Lattice QCD
Challenges and Opportunities

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The role of Lattice QCD

- Provide quantitative input on hadron structure and spectroscopy and the properties of matter under extreme conditions

- Constrain hadronic contributions to processes and matrix elements relevant for BSM physics searches:
  - Flavour physics — CKM matrix elements
  - Nucleon matrix elements: axial, scalar and tensor charges, $\sigma$-terms
  - Muon $g - 2$

- Provide input for experiments and phenomenology
  - Parton Distribution Functions — joint fits of experimental and lattice data
  - Nucleon form factors — neutrino-nucleus cross sections, proton radius puzzle

- Validate model calculations and EFT descriptions

- Investigate BSM scenarios
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Lattice QCD then and now

“The lattice has produced numbers with small error bars of unmeasured quantities…”

[Guido Altarelli, DESY Theory Workshop 1990]

Lattice QCD makes predictions for a large variety of quantities in strong interaction physics with controlled errors:

• Hadron masses and matrix elements
• SM parameters
• Quantities characterising strongly interacting matter under extreme conditions
• Precision observables
**Example:** $B$-meson decay constant in Lattice QCD

→ relevant for $V_{ub}$, no experimental measurement
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Quenched approximation
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   → relevant for $V_{ub}$, no experimental measurement

Overall precision: 15% in 1997 → 0.5% in 2018

Correlated with an increase of $5 \times 10^5$ in computer power
Community efforts in Lattice QCD

FLAG Report:

* Performs PDG-style global analyses and averages [S. Aoki et al., arXiv:1902.08191]
  
  • SM parameters (quark masses, alpha_s)
  • Flavour physics ($K$, $D$, $B$ mesons)
  • Low-energy constants
  • Nucleon matrix elements

White papers:

* Community White Paper on Lattice input for global PDF analyses [H.-W. Lin et al., PPNP 100 (2018) 107]

* The muon $g - 2$ (to appear)

* USQCD 2019 : WPs on Lattice QCD in nuclear, hadron, flavour, BSM physics, QCD phase diagram, Exascale computing
Technical and conceptual challenges

* Observables have become more “complex”:
  - Baryon correlation functions
  - Quark-disconnected diagrams (→ flavour-singlet quantities)
  - Multi-hadron systems and light nuclei
  - Four-point functions required for hadronic light-by-light scattering
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* Exponential increase of statistical noise in correlation functions
  - e.g. nucleon matrix elements, muon g – 2

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![Graphs showing correlation functions and statistical noise](image-url)

**Figure 1**: Examples of correlation functions and statistical noise. The graphs illustrate the exponential increase of statistical noise in correlation functions, particularly evident for nucleon matrix elements and muon g – 2 measurements.
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* Sign problem in simulations of QCD with $\mu_B \neq 0$

* Incorporation of isospin breaking effects: QCD+QED, $m_u \neq m_d$

* $n$-particle decays of resonances, $n > 2$
Outline

Hadron structure

Hadron spectroscopy and hadronic interactions

Precision observables

QCD phase diagram

Resources

Outlook; Recommendations
Hadron structure calculations

* Confront experimental determinations with lattice QCD results

⇒ Consistency checks:
  proton radius and magnetic moment, axial charge

* Provide input for experiments and phenomenology:
  • Nucleon scalar and tensor charges $g_S, g_T$: constrain BSM effects
  • Axial form factor: $\nu$-nucleus cross sections (DUNE/LBNF)
  • (Generalised) Parton Distribution Functions; transversity (EIC)
  • Decomposition of the nucleon spin

* Additional systematic effects:
  • Bias from unsuppressed excited states:
    multi-state fits, summed operator insertions
  • Finite-volume effects
  • Renormalisation and matching
Axial, scalar and tensor charges

**FLAG averages:**

\[
\begin{align*}
    g_A^{u-d} &= 1.251 \pm 0.033 \\
    &\quad [1.2724 \pm 0.0023 \text{ (exp.)}] \\
    g_S^{u-d} &= 1.02 \pm 0.10 \\
    g_T^{u-d} &= 0.989 \pm 0.034 \\
\end{align*}
\]

[Aoki et al., arXiv:1902.08191]
Nucleon form factors

* **Electromagnetic form factors**: proton charge radius, magnetic moment
  → proton radius puzzle

* **Axial form factor**: constrain $\nu$-nucleus interaction and cross sections
  → long baseline neutrino experiments (DUNE/LBNF)

* **Strange form factors**: probe contribution from quark sea
  → parity-violating $ep$ scattering

* Quark-disconnected diagrams:
  * required to determine iso-scalar form factors
  * strange form factor
Electromagnetic form factors

Recent results: [Alexandrou et al., arXiv:1812.10311]

- Charge radii smaller than observed in $ep$ scattering experiments
- Focus on systematics and better statistical precision
Axial form factors

Definition:

\[
\langle N(p', s') | A^a_\mu(0) | N(p, s) \rangle = \bar{u}(p', s') \left[ \gamma_\mu \gamma_5 G_A(Q^2) - i\gamma_5 \frac{Q_\mu}{2m_N} G_P(Q^2) \right] \frac{1}{2} \tau^a u(p, s)
\]

Slope of \( G_A(Q^2) \) underestimated in calculations

\[ \rightarrow \text{focus on excited states systematics} \]
Strange electromagnetic form factors

- Signal entirely given by disconnected diagrams

- Good agreement among different calculations

- Increase statistical precision further

[Djukanovic et al., arXiv:1903.12566]
PDFs from Lattice QCD

∗ Workflow diagram for PDF determination

[Monahan, arXiv:1811.00678]

∗ Control over lattice systematics
∗ Non-local operator matching and renormalisation
∗ Higher-twist effects
∗ Inverse transform problems
PDFs from Lattice QCD

* Isovector unpolarised and transversity distributions  
  [Monahan, arXiv:1811.00678]


Hadron spectroscopy and hadronic interactions

- Lattice spectroscopy: beyond naïve extraction of ground state mesons and baryons
- Focus on
  - excitation spectrum
  - electromagnetic mass splittings
  - resonance properties
  - multi-hadron states

[Figure showing data comparison between experiment and QCD+QED predictions]

**Hadron spectroscopy and hadronic interactions**

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* Methods:
  - Lüscher’s finite-volume quantisation condition: energy levels in finite volume related to scattering phase shift
  - Correlator matrices and Generalised Eigenvalue problem
  - “Distillation”, smearing

[Image: BMW 2014    HCH
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[Andersen et al., NPB 939 (2019) 145]
Charmed resonances and baryons

* The $\Omega_c$ (css) in lattice QCD

![Graph showing the distribution of $\Omega_c$ candidates in the LHCb experiment.](image)
Charmed resonances and baryons

* The $\Omega_c$ ($css$) in lattice QCD

[Padmanath et al., PRL 119 (2017) 042001]

(strong decays ignored)
Charmed resonances and baryons

* The $\Omega_c$ (css) in lattice QCD

\begin{align*}
\Omega_{c\eta} & \quad 3/2^- \\
\Xi_D & \quad 1/2^- \\
\Xi_{c} & \quad 5/2^- \\
\Xi'_{c} K (S) & \quad 3/2^- \\
\Xi_{c} K (S) & \quad 3/2^- \\
\Xi_{c} K & \quad 1/2^- \\
\Xi_{c} K' & \quad 1/2^- \\
\end{align*}

\begin{align*}
E - E_{1/2^+} & \quad [\text{GeV}] \\
\text{Expt.} & \quad \text{Lattice} \\
\end{align*}

[Padmanath et al., PRL 119 (2017) 042001]

(strong decays ignored)

* Good agreement between conventional charmonia and recently discovered $X(3842)$ resonance

$X(3842)$ \quad $\phi (3842)$ \quad $\psi (3770)$ \quad $2m_D$ \quad $2m_{D^{\ast}}$ \quad $2m_{D^{0}}$

$\psi (2S)$

The H Dibaryon

* Conjectured scalar, SU(3)-singlet hexaquark state with quark content \( udsuds \)

* Binding energy from pole of scattering amplitude:

\[
\mathcal{A} \propto \frac{1}{p \cot \delta(p) - ip}
\]

\[
p \cot \delta(p) = \frac{2}{\gamma L \sqrt{\pi}} \mathcal{Z}_{00}(1, (pL/2\pi)^2)
\]

\[
p^2 = \frac{1}{4}(E^2 - \mathbf{P} \cdot \mathbf{P}) - m^2_{\Lambda}
\]

[Francis et al., PRD 99 (2019) 074505]
Tetraquarks

* Prediction of doubly-bottom tetraquark states:

\[ \bar{b}b\bar{u}d, \quad (J^P = 1^+, I = 0) \]

* Apply Lüscher finite-volume quantisation condition to extract binding energy

\[ \Delta E = m_{\text{tetra}} - m_B - m_{B^*} \]

[Leskovec et al., arXiv:1904.04197]

* Tetraquark also expected for \( \bar{b}\bar{b}u\bar{s}, \quad (J^P = 1^+) \)
Precision observables

* Muon $g - 2$ : intriguing hint of a deviation from the SM

* Uncertainty of the SM prediction dominated by strong interaction
Precision observables

- Muon $g - 2$: intriguing hint of a deviation from the SM
- Uncertainty of the SM prediction dominated by strong interaction
- Determine hadronic vacuum polarisation and light-by-light scattering contributions in Lattice QCD
  - High overall statistical accuracy
  - Quark-disconnected diagrams
  - Long-distance regime dominated by multi-hadron states
  - Isospin breaking contributions (QCD+QED) must be included

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* Target precision: $\frac{\delta a_{\mu}^{\hvp}}{a_{\mu}^{\hvp}} < 0.5\%$, $\frac{\delta a_{\mu}^{\hlbl}}{a_{\mu}^{\hlbl}} \lesssim 10\%$
Hadronic contributions to the muon $g - 2$

* Hadronic vacuum polarisation:

![Graph showing hadronic vacuum polarisation contributions from various collaborations.]

[Gerardin et al., arXiv:1904.03120]
Hadronic contributions to the muon $g - 2$

* Hadronic vacuum polarisation:

![Graph showing hadronic vacuum polarisation]

- Mainz/CLS 19
- FNAL-HPQCD-MILC 19
- PACS 19
- ETMC 19
- RBC/UKQCD 18
- BMW 17
- Mainz/CLS 17 ($N_f = 2$)

* Light-by-light scattering

![Diagram of light-by-light scattering]

$(a_\mu^{hlbl})_{con} = (116.0 \pm 9.6) \cdot 10^{-11}$

$(a_\mu^{hlbl})_{disc} = (-62.5 \pm 8.0) \cdot 10^{-11}$

[Blum et al., PRD 93 (2016) 014503, PRL118 (2017) 022005]

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**Hadronic contributions to the muon $g - 2$**

**Hadronic vacuum polarisation:**

![Graph showing hadronic vacuum polarisation](image)

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**Light-by-light scattering**

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**Pion pole contribution: transition form factor** $\pi^0 \to \gamma^* \gamma^*$

\[
(a_\mu^{\text{hlbl}})^{\pi^0} = \begin{cases} 
(59.7 \pm 3.6) \cdot 10^{-11} & \text{(Lattice QCD)} \\
(62.6^{+3.0}_{-2.5}) \cdot 10^{-11} & \text{(Dispersion Theory)}
\end{cases}
\]

Hadronic contributions to the muon $g - 2$

* Convolution integral over Euclidean momenta:

$$a_{\mu}^{hvp} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^{\infty} dQ^2 f(Q^2) \hat{\Pi}(Q^2), \quad \hat{\Pi}(Q^2) = 4\pi^2 \left( \Pi(Q^2) - \Pi(0) \right)$$

* Hybrid method: use experimental data in the low-momentum region

MUonE experiment:

Determine $a_{\mu}^{hvp}$ in elastic $\mu e$ scattering
QCD at non-zero temperature and density

Open questions

- Existence of a critical point in the phase diagram
- Where is the transition line at high density?
- Are we creating a thermal medium in experiments?
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Role of Lattice QCD

🌟 Equation of State → hydrodynamic description of the QG plasma
🌟 QCD phase diagram: transition line and critical endpoint
🌟 Fluctuation of conserved charges → evolution of heavy-ion collisions
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Quark determinant complex for $\mu_B \neq 0$; conventional MC sampling fails

→ Taylor expansion, imaginary $\mu_B$, reweighting
Equation of State

Equation of State: Lattice QCD vs. Hadron Resonance Gas

[C Ratti @ Lattice2018; Bazavov et al., arXiv:1407.6387; Borsanyi et al., arXiv:1309.5258]

Extension to non-zero baryon density via Taylor expansion:

\[
p(T, \mu_B) \frac{T^4}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left( \frac{\mu_B}{T} \right)^{2n}
\]

Direct lattice calculation of coefficients; imaginary chemical potential
Equation of state for $\mu_B \neq 0$

* Pressure coefficients — strangeness neutrality

[Bazavov et al. (HotQCD), arXiv:1701.04325]

[Günther et al. (BW), arXiv:1607.02493]
Fluctuation observables

* Study evolution of heavy-ion collisions — freeze-out properties
* Fluctuations of conserved charges linked to cumulants, e.g.

\[ \chi_n^B = T^{n-4} \frac{\partial^n p(T, \mu_B)}{\partial \mu_B^n}, \quad \chi_2^B \sim \langle (\Delta N_B)^2 \rangle \]

* Compare cumulant ratios to experiment and HRG model

[Bazavov et al. (HotQCD), arXiv:1708.04897]
Sufficient computing resources crucial for continued progress

Estimated required resources for all European groups:

\[ \approx 10 \text{ Gcore-hours p.a.} \]

Lattice QCD has driven the field of High-Performance Computing

(Hitachi’s CP-PACS, IBM’s BlueGene, APE computers, LQCD on GPUs)

Future applications will require Exa-scale computing capabilities

New approaches:
  machine learning, path optimisation, quantum computing,...
Lattice QCD has become a non-perturbative toolkit for strong interactions

* Interaction between lattice calculations and experiment
* Enter “precision era” of lattice QCD

⇒ Crucial for experimental program at LHC, RHIC, EIC, FAIR, NICA, BESIII,...
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Further progress required

- Increase statistical precision → noise reduction methods
- Conceptual progress:
  - inverse problems, $\mu_B \neq 0$, $n$-particle decays, renormalisation
**Outlook**

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All efforts to improve methodology must be accompanied by continued investment and improvement of computing resources