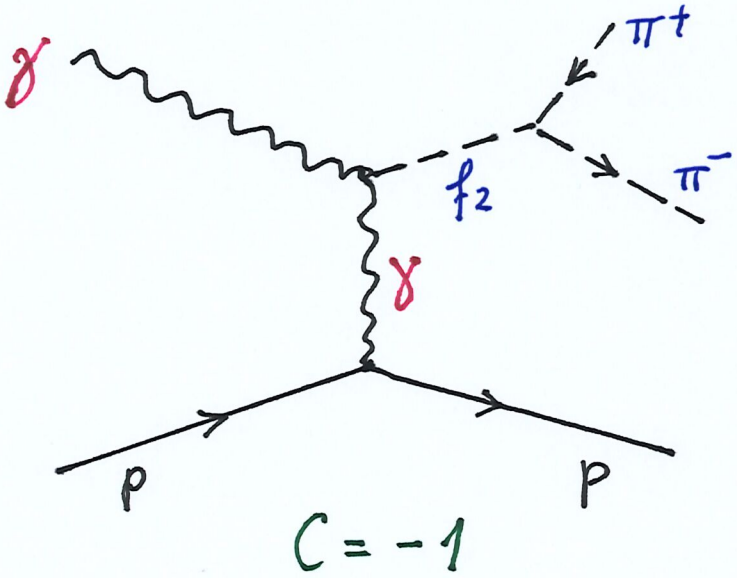
- How large is the effect? Do you have a theoretical model allowing to make predictions?
- Is there a background and how large is it?
- Is there a corresponding event generator?

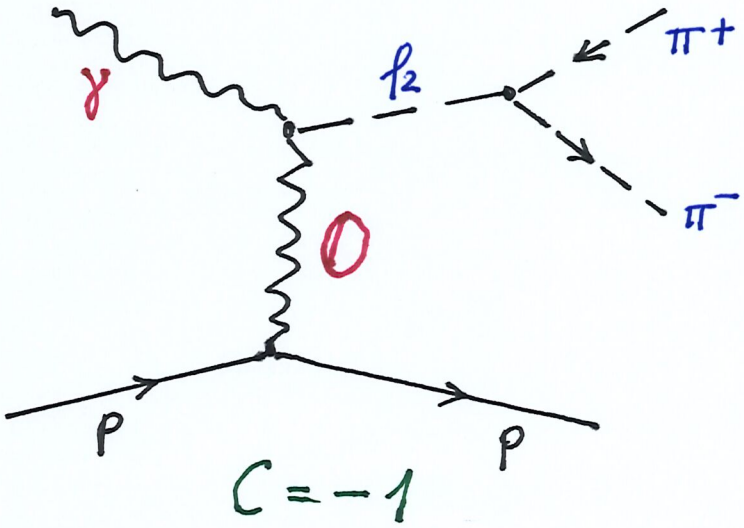
4 Tensor pomeron and Vector odderon

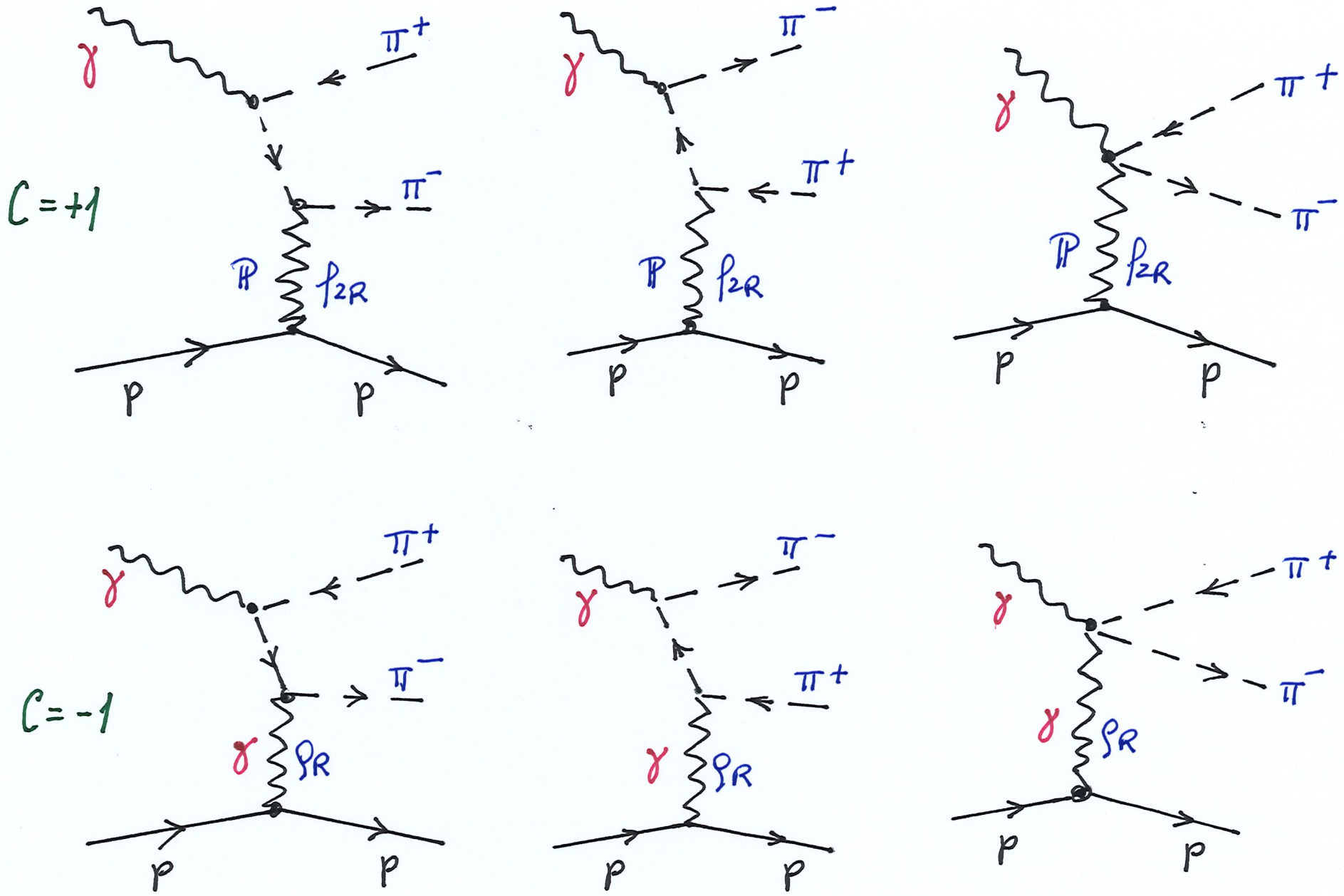
To answer André's questions we drew all the diagrams for high-energy $\pi^+\pi^-$ photoproduction we could think of. There is a great number of such diagrams and it was clear that we needed some effective model in order to treat them all. At first we tried to use for the pomeron $\mathbb{P}$ the Donnachie-Landshoff ansatz where the pomeron is treated as an effective vector exchange. But this led to – for us – unsurmountable problems with gauge invariance.


C = +1
γ
ρ, ω, ρ'
π+
ρ, ω, ρ'
π−
IP
f2R, a2R
p
p


C = −1
γ
ρ, ω
π+
f2
π−
ρR, ωR
p
p


γ
π+
f2
π−
γ
p
p
C = −1


γ
f2
π+
π−
0
p
p
C = −1


C = +1
C = -1
γ
π+
π−
ℙ
f2R
ρR
p
p

Our solution to these problems was to construct an effective theory for high-energy soft reactions where

- all $C=+1$ exchanges $\mathbb{P}$, f_{2R}, a_{2R} are treated as effective spin 2 symmetric tensor exchanges,
- all $C=-1$ exchanges $\mathbb{O}$, ω_R, ρ_R are treated as effective vector exchanges.

(Ewerz, Maniatis, O.N., Ann. Phys. 342 (2014) 31)

The theory is formulated in terms of propagators and vertices. The vertices are derived from Lagrangians for the couplings. In this way inclusion of photons is straightforward and gauge invariance is guaranteed.

The Regge factors are incorporated in the effective propagators.

Now we were equipped with a model allowing us to calculate all diagrams for $\pi^+\pi^-$ photoproduction and our experimental friends also produced an event generator for it.

I shall show you results for

- total cross section $\sigma(\gamma p \to \pi^+\pi^- p)$,
- distribution of the invariant mass $m_{\pi^+\pi^-}$,
- charge asymmetry of π^+, π^- in the proton-Jackson system,
- a detailed study (preliminary) of the shape of the ρ resonance.

Bolz, Ewerz, Maniatis, O.N., Sauter, Schöning, JHEP 01 (2015) 151

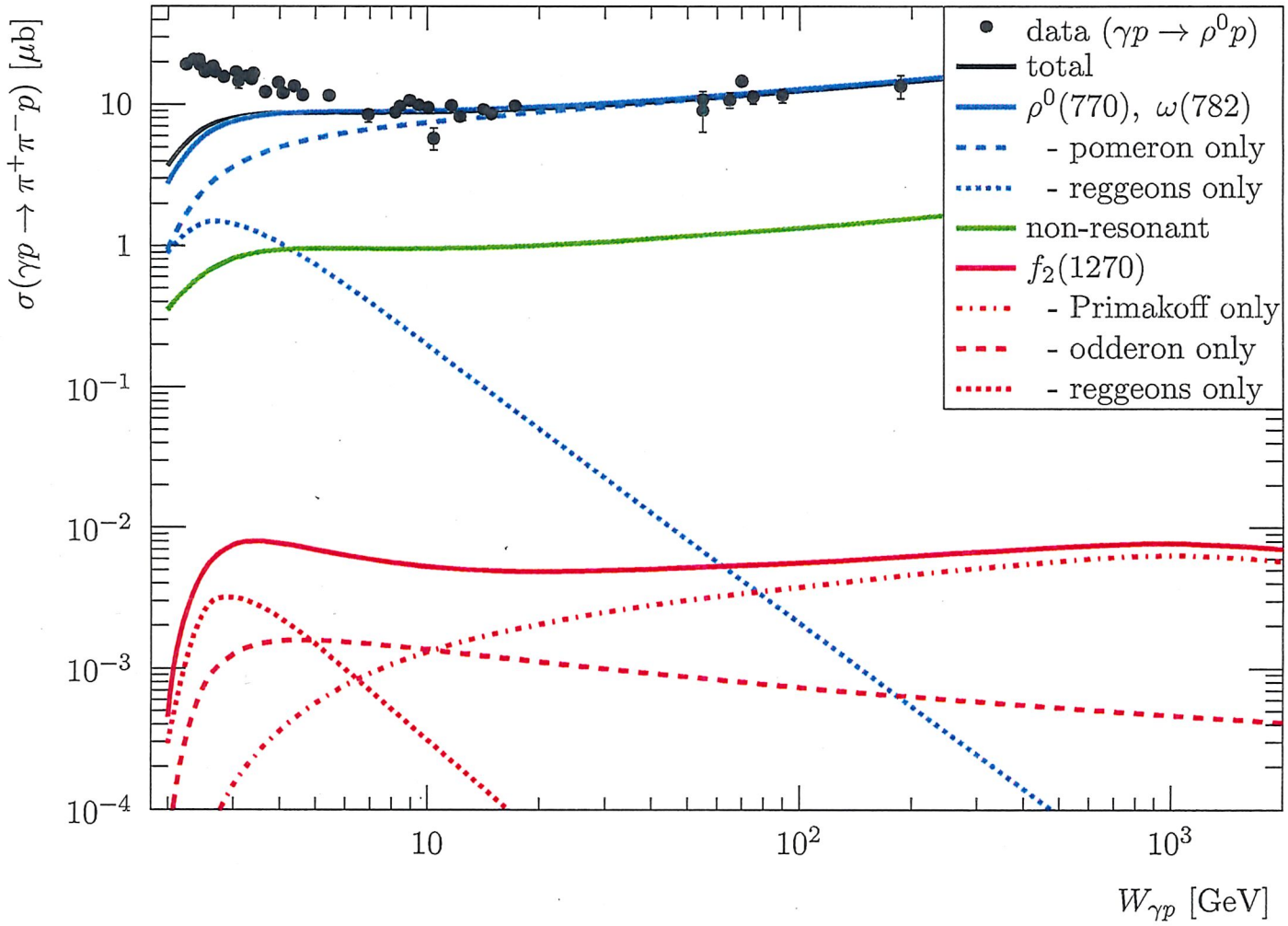
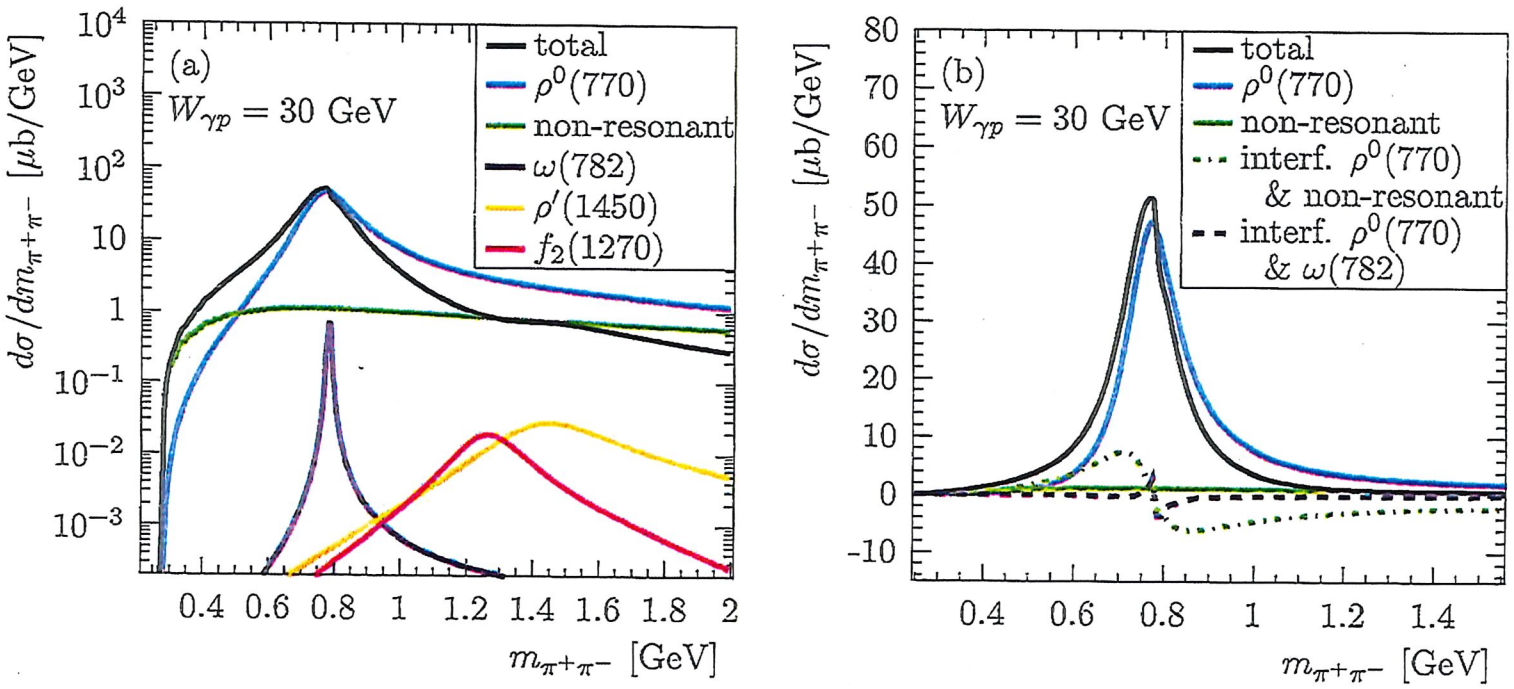
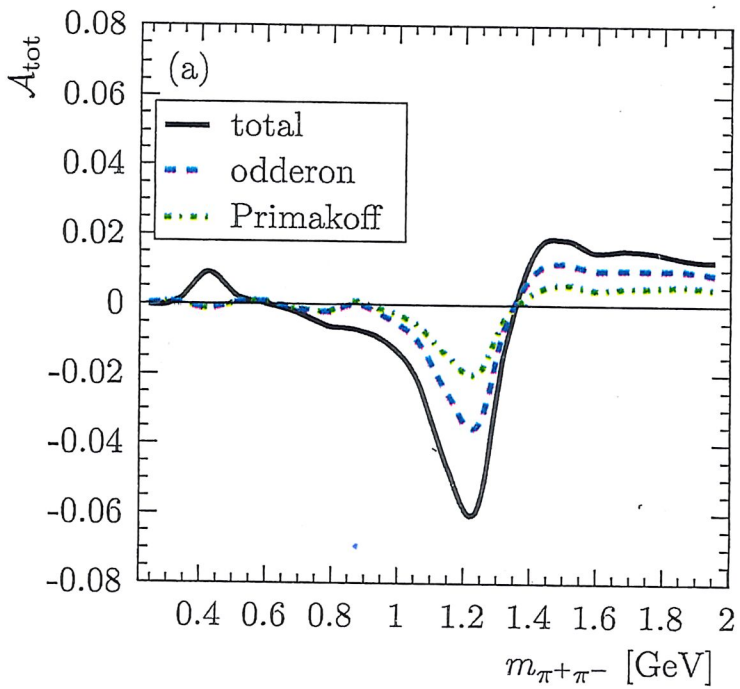
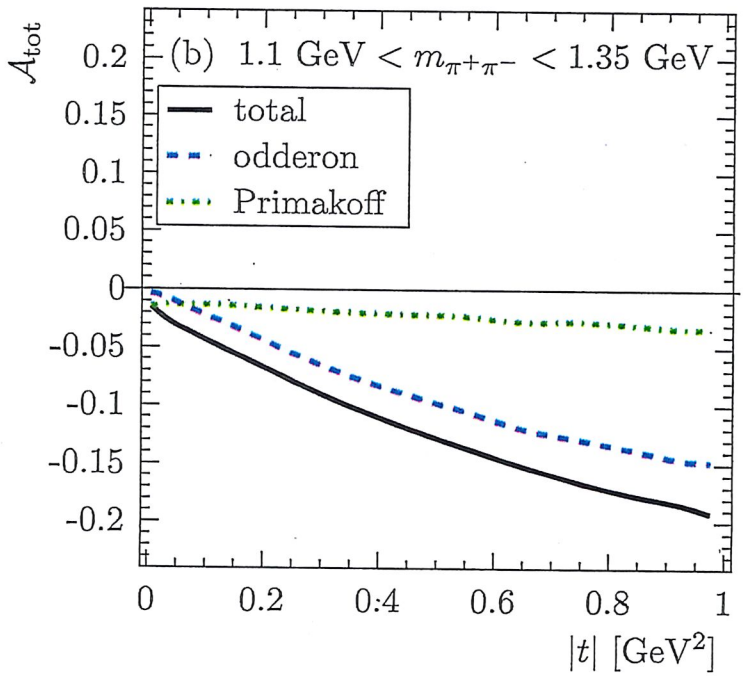


Figure 3. The total cross section $\sigma\left(\gamma p \to \pi^+\pi^- p\right)$ as a function of the center-of-mass energy $W_{\gamma p}$. The cross section is integrated over $2m_\pi \leq m_{\pi^+\pi^-} \leq 1.5\,\text{GeV}$ and $-1\,\text{GeV}^2 \leq t \leq 0$. The full model and individual contributions from vector meson production, non-resonant processes, and f_2 production are shown. The reggeon contributions comprise f_{2R} and a_{2R} in case of vector meson, and ρ_R and ω_R in case of f_2 production. High energy data for $\sigma\left(\gamma p \to \rho^0 p\right)$ from H1 [17] and ZEUS [16, 18] at HERA as well as fixed target data, referenced in [18], are shown for illustration.

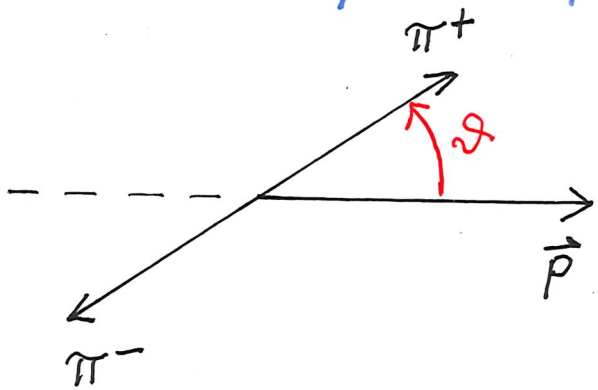


Distribution of the invariant mass $m_{\pi^+\pi^-}$.
Note the "skewing" of the ρ shape due to interference effects. The "pure" ρ parameters and the ρ-ω interference are determined from e^+e^- annihilation.
(Melikhov, D.N., Nikonov, Paulus, EPJC 34 (2004) 345)



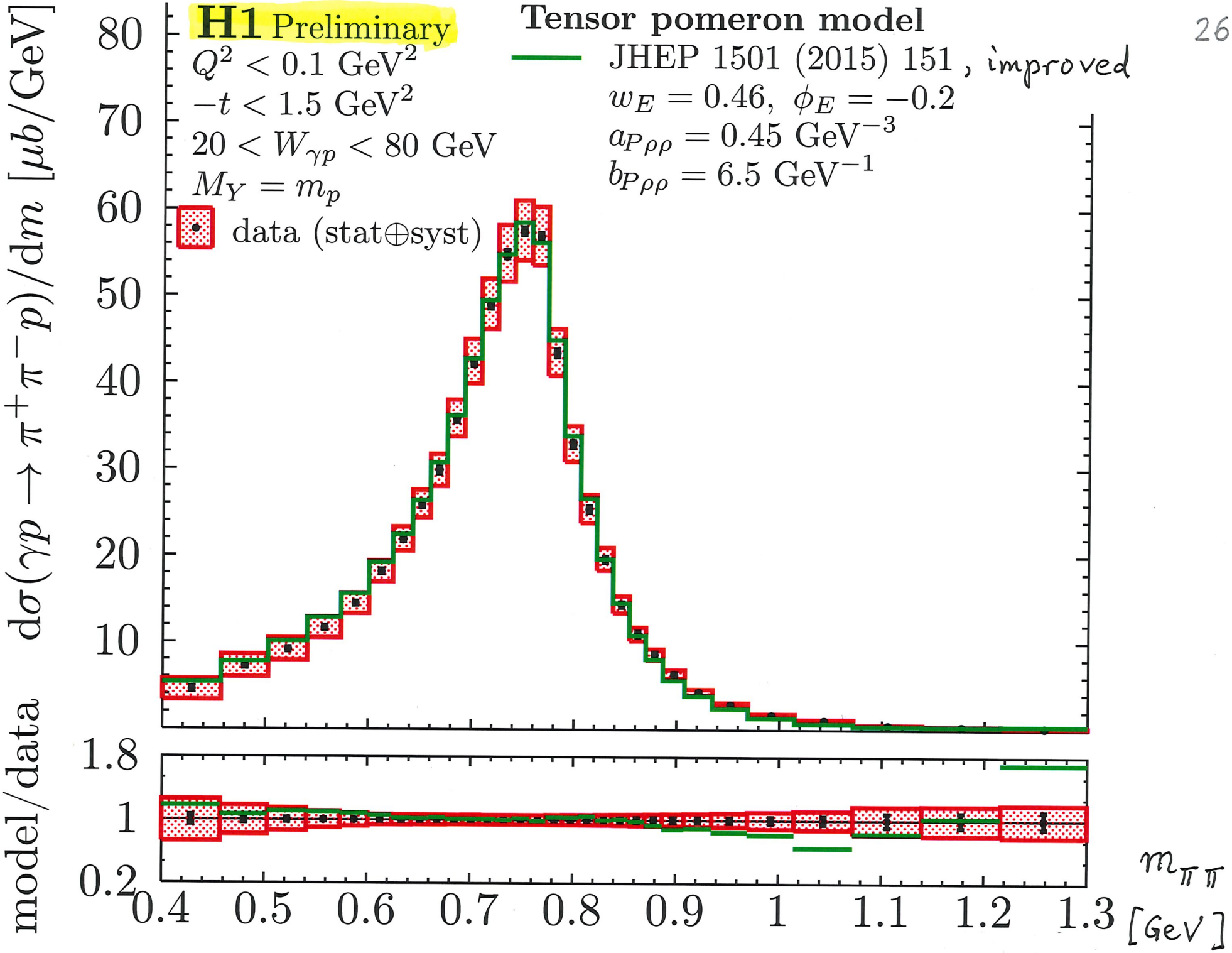


Go to the rest system of $\pi^+\pi^-$



Charge asymmetry:

$$A = \frac{\sigma(\cos\vartheta > 0) - \sigma(\cos\vartheta < 0)}{\sigma(\cos\vartheta > 0) + \sigma(\cos\vartheta < 0)}$$


H1 Preliminary
$Q^2 < 0.1$ GeV^2
$-t < 1.5$ GeV^2
$20 < W_{\gamma p} < 80$ GeV
$M_Y = m_p$
data (stat⊕syst)
Tensor pomeron model
JHEP 1501 (2015) 151, improved
$w_E = 0.46$, $\phi_E = -0.2$
$a_{P\rho\rho} = 0.45$ GeV^{-3}
$b_{P\rho\rho} = 6.5$ GeV^{-1}
$d\sigma(\gamma p \to \pi^+\pi^- p)/dm$ [μb/GeV]
model/data
$m_{\pi\pi}$ [GeV]

Conclusion: We are looking forward to studies of $\pi^+\pi^-$ CEP in ultraperipheral Ap collisions. Most interesting is the interference of

ρ production (a $\mathbb{P}$ exchange process) with

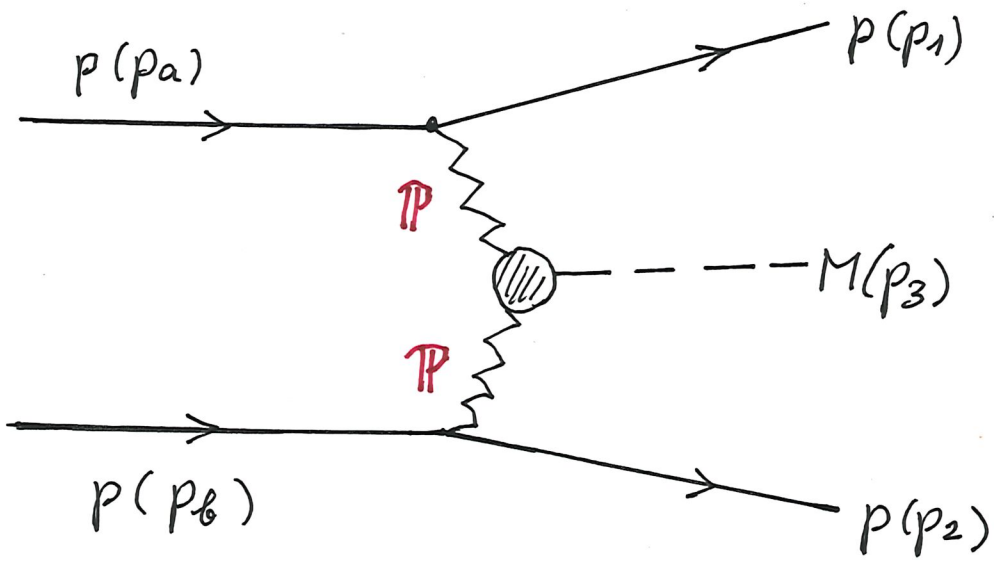
f_2 production (an $\mathbb{O}$ exchange process),

manifesting itself by $\pi^+\pi^-$ charge asymmetries.

5 CEP of mesons

Once we have a model for high-energy soft reactions we clearly can apply it to CEP. This we did extensively together with our friends in Kraków, in particular with A. Szczurek and P. Lebiedowicz. From the many reactions which we have studied I shall only present you some results for single meson M and two meson $\phi\phi$ production.

5.1 CEP of single mesons M



For ℙ ℙ fusion giving M the meson must have $I^G = 0^+$.

But various spin-parity values are possible, e.g.

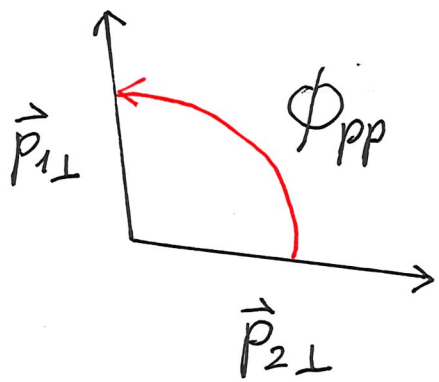
$$J^{PC} = 0^{-+},\ 0^{++},\ 1^{++},\ 2^{++},\ \ldots$$

Since we know the coupling of ℙ to the external protons we can use CEP of mesons to determine the ℙℙM couplings.

Extensive studies of CEP of mesons were done in 1990 - 2000 by the WA76, WA91, and WA102 collaborations. The c.m. energies were $\sqrt{s} = 12.7 - 29.1$ GeV.

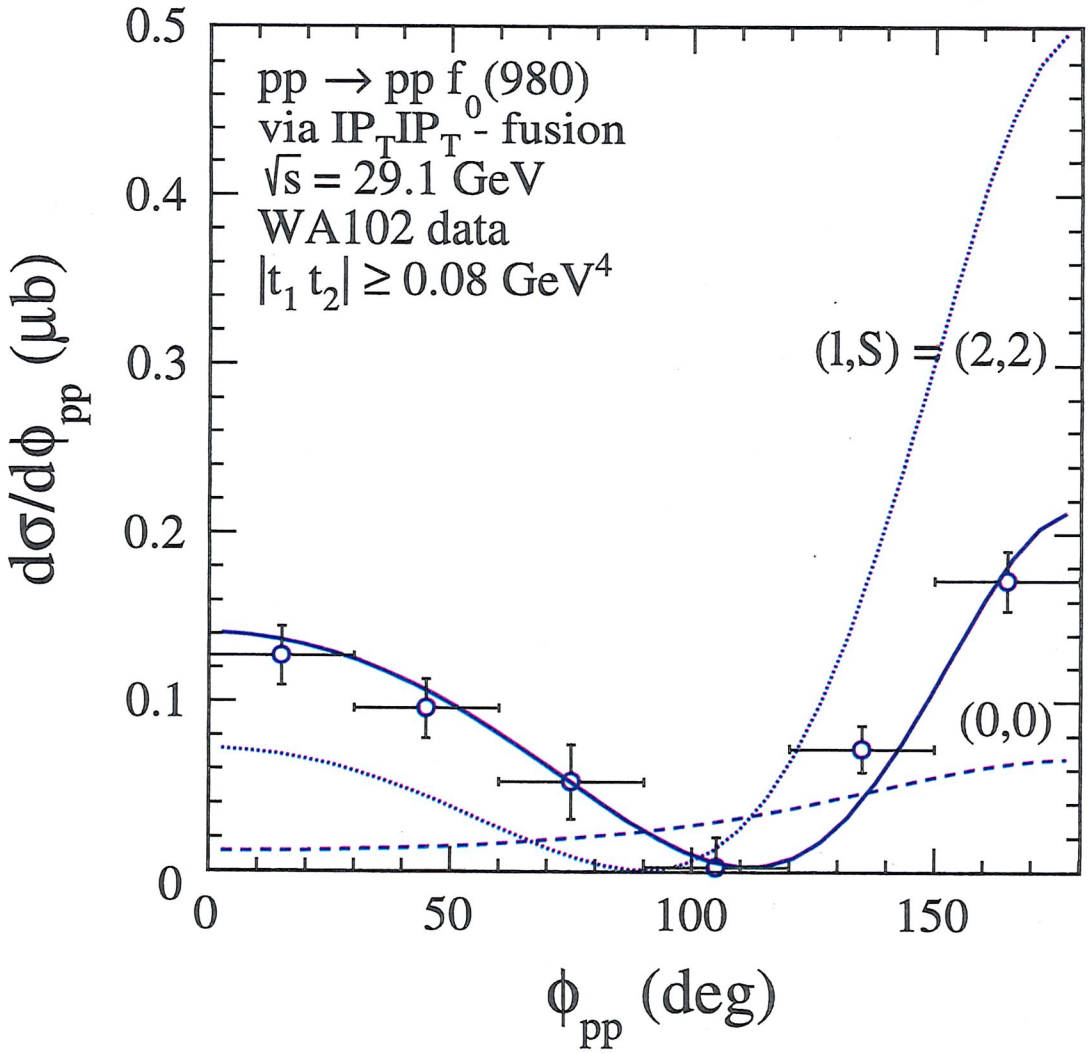
For a review see A. Kirk, Phys. Lett. B489 (2000) 29.

In these experiments the complete final state was detected. Thus, they could measure total cross sections, t and Feynman x_F distributions, but also subtle effects like the distribution in the azimuthal angle ϕ_{pp} between the outgoing protons' transverse momenta.

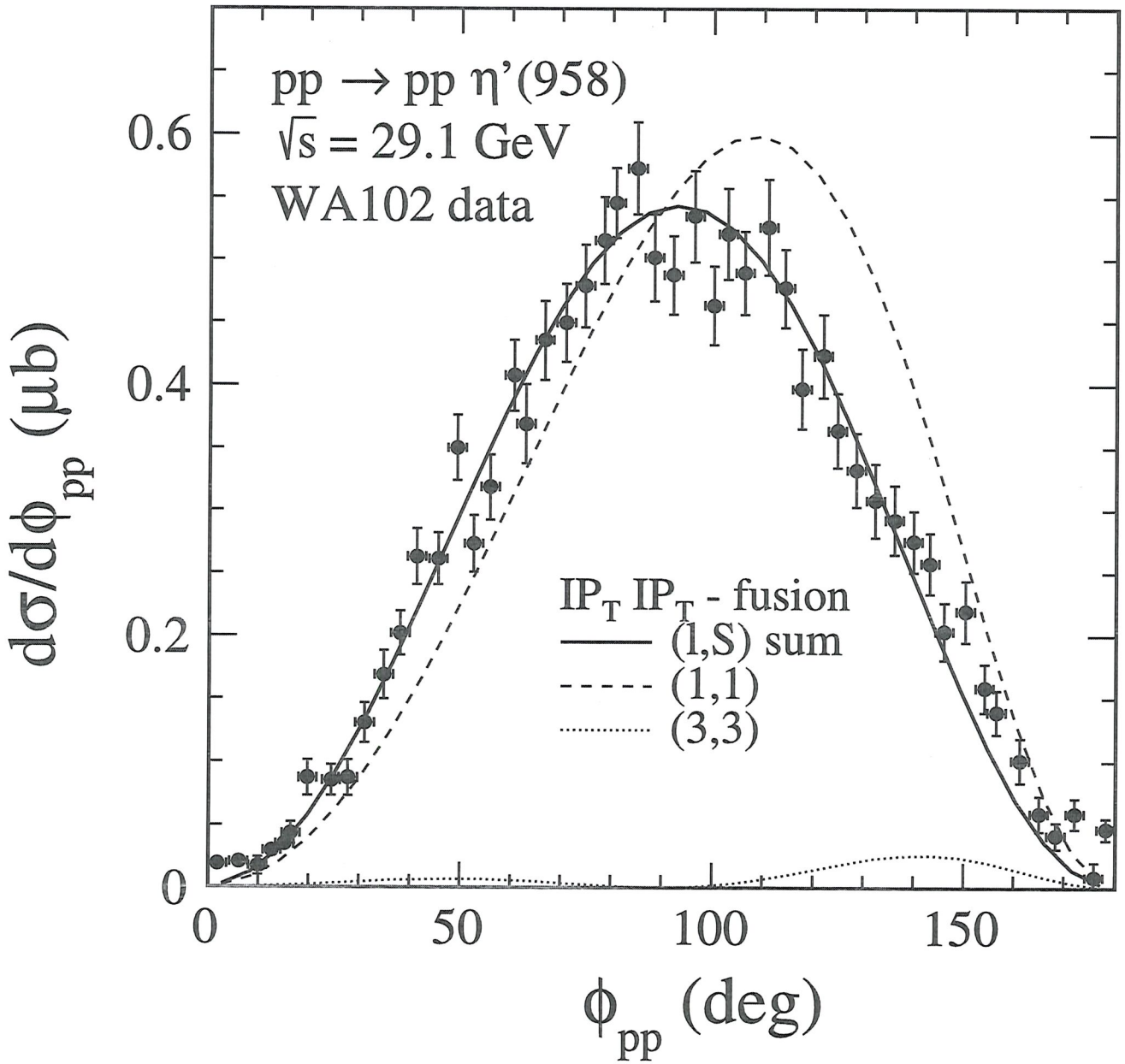


I show you results for CEP of $f_0(980)$, $\eta'(958)$, $f_0(1710)$

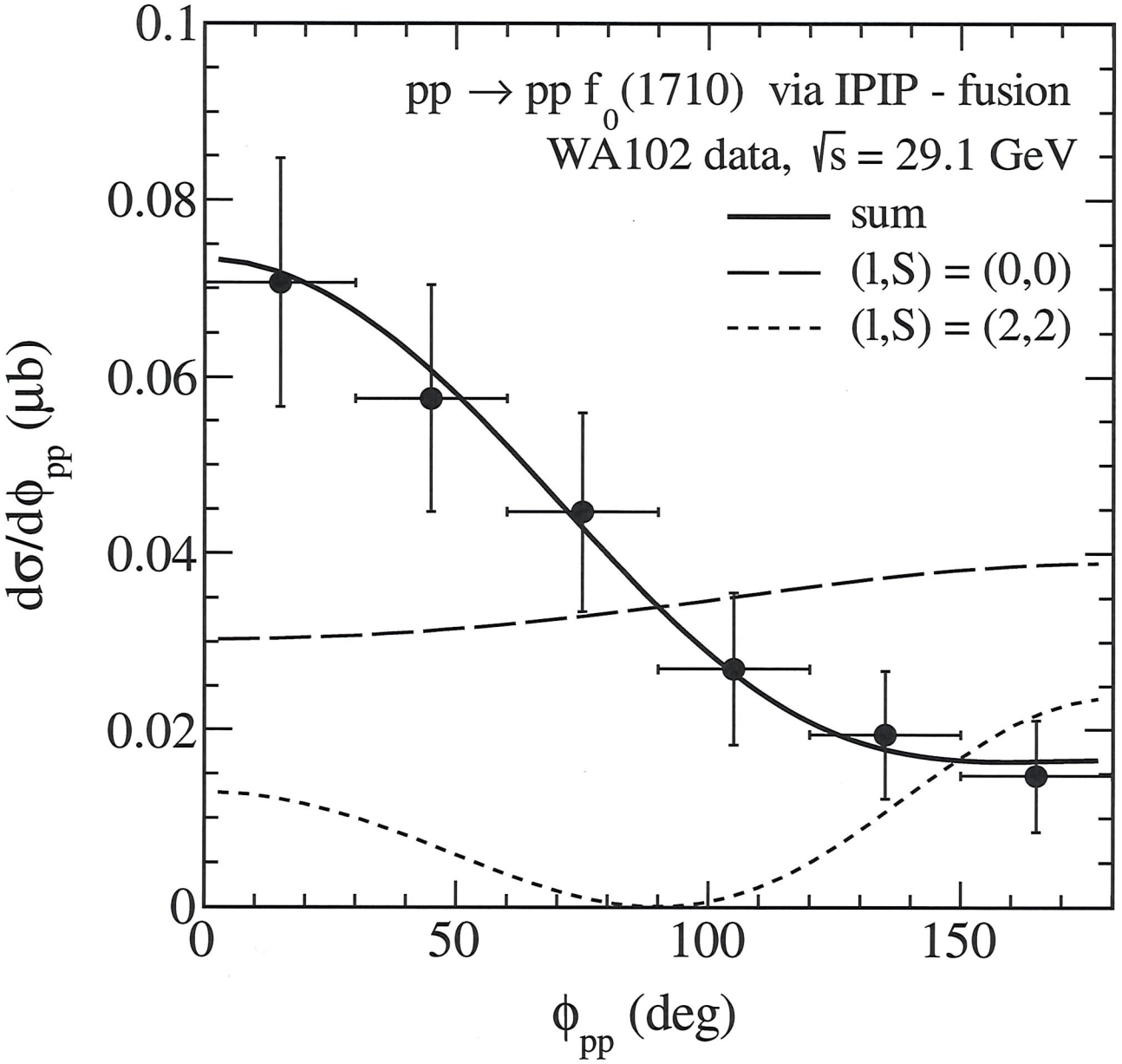
In these cases we find from the general analysis that two $\mathbb{P}\mathbb{P}M$ couplings are possible. In all these cases we find that for a good description of the ϕ_{pp} distributions two couplings are necessary. There are important interference effects between the contributions of these two couplings in each case.



Lebiedowicz, O.N., Szczurek, Ann. Phys. 344 (2014) 301



Lebiedowicz, O.N., Szczurek, Ann. Phys. 344 (2014) 301



Lebiedowicz, O.N., Szczurek, PRD 98 (2018) 014001

FIG. 3. The distribution in azimuthal angle ϕ_{pp} between the outgoing protons for central exclusive production of the $f_0(1710)$

I think that from these examples we can conclude the following for CEP of single mesons at the LHC:

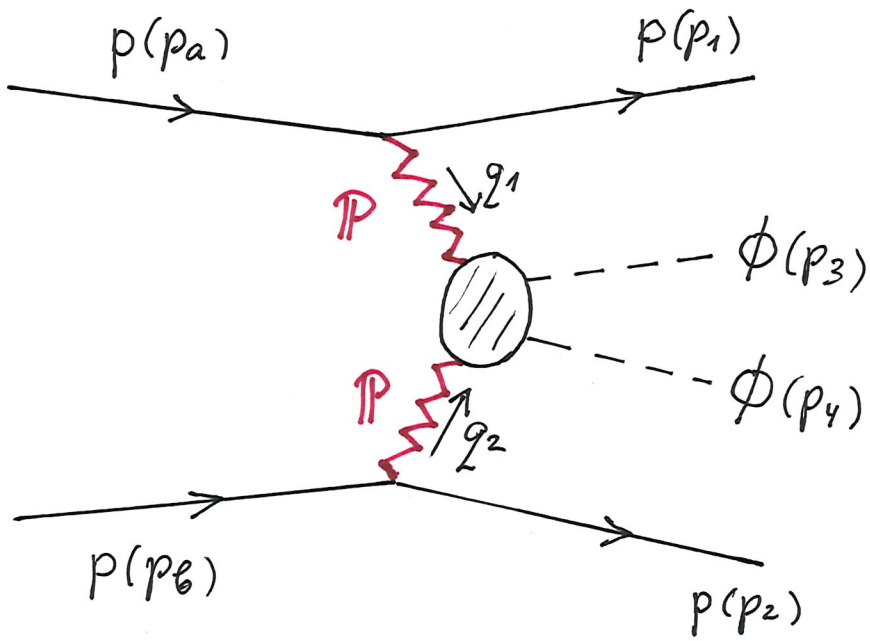
- it would be very useful to measure the outgoing protons,
- the angle ϕ_{pp} gives the information on the $\mathbb{P}\mathbb{P}M$ couplings,
- a further interesting variable is the so-called "glueball-filter variable", $dP_\perp = |\vec{p}_{2\perp} - \vec{p}_{1\perp}|$. Undisputed $q\bar{q}$ states (η, η', $f_1(1285)$ ...) are suppressed for $dP_\perp \to 0$, glueball candidates ($f_0(1500)$, $f_2(1950)$ ...) are prominent for $dP_\perp \to 0$.

Close and Kirk, PL B 397 (1997) 333

5.2 The reaction $p\,p \longrightarrow p\,\phi\,\phi\,p$

$$p(p_a) + p(p_b) \longrightarrow p(p_1) + \phi(p_3) + \phi(p_4) + p(p_2)$$

At high energies we expect this reaction to be dominated by the fusion process $\mathbb{P}\,\mathbb{P} \longrightarrow \phi\,\phi$



$$q_1 = p_a - p_1\,,$$

$$q_2 = p_b - p_2\,.$$

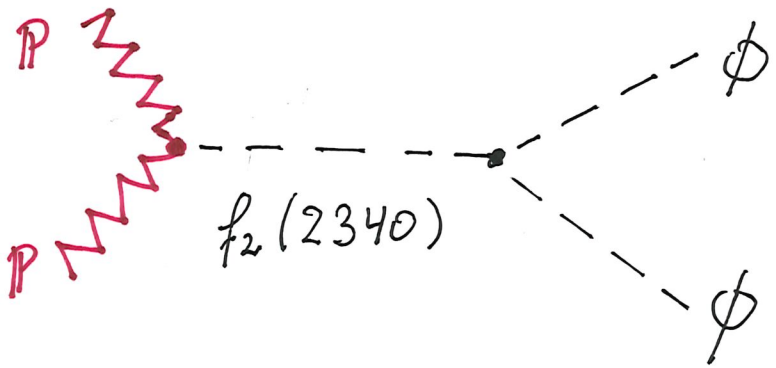
P. Lebiedowicz, O.N., A. Szczurek, PRD 99 (2019) 094034

What can we expect for the process

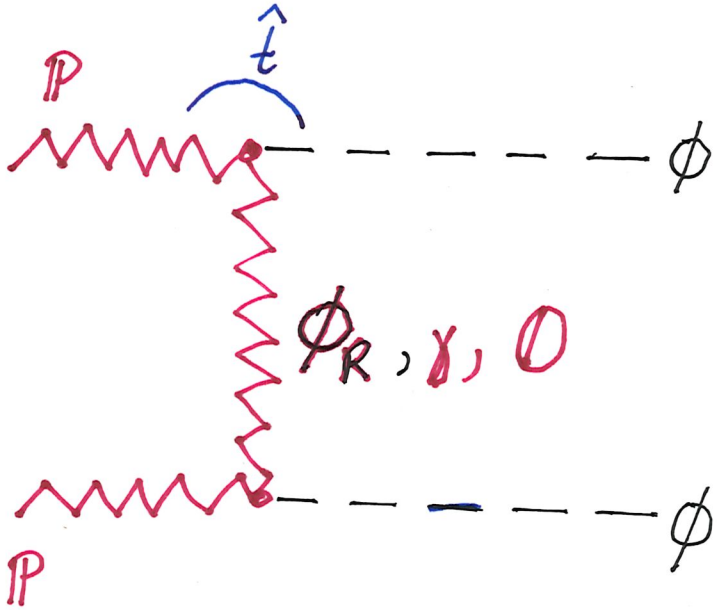
$$\mathbb{P}(q_1) + \mathbb{P}(q_2) \longrightarrow \phi(p_3) + \phi(p_4)$$

as function of its c.m. energy $M_{\phi\phi} = \sqrt{(p_3+p_4)^2}$?

- Resonances at low $M_{\phi\phi}$, e.g.



- Regge exchanges at high $M_{\phi\phi}$. But here there can only be $C = -1$ exchanges.



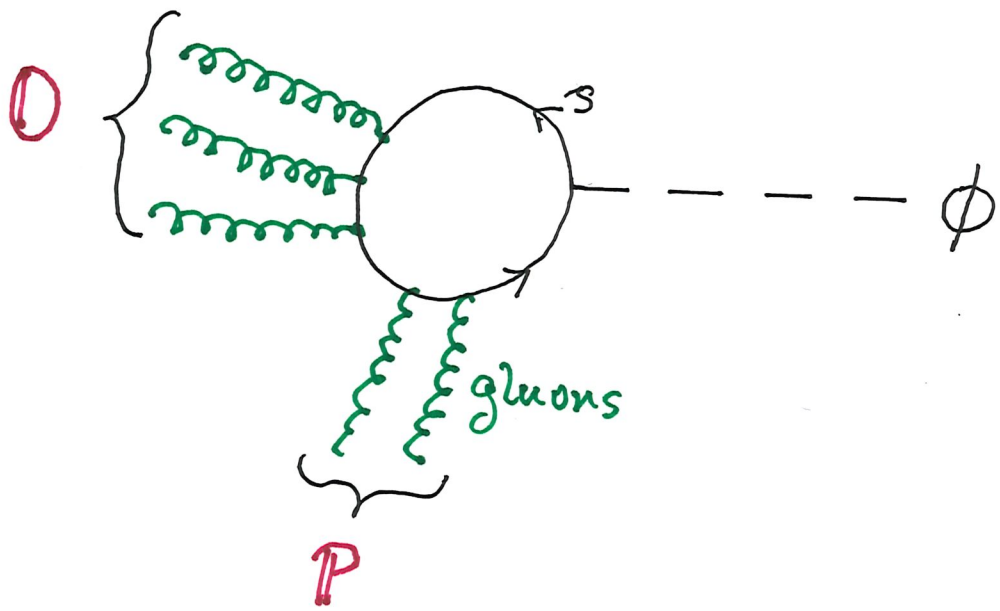
contributions: γ : negligeable

ϕ_R : $\propto (M^2_{\phi\phi})^{\alpha_\phi(\hat{t})-1}$, $\alpha_\phi(\hat{t}) = 0.1 + 0.9\,\hat{t}$

$\mathbb{O}$: $\propto (M^2_{\phi\phi})^{\alpha_{\mathbb{O}}(\hat{t})-1}$, $\alpha_{\mathbb{O}}(0) \approx 1$?

If $\alpha_{\mathbb{O}} \approx 1$, then $\mathbb{O}$ exchange will win for large $M_{\phi\phi}$.

A simple QCD diagram contributing to the $\mathbb{P}\mathbb{O}\phi$ vertex:



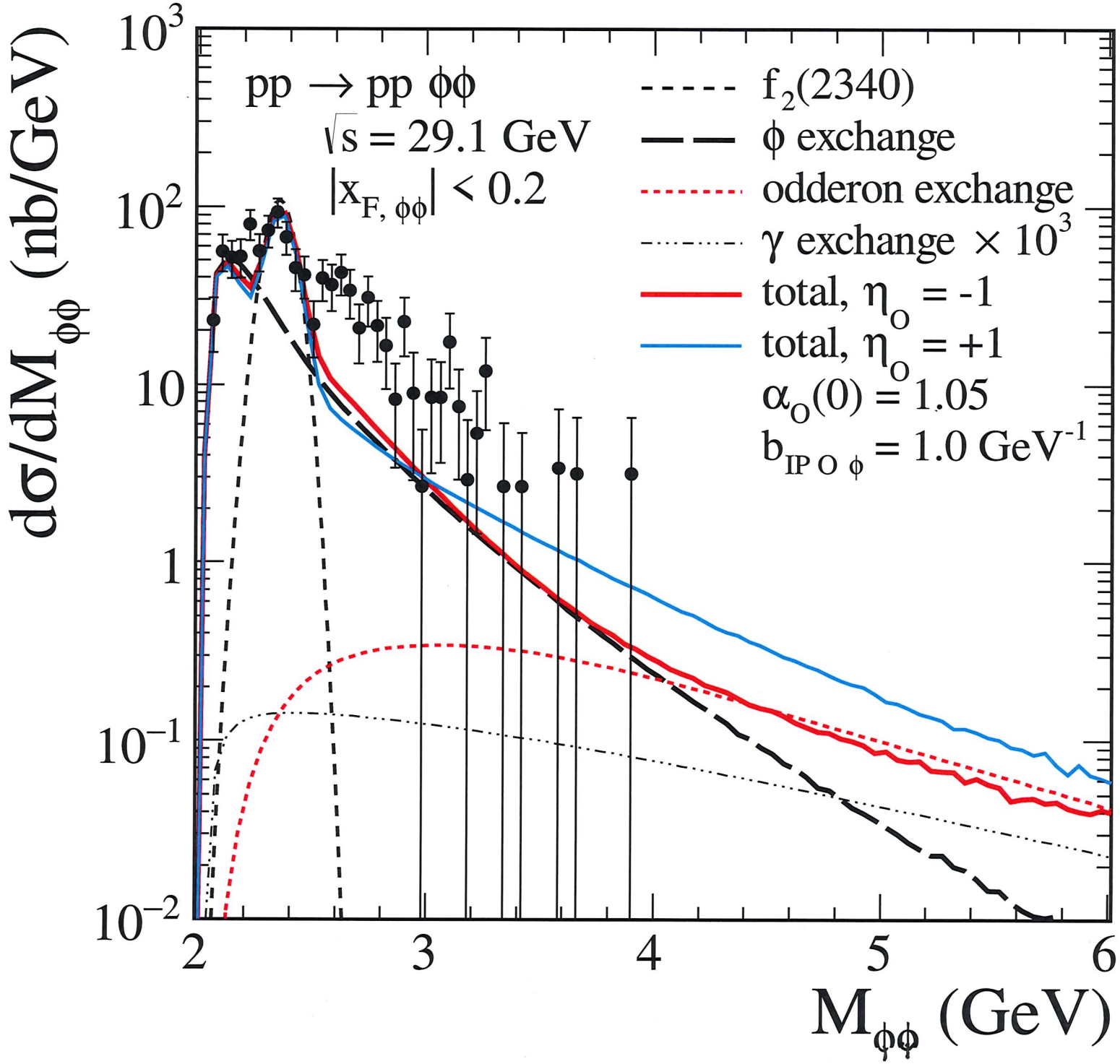
To look for odderon effects in CEP of $\phi\phi$, $J/\psi J/\psi$, $\Upsilon\Upsilon$, was suggested in C. Ewerz, arXiv: hep-ph/0306137.

The signal are <u>three-gap events</u>:

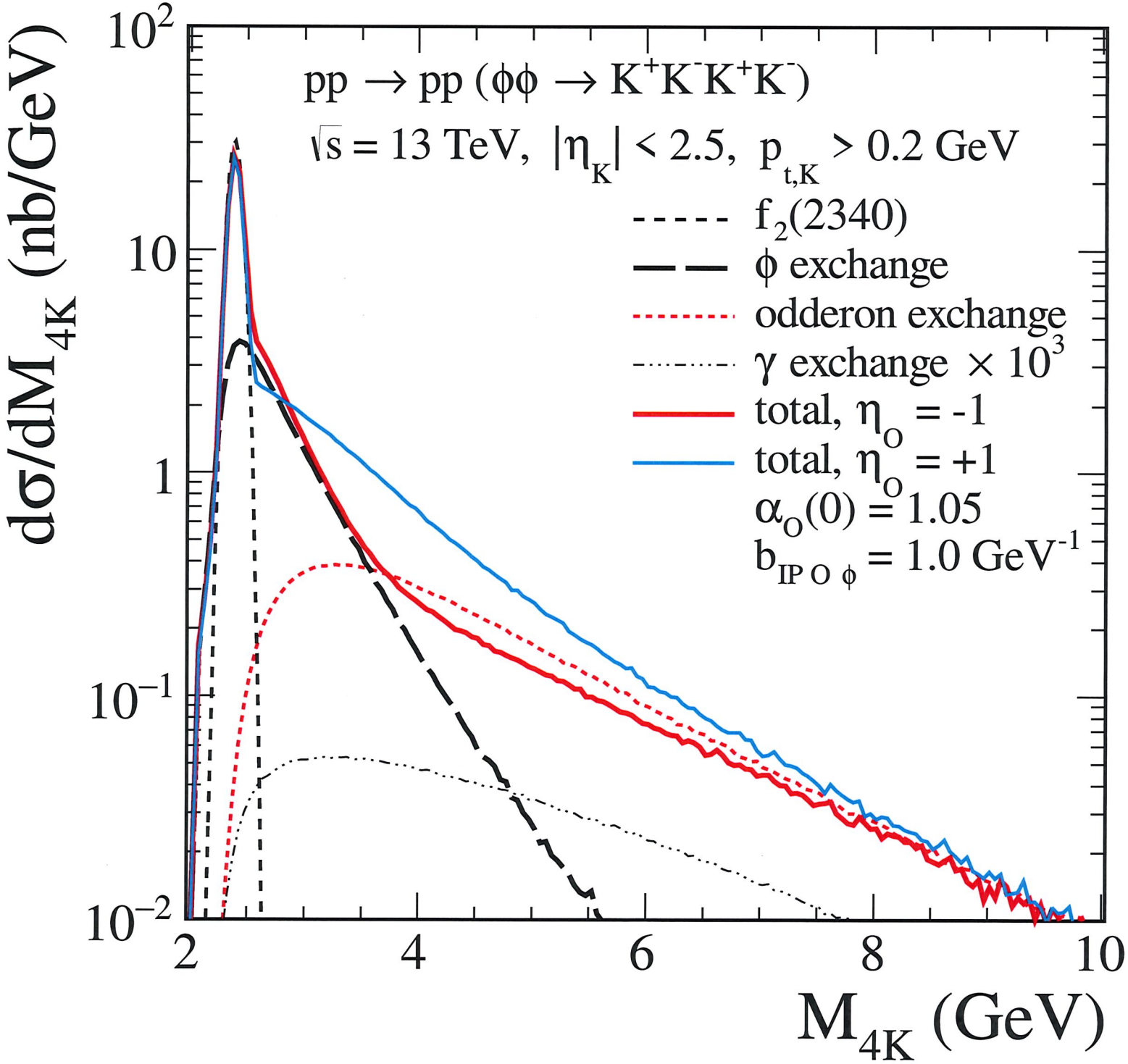
$$p - gap - \phi - gap - \phi - gap - p$$

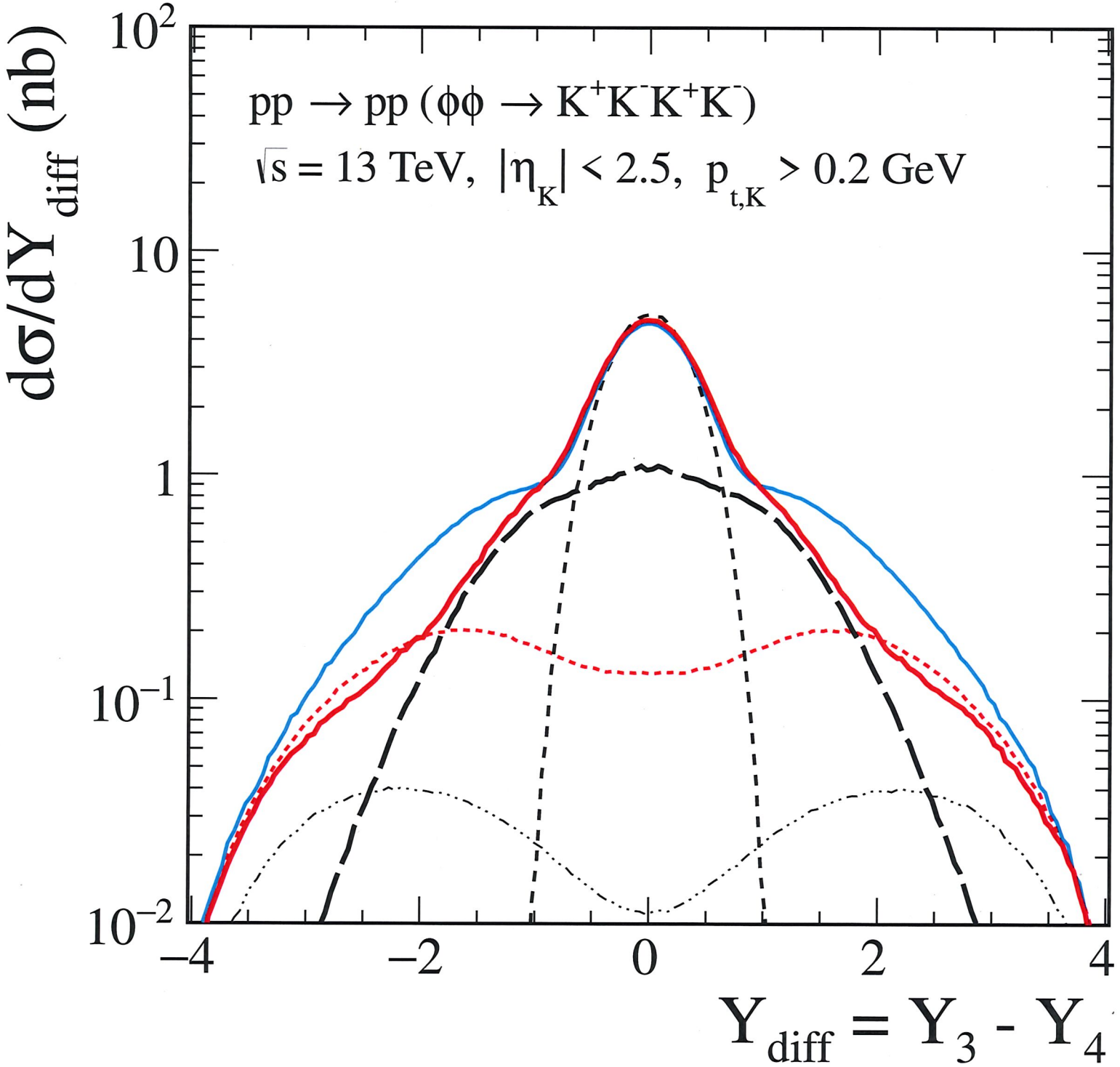
In the following I show you some of our results for the WA 102 energy $\sqrt{s} = 29.1$ GeV and for LHC energies. In the calculations absorption effects have been included.

In principle CEP of $\phi\phi$ offers the possibility to determine the $\mathbb{P}\mathbb{O}\phi$ coupling and the $\mathbb{O}$ regge trajectory.


pp → pp φφ
√s = 29.1 GeV
|x_F, φφ| < 0.2
f_2(2340)
φ exchange
odderon exchange
γ exchange × 10^3
total, η_O = -1
total, η_O = +1
α_O(0) = 1.05
b_IP O φ = 1.0 GeV^-1
dσ/dM_φφ (nb/GeV)
M_φφ (GeV)
10^3
10^2
10
1
10^-1
10^-2
2
3
4
5
6

Data:
WA 102


pp → pp (φφ → K+K-K+K-)
√s = 13 TeV, |η_K| < 2.5, p_t,K > 0.2 GeV
f2(2340)
φ exchange
odderon exchange
γ exchange × 10^3
total, η_O = -1
total, η_O = +1
α_O(0) = 1.05
b_IP O φ = 1.0 GeV^-1
dσ/dM_4K (nb/GeV)
M_4K (GeV)
10^2
10
1
10^-1
10^-2
2
4
6
8
10



Y_3, Y_4 : rapidities of the ϕ mesons

6 Conclusions

- I have discussed how we came to introduce our model with tensor pomeron and vector odderon. It is an effective model where some parameters have to be determined from experiment. But these parameters should then be applicable universally for soft high-energy reactions. I have shown you some applications of the model to CEP.
- Do we really have to describe the pomeron by an effective tensor exchange? I think the answer is yes!
- A "scalar pomeron" can be excluded by comparing such an ansatz with the helicity structure of small $|t|$ proton-proton scattering as measured by the STAR Coll. PL B 719 (2013) 62; see Ewerz et al. PL B763 (2016) 382.

- A "vector pomeron" can be excluded for many reasons. In Britzger et al., arXiv: 1901.08524 (2019) we give one very good reason: a "vector pomeron" cannot couple in the total photo absorption cross section $\sigma_{\gamma p}$. On the other hand, our tensor pomeron gives an excellent description.
- We have also considered low x DIS in our tensor pomeron approach introducing in addition to the "soft pomeron" a "hard pomeron". I think that we get very nice results also there.
- We are looking forward to many checks of the model in CEP at the LHC.

Thank you for
your attention!

Additional Material

A Effective propagators and vertices of the Tensor Pomeron Model

B Helicity in pp elastic scattering and the spin structure of the pomeron

C Photoproduction and low x DIS in the two-tensor-pomeron model

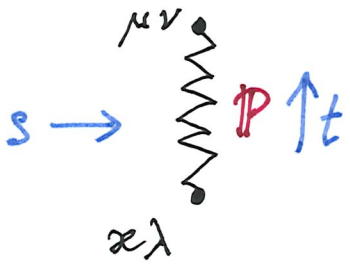
D Various CEP reactions in the tensor-pomeron model

A Effective Propagators and Vertices of the Tensor - Pomeron Model

Here I give some examples of propagator and vertices of our model.

Pomeron exchange: $P^{\mu\nu}(x) = P^{\nu\mu}(x)$, $P^{\mu\nu}(x)\, g_{\mu\nu} = 0$.

- effective propagator

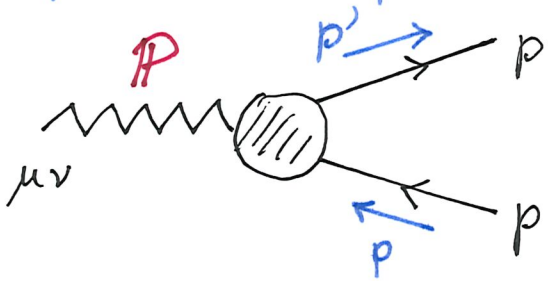


$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)$$

$$(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\alpha_{\mathbb{P}}(t) = 1 + \varepsilon_{\mathbb{P}} + \alpha'_{\mathbb{P}}\, t \quad , \quad \varepsilon_{\mathbb{P}} = 0.0808 \, , \quad \alpha'_{\mathbb{P}} = 0.25\ \mathrm{GeV}^{-2}$$

- pomeron – proton vertex



$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i\,3\beta_{\mathbb{P}NN}\, F_1\left[(p'-p)^2\right]$$

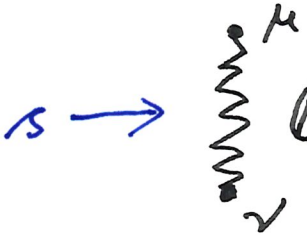
$$\left\{\frac{1}{2}\gamma_\mu(p'+p)_\nu + (\mu\leftrightarrow\nu) - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

$$3\beta_{\mathbb{P}NN} = 3\times 1.87\ \mathrm{GeV}^{-1}$$

Parameters for $\mathbb{P}$ are determined from pp elastic scattering.

Odderon exchange: parameters still to be determined by exp.

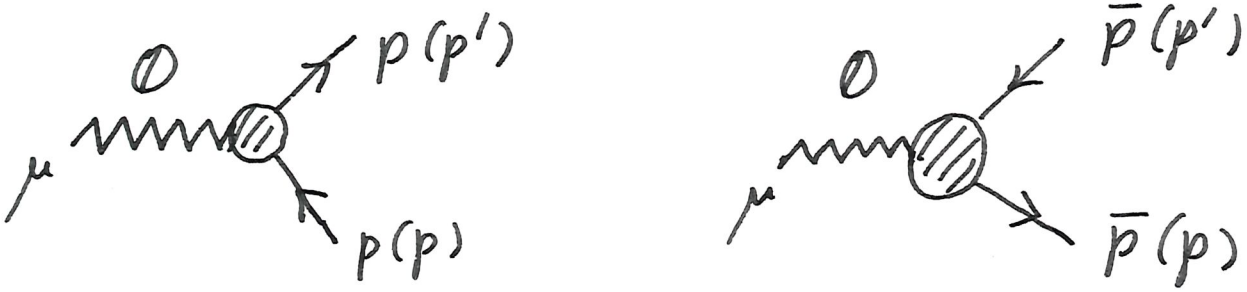
- effective propagator:



$$i\Delta^{(\mathbb{O})}_{\mu\nu} = -i g_{\mu\nu} \frac{\eta_{\mathbb{O}}}{M_0^2} \left(-i s \alpha'_{\mathbb{O}}\right)^{\alpha_{\mathbb{O}}(t)-1} ,$$

$$\eta_{\mathbb{O}} = \pm 1 , \quad M_0 = 1\,\mathrm{GeV}$$

- $\mathbb{O}NN$ vertex:



$$i\Gamma^{(\mathbb{O}pp)}_{\mu}(p',p) = -i\Gamma^{(\mathbb{O}\bar{p}\bar{p})}_{\mu}(p',p) = -i\,3\beta_{\mathbb{O}pp} M_0 F_1(t)\,\gamma_\mu$$

The vector ansatz for $\mathbb{O}$ automatically gives minus sign for the coupling of proton versus antiproton.

Pomeron - pomeron - meson coupling Lagrangians

- 0^{++} meson, field $\chi(x)$, $M_0 \equiv 1\,\mathrm{GeV}$

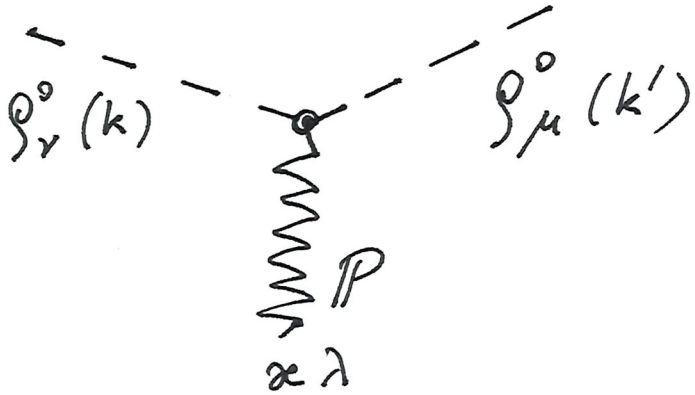
$$\mathcal{L}_{\mathbb{P}\mathbb{P}M(0^{++})} = M_0\, g'_{\mathbb{P}\mathbb{P}M}\, \mathbb{P}_{\mu\nu}\, \mathbb{P}^{\mu\nu}\, \chi$$
$$+ \frac{1}{2M_0}\, g''_{\mathbb{P}\mathbb{P}M} \left[\partial^\mu \mathbb{P}^{\nu\rho} - \partial^\nu \mathbb{P}^{\mu\rho}\right]\left[\partial_\mu \mathbb{P}_{\nu\rho} - \partial_\nu \mathbb{P}_{\mu\rho}\right]\chi$$

- 0^{-+} meson, field $\tilde{\chi}(x)$

$$\mathcal{L}_{\mathbb{P}\mathbb{P}\tilde{M}(0^{-+})} = -\frac{2}{M_0}\, g'_{\mathbb{P}\mathbb{P}\tilde{M}} \left[\partial_\rho \mathbb{P}_{\mu\nu}\right]\left[\partial_\sigma \mathbb{P}_{\varkappa\lambda}\right] g^{\mu\varkappa} \varepsilon^{\nu\lambda\rho\sigma}\, \tilde{\chi}$$
$$- \frac{g''_{\mathbb{P}\mathbb{P}\tilde{M}}}{M_0^3}\, \varepsilon^{\mu_1\mu_2\nu_1\nu_2} \left(\partial_{\mu_1}\tilde{\chi}\right)\Big[\left(\partial_{\mu_3}\mathbb{P}_{\mu_4\nu_1} - \partial_{\mu_4}\mathbb{P}_{\mu_3\nu_1}\right)\overleftrightarrow{\partial}_{\mu_2}$$
$$\left(\partial^{\mu_3}\mathbb{P}^{\mu_4}{}_{\nu_2} - \partial^{\mu_4}\mathbb{P}^{\mu_3}{}_{\nu_2}\right)\Big]$$

- all vertices respect standard QFT rules.

Example: $\mathbb{P}\rho\rho$ vertex $\left(t = (k'-k)^2 \right)$



$$i F_M(t) \Big[2 a_{\mathbb{P}\rho\rho} \, \Gamma^{(0)}_{\mu\nu\varkappa\lambda}(k', -k) - b_{\mathbb{P}\rho\rho} \, \Gamma^{(2)}_{\mu\nu\varkappa\lambda}(k', -k) \Big]$$

$$F_M(t) = \frac{m_0^2}{m_0^2 - t} \, , \qquad m_0^2 = 0.50 \ \mathrm{GeV}^2$$

$a_{\mathbb{P}\rho\rho}$, $b_{\mathbb{P}\rho\rho}$ coupling constants

$$\Gamma^{(0)}_{\mu\nu\varkappa\lambda}(k_1, k_2) = \left[(k_1 k_2) g_{\mu\nu} - k_{2\mu} k_{1\nu}\right]$$

$$\left[k_{1\varkappa} k_{2\lambda} + k_{2\varkappa} k_{1\lambda} - \frac{1}{2}(k_1 k_2) g_{\varkappa\lambda}\right]$$

$$\Gamma^{(2)}_{\mu\nu\varkappa\lambda}(k_1, k_2) = (k_1 k_2)(g_{\mu\varkappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\varkappa})$$

$$+ g_{\mu\nu}(k_{1\varkappa} k_{2\lambda} + k_{2\varkappa} k_{1\lambda})$$

$$- k_{1\nu} k_{2\lambda} g_{\mu\varkappa} - k_{1\nu} k_{2\varkappa} g_{\mu\lambda}$$

$$- k_{2\mu} k_{1\lambda} g_{\nu\varkappa} - k_{2\mu} k_{1\varkappa} g_{\nu\lambda}$$

$$- \left[(k_1 k_2) g_{\mu\nu} - k_{2\mu} k_{1\nu}\right] g_{\varkappa\lambda}$$

This $\mathbb{P}\rho\rho$ vertex is obtained from the coupling Lagrangian:

$$\mathcal{L}'_{\mathbb{P}\rho\rho} = \Big[a_{\mathbb{P}\rho\rho} \left(\partial_\varkappa F^{(\rho)}_{\mu\nu}\right)\left(\partial_\lambda F^{(\rho)\mu\nu}\right)$$

$$+ b_{\mathbb{P}\rho\rho} F^{(\rho)}_{\mu\varkappa} F^{(\rho)\mu}{}_{\lambda} \Big] \left(g^{\varkappa\varkappa'} g^{\lambda\lambda'} - \frac{1}{4} g^{\varkappa\lambda} g^{\varkappa'\lambda'}\right) \mathbb{P}_{\varkappa'\lambda'} ,$$

$$F^{(\rho)}_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu .$$