Discoveries and Open Puzzles in Particle Physics and Gravitation, Kitzbühel, June 2019

Tensor Networks and Quantum Simulation Methods for Gauge Theories

EREZ ZOHAR

MPQ

Theory Group, Max Planck Institute of Quantum Optics (MPQ, Garching, Germany)
→ **Racah Institute of Physics, Hebrew University of Jerusalem, Israel**

<u>Based on works with</u> (in alphabetical order):

Julian Bender (MPQ)
J. Ignacio Cirac (MPQ)
Patrick Emonts (MPQ)
Alessandro Farace (MPQ)
Daniel Gonzalez Cuadra (MPQ→ICFO)
Ilya Kull (MPQ → Vienna)
Andras Molnar (MPQ)
Benni Reznik (TAU)
Thorsten Wahl (MPQ → Oxford)

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Gauge Theories are challenging:

- Local symmetry → many constraints
- Involve non-perturbative physics
  - Confinement of quarks
  - Exotic phases of QCD

→ Hard to treat experimentally (strong forces)

→ Hard to treat analytically (non perturbative)

→ Lattice Gauge Theory (Wilson, Kogut-Susskind…)

→ Lattice regularization in a gauge invariant way

Conventional LGT techniques

- Discretization of both space and time
- Monte Carlo computations on a Wick-rotated, Euclidean lattice

$$\left\langle \hat{A}\left(\hat{\Phi}\right)\right\rangle = \frac{\int \mathcal{D}\phi A(\phi) e^{iS_M}}{\int \mathcal{D}\phi e^{iS_M}}$$

$$\xrightarrow[t\rightarrow -i\tau]{} \frac{\int \mathcal{D}\phi A(\phi) e^{-S_E}}{\int \mathcal{D}\phi e^{-S_E}} \equiv \int \mathcal{D}\phi A\left(\phi\right) p\left(\phi\right)$$

- **Very (very) successful for many applications, e.g. the hadronic spectrum**
- **Problems:**
  - **Real-Time evolution:**
    - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - **Sign problem:**
    - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime

Quantum Information Methods for LGTs

- An active, rapidly growing research field

- **Quantum Simulation for LGTs** (around 8 years):
  - MPQ Garching & Tel Aviv University (Cirac, Reznik, EZ)
  - IQOQI Innsbruck, Bern, Trieste, Waterloo (Zoller, Wiese, Blatt, Dalmonte, Muschik)
  - Barcelona (Lewenstein, Tagliacozzo, Celi)
  - Heidelberg (Berges, Oberthaler, Jendrzejewski, Hauke …)
  - Iowa (Meurice)
  - Bilbao (Solano, Rico)
  - …

- **Tensor Networks for LGTs** (around 6 years):
  - MPQ Garching & DESY (Cirac, Jansen, Banuls, EZ…)
  - Ghent (Verstraete, Haegeman)
  - Barcelona (Lewenstein, Tagliacozzo, Celi)
  - IQOQI, Bern, Trieste, Ulm (Zoller, Wiese, Dalmonte , Montangero,…)
  - Iowa (Meurice)
  - Mainz (Orus)
  - …

- **Quantum Computation for LGTs** (relatively new):
  - Seattle (Kaplan, Savage)
  - Fermilab, …
  - Bilbao (Solano, Rico)
  - …

Quantum Simulation

- Take a model, which is either
  - Theoretically unsolvable
  - Numerically problematic
  - Experimentally inaccessible
- Map it to a fully controllable quantum system – quantum simulator
- Study the simulator experimentally

Quantum Simulation of LGTs

- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Exists by default in a real experiment done in a **quantum simulator**: prepare some initial state and the appropriate Hamiltonian (in terms of the simulator degrees of freedom), and let it evolve
- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - In real experiments, as those carried out by a **quantum simulator**, fermions are simply fermions, and no path integral is calculated: **nature does not calculate determinants**.

Tensor Networks

- The **number of variables** needed to describe states of a many-body system **scales exponentially** with the system size. This makes it hard to simulate large systems (classically).

- **Tensor networks** are Ansätze for describing and solving **many body states**, mostly on a lattice, for either **analytical or numerical** studies, based on contractions of **local tensors that depend on few parameters**.

- In spite of their **simple description**, tensor network states describe and approximate **physically relevant states of many-body systems**.

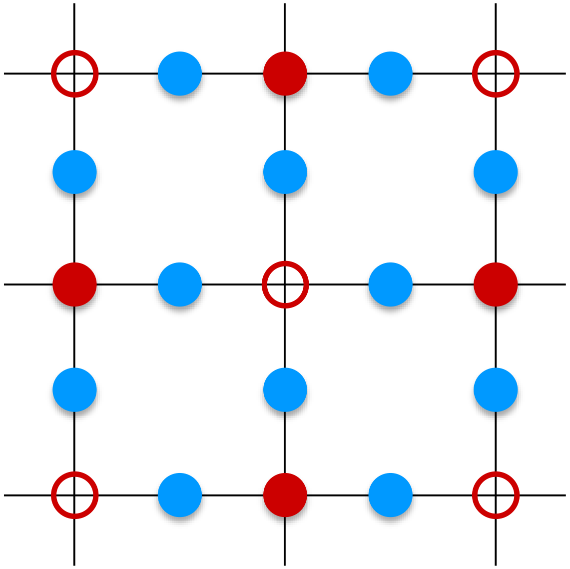
Tensor Network Studies of LGTs

- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Calculations in **quantum Hilbert spaces**, where states evolve in real time, instead of in Wick-rotated statistical mechanics analogies.

- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - Calculations in **quantum Hilbert spaces**: fermions are fermions, no integration over time dimension. If the problem arises, it can be the result of using a particular method, nothing general.

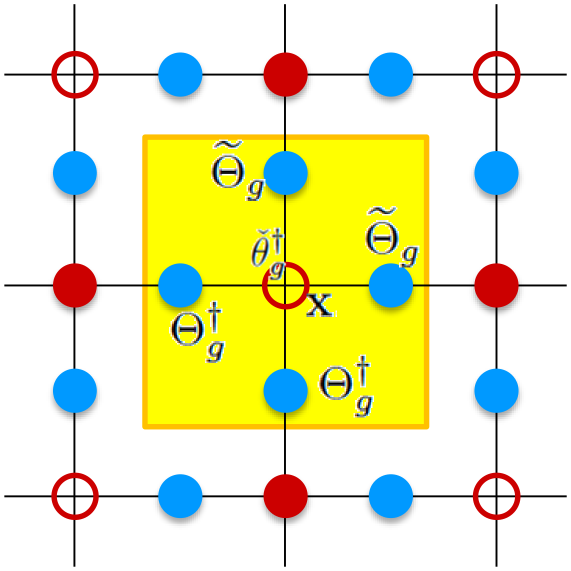
Hamiltonian LGT - Degrees of Freedom

- **The lattice is spatial**: time is a continuous, real coordinate.
- **Matter particles** (fermions) – on the **vertices**.
- **Gauge fields** – on the lattice's **links**



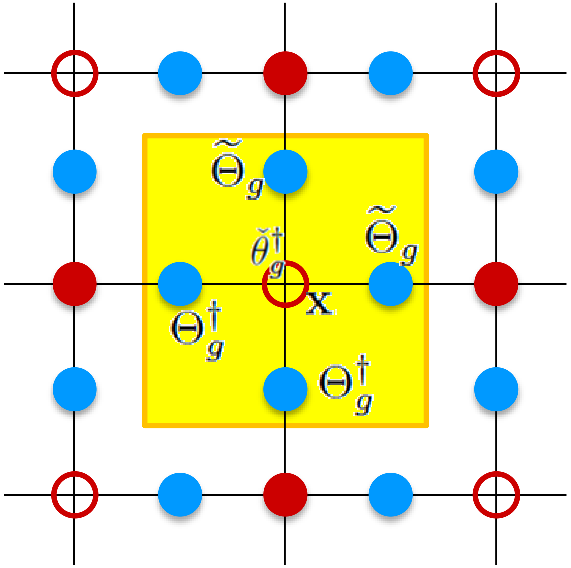
Gauge Transformations

- Act on both the matter and gauge degrees of freedom.
- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site
- The states are **invariant under each local transformation separately**.



$$\hat{\Theta}_g\left(\mathbf{x}\right)=\prod_{k=1\dots d}\left(\widetilde{\Theta}_g\left(\mathbf{x},k\right)\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^\dagger\left(\mathbf{x}\right)$$

Symmetry → Conserved Charge



– **Transformation rules on the links**

$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g |h\rangle = |hg^{-1}\rangle \quad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g |h\rangle = |g^{-1}h\rangle \quad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$

– **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\dots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^\dagger(\mathbf{x})$$

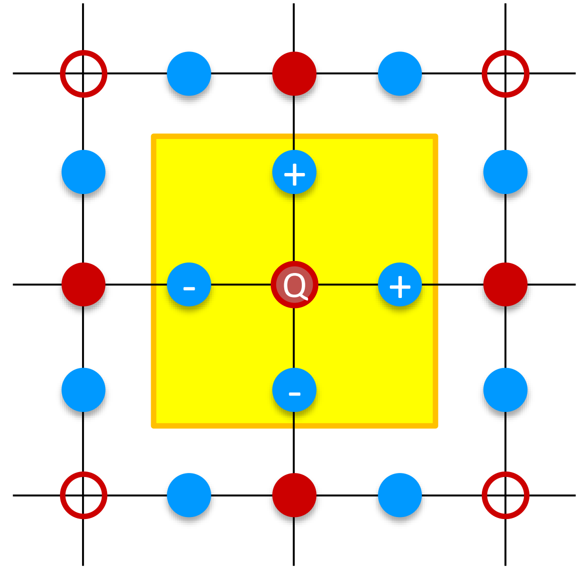
$$\hat{\Theta}_g(\mathbf{x})|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

– **Generators → Gauss law , left and right E fields:**

$$G_a(\mathbf{x}) = \sum_{k=1\dots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})|\Psi\rangle = 0 \quad [G_a(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}, a$$

Structure of the Hilbert Space



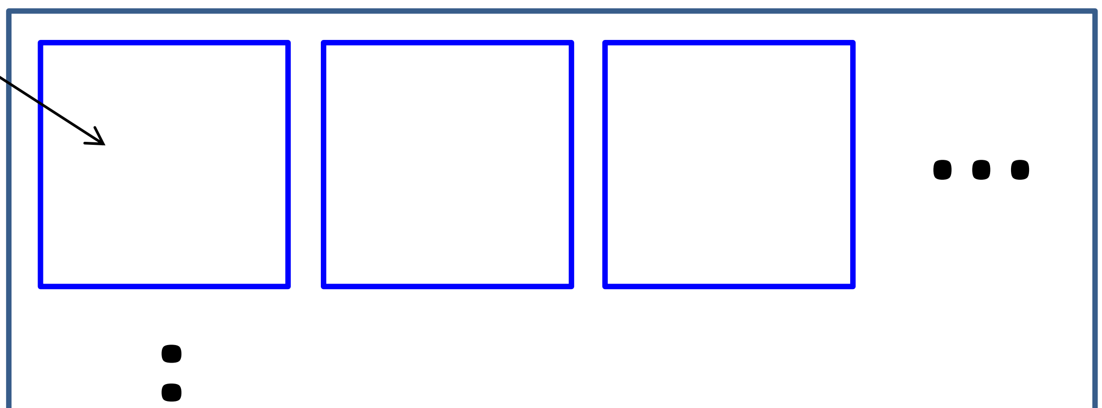
- Generators of gauge transformations (cQED):

$$G\left(\mathbf{x}\right) = \mathrm{div}L\left(\mathbf{x}\right) - Q\left(\mathbf{x}\right)$$

$$\equiv \sum_k \left(L_k\left(\mathbf{x}\right) - L_k\left(\mathbf{x} - \hat{\mathbf{e}}_k\right)\right) - Q\left(\mathbf{x}\right)$$

Gauss' Law $\quad G\left(\mathbf{x}\right)\left|\psi\right\rangle = q\left(\mathbf{x}\right)\left|\psi\right\rangle$

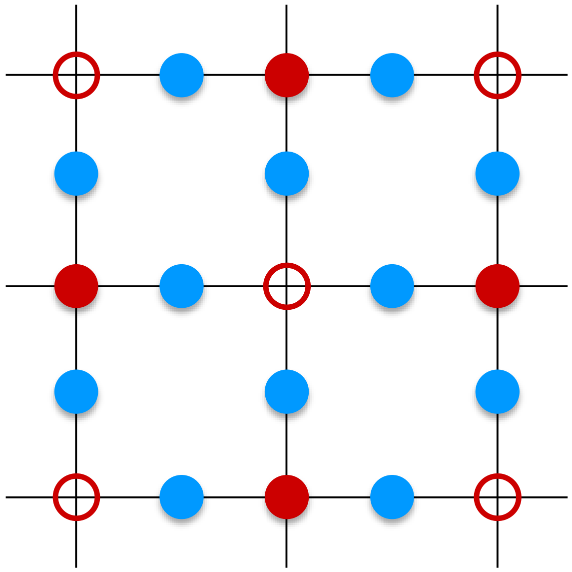
Sectors with fixed Static charge configurations $\quad \left[G\left(\mathbf{x}\right), H\right] = 0 \quad \forall \mathbf{x}$



$$\mathcal{H} = \oplus \mathcal{H}\left(\left\{q\left(\mathbf{x}\right)\right\}\right)$$

Allowed Interactions

- Must preserve the symmetry – commute with the “Gauss Laws” (generators of symmetry transformations)

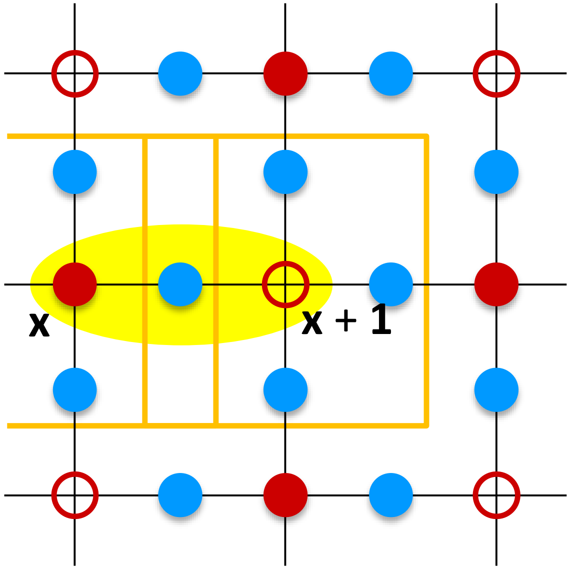


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>First option</u>: Link (**matter**-**gauge**) interaction:

$$\psi_m^\dagger\left(\mathbf{x}\right) U_{mn}\left(\mathbf{x},k\right) \psi_n\left(\mathbf{x}+\hat{\mathbf{k}}\right)$$

  – A **fermion** hops to a **neighboring site**, and the **flux on the link in the middle changes** to preserve **Gauss laws on the two relevant sites**

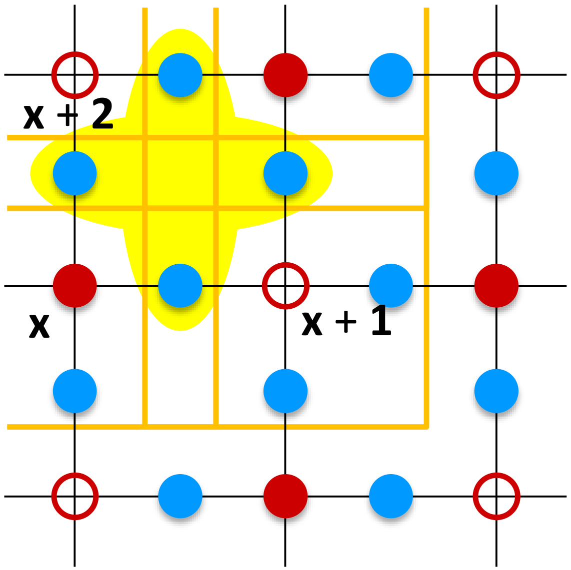


Allowed Interactions

- Must preserve the symmetry – commute with the “Gauss Laws” (generators of symmetry transformations)
- Second option: plaquette interaction:

$$\mathrm{Tr}\left(U(\mathrm{x},1)U(\mathrm{x}+\hat{1},2)U^{\dagger}(\mathrm{x}+\hat{2},1)U^{\dagger}(\mathrm{x},2)\right)$$

  - The **flux on the links of a single plaquette changes** such that the **Gauss laws on the four relevant sites** is preserved.
  - **Magnetic interaction.**



Quantum Simulation of LGT

- Theoretical Proposals:
  - Various gauge groups:
    - Abelian (U(1), $\mathbf{Z}_N$)
    - non-Abelian (SU(N)…)
  - Various simulating systems:
    - Ultracold Atoms
    - Trapped Ions
    - Superconducting Qubits
  - Various simulation approaches:
    - Analog
    - Digital

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- Experiments: Innsbruck (2016) → …

Basic Requirements from a GT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in an appropriate continuum limit.

- Manifest **Local** (Gauge) Invariance on top of the natural global atomic symmetries (number conservation)

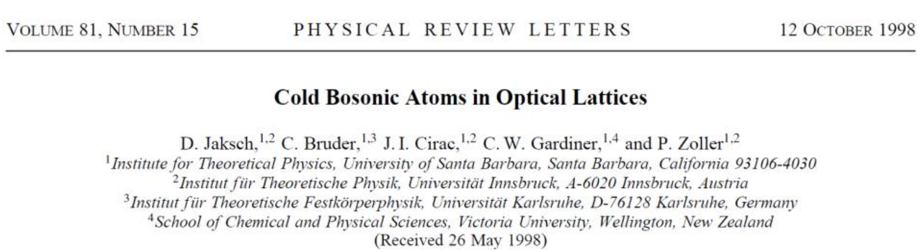
  Local (gauge) symmetries may be introduced to the atomic simulator using several methods.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

Ultracold Atoms in Optical Lattices

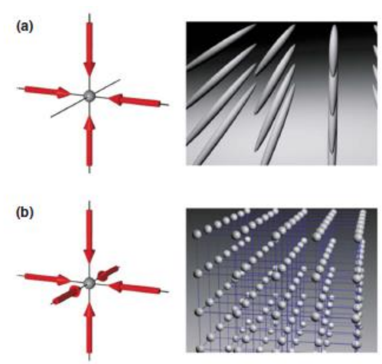
- Atoms are cooled and trapped in periodic potentials created by laser beams.
- Highly controllable systems:
  - Tuning the laser beams → shape of the potential
  - Tunable interactions (S-wave collisions among atoms in the ultracold limit tunable with Feshbach resonances, external Raman lasers)
  - Use of several atomic species →
    different internal (hyperfine) levels $\mathbf{F} = \mathbf{I} + \mathbf{L} + \mathbf{S}$ may be used, experiencing different optical potentials
  - Easy to measure, address and manipulate



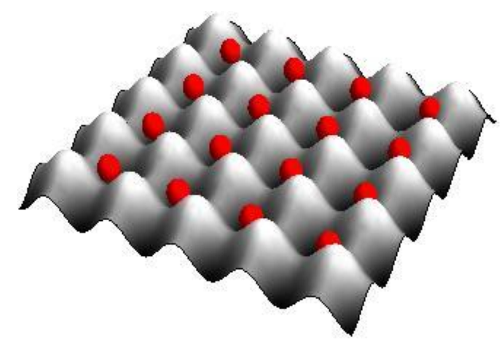
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]
[1]*Institute for Theoretical Physics, University of Santa Barbara, Santa Barbara, California 93106-4030*
[2]*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[3]*Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[4]*School of Chemical and Physical Sciences, Victoria University, Wellington, New Zealand*
(Received 26 May 1998)

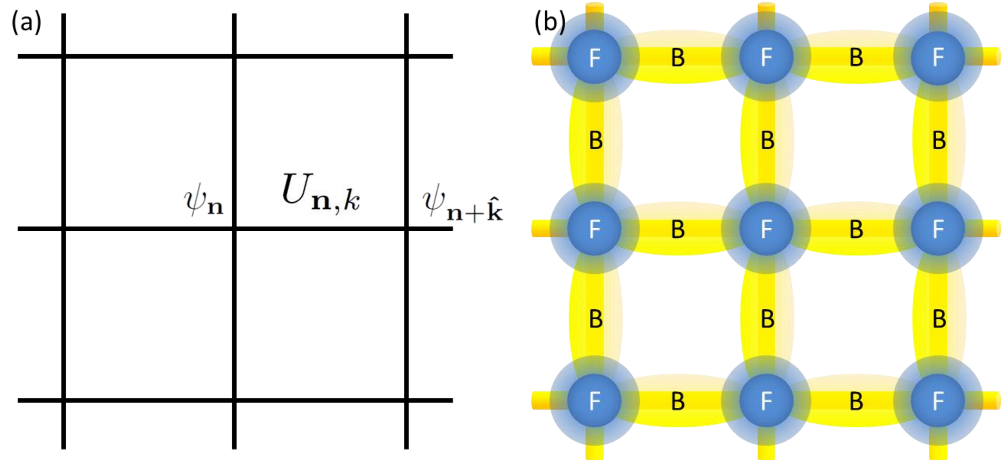


RMP 80, 885 (2008)

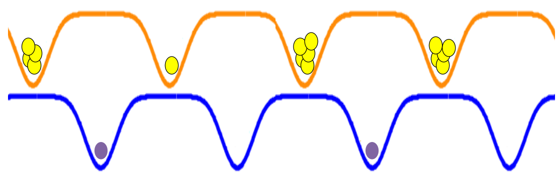


QS of LGTs with Ultracold Atoms in Optical Lattices

- **Fermionic** matter fields
- (Bosonic) gauge fields



Super-lattice:



Atomic internal (**hyperfine**) levels

0 (ultracold)

$$\mathbf{F} = \mathbf{I} + \cancel{\mathbf{L}} + \mathbf{S} \qquad \mathbf{F}^2|F, m_{\mathrm{F}}\rangle = F(F+1)|F, m_{\mathrm{F}}\rangle \qquad F_z|F, m_{\mathrm{F}}\rangle = m_{\mathrm{F}}|F, m_{\mathrm{F}}\rangle$$

$$\mathcal{H} = \sum_{\alpha,\beta} \Phi_\alpha^\dagger(\mathbf{x})\left(\delta^{\alpha\beta}\left(-\frac{\nabla^2}{2m} + V_{\mathrm{op}}^{\alpha}(\mathbf{x}) + V_{\mathrm{T}}(\mathbf{x})\right) + \Omega^{\alpha\beta}(\mathbf{x})\right)\Phi_\beta(\mathbf{x})$$
$$+ \sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x' \Phi_\alpha^\dagger(\mathbf{x}')\Phi_\beta^\dagger(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x}-\mathbf{x}')\Phi_\gamma(\mathbf{x})\Phi_\delta(\mathbf{x}')$$

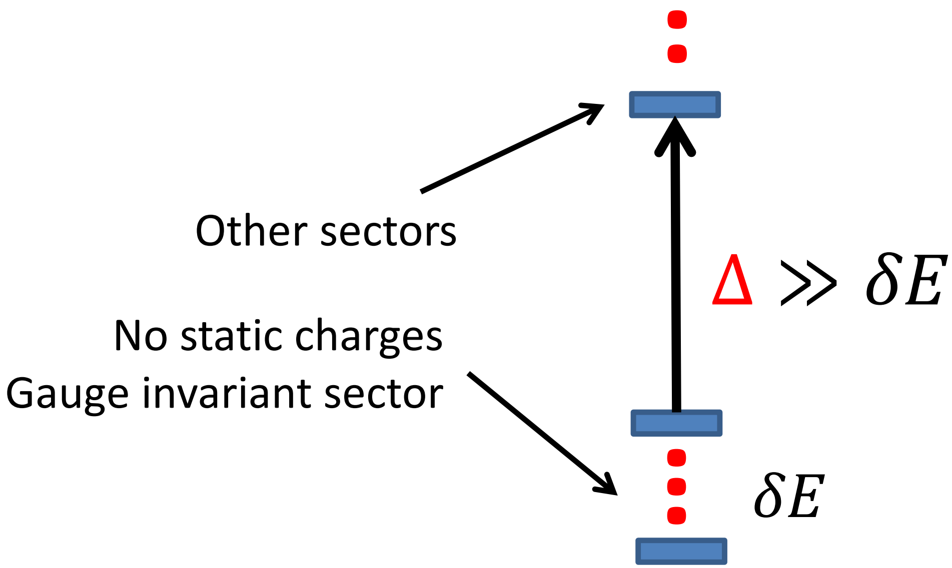
Analog Approach I: Effective Local Gauge Invariance

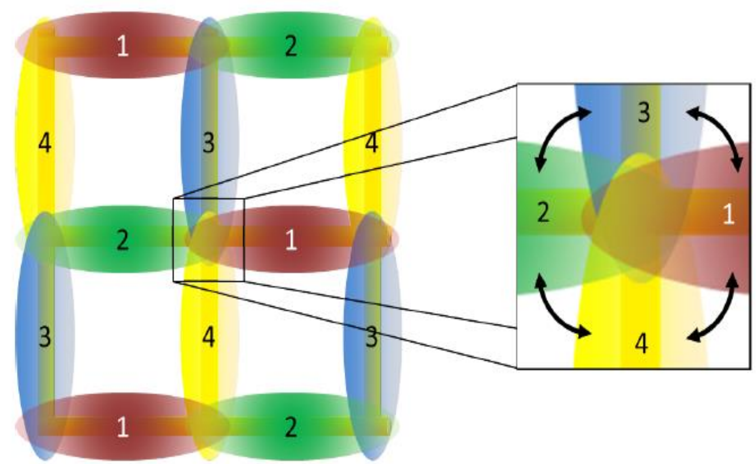
Gauss law is added to the Hamiltonian as a constraint (penalty term).

Leaving a gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with an effective gauge invariant Hamiltonian.

Emerging plaquette interactions (second order perturbation theory).





E. Zohar, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)

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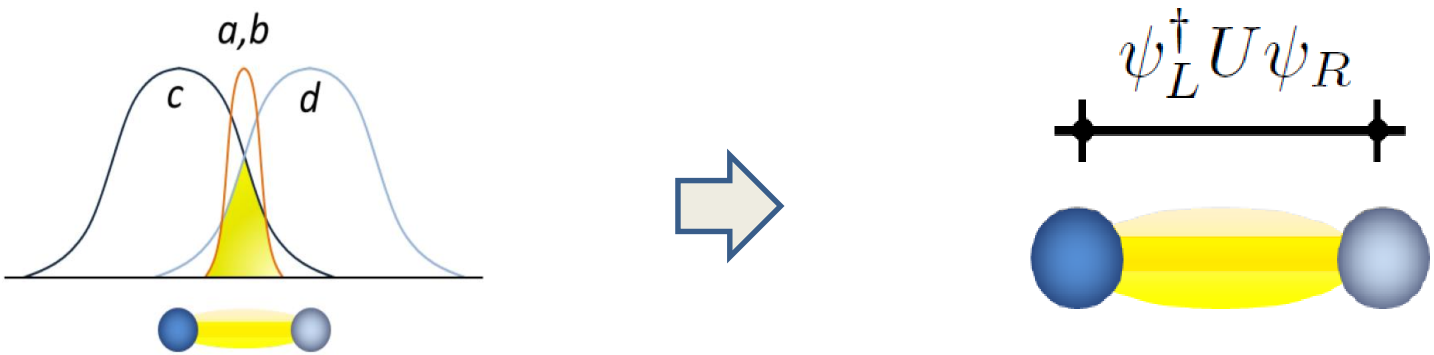
Analog Approach II:
Atomic Symmetries → Local Gauge Invariance



Atomic boson-fermion collisions

Hyperfine angular momentum conservation

Fermionic atoms c,d (or more)

(Generalized) Schwinger algebra, constructed out of the bosonic atoms a,b (or more)

Link gauge-matter interactions

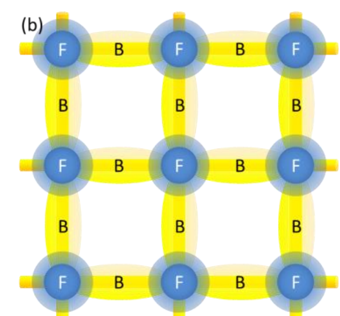
Gauge invariance / charge conservation

Fermionic matter

Gauge field operator *U*

Gauge invariance is a fundamental symmetry of the quantum simulator.

Applicable for U(1), SU(N) etc. with truncated local Hilbert spaces.



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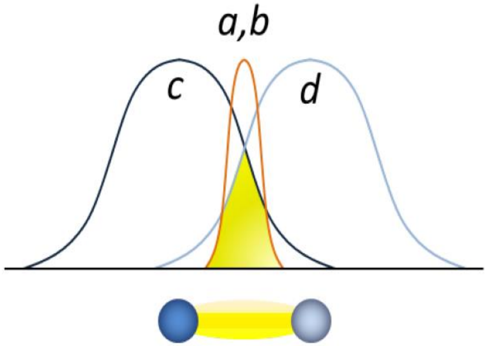

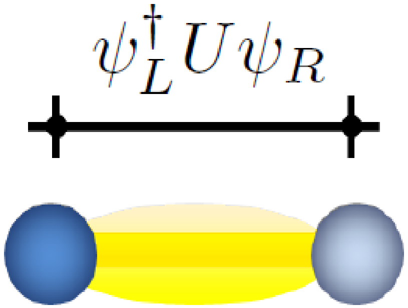

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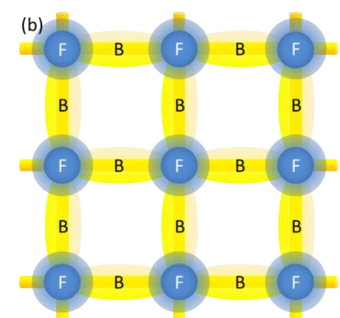
Calculations applying our scheme towards an experiment: **Kasper, Hebenstreit, Jendrzejewski, Oberthaler, Berges**, NJP 19 023030 (2017) – very exciting results



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Further Dimensions → Plaquette Interactions

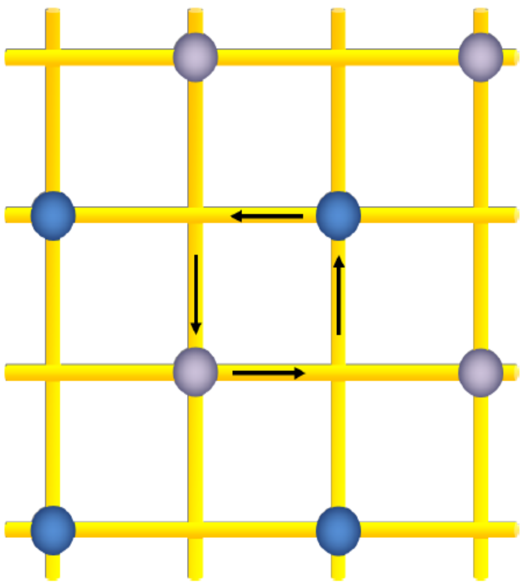
$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

1d elementary link interactions are **already gauge invariant**

Auxiliary fermions:

Heavy,
constrained to “sit”
on special vertices

- Virtual processes
- Valid for any gauge group, once the link interactions are realized



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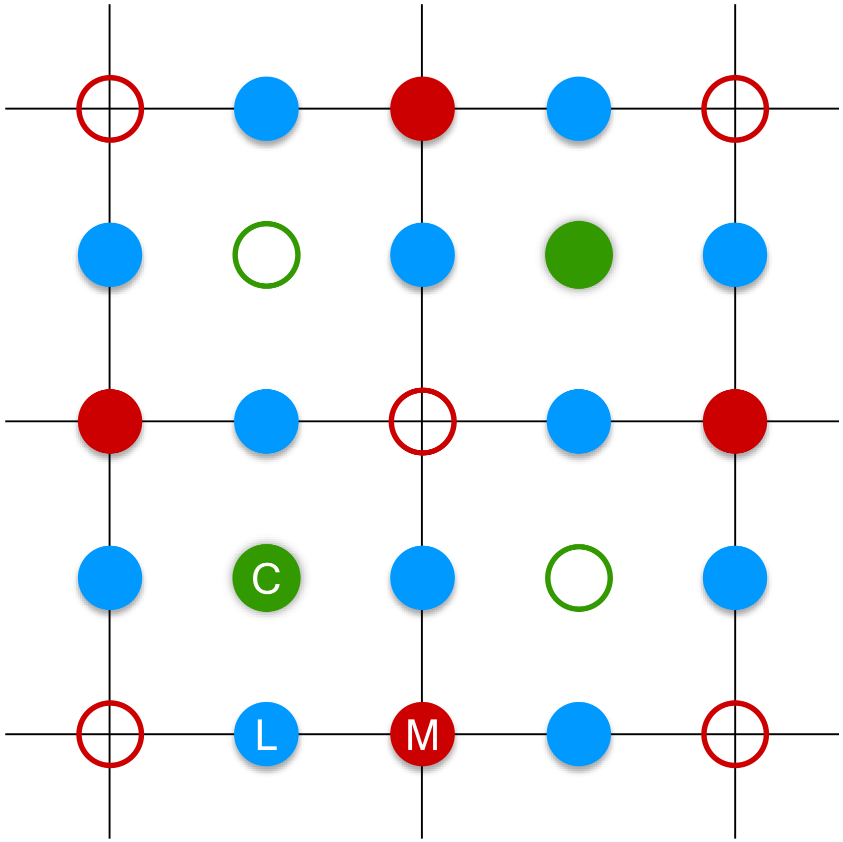
D. González Cuadra, E. Zohar, J. I. Cirac, New J. Phys. 19 063038 (2017)

Digital Lattice Gauge Theories

Trotterized time evolution: $$e^{-i\Sigma_j H_j T} = \lim_{M\to\infty} \left(\Pi_j e^{-iH_j \frac{T}{M}}\right)^M$$



Matter Fermions

Link (Gauge) degrees of freedom

Control degrees of freedom

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E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)
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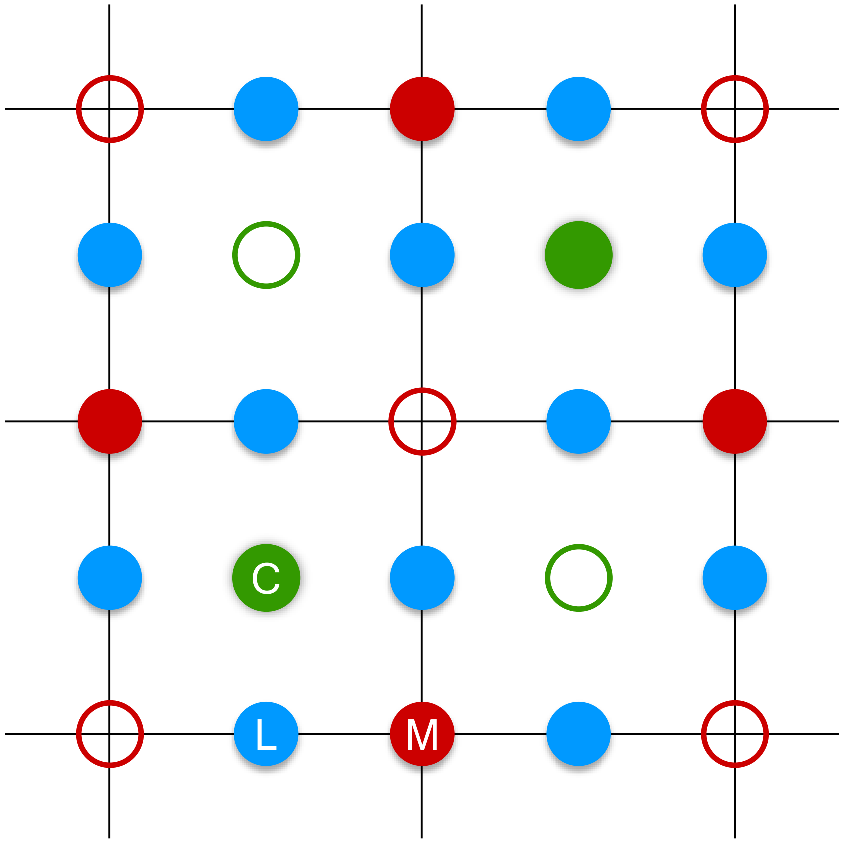

Matter Fermions

Link (Gauge) degrees of freedom

Control degrees of freedom

Entanglement is created and undone between the control and the physical degrees of freedom.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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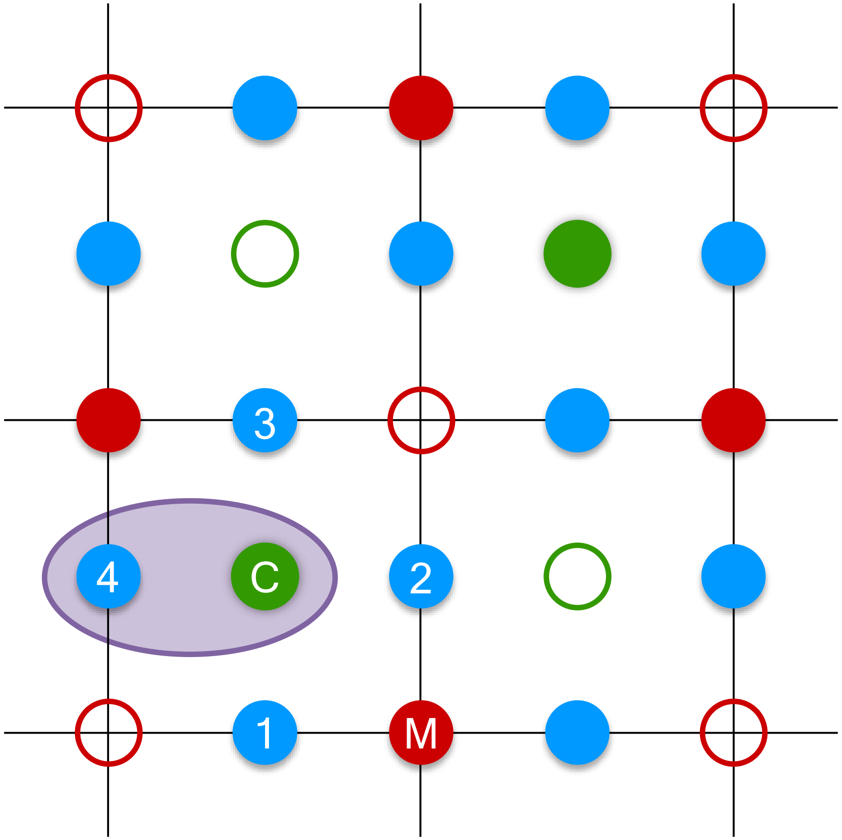
Plaquettes: Four-body Interactions

Two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^{\dagger} = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^x \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$



$$|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

$$\mathcal{U}_4^{\dagger}|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

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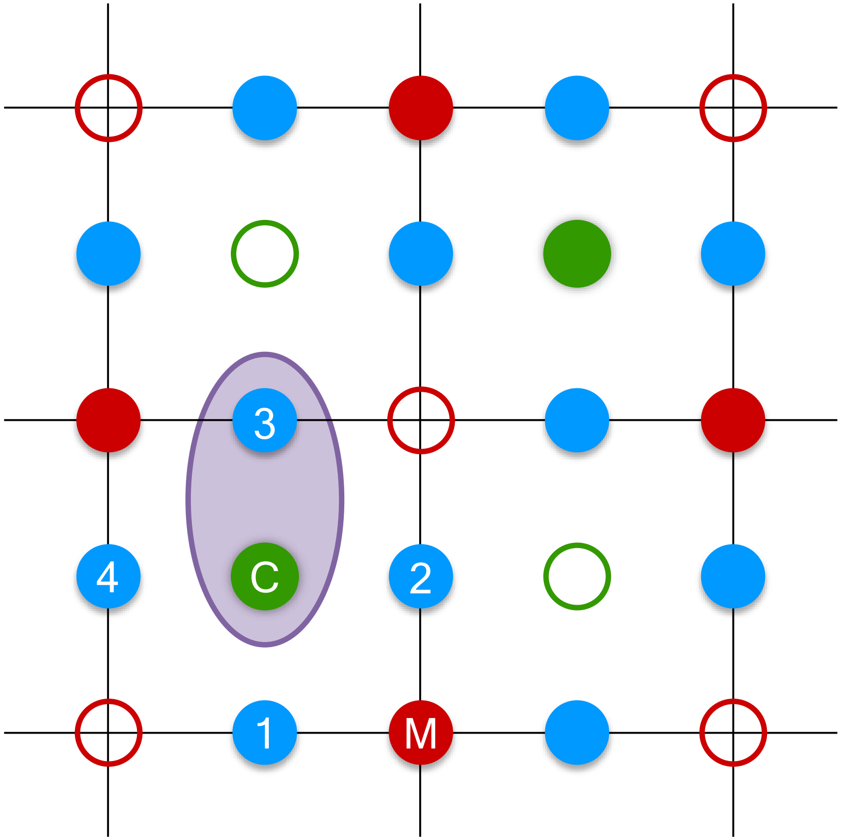

$$|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

$$\mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

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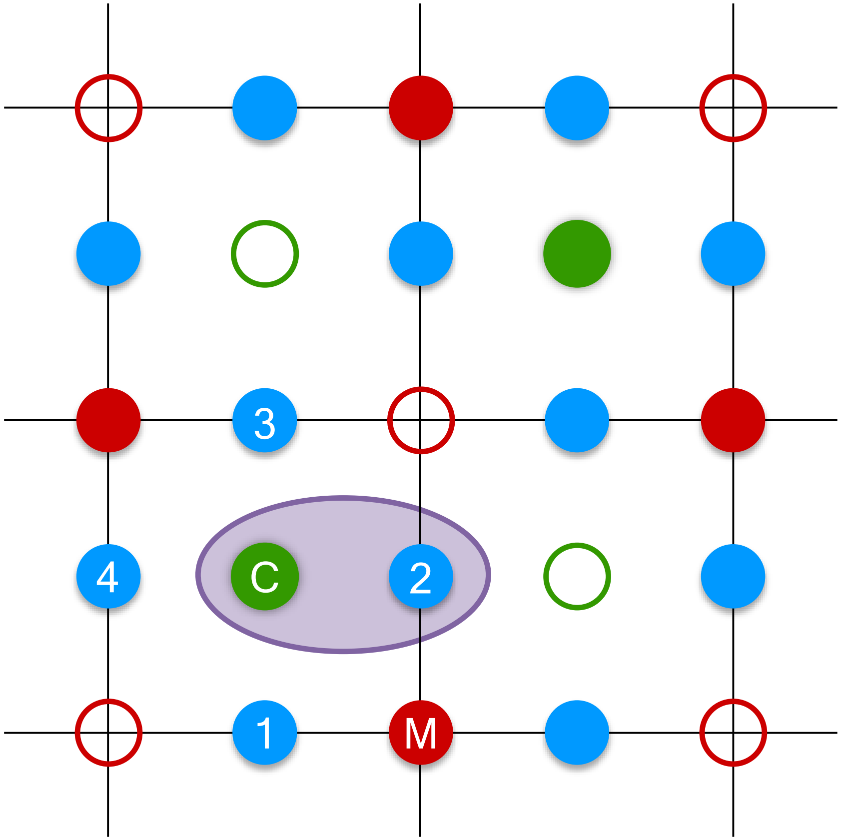
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)
J. Bender, E. Zohar, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

Plaquettes: Four-body Interactions

Two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^{\dagger} = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^{x} \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$



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$$\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger}|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_2^x\sigma_3^x\sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

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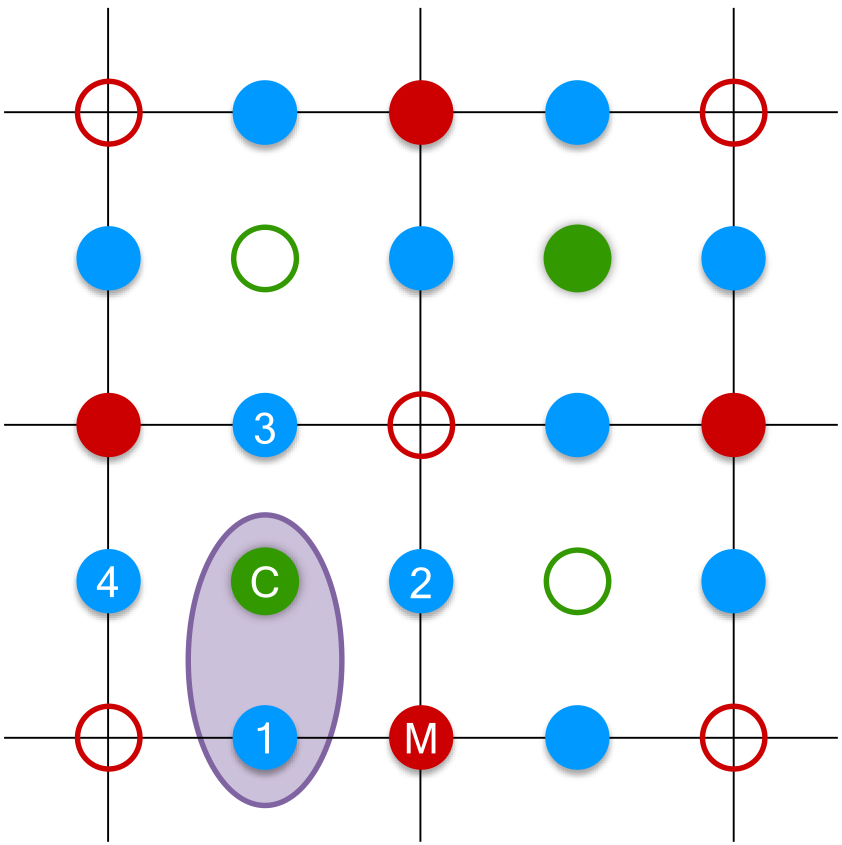

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$$\mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
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Two-body interactions → four-body interactions

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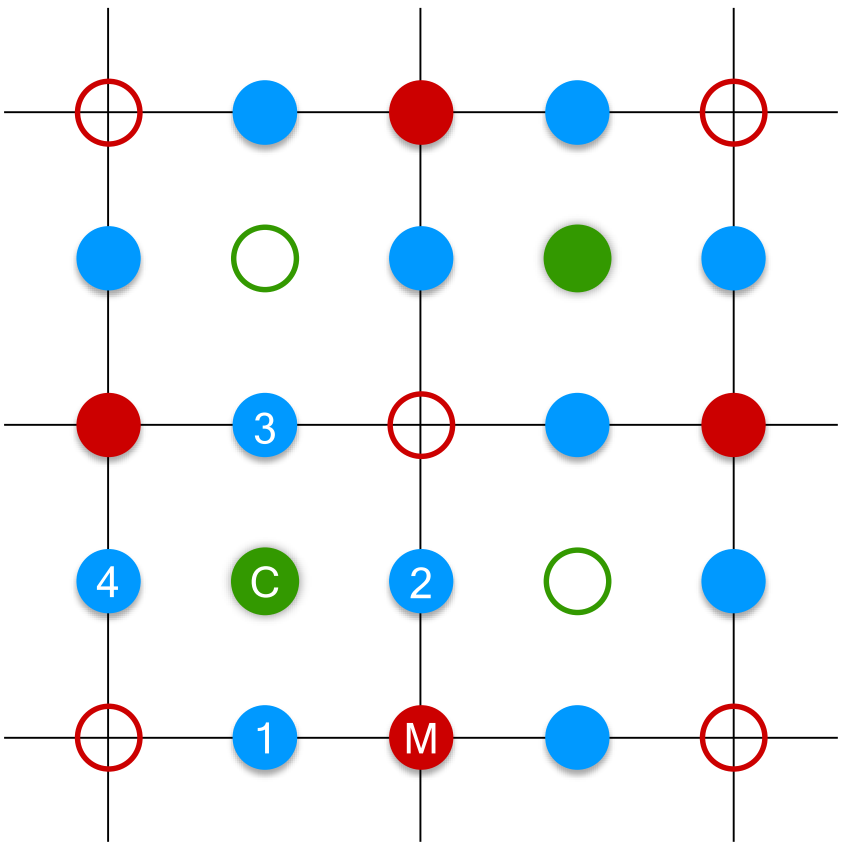

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$$\mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

$$S_\square = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_\square^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)
J. Bender, E. Zohar, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

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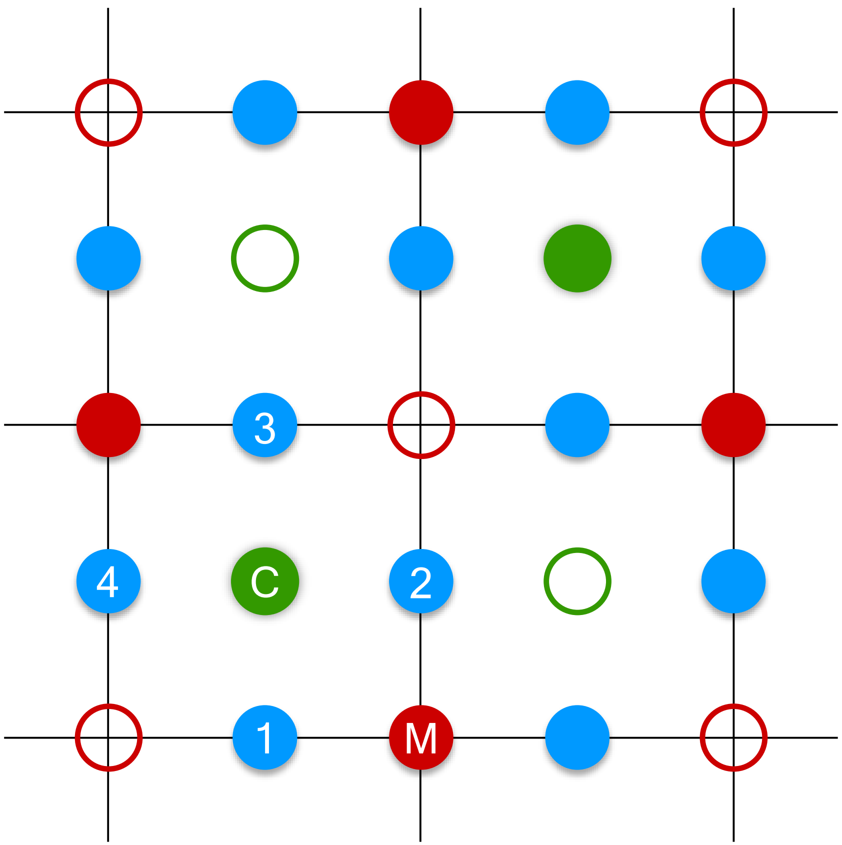


$$S_{\square} = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma^{x}_{\square} \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\widetilde{\sigma}^{x} S_{\square} = S_{\square}\sigma^{x}_{\square}$$

$$e^{-i\lambda\widetilde{\sigma}^{x}\tau} S_{\square} = S_{\square} e^{-i\lambda\sigma^{x}_{\square}\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^{\dagger}\mathcal{U}_1^{\dagger} e^{-i\lambda\widetilde{\sigma}^{x}\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger}\left|\widetilde{in}\right\rangle = \left|\widetilde{in}\right\rangle e^{-i\lambda\sigma^{x}_{\square}\tau}$$

- A "Stator" (state-operator)

B. Reznik, Y. Aharonov, B. Groisman, Phys. Rev. A 6 032312 (2002)

E. Zohar, J. Phys. A. 50 085301 (2017)

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

J. Bender, E. Zohar, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

Further generalization

Any gauge group

$$S = \int dg \left|g_A\right\rangle \left\langle g_A\right| \otimes \left|g_B\right\rangle$$

$$\left(U^j_{mn}\right)_B S = S \left(U^j_{mn}\right)_A$$

$$S_\square = \mathcal{U}_\square \left|\widetilde{in}\right\rangle \equiv \mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle$$

$$\mathrm{Tr}\left(\widetilde{U^j} + \widetilde{U}^{j\dagger}\right) S_\square = S_\square \mathrm{Tr}\left(U_1^j U_2^j U_3^{j\dagger} U_4^{j\dagger} + H.c.\right)$$

Feasible for finite or truncated infinite groups

E. Zohar, J. Phys. A. 50 085301 (2017)

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Is it necessary to use cold atoms?

- **Cold atoms** offer a **combination of fermionic and bosonic degrees of freedom**, which makes them useful for the quantum simulation of gauge theories with **fermionic matter in 2+1d and more**.

- Using systems that **do not offer fermionic degrees of freedom**, one can simulate
  - **Pure gauge theories** could be simulated using other architectures – e.g. trapped ions (Innsbruck), superconducting qubits (Bilbao),…
  - **1+1d gauge theories with matter**, using Jordan-Wigner transformations (like in the trapped ions **Innsbruck experiment**).
  - **Something else?!**

Do we really need fermions?

- Fermions are subject to a **global $\mathbf{Z}_2$ symmetry** (parity superselection)
- If this symmetry is **local** (which happens naturally in a lattice gauge theory whose gauge group contains $\mathbf{Z}_2$ as a normal subgroup), it can be used for **locally transferring the statistics information to the gauge field**
- One is left with **hard-core bosonic matter (spins),** with **fermionic statistics taken care of by the gauge field**

$$\psi^{\dagger}\left(\mathbf{x}\right)=\underbrace{c\left(\mathbf{x}\right)}_{\text{Majorana Fermion: Statistics}}\underbrace{\eta^{\dagger}\left(\mathbf{x}\right)}_{\text{Hardcore Boson: Physics}}$$

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Do we really need fermions?

- With a **local unitary transformation** which is independent of the space dimension, one can remove the fermions from the Hamiltonian, and stay with **hard-core bosonic matter** and **electric field dependent signs** that preserve the fermionic statistics.

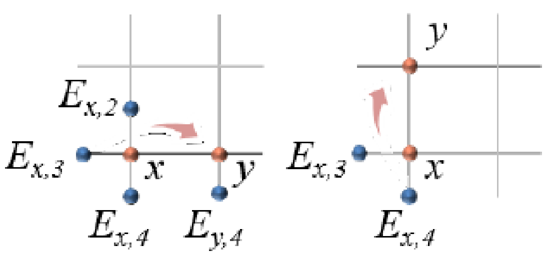
$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\psi^{\dagger}(\mathbf{x})\, U(\mathbf{x},i)\, \psi(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

$$\psi^{\dagger}(\mathbf{x}) = c(\mathbf{x})\, \eta^{\dagger}(\mathbf{x})$$

$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\eta^{\dagger}(\mathbf{x})\, c(\mathbf{x})\, U(\mathbf{x},i)\, c(\mathbf{x}+\hat{\mathbf{e}}_i)\, \eta(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

Unitary transformation

$$-i\epsilon \sum_{\mathbf{x},i=1,2} \left(\xi_i \sigma_+(\mathbf{x})\, U(\mathbf{x},i)\, \sigma_-(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$



$$\xi_h = e^{i\pi(E_{x,2}+E_{x,3}+E_{x,4}+E_{y,4})}$$

$$\xi_v = e^{i\pi(E_{x,3}+E_{x,4})}$$

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Do we really need fermions?

- This procedure opens the way for **quantum simulation of lattice gauge theories with fermionic matter in 2+1d and more**, even with **simulating systems that do not offer fermionic degrees of freedom**.

- In the U(N) case **the matter can be removed completely**!

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)
E. Zohar, J. I. Cirac, arXiv:1905.00652 [quant-ph], accepted to PRD

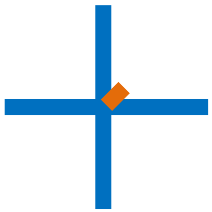
PEPS

- **Projected Entangled Pair States**: a particular tensor network construction, that
  - Allows to **encode and treat symmetries** in a very natural way.
  - Has, by construction, a **bipartite entanglement area law**, and therefore is suitable for describing “physically relevant” states.
  - Offers new approaches for the **study of phase diagrams and other properties of many body systems**.

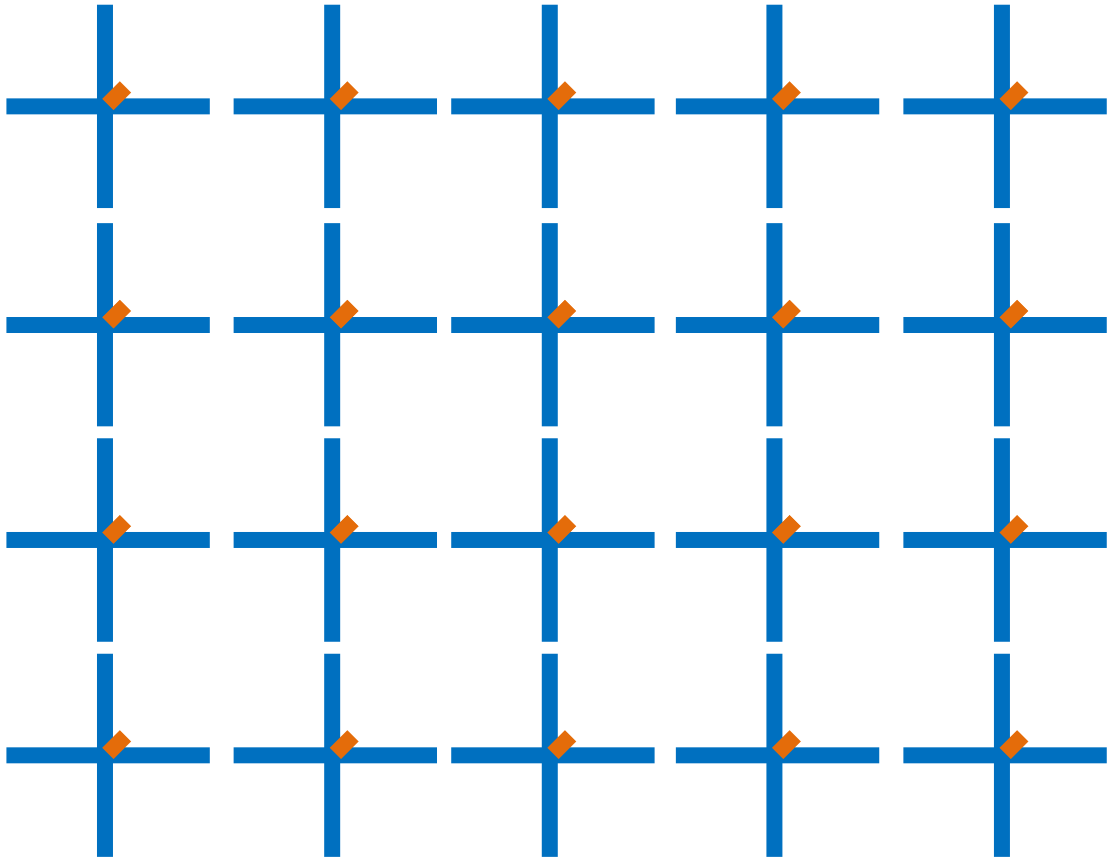
- In 1 space dimension – **MPS (Matrix Product States)**

PEPS

- Constructed out of local ingredients that include **physical** and **auxiliary** degrees of freedom.

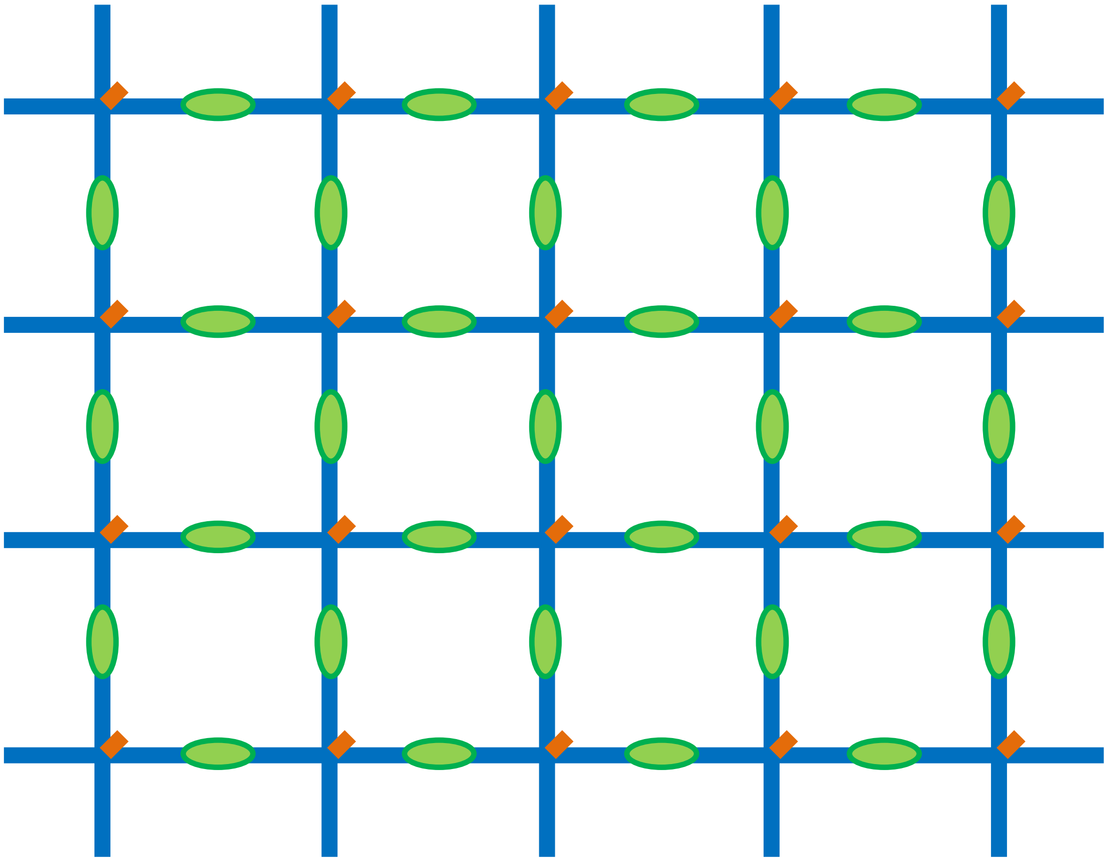


$$A\left(\mathbf{x}\right)|\Omega\rangle$$



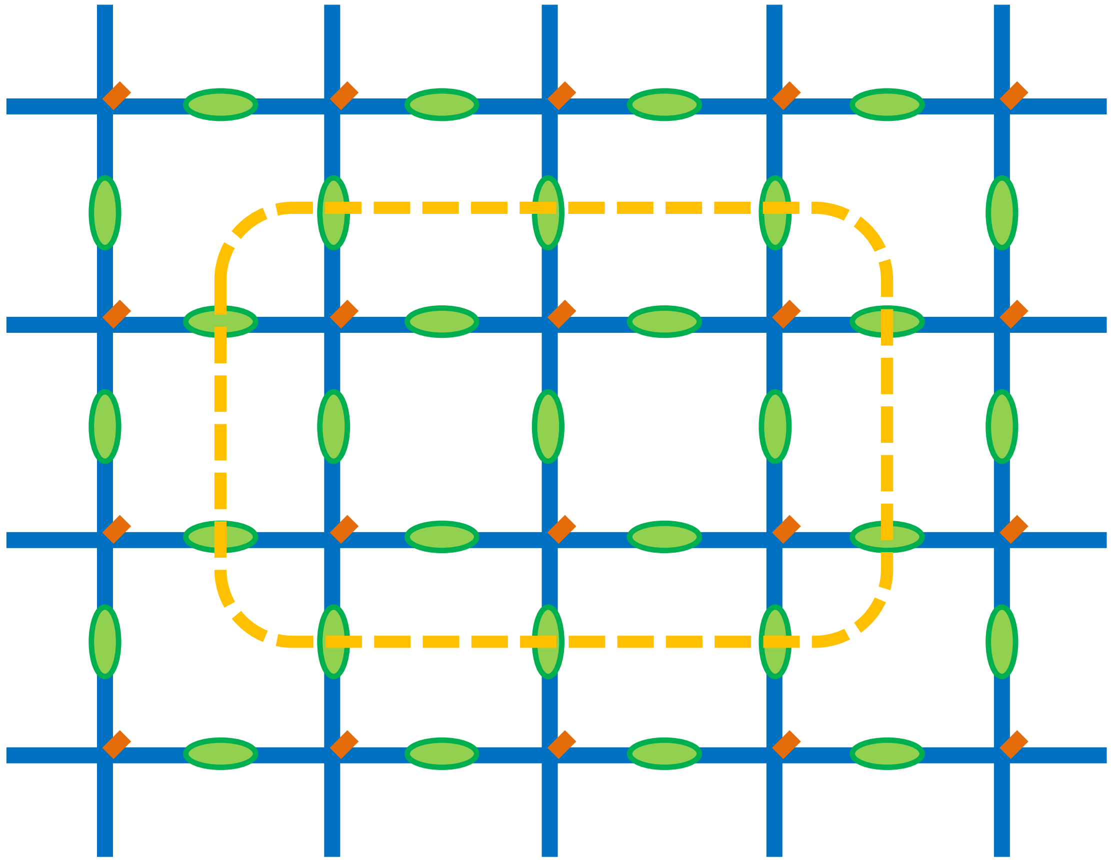
$$\prod_{\mathbf{x}} A(\mathbf{x})\,|\Omega\rangle$$

- A physical only state remains out of projecting pairs of auxiliary degrees of freedom, on the two sides of a link, onto maximally entangled states.

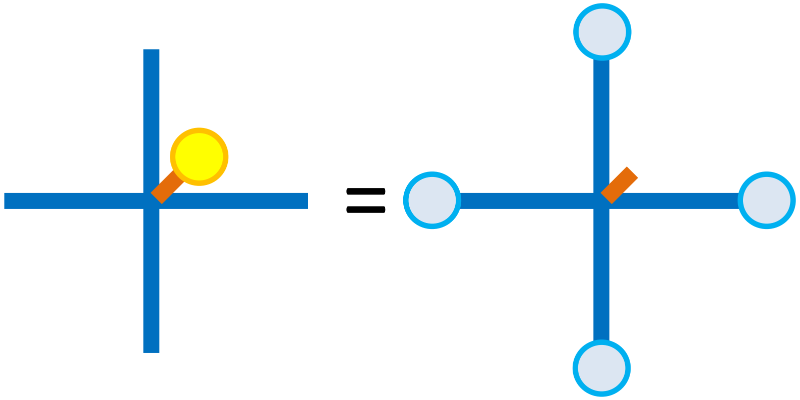


$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

- An entanglement area law is satisfied by construction.

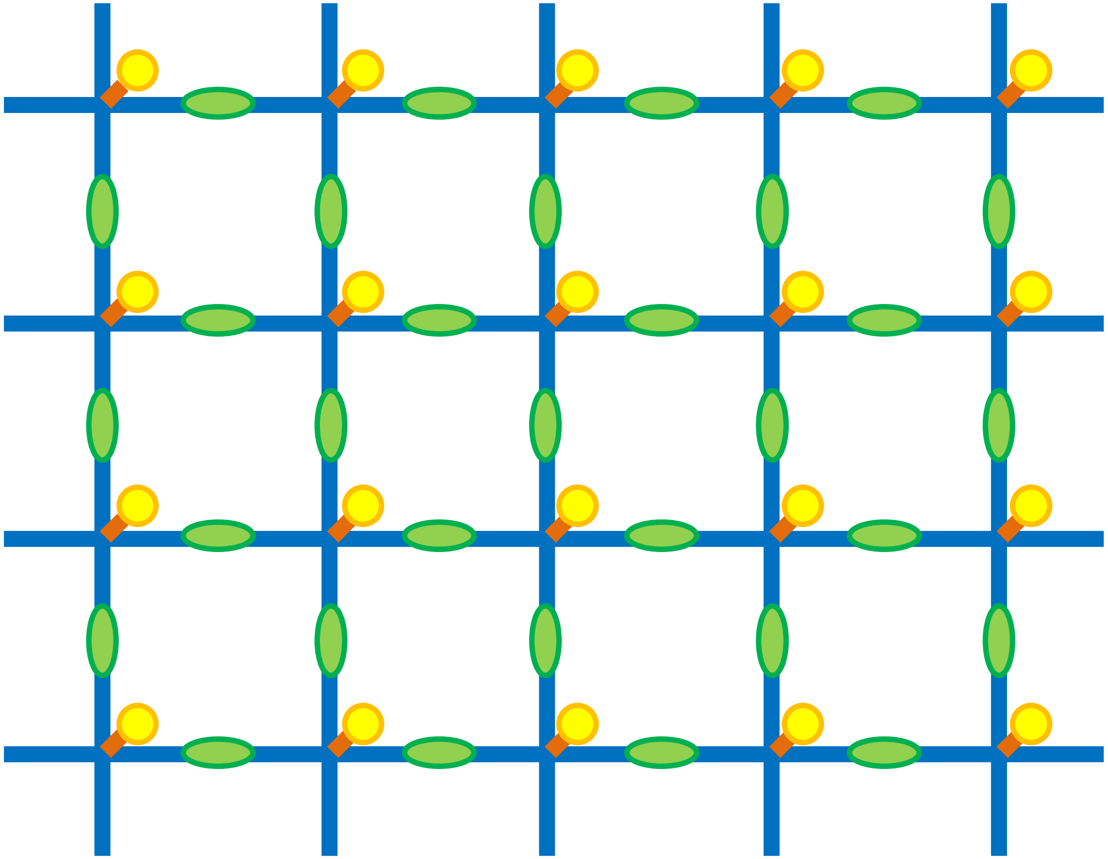


- Demanding global symmetry:
  - Acting with a group transformation on the **physical** degrees of freedom is equivalent to acting on the **auxiliary** ones.

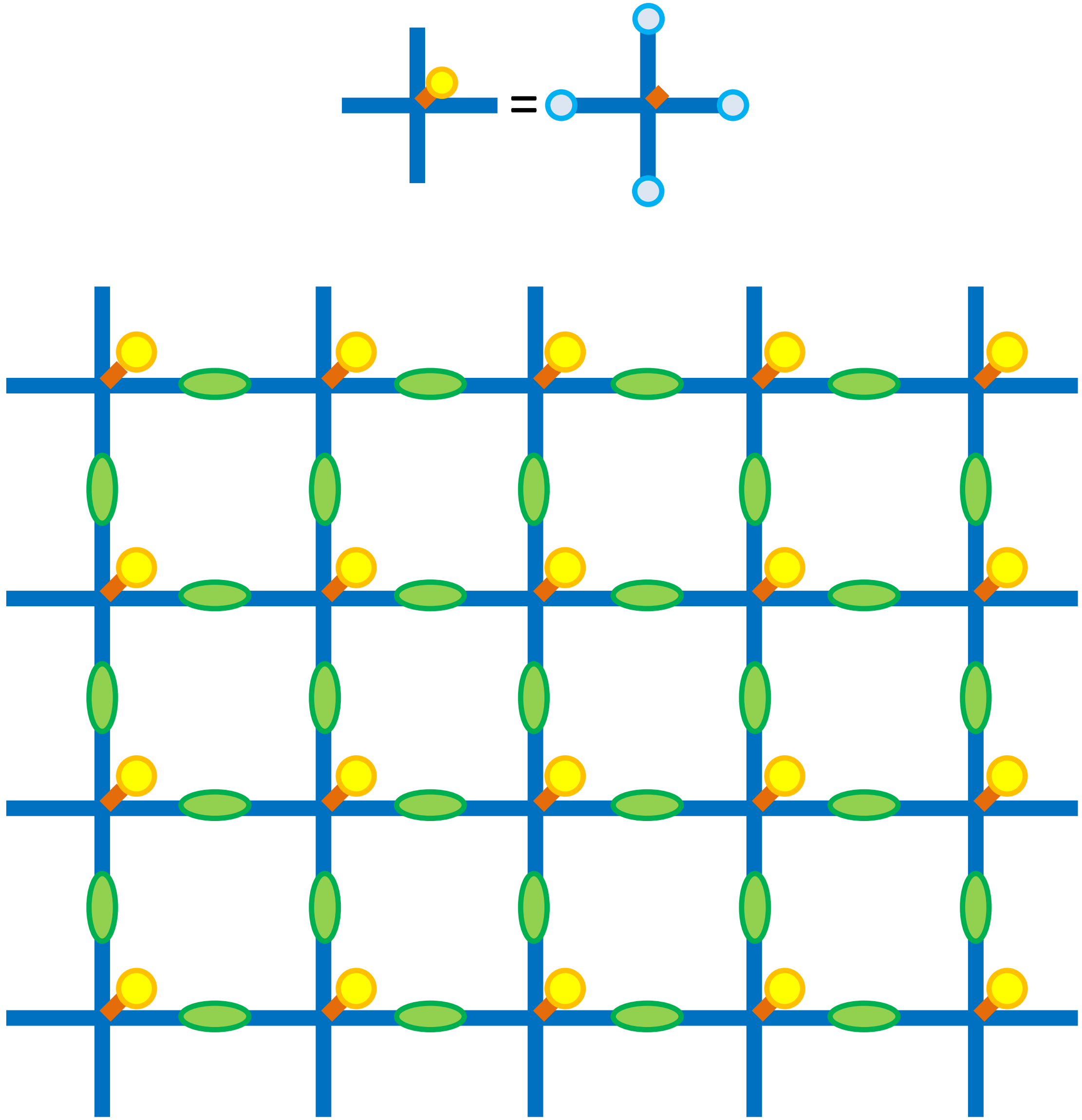


  - Projectors are invariant under group actions from both sides.

Global Transformation: $e^{i\Lambda \sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle$



$$|\psi_0\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$



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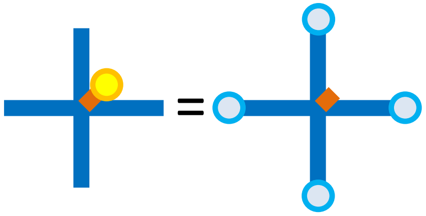

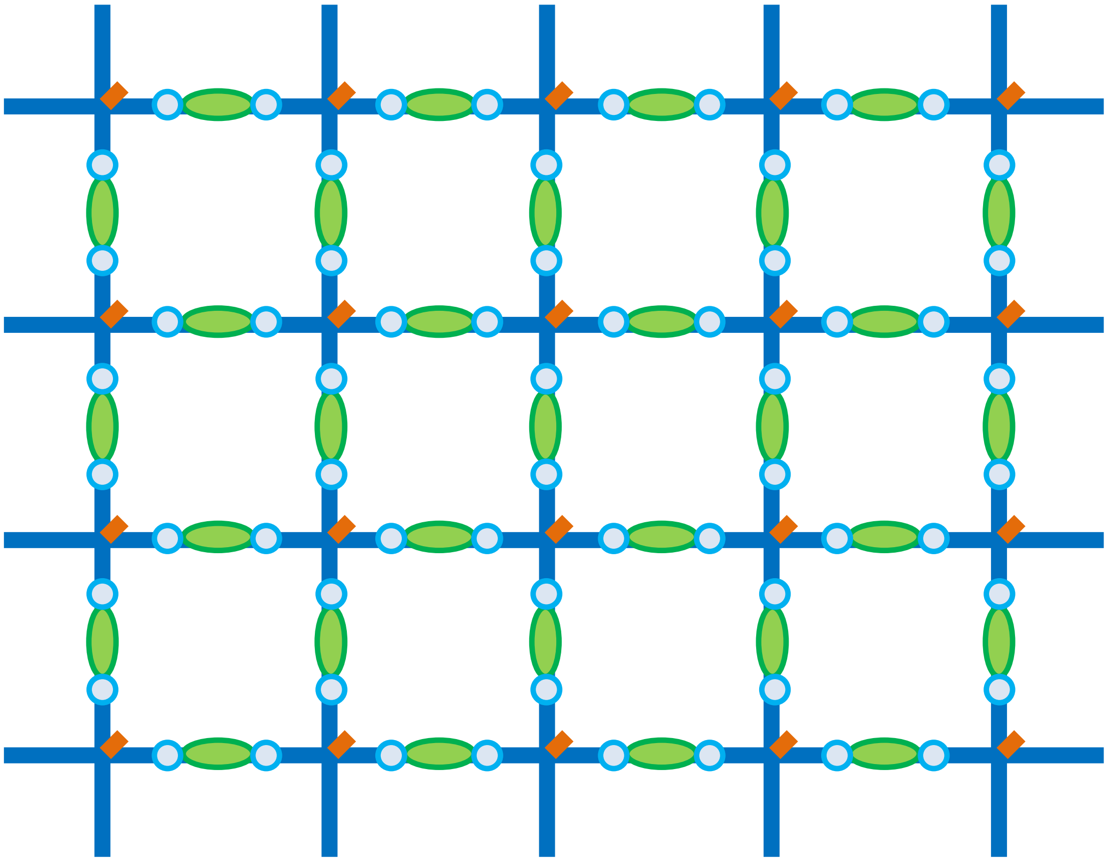

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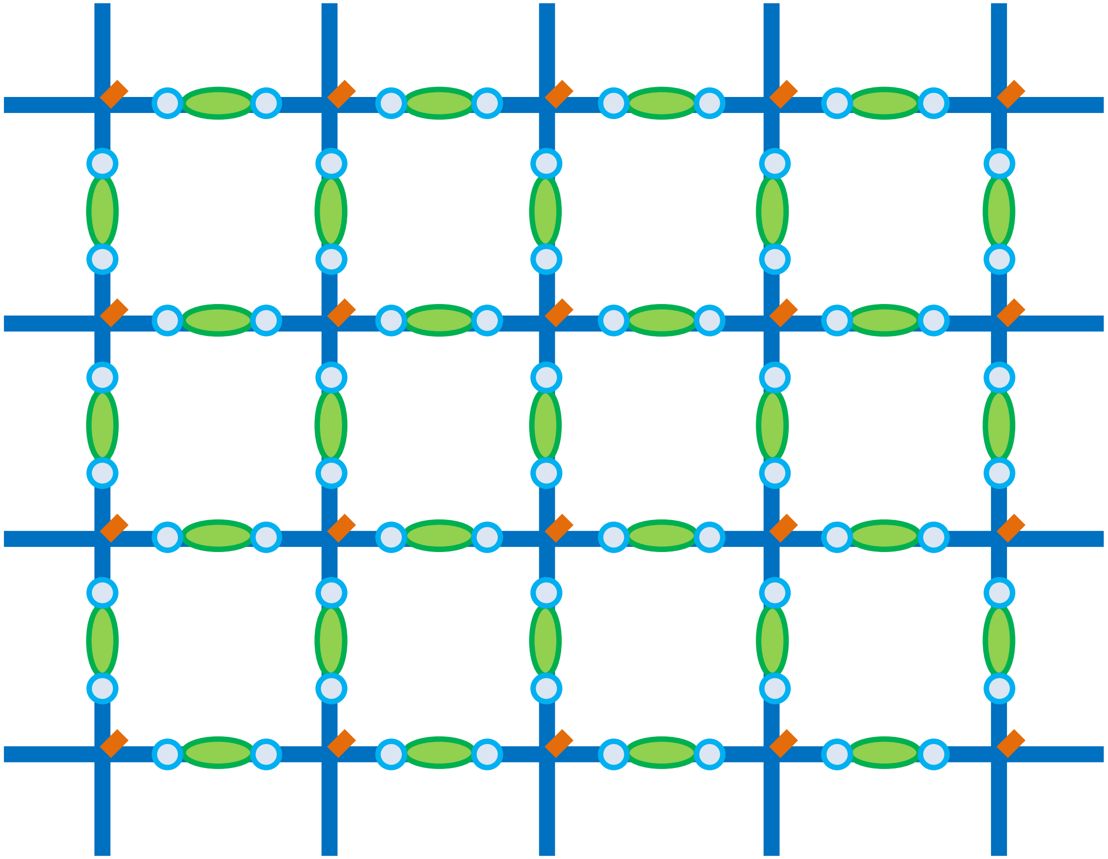

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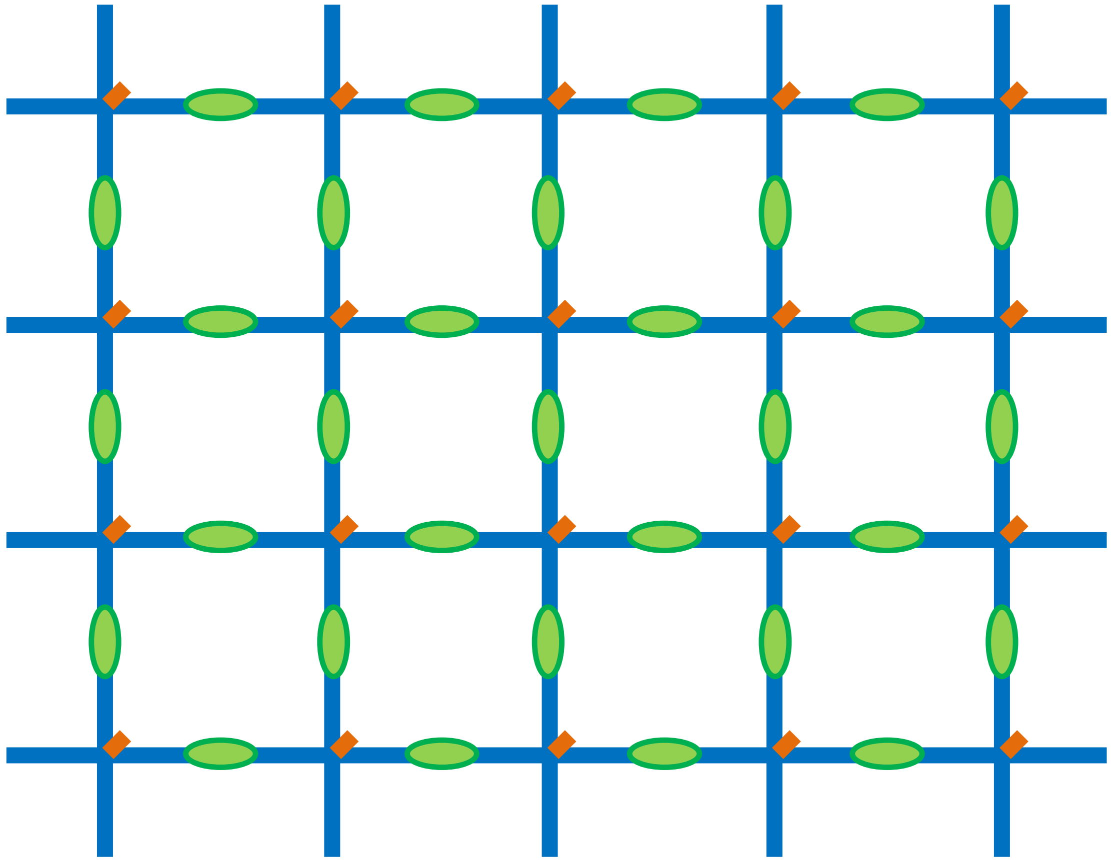
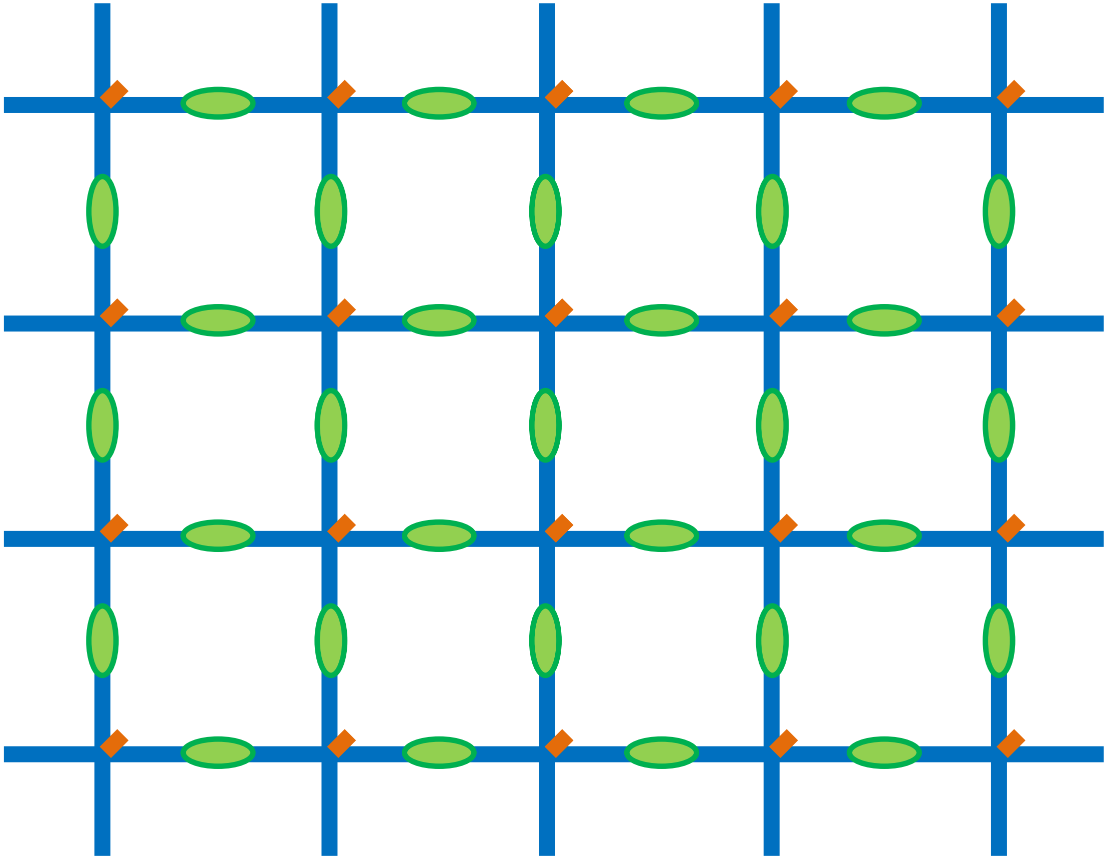


$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

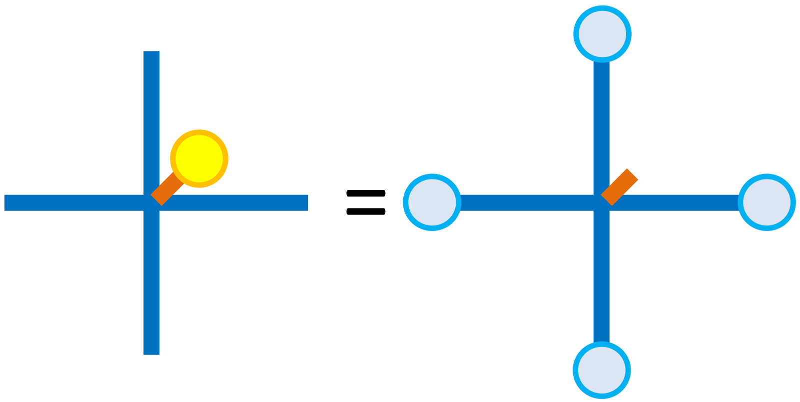
Global Symmetry: $e^{i\Lambda \sum_{\mathbf{x}} Q(\mathbf{x})} |\psi_0\rangle = |\psi_0\rangle$



$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

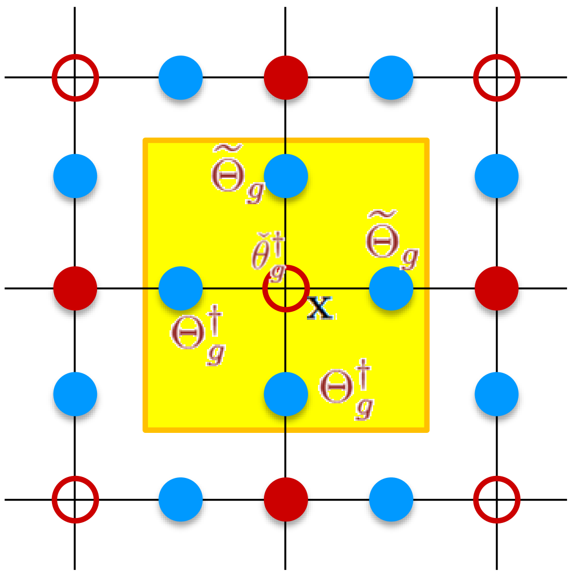
Virtual vs. Physical Gauge Invariance

Virtual- PEPS



Physical charge, but auxiliary electric fields: local symmetry exists, but it auxiliary/virtual. The physical symmetry is global, after the bonds projection.

Physical – LGT states



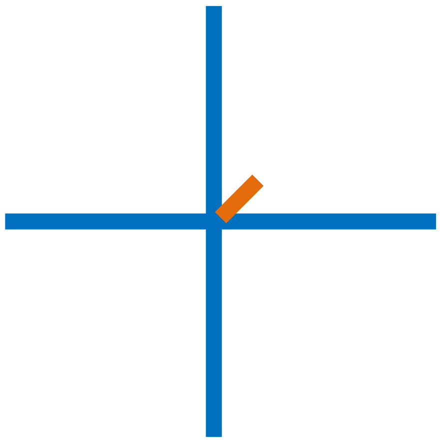
$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d} \left(\widetilde{\Theta}_g(\mathbf{x}, k)\, \Theta_g^\dagger\left(\mathbf{x} - \hat{\mathbf{k}}, k\right)\right) \breve{\theta}_g^\dagger(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x}) |\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  The global to local.

$$|\psi_0\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$



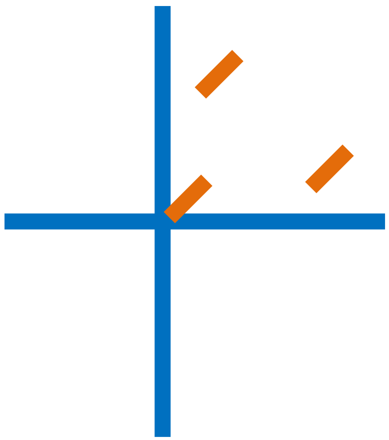
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  The global to local.
- Step 1: Introduce gauge field Hilbert spaces on the links. Add (by a tensor product) the gauge field singlet states:

$$|\psi_0\rangle = \langle \Omega_v | \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

$$\langle \Omega_v | \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$



E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

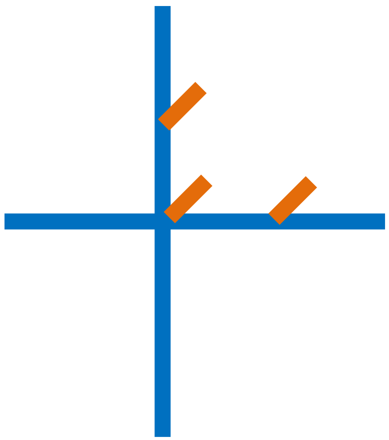
Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  The global to local.
- Step 2: Entangle the auxiliary degrees on the outgoing links with the gauge fields, by a unitary gauging transformation (map the auxiliary electric field information to the physical one)

$$|\psi_0\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

$$\downarrow$$

$$\langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$



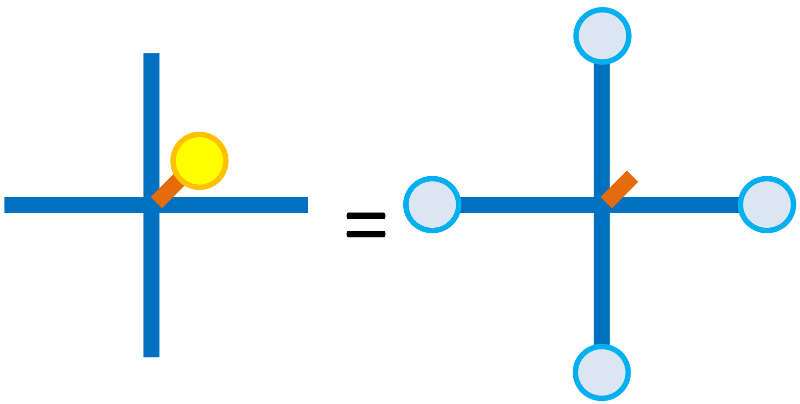
$$\downarrow$$

$$|\psi\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\, \mathcal{U}_G(\mathbf{x},2) A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

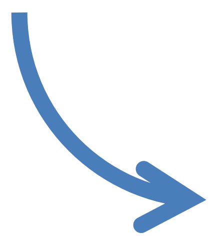
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Gauging the PEPS: minimal coupling of a state

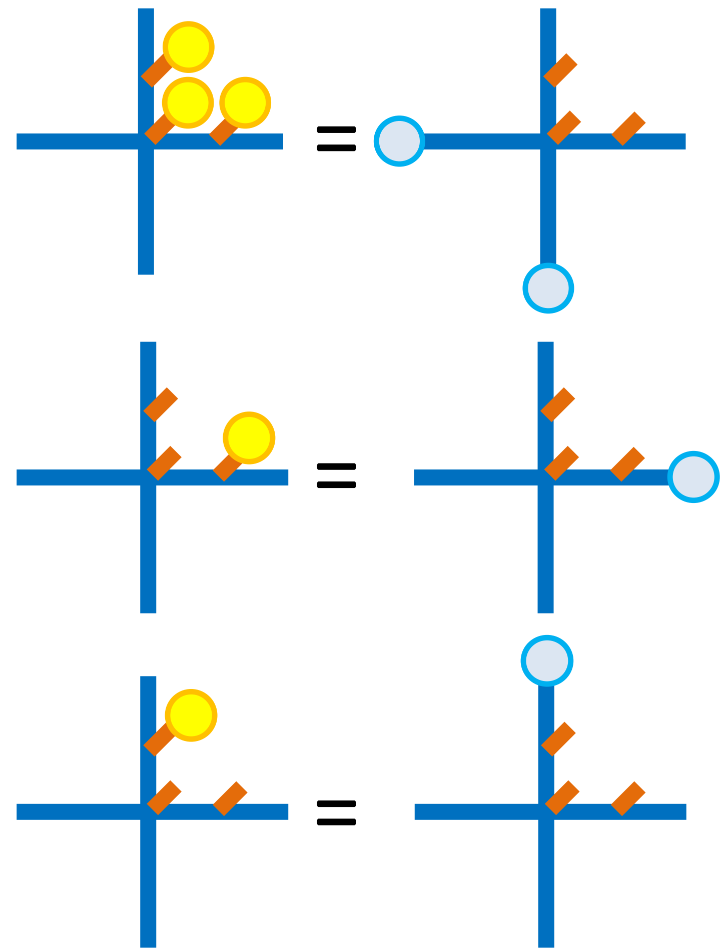
Building block of a globally invariant PEPS



Gauging Transformation

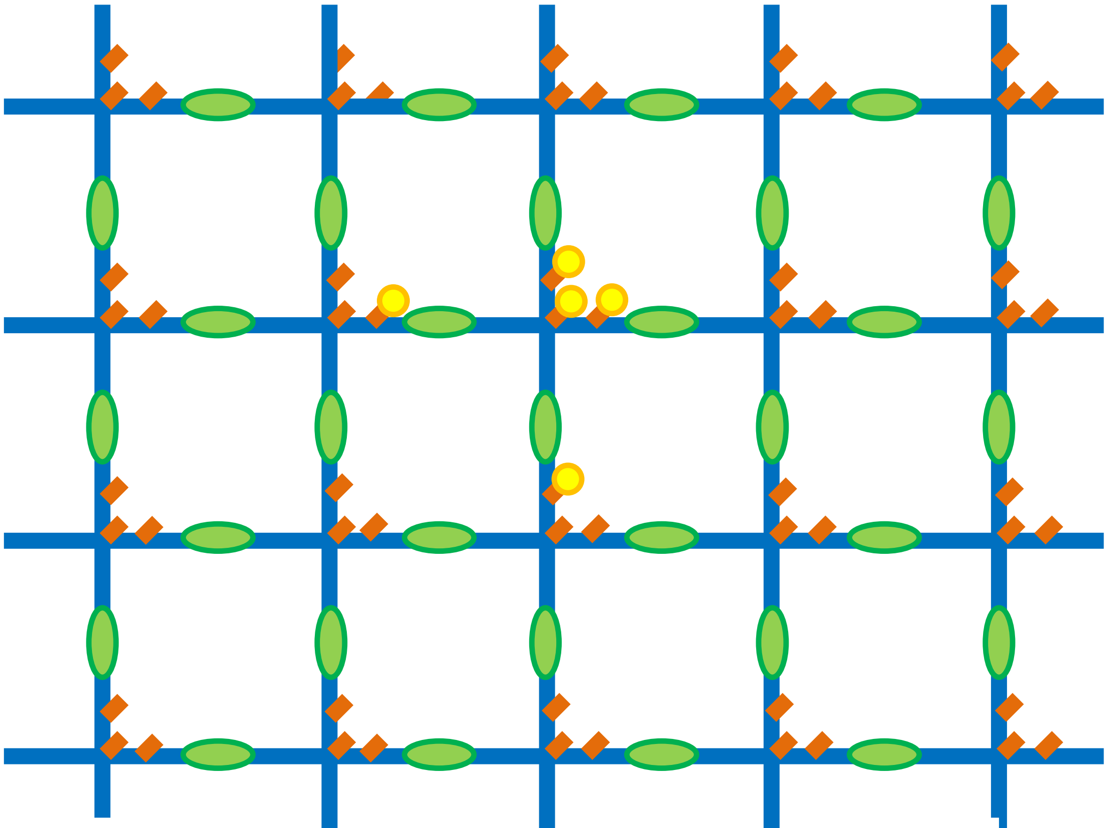


Building block of a globally invariant PEPS (gluing together the matter and gauge field tensors)



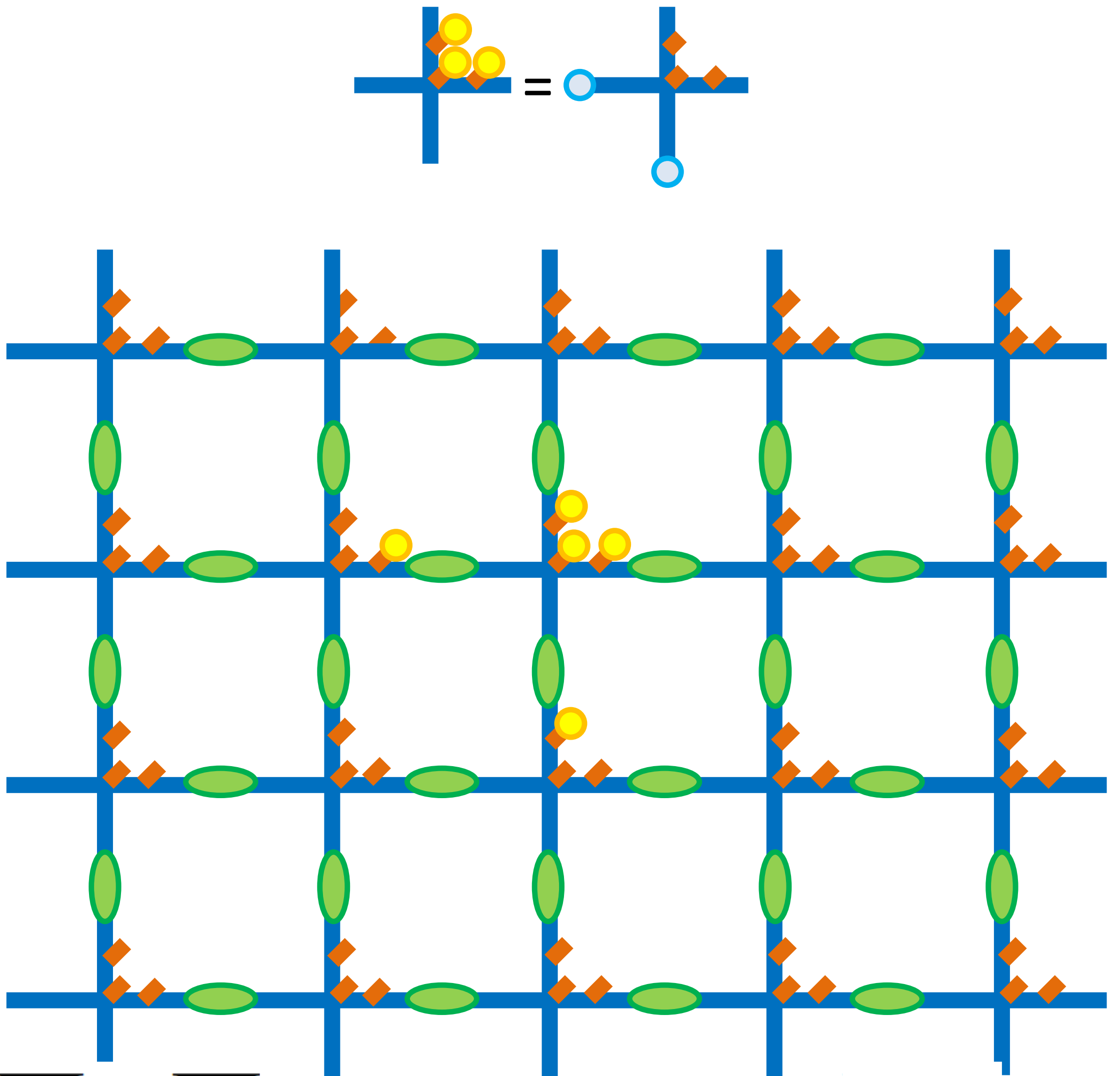
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Local Transformation: $e^{i\Lambda \mathcal{G}(\mathbf{x}_0)} \left|\psi\right\rangle$



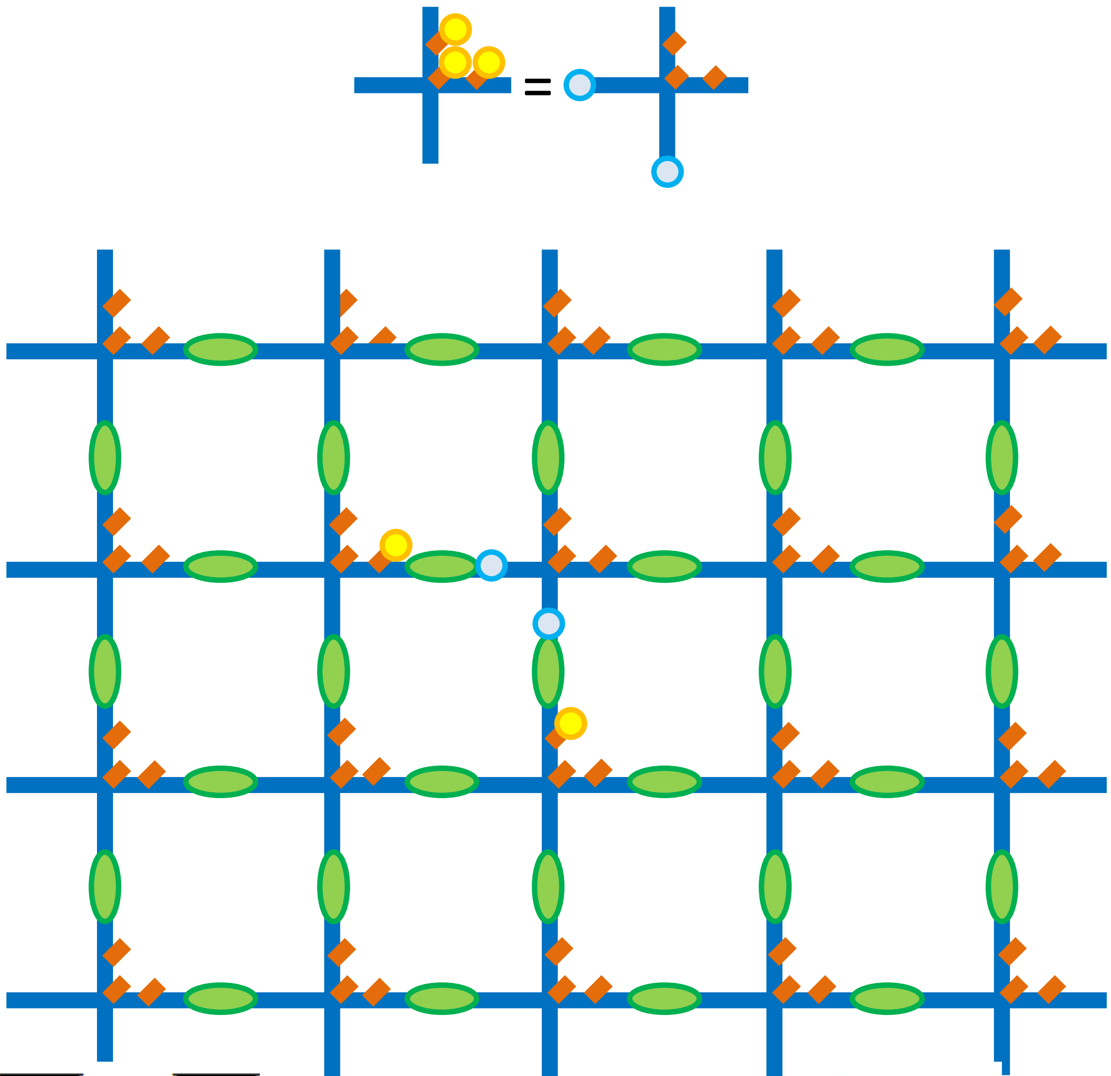
$$\left|\psi\right\rangle = \left\langle \Omega_v \right| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G\left(\mathbf{x},1\right) \mathcal{U}_G\left(\mathbf{x},2\right) A\left(\mathbf{x}\right) \left|0\right\rangle_{\mathbf{x},1} \left|0\right\rangle_{\mathbf{x},2} \left|\Omega\right\rangle$$

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



$$|\psi\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\, \mathcal{U}_G(\mathbf{x},2) A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)



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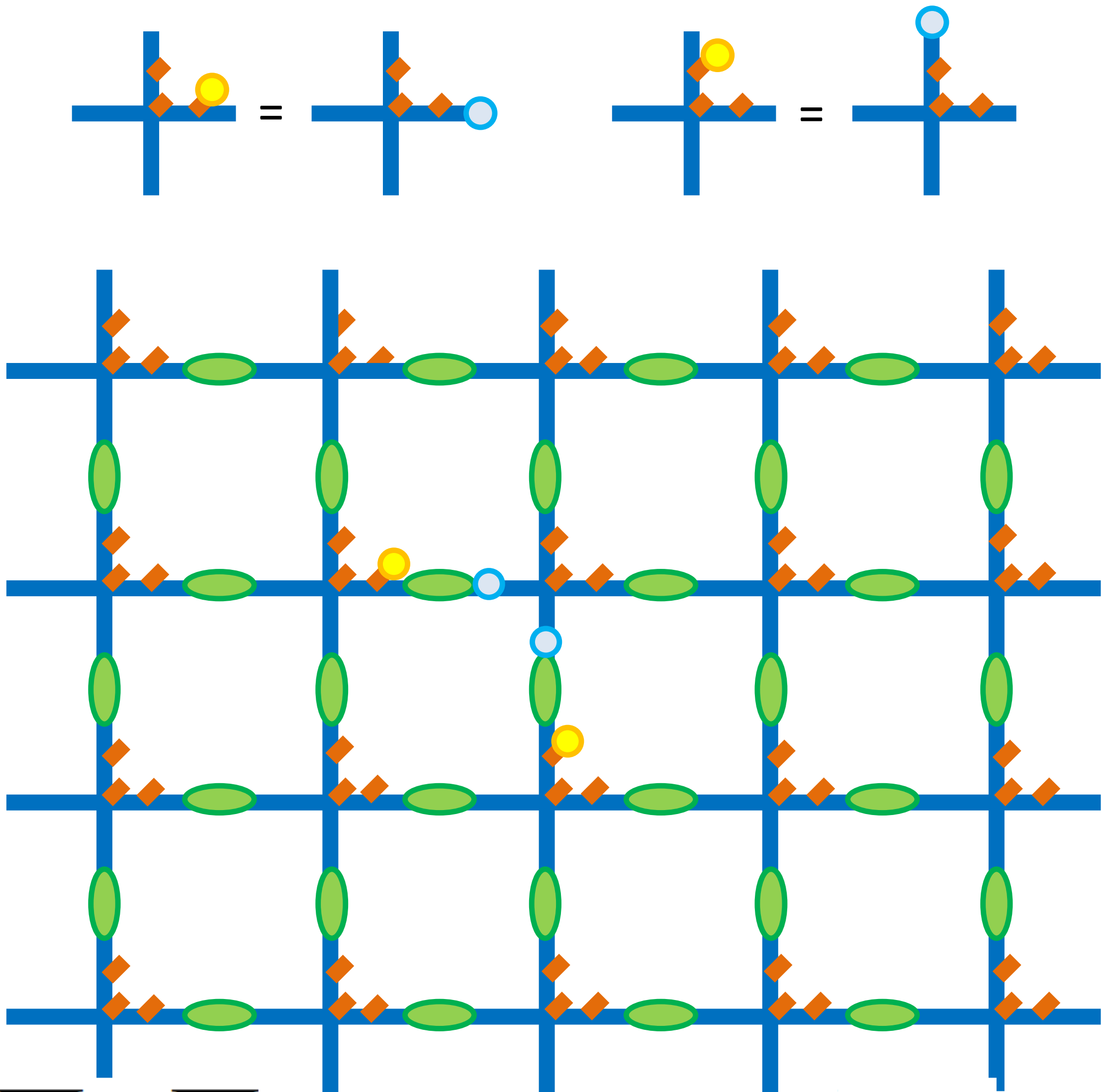

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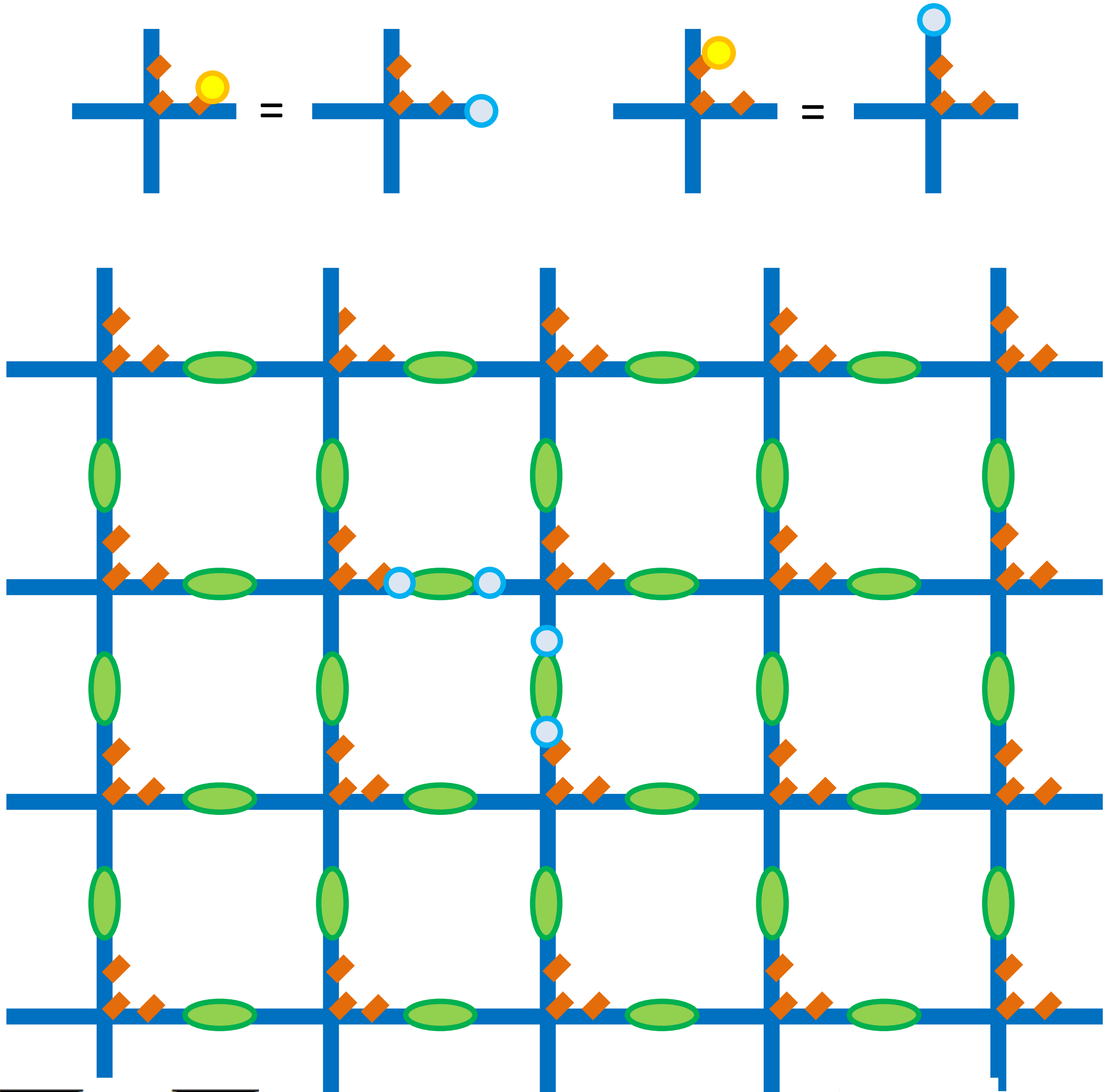

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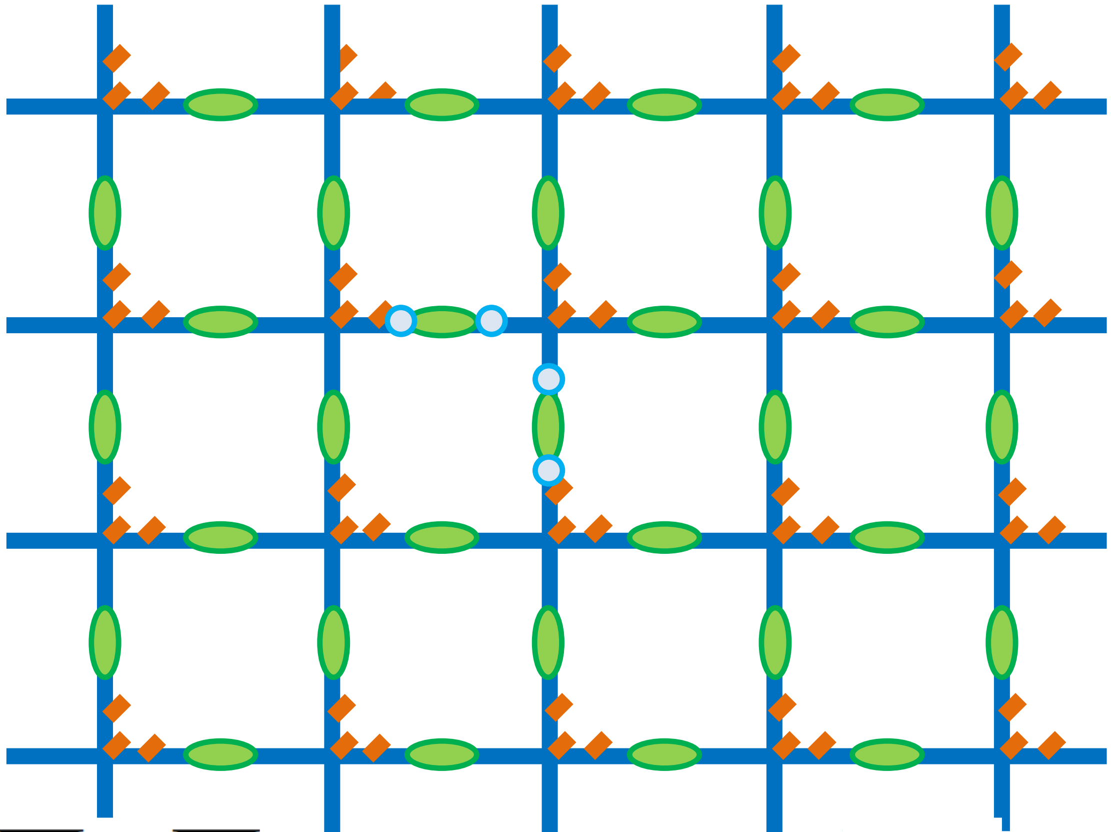

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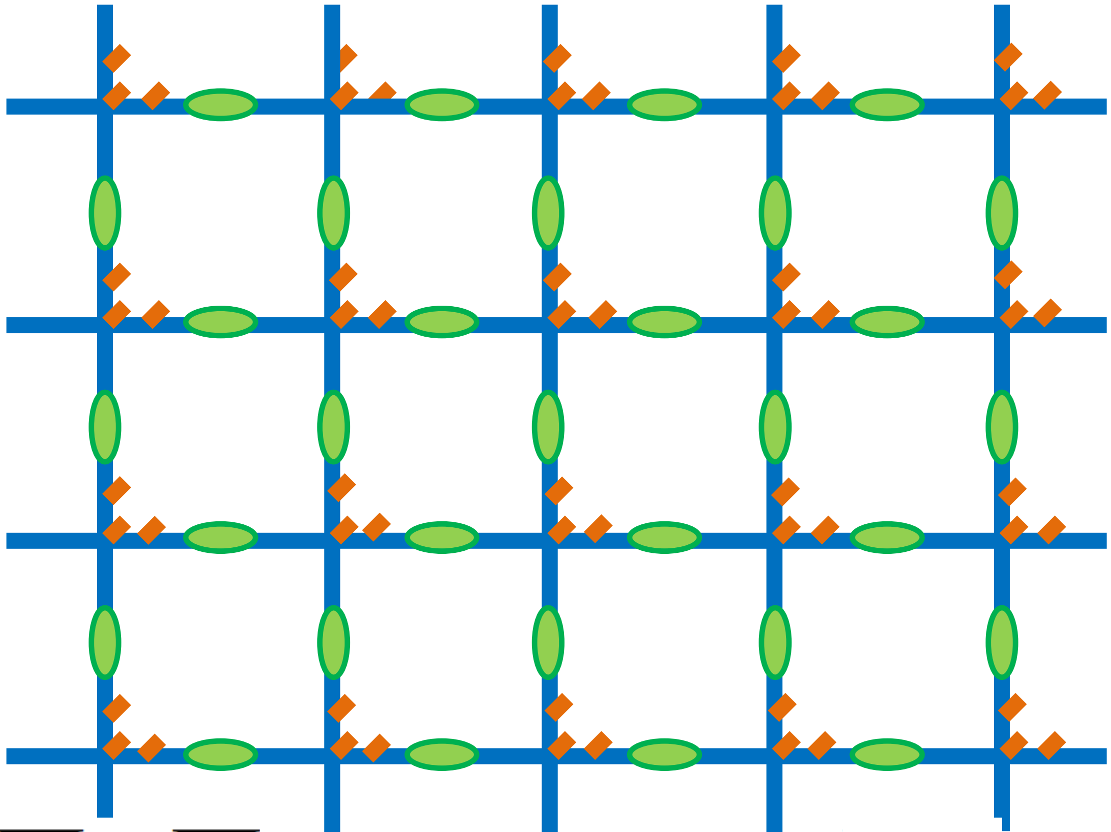
E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Local Symmetry: $e^{i\Lambda\mathcal{G}(\mathbf{x}_0)}\left|\psi\right\rangle = \left|\psi\right\rangle$



$$\left|\psi\right\rangle = \left\langle\Omega_v\right|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}\mathcal{U}_G\left(\mathbf{x},1\right)\mathcal{U}_G\left(\mathbf{x},2\right)A\left(\mathbf{x}\right)\left|0\right\rangle_{\mathbf{x},1}\left|0\right\rangle_{\mathbf{x},2}\left|\Omega\right\rangle$$

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Locally gauge invariant fermionic PEPS

- We We wish to describe PEPS of fermionic matter coupled to dynamical gauge fields.

- Starting point – **Gaussian fermionic PEPS** with a global symmetry.
  - **Gaussian states** – ground states of quadratic Hamiltonians, completely described by their covariance matrix. Very easy to handle analytically with the use of the Gaussian formalism.
  - **Fermionic PEPS** – defined with fermionic creation operators acting on the Fock vacuum. Easy to parameterize if they are Gaussian.

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

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- **Start with these, then make the symmetry local and add the gauge field.**

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E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
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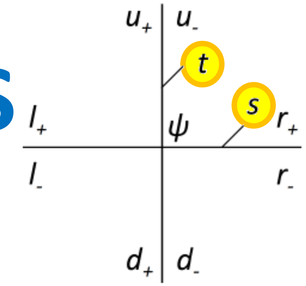
- **Start with these, then make the symmetry local and add the gauge field.** Similar to **minimal coupling**: **Gauge a free matter state → obtain an interacting matter-gauge field state.**

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Gauging the Gaussian fermionic PEPS



- The state is not Gaussian anymore,
  but rather a "generalized Gaussian state"
- Gaussian mapping and formalism are generally not valid, but **the parameterization of the original states "survives"**:
  – Translation invariance → Charge conjugation
  – Rotation invariance → Rotation invariance
  – Global invariance → Local gauge invariance:
    - "Virtual Gauss law" → Physical Gauss laws

$$\widetilde{\Theta}^r_g \widetilde{\Theta}^u_g \Theta^{l\dagger}_g \Theta^{d\dagger}_g \Theta^{\dagger p}_g$$

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

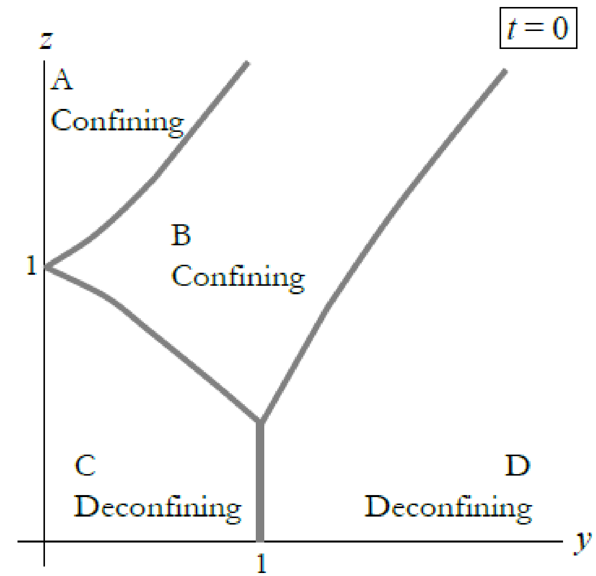
$$\hat{\Theta}_g(\mathbf{x})\,|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

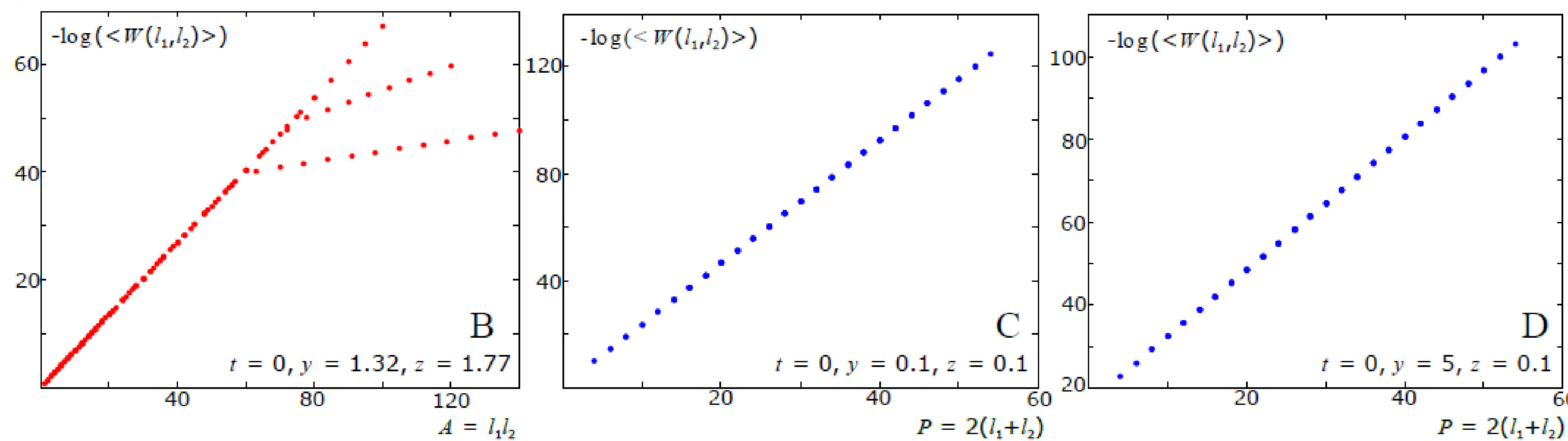
E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)

Example: The phases of the pure gauge theory – $U(1)$

B,C,D – clear results from the Wilson loops (also from other computations, such as the Creutz parameter)

A,D – also some analytical results from $1/z$ or $1/y$ expansions.





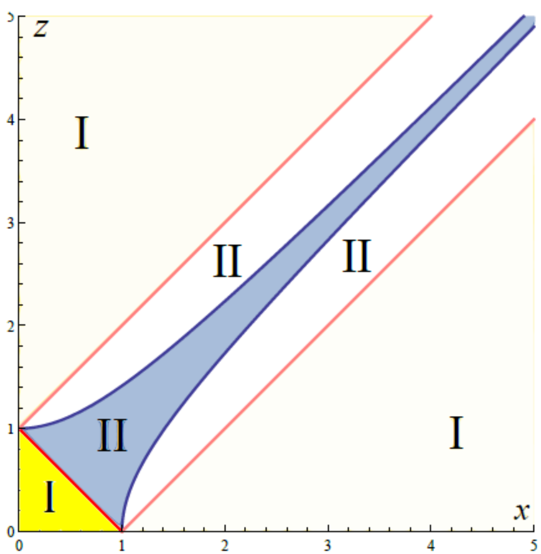
E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)

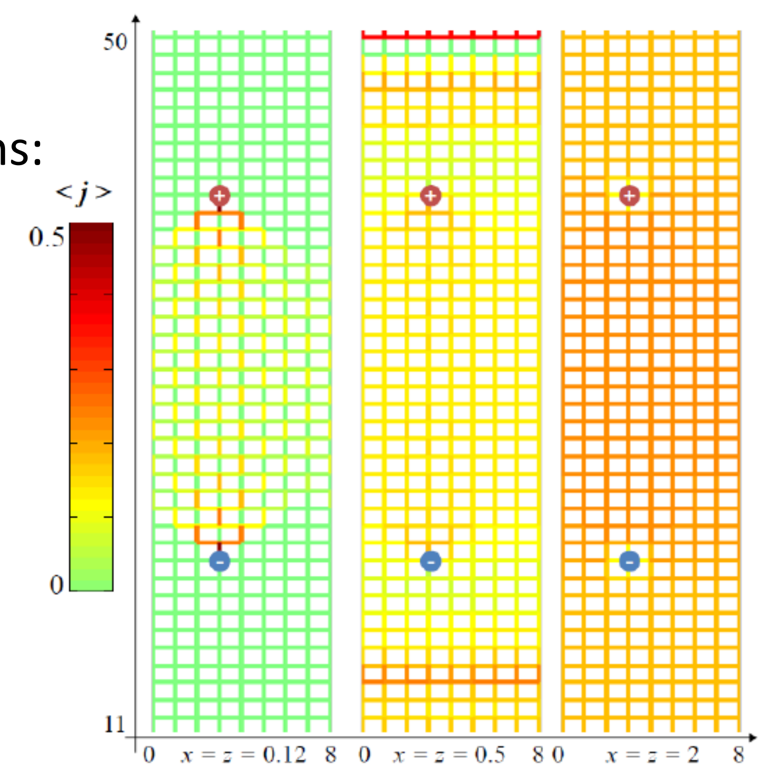
Example: The phases of the pure gauge theory – *SU*(2)

Perimeter law everywhere (Numerical calculation + perturbative expansions where applicable)

I – gapped – "Higgs"-like
II – gapless – "Coulomb"-like

Supported by flux line configuration observations:





E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)

MPS – Numerical Approach

- **Mostly in 1+1d**, combining MPS (Matrix Product States) with White's **DMRG (Density Matrix Renormalization Group)**; have been widely and successfully used for various many body models, mostly from condensed matter, for
  - Variational studies of ground states
  - Thermal equilibrium properties
  - Dynamics

- Very successfully applied to 1+1d lattice gauge theories

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Published: November 20, 2013

The mass spectrum of the Schwinger model with matrix product states

M.C. Bañuls,[a] K. Cichy,[b,c] J.I. Cirac[a] and K. Jansen[b,d]

PRL 118, 071601 (2017) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2017

Density Induced Phase Transitions in the Schwinger Model: A Study with Matrix Product States

Mari Carmen Bañuls,[1] Krzysztof Cichy,[2,3] J. Ignacio Cirac,[1] Karl Jansen,[4] and Stefan Kühn[1]
[1]Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
[2]Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany
[3]Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[4]NIC, DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany
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Non-Abelian string breaking phenomena with matrix product states

Stefan Kühn, Erez Zohar, J. Ignacio Cirac and Mari Carmen Bañuls

- High dimensional generalizations: challenging and demanding scaling, generally unavailable (see, however, recent works by Corboz)

Monte Carlo with gauged Gaussian fPEPS

- It is possible to express our states in **a basis**, that allows one to perform **efficient Monte-Carlo calculations**

$$|\Psi\rangle = \int \mathcal{D}\mathcal{G} \, |\mathcal{G}\rangle \, |\psi(\mathcal{G})\rangle$$

  - $|\mathcal{G}\rangle$ is a **fixed configuration state of the gauge field on the links**.

$$|\mathcal{G}\rangle \equiv \bigotimes_{\mathbf{x},k} |g(\mathbf{x},k)\rangle \qquad \mathcal{D}\mathcal{G} = \prod_{\mathbf{x},k} dg(\mathbf{x},k)$$

$$\langle \mathcal{G}' | \mathcal{G} \rangle = \delta(\mathcal{G}', \mathcal{G})$$

  - $|\psi(\mathcal{G})\rangle$ is a **fermionic Gaussian state**, representing **fermions coupled to a static, background gauge field** $\mathcal{G}$.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510

Monte Carlo with gauged Gaussian fPEPS

- It is possible to express our states in **a basis**, that allows one to perform **efficient Monte-Carlo calculations**

$$\left|\Psi\right\rangle = \int \mathcal{D}\mathcal{G} \left|\mathcal{G}\right\rangle \left|\psi\left(\mathcal{G}\right)\right\rangle$$

- Configuration states are eigenstates of functions of group element operators:

$$U^{j}_{mn}\left|g\right\rangle = D^{j}_{mn}\left(g\right)\left|g\right\rangle \qquad \left|\mathcal{G}\right\rangle \equiv \bigotimes_{\mathbf{x},k}\left|g\left(\mathbf{x},k\right)\right\rangle$$

$$F\left(\left\{U^{j}_{mn}\left(\mathbf{x},k\right)\right\}\right)\left|\mathcal{G}\right\rangle = F\left(\left\{D^{j}_{mn}\left(g\left(\mathbf{x},k\right)\right)\right\}\right)\left|\mathcal{G}\right\rangle$$

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $W\left(C\right) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U\left(\mathbf{x},k\right)\right)$

  - exp. value for $\left|\Psi\right\rangle = \int \mathcal{D}\mathcal{G}\left|\mathcal{G}\right\rangle\left|\psi\left(\mathcal{G}\right)\right\rangle$:

$$\left\langle W\right\rangle = \frac{\int \mathcal{D}\mathcal{G}\mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D\left(g\left(\boldsymbol{x},k\right)\right)\right)\left\langle\psi\left(\mathcal{G}\right)|\psi\left(\mathcal{G}\right)\right\rangle}{\int \mathcal{D}\mathcal{G}\left\langle\psi\left(\mathcal{G}\right)|\psi\left(\mathcal{G}\right)\right\rangle}$$

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{D}\mathcal{G} \, |\mathcal{G}\rangle \, |\psi(\mathcal{G})\rangle$:

$$\langle W\rangle = \frac{\int \mathcal{D}\mathcal{G} \mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D(g(\boldsymbol{x},k))\right) \langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{D}\mathcal{G} \langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}$$

- The function

$$p(\mathcal{G}) = \frac{\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{D}\mathcal{G}' \langle\psi(\mathcal{G}')|\psi(\mathcal{G}')\rangle}$$

is a probability density.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{DG} \left|\mathcal{G}\right\rangle \left|\psi(\mathcal{G})\right\rangle$:

$$\langle W\rangle = \frac{\int \mathcal{DG}\,\mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D\left(g\left(\boldsymbol{x},k\right)\right)\right)\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{DG}\,\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}$$

- **The fermionic calculation is easy, through the gaussian formalism: very efficient, no sign problem**

→ **Monte Carlo integration!**

E. **Zohar,** J.I. Cirac, Phys. Rev. D 97, 034510

Monte Carlo with gauged Gaussian fPEPS

- The method is extendable to further physical observables (e.g. mesonic operators and electric energy operators), always involving the **probability density function**

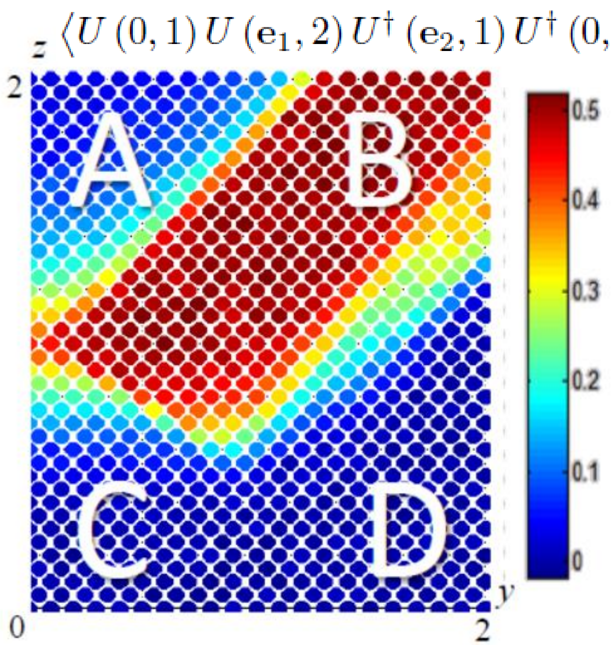
$$p\left(\mathcal{G}\right)=\frac{\left\langle\psi\left(\mathcal{G}\right)|\psi\left(\mathcal{G}\right)\right\rangle}{\int\mathcal{D}\mathcal{G}'\left\langle\psi\left(\mathcal{G}'\right)|\psi\left(\mathcal{G}'\right)\right\rangle}$$

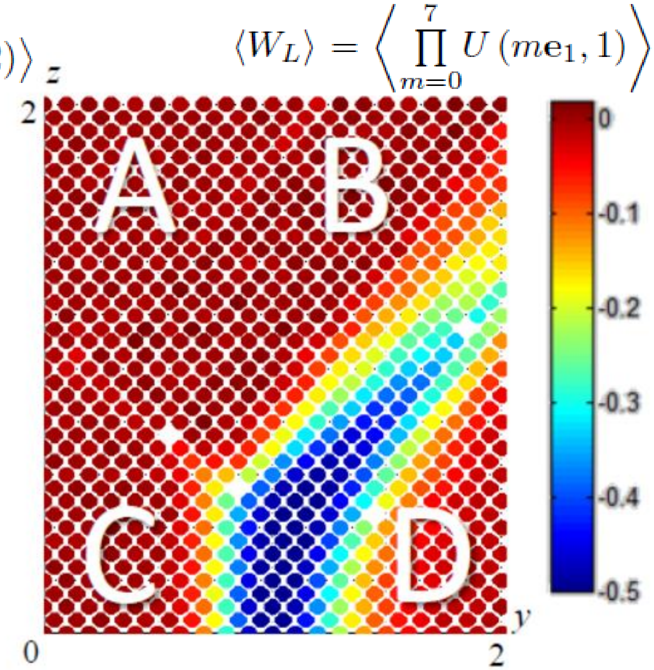
  and possibly elements of the **covariance matrix of the Gaussian state** $\left|\psi\left(\mathcal{G}\right)\right\rangle$, which could be calculated very efficiently.

- **It is possible to contract gauged Gaussian fPEPS beyond 1+1d, and without the sign problem of conventional LGT methods (it is not a Euclidean path integral).**

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510

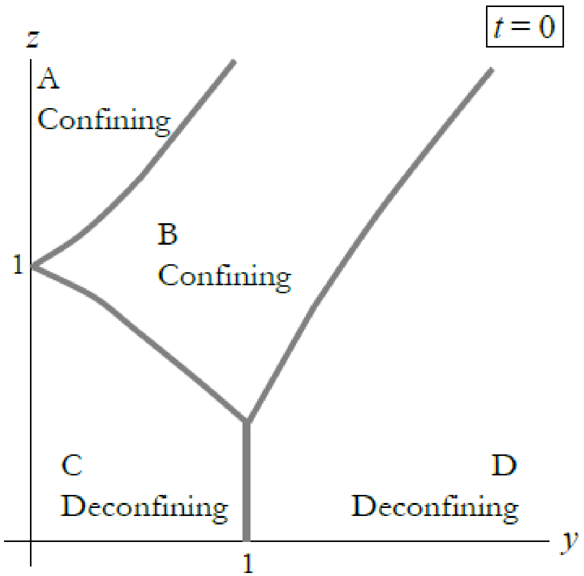
Illustration: phase diagram of pure gauge Z_3 PEPS in 2+1d





MC, 8x8

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510



Exact contraction

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)

Summary

- **Lattice gauge theories** may be simulated by **ultracold atoms** in optical lattices. **Gauge invariance** may be obtained using several methods.
- **PEPS** are very useful for the study of many body systems with symmetries – **even when the symmetries are local**.
- The **gauged gaussian fermionic PEPS** construction could be **combined with Monte Carlo methods** for numerical studies in **larger systems and higher dimensions**, without the sign problem, and overcoming the scaling problems of extending MPS+DMRG to more than 1+1d.

For detailed lecture notes on the topics discussed in this talk, see
Gauss law, Minimal Coupling and Fermionic PEPS for Lattice Gauge Theories
E. Zohar, arXiv:1807.01294 (2018)