

# Functional methods in QCD



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Humboldt Kolleg - Discoveries and open puzzles in particle physics and cosmology  
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# Elementary particles

Standard model of particle physics:

Elementary particles that make up the universe (or at least 5% of it)

	Materie (Fermionen)			Bosonen	
	I	II	III		
Quarks	u up	c charm	t top	$\gamma$ Photon	H Higgs Boson
	d down	s strange	b bottom	g Gluon	
Leptonen	$\nu_e$ Elektron- Neutrino	$\nu_\mu$ Myon- Neutrino	$\nu_\tau$ Tau- Neutrino	$Z^0$ Z Boson	Eichbosonen
	e Elektron	$\mu$ Myon	$\tau$ Tau	$W^\pm$ W Boson	

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Strong interaction: **Quarks** and **gluons** (u,d,c,s,t,b,g) described by quantum chromodynamics

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \sum_j \bar{\psi}_j [i \gamma^\mu D_\mu - m_j] \psi_j$$

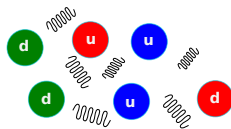
$$\text{WODEI} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$\text{UND} \quad D_\mu = \partial_\mu + igA_\mu$$

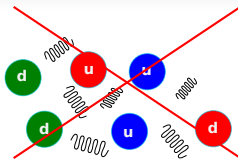
Images: Alexander Gorfer ([quant.uni-graz.at](mailto:quant.uni-graz.at)), (CC-BY-SA 4.0)

# Bound states of QCD

Quarks and gluons:



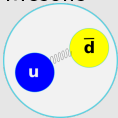
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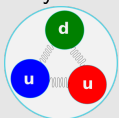
Quarks and gluons:

## Hadrons

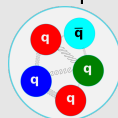
Mesons



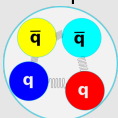
Baryons



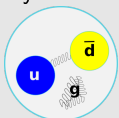
Pentaquarks



Tetraquarks



Hybrids



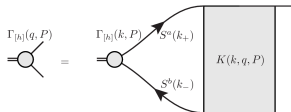
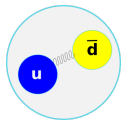
Glueballs



Calculate their properties?

# Hadronic bound states from bound state equations

Example: Meson

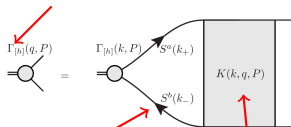
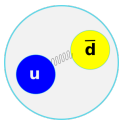


$$\text{Integral equation: } \Gamma(q, P) = \int dk \Gamma(k, P) S(k_+) S(k_-) K(k, q, P)$$

# Hadronic bound states from bound state equations

## Bethe-Salpeter amplitude

Example: Meson



$$\text{Integral equation: } \Gamma(q, P) = \int dk \Gamma(k, P) S(k_+) S(k_-) K(k, q, P)$$

Ingredients:

- Quark propagator  $S$
- Interaction kernel  $K$

$$S(p) = S_0(p) + \int dq \gamma_\mu S(q) D_{\mu\nu}(p-q) \Gamma_\nu(p, q)$$

Nonperturbative diagram: full momentum dependent dressings  
 → numerical solution

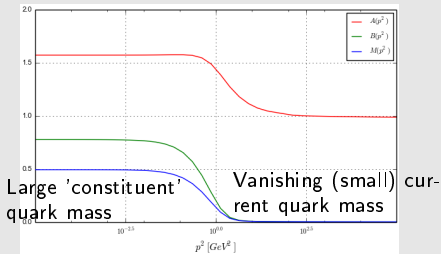
# Solving the quark gap equation

## Generic solution

Momentum dependent mass:

$$M(p^2) = B(p^2)/A(p^2)$$

→ Breaking of chiral symmetry creates mass.





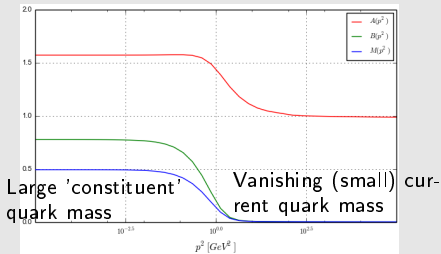
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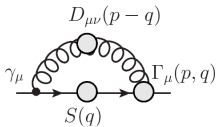
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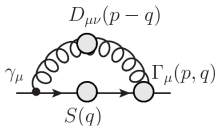
For given interaction and gluon propagator:

- Euclidean momenta: Student 'warm-up'
- Analytic behavior: Depends on input, tricky, open questions

# The elementary pieces: Bottom-up



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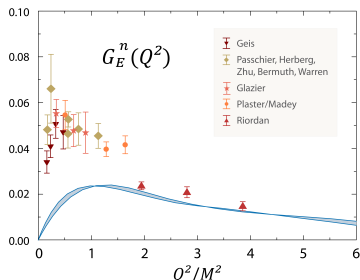
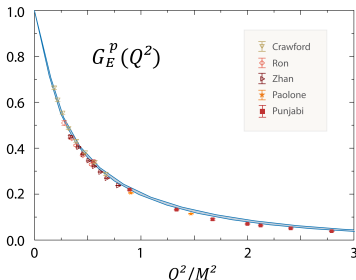
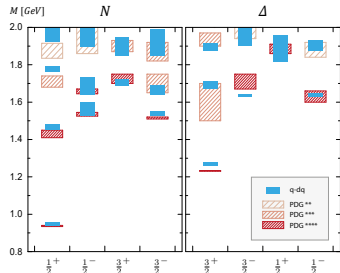
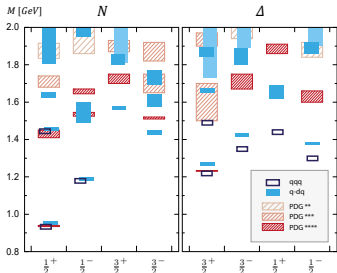
Use models crafted such that phenomenology comes out right.

Use symmetries as guidelines, e.g., chiral symmetry  $\rightarrow$  axial-vector WTl.

## Example

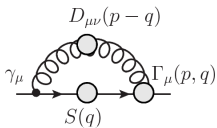
Effective interaction via  $g^2 D_{\mu\nu}(p) \Gamma_\mu(p, q) \rightarrow Z_2 \tilde{Z}_3 D_{\mu\nu}^{(0)}(p) \gamma_\mu \mathcal{G}((p + q)^2)$

# Bottom-up example: Baryons from rainbow-ladder

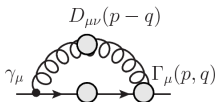


[Eichmann '11; Eichmann, Fischer, Sanchis Alepuz '16]

# The elementary pieces: Top-down



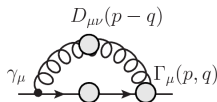
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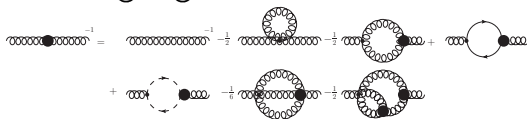
Gluon propagator  $D_{\mu\nu}(p^2)$ :

- Still tricky, normally truncated equation solved
- Untruncated equation (incl. two-loops) recently [Meyers, Swanson '14; MQH '17]

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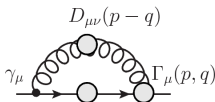
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Quark-gluon vertex  $\Gamma_\mu(p, q)$ :

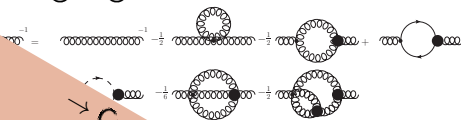


- Technically demanding, handful of results, e.g., [Hopfer, Windisch, Alkofer 13; Aguilar, Binosi, Papavassiliou '14; Mitter, Pawłowski, Strodthoff '14; Williams, Fischer, Heupel '15; Cyrol et al. '17; Aguilar, Cardona, Ferreira, Papavassiliou '18]

# The elementary pieces: Top-down

$$D_{\mu\nu}(p-q)$$


⇒ Gluon propagator



- Still tricky, normally truncated
- Untruncated equation (incl. two-loop) → Coupled Dyson-Schwinger equations [Meyers, Swanson '14; MQH '17]

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⇒ Understanding of pure QCD required (Yang-Mills theory).  
 → Couple to infinity of equations.  
 → Gluonic part is crucial.



# Yang-Mills theory

Consider quarks to be infinitely heavy.

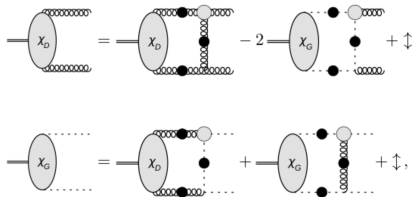
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Bound states of Yang-Mills theory: Glueballs

Similar bound state equation:



[Sanchis-Alepuz, Fischer, Kellermann, von Smekal '15]

Ingredients: Gluon and ghost propagators, gluonic vertices, interaction kernels

# Bottom-up vs. top-down

## Bottom-up:

- **Modeling** to describe certain quantities, symmetries as guiding principles
- Example: Rainbow-ladder truncation with Maris-Tandy interaction:

1 function, 2 parameters  
 $\mathcal{G}(k^2)$

→ Good description of, e.g., pseudoscalars

## Top-down:

- **9 dressings** for gluon propagator and quark-gluon vertex:  
 $D(k^2), \Gamma_i^{A\bar{q}q}(p, q, r), i = 1, \dots, 8$   
→ Technically complex
- Maximal flexibility  $\leftrightarrow$  consistency not easy to achieve
- Parameters of **QCD** only

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## Upgrades:

More parameters?

## Top-down:

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Include more terms of *known equations*.

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- Glueballs: Limited information for modeling (equivalent to Maris-Tandy interaction not known)

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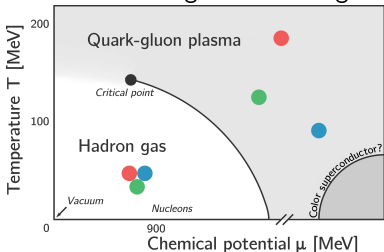
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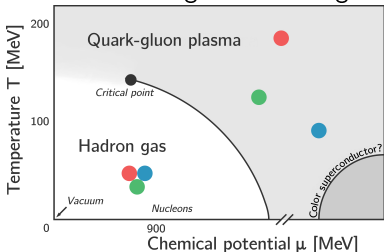
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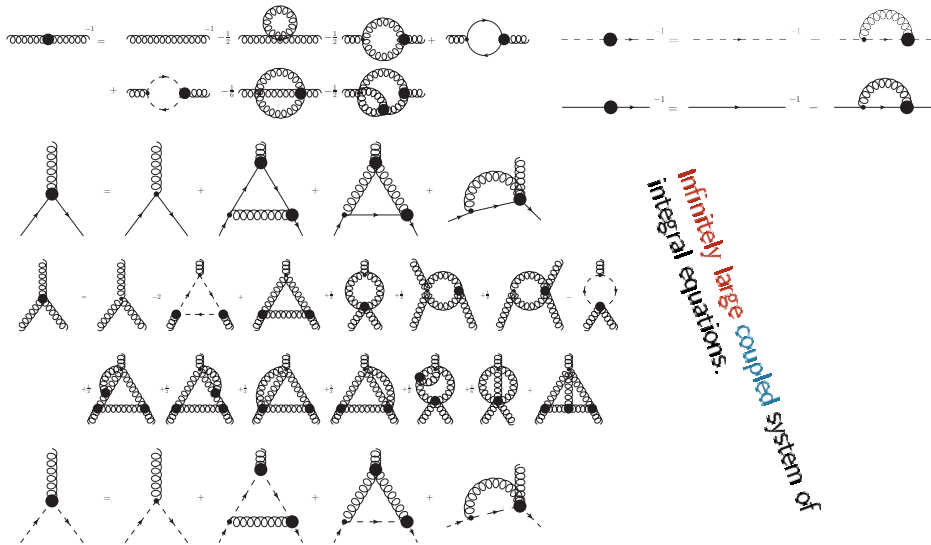


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Error estimation difficult!

# Dyson-Schwinger equations



**Infinitely large coupled system of integral equations.**

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- What is needed for specific problems?  
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- Systematics and tests?  
comparison to other methods? [self-tests](#)? necessary conditions?

# Perturbative resummation for propagators from DSEs

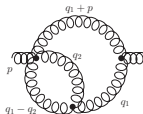
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Emergence from DSEs [MQH '17, '18]:

- Squint diagram (sunset has no  $g^4 \ln^2 p^2$ )
- Correct anomalous dimensions of three-point functions
- Correct renormalization (constants)



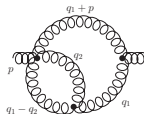
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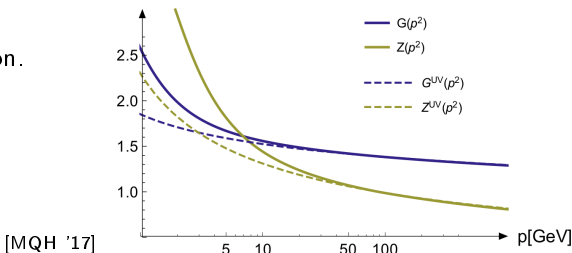
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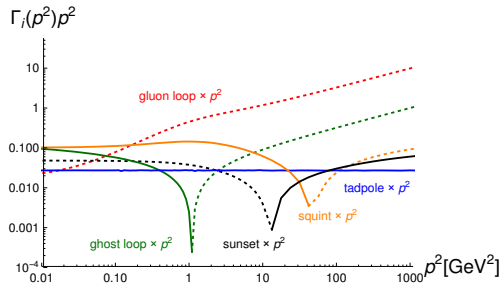


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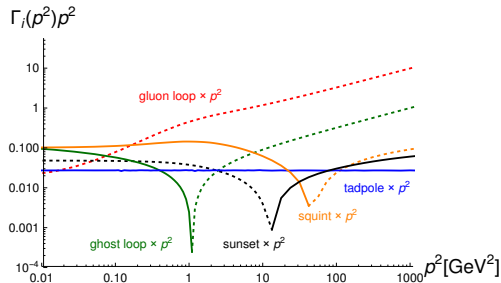
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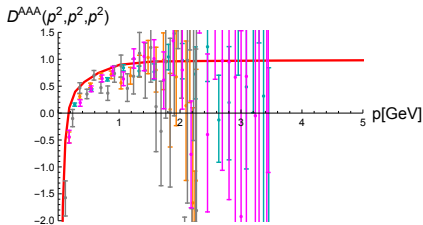
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- We cannot expect to have a clear hierarchy of diagrams, since we consider **all scales**.

Truncated DSEs *cannot* be assigned a concrete order of the coupling. They contain all contributions up to a certain order and some beyond.

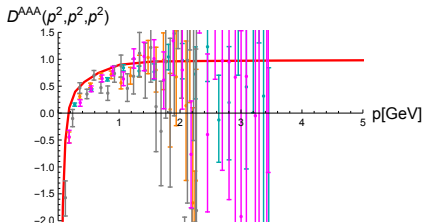
# Diagrams of the three-gluon vertex



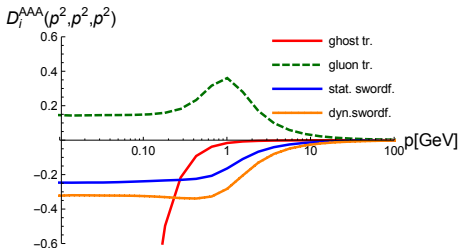
[lattice: Cucchieri, Maas, Mendes '08]

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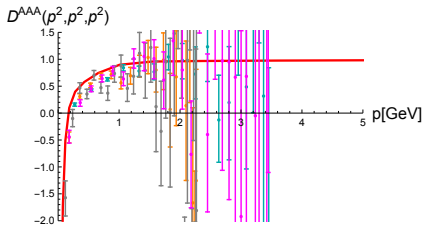


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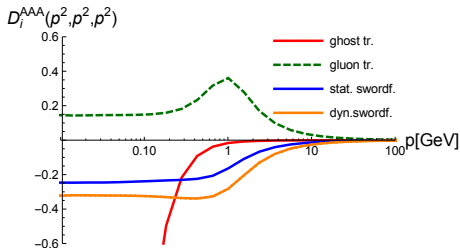
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→ In four dimensions similar qualitative effects, but renormalization complicates things.

# Three-gluon vertex DSE

[Cucchieri, Maas, Mendes '08; Alkofer, MQH, Schwenzer '09; Pelaez, Tissier, Wschebor '13; Aguilar et al. '13; Blum, et al. '14; Eichmann, Alkofer, Vujanovic '14; Cyrol et al. '16; Williams, Fischer, Heupel '16; Sternbeck '16; Athenodorou et al. '16; Duarte et al. '16; Boucaud et al. '17; Aguilar, Ferreira, Figueiredo, Papavassiliou '19]



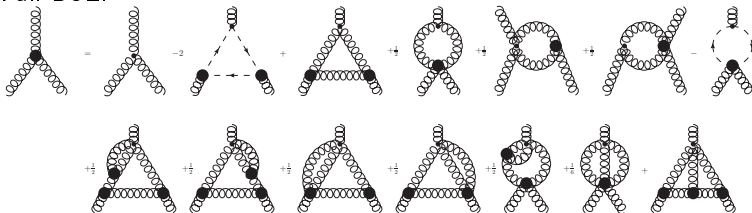




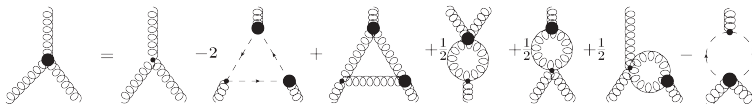
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Full DSE:

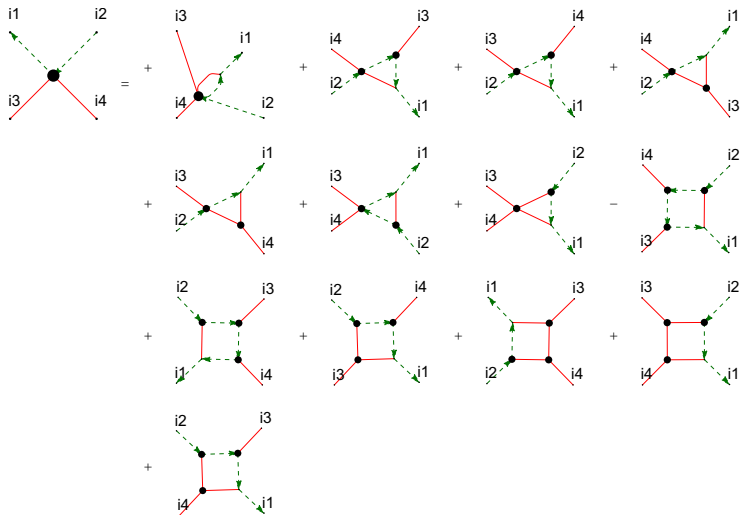


Nonperturbative one-loop truncation [MQH '17]:



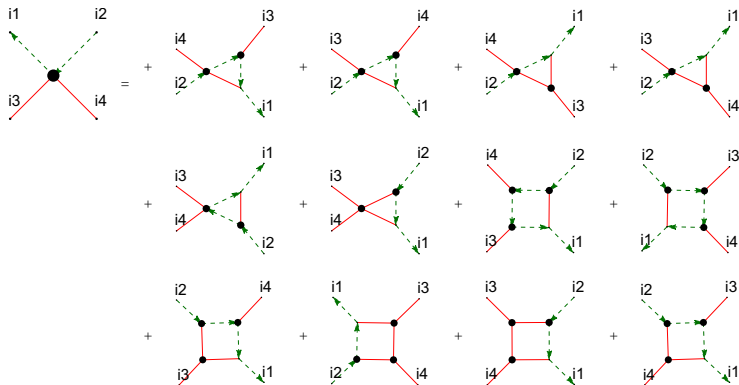
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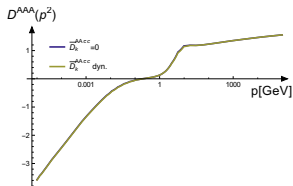
# Influence of two-ghost-two-gluon vertex



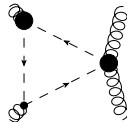
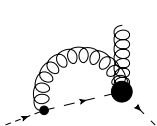
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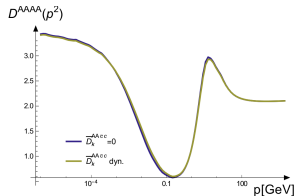
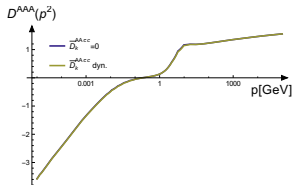
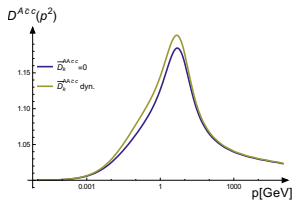
Coupled system of ghost-gluon, three-gluon and four-gluon vertices **with and without** two-ghost-two-gluon vertex [MQH '17]:



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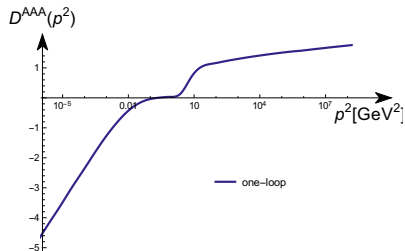


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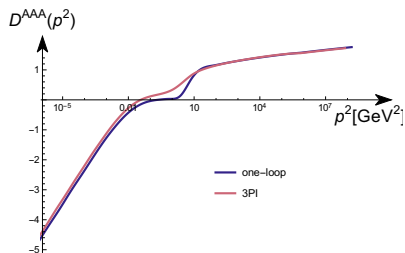


- Color structure: only small dressings in the three-gluon DSE  $\rightarrow$  no change.
- **Small** influence on ghost-gluon vertex ( $< 1.7\%$ )

# Three-gluon vertex: Equations and truncations

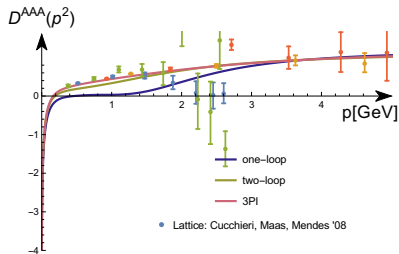
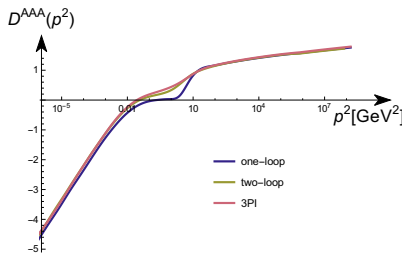


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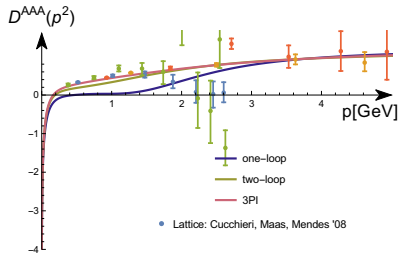
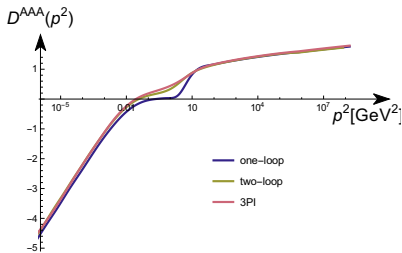




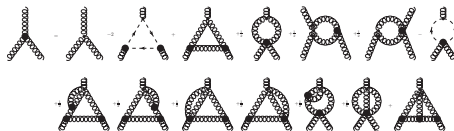
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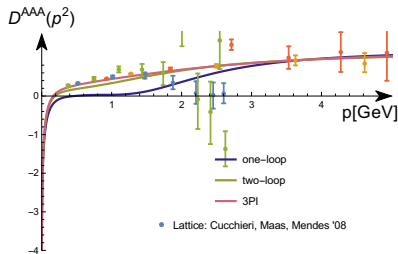
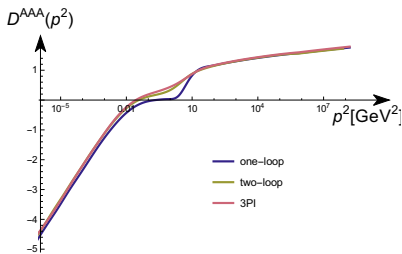
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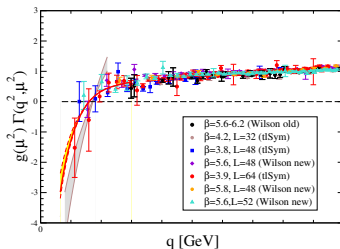
- **Two-loop truncation:** All diagrams except the one with a five-point function.
- One-momentum configuration approximation.



# Three-gluon vertex: Equations and truncations



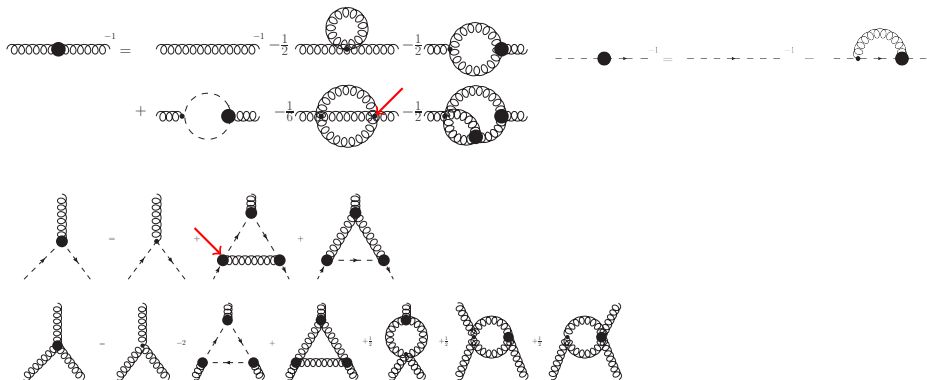
- Difference between two-loop DSE and 3PI smaller than lattice error.
- Zero crossing in agreement with other approaches, e.g., [Pelaez et al. '13; Aguilar et al. '13; Cyrol et al. '16; Athenodorou et al. '16; Duarte et al. '16; Sternbeck et al. '17]



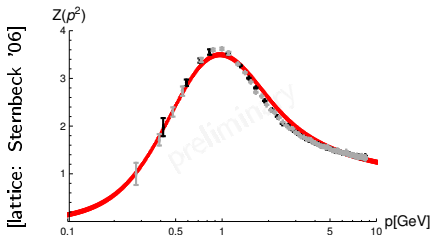
[Athenodorou et al. '16, '18]

# 3PI system of primitively divergent correlation functions

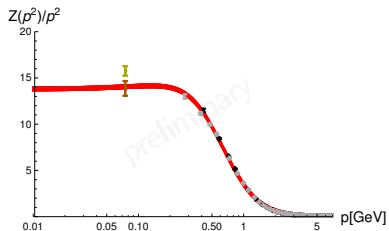
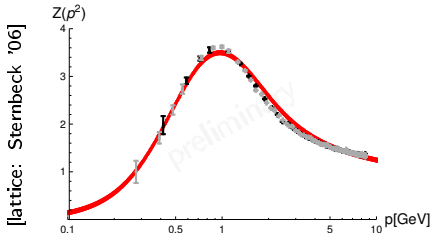
Three-loop expansion of 3PI effective action [Berges '04]:  
Expansion in dressed three-point functions



# Results for fully coupled 3PI system

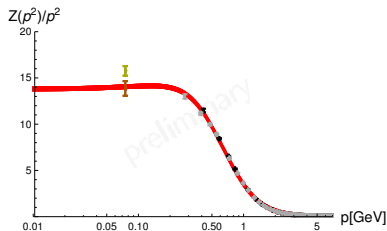
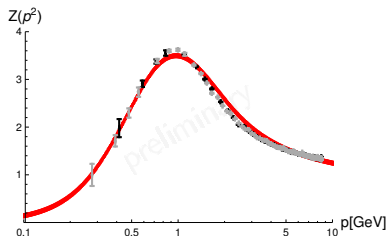


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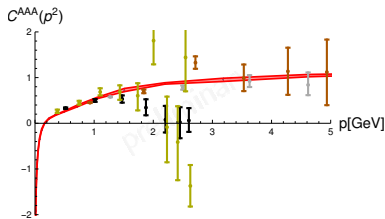


# Results for fully coupled 3PI system

[lattice: Sternbeck '06]



- Details of renormalization crucial!
- Very small angle dependence of three-gluon vertex.
- Slight bending down of gluon propagator in IR.

[lattice: Cucchieri,  
Maas, Mendes '08]

# Open checks

- Effects of larger tensor bases, in particular of the three-gluon vertex
- Renormalization

What tests can be done?



# Couplings

Couplings can be defined from every vertex, e.g., [Allés et al. '96; Alkofer et al., '05; Eichmann et al. '14]:

$$\alpha_{\text{ghg}}(p^2) = \alpha(\mu^2) (D^{A\bar{c}c}(p^2))^2 G^2(p^2) Z(p^2),$$

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- Must agree perturbatively (STIs).  
Important in coupled systems of functional equations. → Highly non-trivial **check of a truncation**  
[Mitter, Pawłowski, Strodthoff '14].
- Scales must match:  
 $\Lambda_{\text{QCD}}^2 = s e^{-\frac{1}{4\pi\alpha(s)\beta_0}}$ ,  $s$  pert. scale:  
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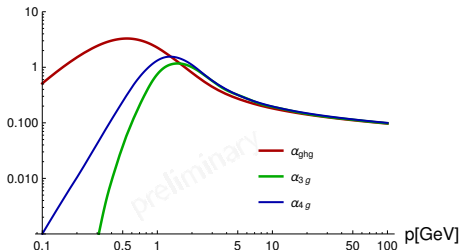
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Ghost-gluon vs. other couplings: Further checks required.

# Renormalization with a hard UV cutoff

Introduces quadratic divergences.

Note: Appears already **perturbatively!**

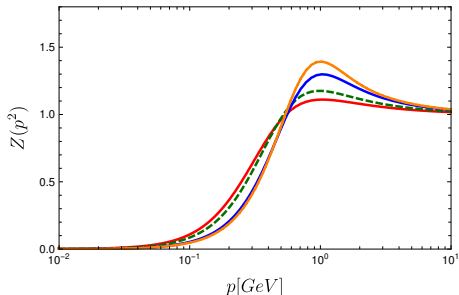
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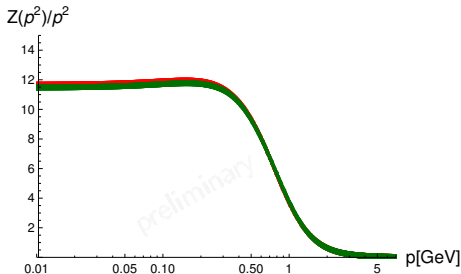
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The breaking of gauge covariance by the UV regularization leads to spurious (quadratic) divergences.

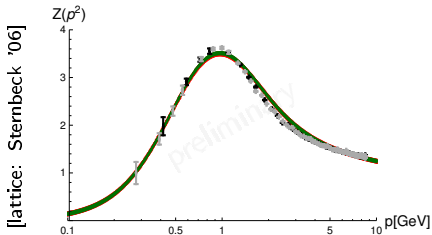
Extreme example: One-loop truncation with bare vertices in three dimensions [MQH '16].



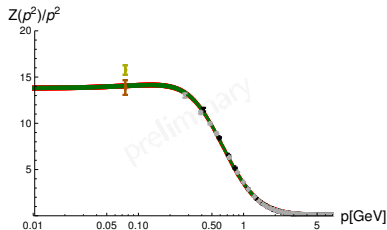
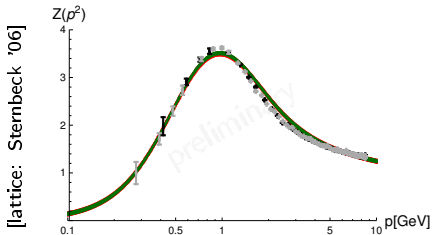
Better example: Full system with one-momentum configuration approximation.



# Results for fully coupled 3PI system revisited



# Results for fully coupled 3PI system revisited



- Two solutions on top of each other. **No model dependence anymore!**
- Provides a **self-test** of a truncation.

# Summary and conclusions

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Outlook and possibilities:

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|---|------------------|
| • Non-classical tensors in gluonic vertices | • Bound states   |
| • Add quarks                                | • Finite density |
| • Finite temperature                        | • <u>Fill in</u> |

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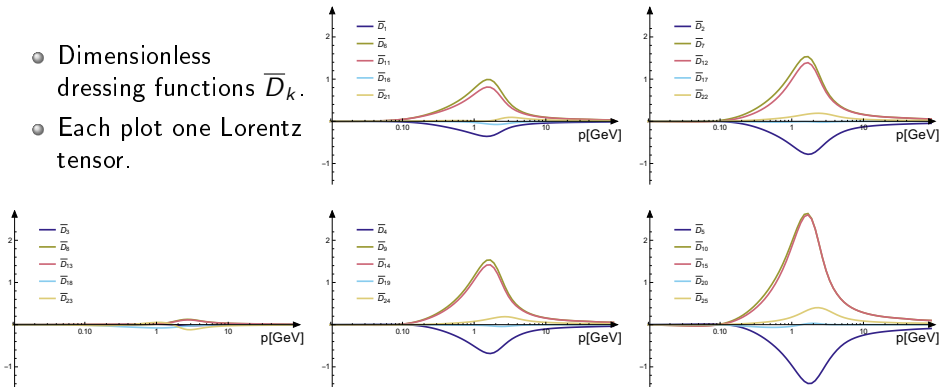
- Fill in

Thank you for your attention!

# Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions  $\bar{D}_k$ .
- Each plot one Lorentz tensor.



→ Two classes of dressings: 13 very small, 12 not small

→ No nonzero solution for  $\{\sigma_6, \sigma_7, \sigma_8\}$  found.

[MQH '17]