Unitarity and Entropy

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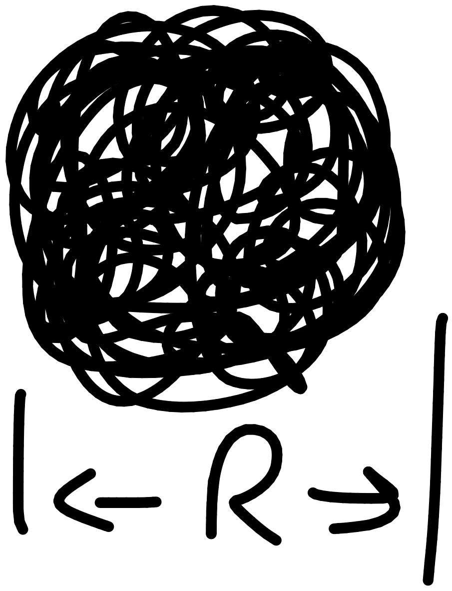
Bekenstein-Bremermann Bound



$$S \leq M R$$

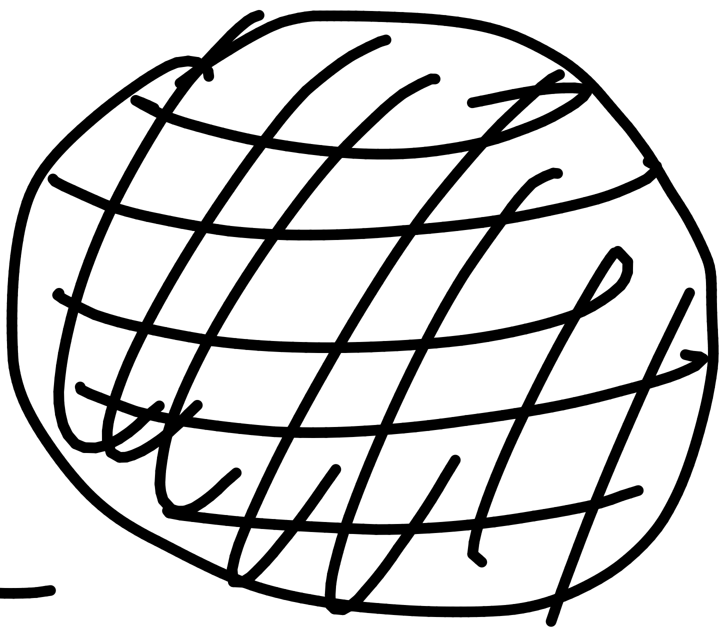
In gravity is saturated by black holes.

Bekenstein entropy:

$$S_{BH} = M_{BH} R = (R M_P)^2$$

(*) What is physical meaning of bound?

(*) What is physical meaning of Area-Law?



$$S = (R M_p)^2$$

The bound has (mostly) been discussed in gravity.

What happens beyond gravity?

In renormalizable theories?

Our main results:

- Bekenstein bound is saturated when theory saturates the bound on <u>unitarity</u>.

- Simultaneously the entropy assumes the area law:

$$S = MR = (Rf)^2$$

$\uparrow$ <u>scale</u>

Bekenstein = Unitarity = Area

$$S_{max} = \frac{1}{g^2} = \text{Area}$$

Quantum Coupling

We shall demonstrate for:

① 't Hooft - Polyakov monopole:

$$S = M_m R_m = \frac{1}{g^2_{gauge}} = (R_m v)^2$$

② Baryon:

$$S \approx M_B R_B = \frac{1}{g^2_{QCD}} = (R_B f_\pi)^2$$

Monopole. SO(3) gauge symmetry Higgsed by

$$\phi^a \quad (a = 1, 2, 3)$$

$$\mathcal{L} = \mathcal{D}_\mu \phi^a \mathcal{D}^\mu \phi^a$$

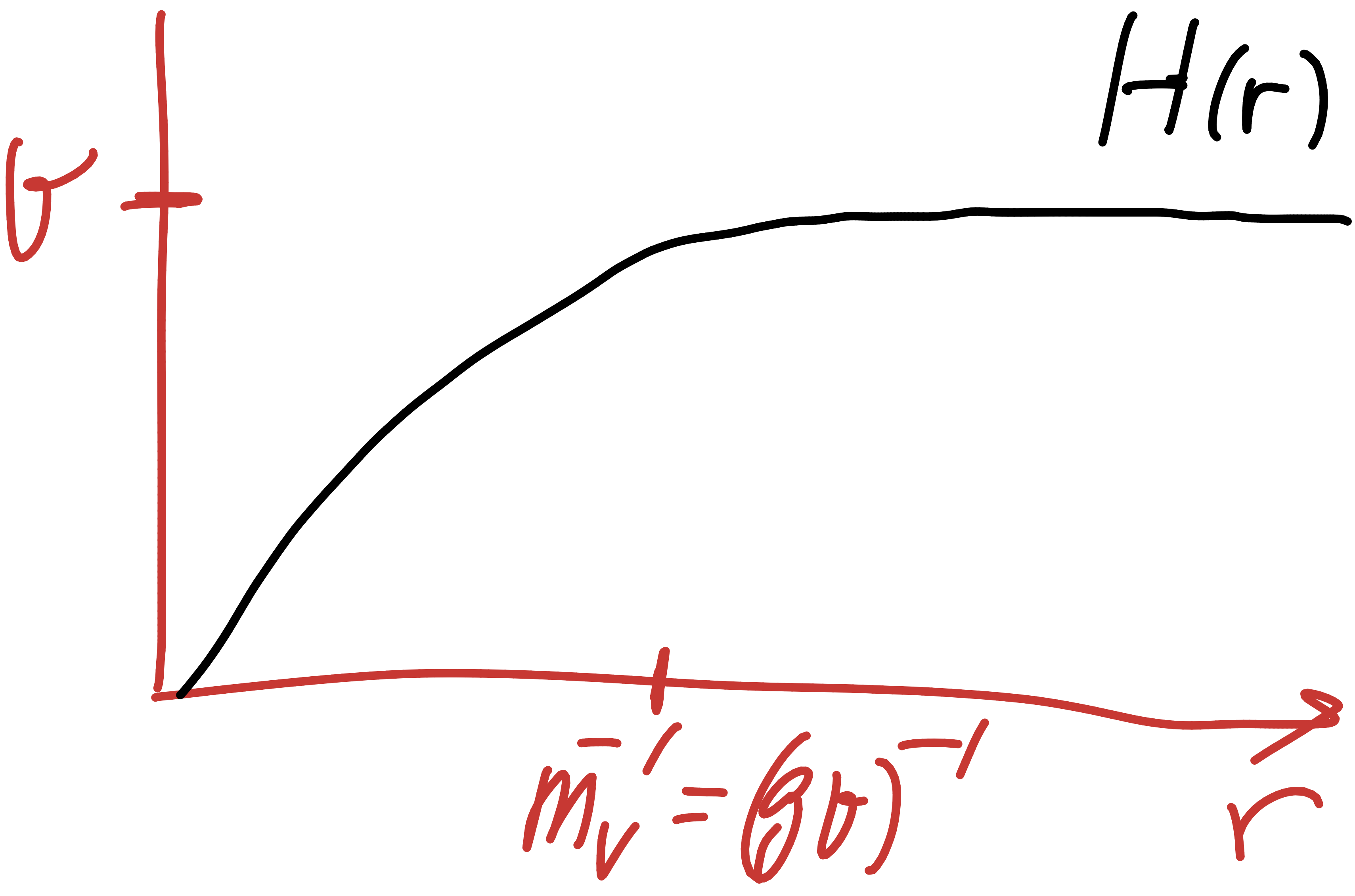
$$- \lambda^2 \left(\phi^a \phi^a - v^2 \right)^2$$

$$- \quad F^a_{\mu\nu} F^{a\mu\nu}$$

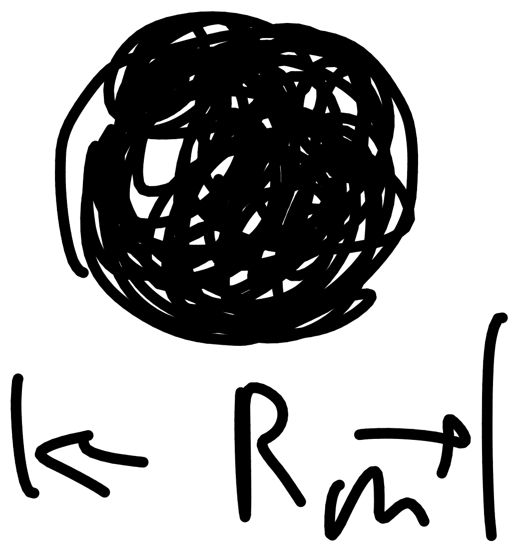
Monopole:

$$\phi^a = \frac{x^a}{r} H(r)$$

$$A^a_\mu = \varepsilon_{0a\mu\nu} \frac{x^\nu}{g r^2} F(r)$$



Monopole mass and size



$$R_m = (m_v)^{-1} = (gv)^{-1}$$

$$M_m = \frac{m_v}{g^2}$$

Entropy bound on monopole:

$$S \leq M_m R_m = \frac{1}{g^2}$$

Can it be saturated?

Entropy from Goldstones

$\sigma_\alpha \quad \alpha = 1, 2, \ldots N$

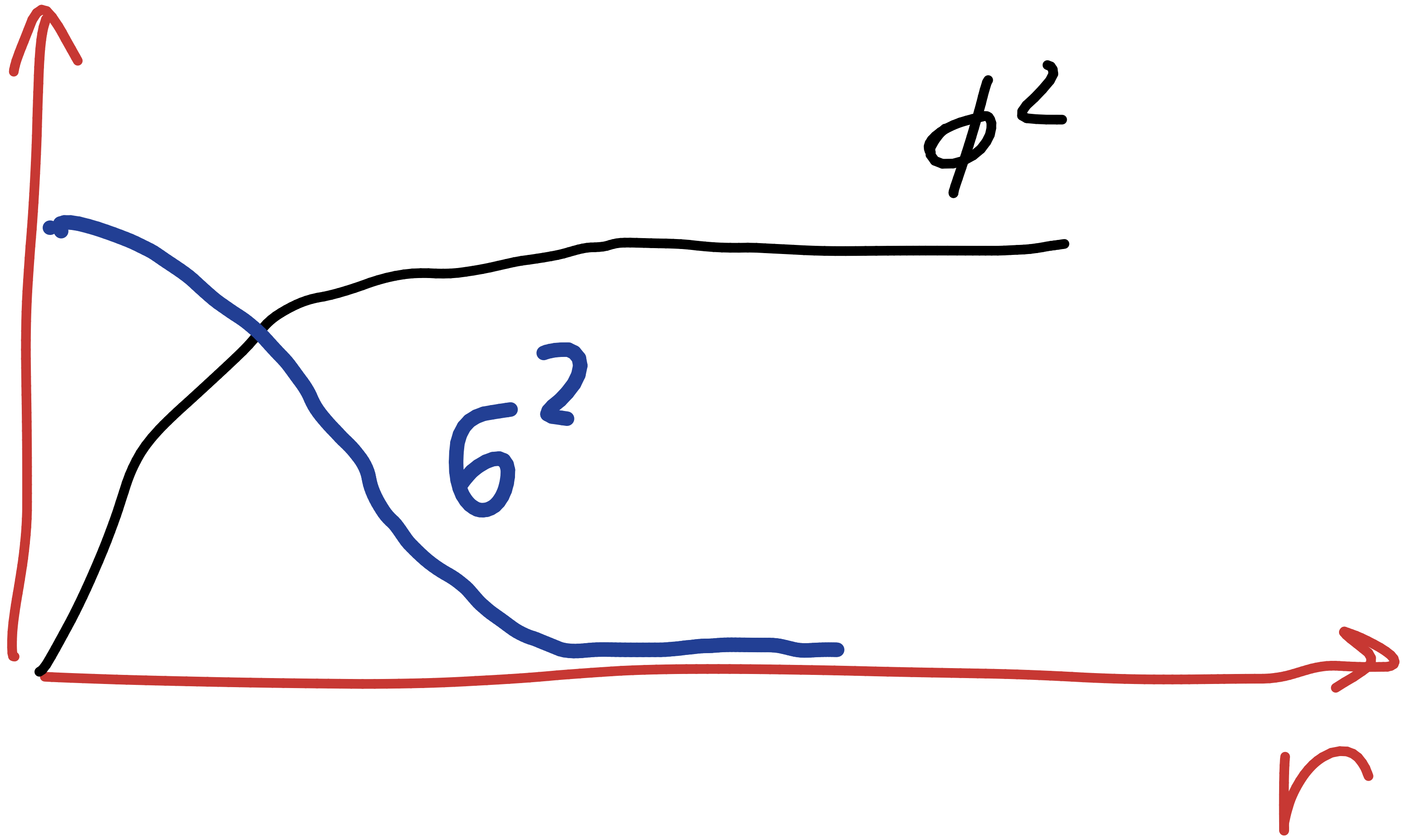
$\leftarrow$ scalar

$SO(N)$ - global symmetry

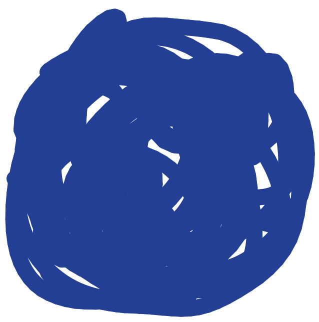
spontaneous breaking

in monopole:

$$\mathcal{L} = \partial_\mu \sigma_\alpha \partial^\mu \sigma_\alpha$$
$$- (g^2 \phi^2 - m^2) \sigma_\alpha \sigma_\alpha$$
$$- g_\sigma^2 (\sigma_\alpha \sigma_\alpha)^2$$



$\langle \sigma \rangle \neq 0$



$SO(N) \to SO(N-1)$

$\sim N$ localized Goldstones!

Number of degenerate micro-states

$$n_{st} \sim \binom{2N}{N} \sim 2^{2N}$$

Monopole entropy:

$$S_{mon} \simeq \ln(n_{st}) \sim N$$

Unitarity bound:

$$g^2 N \leq 1$$

For $N = \frac{1}{g^2}$ entropy bound is saturated by area-law!

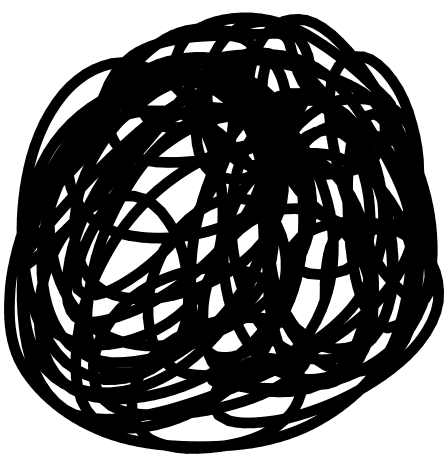
$$S_{mon} = N = \frac{1}{g^2} = M_m R_m = (R_m v)^2$$

The same is achieved by coupling to fermion flavors:

$$\phi^a \psi^b_\alpha \lambda^c_\alpha \varepsilon_{abc}$$

$\alpha = 1, 2 .. N$

$SO(N)$



~N localized fermion zero modes!

Again:

$$n_{st} \sim 2^N$$

Entropy: $S_{mon} \sim N$

Unitarity bound

$$g^2 N \leq 1$$

$$S_{mon} = N = \frac{1}{g^2} = M_m R_m = (R_m G)^2$$

Baryons in $SU(N_c)$ QCD with N-flavors of quarks.

't Hooft limit

$N_c \to \infty$

$g^2 N_c = \text{fixed}$

$\Lambda = \text{fixed}$

Spontaneous chiral symmetry breaking

$$U(N)_L \otimes U(N)_R \to U(N)_F$$

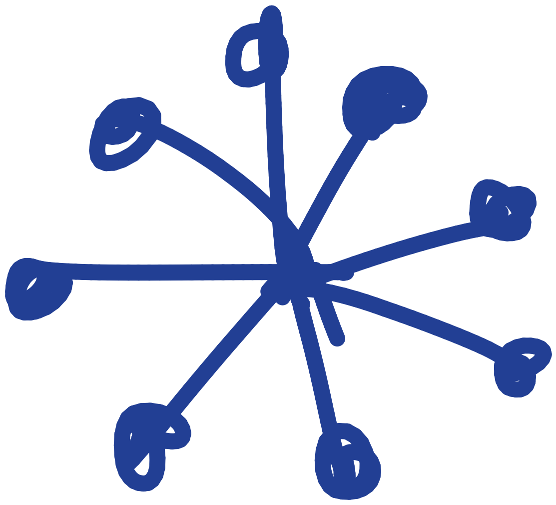
N^2-1 Goldstones (pions)

$+\ \eta'$-meson

Pion decay constant:

$$f_\pi = \sqrt{N_c}\,\Lambda$$

Baryons (Witten):



N_c quarks

$R_B \sim \Lambda^{-1}$

Mass: $M_B \sim N_c \Lambda$

Entropy bound:

$$S_{MAX} = M_B R_B = N_c$$

Is saturated at the unitarity bound:

$$N_c \sim N \sim \frac{1}{g^2}$$

Baryon entropy:

$$S_B = \ln \binom{N_c + N}{N_c} \sim N$$

Thus, at the unitarity bound we have:

$$\rho_B = M_B R_B = \frac{1}{g^2} = N =$$

$$= \left(R_B f_\pi\right)^2$$

Area law!

Conclusions:

The universal phenomenon:

B.B. = unitarity = Area

$$\downarrow$$

$$\text{Bound} = \frac{1}{\text{coupling}} = \text{Area}$$

Can confinement in QCD (at least for large N_c) be understood as preventive mechanism against violation of Bekenstein bound by free colored states?

Entropy of a free quark

$$S_Q = \ln(2 N_c N_f)$$

$$S_{max} = M_Q R$$

?