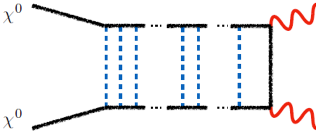
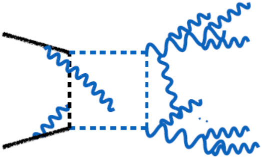
Non-perturbative effects and resummation in weakly interacting dark matter annihilation

Humboldt Kolleg "Discoveries and Open Puzzles in Particle Physics and Gravitation", Kitzbühel, June 24-28 2019

M. Beneke





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MB, Bharucha, Hryczuk, Recksiegel, Ruiz-Femenia [1611.00804] + Dighera, Hellmann [1601.04718] – Sommerfeld, relic density
MB, Broggio, Hasner, Vollmann [1805.07367] + Urban [1903.08702] – EW Sudakov, cosmic rays
MB, Hellmann, Ruiz-Femenia [1210.7928, 1411.6924, 1411.6930] – NREFT formalism, Sommerfeld, MSSM

Some figures courtesy M. Vollmann

Motivation

A TeV scale particle with electroweak SU(2) [$\times$ U(1)] charge provides an attractive dark matter candidate

- No new interaction is required.
- Connection to electroweak scale, where one hopes for New Physics anyway (to solve the hierarchy problem).
- A single new SU(2) mulitplet is enough.
  Example: triplet ("pure wino") provides for the observed relic density for $m_\chi \simeq 2.8$ TeV and is not excluded.

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Despite weak coupling, annihilation cross section cannot be computed accurately at Born level.
Non-perturbative effects and all-order resummations are required.
Highly non-trivial (effective) quantum field theory.

Observables

Relic density

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle_{\rm eff}\,(n^2 - n_{\rm eq}^2)$$

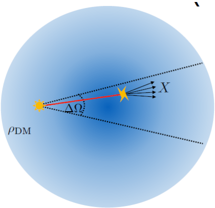
Thermally averaged, inclusive cross section, including co-annihilating particles at the time of freeze-out, $T \sim m_\chi/25$.

Cosmic rays

$$\Phi(E_i) = \frac{1}{8\pi m_\chi^2} \int_{\Delta\Omega} d\Omega \int_{\rm l.o.s.} ds\, \rho_{\rm DM}^2(\mathbf{r}(s))\, \frac{d}{dE_i} [\sigma v]_{\chi\chi\to i+X}$$

Exclusive final state, $v \approx 10^{-3}$ or less.

Note: relevant annihilation processes always when DM is non-relativistic.

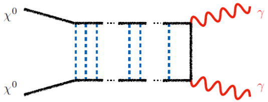


Large quantum (loop) effects

Focus is on DM particle with SU(2) electroweak gauge interactions.
Classic WIMPs.

Sommerfeld effect

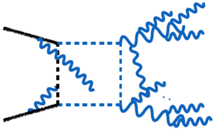
$$\left(\alpha_2 \frac{m_\chi}{m_W}\right)^n$$



$\mathcal{O}(1)$ changes of relic density, huge resonant $\mathcal{O}(10^3)$ effects for annihilation in the present Universe possible.

Electroweak Sudakov logarithms

$$\left(\alpha_2 \ln^2 \frac{m_\chi}{m_W}\right)^n$$



Only for exclusive final states. $\mathcal{O}(1)$ changes of the flux of cosmic rays.

Must sum these enhanced terms to any order in the loop expansion → effective field theory and renormalization group.

Models

Minimal Models [Cirelli et al. (2007)]

$$\mathcal{L}_{\rm DM} = \frac{1}{2}\bar{\chi}(\not{D} - m_\chi)\chi$$

Fermionic electroweak doublet ("pure Higgsino"), triplet ("pure Wino"), quintuplet, ...
Correct relic density for masses 1-10 TeV.

Simplified Models

$$\mathcal{L}_{\rm DM} = \frac{1}{2}\bar{\chi}(\not{D} - m_\chi)\chi + \mathcal{L}_{\rm mediator} + \mathcal{L}_{\rm int}$$

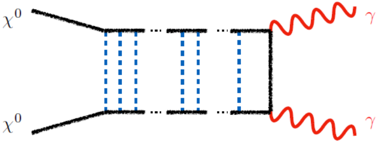
DM scalar or fermionic EW multiplet
+ a single mediator multiplet,
e.g. fermionic singlet DM and doublet scalar mediator.

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{f}\left(i\not{D} - m_f\right)f + \frac{1}{2}\bar{\chi}\left(i\not{\partial} - m_\chi\right)\chi + (D_\mu\phi)^\dagger(D^\mu\phi) - m_\phi^2\phi^\dagger\phi + \left(\lambda\bar{\chi}P_L f^-\phi^+ + h.c.\right)$$

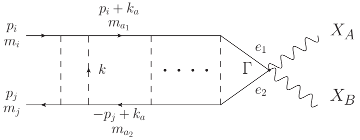
Minimal Supersymmetric Standard Model (MSSM)

Includes some of the above in corners of the MSSM parameter space.

A. Sommerfeld effect



EW Sommerfeld enhancement [Hisano et al. (2004,2006)]



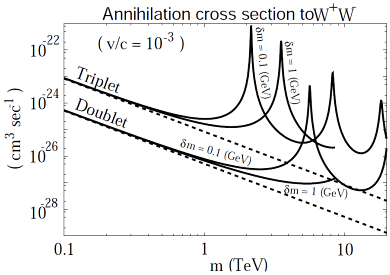
Contribution from one ladder rung from the loop momentum region $k^0 \ll \vec{k} \ll m_\chi$ (non-relativistic scattering) is $O(1)$ effect [$\rightarrow$ summation] for

$$m_\chi \geq \frac{m_W}{\pi\alpha_2} \sim \text{TeV} \qquad \delta m_{\chi^+\chi^0} \lesssim \frac{m_W^2}{m_\chi} \sim \text{GeV}$$

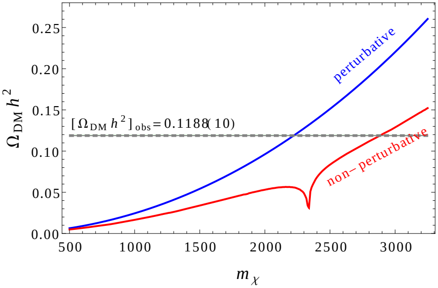
- Summation by solving (numerically) a Schrödinger equation for the wave-function at the origin of a two-particle scattering state.
- Example: for wino $\chi^0\chi^0$ and $\chi^+\chi^-$ can scatter into each other. Potential

$$V(r) = \begin{pmatrix} 0 & -\frac{\sqrt{2}\alpha_2}{r}\, e^{-m_W r} \\ -\frac{\sqrt{2}\alpha_2}{r}\, e^{-m_W r} & 2\delta m - \frac{\alpha_{\text{EM}}}{r} - \frac{\alpha_2 c_W^2}{r}\, e^{-m_Z r} \end{pmatrix}$$

Example: minimal models, in particular triplet ("pure wino")



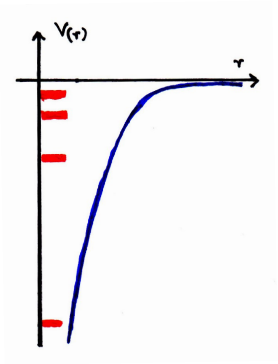
[Hisano et al., 2004]



Pure Wino, following [Hisano et al., 2006]

Resonance effect is a peculiar feature of the Yukawa potential, does not occur for the classic Coulomb Sommerfeld effect.

Resonance effect for the Yukawa potential



Range $r \sim 1/m_W$ cuts off Rydberg states $[r_{\rm Ryd} \sim n^2/(M_\chi \alpha_2)]$

Finite number of levels

$$n^2 \lesssim \frac{m_\chi \alpha_2}{m_W}$$

Increasing M_χ adds levels from above. Zero-energy bound states for certain m_χ. Then

$$S \propto \frac{1}{E - E_{\rm bind}} \sim \frac{1}{v^2}$$

stronger than $1/v$ Coulomb enhancement.

Resonant enhancement at certain values of m_χ starting in TeV range.

Sommerfeld enhancement in the general MSSM

[MB, Hellmann, Ruiz-Femenia, 1210.7928, 1411.6924, 1411.6930; Hellmann, Ruiz-Femenia 1303.0200]

MSSM with $M_\chi \gg M_Z$: degeneracies are natural (electroweak multiplets) $\rightarrow$ co-annihilation of up to four neutralinos and two charginos. In general:

$$\begin{aligned} &14\ \chi_i^0\chi_j^0,\ \chi_i^+\chi_i^-\ \text{charge-0 states},\\ &8\ \chi_i^0\chi_j^+\ \text{charge +1 [+ conjugates]},\\ &3\ \chi_i^+\chi_j^+\ \text{charge +2 states [+ conjugates]}. \end{aligned}$$

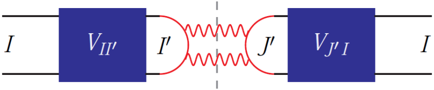
depending on M_1 (bino), M_2 (wino), μ (Higgsino) + sfermion mediators.

Example: Dominantly Wino
$[M_2 < |\mu| \ll |M_1|$ with $m_W \ll |\mu| - M_2]$

$$\delta m_{\tilde{\chi}_1^+} \equiv m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} \approx \frac{12 m_W^4 M_2\, c_{2\beta}^2}{(\mu^2 - M_2^2)^2} \text{ “+” } \underbrace{\frac{1-c_w}{2}\,\alpha_2 m_W}_{\text{radiative, } \approx\ 158\ \text{MeV}}$$

In the charge-0 sector two highly degenerate states $\chi_1^0\chi_1^0$, $\chi_1^+\chi_1^-$,
followed by four states $\chi_1^0\chi_{2,3}^0$, $\chi_1^\pm\chi_2^\mp$, then the four two-Higgsino-like states $\chi_{2,3}^0\chi_{2,3}^0$, $\chi_2^+\chi_2^-$
and finally four heavy bino-like states $\chi_{1,2,3}^0\chi_4^0$, $\chi_4^0\chi_4^0$.

Scatter into one another through Yukawa interaction. Each annihilates into a multitude of SM final states. Systematic calculation in (potential-) non-relativistic effective field theory.



$$\sigma_I(v) = \sum_i \Gamma_i({}^{2S+1}L_J)_{I'J'} \, \langle [\chi\chi]_I | \mathcal{O}({}^{2S+1}L_J)_{I'J'} | [\chi\chi]_I \rangle \stackrel{\text{Born}}{=} a_I + b_I v^2$$

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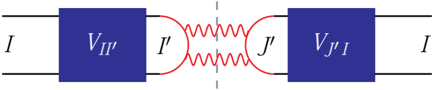

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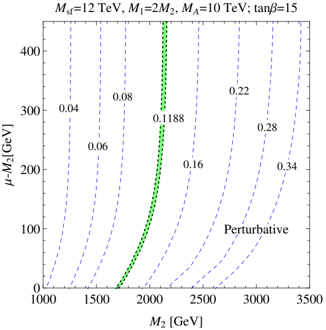
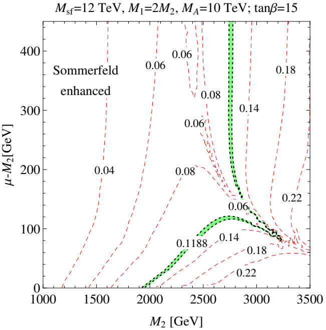
- I Compute the potentials from Z, W, Higgs exchange (14 x 14 matrix etc.)
- II Compute the tree-level coefficients of *off-diagonal* partial wave forward-amplitudes
- III Solve Schrödinger equation for operator matrix elements (wave-functions + derivatives at origin) for given external velocity and partial wave L.
- IV Tabulate $\sigma_I(v)$ for every two-particle state I and compute the thermally averaged + co-annihilation summed effective annihilation cross section $\langle \sigma_{\rm eff} v \rangle$)(T) for $x = m_\chi/T \sim 10 \ldots 10^8$.
- V Solve Boltzmann equation for relic density
- VI Compute cross sections for exclusive two-particle final states + fragmentation into stable cosmic ray particles.

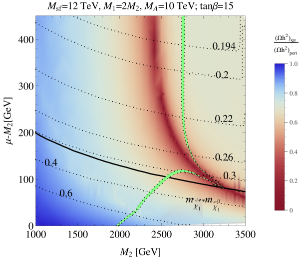
Previous work [Hisano et al. (2004, 2006); Cirelli et al. (2007, 2008, 2009), Hryczuk et al. (2010, 2014)]: pure-Wino and/or -Higgsino LSP limit; no off-diagonals away from pure-W/H limits; no partial-wave separation.

Mixed Wino-Higgsino [MB, Bharucha, Dighera, Hellmann, Hryczuk, Recksiegel, Ruiz-Femenia, 1601.04718]



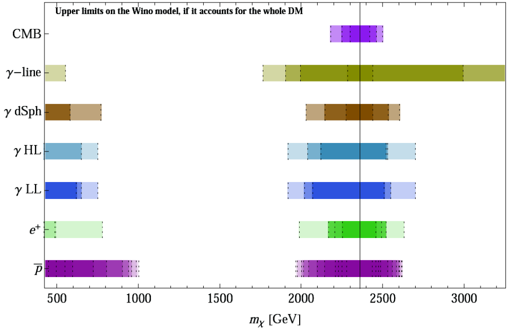


- SE shifts correct relic density to larger masses, above the Sommerfeld resonance.
- Correct relic density line is pulled into the resonance (mass splitting effect)
- Correct relic density for a wide range of wino-like LSP masses 2.0 . . . 3.3 TeV.



Indirect detection (cosmic ray) constraints

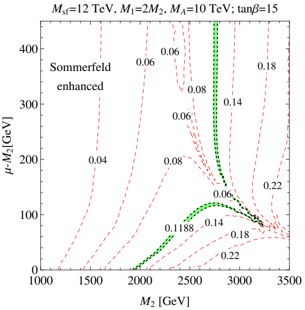
Sommerfeld effect/resonance leads to large annihilation cross sections in the late Universe. Pure wino often said to be excluded by non-observation of cosmic ray signals.

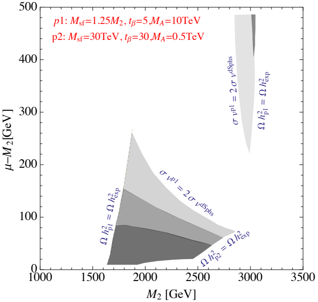


[from Hryczuk et al., 1401.6212]

- Diffuse γ flux from dwarf spheroidal galaxies [FERMI-LAT, MAGIC]; galactic positrons, protons, B/C, Helium [AMS-02 data, DRAGON propagation code]; energy deposition into CMB before and after re-combination [PLANCK].

Indirect (cosmic ray) constraints on the mixed wino-Higgsino [MB, Bharucha, Hryczuk, Recksiegel, Ruiz-Femenia, 1611.00804]



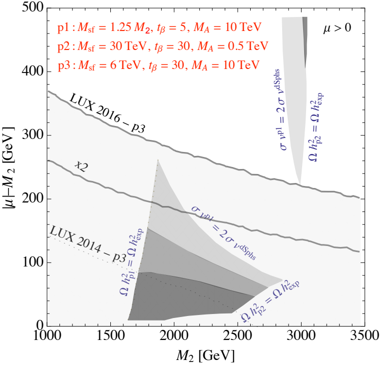


- Strongest constraint from diffuse γs from dwarf spheroidal galaxies and correct relic density.

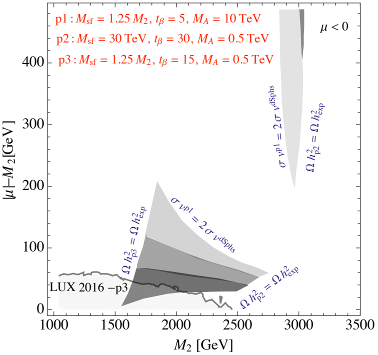
→ Previously studied pure-Wino/Higgsino limits of the MSSM do not nearly capture the full MSSM parameter space where correct relic density is attained for wino-like dark matter.

→ Even the pure-wino is not ecluded under the conservative assumption of a cored DM density profile in the Milky Way.

Direct detection constraints on mixed wino-Higgsino dark matter



$\mu > 0$

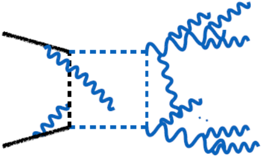


$\mu < 0$

The mixed region is ruled out for $\mu > 0$ and constrained for $\mu < 0$ – even more with recent XENON1T results.

B. Electroweak Sudakov logarithms

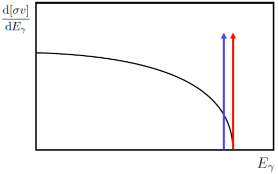
[MB, Broggio, Hasner, Vollmann [1805.07367] + Urban [1903.08702]]



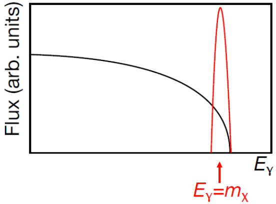
The photon "line" signal

$\chi^0\chi^0 \to \gamma\gamma, \gamma Z$ produces a mono-chromatic photon excess in (here TeV energy) cosmic rays with $E_\gamma \approx m_\chi$.

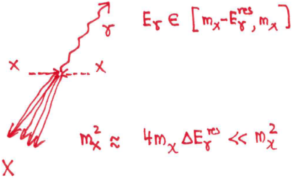
The true observable is the single-inclusive photon-energy spectrum in $\chi^0\chi^0 \to \gamma + X$ with an unobserved final state X, smeared over the energy resolution $E^\gamma_{\rm res}$ of the instrument.



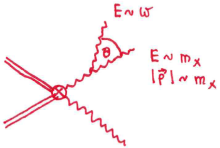
$\Longrightarrow$



For E_γ near the endpoint, the unobserved final state is a "jet". Electroweak perturbation theory develops large double logarithms $\ln \frac{m_\chi}{m_W} \times \ln \frac{m_\chi}{E^\gamma_{\rm res}}$.



Origin of EW Sudakov logs



- Soft and collinear radiation (incomplete cancellation of real and virtual effects)

$$I \propto \frac{g^2}{16\pi^2} \int_{\Delta E_\gamma}^{m_\chi} \frac{d\omega}{\omega} \int d\cos\theta \, \frac{E(\vec{p})}{\sqrt{m_W^2 + \vec{p}^{\,2}} - |\vec{p}| \cos\theta} \propto \frac{\alpha}{4\pi} \ln \frac{m_\chi}{\Delta E_\gamma} \ln \frac{m_\chi}{m_W}$$

- Resummation of large logs $L = \ln(4m_\chi^2/m_W^2)$

$$\sigma v \propto (1 + C_1 \hat{\alpha}_2 + \ldots) \exp\left[L f_0(\hat{\alpha}_2 L) + f_1(\hat{\alpha}_2 L) + \hat{\alpha}_2 f_2(\hat{\alpha}_2 L) \ldots\right]$$

LL	f_0, 1
NLL	$+f_1$
NLL'	$+C_1$
NNLL	$+f_2$

Energy resolution

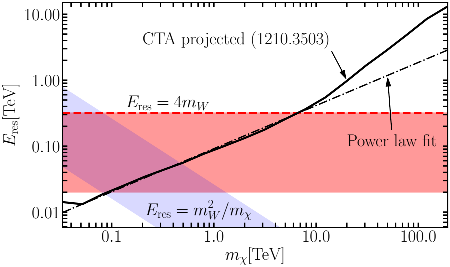
- Large logs depend on energy resolution

→ $E_{\rm res}^\gamma \sim 0$, "line", [Bauer et al. 1409.7392, Ovanesyan et al. 1409.8294, 1612.04814, NLL']

→ $E_{\rm res}^\gamma \sim m_W^2/m_\chi$, "narrow", [MB, Broggio, Hasner, Vollmann, 1805.03767, NLL']

→ $E_{\rm res}^\gamma \sim m_W$, "intermediate", [MB, Broggio, Hasner, Urban, Vollmann, 1903.08702, NLL']

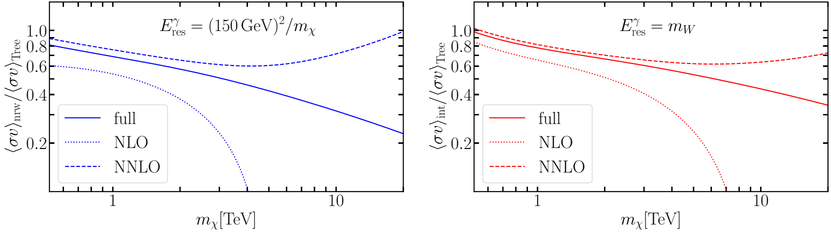
→ $E_{\rm res}^\gamma \sim m_\chi(1-z) \gg m_W$, "wide", [Baumgart et al. 1712.07656 (LL), 1808.08956 (NLL)]



Intermediate resolution theory covers the interesting mass range up to 7 TeV.

Breakdown of electroweak perturbation theory

Minimal triplet ("pure wino") model:



NLO (blue), NNLO (magenta), resummed (black) after Sommerfeld resummation ("tree")
Left: narrow, right: intermediate resolution

Factorization

- Non-relativistic and soft-collinear effective field theory to separate the scales m_χ, $\sqrt{m_\chi m_W}$, m_W, $E^\gamma_{\rm res}$.

$$\frac{d(\sigma v_{\rm rel})}{dE_\gamma} = \underbrace{\sum_{I,J} S_{IJ}}_{\text{Sommerfeld}}\, \Gamma_{IJ}(E_\gamma) = \sum_{I,J} S_{IJ}\, \underbrace{\sum_{i,j=1,2} C_i(\mu)C_j^*(\mu)}_{\text{short distance annihilation}}\, \gamma^{ij}_{IJ}(E_\gamma,\mu)$$

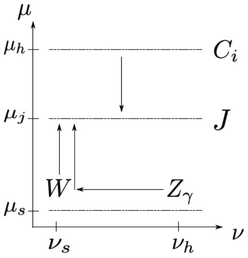
$$\gamma^{ij}_{IJ}(E_\gamma,\mu) = \frac{1}{(\sqrt{2})^{n_{id}}}\,\frac{1}{4}\,\frac{2}{\pi m_\chi}\, V_{\rm int}(\mu,\nu)\, Z^{33}_\gamma(\mu,\nu) \times \int d\omega\, J_{\rm int}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, W^{ij}_{IJ}(\omega,\mu,\nu)\,.$$

$$\gamma^{ij}_{IJ}(E_\gamma,\mu) = \frac{1}{(\sqrt{2})^{n_{id}}}\,\frac{1}{4}\,\frac{2}{\pi m_\chi}\, V_{\rm nrw}(\mu,\nu)\, Z^{33}_\gamma(\mu,\nu) \times D^i_{I,33}(\mu,\nu) D^{j\,*}_{J,33}(\mu,\nu) J^{33}_{\rm nrw}(4m_\chi(m_\chi - E_\gamma),\mu,\nu)\,.$$

- Soft functions (green) at amplitude level for narrow (nrw) resolution, since soft radiation is kinematically forbidden.

Resummation

- NLL' accuracy
- One-loop electroweak corrections in every function
- Renormalization group evolution in virtuality μ and rapidity ν sums logarithms at NLL (one-loop anomalous dimensions + two-loop cusp)

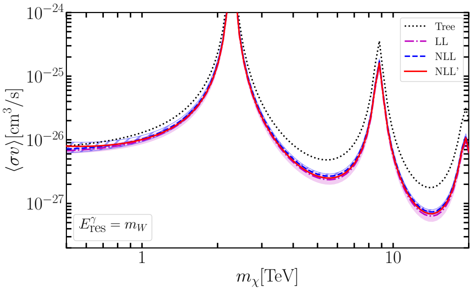


- E.g., function for the electroweak jet balancing the photon momentum

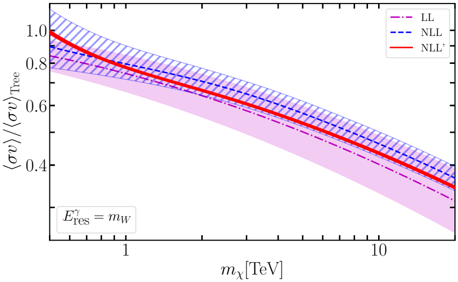
$$j\left(\ln\frac{\Lambda^2}{\mu^2},\mu\right) = \exp\left[-\int_{\ln\mu_j}^{\ln\mu} d\ln\mu'\left(4\gamma_{\rm cusp}(\alpha_2)\ln\frac{\Lambda^2}{\mu'^2}+2\gamma_J(\alpha_2)\right)\right] j\left(\ln\frac{\Lambda^2}{\mu_j^2},\mu_j\right)$$

- Matrix evolution for hard and soft functions.

Resummed, smeared energy spectrum (pure wino model)



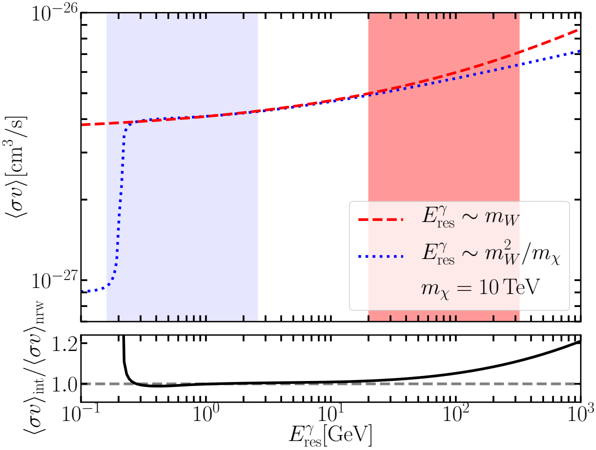
$$\langle \sigma v \rangle (E^{\gamma}_{\rm res}) = \int_{m_\chi - E^{\gamma}_{\rm res}}^{m_\chi} dE_\gamma \, \frac{d(\sigma v)}{dE_\gamma}$$



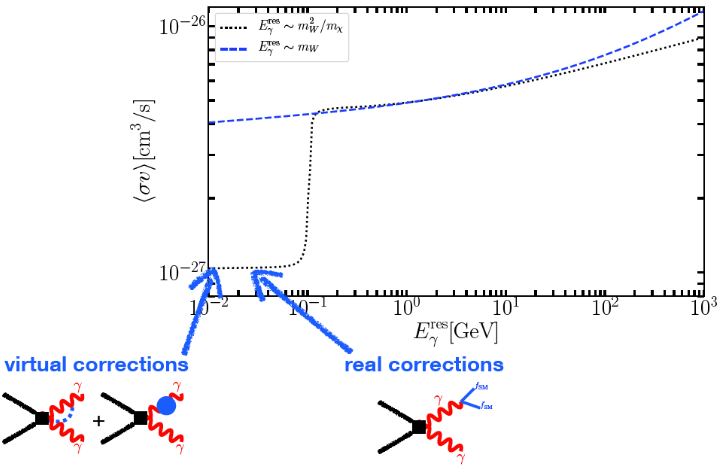
Factor 2 flux reduction from quantum corrections

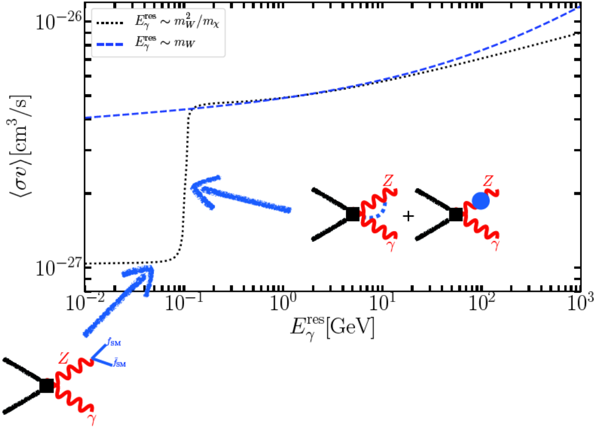
NLL' eliminates theoretical uncertainty for all practical purposes (residual < 1%)

Matching narrow and intermediate resolution

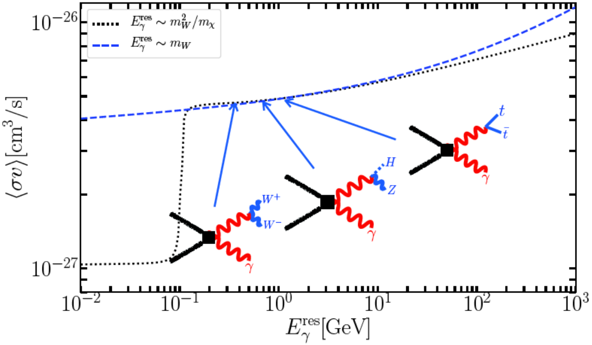


Recall jet mass $\leq 4 m_\chi E_\gamma^{\rm res}$
(plot for $m_\chi = 2$ TeV)

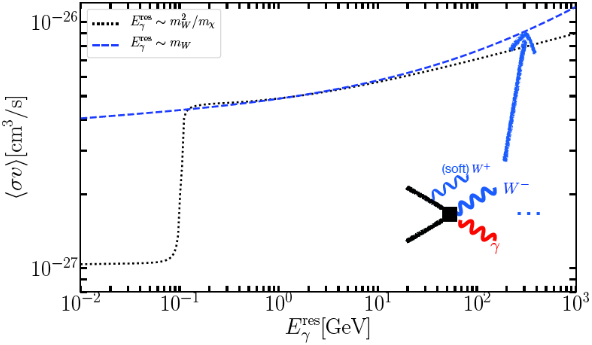




$E_\gamma^{\rm res} \sim m_W^2/m_\chi$
$E_\gamma^{\rm res} \sim m_W$
$\langle \sigma v \rangle\,[{\rm cm}^3/{\rm s}]$
10^{-26}
10^{-27}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
$E_\gamma^{\rm res}\,[{\rm GeV}]$
Z
γ
+
Z
γ
Z
γ
$f_{\rm SM}$
$\bar f_{\rm SM}$



$E_\gamma^{\rm res} \sim m_W^2/m_\chi$
$E_\gamma^{\rm res} \sim m_W$
$\langle \sigma v \rangle [\mathrm{cm}^3/\mathrm{s}]$
10^{-26}
10^{-27}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
$E_\gamma^{\rm res}$[GeV]
W^+
W^-
γ
H
Z
t
$\bar{t}$


$E_\gamma^{\rm res} \sim m_W^2/m_\chi$
$E_\gamma^{\rm res} \sim m_W$
10^{-26}
10^{-27}
$\langle \sigma v \rangle [\mathrm{cm}^3/\mathrm{s}]$
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
$E_\gamma^{\rm res}[\mathrm{GeV}]$
(soft) W^+
W^-
γ

Summary

- The traditional electroweak WIMP with $\mathcal{O}$(TeV) mass is still alive. The pure wino will be excluded by CTA even under most conservative astrophysical assumptions.

- Computing the annihilation cross section to better than a factor of 2 accuracy is a highly non-trivial QFT problem
  Non-perturbative due to electroweak Yukawa force and soft-collinear radiation.

- Modern EFT techniques originating from high-energy physics have solved the problem.
  Sommerfeld + NLL' Sudakov resummation results in few percent accurate inclusive annihilation cross sections, and high-energy photon rates (E_γ near m_χ) from annihilation.