Effective Field Theories in R_ξ gauges[1]

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[1] *M. Misiak, MP, J. Rosiek, K. Suxho and B. Zglinicki, JHEP* **1902**, *051 (2019), [arXiv:1812.11513]*

Motivation

- As current experimental evidence indicate, a sizeable energy gap between the new physics scale and the electroweak scale is present.
- In this region, the most convenient calculational framework is an Effective Field Theory with only the SM degrees of freedom, the so-called SMEFT[2,3].

[2] W. Buchmuller and D. Wyler, (1986).
[3] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, (2010).

- Practical calculations with the (dim-6) SMEFT require introducing convenient gauge-fixing terms.

- In particular, it has been shown[4,5] that effects of higher-dimensional operators should be taken into account in the definition of R_ξ-gauges. Otherwise one can end up with tree-level mixing in the gauge bosons, goldstones and ghosts propagators.

$$\text{e.g.,}\ \frac{C^{\varphi WB}}{\Lambda^2}(\varphi^\dagger \sigma^A \varphi) W^A_{\mu\nu} B^{\mu\nu} \to \left(\frac{C^{\varphi WB} v^2}{\Lambda^2}\right)(\partial_\mu W^3_\nu)(\partial_\mu B_\nu) + \dots$$

$\Rightarrow$ Z-A mixing at tree level (ξ-dependent)!

[4] A. Dedes, W. Materkowska, MP, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143
[5] A. Helset, MP and M. Trott, Phys. Rev. Lett. 120 (2018) 251801

- **Result of R_ξ-SMEFT:** All propagators keep their SM-form (ie., no tree-level mixing) and the effect of dim-6 operators appears only in interactions.

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- **Purpose of R_ξ-EFT:** *Apply R_ξ* **beyond dim-6** *level and* **beyond SM** *content.*

The EFT framework

Consider an EFT that arises after decoupling[6] of heavy particles at scale Λ and assume that the UV-theory at that scale is perturbative.

The dynamics of light fields at low energy scales ($m, E \ll \Lambda$) are described by the effective Lagrangian,

$$\mathcal{L} \;=\; \mathcal{L}^{(4)} \;+\; \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}.$$

- $\mathcal{L}^{(4)}$ is the dimension-four (renormalizable) part of $\mathcal{L}$,
- $Q_i^{(k+4)}$ stand for dimension-$(k+4)$ local operators built out of light fields and their derivatives.
- $C_i^{(k+4)}$ are their respective couplings, known as Wilson coefficients.

The EFT expansion is truncated at arbitrary order N, i.e., $\mathcal{O}(1/\Lambda^{N+1})$ are neglected.

[6] T. Appelquist and J. Carazzone,(1975).

Fundamental blocks of an EFT Lagrangian

The fundamental blocks of a general gauge invariant EFT Lagrangian are [7,8]

$$\mathcal{L} = \mathcal{L}[\Phi, F_{\mu\nu}, D_\mu, (\Psi)]$$

- All scalars in one possibly reducible *real* multiplet:

$$\begin{aligned} \Phi_i &= \varphi_i + v_i \\ D_\mu \Phi &= (\partial_\mu + i A_\mu^a T^a)\Phi \end{aligned}$$

- One field strength tensor in adjoint of the group - reducible if not simple:

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\mu - \partial_\nu A^a_\mu - f^{abc} A^b_\mu A^c_\nu, \\ (D_\rho F_{\mu\nu})^a &= \partial_\rho F^a_{\mu\nu} - f^{abc} A^b_\rho F^c_{\mu\nu} \end{aligned}$$

[7] *footnote in B. Grzadkowski et al., (2010).*

[8] *proof in M. Iskrzyński, MSc thesis.*

Main steps in R_ξ-EFT

1. Distinguish which operators are irrelevant to gauge fixing, which are relevant and which of them are dangerous.

2. Eliminate the dangerous ones with Equations of Motion, ie., "send" them beyond truncation order N.

3. Introduce a gauge fixing term and a corresponding ghost sector which gives perturbation friendly Feynman Rules.

1. Distinguishing (ir-)relevant and dangerous operators

An operator potentially relevant for gauge-fixing has the form,

$$Q^{(n+2m+k)} = \Phi^n F^m D^k$$

It is irrelevant if it has 3 or more objects with vanishing VEVs,

$$\text{e.g.,} \;\; (F^T_{\mu\nu} F^{\mu\nu})^2 \rightarrow \text{pure interactions}$$

It is relevant if it contributes to gauge and scalar boson bilinears,

$$\text{e.g.,} \;\; (\Phi^T \Phi)^2 [(D^\mu \Phi)^T D^\mu \Phi] \;\rightarrow\; v^4 [(D^\mu \Phi)^T D^\mu \Phi])$$

but it is dangerous if it contains higher derivative bilinears,

$$\text{e.g.,} \;\; (D^\mu D_\mu \Phi)^T (D^\nu D_\nu \Phi) \;\rightarrow\; (\partial^\mu \partial_\mu \Phi)^T (\partial^\nu \partial_\nu \Phi).$$

The latter affect the form of the propagators - have to be removed!

2. Eliminating dangerous operators

One can remove the dangerous operators applying (perturbative) field redefinitions making use of the equivalence theorem of S-matrix[9,10].

Equivalently, for the purpose here using the classical Equations of Motion (EOM).

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[9] H. D. Politzer (1980), C. Arzt (1995), H. Simma (1994).

[10] J. C. Criado and M. Pérez-Victoria, JHEP **1903**, 038 (2019).

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Operator basis reduction in a nutshell:

$$D_\mu D^\mu \Phi = [\text{Lower-D}] + \mathcal{O}(\Lambda^{-1}) \ , \quad D_\mu F^{\mu\nu} = [\text{Lower-D}] + \mathcal{O}(\Lambda^{-1})$$

together with algebraic identities, $D_{[\mu}F_{\nu\rho]} = 0$, $[D_\mu, D_\nu] \sim F^a_{\mu\nu} T^a$,etc.
Apply order by order, dim-5 $\to$ dim-N, and practically eliminate the dangerous operators i.e., $\mathcal{O}(1/\Lambda^{N+1})$.

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The only relevant operators that remain after the reduction are of the form,

$$\Phi^n F^m D^k \to \Phi^n D^2, \Phi^n F^2, \Phi^n \ .$$

[9] H. D. Politzer (1980), C. Arzt (1995), H. Simma (1994).
[10] J. C. Criado and M. Pérez-Victoria, JHEP **1903**, 038 (2019).

It can be shown[11], that these relevant operators can be expressed more conveniently as $(A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$,

$$\mathcal{L}_C = \frac{1}{2}(D_\mu\Phi)_i \, K_{ij} \, (D^\mu\Phi)_j - \frac{1}{4}A^a_{\mu\nu} \, J^{ab} \, A^{b\,\mu\nu} + \ldots (\text{Interactions or } V[\Phi]) \, .$$

where J, K are symmetric and positive definite - possess **inverse** and **square-root**,

$$K_{ij} = \mathbf{1}_{ij} + \mathcal{O}_{ij}\,(Cv/\Lambda) \, , \quad J^{ab} = \mathbf{1}^{ab} + \mathcal{O}^{ab}(Cv/\Lambda).$$

[11] A. Helset, MP and M. Trott, (2018)

3. Introducing gauge-fixing

This gives the (usual) "unwanted" gauge-goldstone boson mixing term,

$$\frac{1}{2}(D_\mu \Phi)^T K (D^\mu \Phi) \to -i \left(\partial^\mu A_\mu^a\right) \left[\varphi^T K T^a v\right],$$

modified by the presence of the matrix K.

To compensate for the presence of J, K in the Lagrangian, the gauge-fixing (GF) and the Fadeev-Popov (FP) ghost term need to be modified accordingly:

$$\begin{aligned} \mathcal{L}_{GF} + \mathcal{L}_{FP} &= -\frac{1}{2\xi} \mathcal{G}^a J^{ab} \mathcal{G}^b + \bar{N}^a J^{ab} M_F^{bc} N^c \ , \\ \mathcal{G}^a &= \partial^\mu A_\mu^a - i\xi (J^{-1})^{ac} \left[\varphi^T K T^c v\right] \ , \end{aligned}$$

with $\mathcal{G}^a$ **linear** in the fields and M_F obtained as usual,

$$\delta_{\mathrm{BRST}} \mathcal{G}^a = \epsilon M_F^{ab} N^b \ .$$

- The unwanted gauge-goldstone mixing is eliminated.

By redefining the fields as follows:

$$\tilde{\varphi} = K^{\frac{1}{2}}\varphi \ , \quad \tilde{A}_\mu = J^{\frac{1}{2}} A_\mu \ , \quad \eta = J^{\frac{1}{2}} N \ , \quad \bar{\eta} = J^{\frac{1}{2}} \bar{N} \ ,$$

- all kinetic terms become canonical.

$$\begin{aligned}
\mathcal{L}_C + \mathcal{L}_{GF} &= -\frac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu \tilde{A}_\mu)^T(\partial^\nu \tilde{A}_\nu) + \frac{1}{2}\tilde{A}^T_\mu (M^T M)\tilde{A}^\mu \\
&\quad + \frac{1}{2}(\partial_\mu \tilde{\varphi})^T(\partial^\mu \tilde{\varphi}) - \frac{\xi}{2}\tilde{\varphi}^T(MM^T)\tilde{\varphi} \ , \\
\mathcal{L}_{FP} &= \bar{\eta}^T \partial^\mu \partial_\mu \eta + \xi\, \bar{\eta}^T (M^T M)\eta + \ldots\text{(interactions)}
\end{aligned}$$

with the (non-square in general), ${M_j}^b \equiv [K^{\frac{1}{2}}(iT^a)\langle\Phi\rangle]_j\,(J^{-\frac{1}{2}})^{ab}$.

With Singular Value Decomposition one can further show,

- for all gauge bosons and ghosts, $(m^2_\eta)^a = \xi (m^2_A)^a$
- for massive gauge and (would-be) goldstone bosons: $(m^2_\phi)^i = \xi (m^2_A)^i$

This is the convenient R_ξ framework of SM(EFT)!

Conclusions

- When generalizing R_ξ to EFTs one confronts,

$$\text{dangerous } Q_i \ : \quad (D_\mu D^\mu \Phi)^T (D_\nu D^\nu \Phi) \xrightarrow{EOM} \text{push to } \mathcal{O}(1/\Lambda^{N+1})$$

$$\text{relevant } Q_i \ : \quad v^2 F^T_{\mu\nu} F^{\mu\nu} \xrightarrow{J,K} \text{include in } \mathcal{L}_{GF+FP}$$

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- The case of common ξ was discussed here but it is also possible to apply different ξ's - this is useful for practical calculations (e.g., in SMEFT ξ_W, ξ_Z, ξ_A).

Backup - SVD

To diagonalize the mass matrices, one can apply the Singular Value Decomposition

$$M = U^T \Sigma V$$

with orthogonal $U_{m\times m}$, $V_{n\times n}$ and diagonal $\Sigma_{m\times n}$, (i.e., a non-square matrix with ${\Sigma_j}^b = 0$ for $j \neq b$). Then,

$$\begin{aligned} VM^TMV^T &= \Sigma^T\Sigma = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{n\times n} \\ (\xi\times) \qquad UMM^TU^T &= \Sigma\Sigma^T = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{m\times m} \end{aligned}$$

with $p = min(m, n)$.
This suggests that the non-vanishing eigenvalues of gauge-bosons and goldstones are proportional, with ξ being the proportionality factor.

Backup - pedagogical EOM

Understand the logic of EOM reduction through a toy-example:

$$\mathcal{L}_{\rm toy} = (\partial\phi)^2 + m^2\phi^2 + \frac{C^{(6)}}{\Lambda^2}(\partial^2\phi)^2$$

giving the EOM,

$$\partial^2\phi = m^2\phi + \frac{C^{(6)}}{\Lambda^2}\partial^2(\partial^2\phi)$$

Applying EOM one can trade,

$$\frac{C^{(6)}}{\Lambda^2}(\partial^2\phi)^2 = \frac{C^{(6)}}{\Lambda^2}m^2\phi(\partial^2\phi) + \frac{(C^{(6)})^2}{\Lambda^4}(\partial^4\phi)(\partial^2\phi)$$

Both **higher** and **lower** derivative operators can be obtained.

But **higher** derivatives are **always suppressed** by extra powers of $1/\Lambda$.

Backup - EOM beyond the nutshell

1. $D_{\mu_1}...D_{\mu_k}\Phi$ with internal contractions.
2. $D_{\mu_1}...D_{\mu_k}\Phi$ without internal contractions must be contracted with $(...)D^{\mu_{\sigma(1)}}...D^{\mu_{\sigma(k)}}\Phi$ or $(...)D^{\mu_{\sigma(1)}}...D^{\mu_{\sigma(k-2)}}F^{\mu_{\sigma(k-1)}\mu_{\sigma(k)}}$.
3. $D_\mu\Phi$ contracted with $(...)D_\nu F^{\nu\mu}$.
4. $P^{ab}(\Phi)[(...)D_\mu F_{\nu\rho}]^a[(...)D^\mu F^{\nu\rho}]^b$ or $P^{ab}(\Phi)[(...)D_\mu F_{\nu\rho}]^a[(...)D^\nu F^{\mu\rho}]^b$

some steps involving $\tilde{F}$ not shown here.

Backup - R_ξ bilinears

The three classes $\Phi^n F^m D^k \to \Phi^n D^2, \Phi^n F^2, \Phi^n$ can be expressed as[12],

$$\mathcal{L}_C = \frac{1}{2}(D_\mu \Phi)_i \; K_{ij}[\Phi] \; (D^\mu \Phi)_j - \frac{1}{4} F^a_{\mu\nu} \; J^{ab}[\Phi] \; F^{b\,\mu\nu} - V[\Phi],$$

Bilinear terms arise when $J[\Phi]$ and $K[\Phi]$ are set to their expectation values,

$$\begin{aligned} K_{ij}[\Phi] \quad &\to \quad K_{ij} = \mathbf{1}_{ij} + \mathcal{O}_{ij}(Cv/\Lambda), \\ J^{ab}[\Phi] \quad &\to \quad J^{ab} = \mathbf{1}^{ab} + \mathcal{O}^{ab}(Cv/\Lambda). \end{aligned}$$

with J, K being symmetric and positive definite - possess **inverse** and **square-root**. Then $\mathcal{L}_C$ becomes $(A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$,

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