Two-loop corrections to μe scattering in QED

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Discoveries and Open Puzzles in Particle Physics and Gravitation - Kitzbühel June 25, 2019

Outline

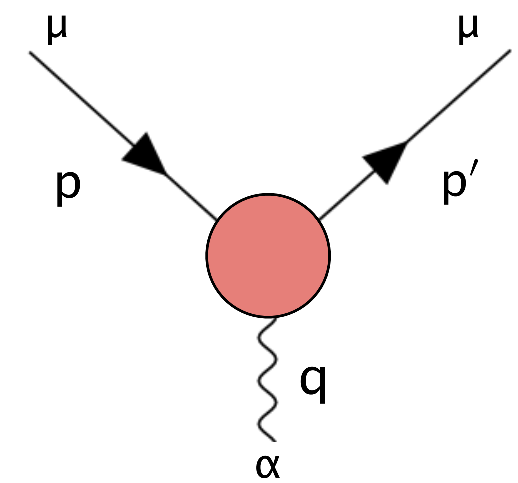
- Motivation: the muon (g-2) and the MUonE experiment
- Advances in NNLO QED corrections to μe-scattering
  - Double-virtual amplitude
  - Master integrals evaluation
- Conclusions and perspectives

based on: 1904.10964, 1806.08241, 1709.07435,

in collaboration with: S.Di Vita, T.Gehrmann, S.Laporta, M.Passera, P.Mastrolia, U.Schubert, W.Torres

The $(g-2)_\mu$

- Muon anomalous magnetic moment



$$\vec{m} = 2(1 + a_\mu)\frac{Qe}{2m_\mu}\vec{s}$$

- Experimentally measured at BNL

$$a_\mu^{exp} = 116\,592\,091\,(63) \times 10^{-11}$$ [E821 06]

  - E989 aims at 0.14ppm

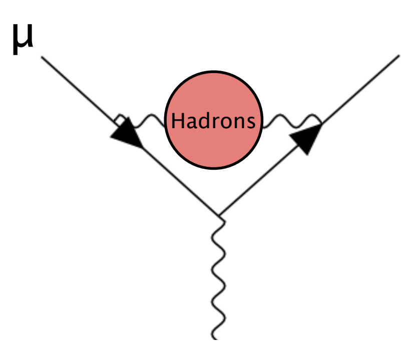
- Longest standing deviation from the SM prediction $(3.5-4)\sigma$

- Largest uncertainty from the hadronic contribution

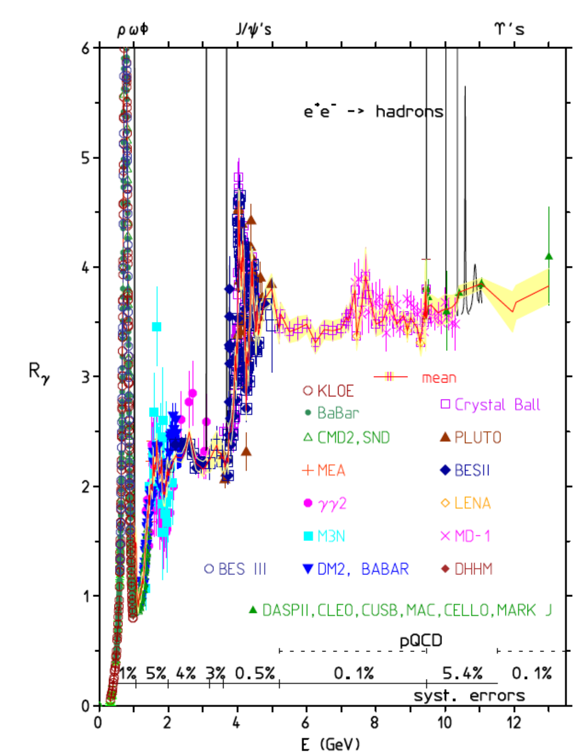
$$a_\mu^{HLO} = (6949.1 \pm 42.7) \times 10^{-11}$$ [Keshavarzi, Nomura, Teubner 18]

$$a_\mu^{HLO} = (6880.7 \pm 41.4) \times 10^{-11}$$ [Jegerlehner 17]

  - leading term from dispersive approach



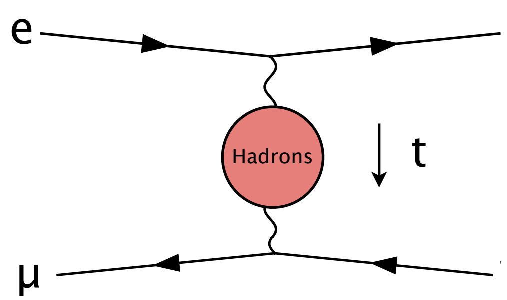
$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m^2}\,\sigma_{e^+e^-\to Had}(s)$$



[Jegerlehner 15]

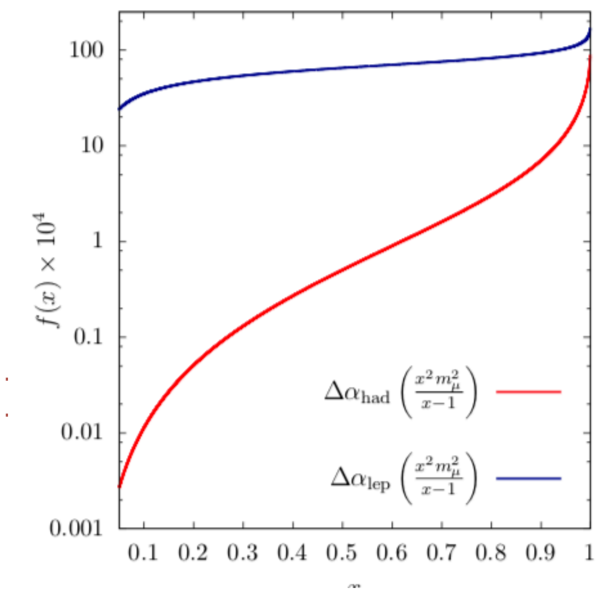
a_μ^{HLO} from μe scattering

- Alternative approach: a_μ^{HLO} from space-like data

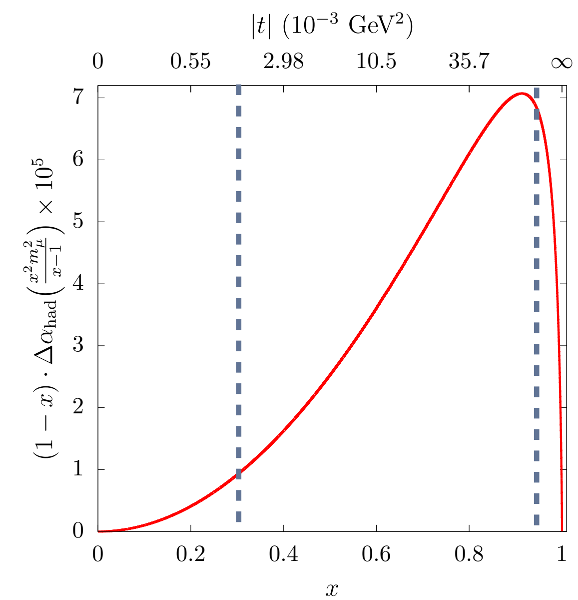


$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\Delta\alpha_{Had}[t(x)] \qquad t = \frac{x^2 m^2}{x-1} < 0$$

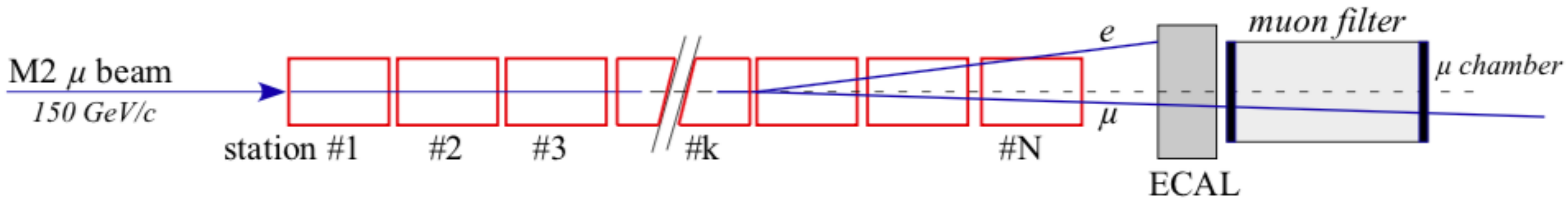
[Carloni Calame, Passera, Trantadue, Venanzoni 15]



- MUonE: proposal for a new experiment at CERN



  - 150 GeV μ-beam on atomic electrons
  - differential measurement of $d\sigma_{\mu e}$



[Abbiendi, Carloni Calame, Marconi et al 16]

[LOI, MUonE collaboration 19]

- Theory input: extract $\Delta\alpha_{Had}$ from the knowledge of the EW contribution to $d\sigma_{\mu e}$

MuonE: theory program

Goal: differential MC generator for the EW cross section

- target systematics: signal/normalization ≲ 10ppm

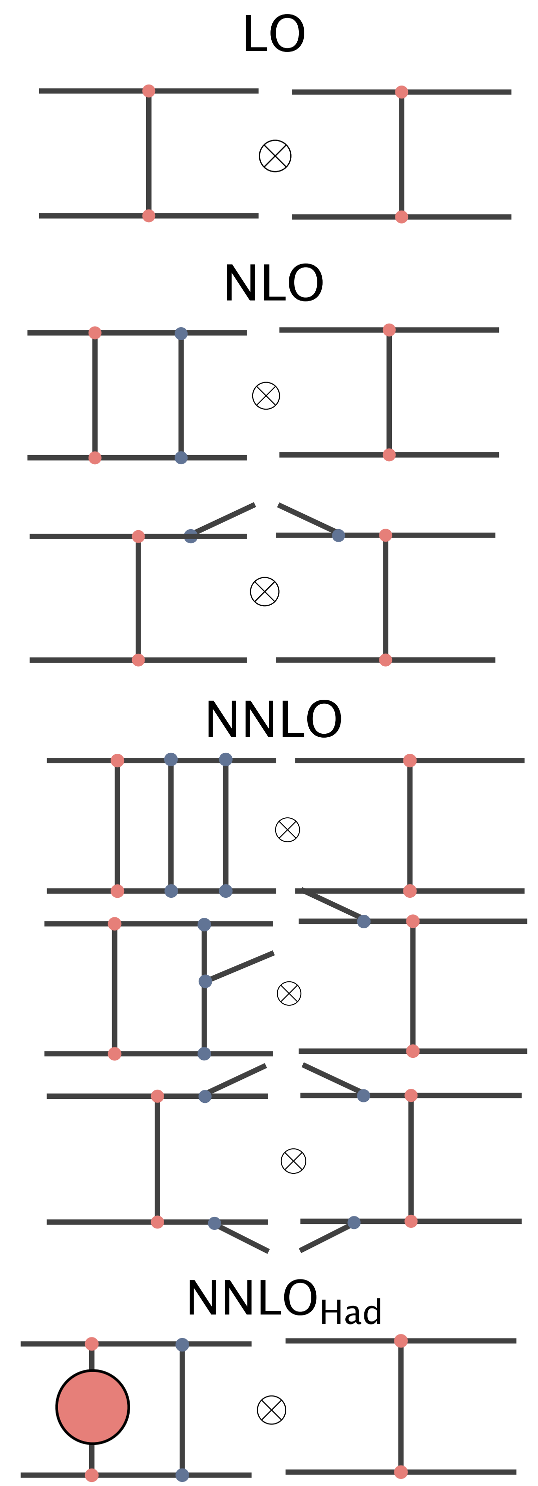
Bulk: fixed-order radiative corrections in QED

$$d\sigma = \underbrace{d\sigma^{(0)}}_{\text{LO}} + \underbrace{\alpha\, d\sigma^{(1)}}_{\text{NLO}} + \underbrace{\alpha^2 d\sigma^{(2)}}_{\text{NNLO}} + \underbrace{\alpha_i^3 d\sigma^{(3)}}_{\text{N}^3\text{LO}} + \mathcal{O}(\alpha_i^4)$$

- NLO QED+EW: available MC ($m_e \neq 0$)
  [Alacevich, Carloni Calame, Chiesa et al 19]
- NNLO QED: virtual matrix element (m_e=0)
  [Di Vita, Laporta, Passera, AP, Mastrolia, Schubert, Torres 17,18, xx]
  single and double radiation contributions
  [Fael, Mastrolia, Ossola, Passera, Signer,Torres, in progress]
- hadronic NNLO from space-like data [Fael, Passera 19]

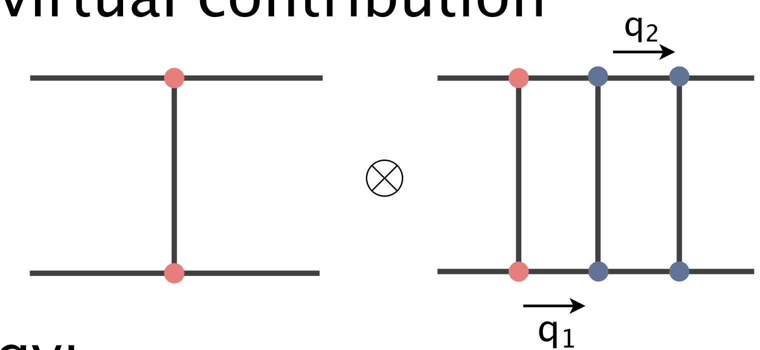
Physical matrix element

- "massification": leading m_e^2 effects at NNLO
- resummation of leading logarithmic effects



NNLO virtual amplitude

NNLO virtual contribution



Strategy:

$$\mathcal{M}^{(0)*}\mathcal{M}^{(2)} = \sum_k c_k(s,t,m^2) I_k(s,t,m^2)$$

- c_k: rational coefficients
- I_k: two-loop Feynman integrals

$$\mathcal{I} = \int d^d q_1 d^d q_2 \frac{1}{D_1^{a_1} \ldots D_n^{a_n}} \qquad D_j = l_j^2 - m_j^2$$

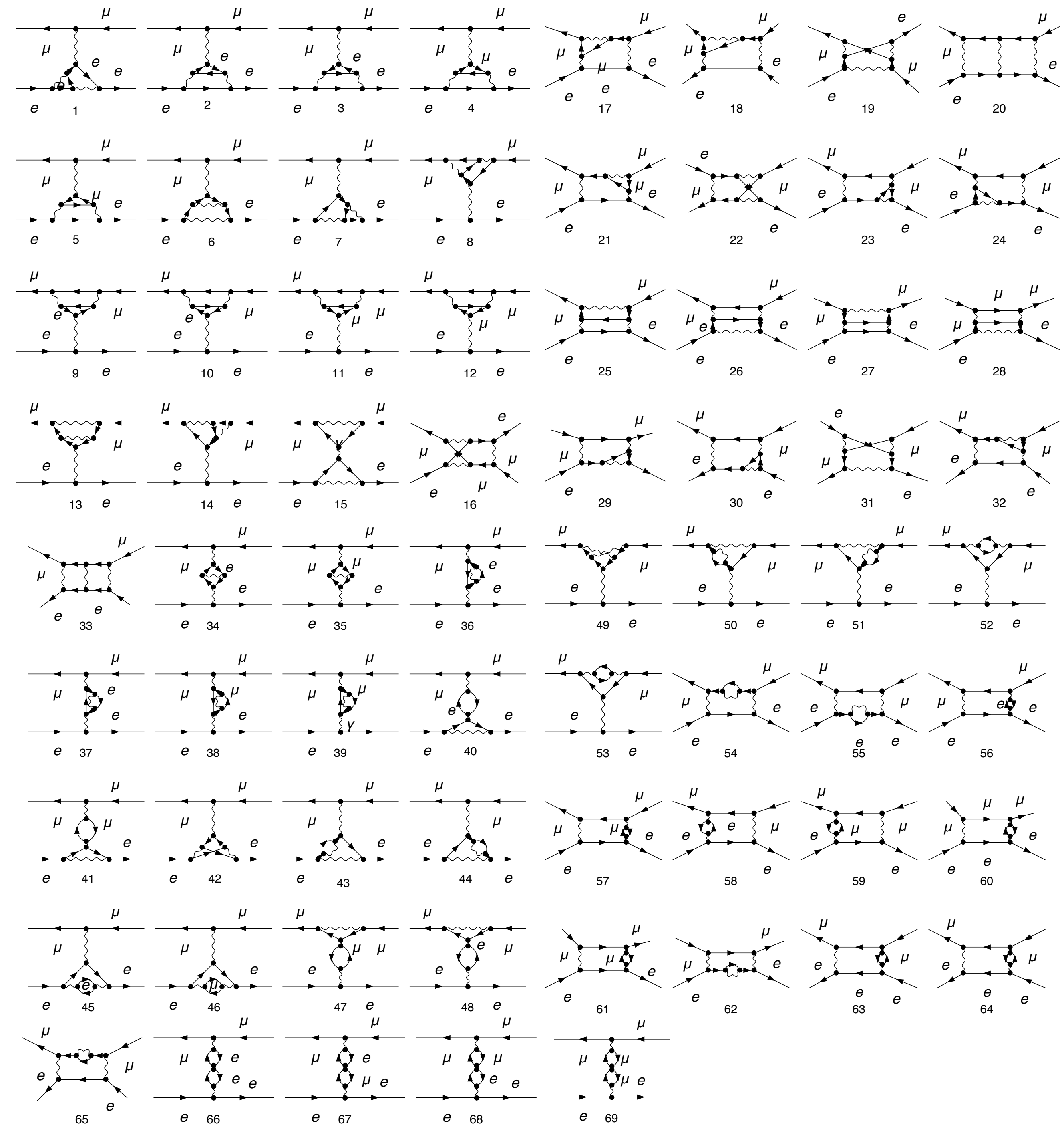
Goal:

- extract c_k for a minimal basis of I_k
- analytically compute the master integrals I_k

Assumptions:

- $m_e = 0$
- dimensional regularisation $d = 4 - 2\varepsilon$

$\mathcal{M}^{(2)}$



c_k: integration by parts

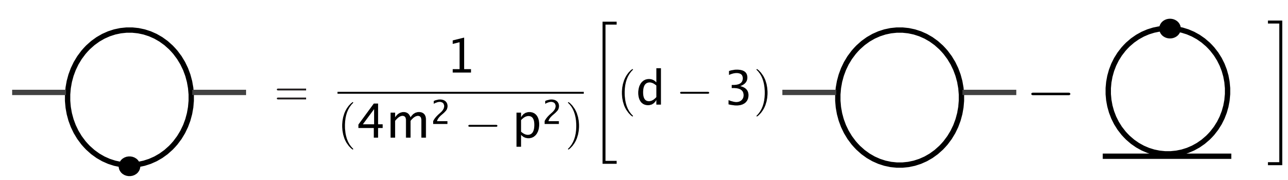
- Feynman integrals obey integration by parts

$$\int \prod_{j=1}^{\ell} d^d q_j \, \frac{\partial}{\partial q_i^\mu} \left(v^\mu \frac{1}{D_1^{a_1} \cdots D_n^{a_n}} \right) = 0 \qquad v^\mu \in \{q_i^\mu, p_i^\mu\}$$

[Chetyrkin, Tkachov 81]

- IBPs produce linear relations between different Feynman integrals

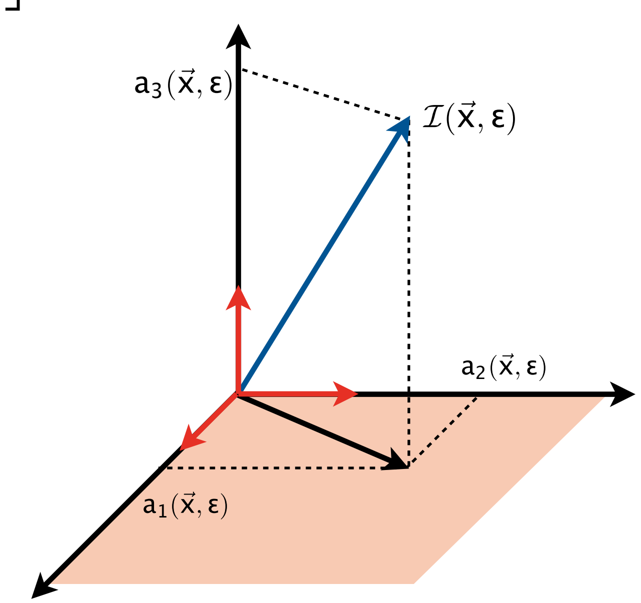
$$0 = \int \frac{\partial}{\partial q^\mu} \left(q^\mu \frac{1}{(q^2 - m^2)((q+p)^2 - m^2)} \right)$$
$$= (d-3) \int \frac{d^d q}{(q^2 + m^2)((q+p)^2 - m^2)} - p^2 \int \frac{d^d q}{(q^2 - m^2)^2} - (4m^2 - p^2) \int \frac{d^d q}{(q^2 - m^2)^2((q-p)^2 - m^2)}$$



- Only a finite number of integrals is independent

- Master integrals are a basis of the space of all $\mathcal{I}(\vec{x}, \varepsilon)$

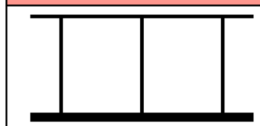
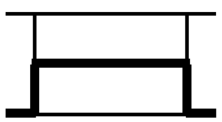
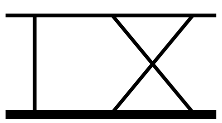
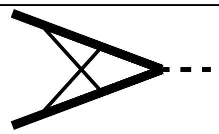
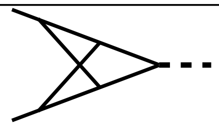
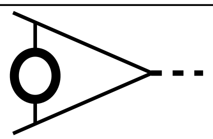
$$\mathcal{I}(\vec{x}, \varepsilon) = \sum_{i=1}^{N} a_i(\vec{x}, \varepsilon) I_i(\vec{x}, \varepsilon) \qquad \vec{x} = (x_1, x_2, \ldots, x_m)$$



NNLO amplitude decomposition

- Use IBPs to minimise the number of unknown integrals

$$\mathcal{M}^{(0)*}\mathcal{M}^{(2)} = \sum_{k=1}^{154} c_k(s,t,m^2,\varepsilon) I_k(s,t,m^2,\varepsilon)$$

Topology	# integrals in $\mathcal{M}^{(2)}$	# master integrals I_k	analytic expression
	2754	34 (+21 crossings)	known[1]
	2359	31 (+16 crossings)	unknown
	1128	19 (9 crossings)	unknown
	654	17	known[2]
	75	3	known[3]
	50	4	known[4]

1 [Bonciani, Ferroglia, Gehrmann et al 08]
[Bonciani, Ferroglia, Gehrmann, Studerus 09]
[Bonciani, Ferroglia, Gehrmann, et al 11-13]

2 [Bonciani, Mastrolia, Remiddi 03]

3 [Gonzalves 83, Kramer, Lampe 87]

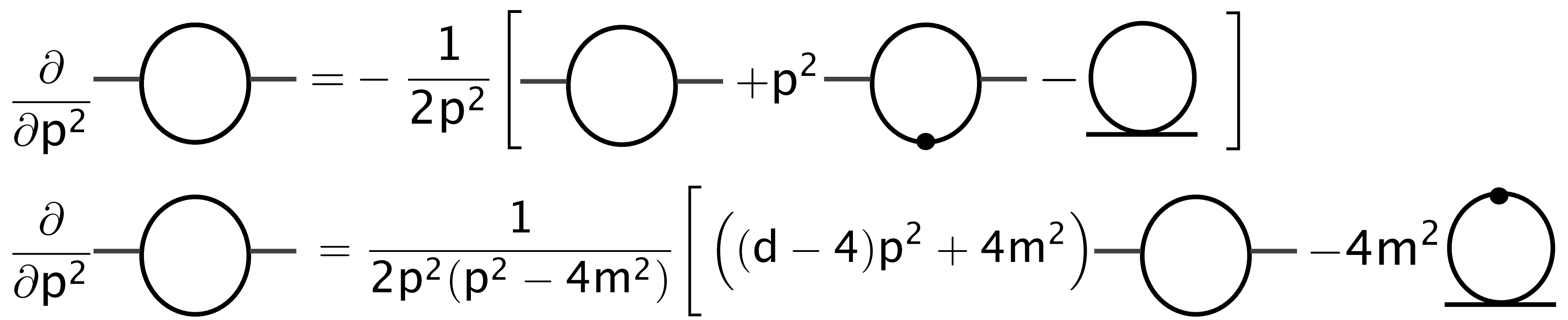
4 [Aglietti, Bonciani 04]

- How do we compute the master integrals?

I_k: differential equations

- Master integrals fulfil 1st order differential equations in the kinematics $\vec{x}$

$$\frac{\partial}{\partial p^2}\int \frac{d^dq}{(q^2-m^2)((q+p)^2-m^2)} = \frac{1}{2s}p^\mu\frac{\partial}{\partial p^\mu}\int \frac{d^dq}{(q^2-m^2)((q+p)^2-m^2)}$$



[Kotikov 91, Remiddi 97,Gehrmann, Remiddi 00 ...]

- Given a basis $\vec{I} = (I_1, I_2\,, \ldots, I_N)$ build closed systems of partial DEs

$$\frac{\partial}{\partial x_i}\vec{I}(\vec{x},\varepsilon) = \mathbf{A}_i(\vec{x},\varepsilon)\vec{I}(\vec{x},\varepsilon)$$

- Master integrals calculation: general solutions + initial conditions

I_k: differential equations

- Solve block-triangular systems of DEs

$$\frac{\partial}{\partial x_i}\vec{I}(\vec{x},\epsilon) = \mathbf{A}_i(\vec{x},\epsilon)\vec{I}(\vec{x},\epsilon)$$

$$\mathbf{A}_i(\vec{x},\epsilon) = \begin{pmatrix} * & & & & & \\ * & * & & & & \\ * & * & * & * & & \\ * & * & * & * & & \\ * & * & * & * & * & \\ * & * & * & * & * & * \end{pmatrix}$$

  - $\mathbf{A}_i(\vec{x},\epsilon)$ are rational in $\vec{x}$ and ϵ

- Master integrals determined by series expansion for $\epsilon \approx 0$

$$\vec{I}(\vec{x},\epsilon) = \sum_{k=0}^{\infty} I^{(k)}(\vec{x})\epsilon^k$$

- Systems of DEs are not unique:
  - change of variables $\vec{x} \to \vec{y}(\vec{x})$

$$\frac{\partial \vec{I}}{\partial y_i} = \left(\frac{\partial x_j}{\partial y_i}\mathbf{A}_j(\vec{y},\epsilon)\right)\vec{I}$$

  - change of basis $\vec{I} = \mathbf{B}(\epsilon,\vec{x})\vec{J}$

$$\frac{\partial \vec{J}}{\partial x_i} = \mathbf{B}^{-1}\left(\mathbf{A}_i\mathbf{B} - \frac{\partial \mathbf{B}}{\partial x_i}\right)\vec{J}$$

- Bottom-up solution simplified by a suitable choice of basis and variables

Canonical differential equations

Iterative structure manifest if DEs are ε–factorised

$$d\vec{I}(\vec{x},\varepsilon) = \varepsilon \left[\sum_{i=1}^{m} \mathbf{M}_i \, d\log \eta_i(\vec{x})\right] \vec{I}(\vec{x},\varepsilon)$$

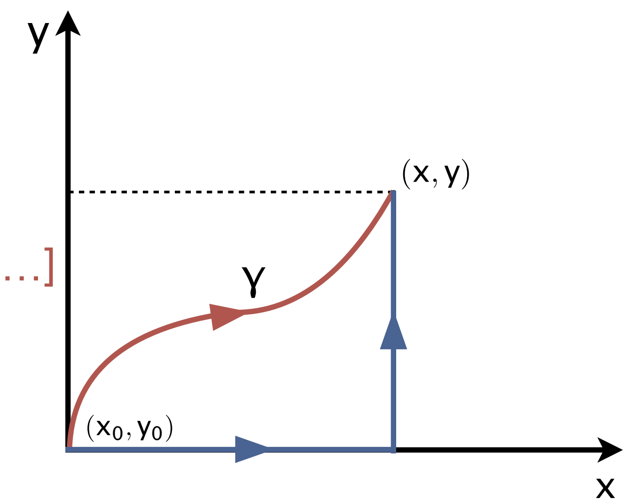
[Henn 13]

- DEs decouple order–by–order $\vec{I}^{(n)}(\vec{x}) = \sum_{k=0}^{n} \int d\mathbf{A} \ldots d\mathbf{A}\, \vec{I}^{(n-k)}(\vec{x}_0)$

For rational log–kernels: iterated integrals are multiple polylogs

$$G(\vec{\omega}_n;x) = \int_0^x \frac{dt}{t-\omega_1} G(\vec{\omega}_{n-1};t) \qquad G(\vec{0}_n;x) = \frac{1}{n!} d\log^n x$$

[Goncharov 98,Remiddi, Vermaseren 99, Gehrmann, Remiddi 00...]



canonical DE obtained trough the Magnus method

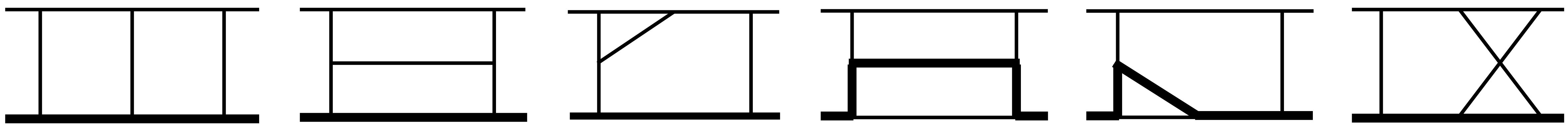
[Argeri, Di Vita, Mastrolia et al 14]

- starting point: $\partial_x \vec{I} = \left(\mathbf{A}^{(0)}(x) + \varepsilon \mathbf{A}^{(1)}(x)\right) \vec{I}$
- $\varepsilon = 0$ term exponentiate $\mathbf{B} = \exp\left(\Omega[\mathbf{A}^{(0)}](x)\right)$

$$\partial_x \vec{J} = \varepsilon \left(\mathbf{B}^{-1}(x)\mathbf{A}^{(1)}(x)\mathbf{B}(x)\right) \vec{J}$$

Master integrals for μe scattering

- Four-point integrals for μe-scattering



F_1 F_2 F_3 F_4 F_5 F_6

  - $F_{1,2,3}$ known
  - $F_{4,5,6}$ previously unknown

[Gehrmann, Remiddi 01, Bonciani Mastrolia Remiddi 04…]
[Bonciani, Ferroglia et al 08, Asatrian, Greub, Pecjak 08…]

- DEs in canonical form through the Magnus exponential

$$d\,\vec{I}(x,y,\varepsilon) = \varepsilon \sum_k M_k \, d\log \eta_k(x,y)\, \vec{I}(x,y,\varepsilon)$$

[Mastrolia, Passera, AP, Schubert 17]
[Di Vita, Laporta, Mastrolia, AP, Schubert 18]

- $F_{4,5}: \quad s = -m^2 x \qquad t = -m^2 \frac{(1-y)^2}{y}$

$$\eta_1 = x \qquad \eta_4 = y \qquad \eta_7 = x + y$$
$$\eta_2 = 1 + x \qquad \eta_5 = 1 + y \qquad \eta_8 = 1 + xy$$
$$\eta_3 = 1 - x \qquad \eta_6 = 1 - y \qquad \eta_9 = 1 - y(1 - x - y)$$

- $F_6: \quad \frac{u - m^2}{s - m^2} = -\frac{x^2}{y} \qquad t = -m^2 \frac{(1-y)^2}{y}$

$$\eta_1 = y \qquad \eta_5 = 1 - x \qquad \eta_9 = x^2 - y$$
$$\eta_2 = 1 + y \qquad \eta_6 = 1 + x \qquad \eta_{10} = 1 - y + y^2 - x^2$$
$$\eta_3 = 1 - y \qquad \eta_7 = x + y \qquad \eta_{11} = 1 - 3y + y^2 + z^2$$
$$\eta_4 = x \qquad \eta_8 = x - y \qquad \eta_{12} = x^2 - y^2 - xy^2 + x^2y^2$$

Master integrals for μe scattering

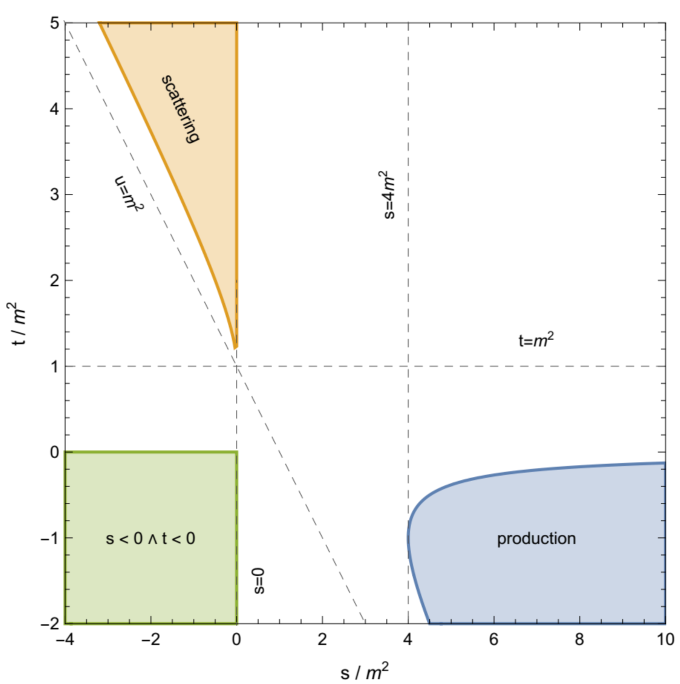
- Complete set of master integrals
  - general solution in terms of GPLs
  - analytic boundary conditions from regularity

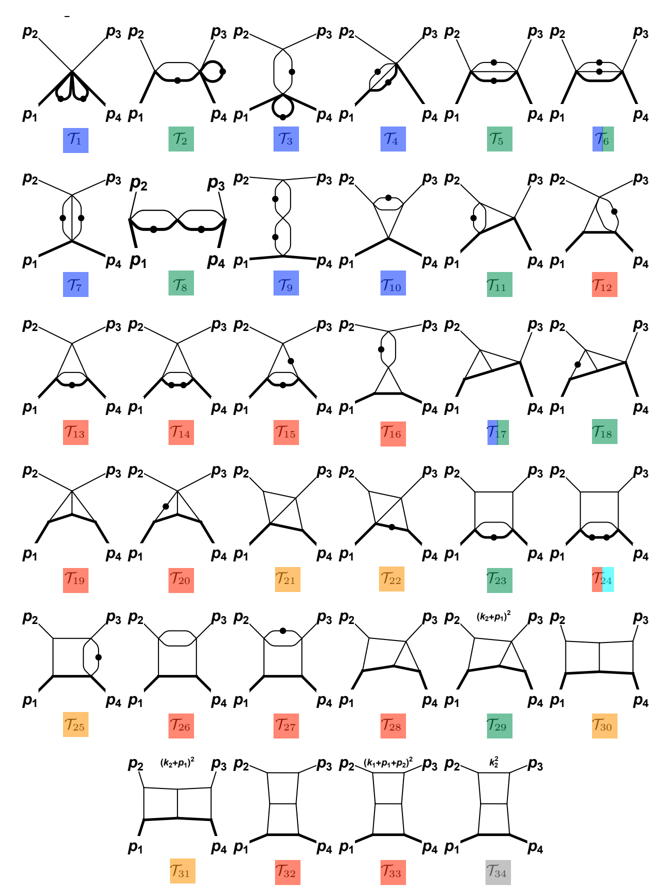
G_1
- Input
- $s \to 0$
- $t \to 4m^2$
- $u \to 2m^2$
- $s \to -m^2$
- $u \to \infty$

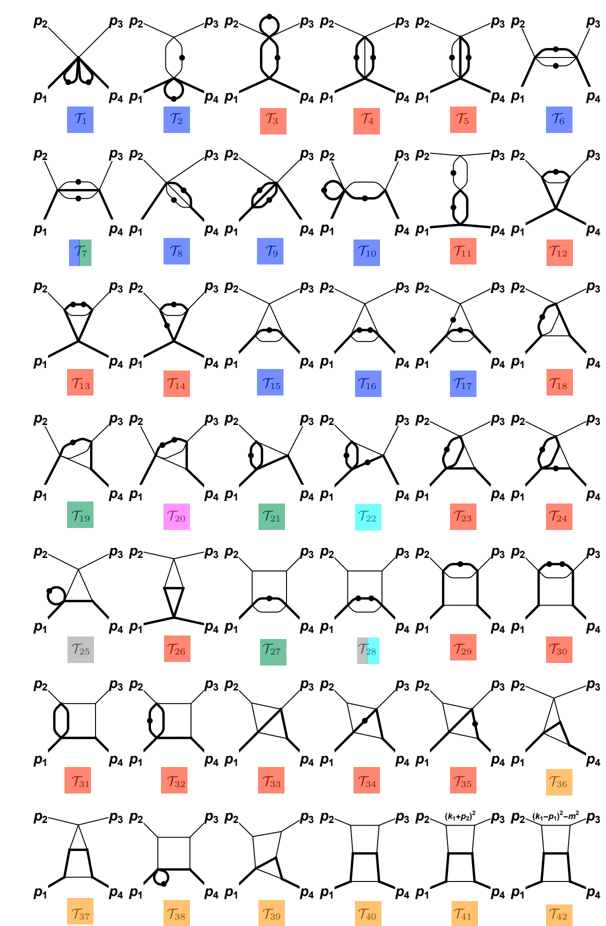
G_2
- planar
- $u \to 0$
- $t \to 0$
- $m^2 \to 0$
- $s \leftrightarrow u$
- $s \to \frac{\sqrt{4m^2 - t} - \sqrt{-t}}{\sqrt{4m^2 - t} + \sqrt{-t}}$

G_3
- Input
- $s \to 0$
- $t \to 0$
- $u \to m^2/2$
- $m^2 \to 0$
- $t \to 4m^2$
- $s \to 2t - m^2 - \lambda_t$

- analytic continuation physical regions

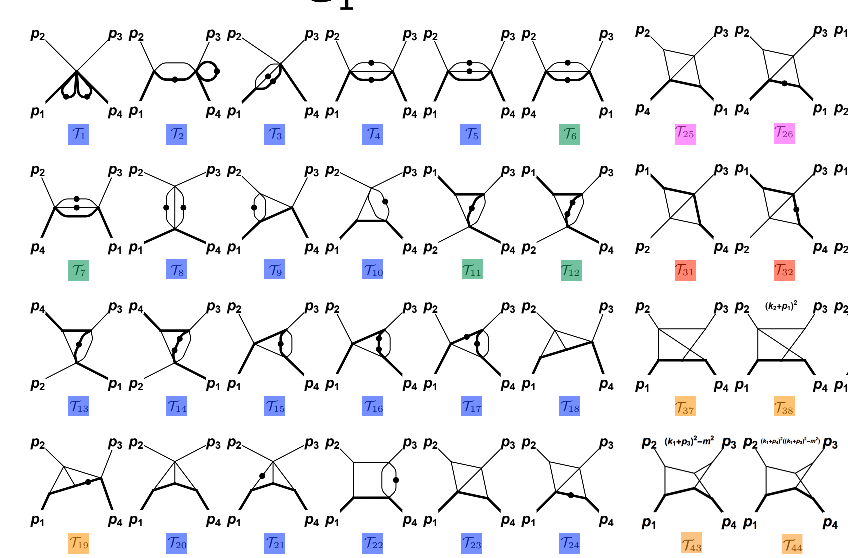






G_1

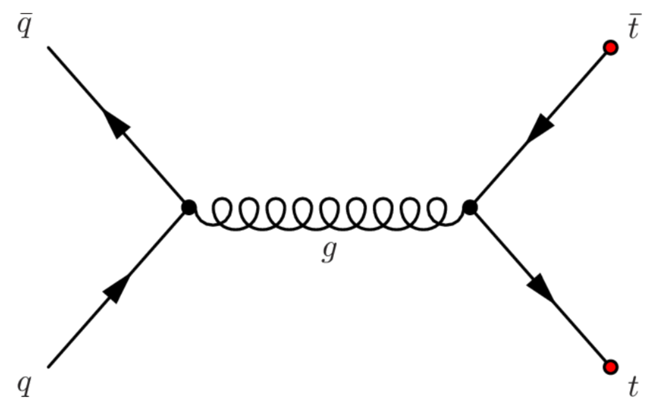
G_2



G_3

Heavy fermions production

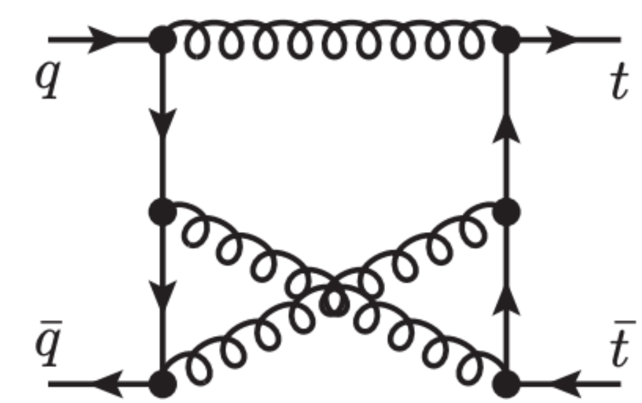
- Analytic continuation to production channels ($s>4m^2$)
  - low energy physics: $e^+e^- \to \mu^+\mu^-$
  - LHC physics: $q\bar{q} \to t\bar{t}$



- top-pair production in the light-quark annihilation channel at $\mathcal{O}(\alpha_s^4)$

$$\mathcal{M}^{(0)*}\mathcal{M}^{(2)} = N_c C_F \left[N_c^2 A + B + \frac{C}{N_c^2} + n_l \left(N_c D_l + \frac{E_l}{N_c} \right) + n_h \left(N_c D_h + \frac{E_h}{N_c} \right) + n_l^2 F_l + n_l n_h F_{lh} + n_h^2 F_h \right]$$

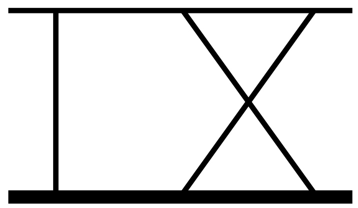
  - the full two-loop contribution known numerically [Czakon 08]
  - the A, D_i, E_i, F_i coefficients known analytically [Bonciani, Ferroglia, Gerhmann, Maitre, Studerus 08-09]
  - the B and C coefficients still unknown



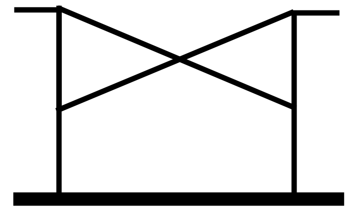
- Non-planar diagrams contribute to B and C

Master integrals for $q\bar{q} \to t\bar{t}$

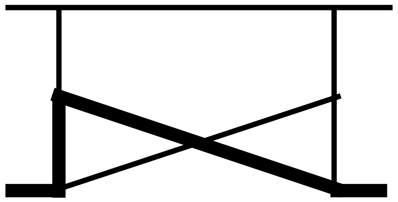
- Non-planar integrals for $q\bar{q} \to t\bar{t}$



F_6



F_7

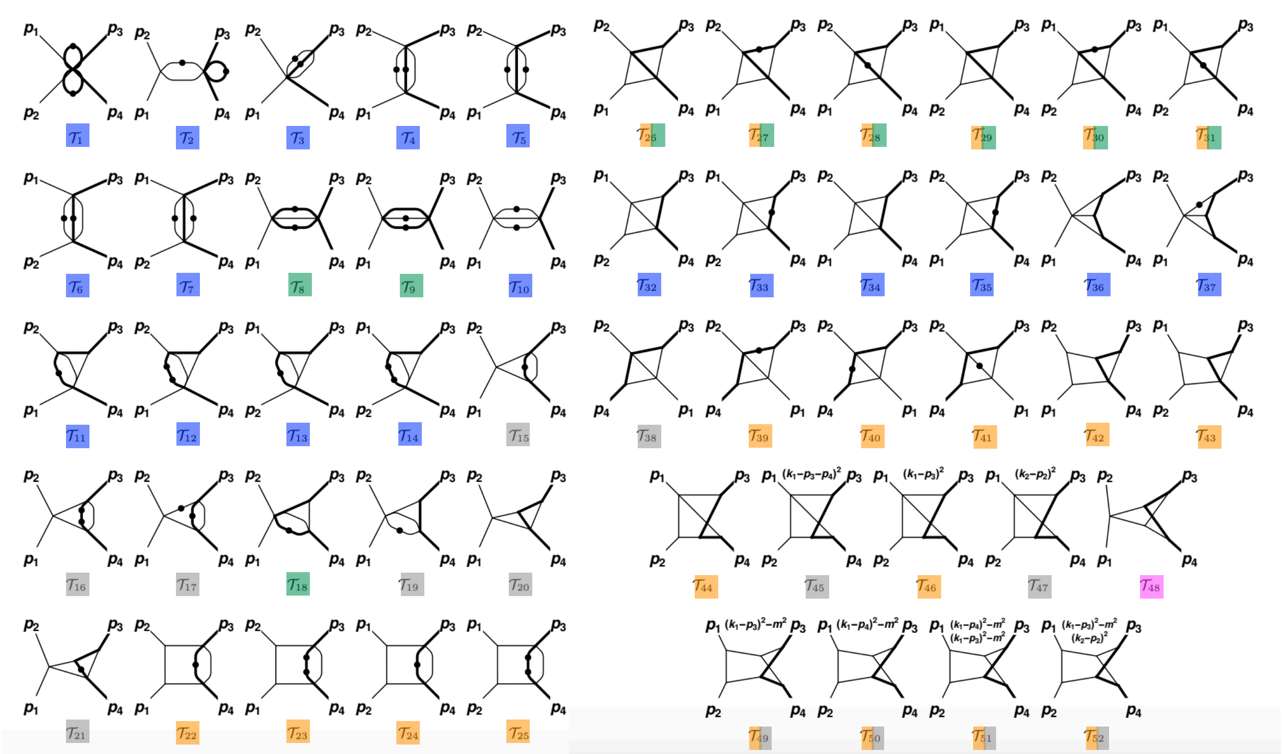


F_8

  - F_6 known from μe
  - F_7 known

[von Manteuffel, Studerus 14]

- Master integrals for F_6 computed in the same setup of μe scattering



from μe

$s \to 0$

$s \to 4m^2$

$t \to m^2 \frac{\sqrt{4m^2 - s} - \sqrt{-s}}{\sqrt{4m^2 - s} + \sqrt{-s}}$

$m^2 \to 0$

[Di Vita, Gerhmann, Laporta, Mastrolia, AP, Schubert 19]

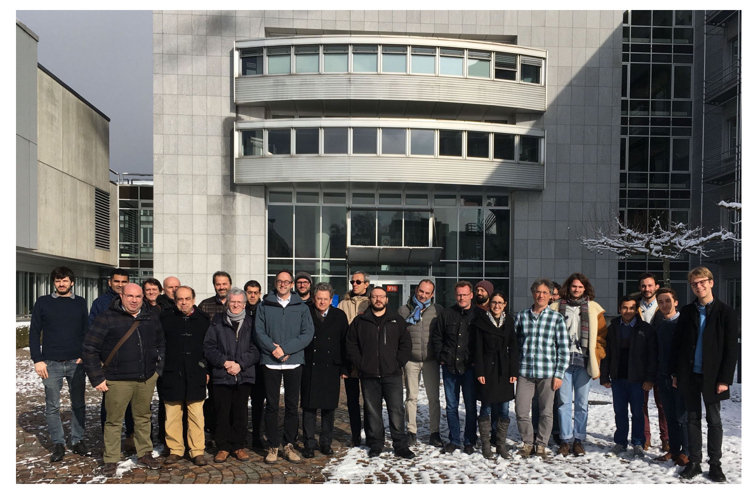
[Becchetti, Bonciani, Casconi Ferroglia, Lavacca, von Manteuffel 19]

Conclusions

- MUonE experiment: new independent assessment of a_μ
  - Letter of intent submitted this month to CERN
  - Plan: Pilot Run in 2021, data taking 2022-24
- Theoretical support:

Padova - Sept 2017

Mainz - Feb 18



Zürich - Feb 19

Mainz - 2020

Theory group: Alacevich, Banerjee, Becher, Broggio, Carloni Calame, Chiesa, Czyż, Di Vita, Engel, Fael, Laporta, Passera, Piccinini, AP, Mastrolia, Montagna, Nicrosini, Ossola, Signer, Schubert, Spira, Torres Bobadilla, Trentadue, Ulrich…

- This talk: modern amplitude methods for double virtual corrections
  - theory go-ahead for MUonE: NNLO corrections soon completed
  - new analytic results for LHC physics: $t\bar{t}$ production