

Discoveries & Open Puzzles in Particle Physics and Gravitation,
Kitzbühel, 2019-06-25



Quantum Simulations of Lattice Models in the Innsbruck *Quantum Cloud*



Lattice Models in Condensed Matter & (Toy Models in) High Energy Physics



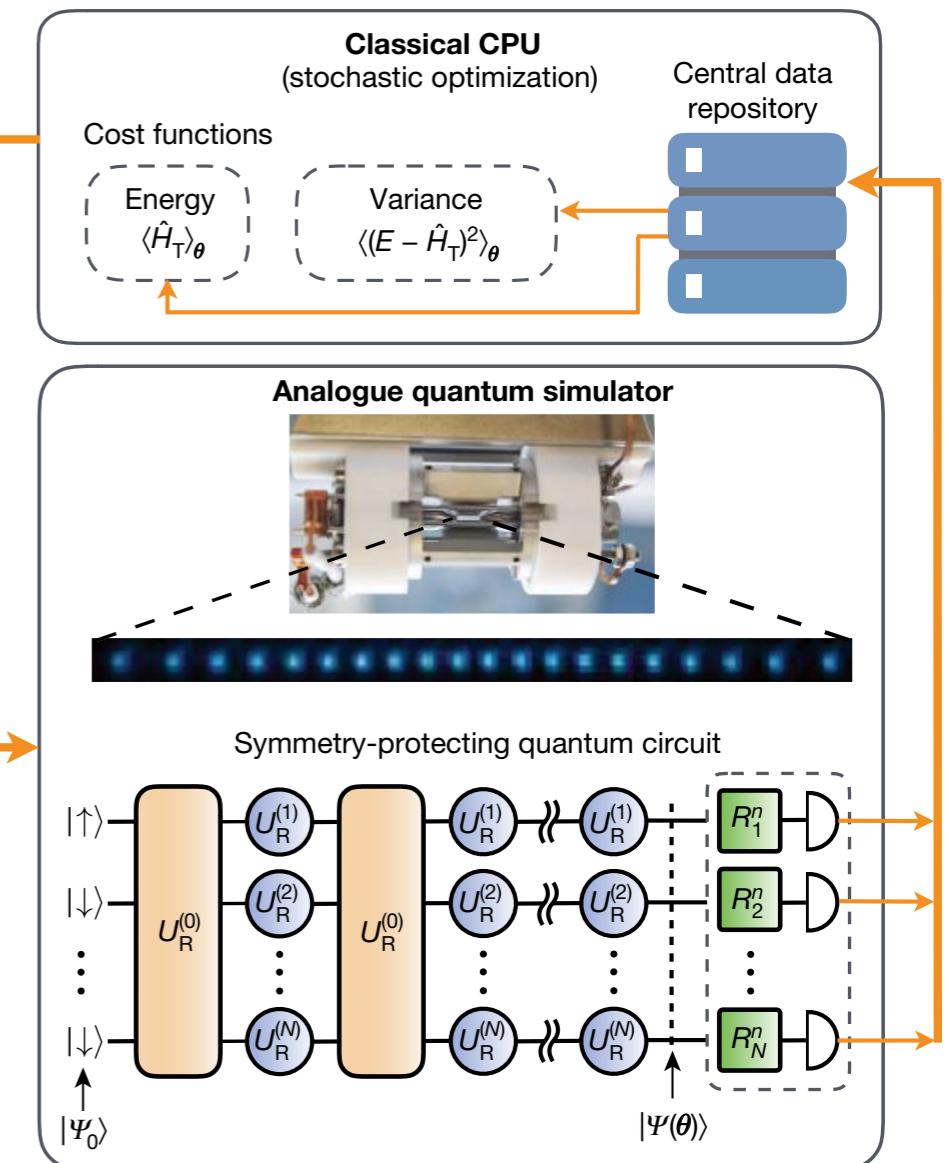
Self-verifying variational quantum simulation of lattice models

C. Kokail^{1,2,3}, C. Maier^{1,2,3}, R. van Bijnen^{1,2,3}, T. Brydges^{1,2}, M. K. Joshi^{1,2}, P. Jurcevic^{1,2}, C. A. Muschik^{1,2}, P. Silvi^{1,2}, R. Blatt¹, C. F. Roos^{1,2} & P. Zoller^{1,2*}

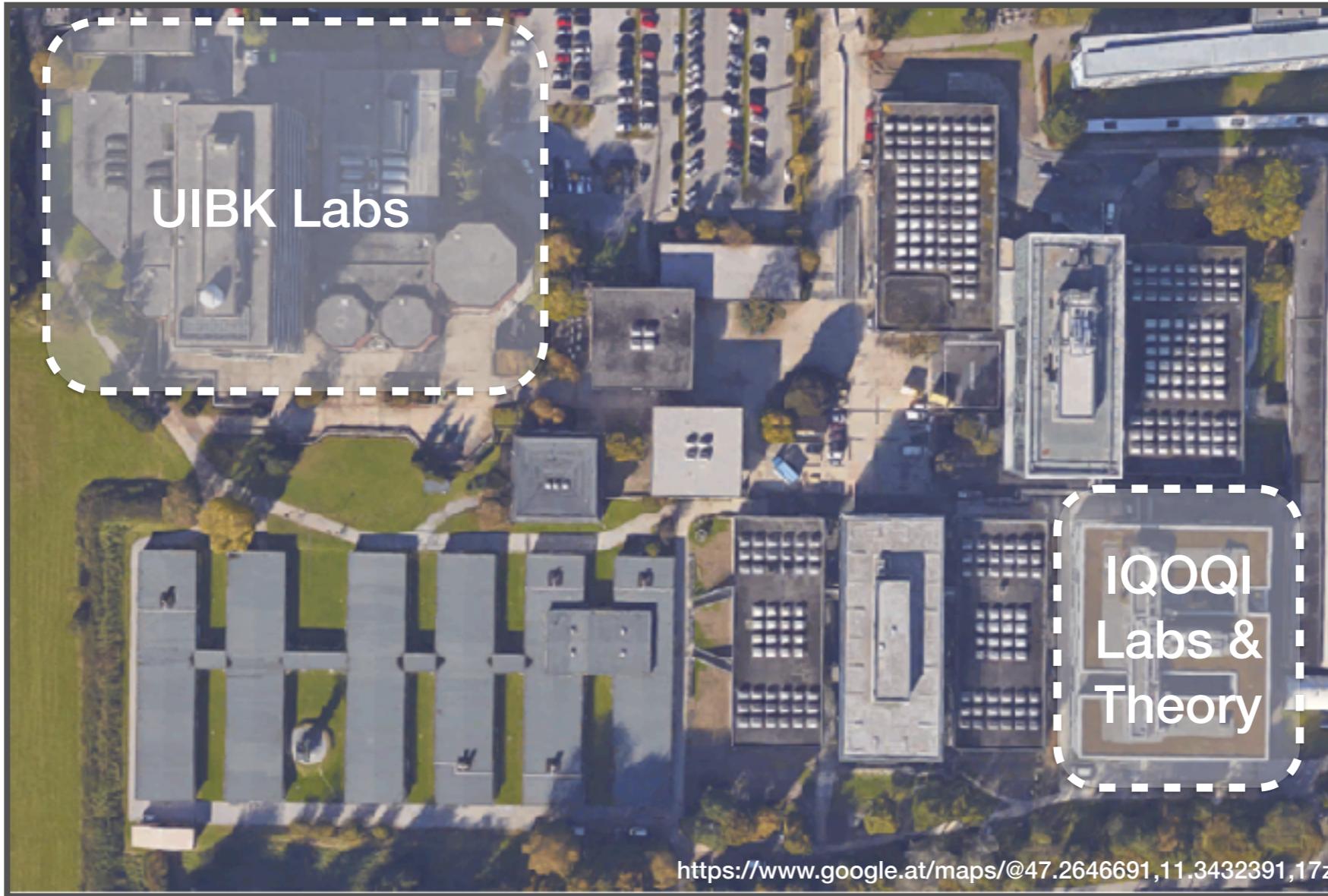


Rick van Bijnen (postdoc-th), Christine Maier (PhD-exp), Christian Kokail (PhD-th)

Classical - Quantum Feedback Loop



Quantum Programming in *Innsbruck Quantum Cloud*

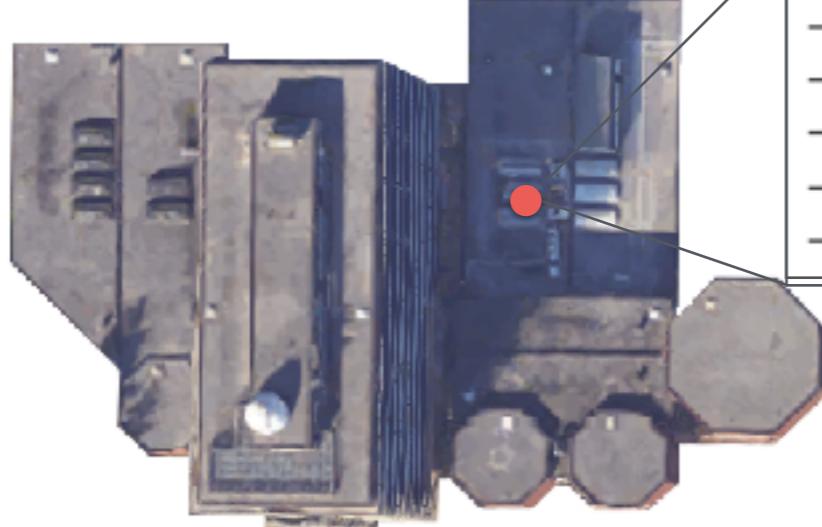


Technik Campus
UIBK

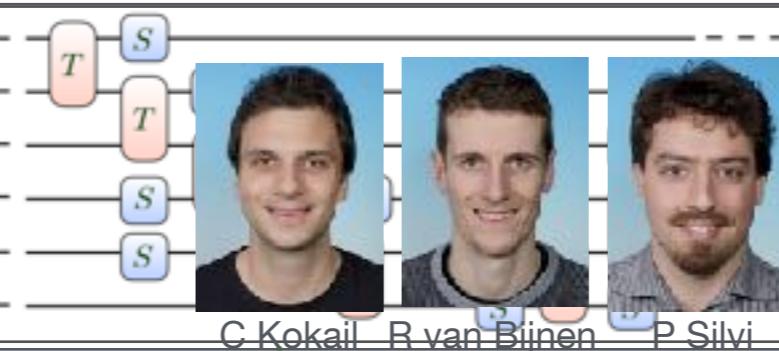
Quantum Programming in *Innsbruck Quantum Cloud*



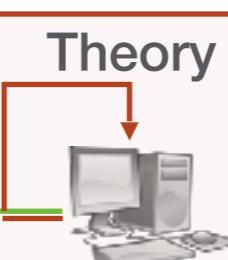
Quantum Programming in Innsbruck Quantum Cloud



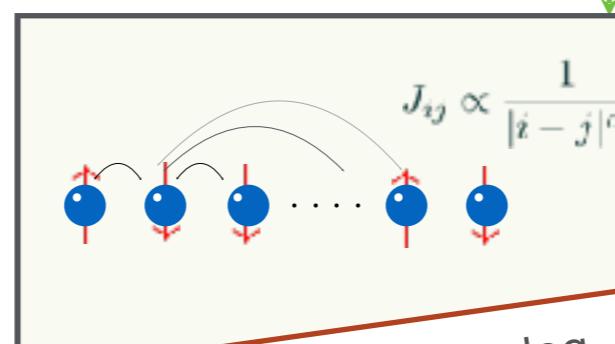
20-Qubit Trapped-Ion
Analog Quantum Simulator
Blatt-Roos-Lanyon



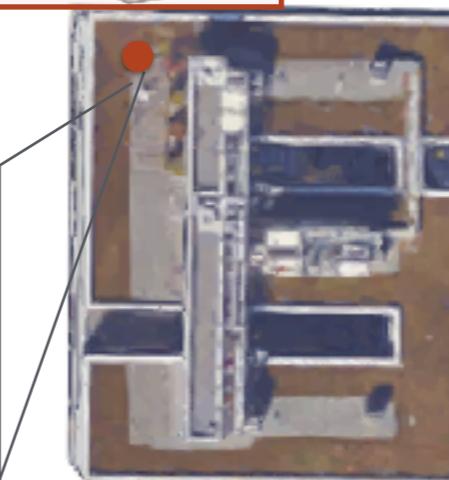
Trapped-Ion
Quantum Computer
Blatt-Monz-Schindler



Flag QuantumPU = False



Programmable Analog
Quantum Simulator



Quantum
Feedback Loop

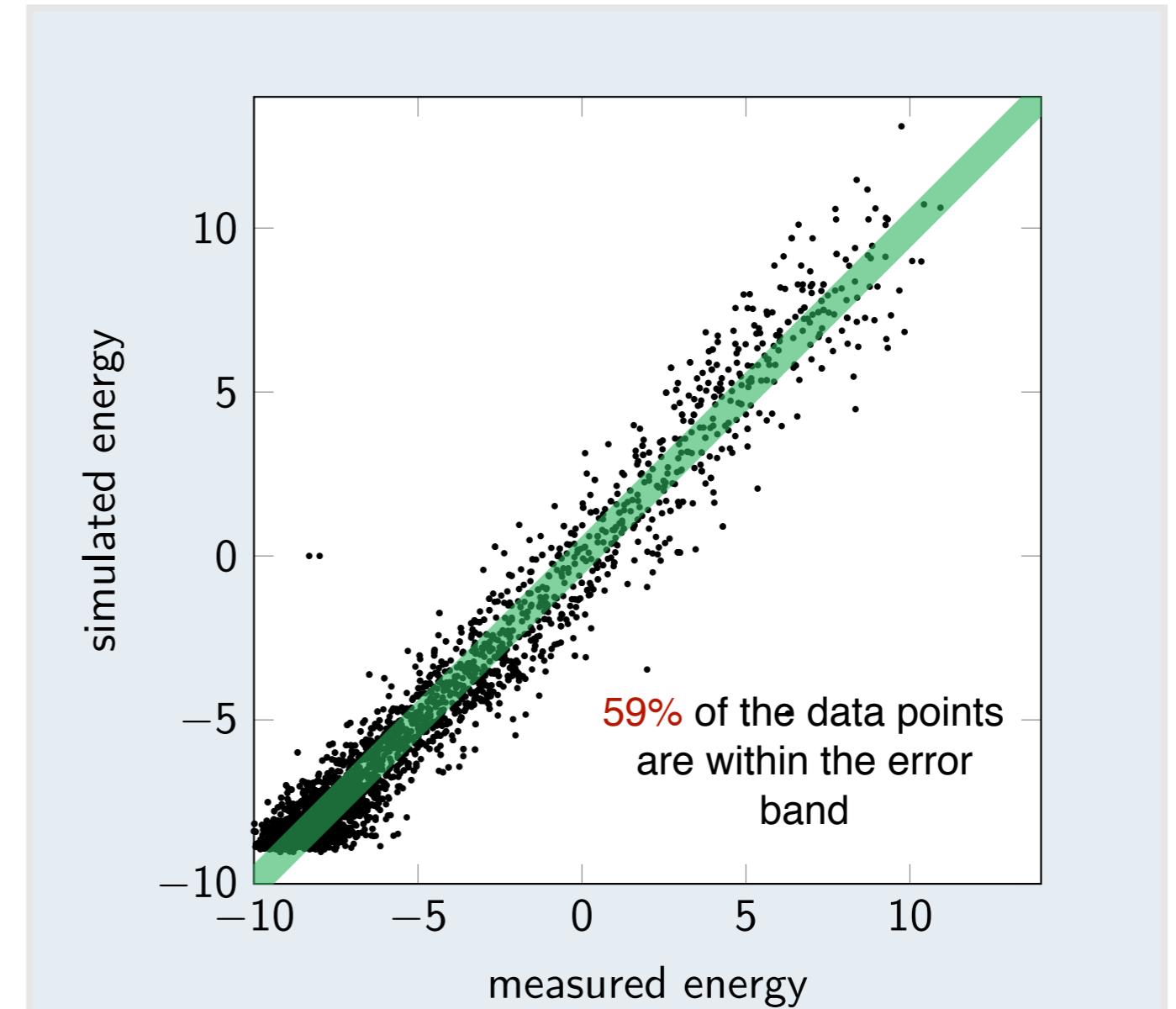
Quantum Programming in *Innsbruck Quantum Cloud*

Theory-Experiment Correlations

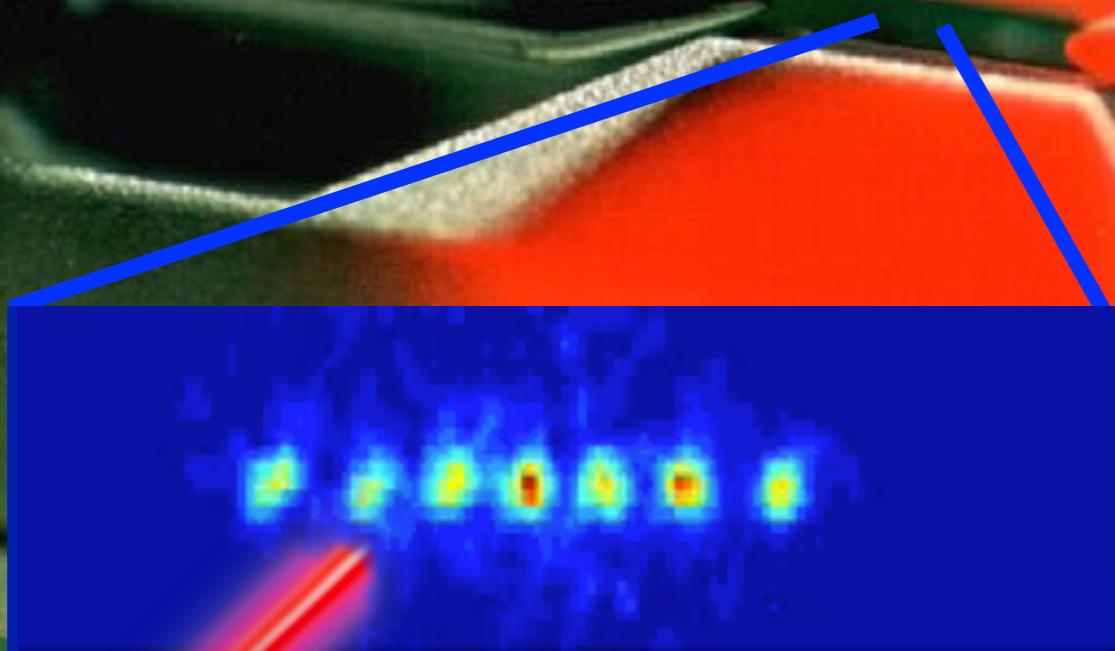
- How good / reliable is 20 ion quantum simulator?

theory predictions
vs.
experimental results

no decoherence



Trapped Ions as Quantum Computers & Simulators

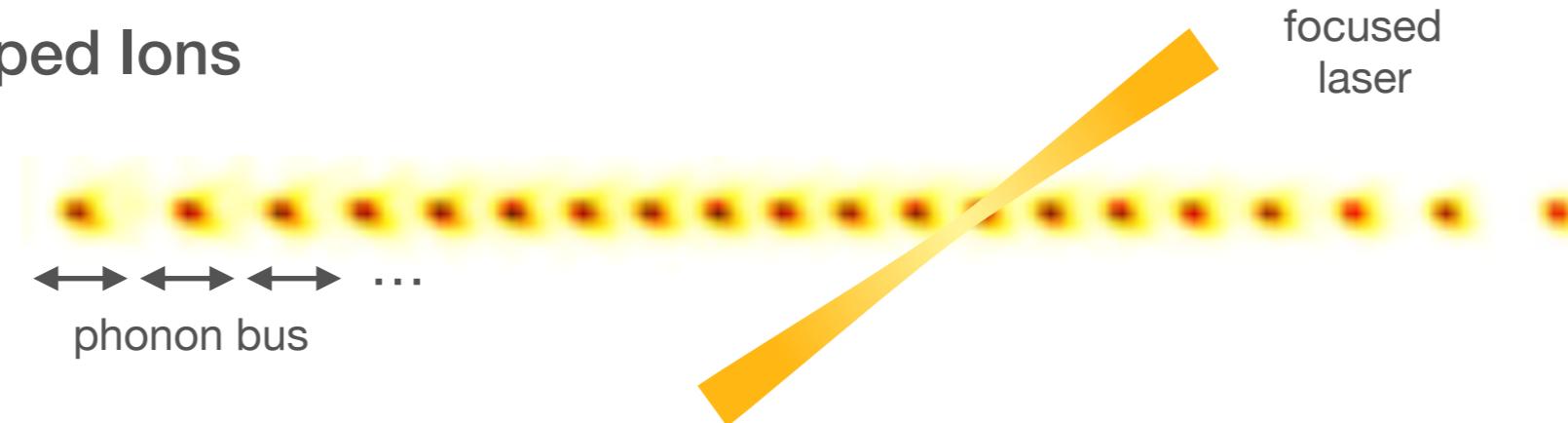


Theory: I.J. Cirac and P. Z. 1995 - ...

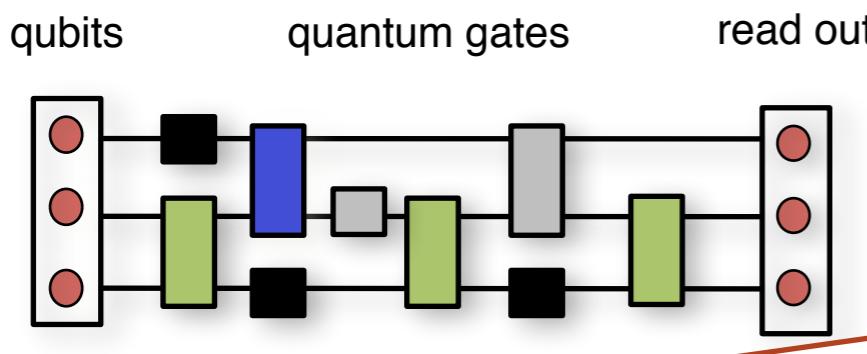
© R. Blatt; Innsbruck, see also: JQI, NIST, Mainz, ...

Trapped Ion Quantum Computers & Simulators

String of Trapped Ions



Quantum Computer [Digital]



- fully programmable / universal
- small # qubits
- error correction

Programmable Quantum Simulator [Analog]

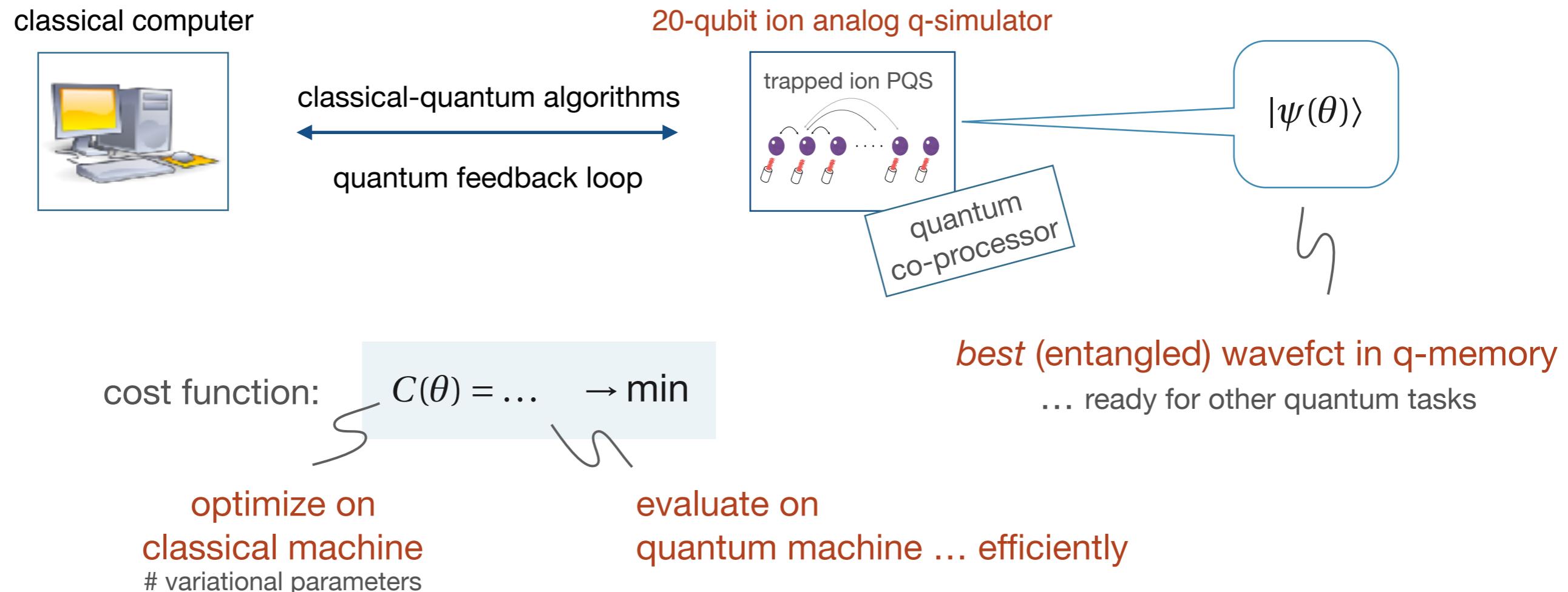
$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$
$$J_{ij} \sim \frac{1}{|i-j|^\alpha} \quad \alpha = 0 \dots 3 \quad \text{long range}$$

... engineered ma

- scalable to large # particles
- restricted class of Hamiltonians
- ... however with high fidelity

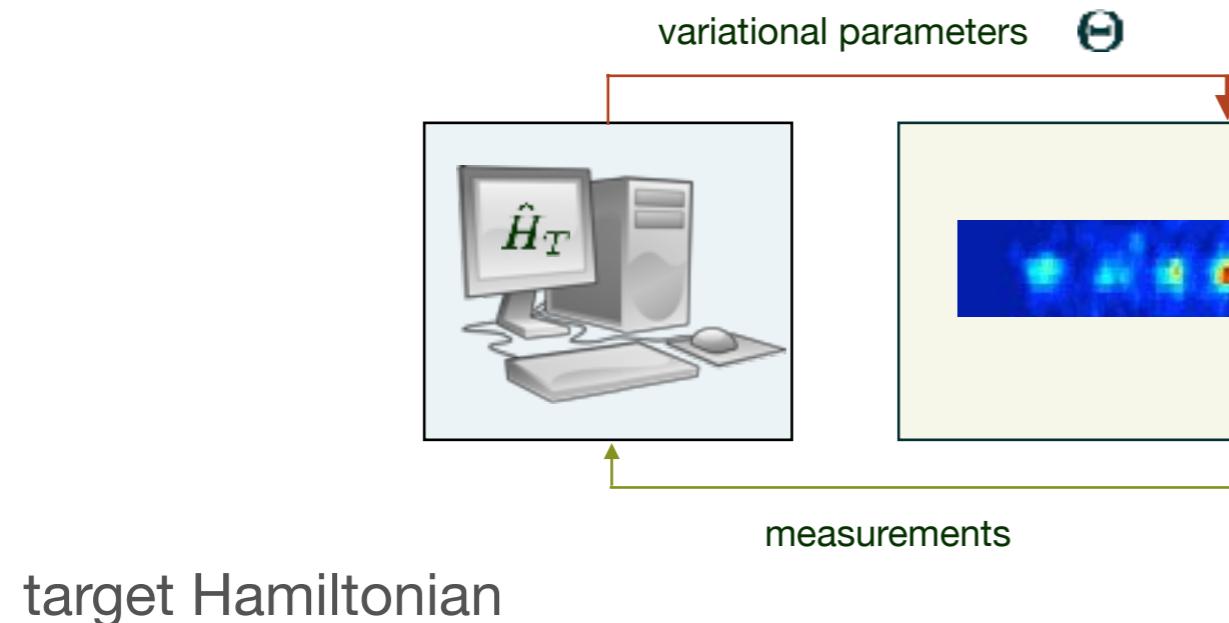
Hybrid Classical-Quantum / Variational Algorithms on PQS

- ‘best answers’ for ‘given (non-universal) quantum resources’



Variational Quantum Simulation / Variational Quantum Eigensolver

The Feedback Loop

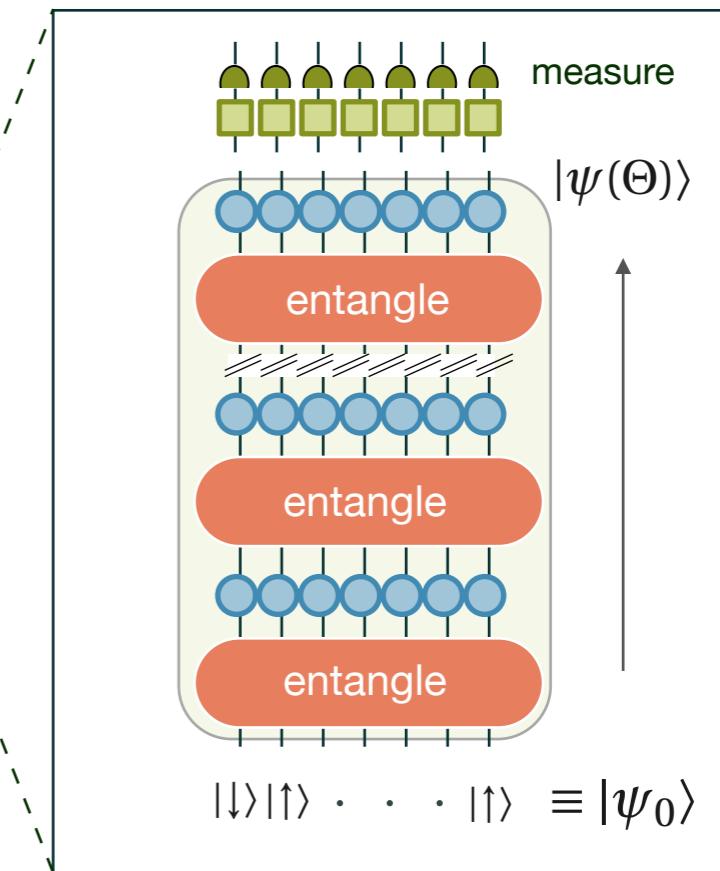


target Hamiltonian

$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \dots$$

cost function

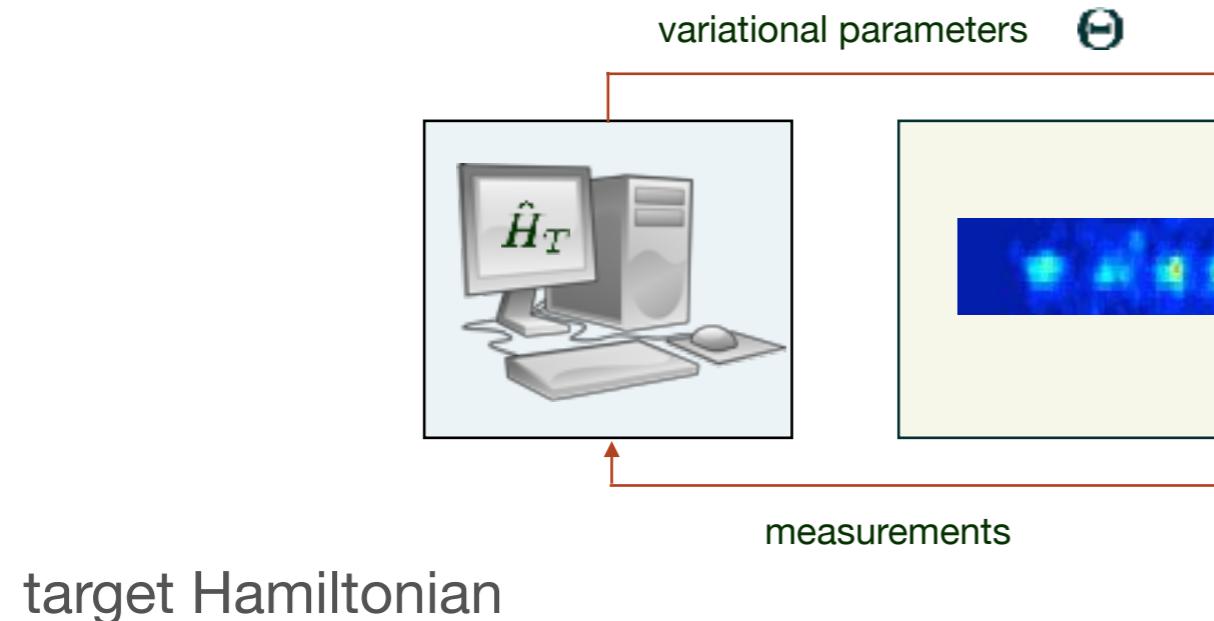
$$C(\theta) = \langle \psi(\theta) | \hat{H}_T | \psi(\theta) \rangle \rightarrow \min$$



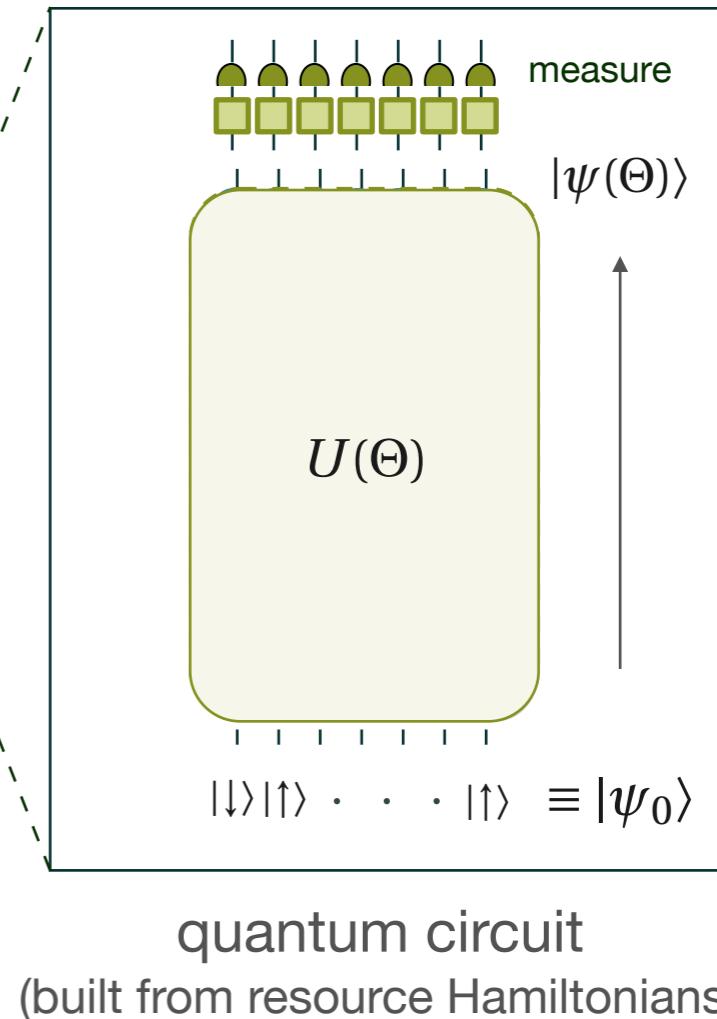
quantum circuit
(built from resource Hamiltonians)

The Idea of Variational Quantum Simulation

The Feedback Loop

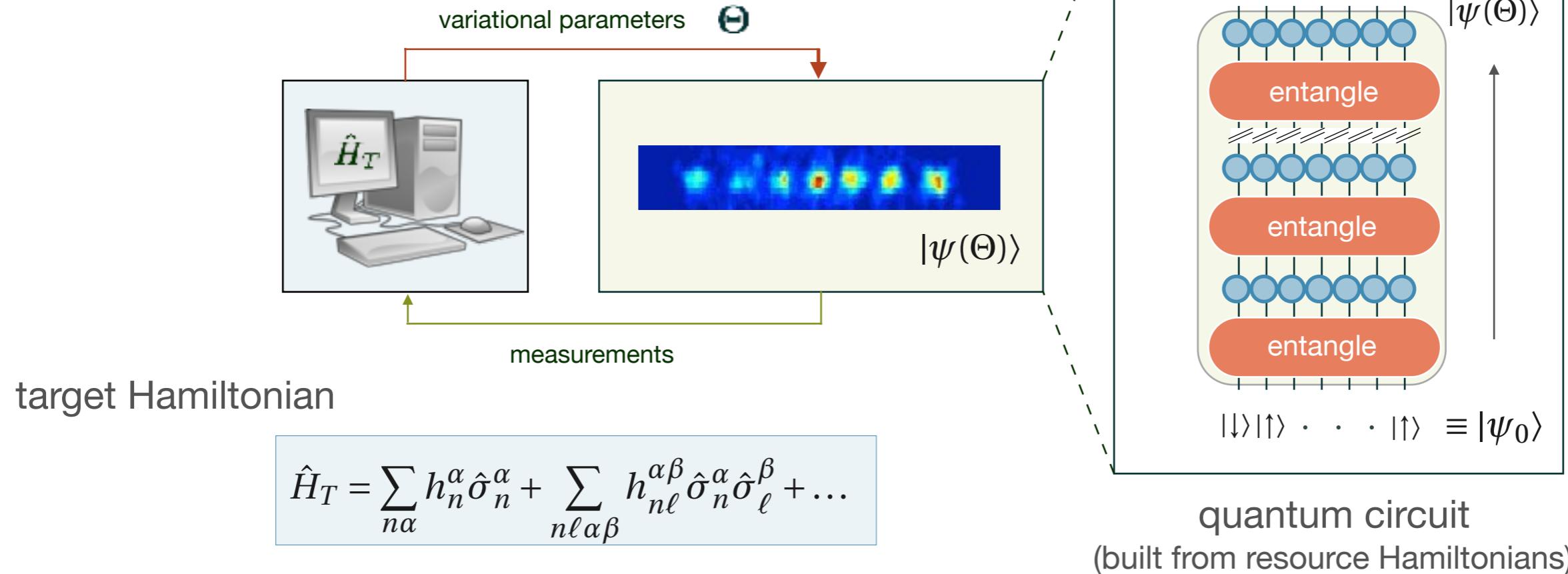


$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \dots$$

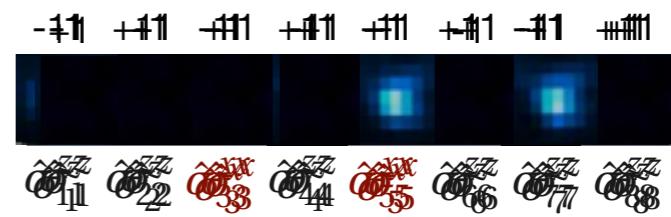


The Idea of Variational Quantum Simulation

The Feedback Loop



The Idea of Variational Quantum Simulation



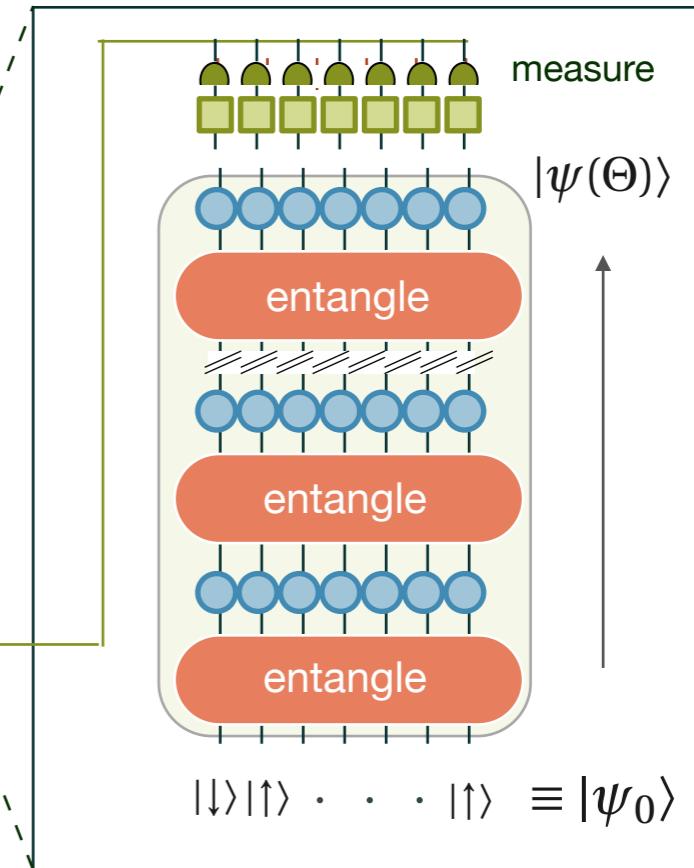
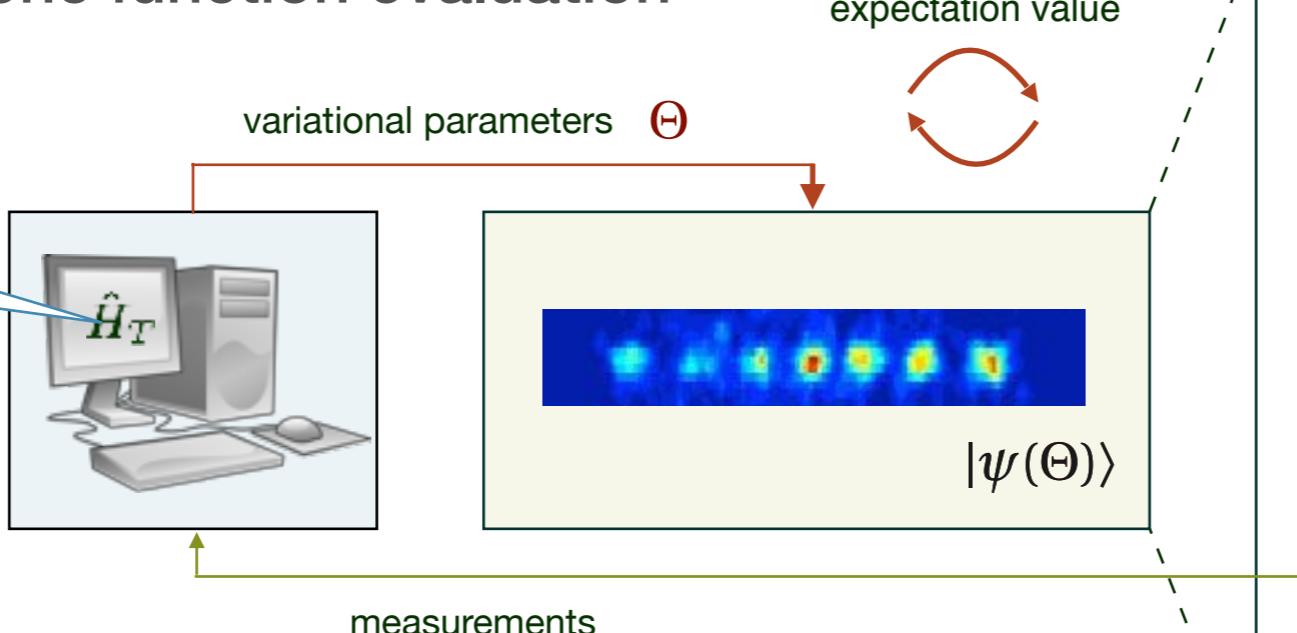
Procedure of one function evaluation

What is $\langle \hat{H}_1 \rangle$ for given Θ ?

target Hamiltonian

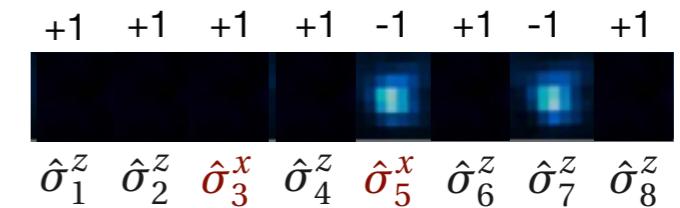
$$\hat{H}_T = \hat{H}_1 + \hat{H}_2 + \dots$$

$$\dots + \dots \hat{\sigma}_3^x \hat{\sigma}_5^x \dots + \dots$$



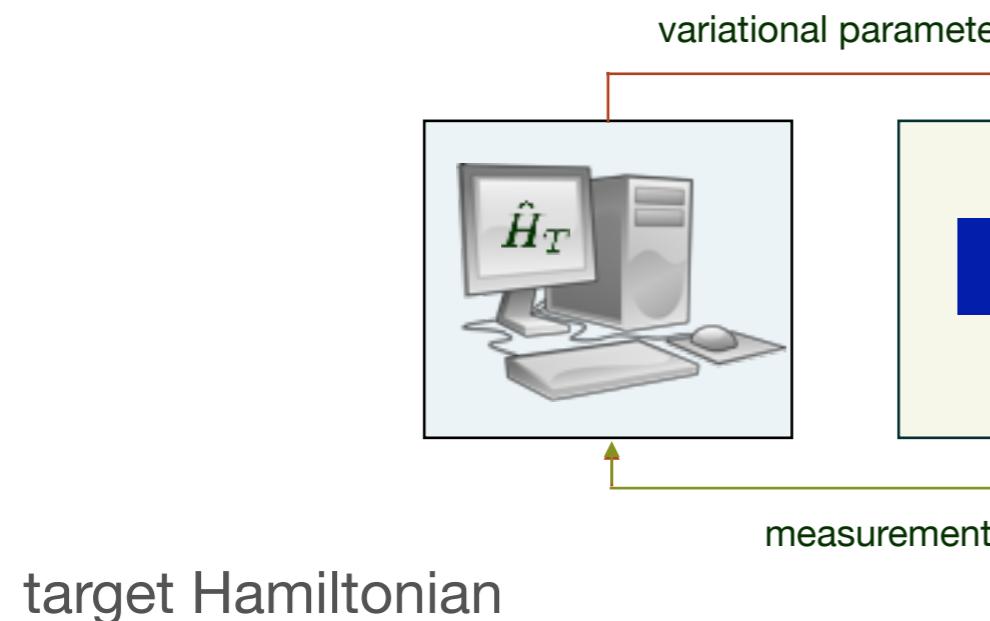
Farhi et al.. arXiv:1411.4028 (2014)

McClean, et al. *NJP* (2016)

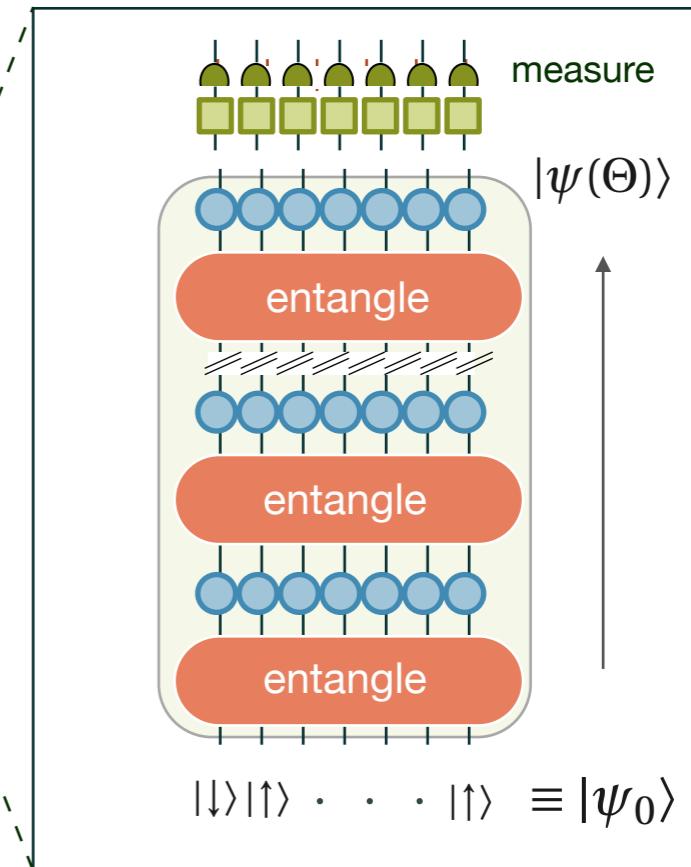


The Idea of Variational Quantum Simulation

Procedure of one function evaluation



... we have evaluated the expectation value
for one point in parameter space

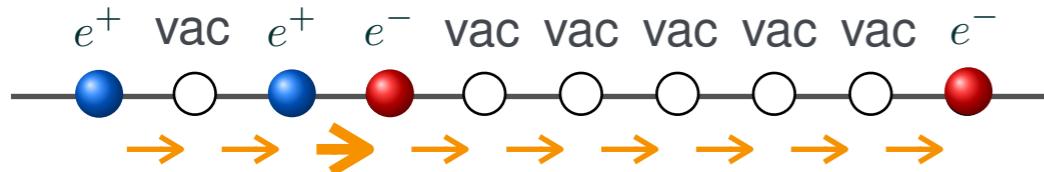


Farhi et al.. arXiv:1411.4028 (2014)

McClean, et al. *NJP* (2016)

The Lattice Schwinger Model

Quantum Electrodynamics in 1D



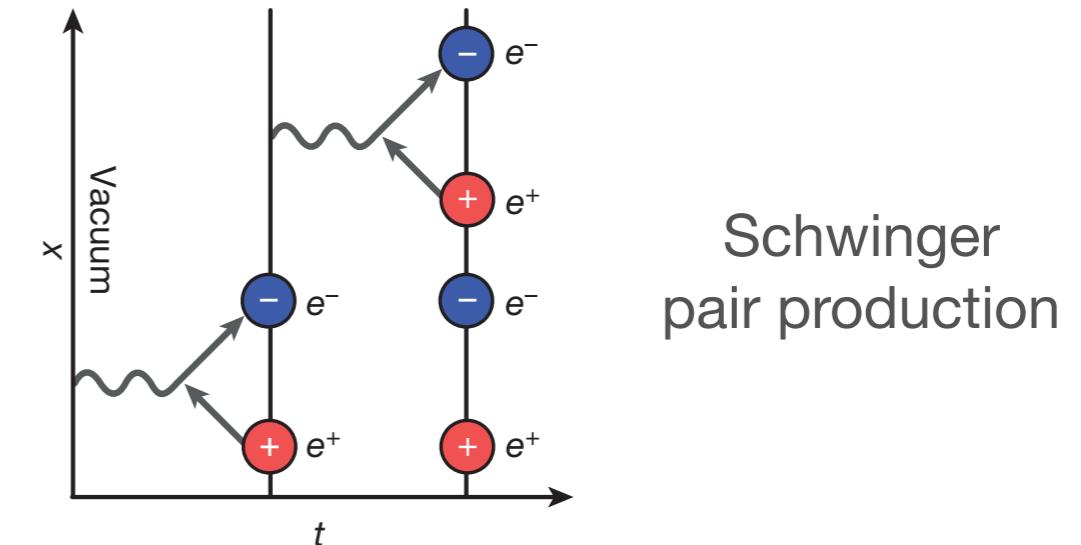
fermionic matter field

e^- ● electrons on even lattice sites

e^+ ● positrons on odd lattice sites

1D: Jordan-Wigner: fermions \rightarrow spins

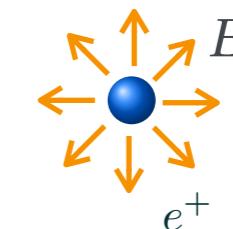
Symmetries of Schwinger Model



quantized electric field

$$\nabla \cdot E = \rho$$

Gauss Law

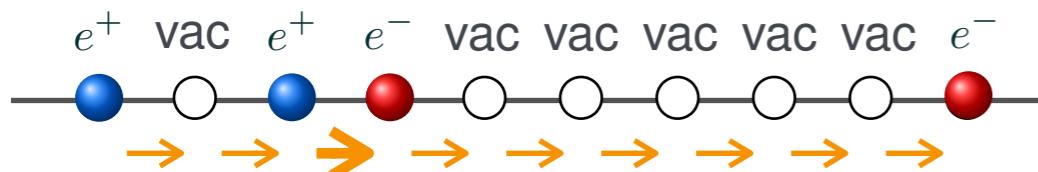


1D: fermion configuration \leftrightarrow field

... and we can eliminate the electric field

The Lattice Schwinger Model

Quantum Electrodynamics in 1D



Schwinger Hamiltonian (Kogut-Susskind)

$$\hat{H}_S = w \sum_{n=1}^{N-1} \left[\sigma_n^+ e^{i\hat{\Theta}_n} \sigma_{n+1}^- + \text{H.c.} \right] + \frac{m}{2} \sum_{n=1}^N (-1)^n \sigma_n^z + J \sum_{n=1}^{N-1} \hat{L}_n^2$$

electron-positron pairs rest mass electric field energy

fermionic matter field

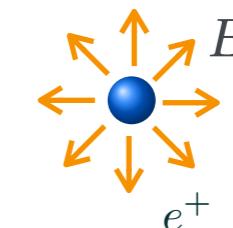
- e^- electrons on even lattice sites
- e^+ positrons on odd lattice sites

1D: Jordan-Wigner: fermions \rightarrow spins

quantized electric field

$$\nabla \cdot E = \rho$$

Gauss Law



1D: fermion configuration \leftrightarrow field

... and we can eliminate the electric field

Symmetries of Schwinger Model

Variational Quantum Simulation

Target

Schwinger Model

$$\hat{H}_S = J \sum_{i < j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

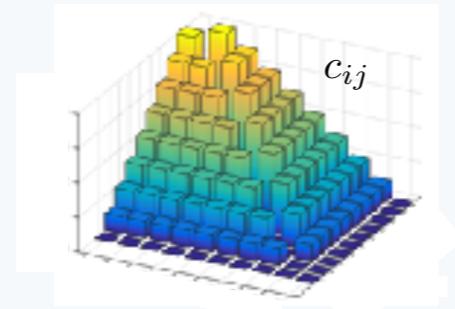
long - range interaction

$$+ w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

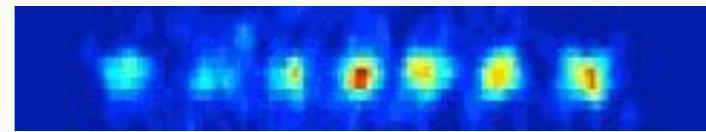
$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses



Quantum Resource (physical)

ions



Analog
QS

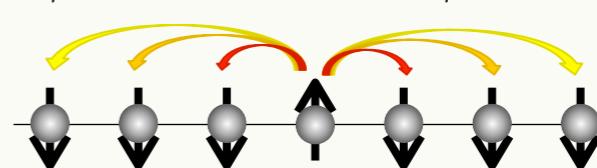
$$\hat{H}_{XY} = \sum_{i,j} J_{ij} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$

flip-flop

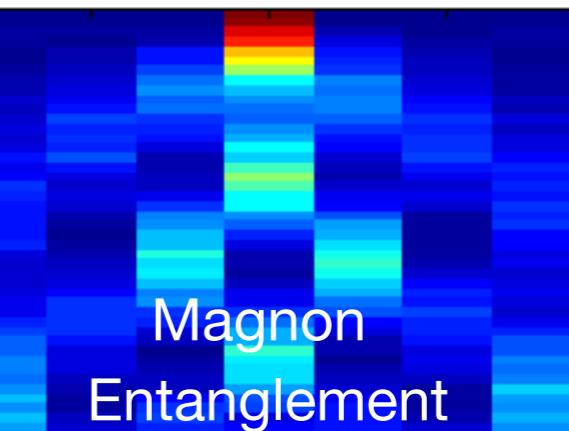
+ single site addressing

quench dynamics:

$$|\psi(t)\rangle = e^{-i\hat{H}_{XY}t} |\psi(0)\rangle$$



time



Magnon
Entanglement

Jurcevic et al.,
Nature 2014

Richmere et al.,
Nature 2014

Variational Quantum Simulation

Target

Schwinger Model

$$\hat{H}_S = J \sum_{i < j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

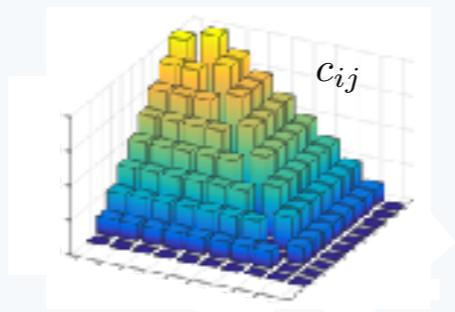
long - range interaction

$$+ w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

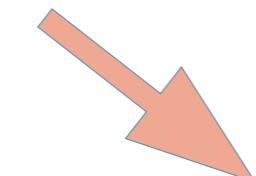
particle - antiparticle creation/annihilation

$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

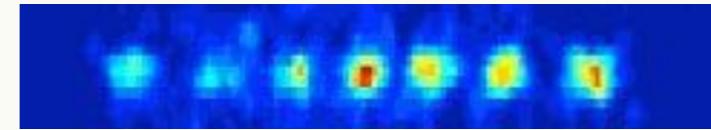


classical computer



Quantum Resource (physical)

ions



Analog
QS

$$\hat{U}_1(\theta) = e^{-i\theta \sum_{ij} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x}$$

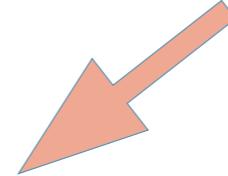
$$\hat{U}_{2,i}(\theta) = e^{-i\theta \vec{n} \cdot \hat{\vec{\sigma}}_i}$$

entangle

local rotations

AQS generates family of entangled states:

$$|\psi(\Theta)\rangle = \hat{U}_N(\theta_N) \dots \hat{U}_2(\theta_2) \hat{U}_1(\theta_1) |\psi_0\rangle$$

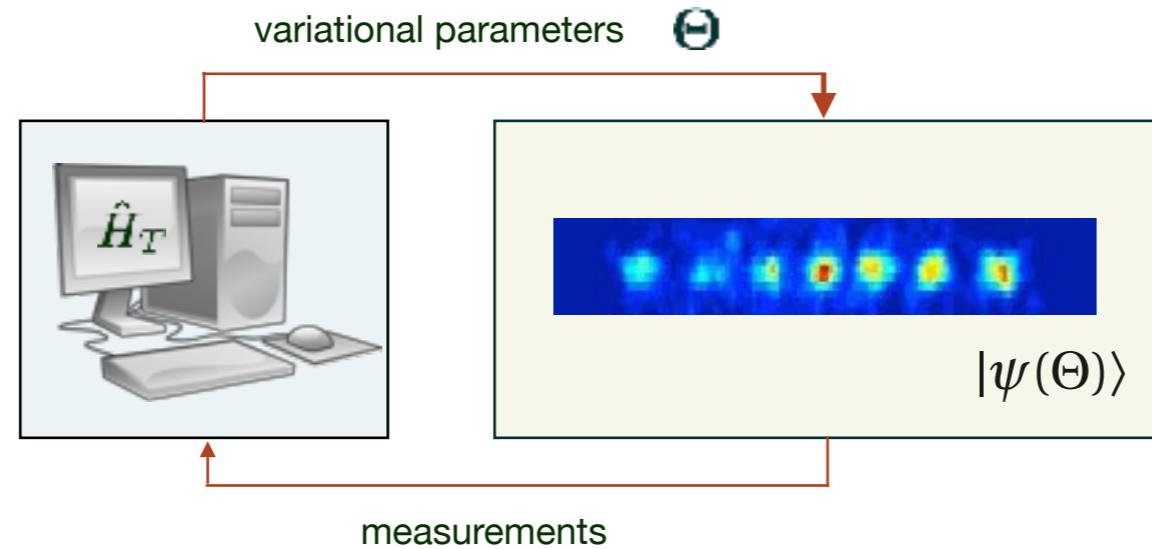


variational parameters
 $\Theta \equiv (\theta_1, \theta_2, \dots, \theta_N)$

depth of quantum circuit

Variational Quantum Simulation with Symmetry-Protecting Circuit

The Feedback Loop

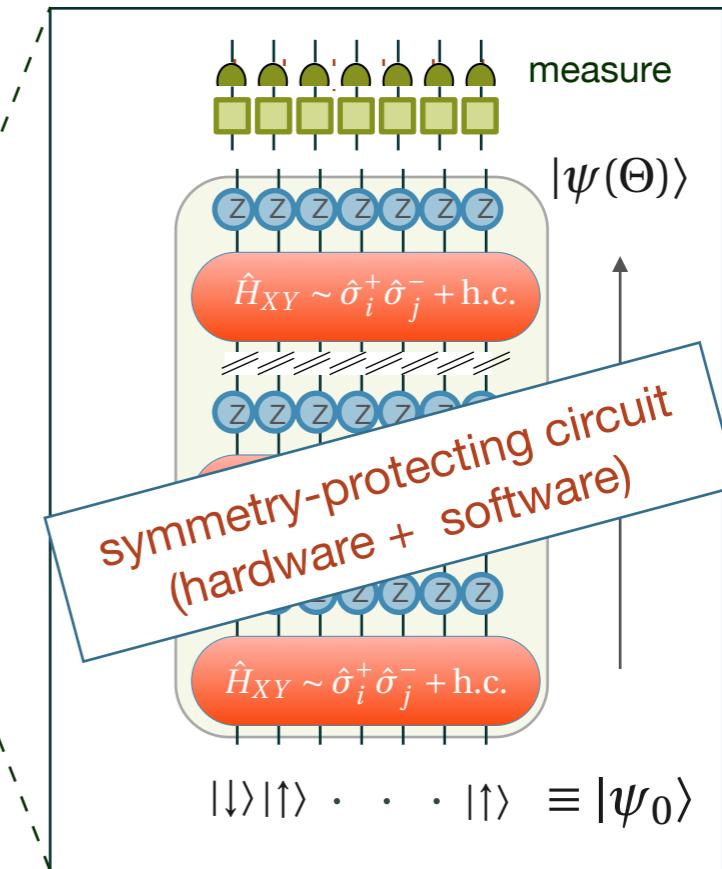


Schwinger Hamiltonian

$$\hat{H}_S = \dots$$

goal: prepare ground state (VQE)

$$\langle\psi(\theta)|\hat{H}_S|\psi(\theta)\rangle \rightarrow \min$$

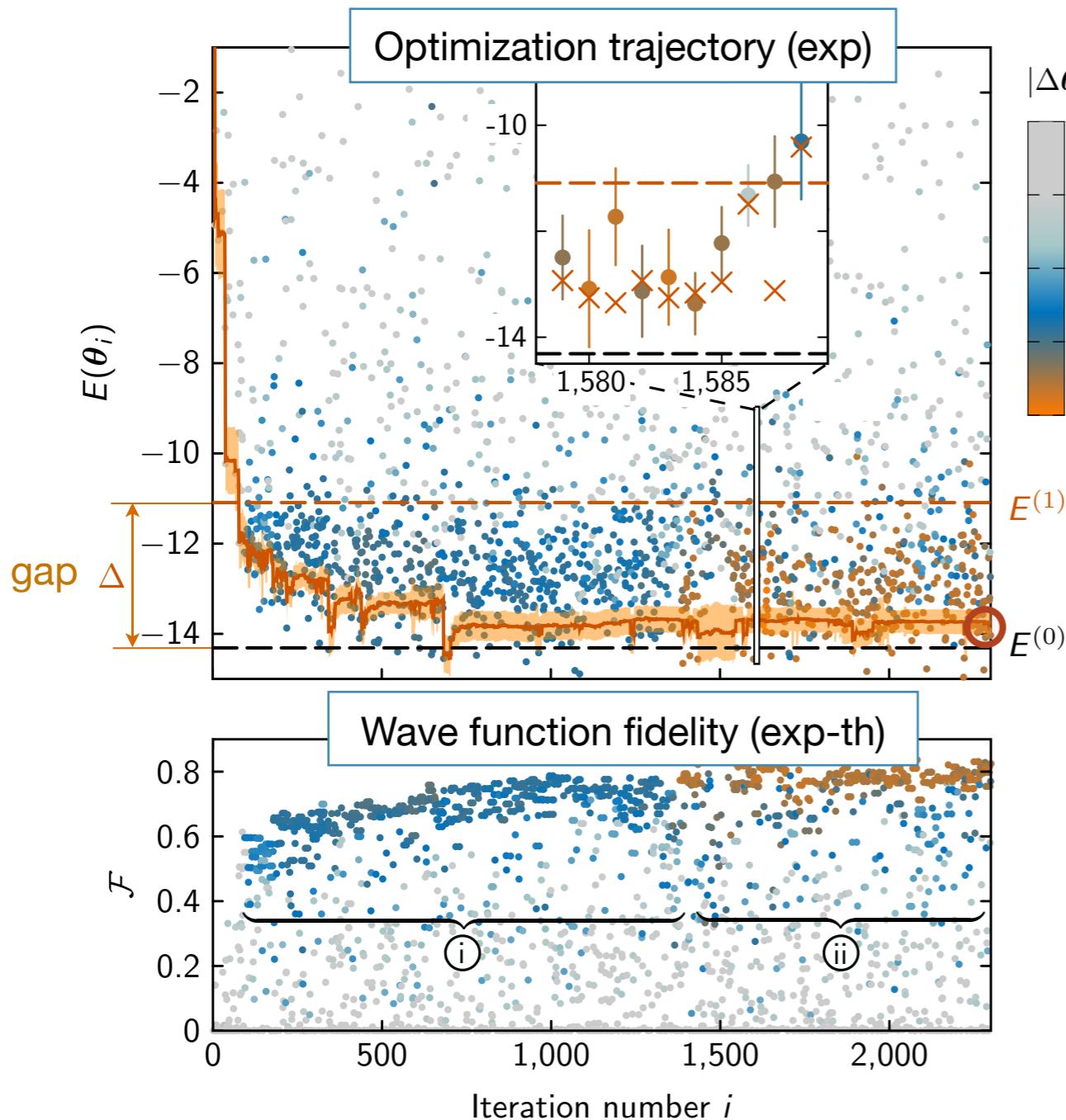


Symmetries of Schwinger Model

- total magnetization \longleftrightarrow q-hardware
- CP - symmetry \longleftrightarrow relation between variational parameters
- approx. translational

Optimization Trajectory for Schwinger Ground State

20 ions, 15 parameters, circuit depth = 6, budget: 10^5 calls to quantum simulator



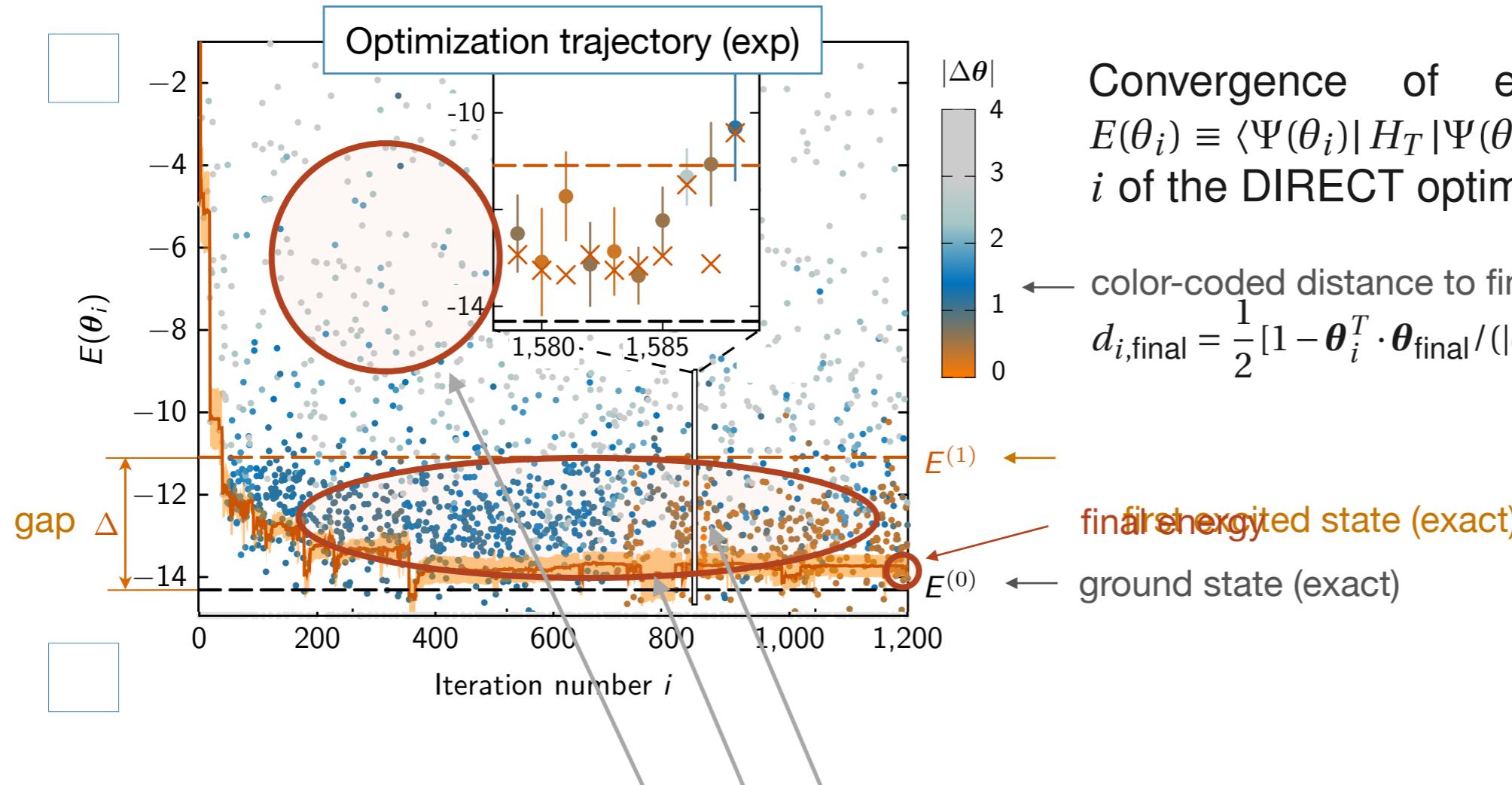
Wave function fidelity (exp-th)

C Kokail, C Maier, R van Bijnen,
T Brydges, MK Joshi, P Jurcevic, CA Muschik, P Silvi,
R Blatt, CF Roos & P. Z., Nature 2019

$m=0.9, w=\bar{g}=1$

Optimization Trajectory for Schwinger Ground State

20 ions, 15 parameters, circuit depth = 6, budget: 10^5 calls to quantum simulator



Convergence of experimental energies
 $E(\theta_i) \equiv \langle \Psi(\theta_i) | H_T | \Psi(\theta_i) \rangle$ vs. iteration number
 i of the DIRECT optimization algorithm

color-coded distance to final state
 $d_{i,\text{final}} = \frac{1}{2} [1 - \boldsymbol{\theta}_i^T \cdot \boldsymbol{\theta}_{\text{final}} / (\|\boldsymbol{\theta}_i\| \|\boldsymbol{\theta}_{\text{final}}\|)]$

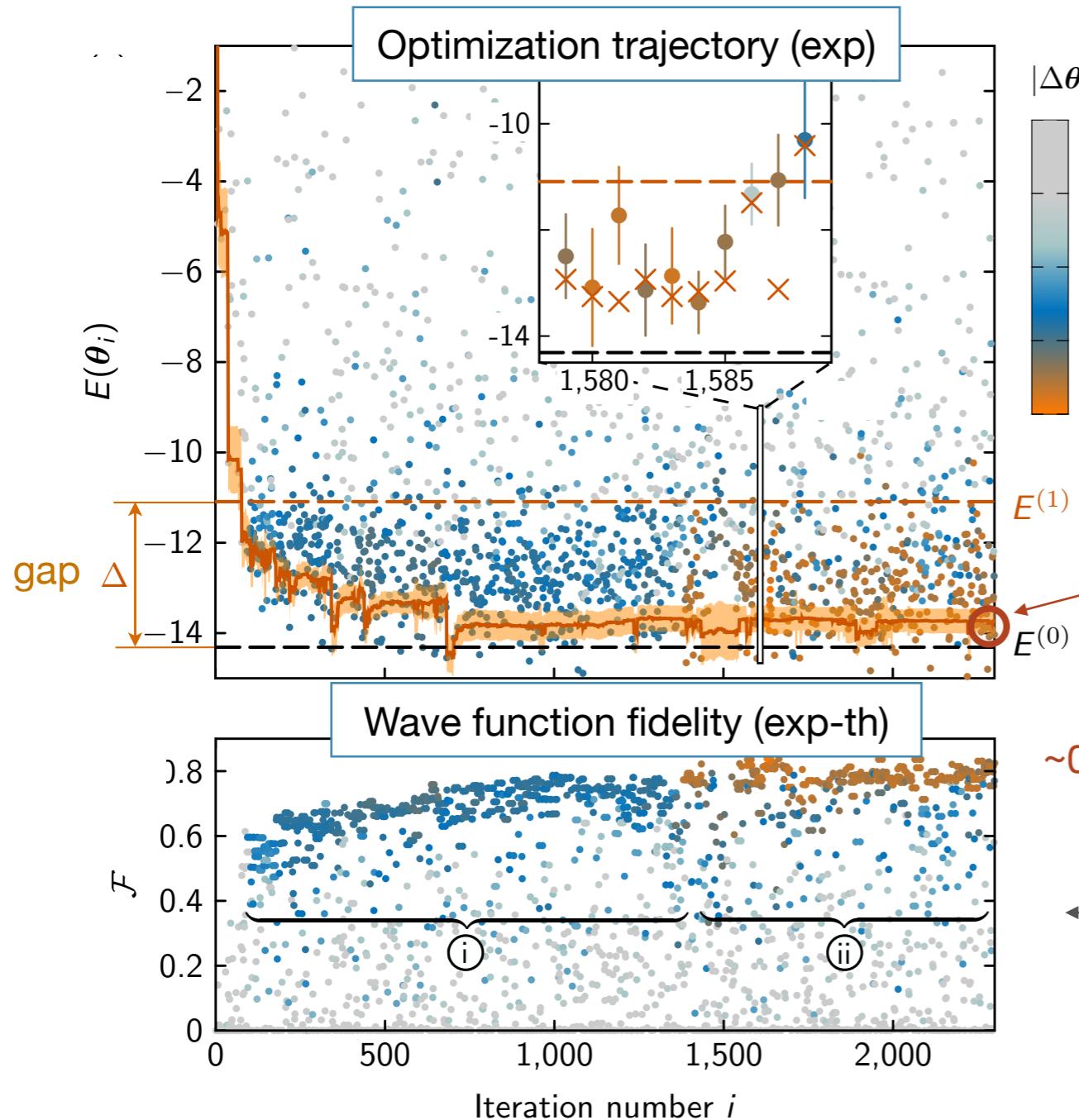
final excited state (exact)
ground state (exact)

exploration vs. refinement

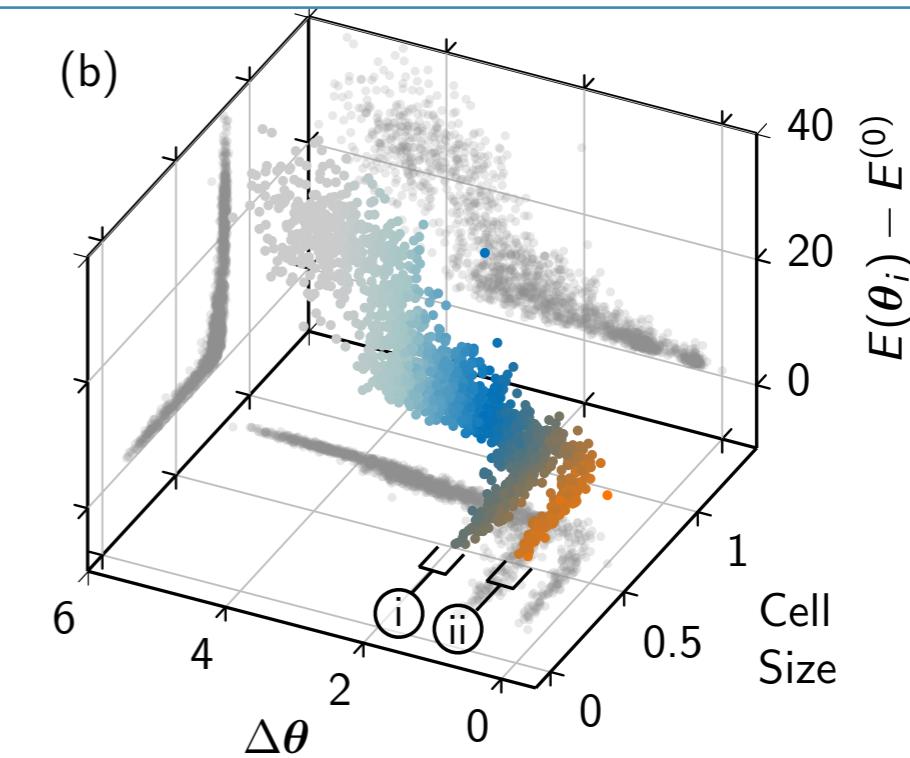
red line: algorithms current estimate

Optimization Trajectory for Schwinger Ground State

20 ions, 15 parameters, circuit depth = 6, budget: 10^5 calls to quantum simulator



Parameter optimization in noisy energy landscape



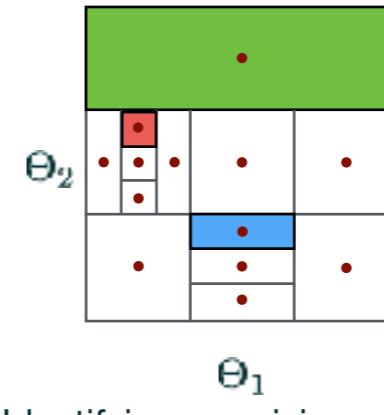
Theoretical fidelities $\mathcal{F} = |\langle \Psi_G | \Psi_{\theta}^{\text{Sim}} \rangle|^2$ computed for θ_i



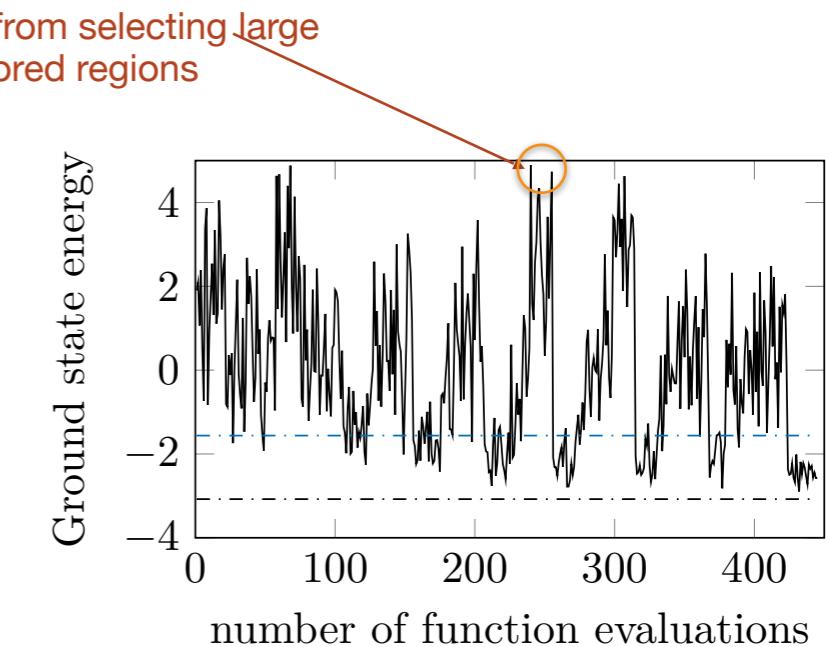
The Classical Optimization Algorithm (Overview)

Stochastic DIRECT search

- Global optimization problem with many local minima
 - Very noisy problem
 - We cannot use gradients
 - Optimization with error bars requires elements from decision theory: **Optimal Computational Budget Allocation (OCBA)**
- **D**IViding **R**ECTangles (**DIRECT**)
global optimization algorithm



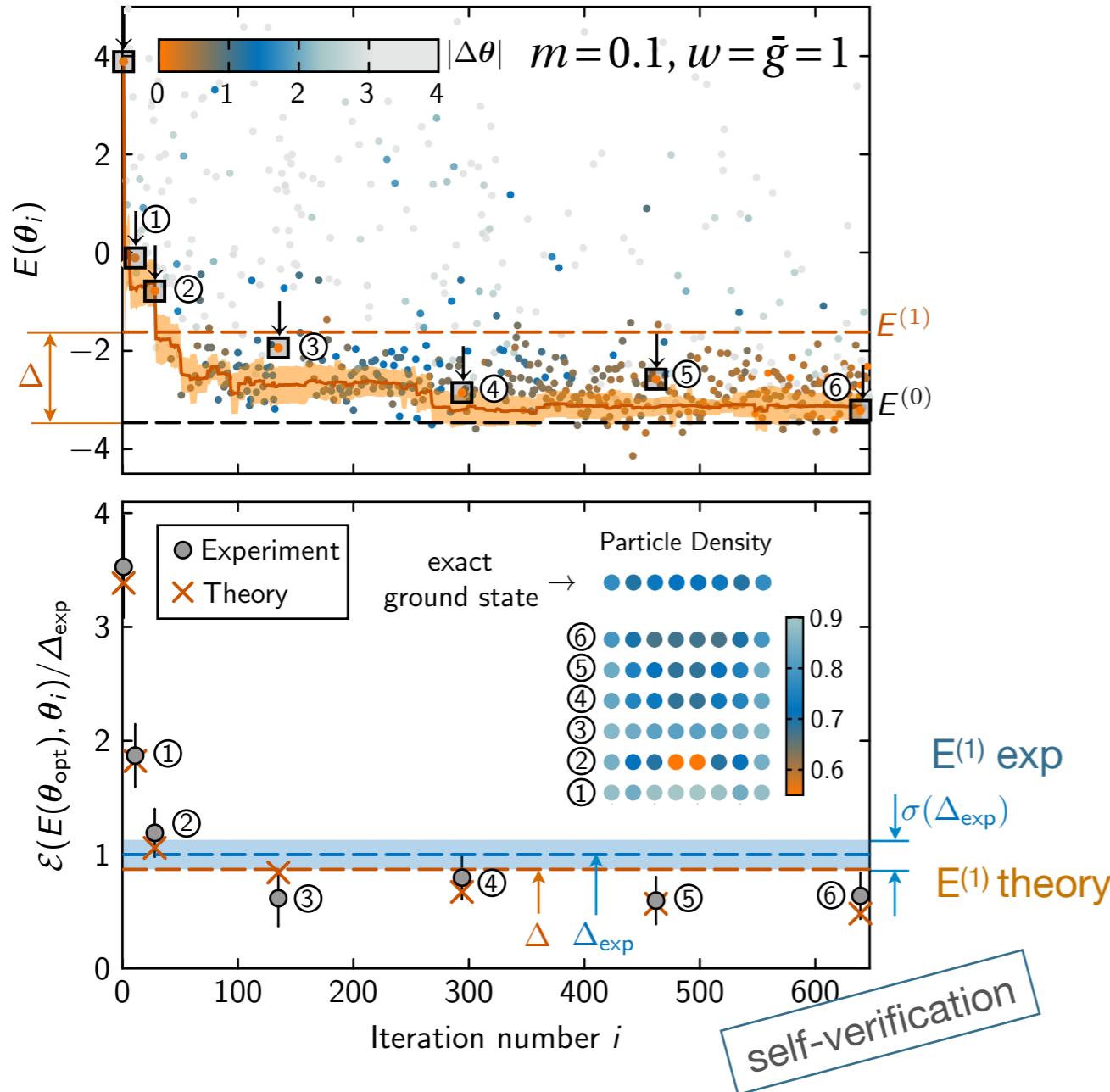
Θ_2
 Θ_1
Identifying promising regions in
a 2D search space



Jones et.al. Lipschitzian optimization without the Lipschitz constant. Journal of Optimization Theory and Applications, 79(1), 157-181. (1993)

Measurement of Error Bars ['Algorithmic Error']

8 ions, 10 parameters



- we can not only compute the optimal energy

$$E_{\boldsymbol{\theta}}^{(0)} = \langle \Psi_{\boldsymbol{\theta}} | \hat{H}_T | \Psi_{\boldsymbol{\theta}} \rangle \rightarrow \min$$

and wave function,

- but also *measure* the error bar as energy variance

$$(\Delta E_{\boldsymbol{\theta}}^{(0)})^2 = \langle \Psi_{\boldsymbol{\theta}} | (E_{\boldsymbol{\theta}}^{(0)} - \hat{H}_T)^2 | \Psi_{\boldsymbol{\theta}} \rangle \geq 0$$

\sim
algorithmic error
(vs. projection noise)

$=0$ for eigenstate
expensive

and monitor convergence with # iterations,
and/or increasing depth of quantum circuit

Quantum Phase Transition in Schwinger Ground State

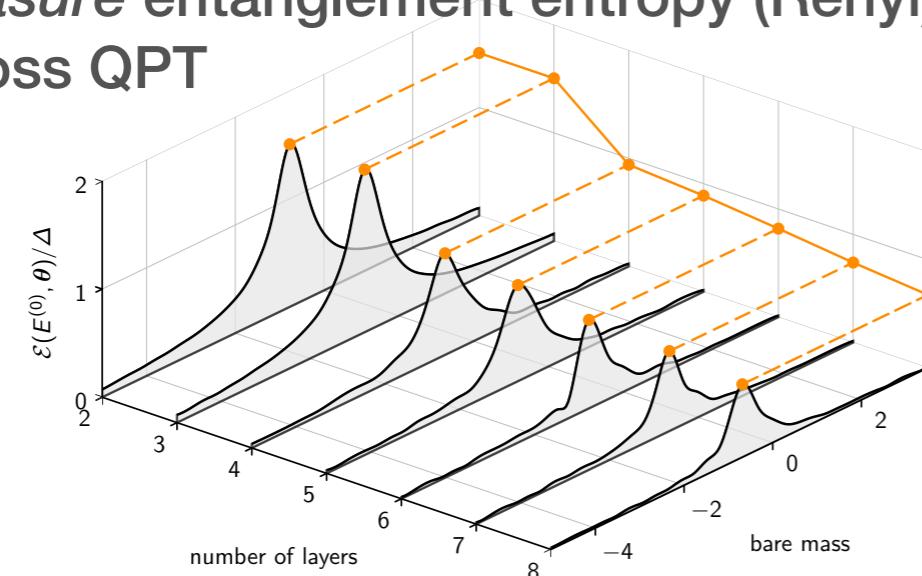
8 ions, 10 parameters

- order parameter

$$\langle \mathcal{O} \rangle \sim \sum_{i,j>i} \langle (1 + (-1)^i \sigma_i^z)(1 + (-1)^j \sigma_j^z) \rangle$$

$$-\infty < m < +\infty$$

- Measure entanglement entropy (Renyi) across QPT



test convergence with circuit depth

