The radion effective potential in 5D warped models

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Based on a work in collaboration with

Marek Olechowski
Stefan Pokorski

Humboldt Kolleg
Kitzbühel, 25th Jun 2019

Introduction: 5D Warped models and the radion

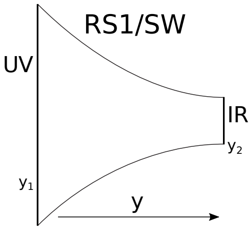
- 5D warped models have been widely used for model building in particle physics.

$$M_P^2 \sim \sqrt{\frac{M_5^9}{\Lambda_5}}, \qquad m_{4d} \sim e^{-(A_{IR}-A_{UV})} m.$$

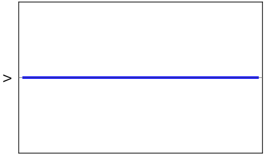
- Some mechanism to stabilize the extra-dimension size (and give mass to the radion):

$$A_{IR} - A_{UV} \sim 30 - 35.$$

- GW mechanism: it requires some fine-tuning in the IR brane tension (small backreaction). [Goldberger, Wise; 99]
- CPR mechanism: no fine-tuning and allows for large backreaction in the IR: soft-wall models.
  [Bellazzini, Csaki, Hubisz, Serra, Terning; 13]
  [Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale; 13]
- Easiest realization: include a GW field with small negative mass squared $m_\phi^2 \sim -10^{-1,-2}\Lambda_5/M_5^3$.
- Soft-wall models can improve tension with EWPD.
  [Carmona, Ponton, Santiago; 11]



$$ds^2 = e^{-2A}dx^2 + dy^2$$



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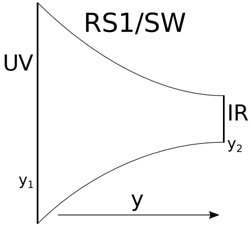

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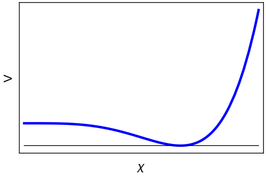


Introduction: 5D Warped models and the radion

- At finite temperature, these models undergo a phase transition to an AdS-Schwarzschild solution.
  [Creminelli, Nicolis, Rattazzi; 01]

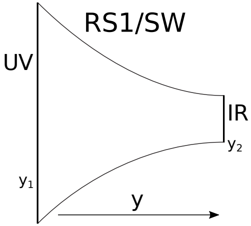
- Phase transition at the early universe: source of gravitational waves. [Randall, Servant; 06]
  [Megías, Quirós; 18]

- If the low energy theory is described by the SM+radion (large gap between the radion and first KK modes):

$$\downarrow$$

- Radion potential correctly describes the phase transition.

Techniques to accurately calculate the radion potential in soft-wall modes.



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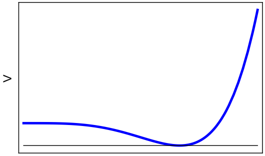

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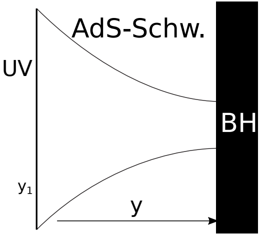

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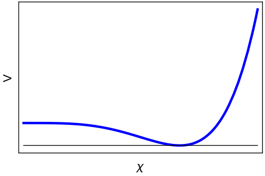

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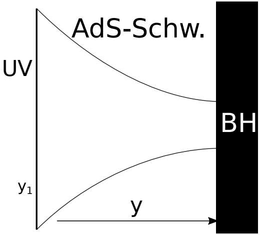

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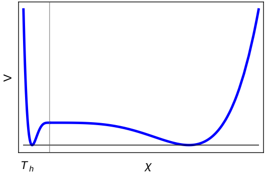

The effective action

- Action:

$$S = \int dx^4\, dy\, \sqrt{|g|}\left[-\frac{1}{2\kappa^2}R + \mathcal{L}_{\rm bulk}(\phi)\right] - \int d^4x \sqrt{|g_1|}\,U_1\Big|_{y_1} - \int d^4x \sqrt{|g_2|}\,U_2\Big|_{y_2}$$

- Effective action for the radion and the 4D metric:

$$\mathrm{e}^{\mathrm{i}S_{\rm eff}[h_{\mu\nu},\chi]} = \int \mathcal{D}\omega\, \delta[h_{\mu\nu} - H_{\mu\nu}(\omega)]\delta[\chi - X(\omega)]\mathrm{e}^{\mathrm{i}S[\omega]}$$

$$\xrightarrow{\text{tree level}} S_{eff}[h_{\mu\nu},\chi] = \min_{\substack{X(\omega)=\chi \\ H(\omega)=h}} S[\omega].$$

- Different choices of the interpolating fields $\rightarrow$ equivalent theories.
- A convenient choice of the interpolating metric is $H_{\mu\nu} = g_{\mu\nu}|_1$.
- $\mathcal{L}_{\rm eff}(h_{\mu\nu},\chi) = -V_{\rm eff}(\chi) - \frac{1}{2}\,C_{\rm eff}(\chi)h^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2M_P^2}K_{\rm eff}(\chi)R[h] + \dots$

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Interpolating radion choice

- Two examples of interpolating radions:

- Using the warp factor:

$$X = \frac{1}{2}\sqrt{g_{\mu\nu}|_2 \; g^{\mu\nu}|_1} = e^{A|_1 - A|_2}$$

- Breaking of the UV and IR BC for A.

$$V_{\rm eff} = \underbrace{\frac{1}{2}\, e^{-4A}\left(U_1 - \frac{6}{\kappa^2}A'\right)\Big|_{y_1}}_{V_{UV}} + \underbrace{\frac{1}{2}\, e^{-4A}\left(U_2 + \frac{6}{\kappa^2}A'\right)\Big|_{y_2}}_{V_{IR}}$$

- Using the physical distance:

$$X = \int dy \sqrt{g_{55}}$$

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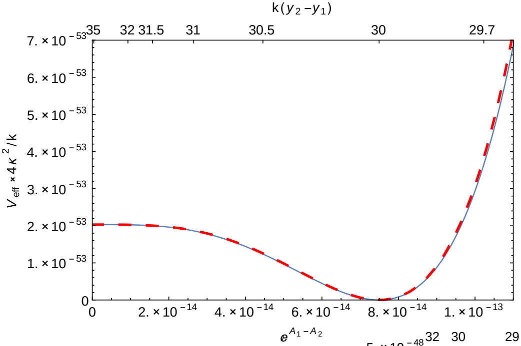

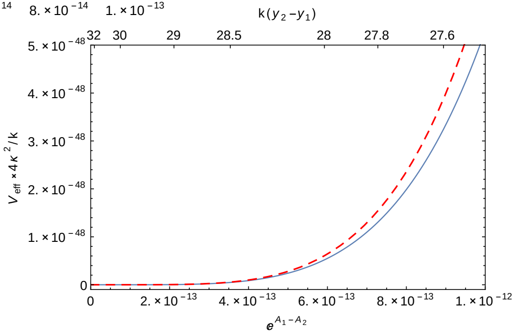

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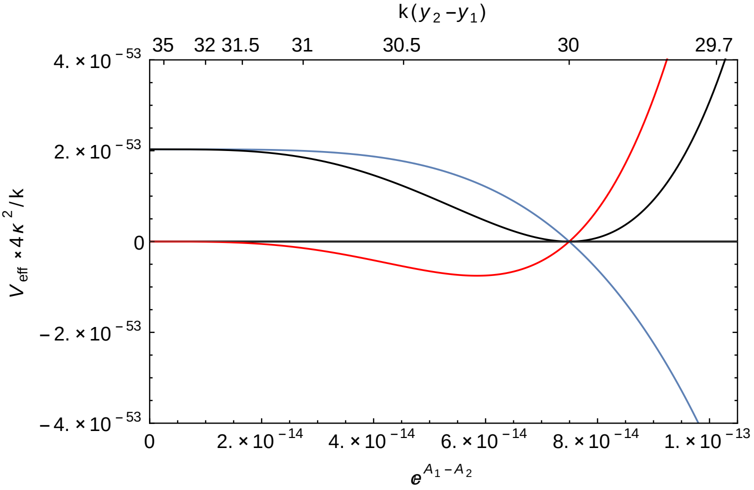
Numerical calculation ($m_\phi^2 = -0.4k^2$)



- High numerical precision to compute V_{UV} (> 50 digits).
- Several approximations used in the literature.

[Bellazzini, Csaki, Hubisz, Serra, Terning; 13], [Megías, Quirós; 18]

The limit $M_P \to \infty$

- If $y_1 \to -\infty$, then $M_P \to \infty$, and gravity is decoupled.
- However, when $y_1 \to -\infty$, the on shell action diverges $\to$ holographic renormalization is required. [de Haro, Solodukhin, Skenderis; 00]
- Result for the effective potential:

$$V_{\rm eff} = \underbrace{\frac{1}{2}\, e^{-4A}\left[\frac{6}{\kappa^2}A' + U_2(\phi)\right]\Bigg|_{y=y_2}}_{V_{IR}} + \underbrace{\frac{k}{2}\Delta_-(\Delta_- - 2)\phi_{(1,0)}\phi_{(0,1)} + \text{const}}_{V_{UV}}\,.$$

- Asymptotic expansion of the fields:

$$\phi(y) = \phi_{(1,0)}e^{\Delta_- ky} + \phi_{(2,0)}e^{2\Delta_- ky} + \ldots + \phi_{(0,1)}e^{\Delta_+ ky} + \phi_{(0,2)}e^{2\Delta_+ ky} + \ldots$$
$$+\phi_{(1,1)}e^{(\Delta_- + \Delta_+)ky} + \ldots + \phi_{(n,m)}e^{(n\Delta_- + m\Delta_+)ky} + \ldots,$$

$$A(y) = ky + A_0 + \ldots + A_{(n,m)}e^{(n\Delta_- + m\Delta_+)ky} + \ldots \qquad \Delta_\pm(\Delta_\pm - 4) = m_\phi^2$$

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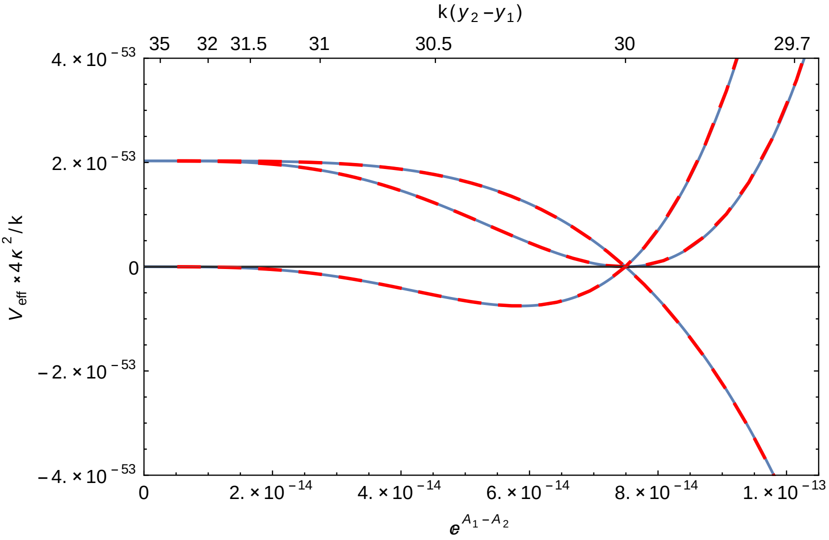

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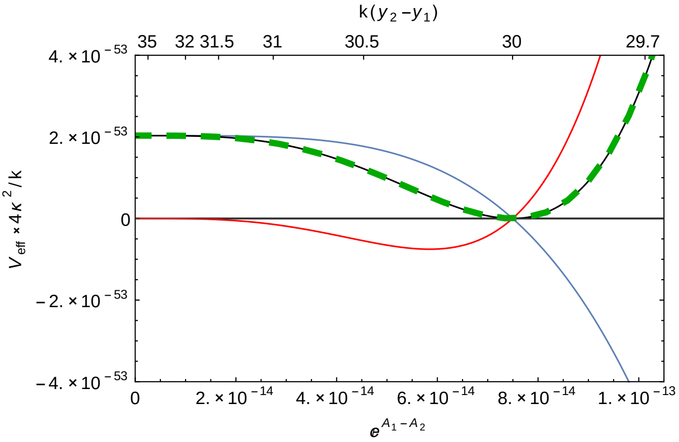
- Playing with the equations, for almost every choice of the interpolating fields, we can express the derivative of the effective potential in terms of IR information:

$$\frac{dV_{\text{eff}}(\chi)}{d\chi} = 4\chi^{-1} V_{IR}(\chi).$$



Conclusions

- The potential for the radion in warped 5D models is relevant, not only to enlighten about the stabilization mechanism, but also to study possible phase transitions that these models predict at the early universe.
- One purpose of this project is to analyze different techniques to compute the effective potential for the radion.
- Naive exact numerical calculations are tedious because they require to keep track of many significant digits.
- Here I have presented two possibilities to simplify exact numeric calculations:
- Taking the limit $M_P \to \infty$: V_{UV} does not vanish in this limit, but gives a relevant contribution to the full potential.
- Relating the derivative of the potential with IR information.