Lattice QCD
Challenges and Opportunities

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HIM
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PRISMA+

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

The role of Lattice QCD

* Provide quantitative input on hadron structure and spectroscopy and the properties of matter under extreme conditions

* Constrain hadronic contributions to processes and matrix elements relevant for BSM physics searches:
  - Flavour physics — CKM matrix elements
  - Nucleon matrix elements: axial, scalar and tensor charges, σ-terms
  - Muon $g-2$

* Provide input for experiments and phenomenology
  - Parton Distribution Functions — joint fits of experimental and lattice data
  - Nucleon form factors — neutrino-nucleus cross sections, proton radius puzzle

* Validate model calculations and EFT descriptions

* Investigate BSM scenarios

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Lattice QCD concerns all aspects of strong interaction physics

* Provide

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Lattice QCD then and now

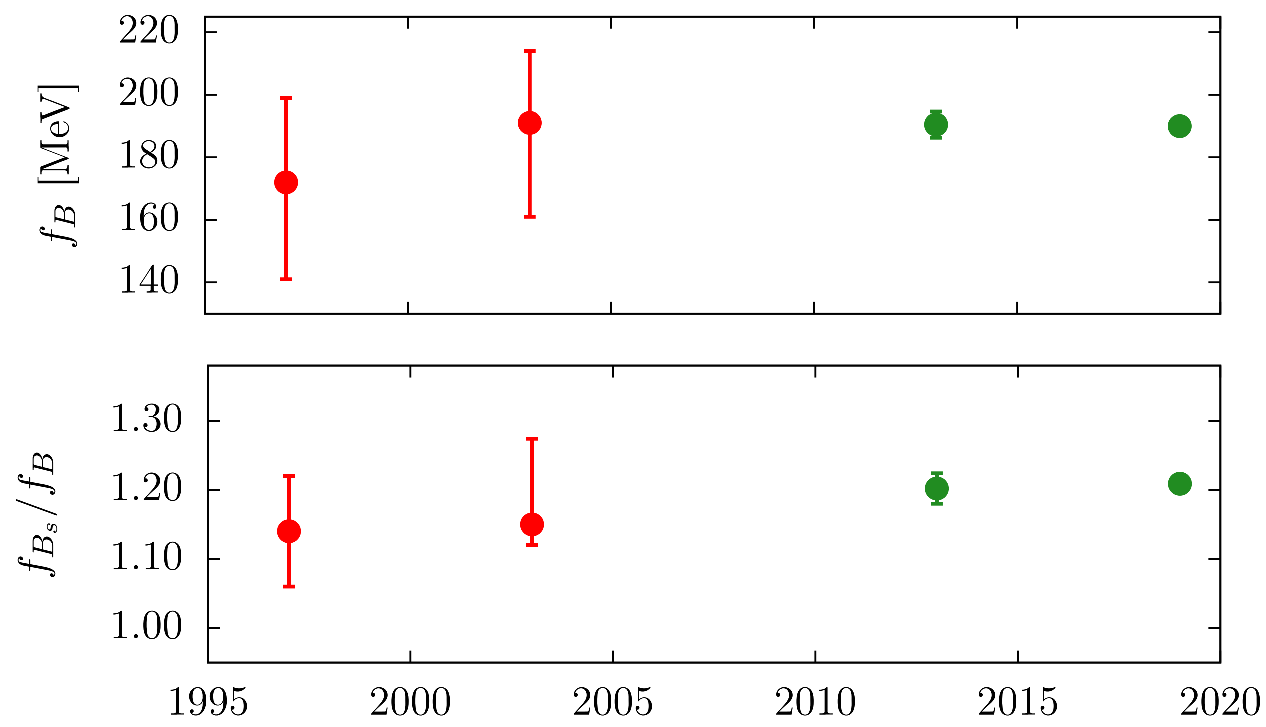
"The lattice has produced numbers with small error bars of unmeasured quantities…"

[Guido Altarelli, DESY Theory Workshop 1990]

* Lattice QCD makes predictions for a large variety of quantities in strong interaction physics with controlled errors:
  - Hadron masses and matrix elements
  - SM parameters
  - Quantities characterising strongly interacting matter under extreme conditions
  - Precision observables

Lattice QCD then and now

* **Example**: B-meson decay constant in Lattice QCD
  $\rightarrow$ relevant for V_{ub}, no experimental measurement



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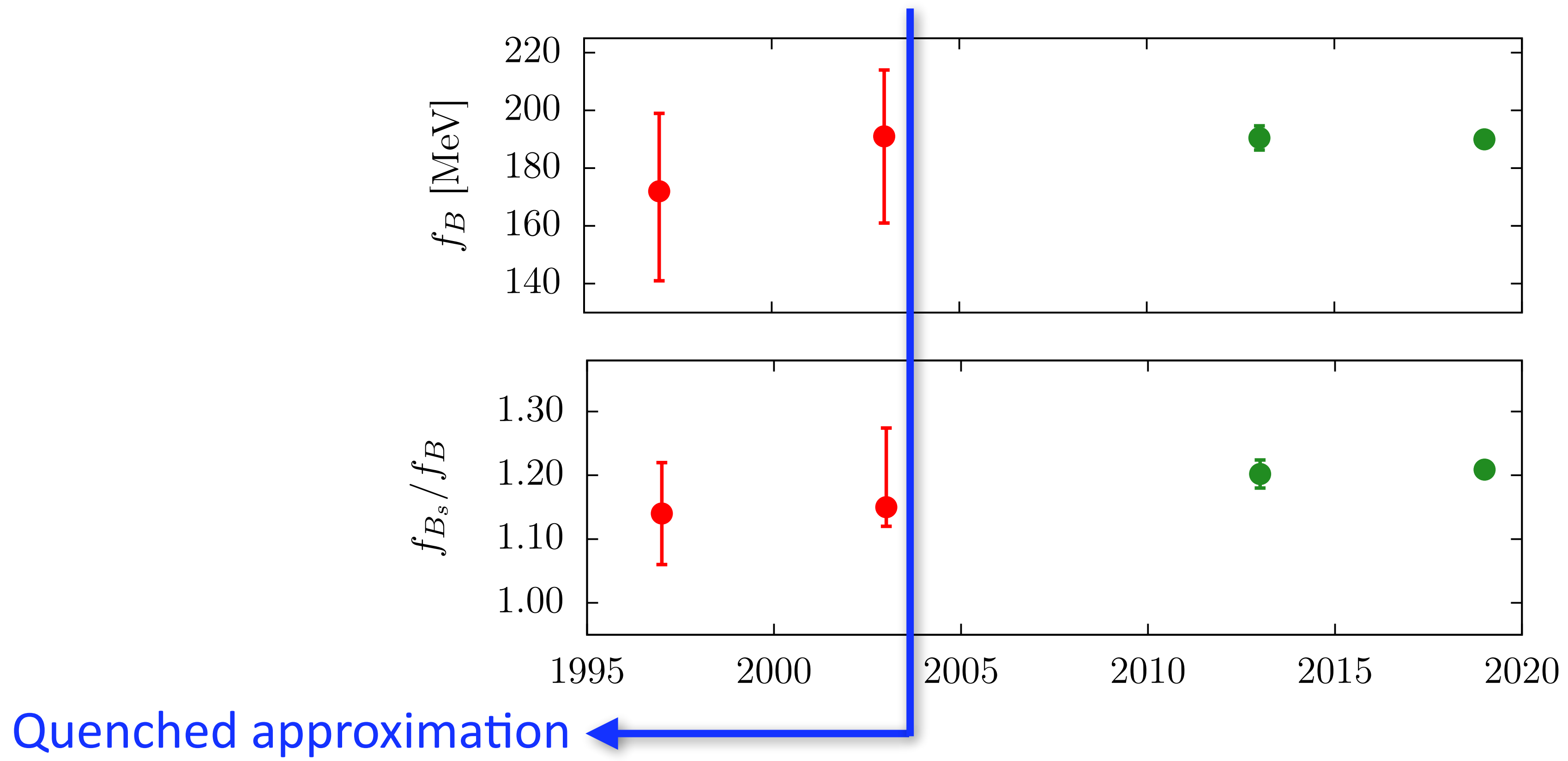

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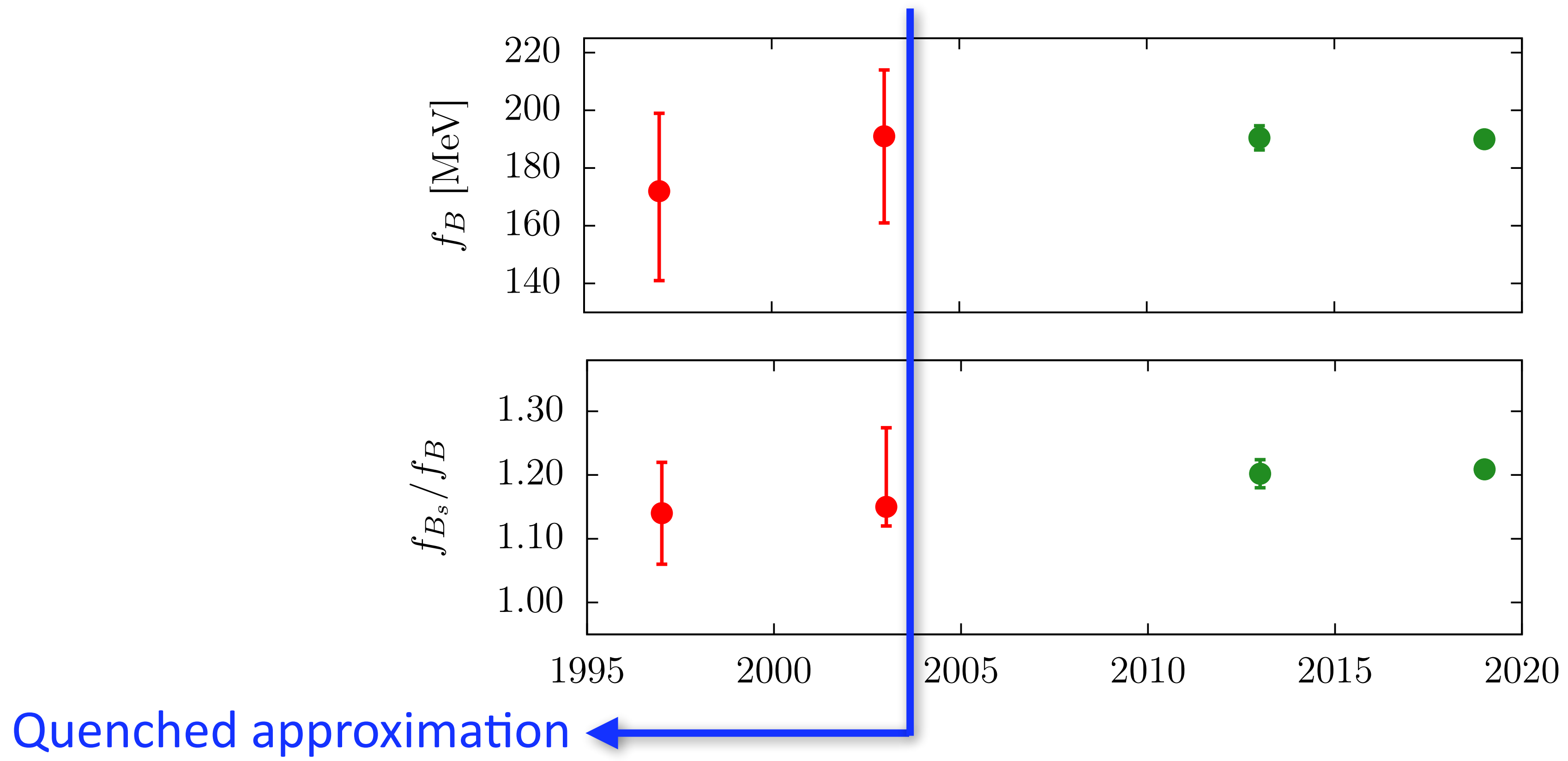


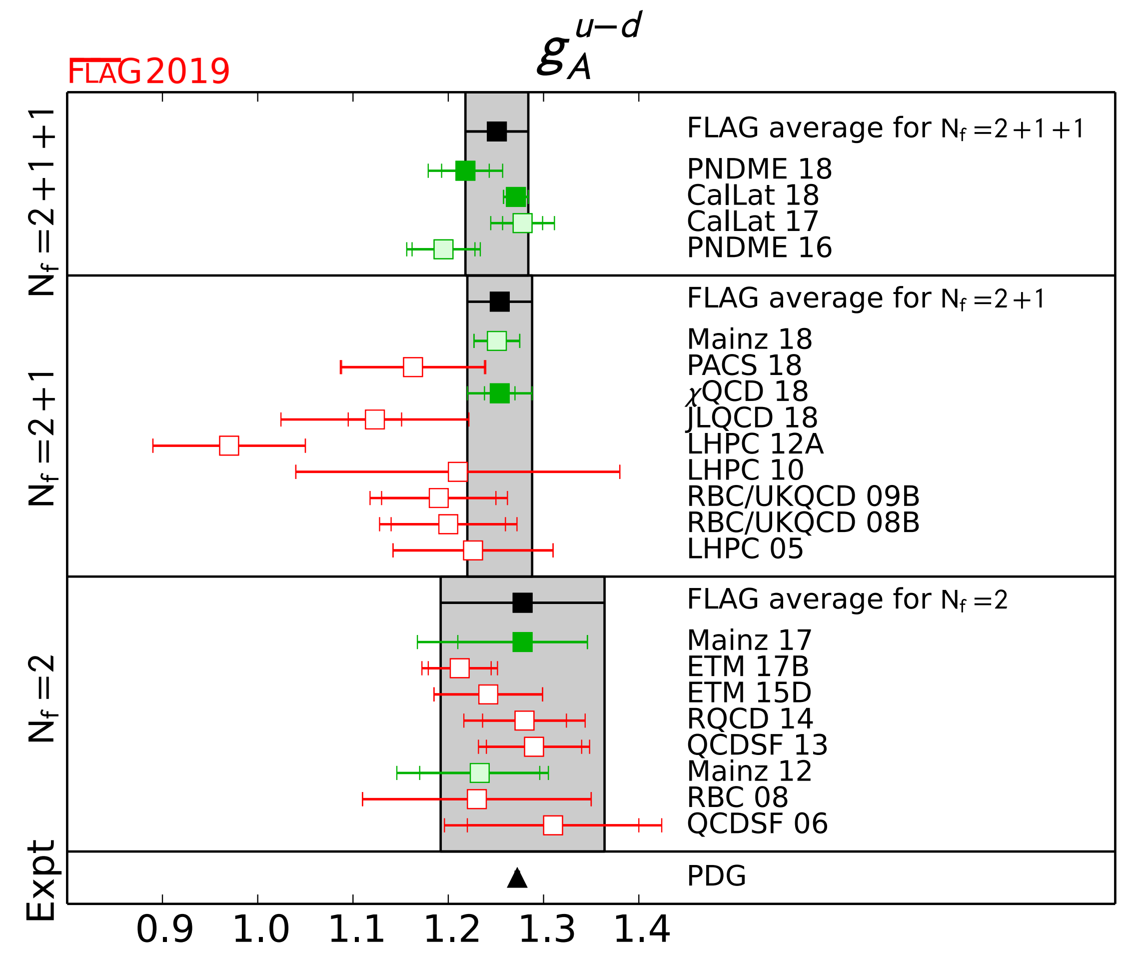
* Overall precision: 15% in 1997 $\rightarrow$ 0.5% in 2018

* Correlated with an increase of 5×10^5 in computer power

Community efforts in Lattice QCD

FLAG Report:

* Performs PDG-style global analyses and averages *[S. Aoki et al., arXiv:1902.08191]*
  - SM parameters (quark masses, alpha_s)
  - Flavour physics (*K, D, B* mesons)
  - Low-energy constants
  - Nucleon matrix elements



White papers:

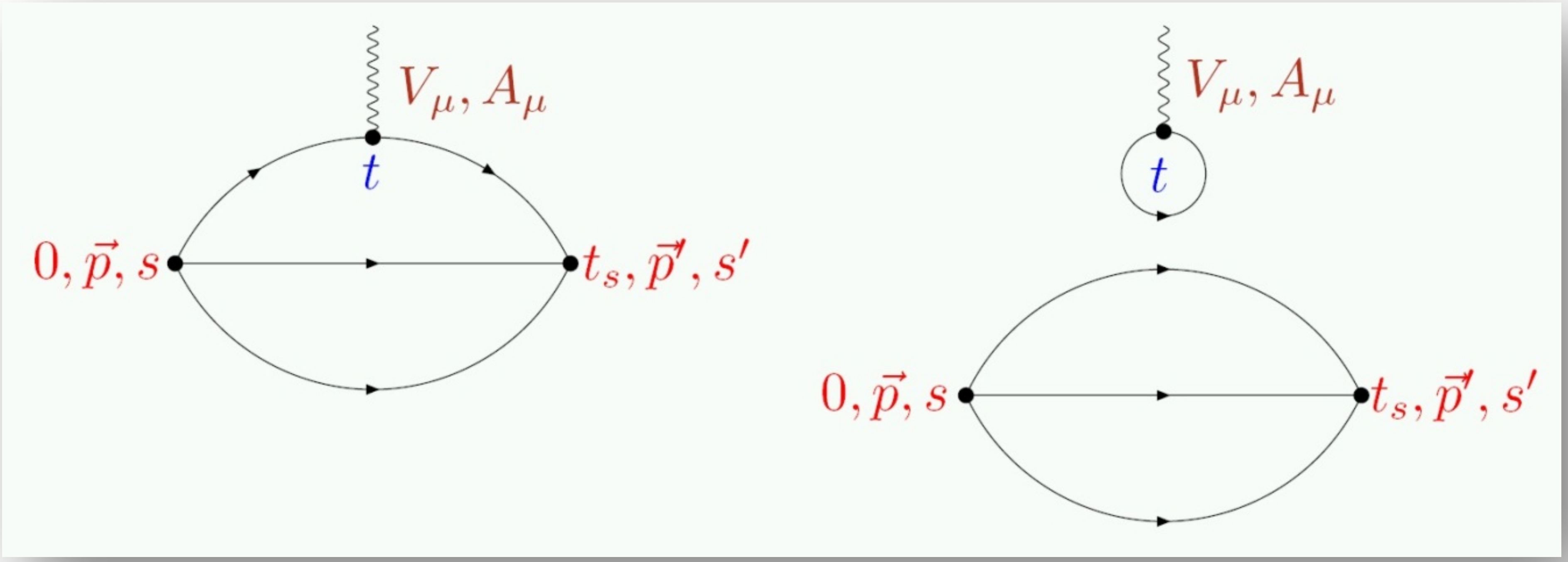
* Community White Paper on Lattice input for global PDF analyses
  [H.-W. Lin et al., PPNP 100 (2018) 107]

* The muon g – 2 (to appear)

* **USQCD** 2019 : WPs on Lattice QCD in nuclear, hadron, flavour, BSM physics, QCD phase diagram, Exascale computing

Technical and conceptual challenges

* Observables have become more "complex":
  - Baryon correlation functions
  - Quark-disconnected diagrams ($\longrightarrow$ flavour-singlet quantities)
  - Multi-hadron systems and light nuclei
  - Four-point functions required for hadronic light-by-light scattering



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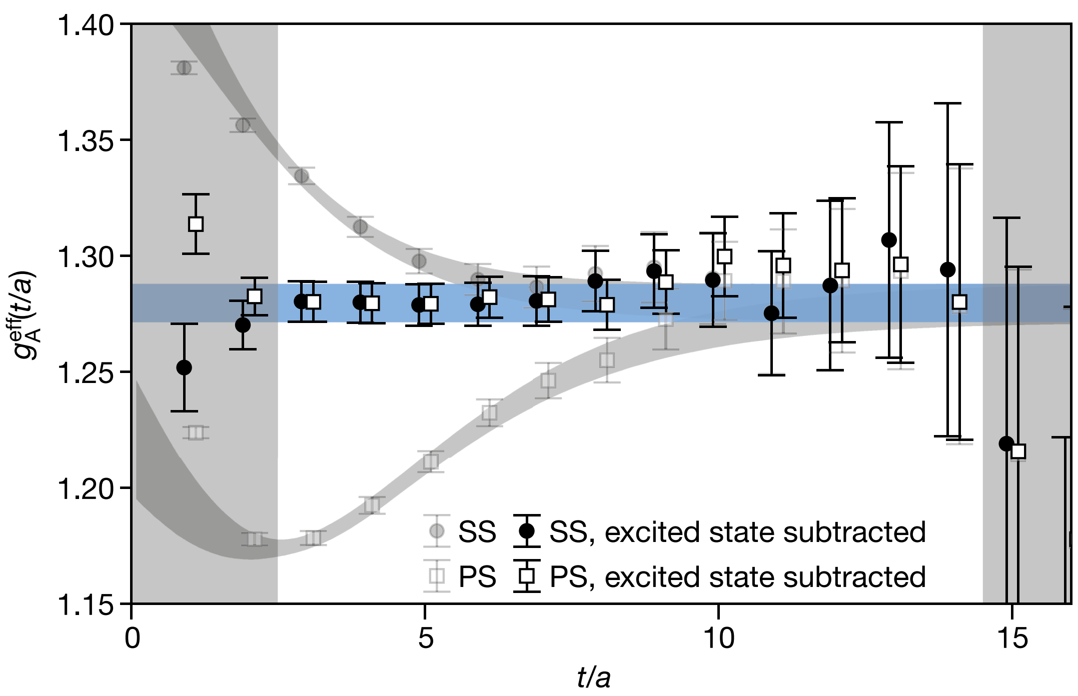
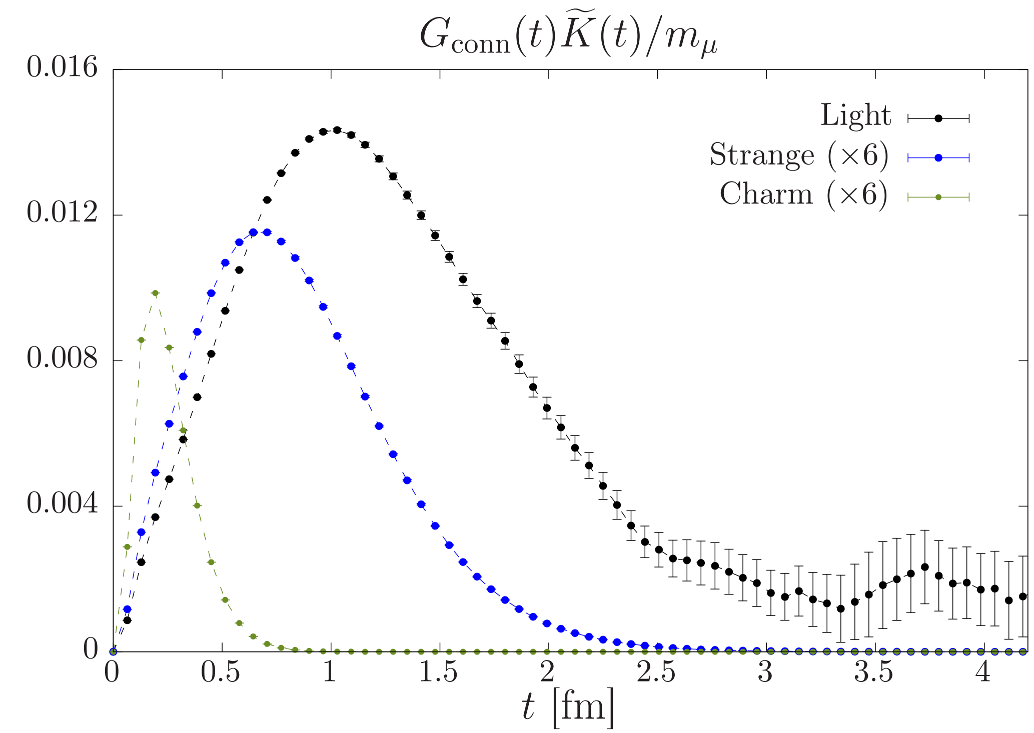
  - e.g. nucleon matrix elements, muon g – 2





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$\Rightarrow$ Bulk of CPU time used for calculation of observables

* Sign problem in simulations of QCD with $\mu_\mathrm{B} \neq 0$
* Incorporation of isospin breaking effects: QCD+QED, $m_\mathrm{u} \neq m_\mathrm{d}$
* n-particle decays of resonances, $n > 2$

Outline

Hadron structure

Hadron spectroscopy and hadronic interactions

Precision observables

QCD phase diagram

Resources

Outlook; Recommendations

Hadron structure calculations

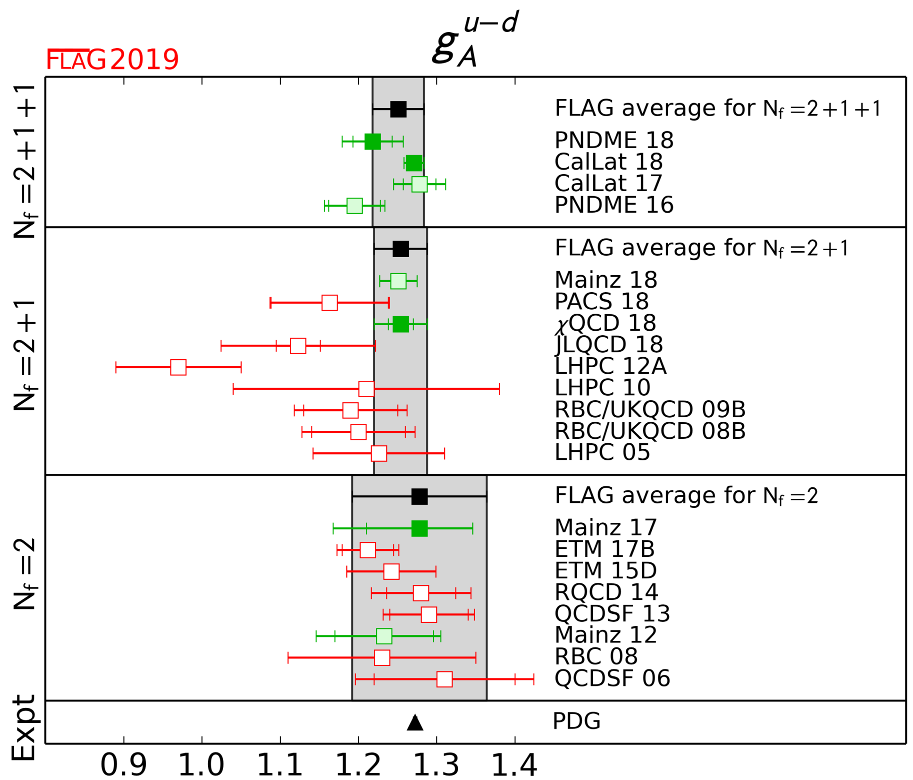
* Confront experimental determinations with lattice QCD results

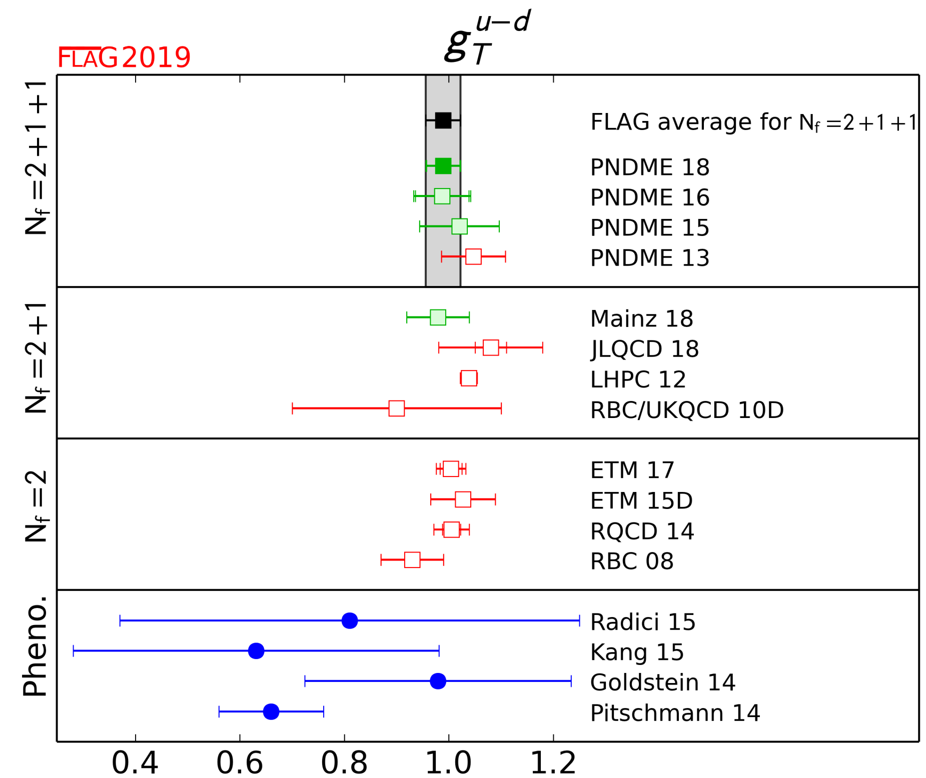
$\Rightarrow$ Consistency checks:
proton radius and magnetic moment, axial charge

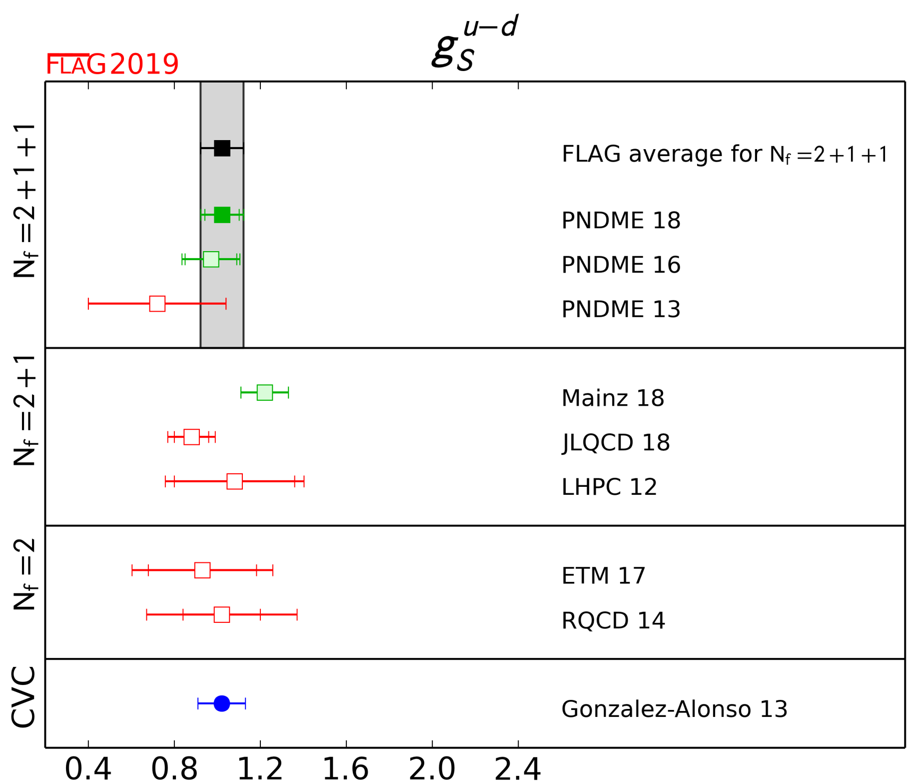
* Provide input for experiments and phenomenology:
  - Nucleon scalar and tensor charges g_S, g_T: constrain BSM effects
  - Axial form factor: ν-nucleus cross sections (DUNE/LBNF)
  - (Generalised) Parton Distribution Functions; transversity (EIC)
  - Decomposition of the nucleon spin

* Additional systematic effects:
  - Bias from unsuppressed excited states:
    multi-state fits, summed operator insertions
  - Finite-volume effects
  - Renormalisation and matching

Axial, scalar and tensor charges







FLAG averages:

$$g_{\mathrm{A}}^{u-d} = 1.251 \pm 0.033$$

$$[1.2724 \pm 0.0023 \text{ (exp.)}]$$

$$g_{\mathrm{S}}^{u-d} = 1.02 \pm 0.10$$

$$g_{\mathrm{T}}^{u-d} = 0.989 \pm 0.034$$

[Aoki et al., arXiv:1902.08191]

Nucleon form factors

* **Electromagnetic form factors**: proton charge radius, magnetic moment
  $\longrightarrow$ proton radius puzzle

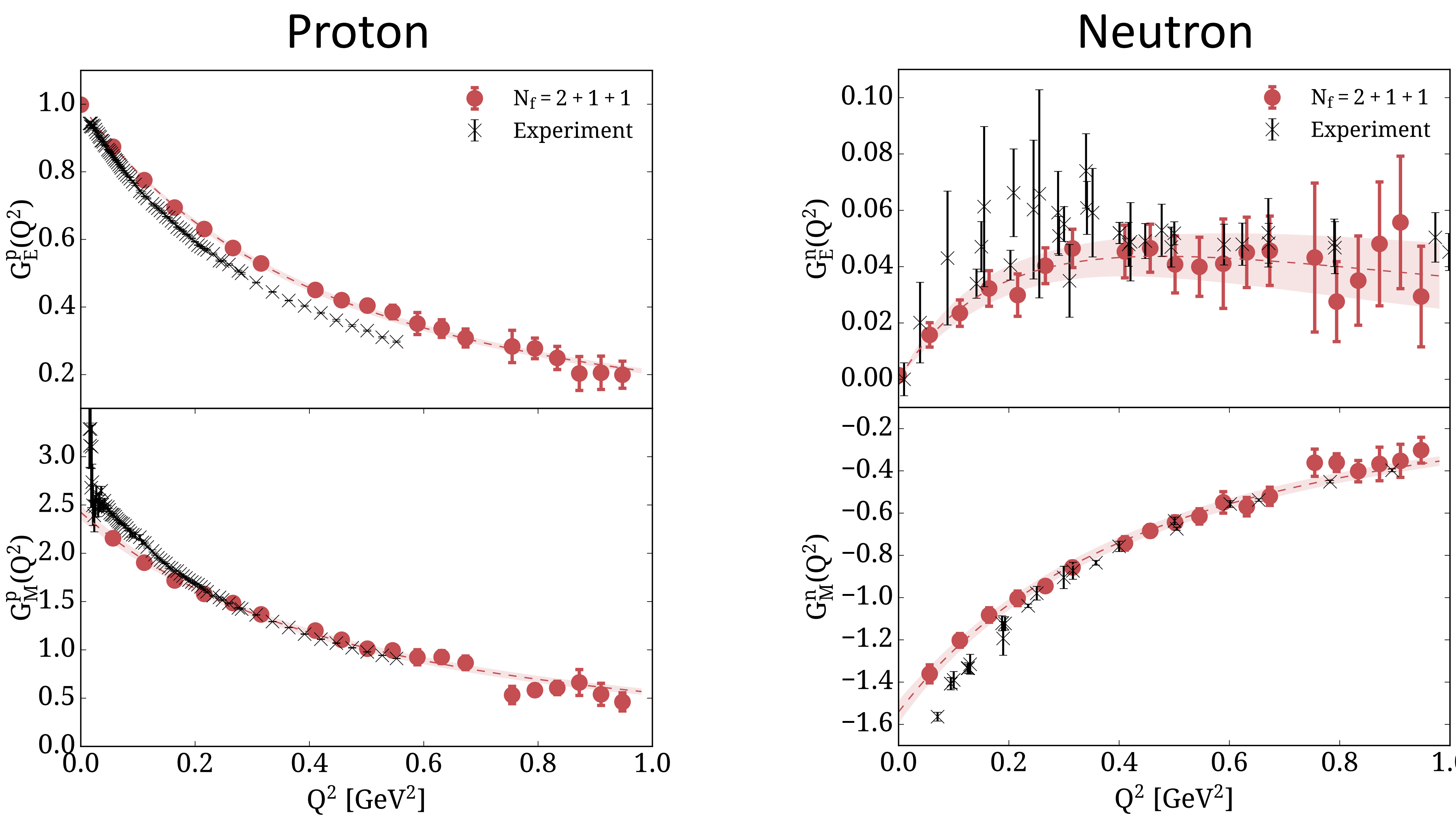
* **Axial form factor**: constrain ν-nucleus interaction and cross sections
  $\longrightarrow$ long baseline neutrino experiments (DUNE/LBNF)

* **Strange form factors**: probe contribution from quark sea
  $\longrightarrow$ parity-violating ep scattering

* Quark-disconnected diagrams:
  - required to determine iso-scalar form factors
  - strange form factor

Electromagnetic form factors

* Recent results: *[Alexandrou et al., arXiv:1812.10311]*



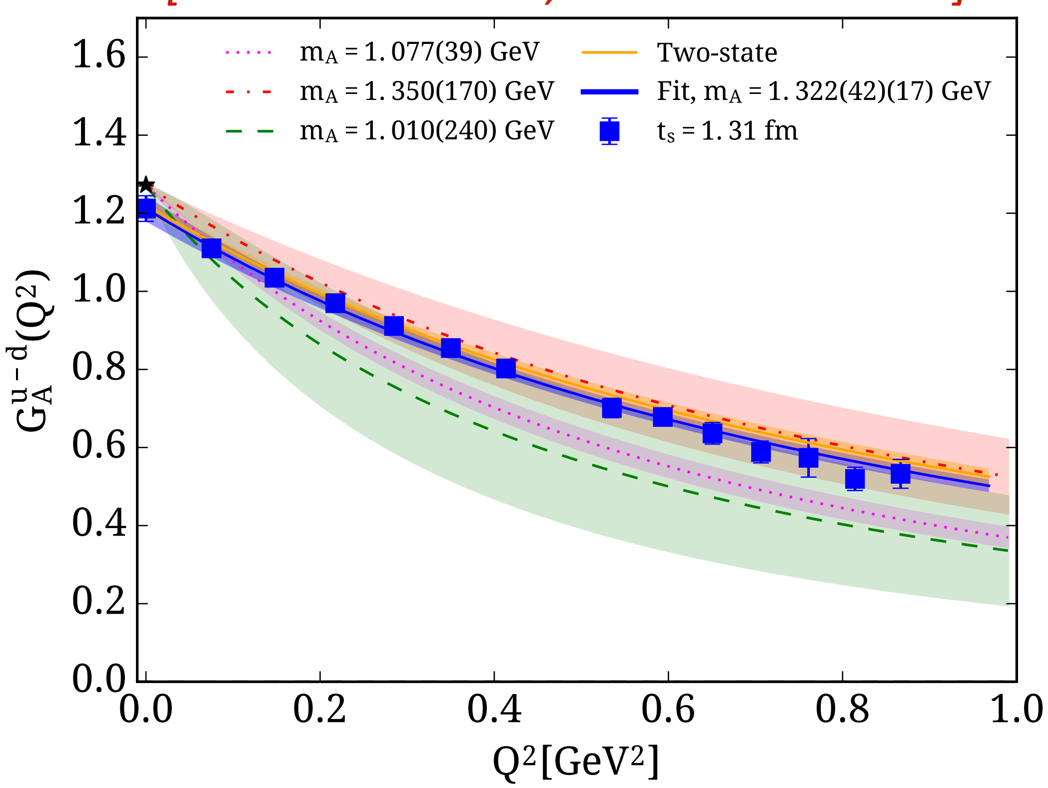
* Charge radii smaller than observed in *ep* scattering experiments
* Focus on systematics and better statistical precision

Axial form factors

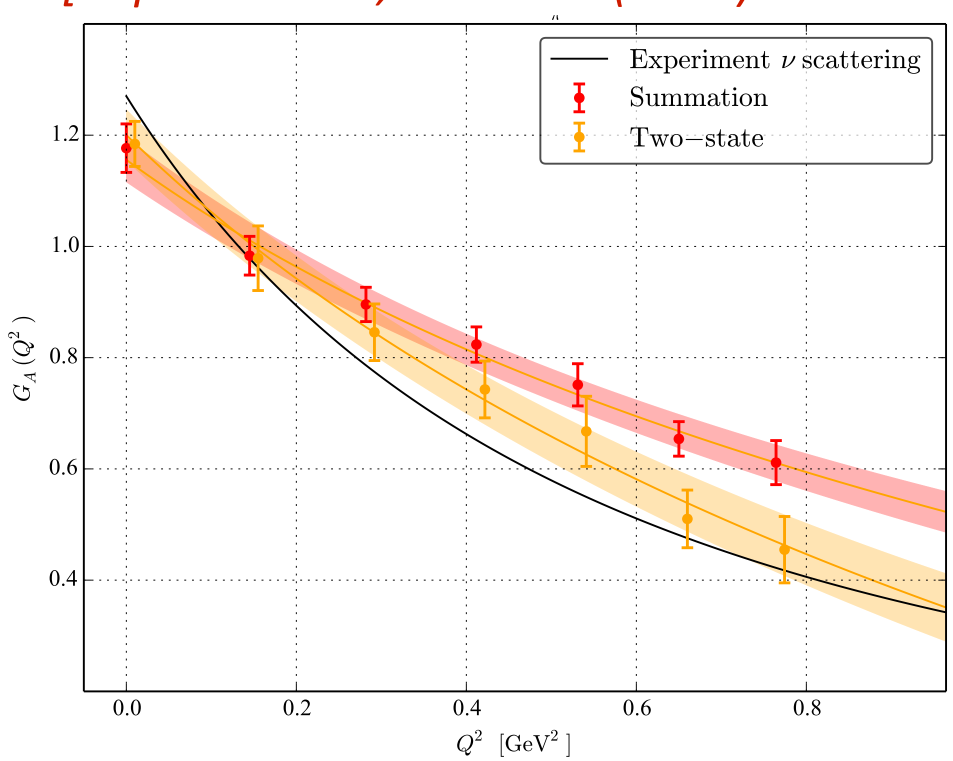
* Definition:

$$\left\langle N(p',s')\left|A_\mu^a(0)\right|N(p,s)\right\rangle = \bar{u}(p',s')\left[\gamma_\mu\gamma_5 G_A(Q^2) - i\gamma_5\frac{Q_\mu}{2m_N}G_P(Q^2)\right]\tfrac{1}{2}\tau^a u(p,s)$$

[Alexandrou et al., arXiv:1807.11203]



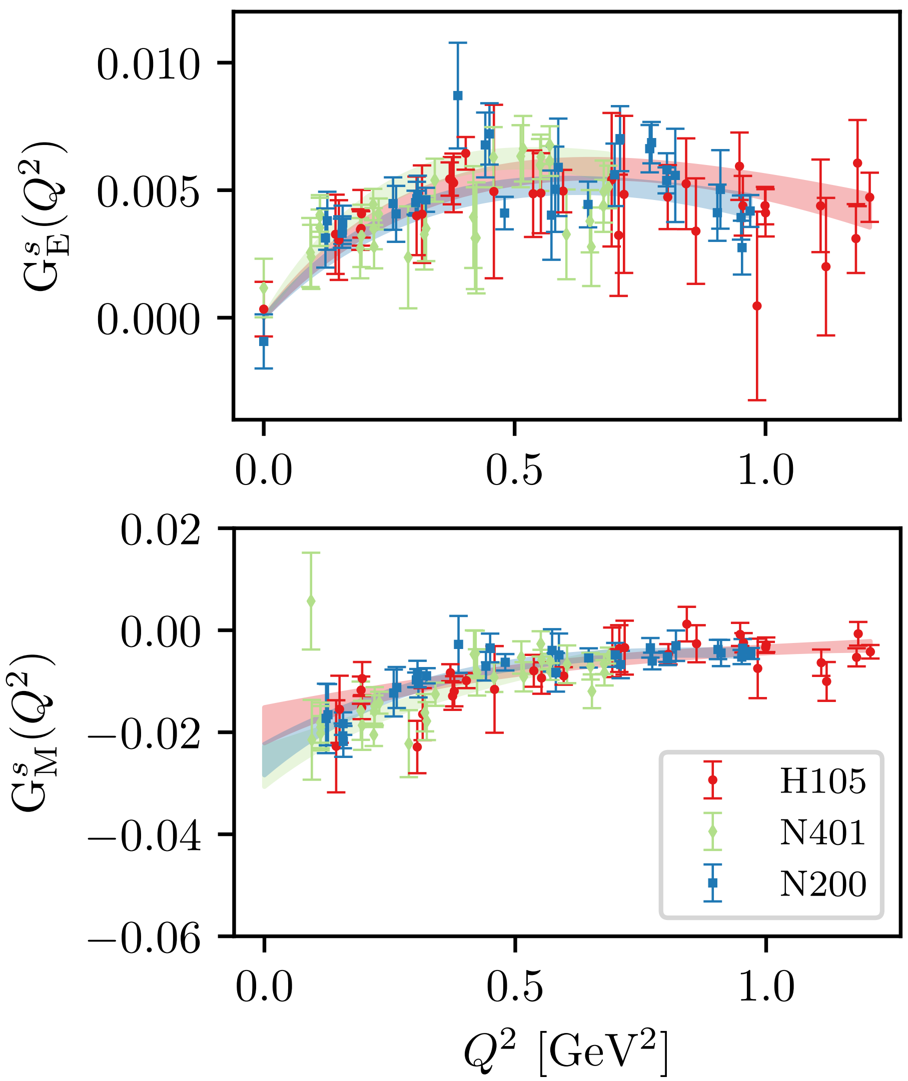
[Capitani et al., IJMPA 34 (2019) 1950009]

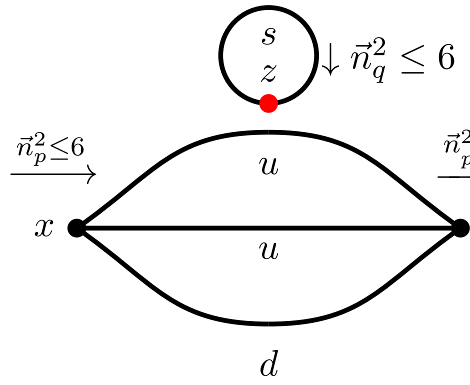


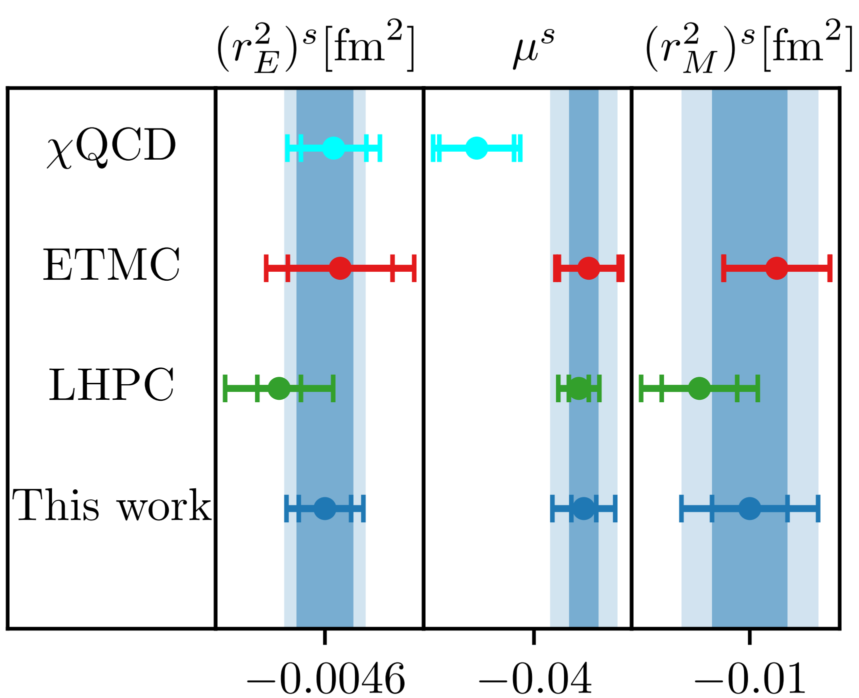
* Slope of $G_A(Q^2)$ underestimated in calculations
  $\longrightarrow$ focus on excited states systematics

Strange electromagnetic form factors

* Signal entirely given by disconnected diagrams





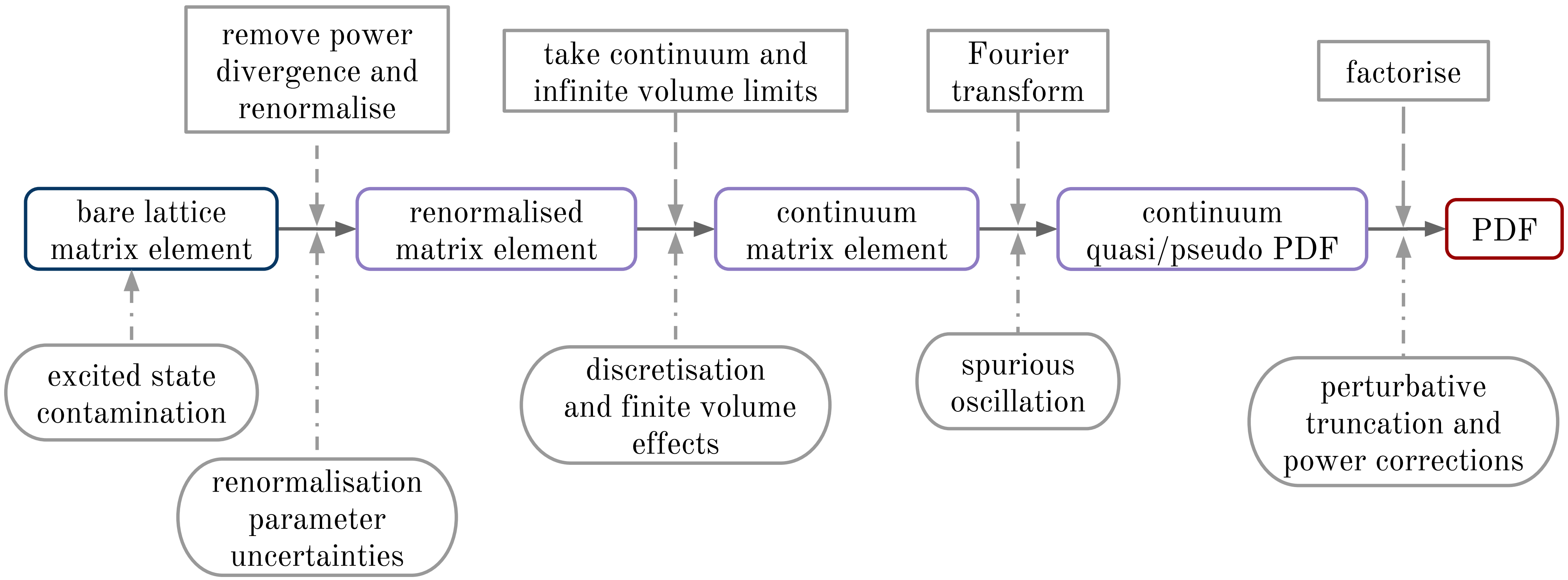


[Djukanovic et al., arXiv:1903.12566]

* Good agreement among different calculations
* Increase statistical precision further

PDFs from Lattice QCD

* Workflow diagram for PDF determination *[Monahan, arXiv:1811.00678]*

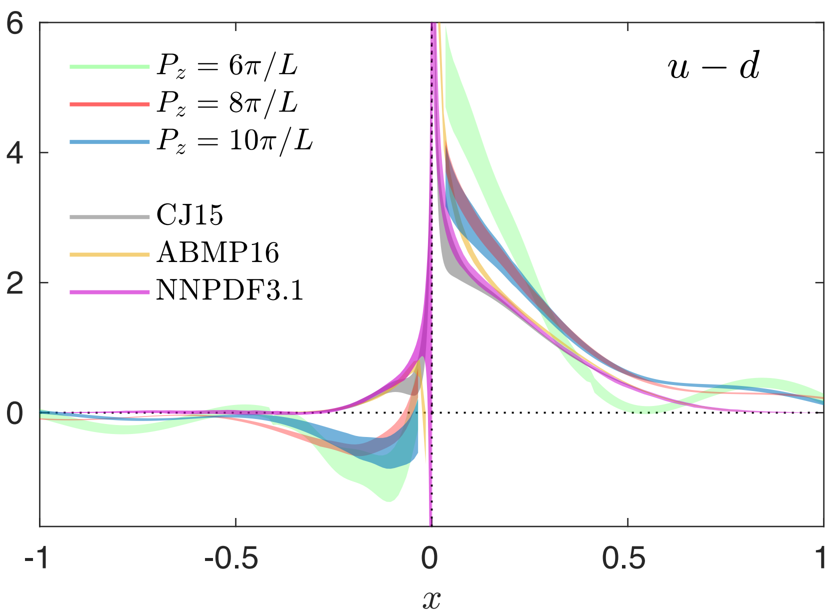


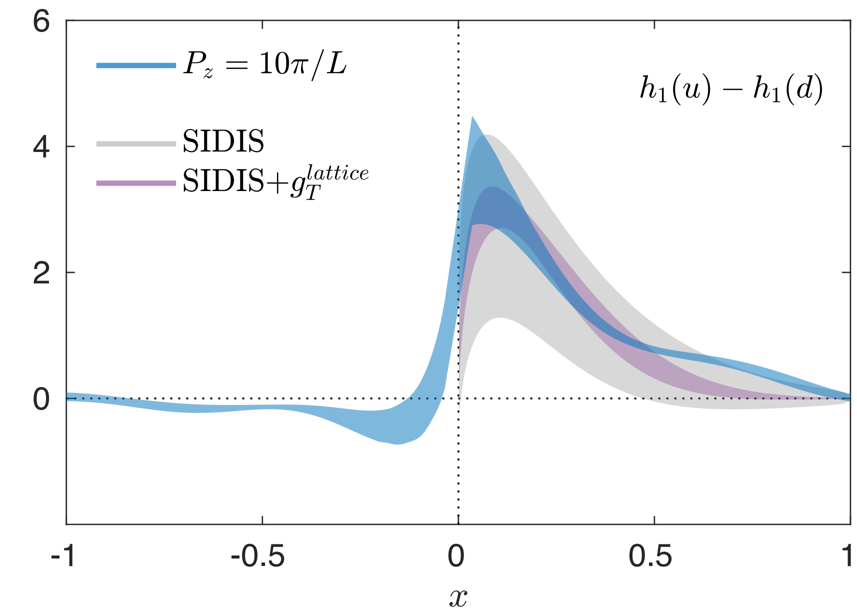
* Control over lattice systematics
* Non-local operator matching and renormalisation
* Higher-twist effects
* Inverse transform problems

PDFs from Lattice QCD

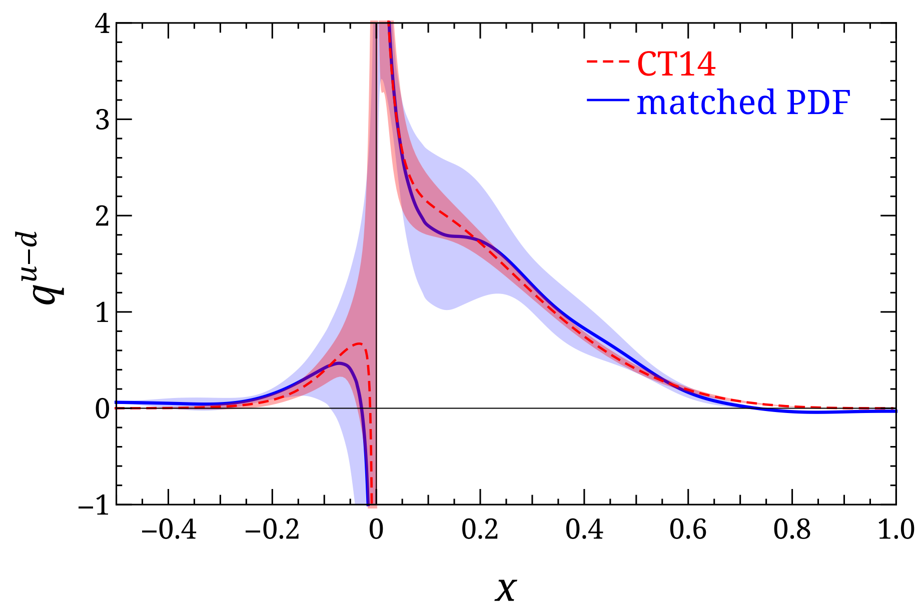
* Isovector unpolarised and transversity distributions *[Monahan, arXiv:1811.00678]*

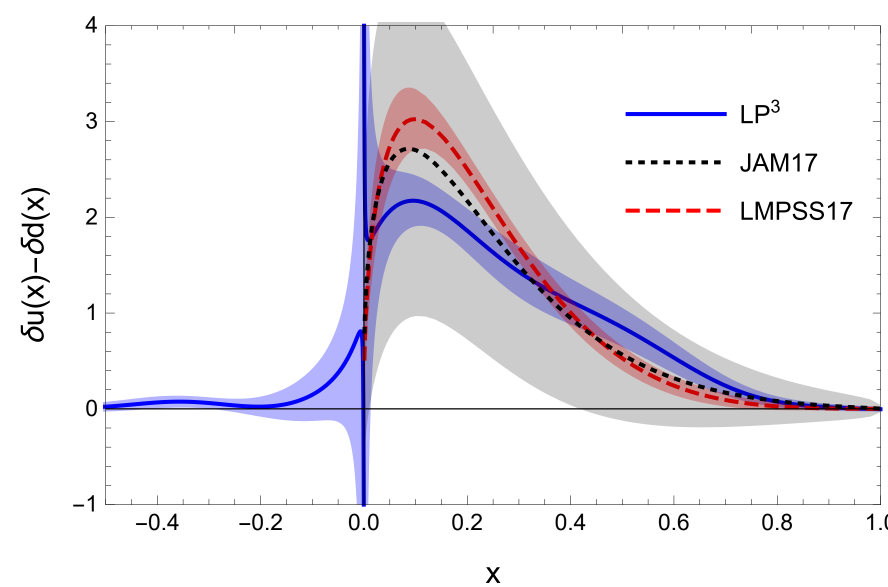
[Alexandrou et al., arXiv:1803.02685, arXiv:1807.00232]





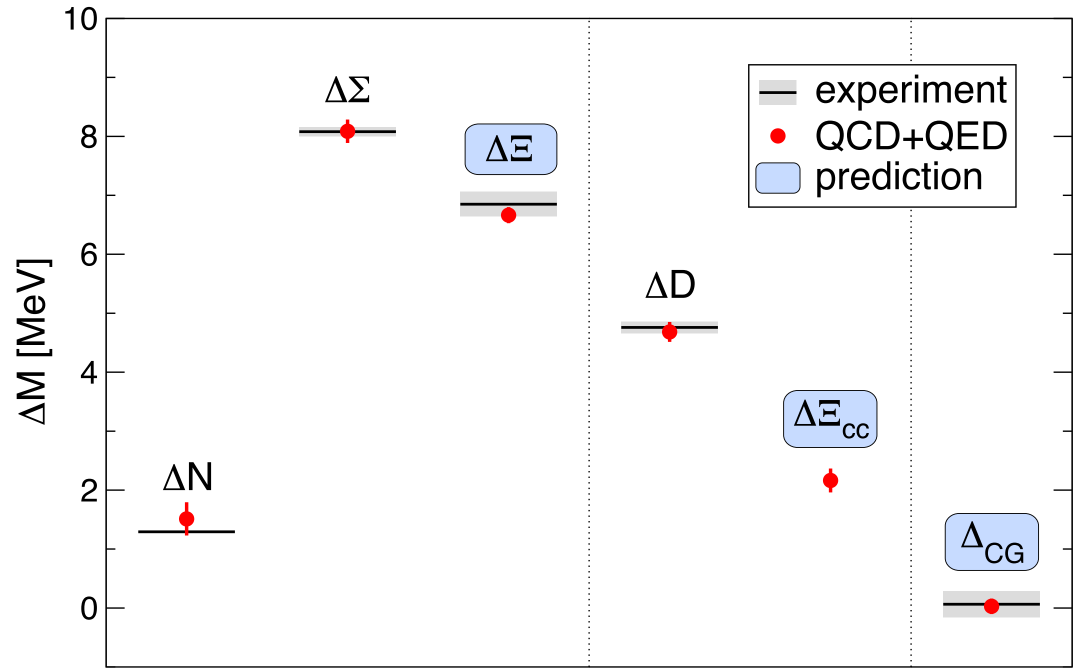
[Chen et al., arXiv:1803.04393, Liu et al., arXiv:1810.05043]





Hadron spectroscopy and hadronic interactions

* Lattice spectroscopy: beyond naïve extraction of ground state mesons and baryons
* Focus on
  - excitation spectrum
  - electromagnetic mass splittings
  - resonance properties
  - multi-hadron states



[Borsanyi et al., Science 347 (2015) 1452]

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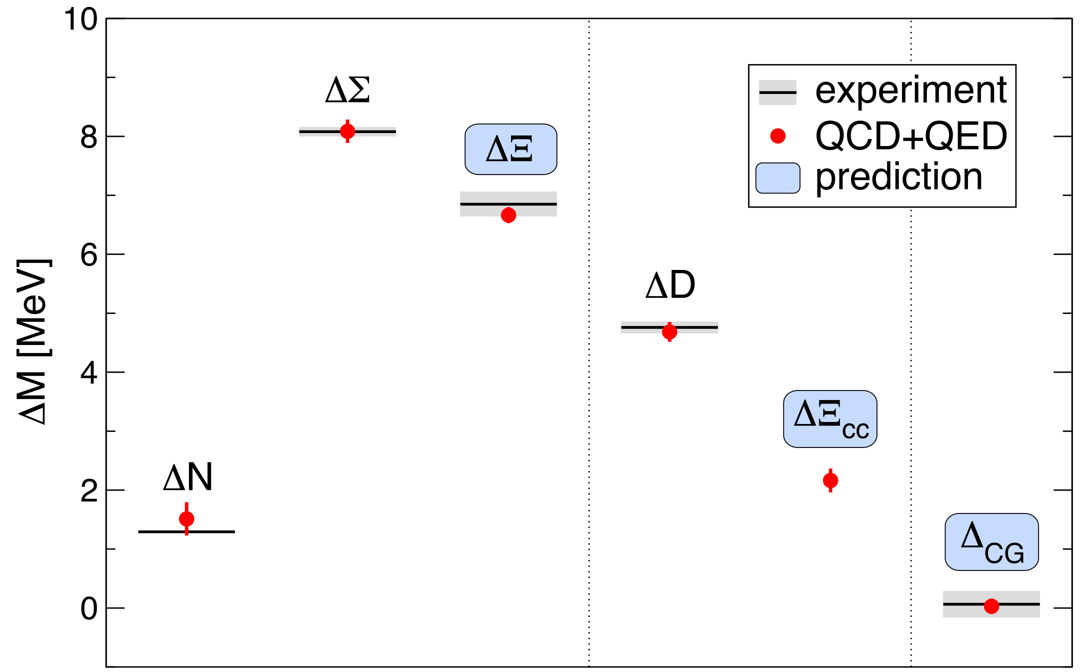

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* Methods:
  - Lüscher's finite-volume quantisation condition:
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  - Correlator matrices and Generalised Eigenvalue problem
  - "Distillation", smearing

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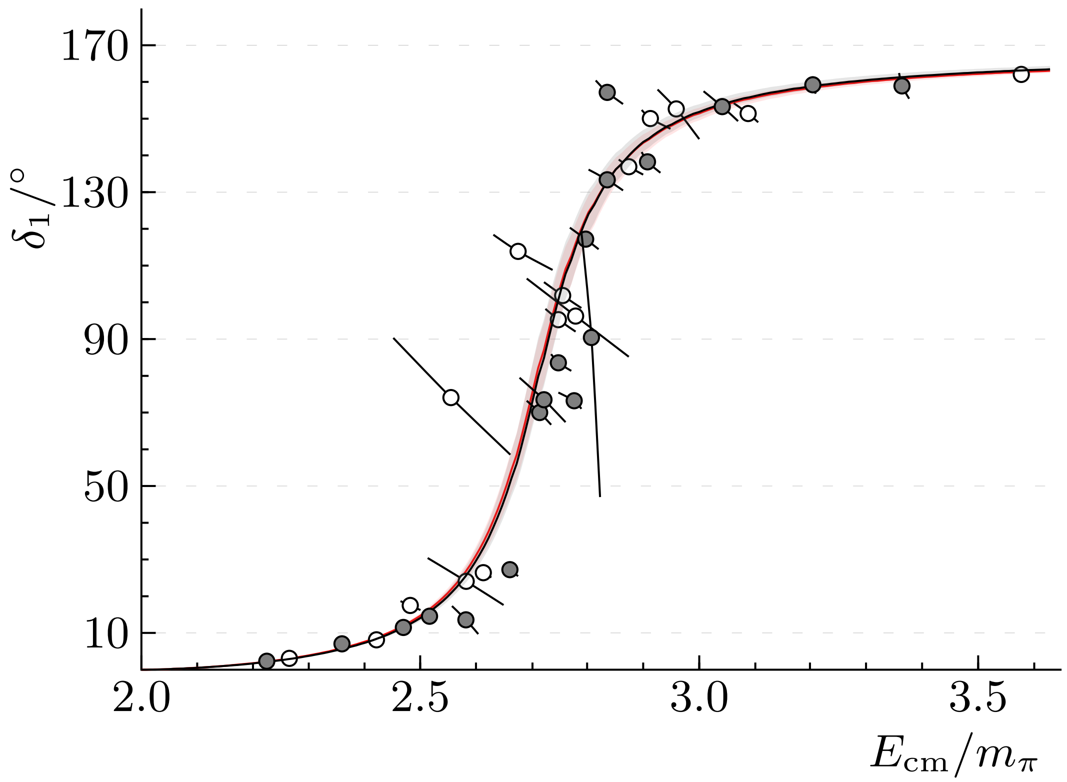

[Andersen et al., NPB 939 (2019) 145]

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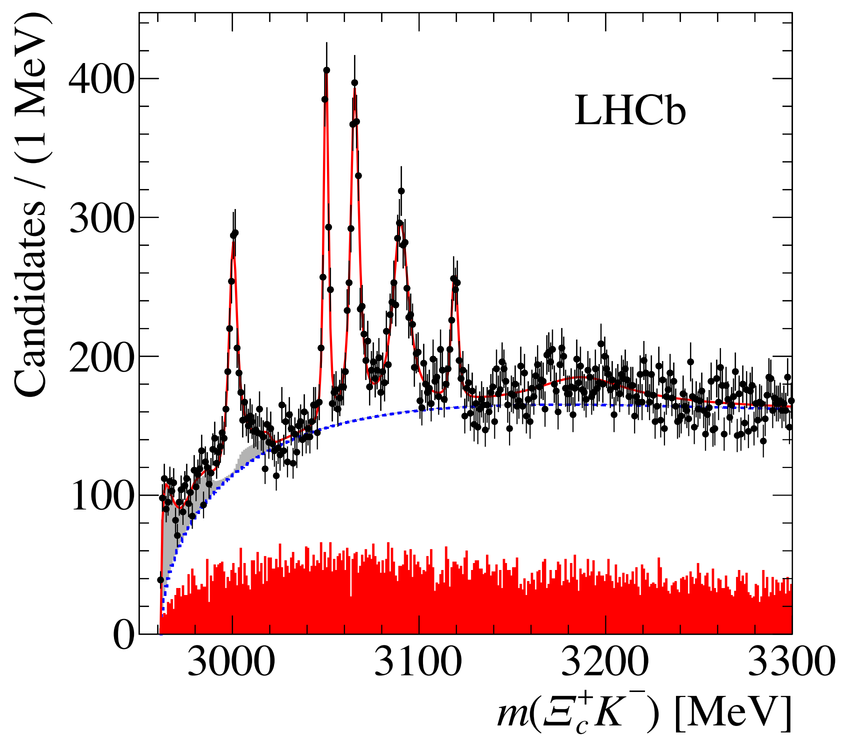
Charmed resonances and baryons

- The Ω_c (css) in lattice QCD



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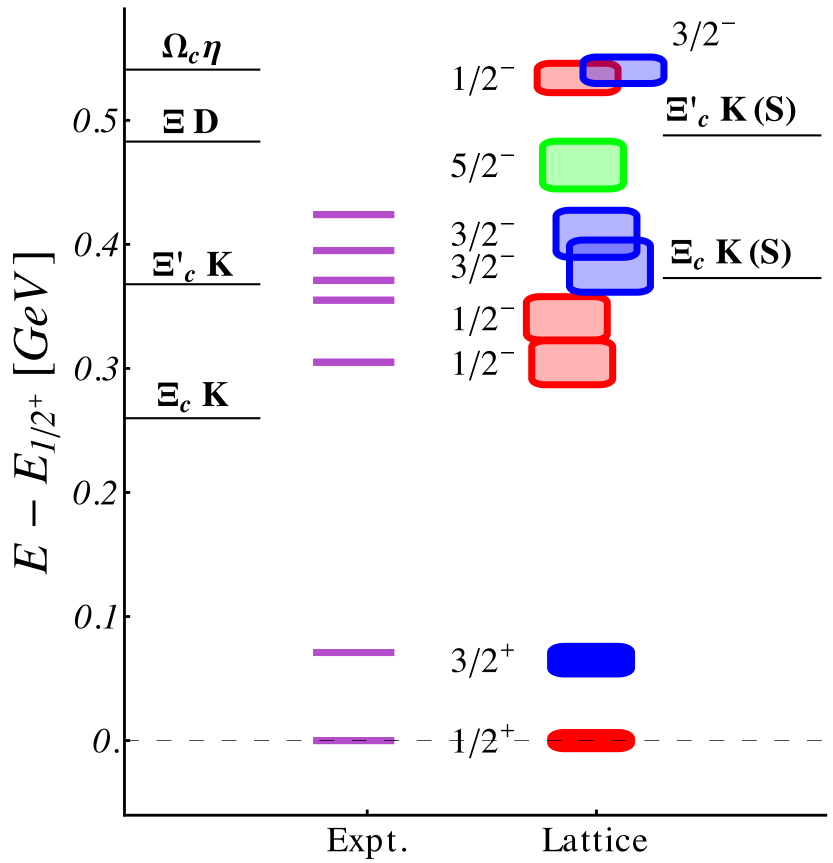

[Padmanath et al., PRL 119 (2017) 042001]

(strong decays ignored)

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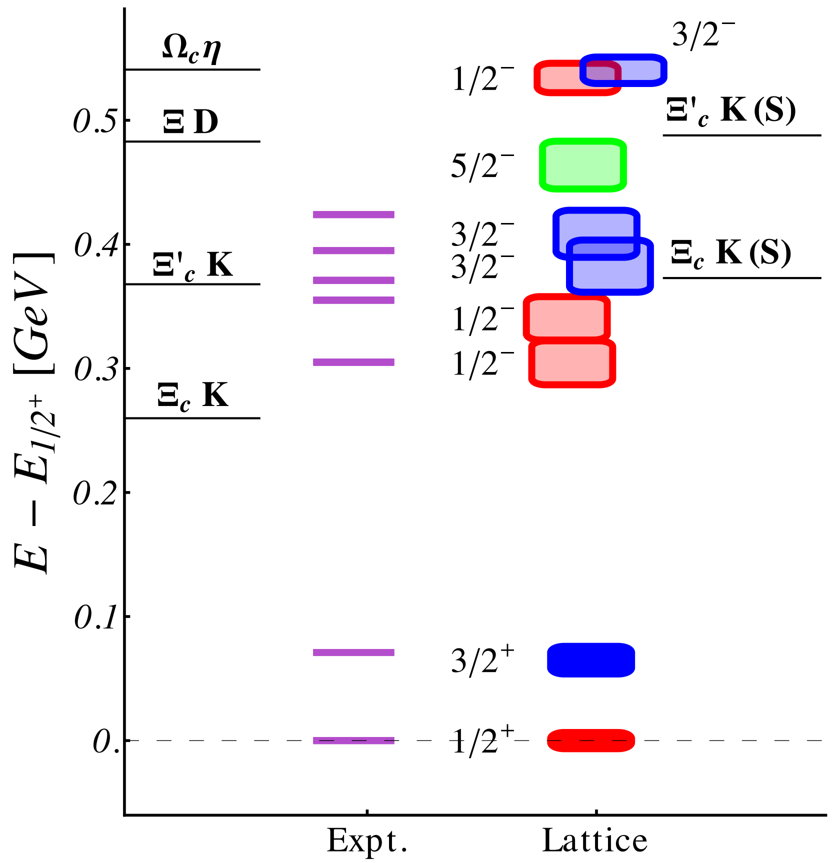

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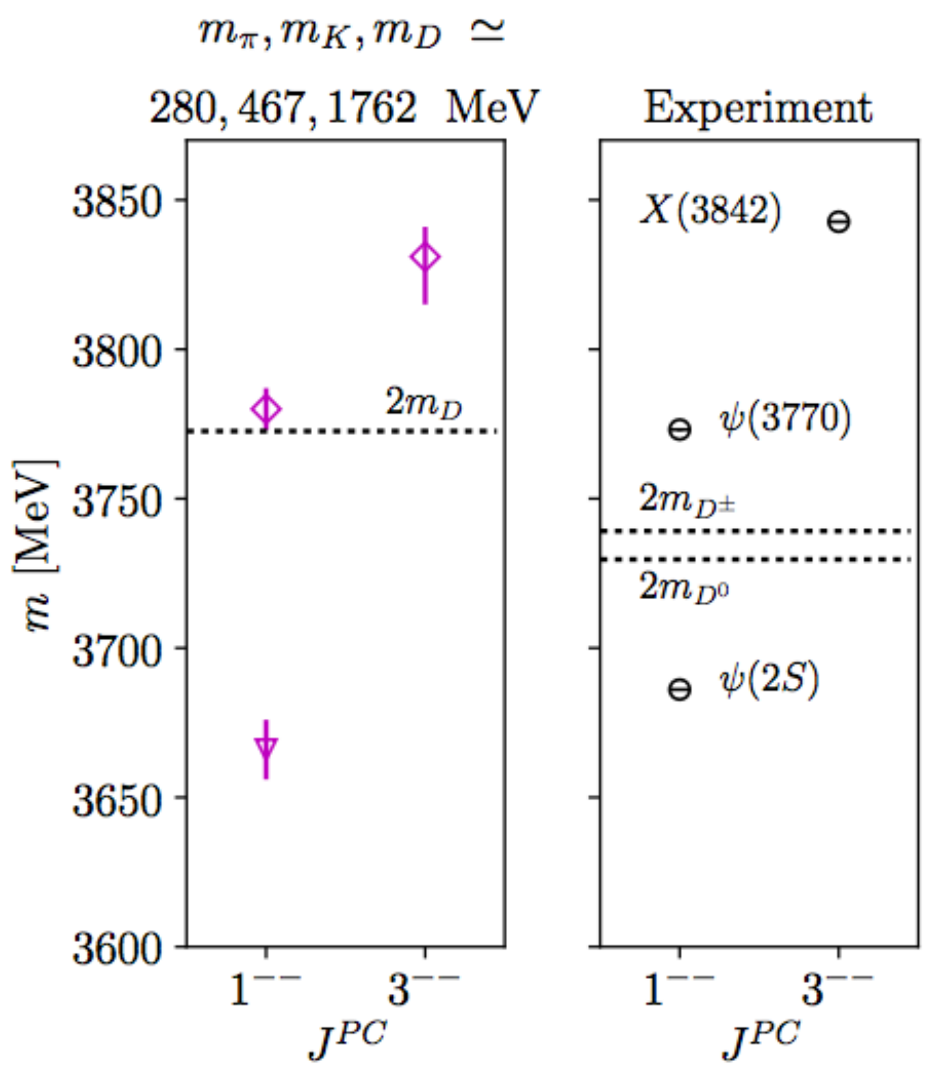
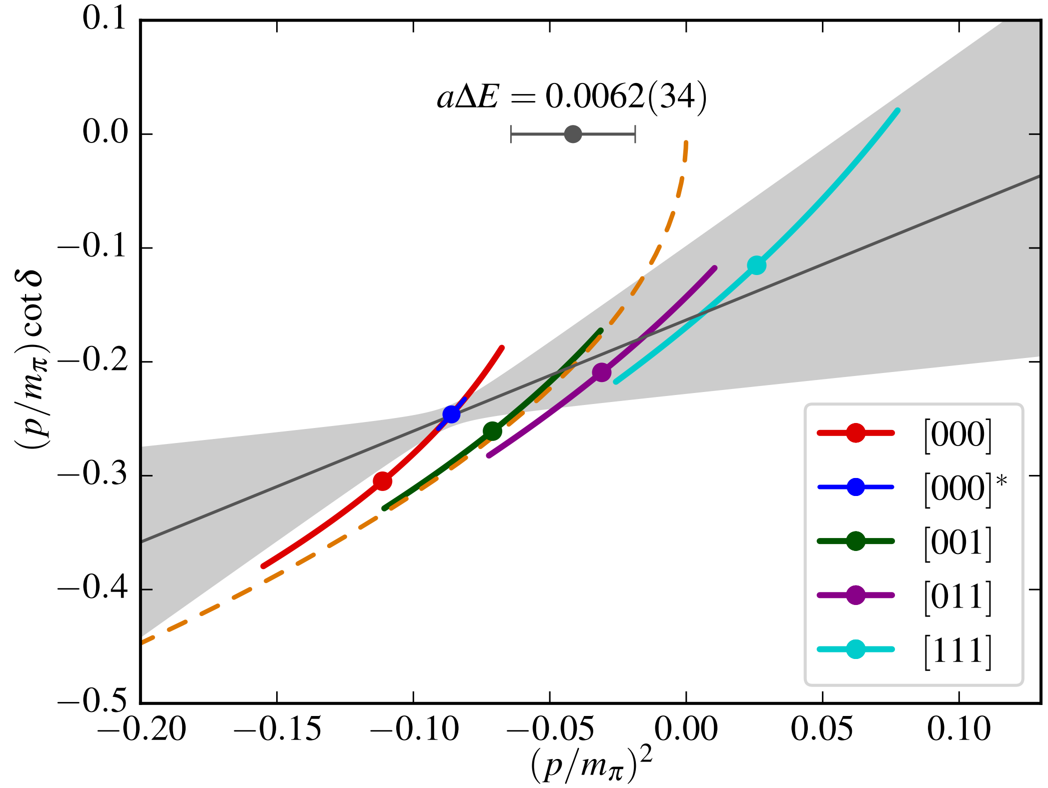


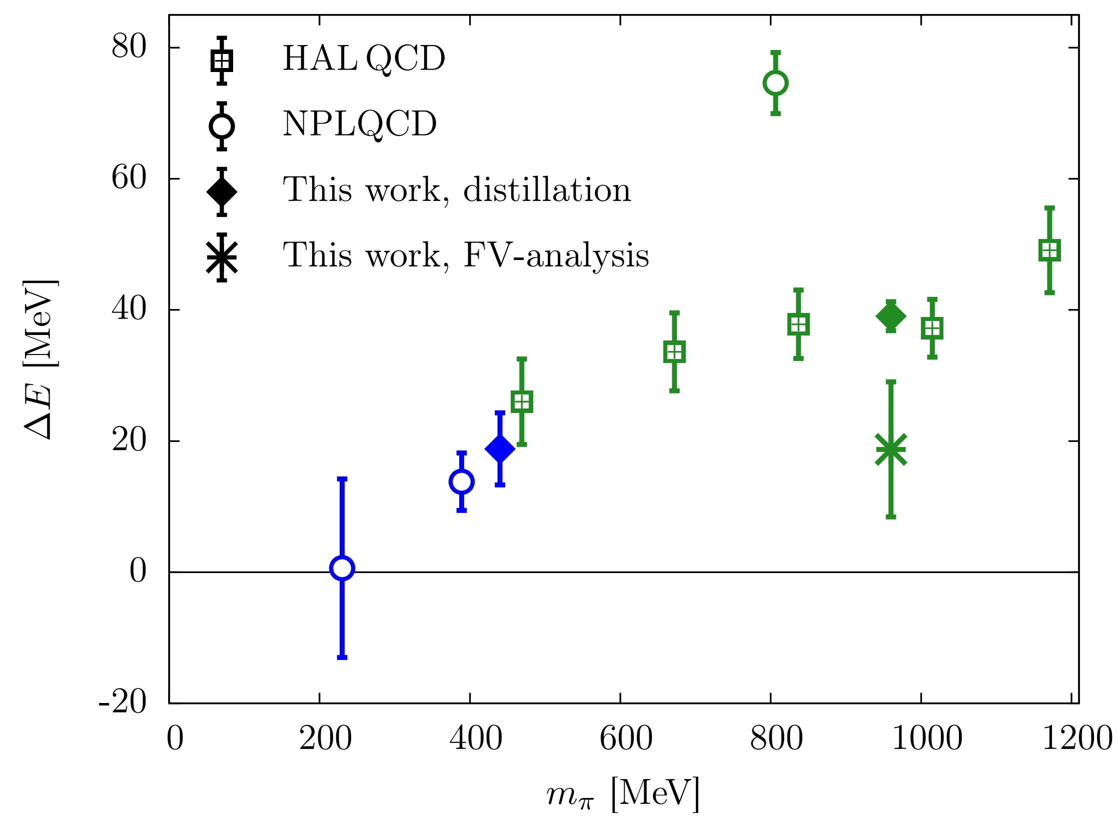
* Good agreement between conventional charmonia and recently discovered $X(3842)$ resonance

[LHCb Collab., arXiv:1903.12240,
Piemonte et al., arXiv:1905.03506]

The H Dibaryon

* Conjectured scalar, SU(3)-singlet hexaquark state with quark content *udsuds*





* Binding energy from pole of scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta(p) - ip}$$

$$p\cot\delta(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{00}(1,(pL/2\pi)^2)$$

$$p^2 = \tfrac{1}{4}(E^2 - \boldsymbol{P}\cdot\boldsymbol{P}) - m_\Lambda^2$$

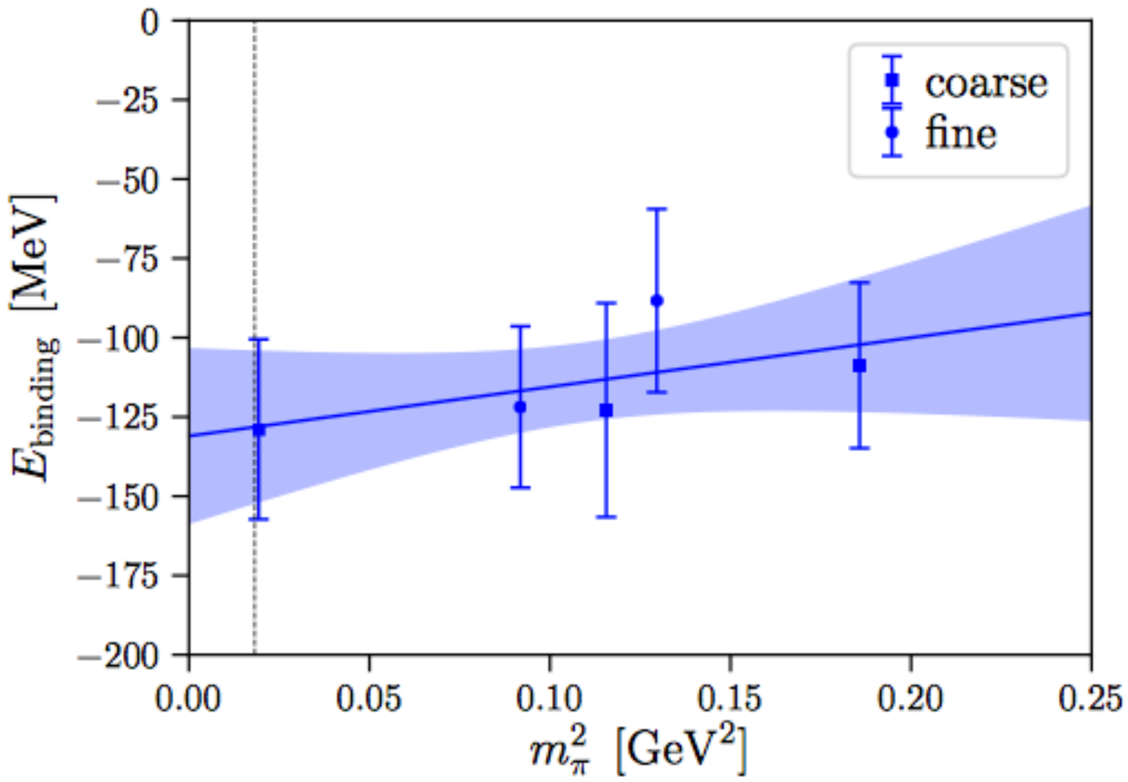
[Francis et al., PRD 99 (2019) 074505]

Tetraquarks

* Prediction of doubly-bottom tetraquark states:

$$\bar{b}\bar{b}ud, \quad (J^P = 1^+, I = 0)$$

* Apply Lüscher finite-volume quantisation condition to extract binding energy



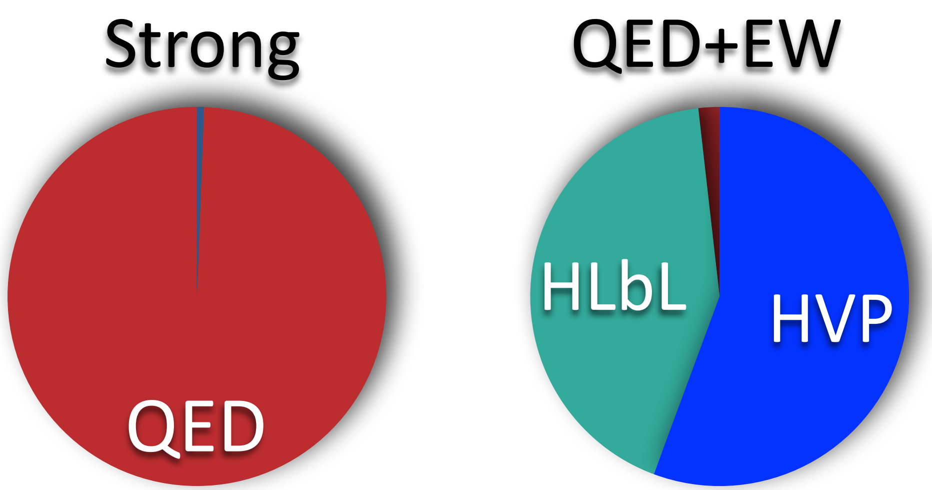
$$\Delta E = m_{\mathrm{tetra}} - m_B - m_{B^*_{(s)}}$$

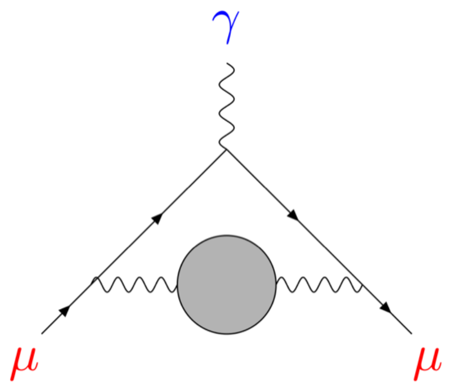
[Leskovec et al., arXiv:1904.04197]

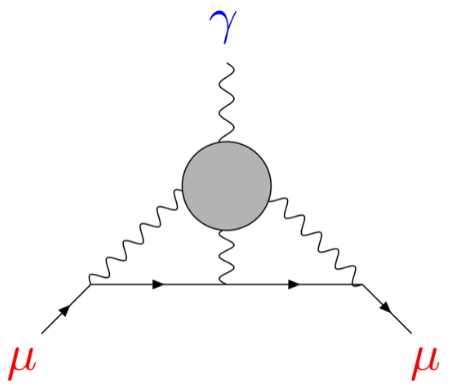
* Tetraquark also expected for $\bar{b}\bar{b}us, \quad (J^P = 1^+)$

Precision observables

- Muon $g-2$: intriguing hint of a deviation from the SM
- Uncertainty of the SM prediction dominated by strong interaction







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* Determine hadronic vacuum polarisation and light-by-light scattering contributions in Lattice QCD
  - High overall statistical accuracy
  - Quark-disconnected diagrams
  - Long-distance regime dominated by multi-hadron states
  - Isospin breaking contributions (QCD+QED) must be included

  [Meyer & HW, PPNP 104 (2019) 46, arXiv:1807.09370]

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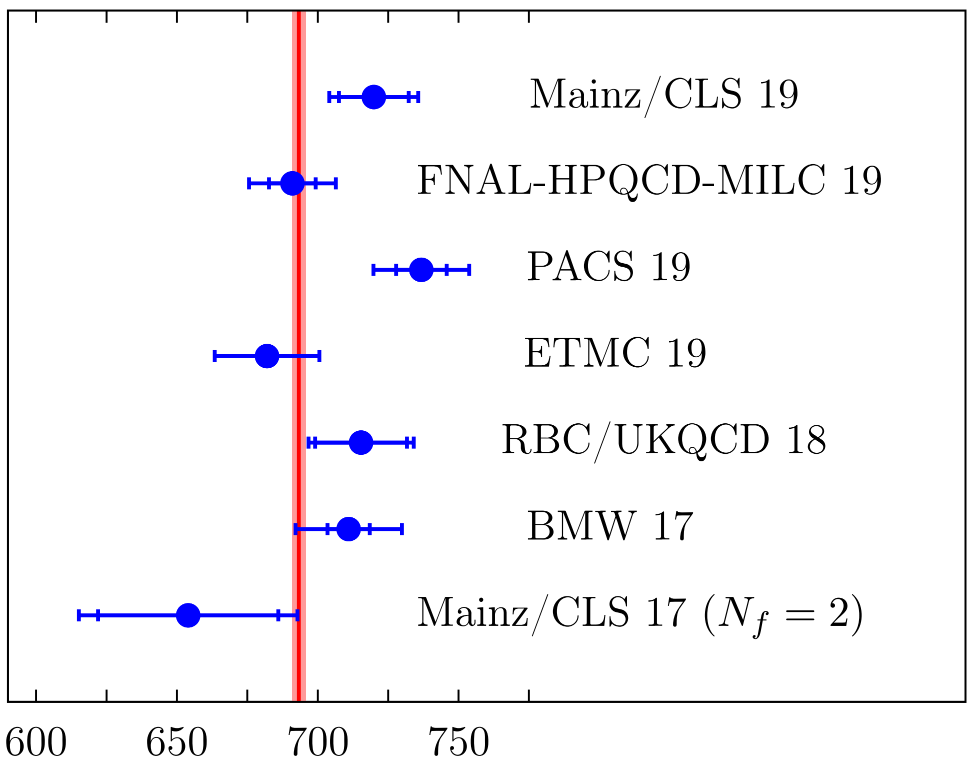
* Target precision: $\delta a_\mu^{\rm hvp}/a_\mu^{\rm hvp} < 0.5\%, \qquad \delta a_\mu^{\rm hlbl}/a_\mu^{\rm hlbl} \lesssim 10\%$

Hadronic contributions to the muon g – 2

* Hadronic vacuum polarisation:



[Gérardin et al., arXiv:1904.03120]

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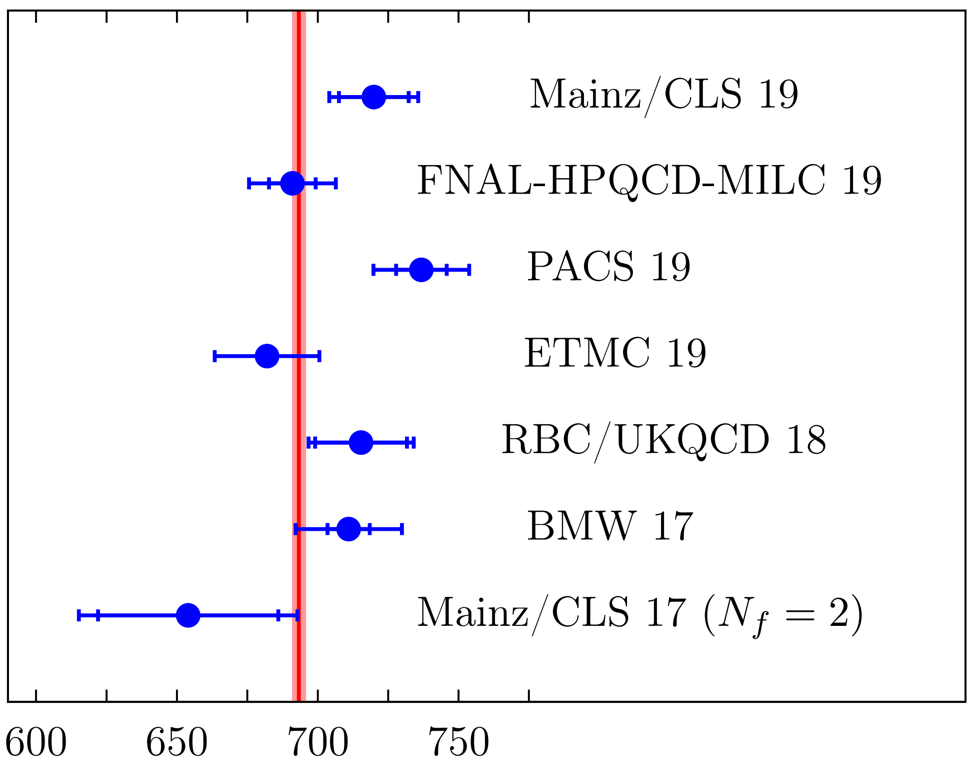
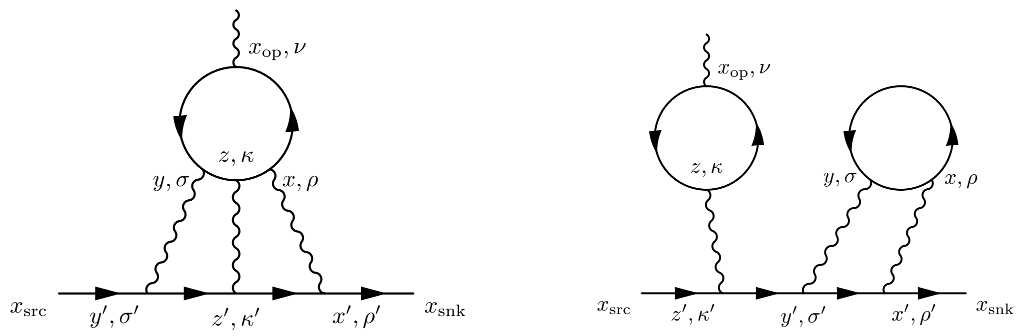


[Gérardin et al., arXiv:1904.03120]

* Light-by-light scattering



$$(a_\mu^{\rm hlbl})_{\rm con} = (116.0 \pm 9.6) \cdot 10^{-11}$$

$$(a_\mu^{\rm hlbl})_{\rm disc}^{\rm LO} = (-62.5 \pm 8.0) \cdot 10^{-11}$$

[Blum et al., PRD 93 (2016) 014503, PRL118 (2017) 022005]

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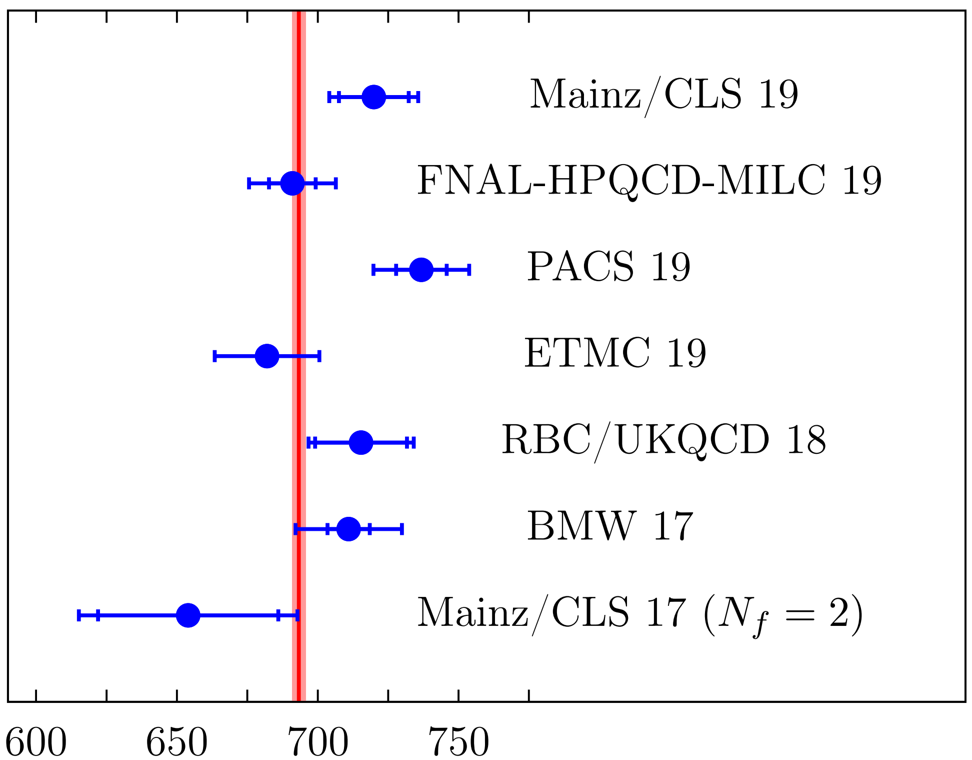

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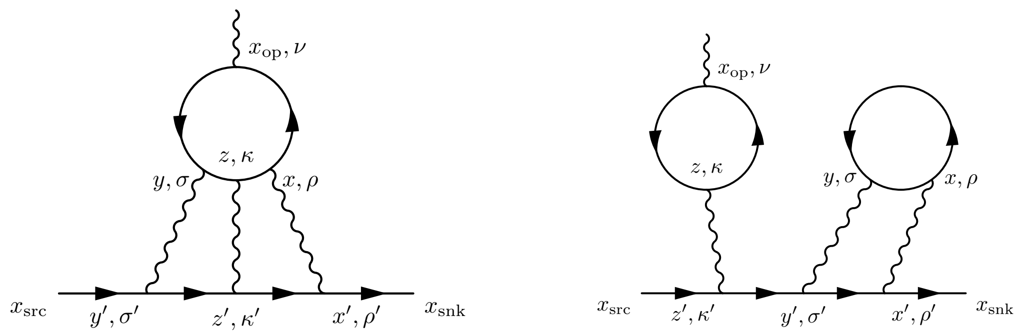

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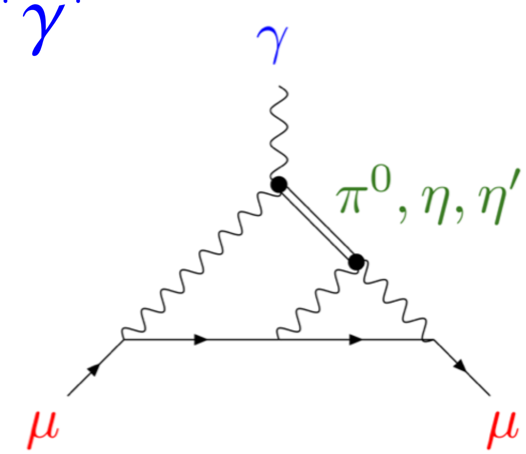
* Pion pole contribution: transition form factor $\pi^0 \to \gamma^*\gamma^*$

$$(a_\mu^{\rm hlbl})^{\pi^0} = \begin{cases} (59.7 \pm 3.6) \cdot 10^{-11} & \text{(Lattice QCD)} \\ (62.6^{+3.0}_{-2.5}) \cdot 10^{-11} & \text{(Dispersion Theory)} \end{cases}$$



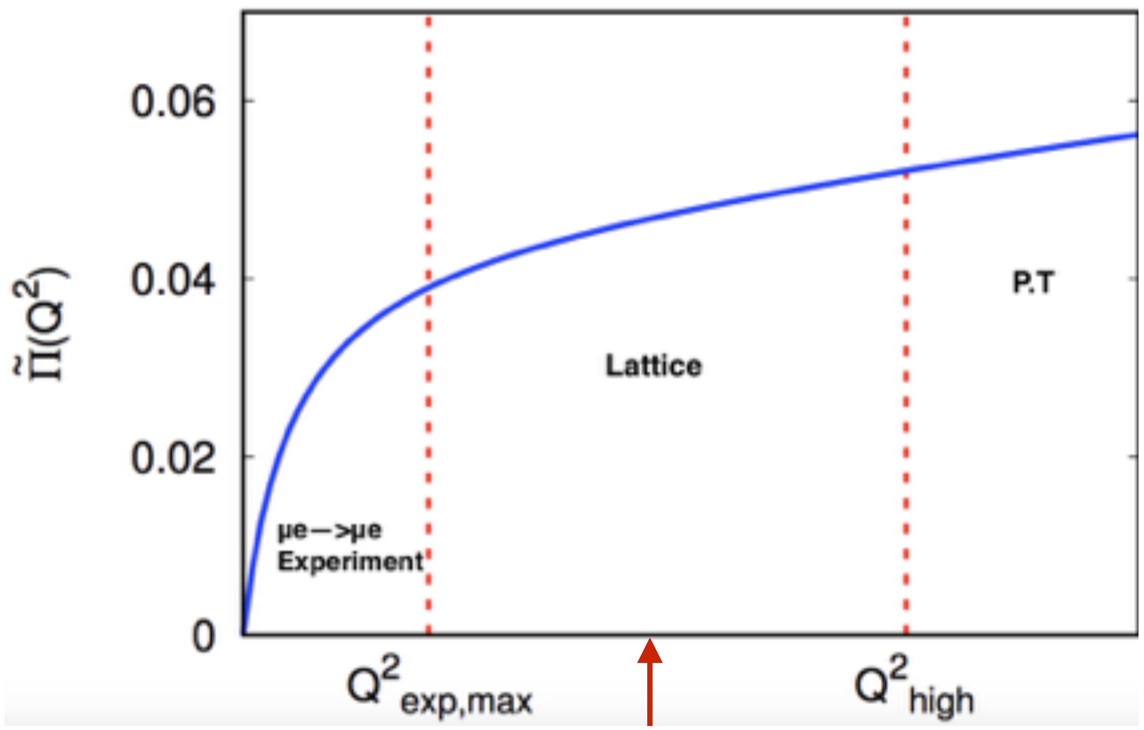
[Gérardin et al., arXiv:1903.09471, Hoferichter at al., arXiv:1808.04823]

Hadronic contributions to the muon g – 2

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

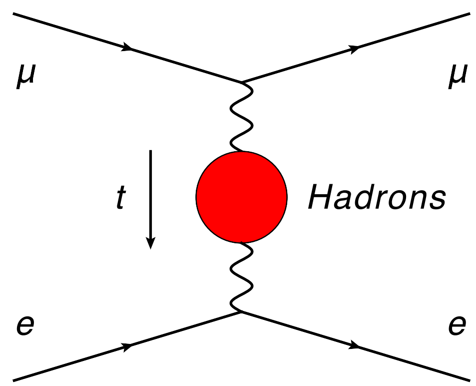
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\hat{\Pi}(Q^2), \quad \hat{\Pi}(Q^2) = 4\pi^2\left(\Pi(Q^2) - \Pi(0)\right)$$

* Hybrid method: use experimental data in the low-momentum region



MUonE experiment:

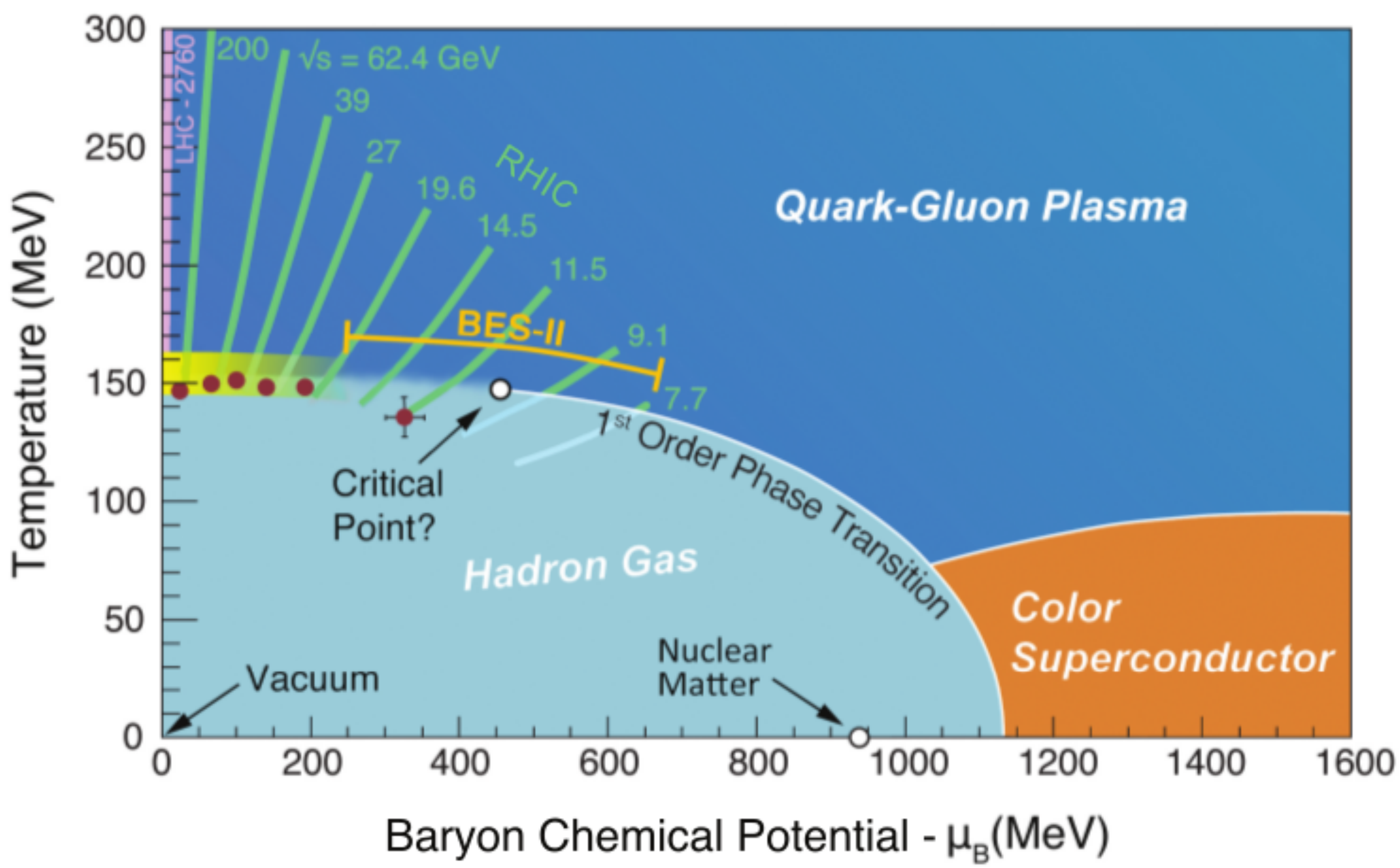
Determine $a_\mu^{\rm hvp}$ in elastic μe scattering



QCD at non-zero temperature and density

Open questions

* Existence of a critical point in the phase diagram
* Where is the transition line at high density?
* Are we creating a thermal medium in experiments?



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Role of Lattice QCD

- Equation of State $\longrightarrow$ hydrodynamic description of the QG plasma
- QCD phase diagram: transition line and critical endpoint
- Fluctuation of conserved charges $\longrightarrow$ evolution of heavy-ion collisions

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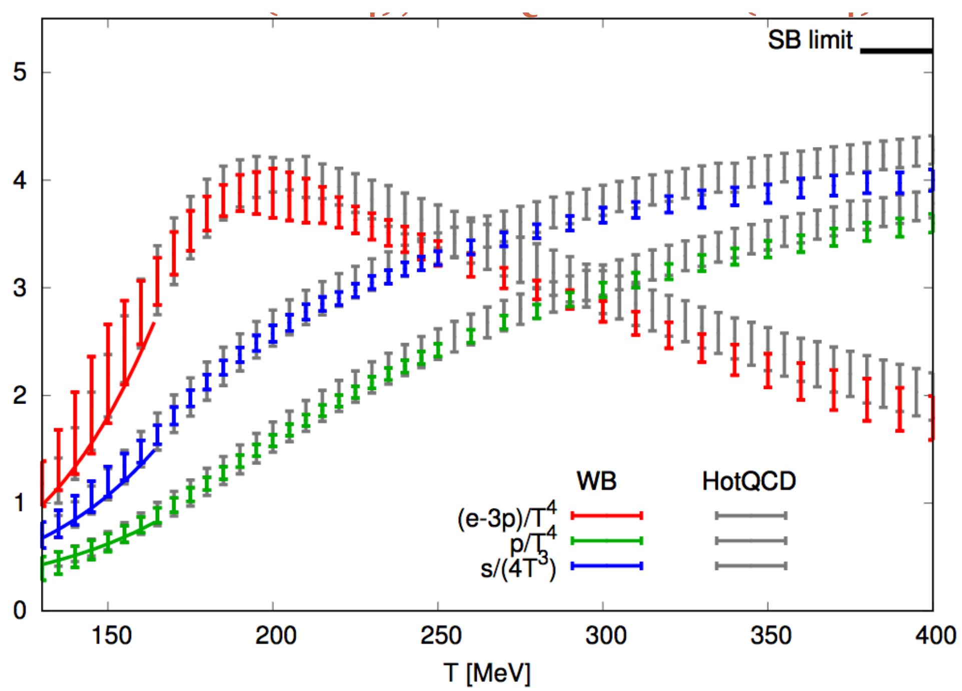
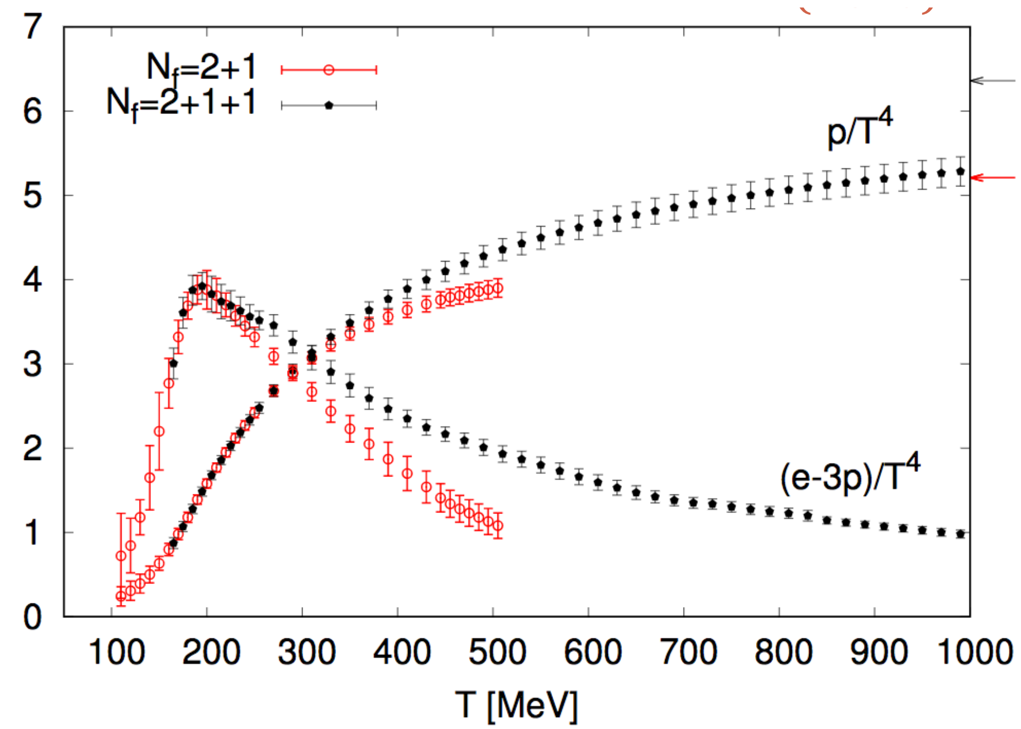
Quark determinant complex for $\mu_{\mathrm{B}} \neq 0$; conventional MC sampling fails

$\longrightarrow$ Taylor expansion, imaginary μ_{B}, reweighting

Equation of State

* Equation of State: Lattice QCD vs. Hadron Resonance Gas
  [C Ratti @ Lattice2018; Bazavov et al., arXiv:1407.6387; Borsanyi et al., arXiv:1309.5258]





* Extension to non-zero baryon density via Taylor expansion:

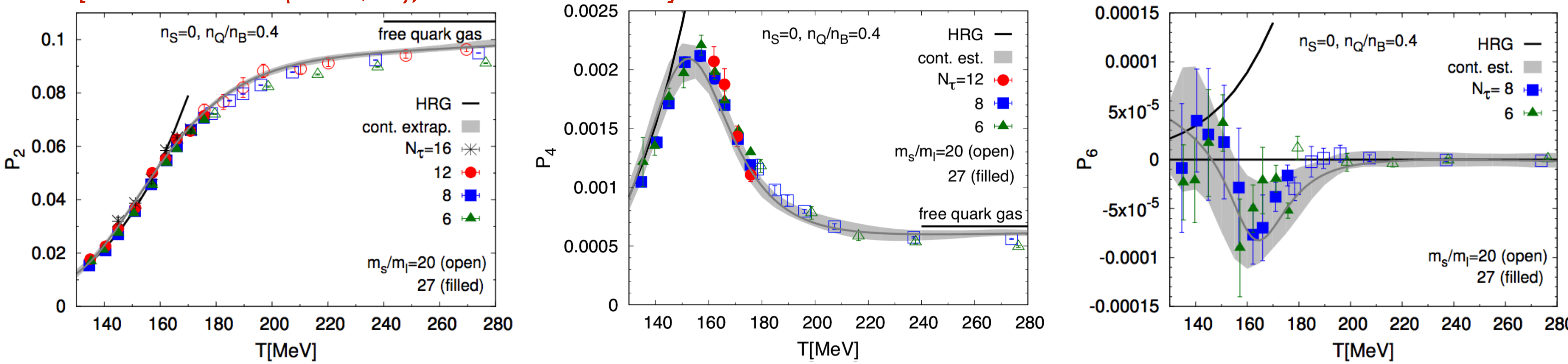
$$\frac{p(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

* Direct lattice calculation of coefficients; imaginary chemical potential

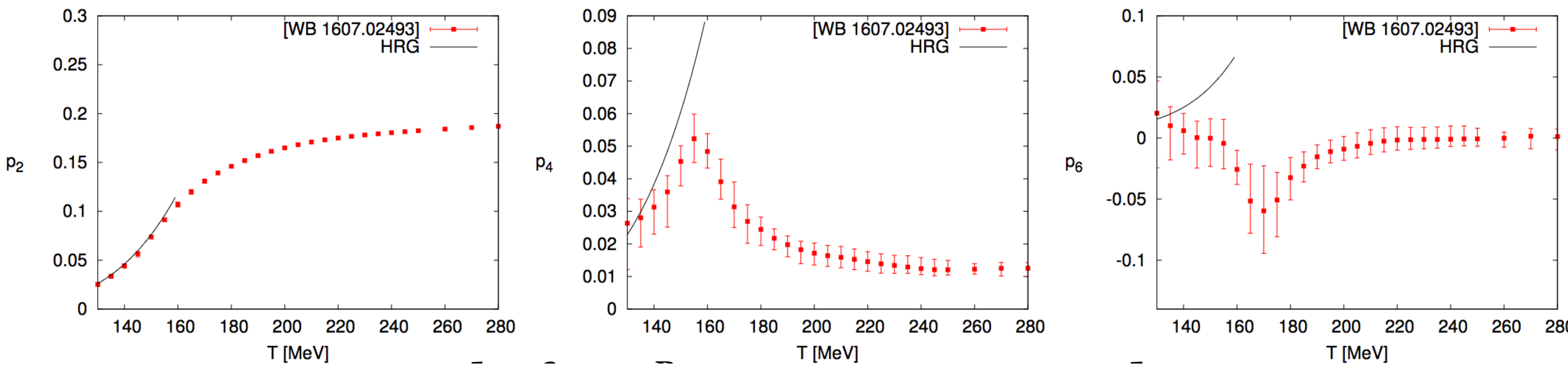
Equation of state for $\mu_B \neq 0$

* Pressure coefficients — strangeness neutrality

[Bazavov et al. (HotQCD), arXiv:1701.04325]



[Günther et al. (BW), arXiv:1607.02493]

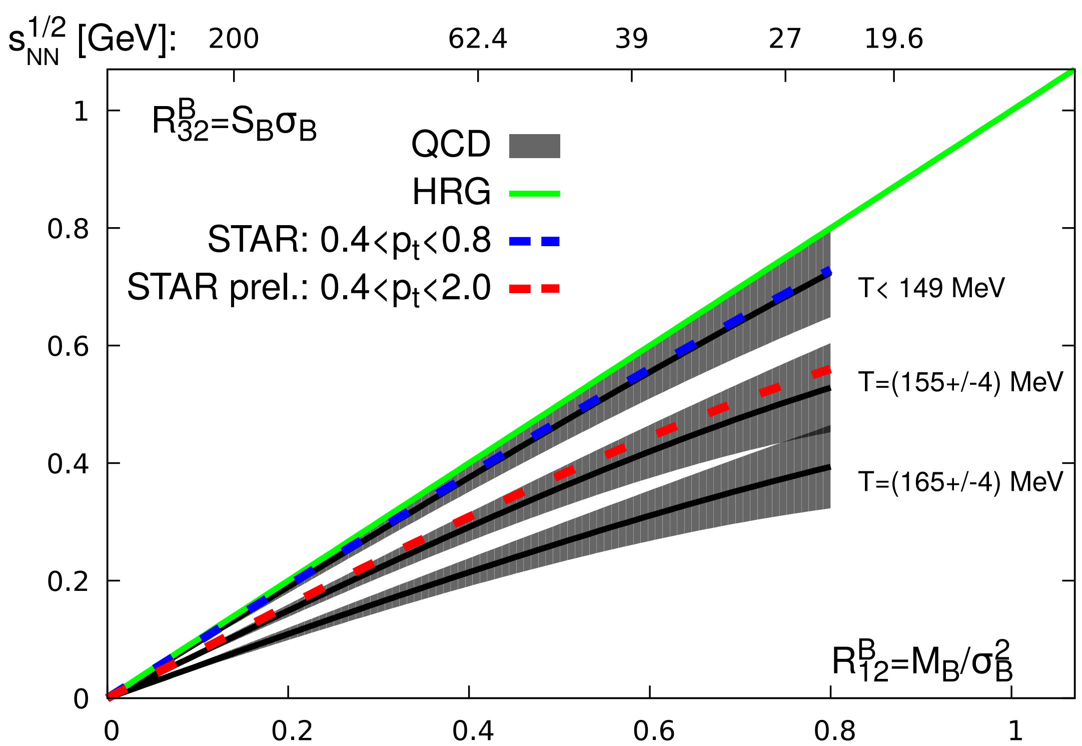


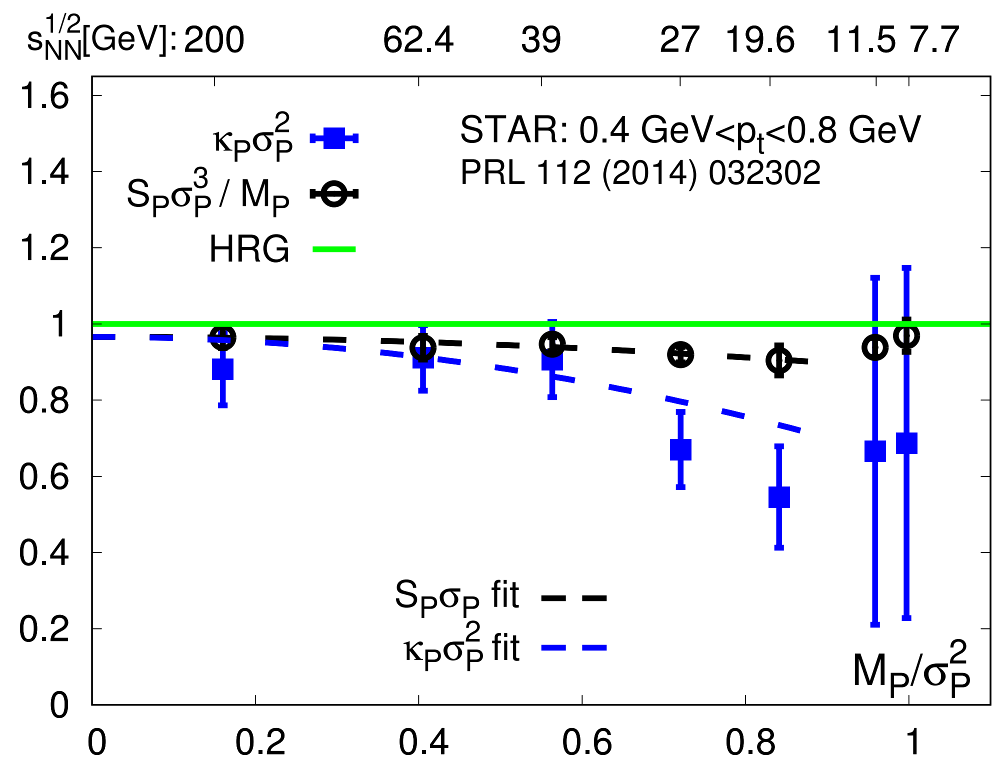
Fluctuation observables

* Study evolution of heavy-ion collisions — freeze-out properties
* Fluctuations of conserved charges linked to cumulants, e.g.

$$\chi_n^B = T^{n-4}\frac{\partial^n p(T,\mu_B)}{\partial\mu_B^n}, \quad \chi_2^B \sim \left\langle(\Delta N_B)^2\right\rangle$$

* Compare cumulant ratios to experiment and HRG model





[Bazavov et al. (HotQCD), arXiv:1708.04897]

Resources

* Sufficient computing resources crucial for continued progress
* Estimated required resources for all European groups:

  ≈ 10 Gcore-hours p.a.

* Lattice QCD has driven the field of High-Performance Computing

  (Hitachi's CP-PACS, IBM's BlueGene, APE computers, LQCD on GPUs)

* Future applications will require **Exa-scale computing capabilities**
* New approaches:
  machine learning, path optimisation, quantum computing,...

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Lattice QCD has become a non-perturbative toolkit for strong interactions

* Interaction between lattice calculations and experiment
* Enter "precision era" of lattice QCD

⇒ Crucial for experimental program at LHC, RHIC, EIC, FAIR, NICA, BESIII,…

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* Conceptual progress:
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All efforts to improve methodology must be accompanied by continued investment and improvement of computing resources