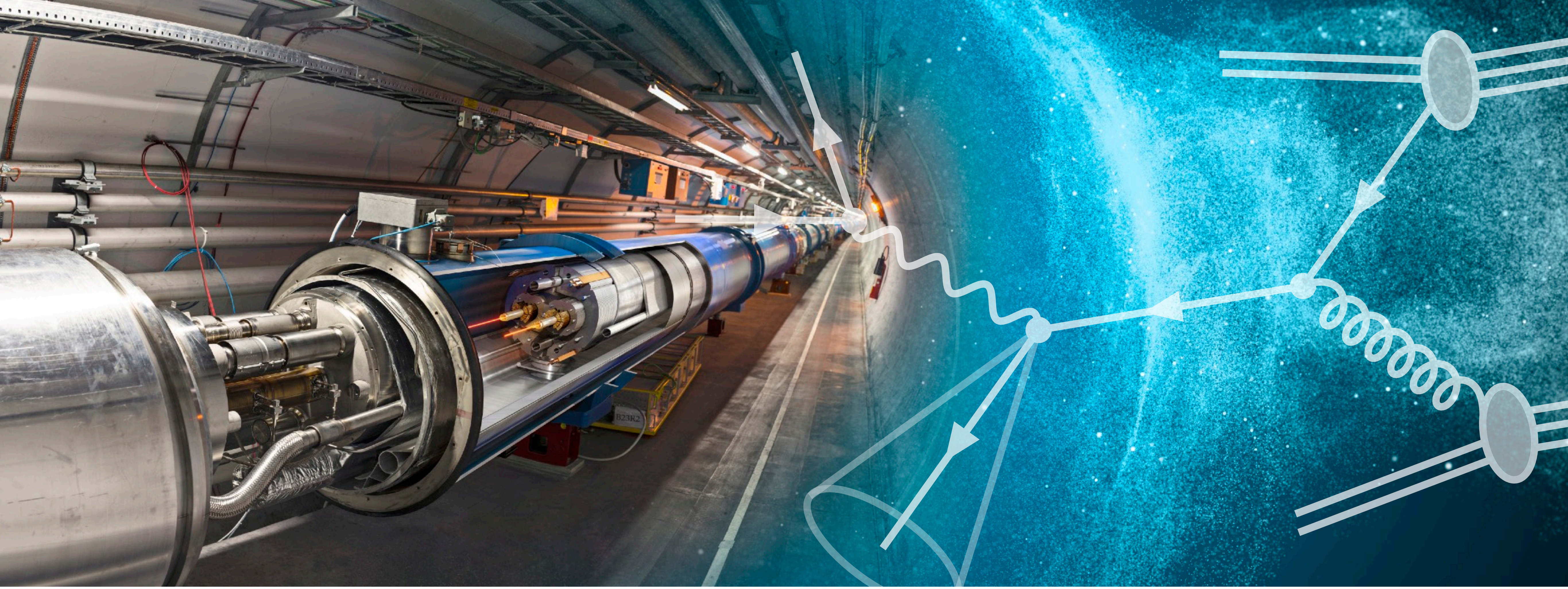


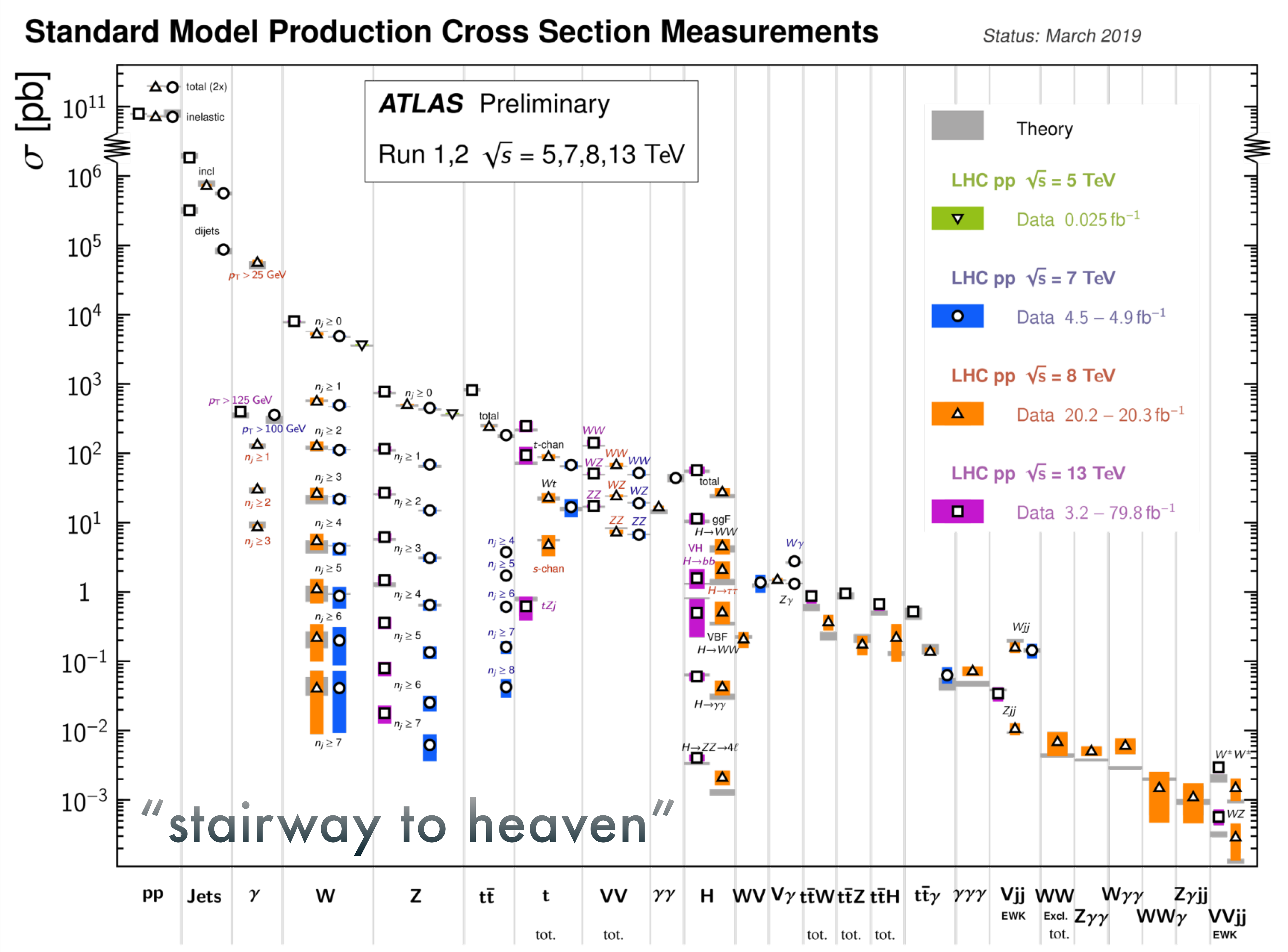
HIGH-PRECISION PREDICTIONS FOR THE LHC

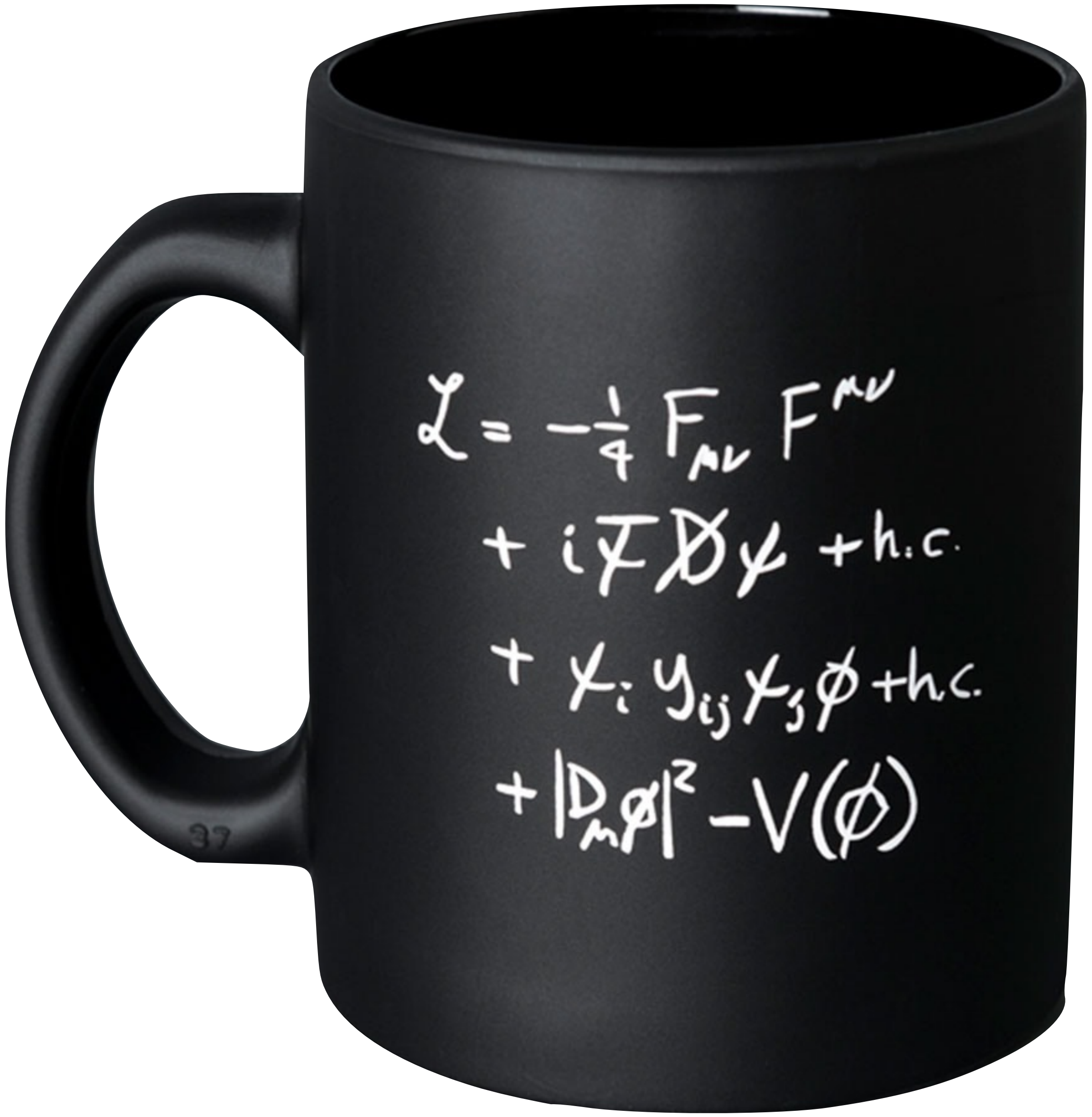
Standard Candles and the Higgs to lighten the path to discoveries

Alexander Huss

Humboldt Kolleg — Kitzbühel, 27th June 2019

A Remarkable Success Story...





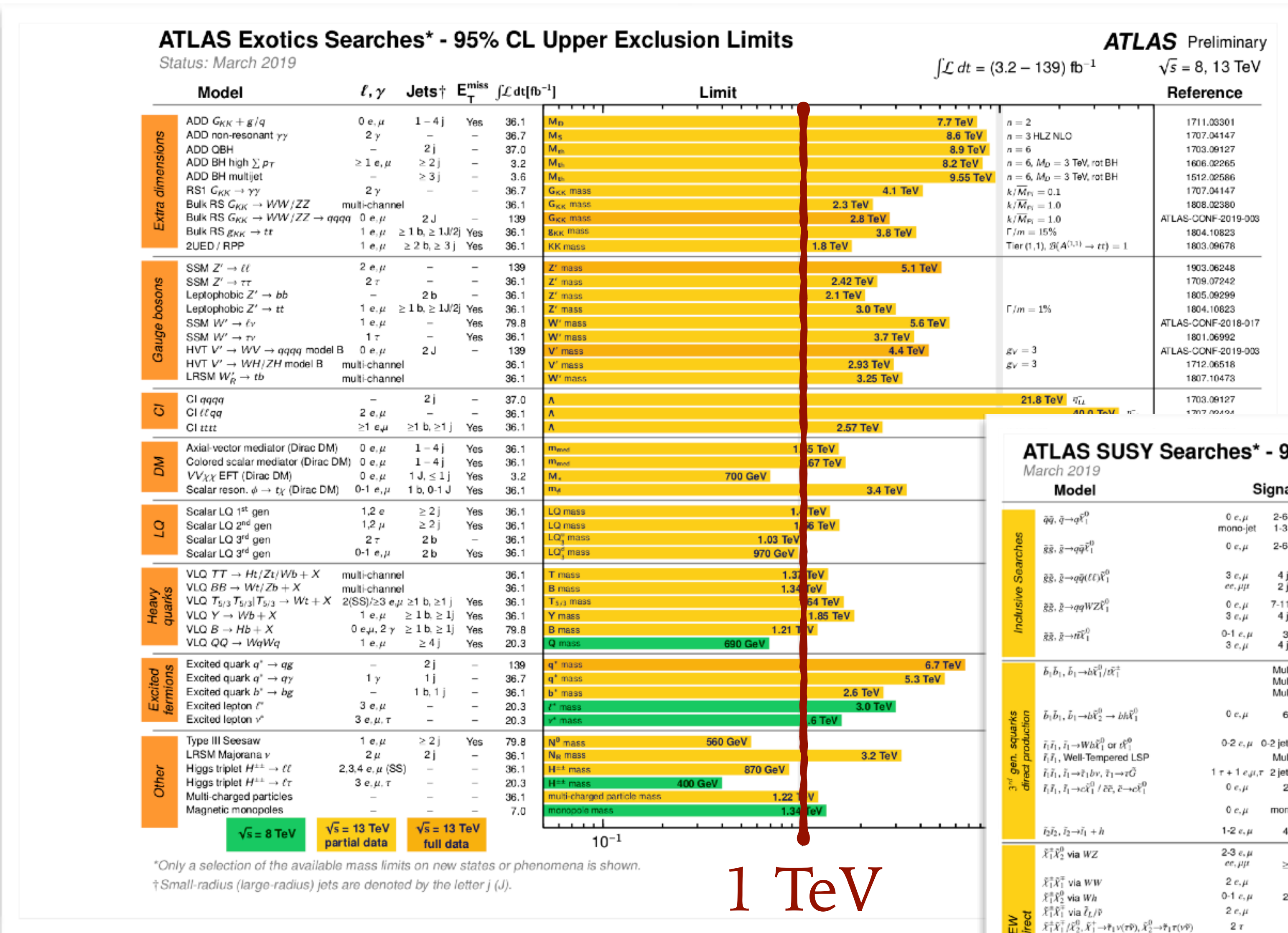
This equation neatly sums up our current understanding of fundamental particles and forces.

...But not the Full Story

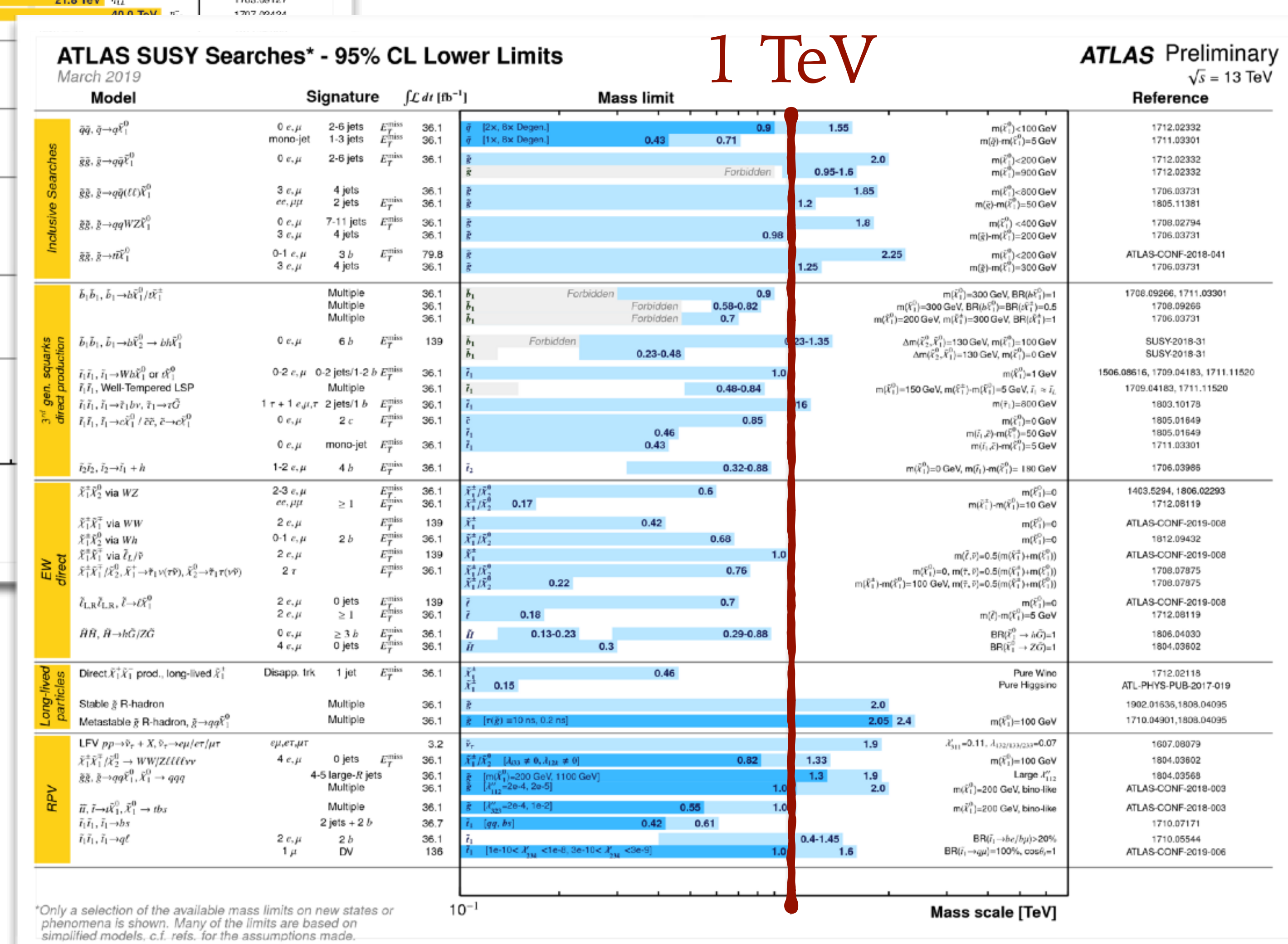
- origin of dark matter
- hierarchy problem
- matter anti-matter asymmetry
- hierarchy of scales (generations)
- unification with gravity
- ...
- what is the Higgs potential?
- establish the Yukawa's Y_{ij}
- ...

New Physics — Direct Searches

No striking signals so far!



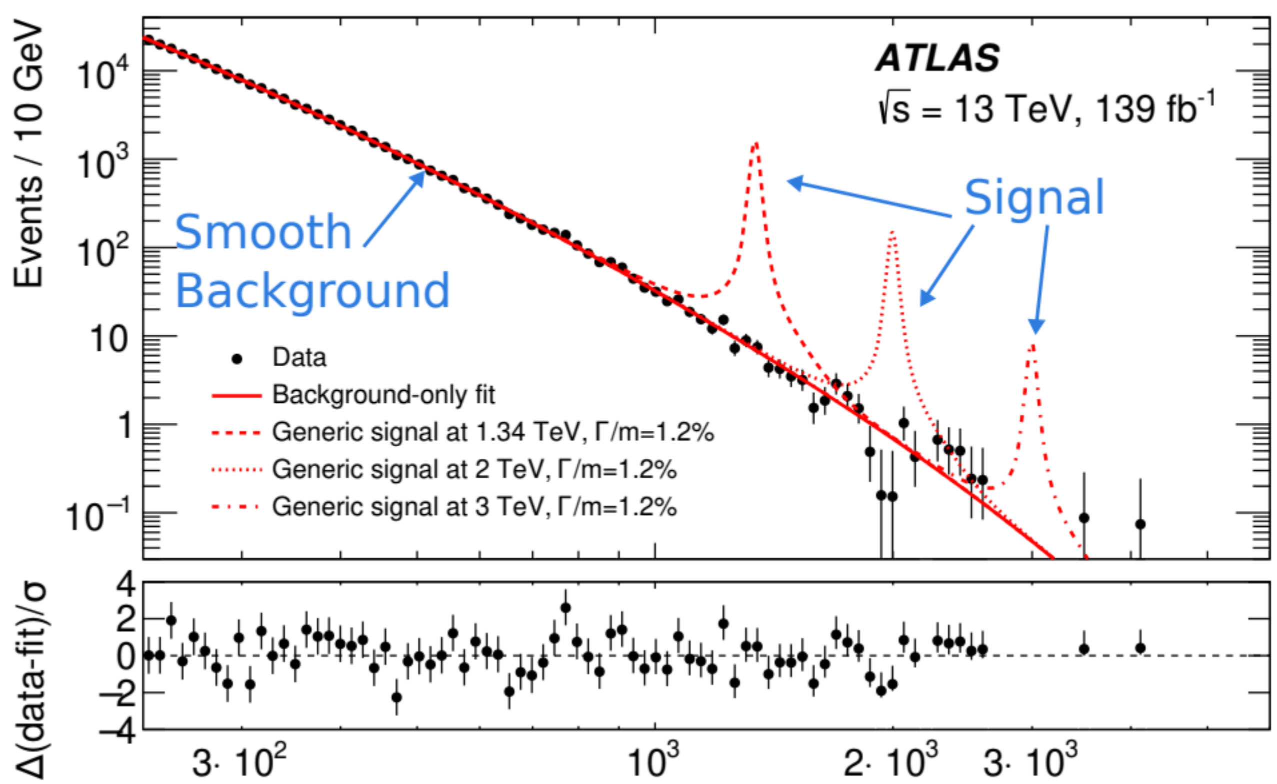
Exotics



SUSY

New Physics — Hiding in Small & Subtle Effects?

Z' in dileptons

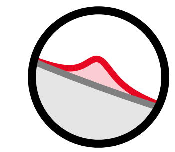
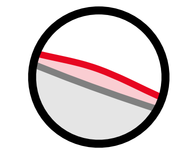
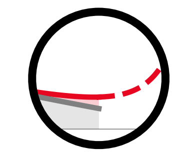
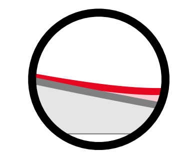


Excluding particles with mass ~ 4-5 TeV

“bump hunting”

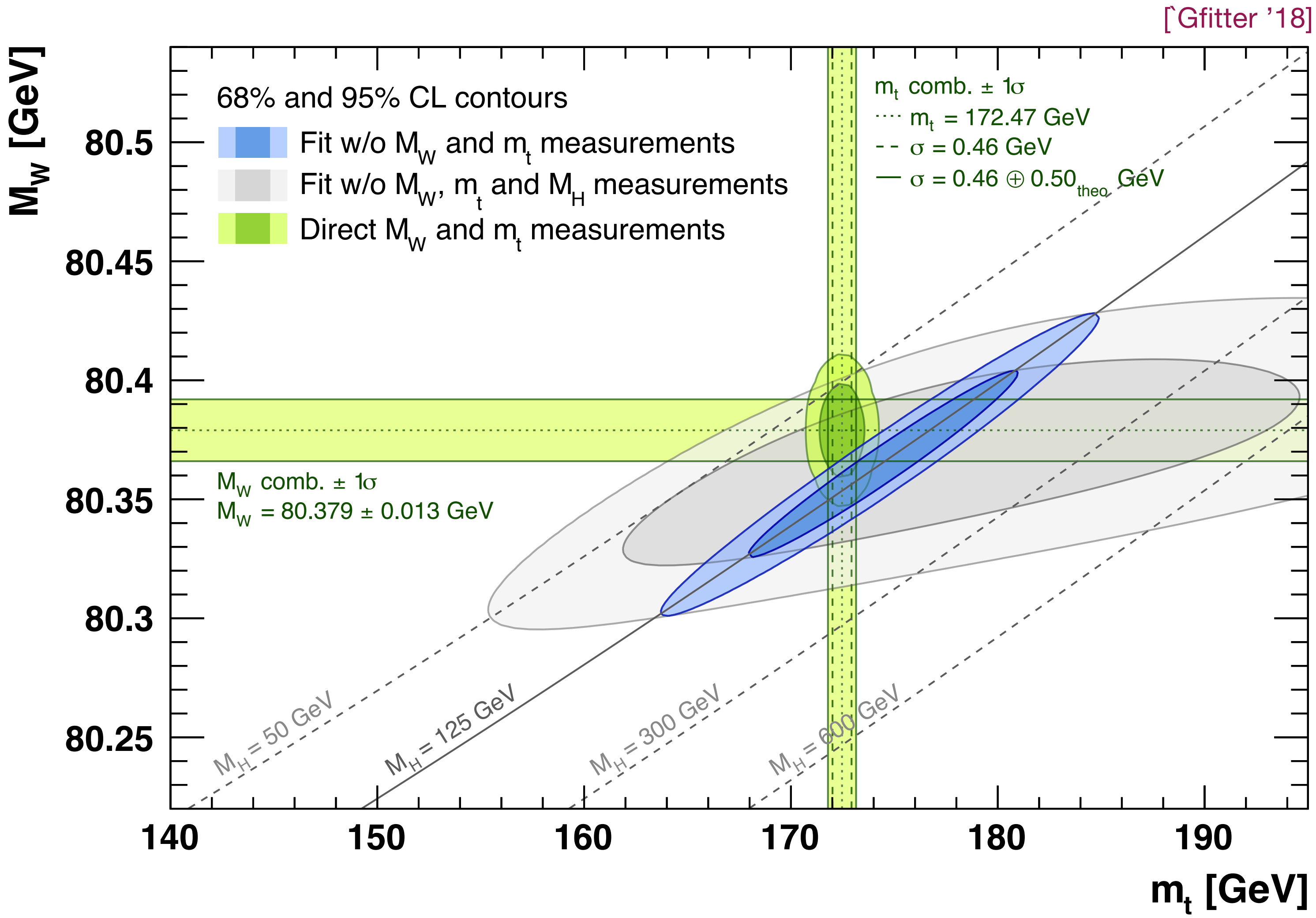
➡ *little to no theory input needed*

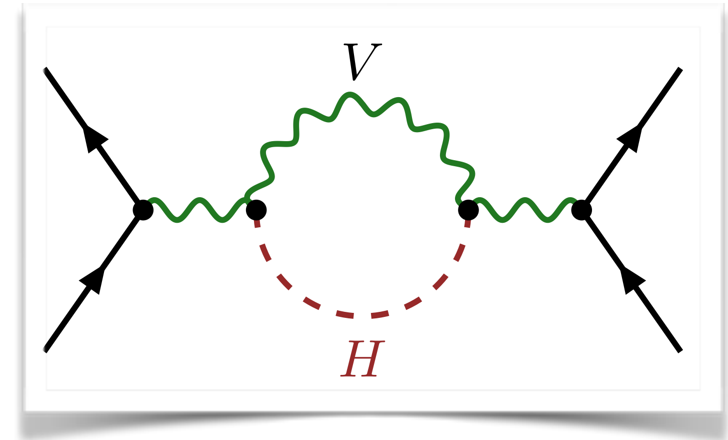
What if?

- *interaction weak* 
- *wide resonance* 
- *too heavy* 
- *shape distortion* 
- *challenging signature*
- …

requires solid understanding and control of SM backgrounds

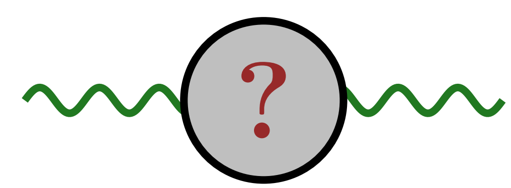
PRECISION MEASUREMENTS & INDIRECT SEARCHES





- constrained system

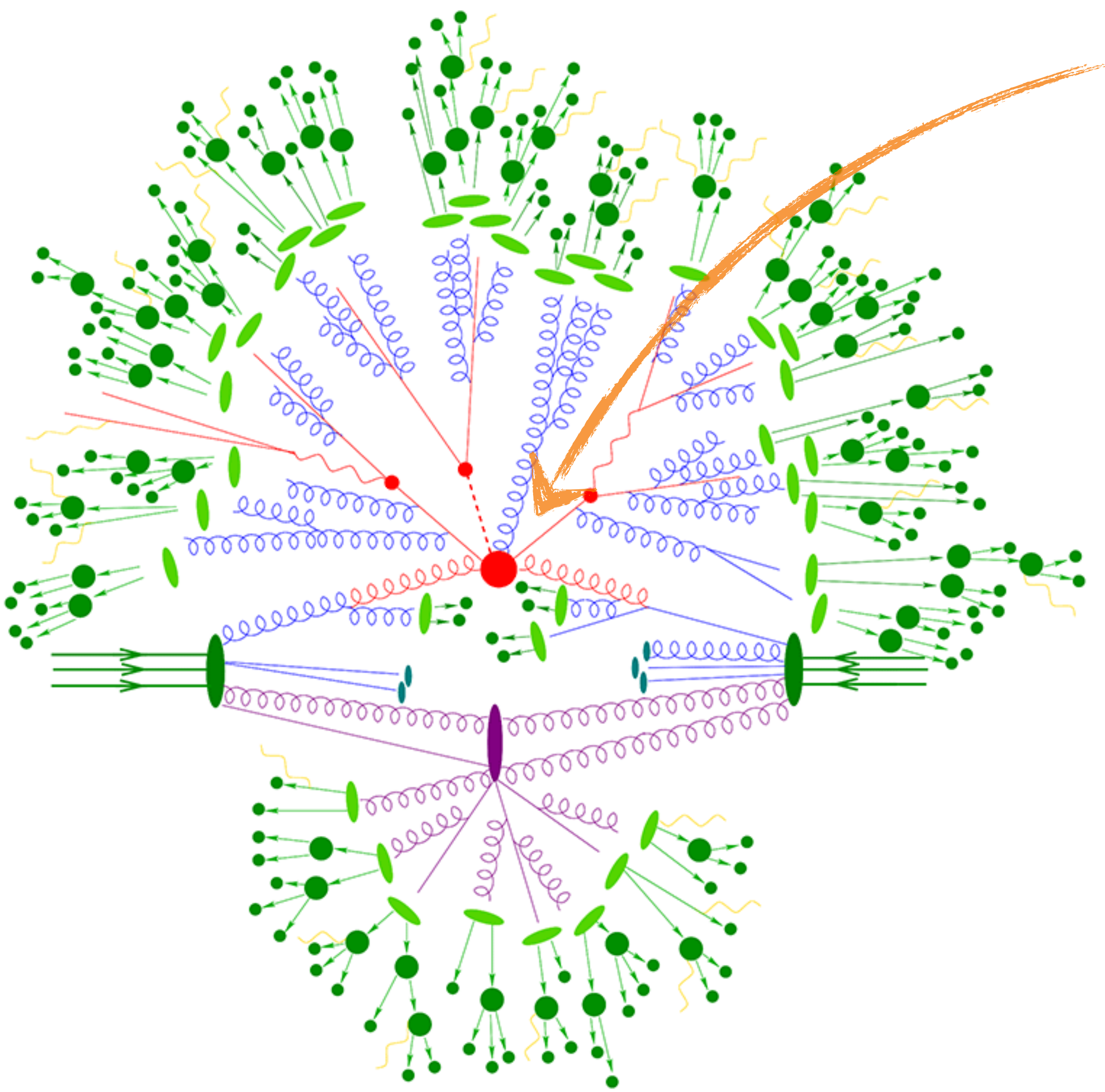
➡ self consistent?

➡ 

m/GeV	measured	fit value
m_{t}	172.47 ± 0.68	176.4 ± 2.1
M_{H}	125.1 ± 0.2	90^{+21}_{-18}
M_{W}	80.379 ± 0.013	80.354 ± 0.007

precision theory
for
"standard candles"

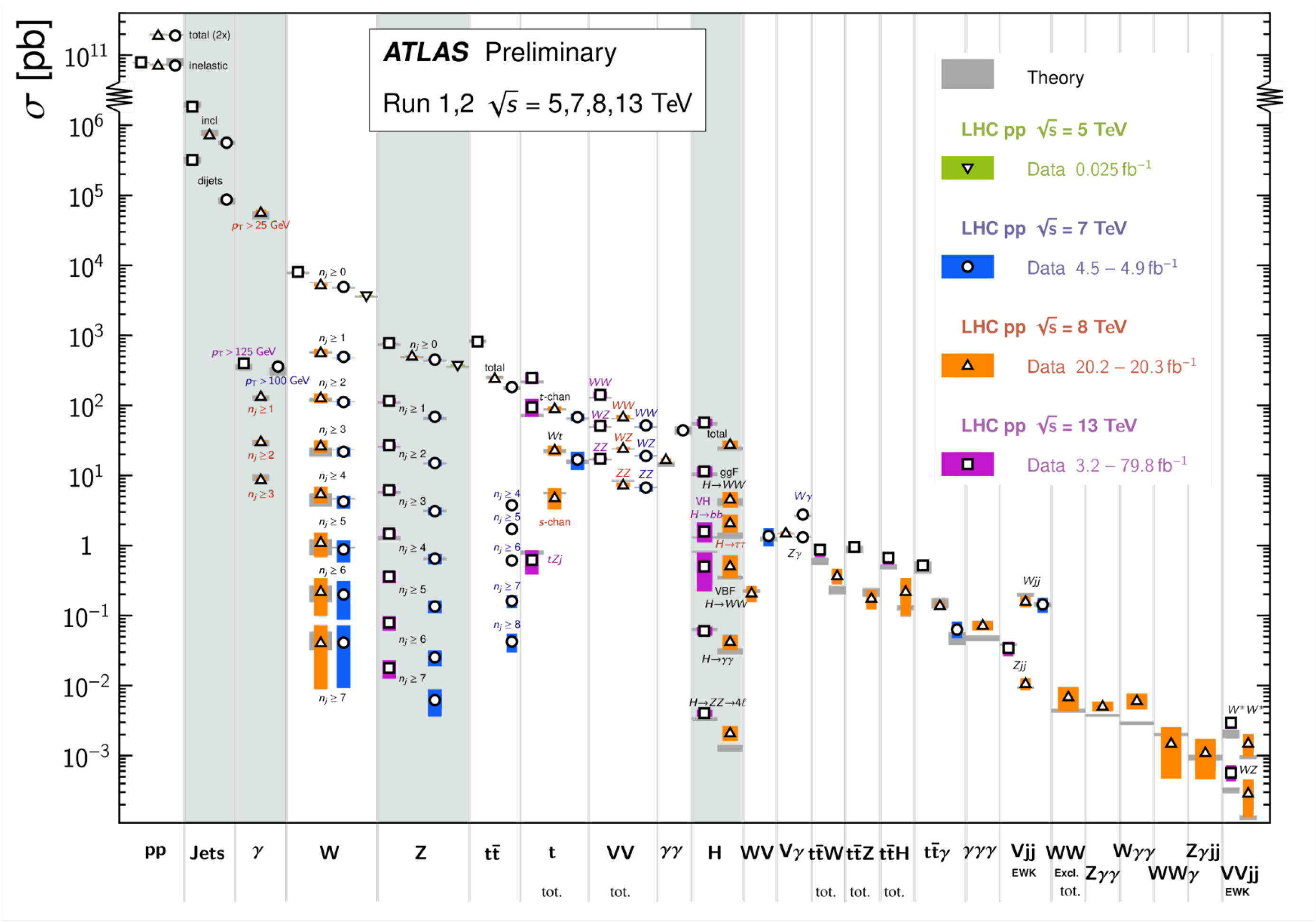
High-Precision Theory Predictions!



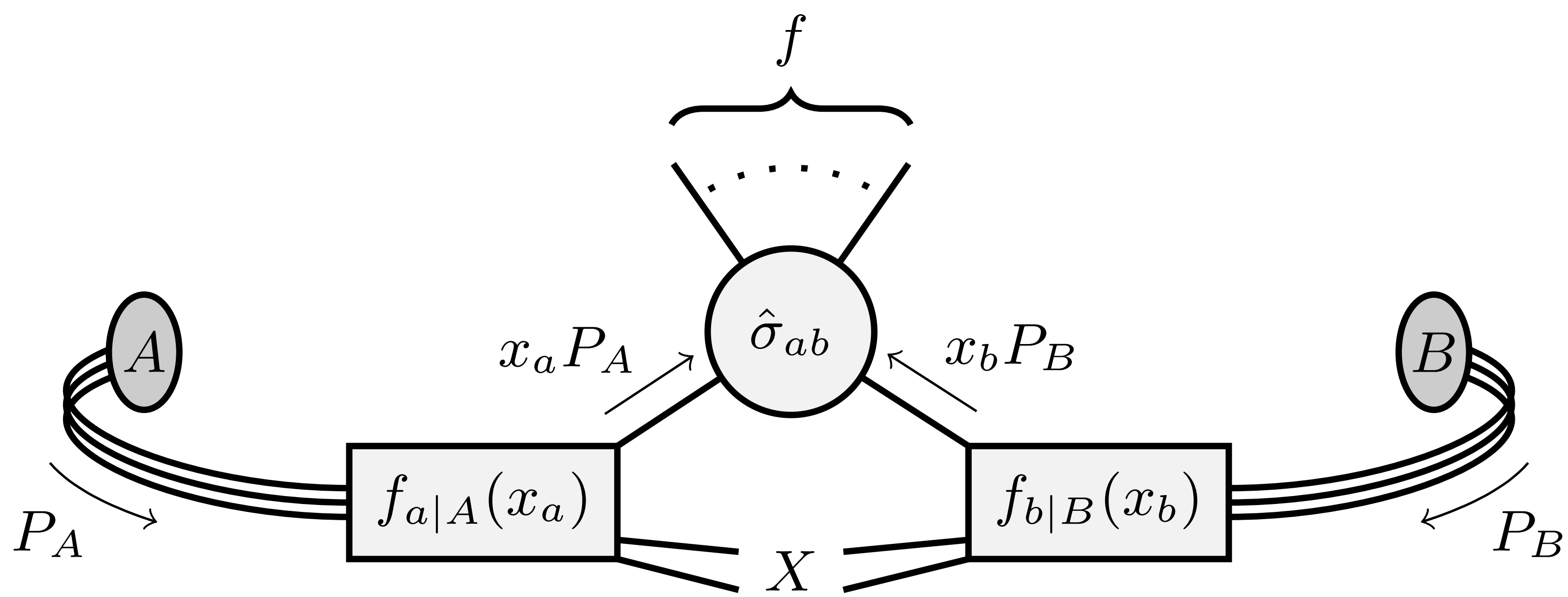
- (HL-)LHC — per-cent level!
- Focus — clean processes with high momentum transfer
  - *perturbative QCD*
- with $\alpha_s \sim 0.1$
  - $NLO \sim \mathcal{O}(10\%),\ NNLO \sim \mathcal{O}(1\%)$
  - *exceptions: Higgs, new channels, …*
- predictions as close as possible to the experiment
  - *fiducial cross sections & differential distributions*

The Plan.

1. *Predictions for the LHC*
2. *Phenomenological Applications*
    - *Jets — Photon — Z-boson — Higgs*



Theory Predictions for the LHC



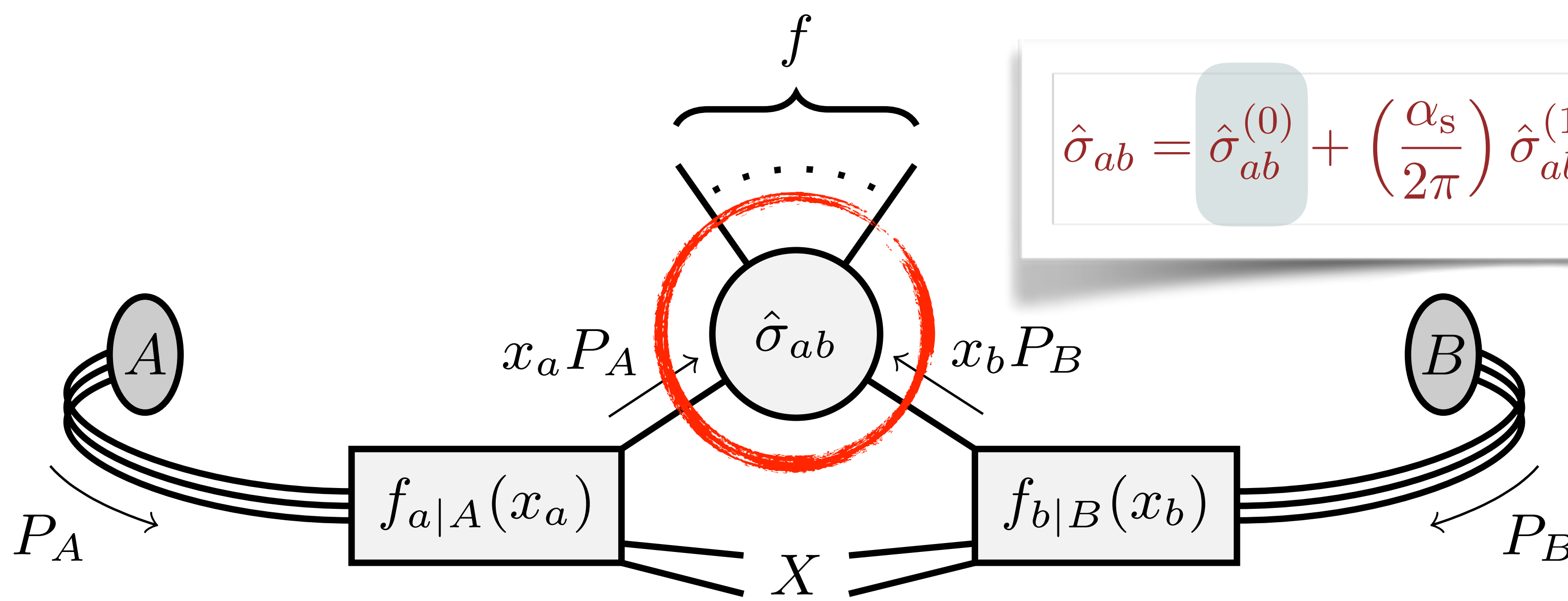
$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\; f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; \left(1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\right)$$

parton distribution functions
(in principle, improvable)
few % at LHC

hard scattering
(systematically improvable)
aim for few % level!

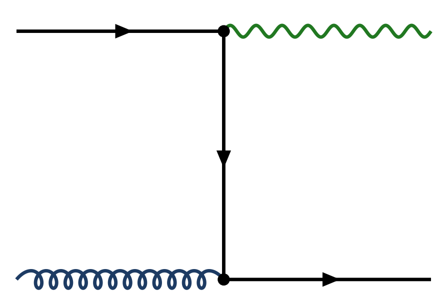
non-perturbative effects
(no good understanding)
$\sim$ few %?

Hard Scattering — Perturbation Theory



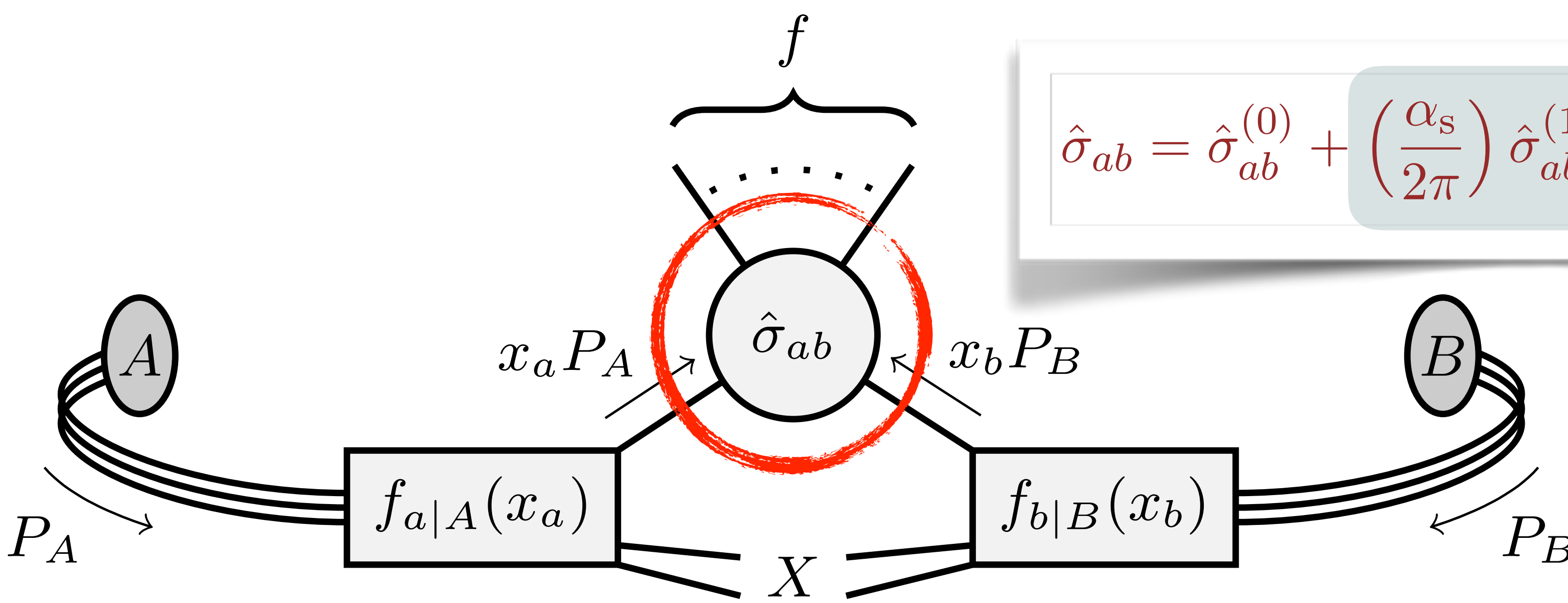
$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}_{ab} + \ldots$$

leading order (LO)



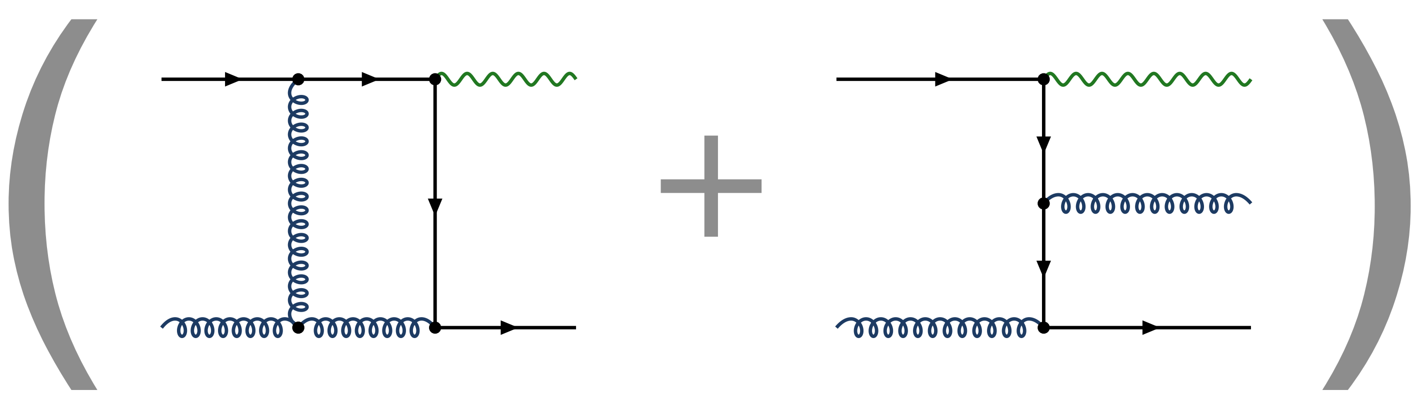
"tree level"

Hard Scattering — Perturbation Theory



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$

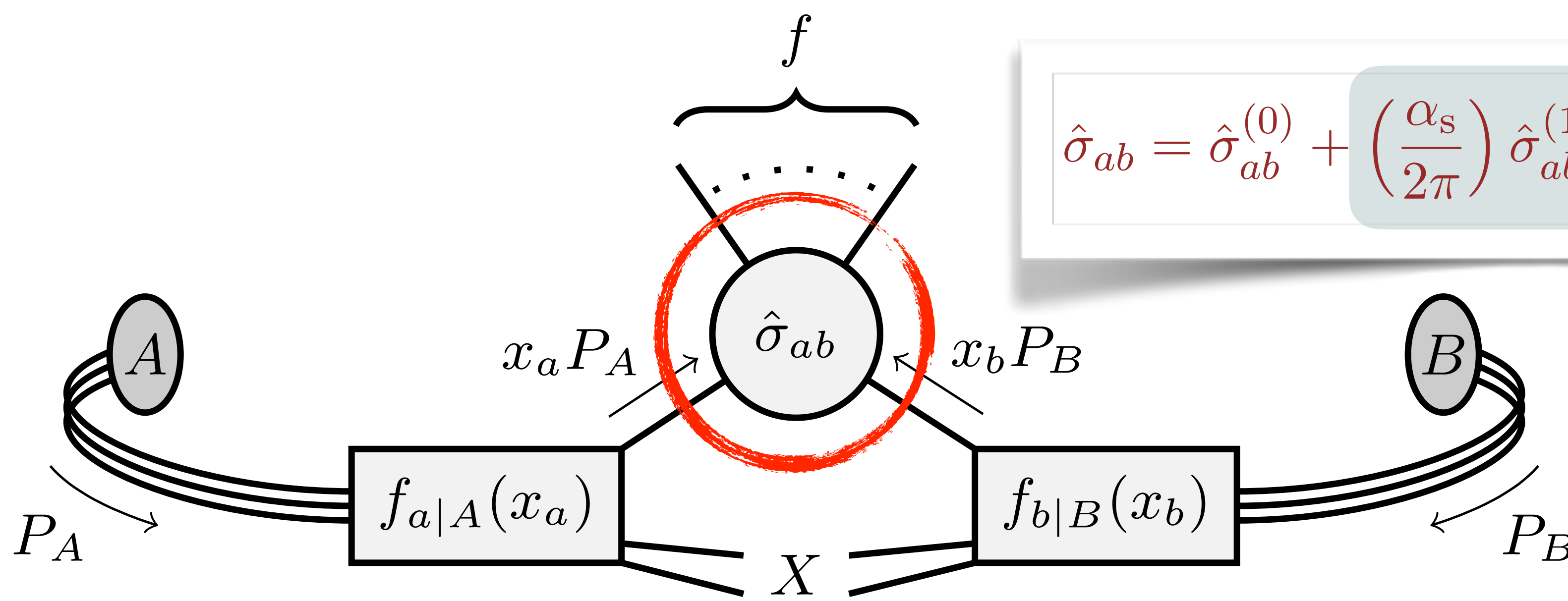
next-to-leading order (NLO)



"virtual" *"real"*

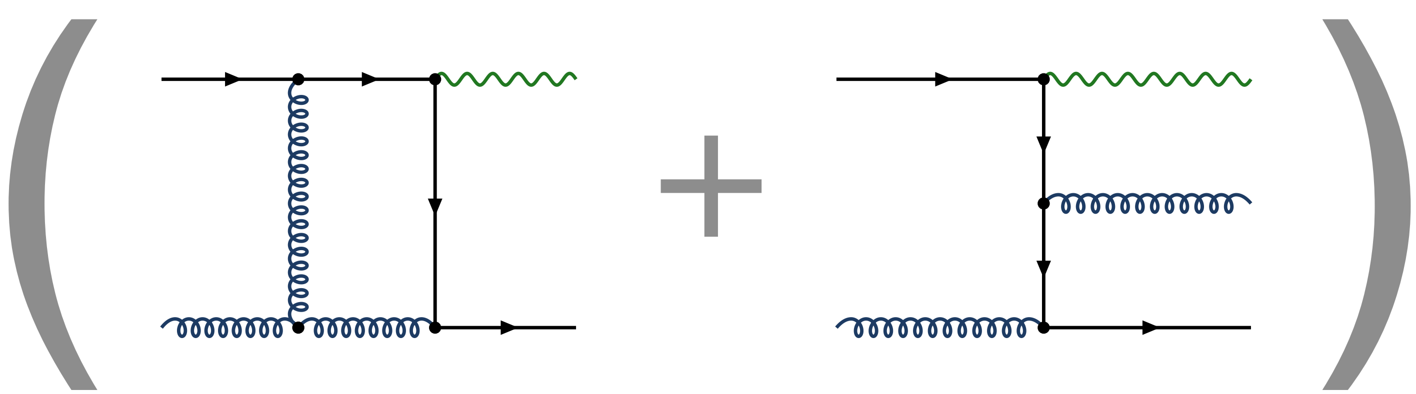
infrared singularities

Hard Scattering — Perturbation Theory



$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}_{ab} + \ldots$$

next-to-leading order (NLO)



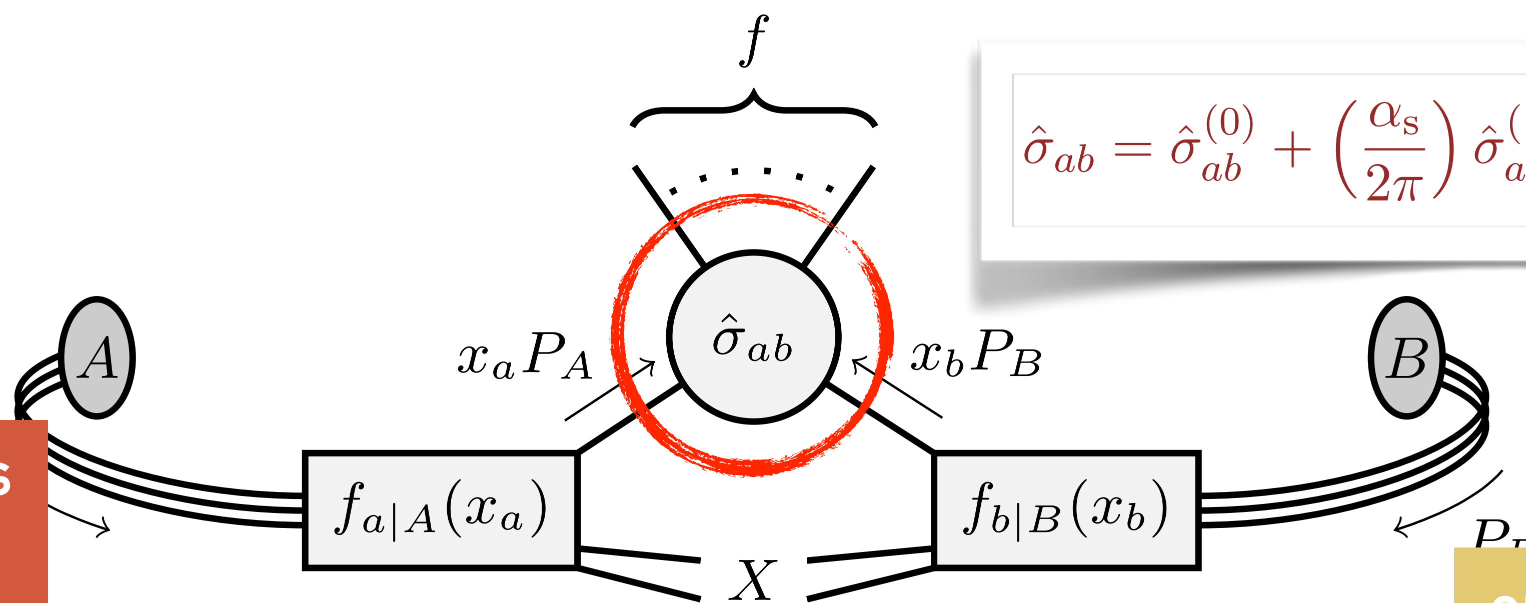
"virtual" *"real"*

one-loop amplitudes
(all master integrals known, well understood: log, Li_2)

infrared singularities

IR subtraction
(conceptually solved: CS, FKS, …)

Hard Scattering — Perturbation Theory

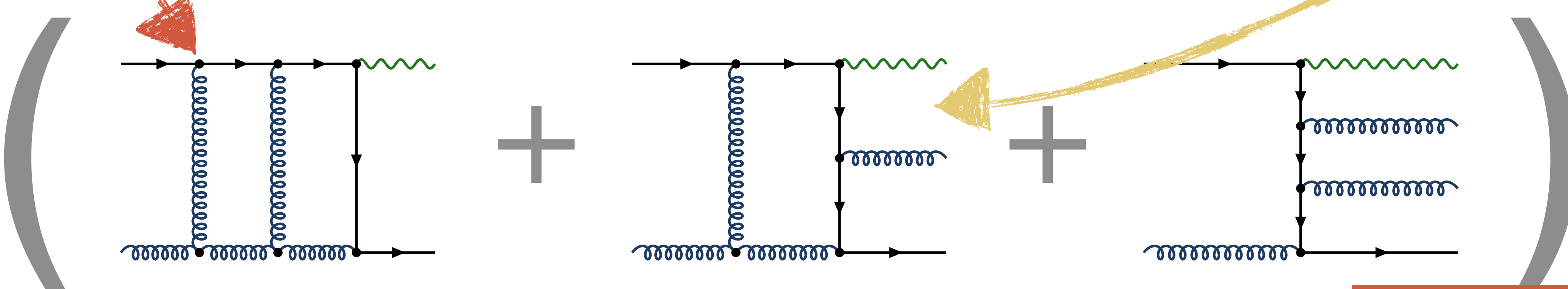


$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots$$

two-loop amplitudes
(new class of functions, combinatoric & algebraic complexity)

one-loop amplitudes
(evaluation in singular & unstable regions)

next-to-next-to-leading order (NNLO)



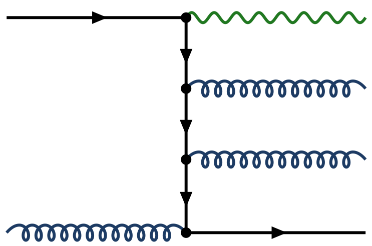
"double virtual" + *"real-virtual"* + *"double re[al]"*

IR subtraction
(involved IR structure, numerical stability, construction)

infrared singularities

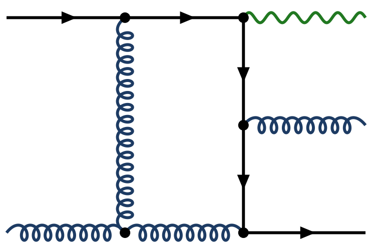
Anatomy of NNLO Calculations

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_{Z+3}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$



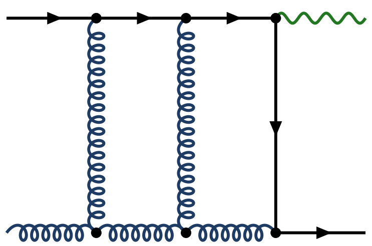
- single-unresolved
- double-unresolved

$$+ \int_{\Phi_{Z+2}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$$



- single-unresolved
- $1/\epsilon^2$, $1/\epsilon$

$$+ \int_{\Phi_{Z+1}} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$



- $1/\epsilon^4$, $1/\epsilon^3$, $1/\epsilon^2$, $1/\epsilon$

$\sum$ finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

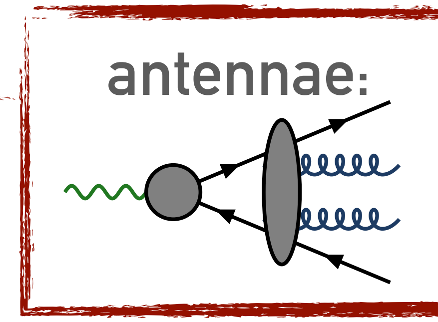
NNLO using Antenna Subtraction

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \left(\mathrm{d}\sigma^{\text{RR}}_{\text{NNLO}} - \mathrm{d}\sigma^{\text{S}}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+2}} \left(\mathrm{d}\sigma^{\text{RV}}_{\text{NNLO}} - \mathrm{d}\sigma^{\text{T}}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+1}} \left(\mathrm{d}\sigma^{\text{VV}}_{\text{NNLO}} - \mathrm{d}\sigma^{\text{U}}_{\text{NNLO}} \right)$$

$$\sum \quad \text{finite} \quad - 0$$



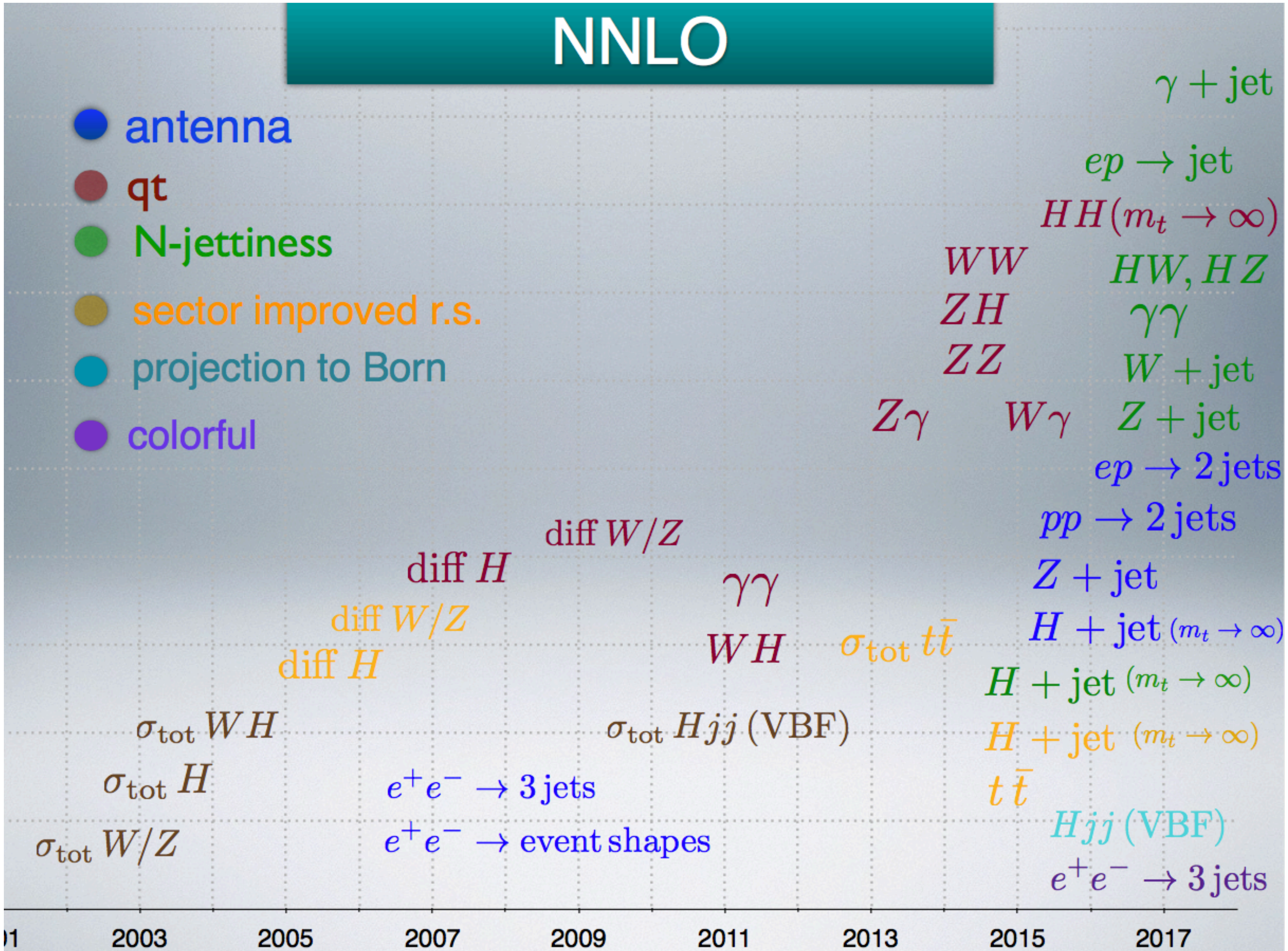
- $\mathrm{d}\sigma^{\text{S}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\text{T}}_{\text{NNLO}}$: mimic $\mathrm{d}\sigma^{\text{RR}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\text{RV}}_{\text{NNLO}}$ in unresolved limits
- $\mathrm{d}\sigma^{\text{T}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\text{U}}_{\text{NNLO}}$: *analytic* cancellation of poles in $\mathrm{d}\sigma^{\text{RV}}_{\text{NNLO}}$, $\mathrm{d}\sigma^{\text{VV}}_{\text{NNLO}}$

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder,
T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues,
J. Pires, D. Walker, J. Whitehead

Processes computed using the antenna subtraction method

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ NNLO
  - $\hookrightarrow V \to \ell\bar{\ell} \qquad (V = \mathrm{Z}/\gamma^*, \mathrm{W}^\pm)$
- $\mathrm{pp} \to$ jets (inc. jets, 2j) @ NNLO
- $\mathrm{pp} \to \gamma + j$ @ NNLO
- $\mathrm{ep} \to 1j$ @ $\mathrm{N^3LO}$
- $\mathrm{ep} \to 2j$ @ NNLO
- $\mathrm{e^+e^-} \to 3$ jets @ NNLO
- $\mathrm{pp} \to \mathrm{H}$ (ggH) @ $\mathrm{N^3LO}$
- $\mathrm{pp} \to \mathrm{H} + j$ (ggH) @ NNLO
- $\mathrm{pp} \to \mathrm{H} + 2j$ (VBF) @ NNLO
  - $\hookrightarrow \mathrm{H} \to \gamma\gamma, \tau\tau, V\gamma, VV$
- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  - $\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...



[slide by Gudrun Heinrich]

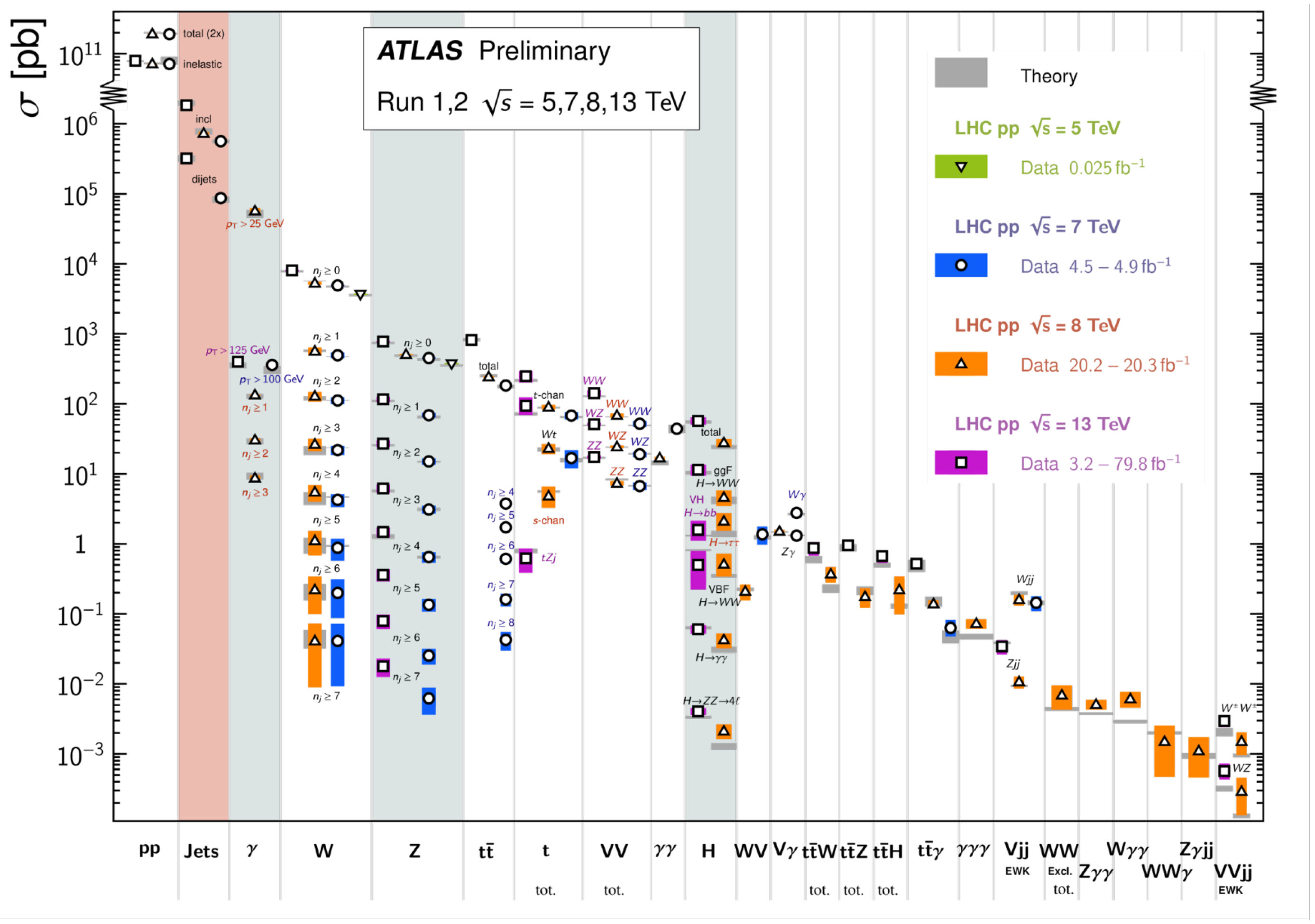
$W + \text{jet}$ ('17)
$\mathrm{H}jj$(VBF) ('18)
$\mathrm{HH}jj$(VBF) ('19)
$\mathrm{t\bar{t}}$ ('19)
$\gamma + \text{jet}$ ('19)

combination of (production) $\times$ (decay):

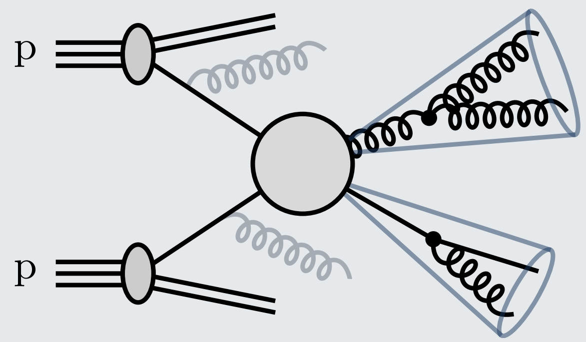
- (t-channel single-t) $\times$ $(\mathrm{t} \to \mathrm{W^+ b})$ .... [Berger, Gao, Yuan, Zhu '16]
- $(V\mathrm{H}) \times (\mathrm{H} \to \mathrm{bb})$ ....[Ferrera, Somogyi, Tramontano '17]
- $(\mathrm{WH}) \times (\mathrm{H} \to \mathrm{bb})$ .... [Caola, Luisoni, Melnikov, Röntsch '17]
- $(\mathrm{t\bar{t}}) \times (\mathrm{t} \to \mathrm{Wb})^2$ .... [Behring, Czakon, Mitov, Papanastasiou, Poncelet, '19]

The Plan.

1. *Predictions for the LHC*
2. *Phenomenological Applications*
   - *Jets — Photon — Z-boson — Higgs*



Jet Production at the LHC

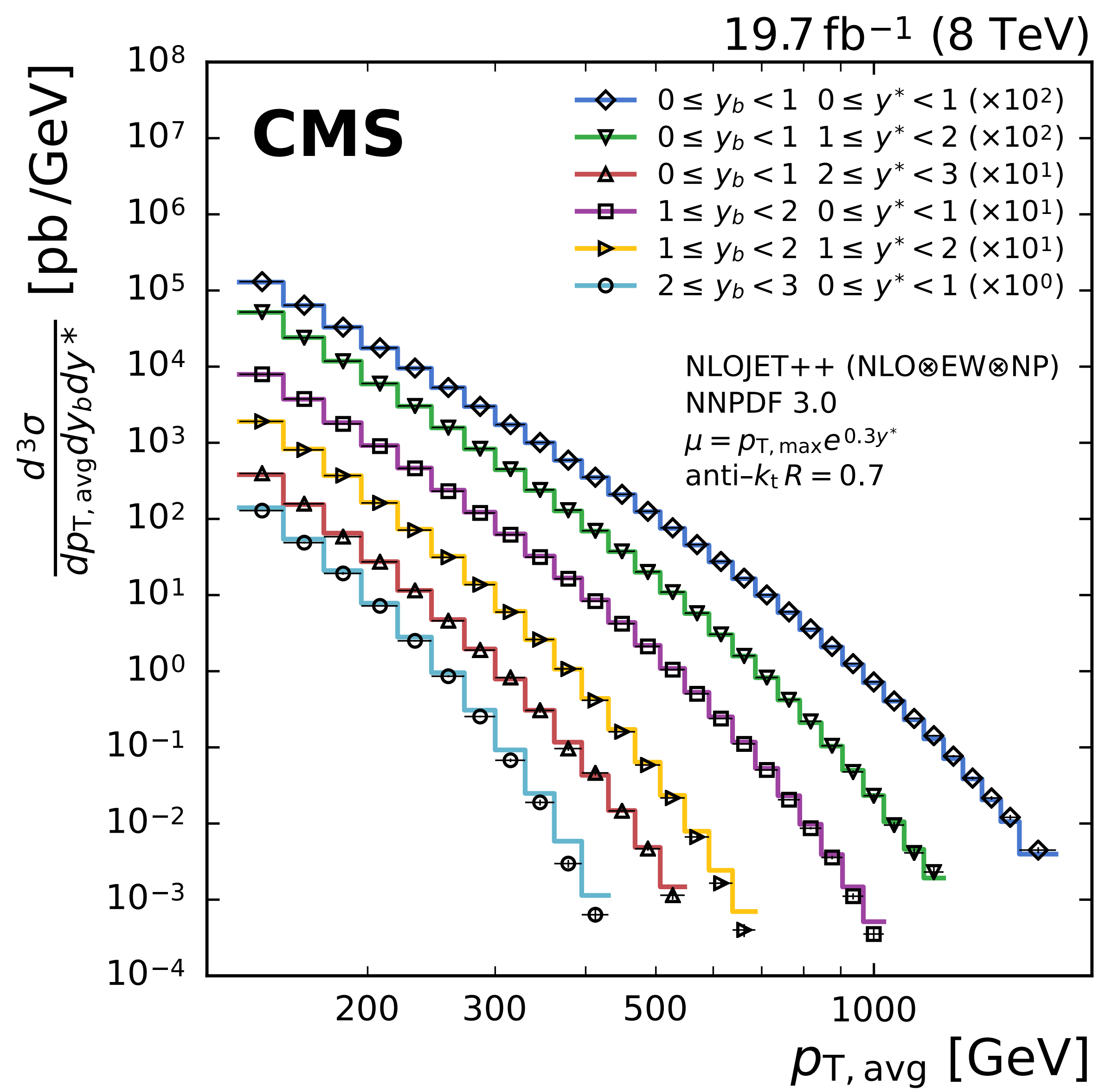


$\mathrm{p} + \mathrm{p} \rightarrow \text{jet(s)} + X$

- jets produced in abundance
- precise measurements $(p_{\mathrm{T},j} \gtrsim 20\ \mathrm{GeV})$
- wide kinematic range accessible

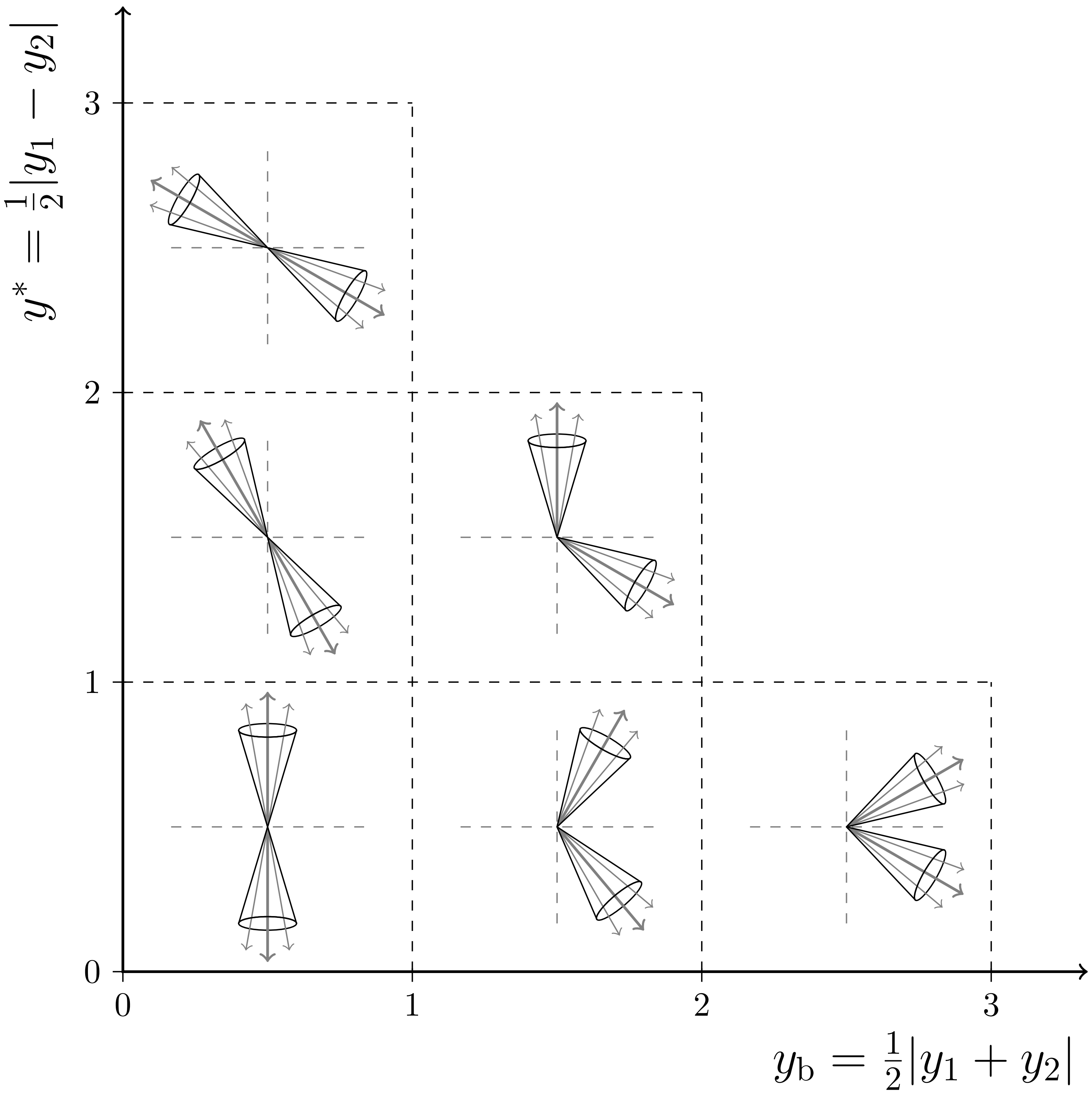
- test perturbative QCD
  ↪ study scale choices
- constrain PDFs
  ↪ sensitive to *gluon*
  ↪ probe wide x-range
- $\alpha_s(M_Z)$ and *running*
- search for BSM physics

high-precision predictions mandatory!



[CMS, Eur. Phys. J. C77, 746 '17]

Triple-Differential Cross Section



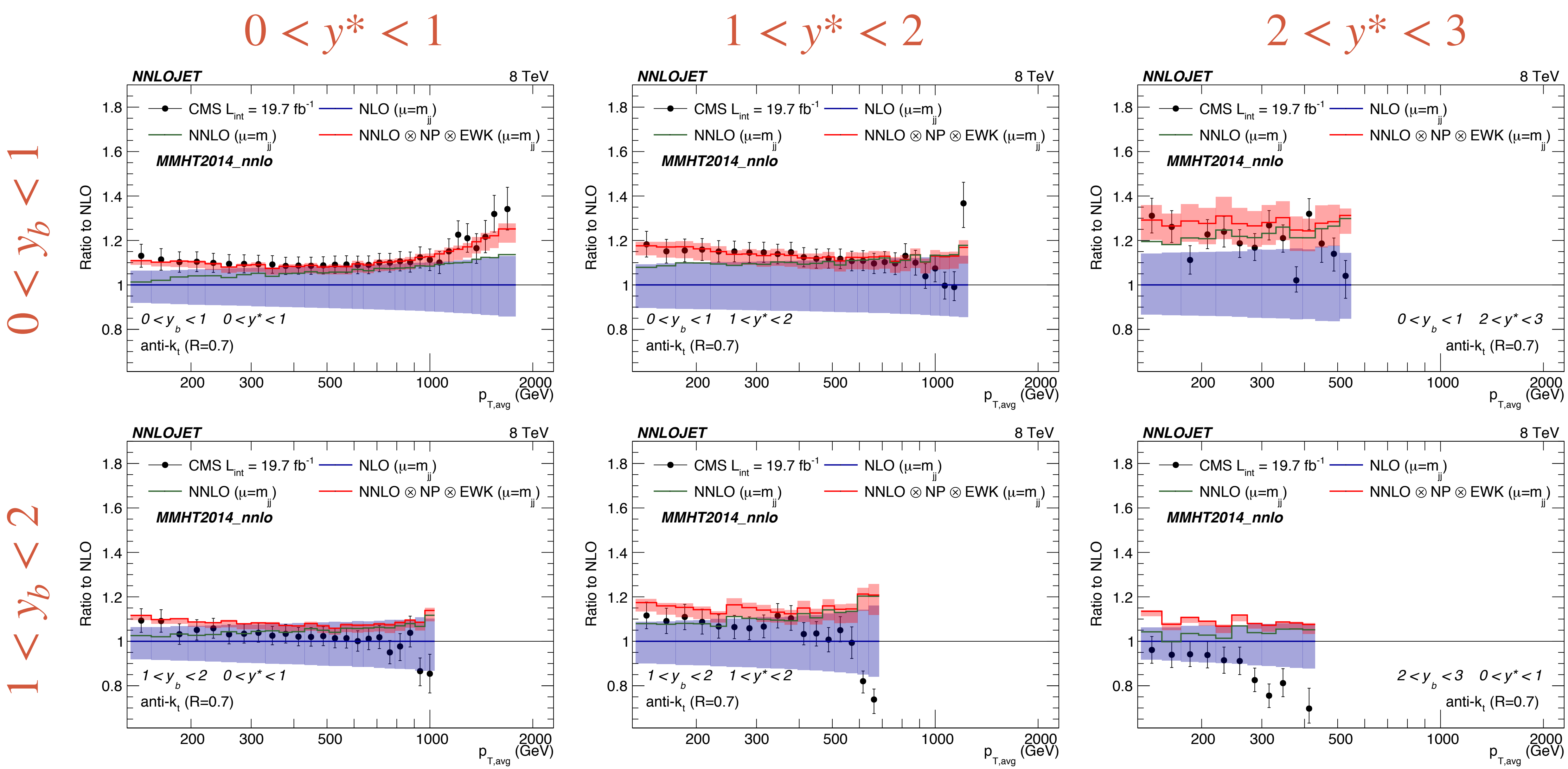
$$\frac{d^3\sigma}{dp_{T,avg}\,dy^*\,dy_b}$$

- study different kinematic regimes
- disentangle momentum fractions x_1 & x_2

Triple-Differential Cross Section @ NNLO

[Currie, Glover, Pires '16]

[Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '19]



NLO NNLO ⊗ NP ⊗ EWK

improved description of data & reduced uncertainties!

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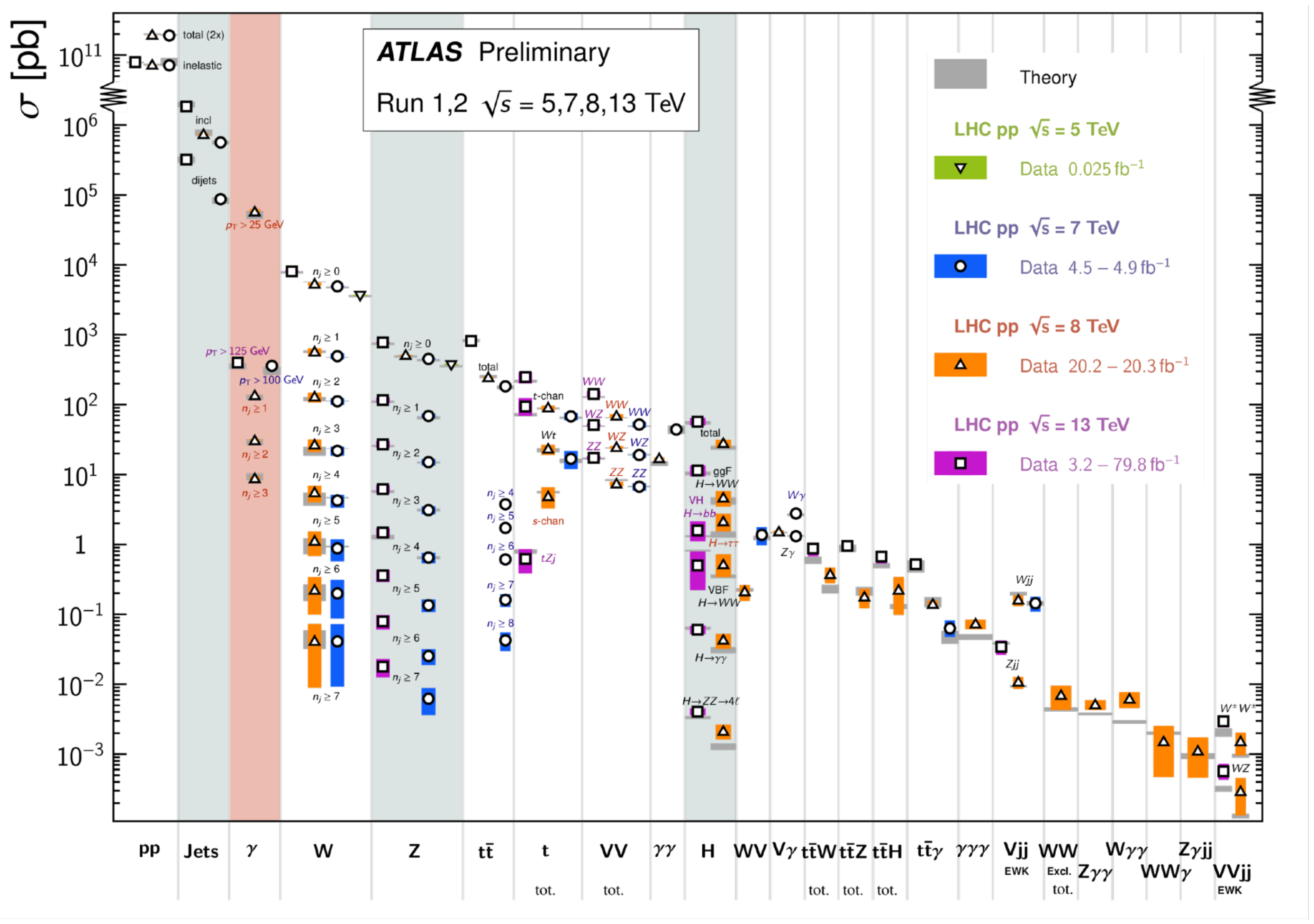
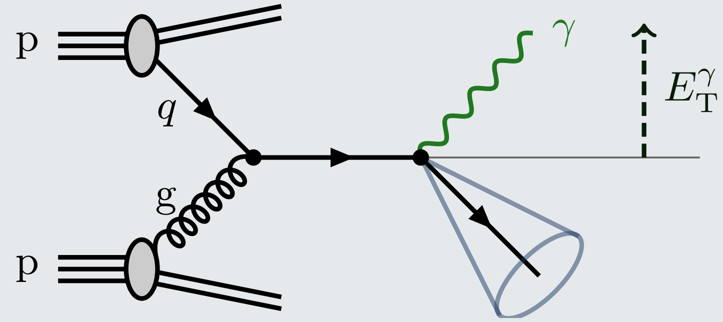
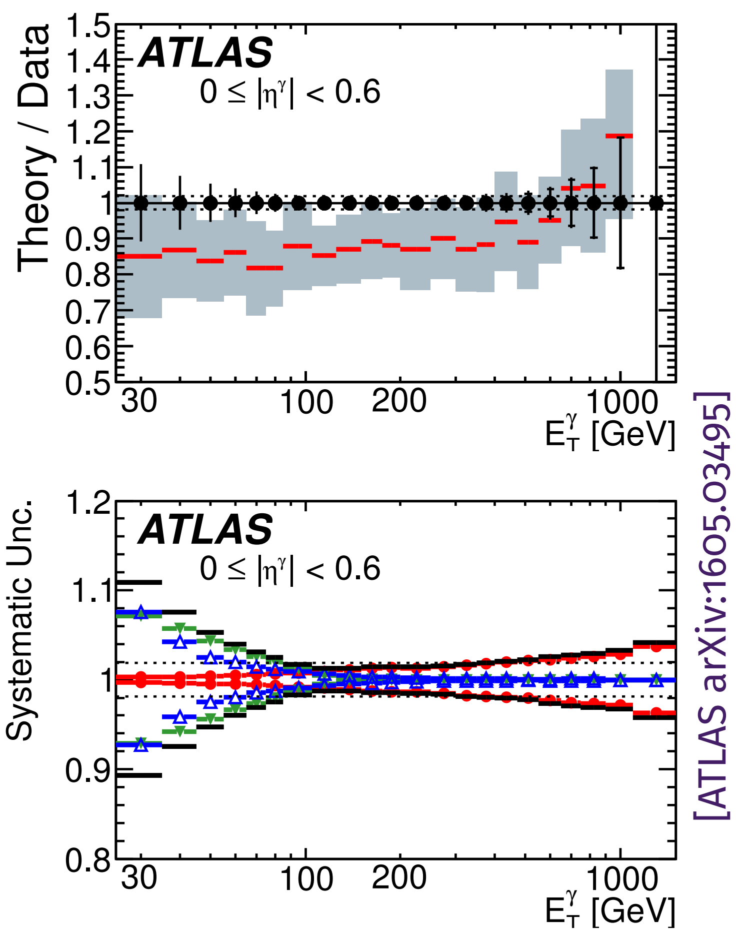


PHOTON PRODUCTION



$\mathrm{p}\,\mathrm{p} \to \gamma + X$

- highest-rate electroweak process @ LHC
- photon as probe of hard scattering
  - $\rightsquigarrow$ sensitivity to α_s, gluon PDF



JetPhox (NLO QCD)

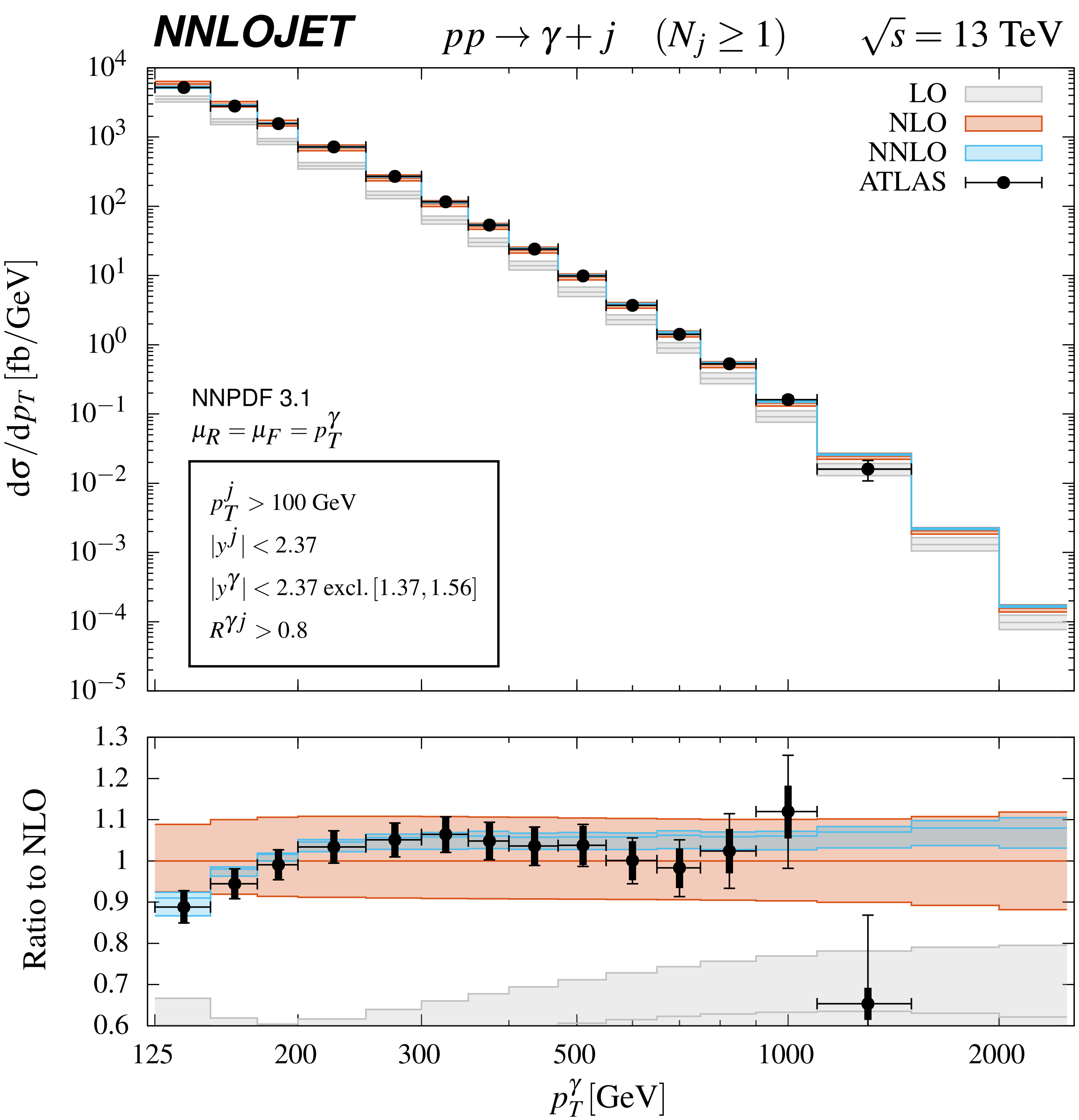
[Catani, Fontannaz, Guillet, Pilon '02]

$\hookrightarrow$ tension between theory vs. data

$\hookrightarrow$ large scale uncertainties: $\sim \pm 10\%$

- experimental uncertainties $\lesssim \pm 3$–5%
  - $\hookrightarrow$ smaller than NLO theory
  - $\Rightarrow$ NNLO QCD needed!

PHOTON + JET @ 13 TEV



[Chen, Gehrmann, Glover, Höfer, AH '19]

hybrid isolation

NLO ($\rightsquigarrow 1$)

- $+40\%$ corrections
- $\pm 10\%$ uncertainties

NNLO

- $\sim 5\%$ corrections
- *shape distortions*
- $\lesssim 5\%$ uncertainties

- previous NNLO calculation

τ_N [Campbell, Ellis, Williams '17]

(dynamical cone isol.)

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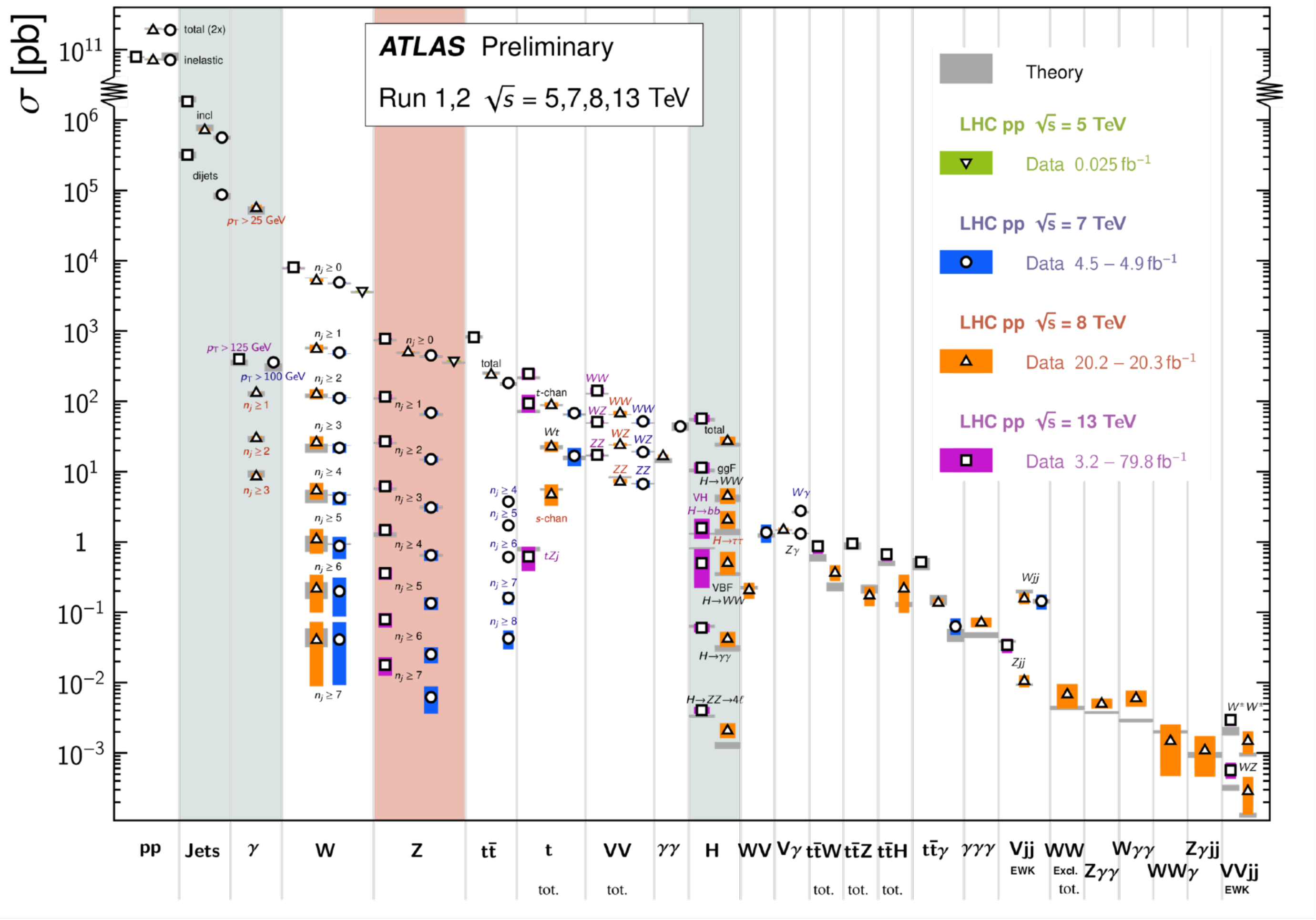
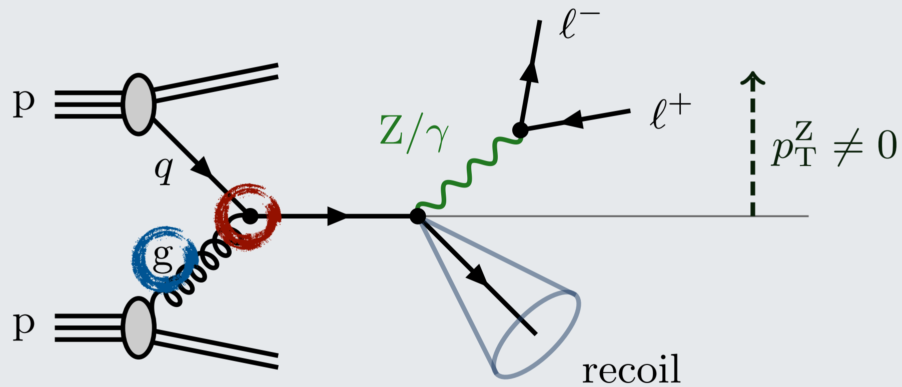


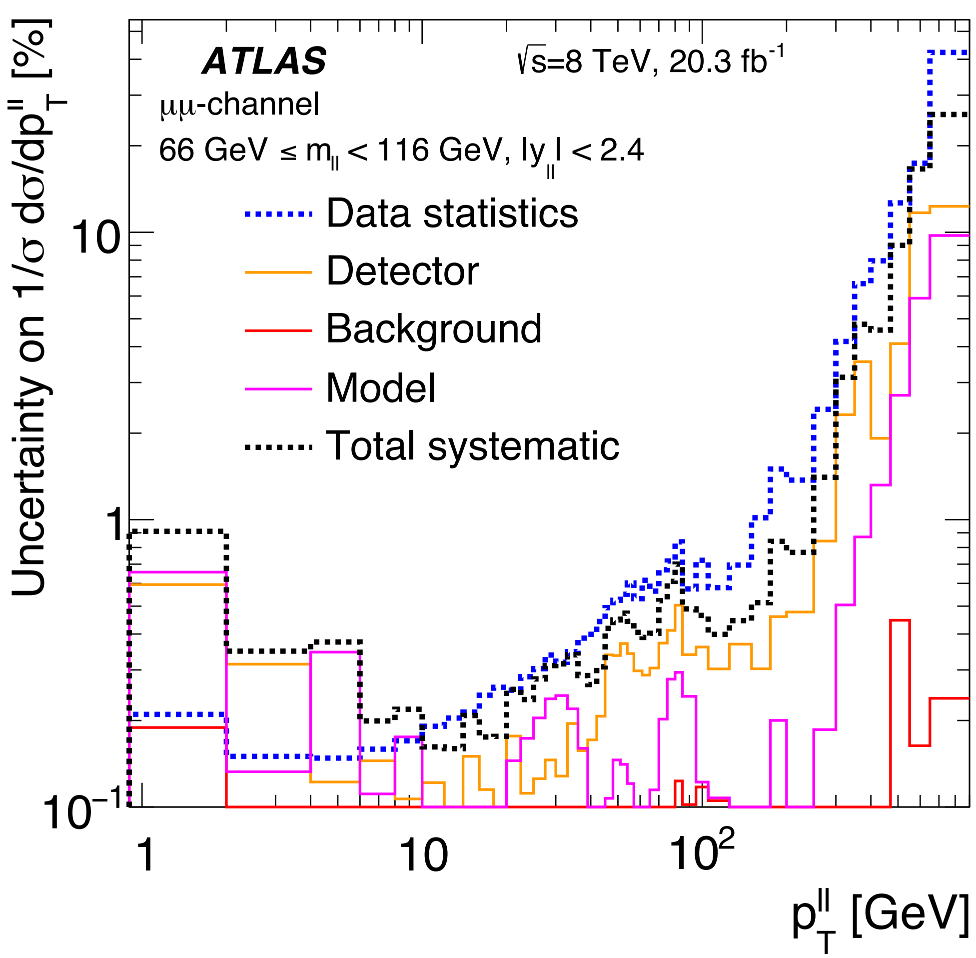
Towards per-cent Phenomenology



$$\mathrm{p\,p} \to \mathrm{Z}/\gamma^* + X \to \ell^-\ell^+ + X$$

- large cross section
- clean leptonic signature

recoil $\rightsquigarrow$ sensitivity to α_s gluon PDF



[Eur. Phys. J. C **76** (2016) no.5, 291]

- only reconstruct ℓ^+, ℓ^-

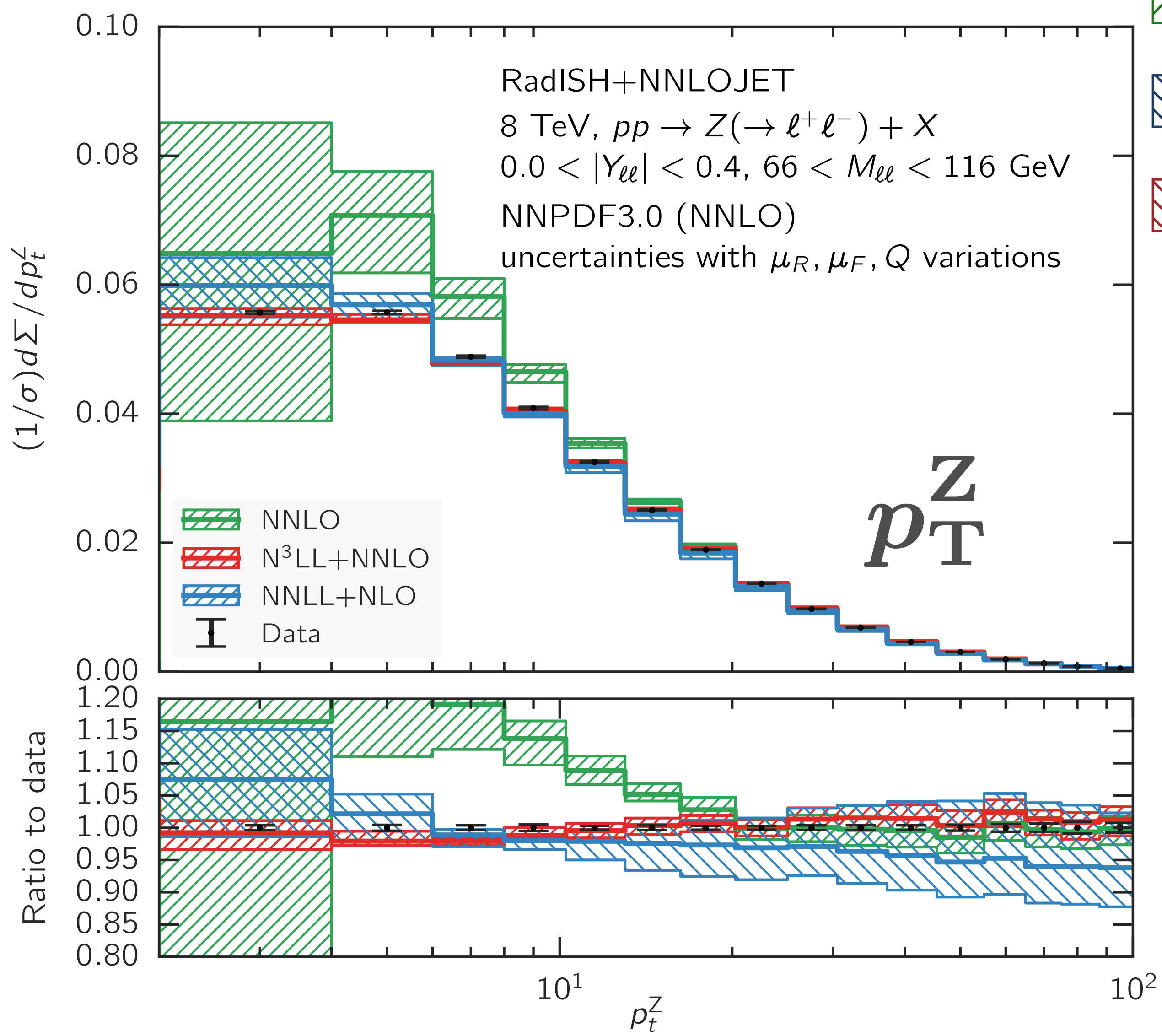
  $\rightsquigarrow$ sub-% accuracy!
- important constraints in PDF fits

  [Boughezal et al. '17]
- probe various theory aspects:

very low p_T	non-pert. effects
low p_T	resummation
interm. p_T	fixed order
high p_T	EW Sudakov logs

Fixed Order + Resummation — NNLO + N^{3}LL

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Torrielli '18]



NNLO fixed order: $\mathcal{O}(\alpha_s^3)$

NNLL+NLO ($\leftrightarrow$ DYRes)

N^{3}LL+NNLO

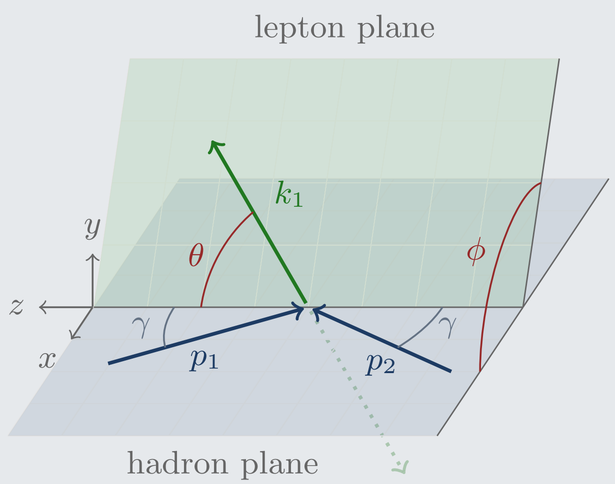
Data (ATLAS)

- substantially reduced uncertainties
  $\hookrightarrow$ 3–5% (more than $\times 2$)
- strong distortion of shape
  $\hookrightarrow$ (data vs. theory) much improved
- similarly: $m_{\ell\ell}$ & y^Z bins, ϕ^*_η

also: p_T^W & p_T^W / p_T^Z (for M_W)

[Bizoń, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Walker `19]

Angular Coefficients



$$\mathrm{p\,p} \to \mathrm{Z}/\gamma^* + X \to \ell^- \ell^+ + X$$

- lepton angular distributions (θ, ϕ)
- probe production dynamics & polarisation
- M_W & $\sin^2\theta_\mathrm{w}$ measurement

Angular coefficients: $A_i(p_\mathrm{T}^\mathrm{Z}, y^\mathrm{Z}, m_{\ell\ell})$ $\quad Y_{lm}(\theta,\phi),\ l = 0,1,2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \frac{3}{16\pi}\frac{\mathrm{d}\sigma^{\text{unpol.}}}{\mathrm{d}^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta)$$
$$+ A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi)$$
$$+ A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi)$$
$$+ A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\}$$

$A_i(q)$ $+\ \sigma^{\text{unpol.}}$

production dynamics

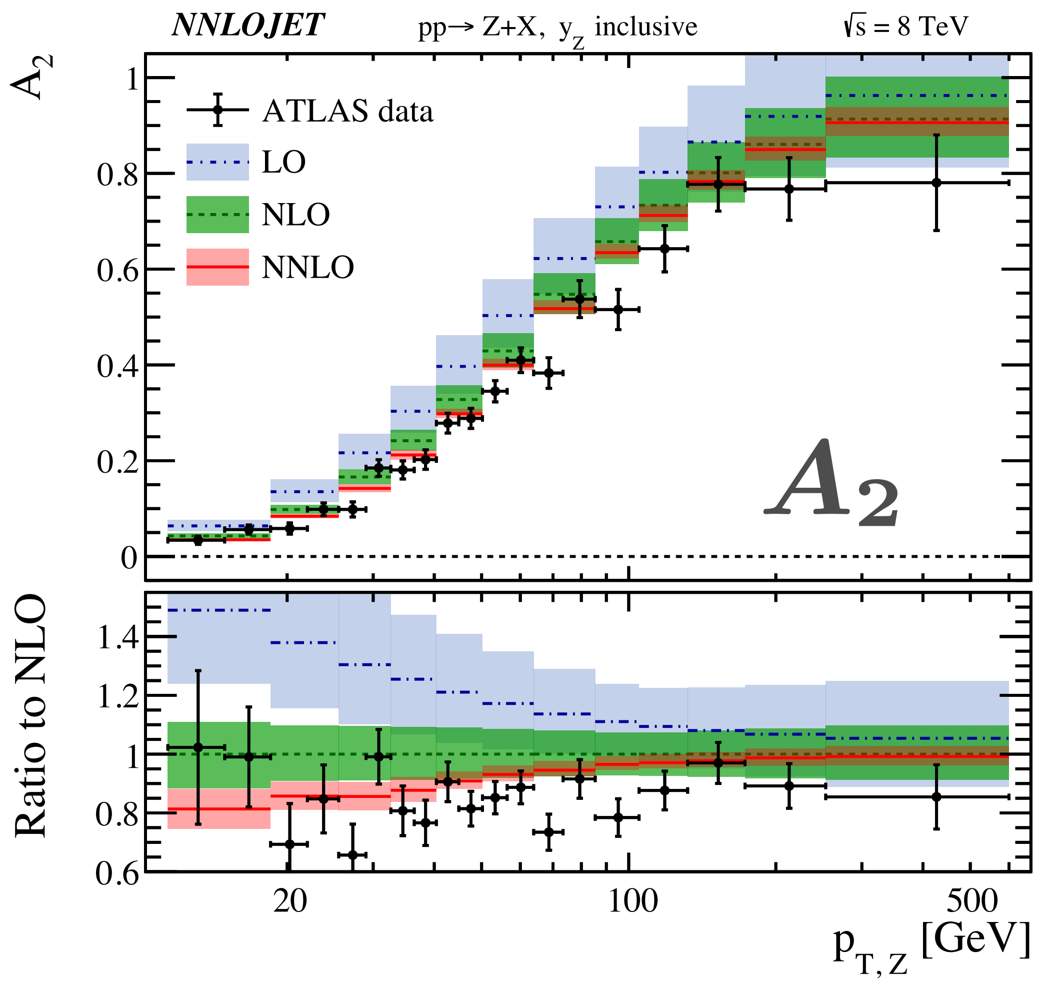
$Y_{lm}(\theta,\phi)$

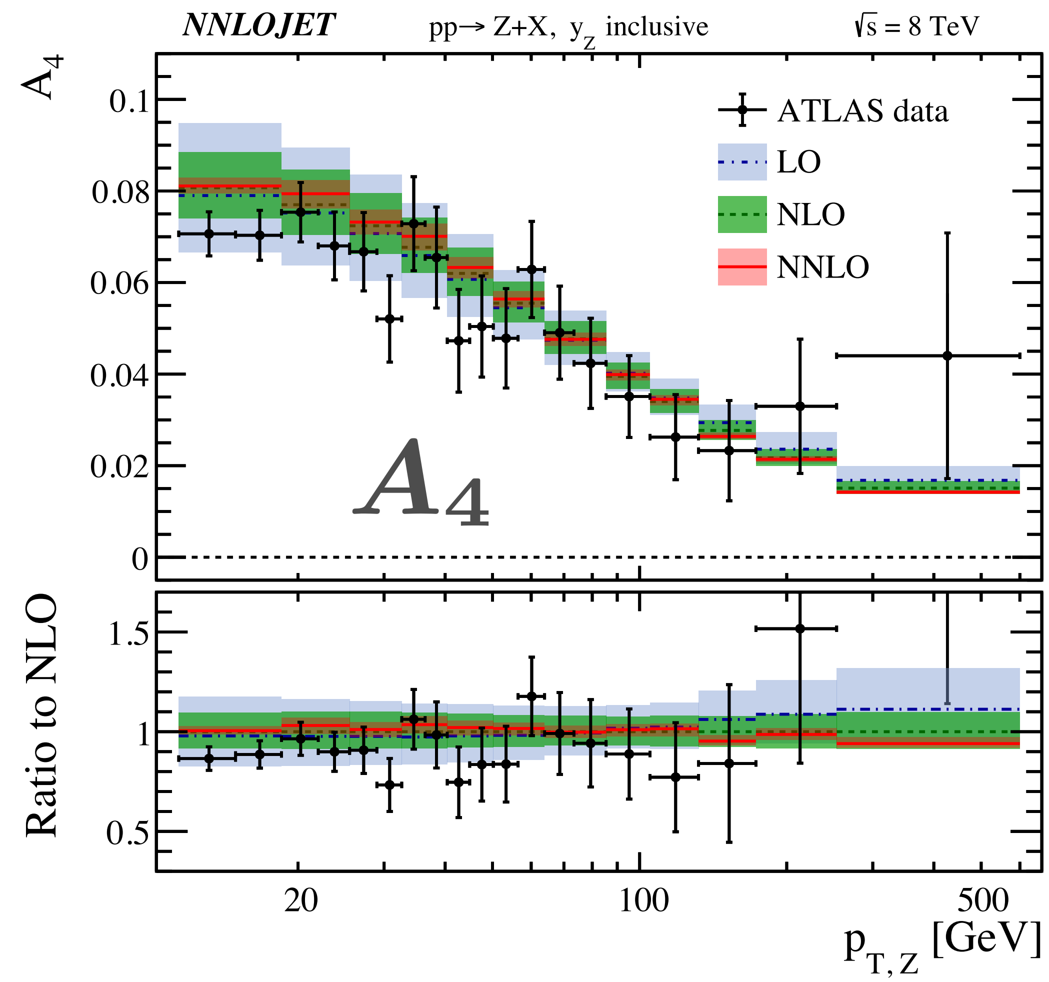
lepton kinematics

$l = 0:$	$m = 0$
$l = 1:$	$m = \pm 1,\ 0$
$l = 2:$	$m = \pm 2,\ \pm 1,\ 0$
total:	9

Angular Coefficients — A_2 & A_4

[Gauld, Gehrmann—De Ridder, Gehrmann, Glover, AH '17]

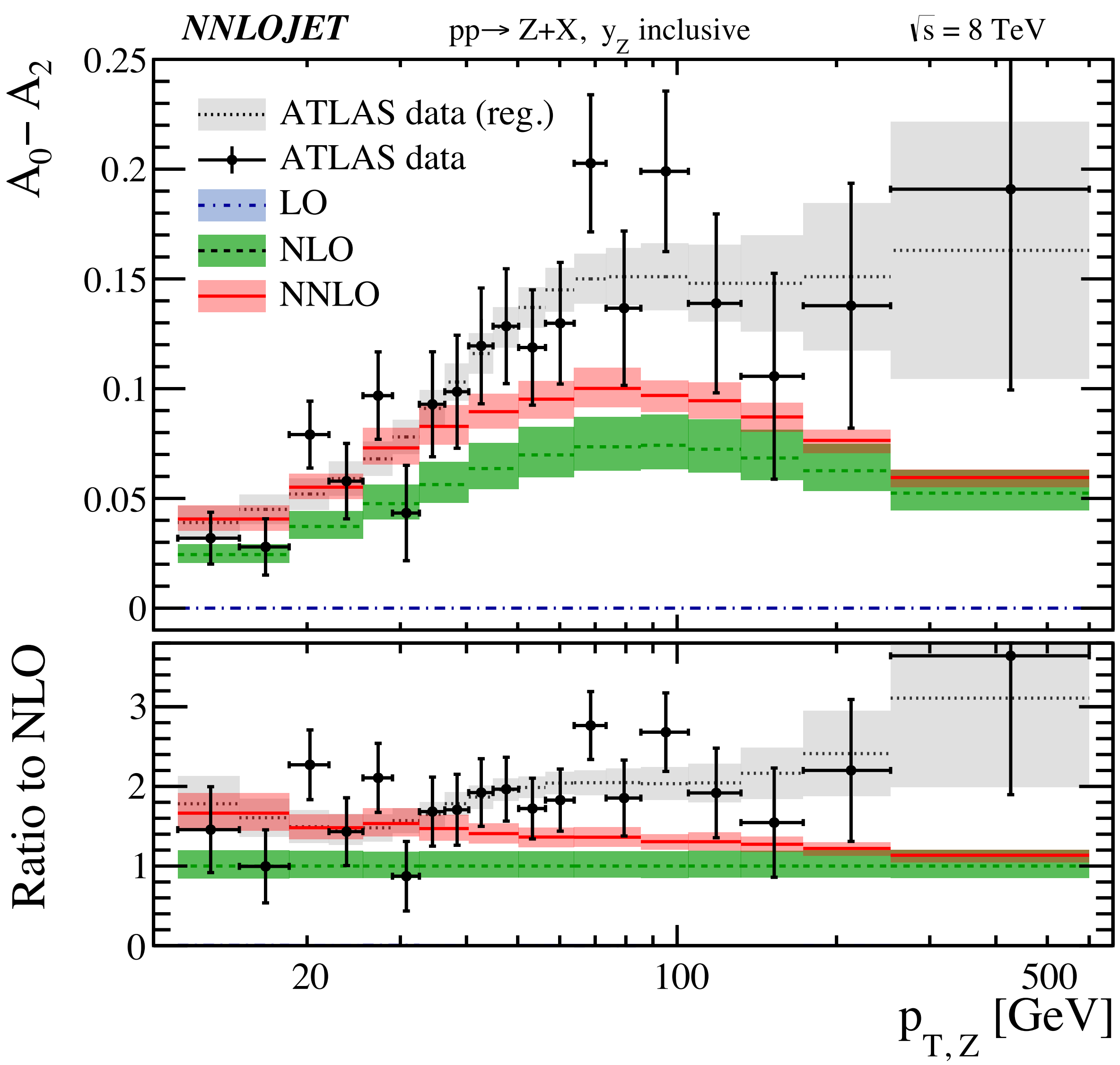




- A_2 drives *Lam–Tung* violation ($A_0 \neq A_2$)
- negative corrections (@ low p_rT)
  $\hookrightarrow$ **NNLO** $\sim -20\%$ (+ scale reduction)
- visible improvement (data vs. theory)

- A_4 sensitive to $\sin^2\theta_\mathrm{w}$
- very stable w.r.t. QCD corrections
  $\hookrightarrow$ **NLO** $\to$ **NNLO** (mostly scale reduction)

ANGULAR COEFFICIENTS — LAM–TUNG



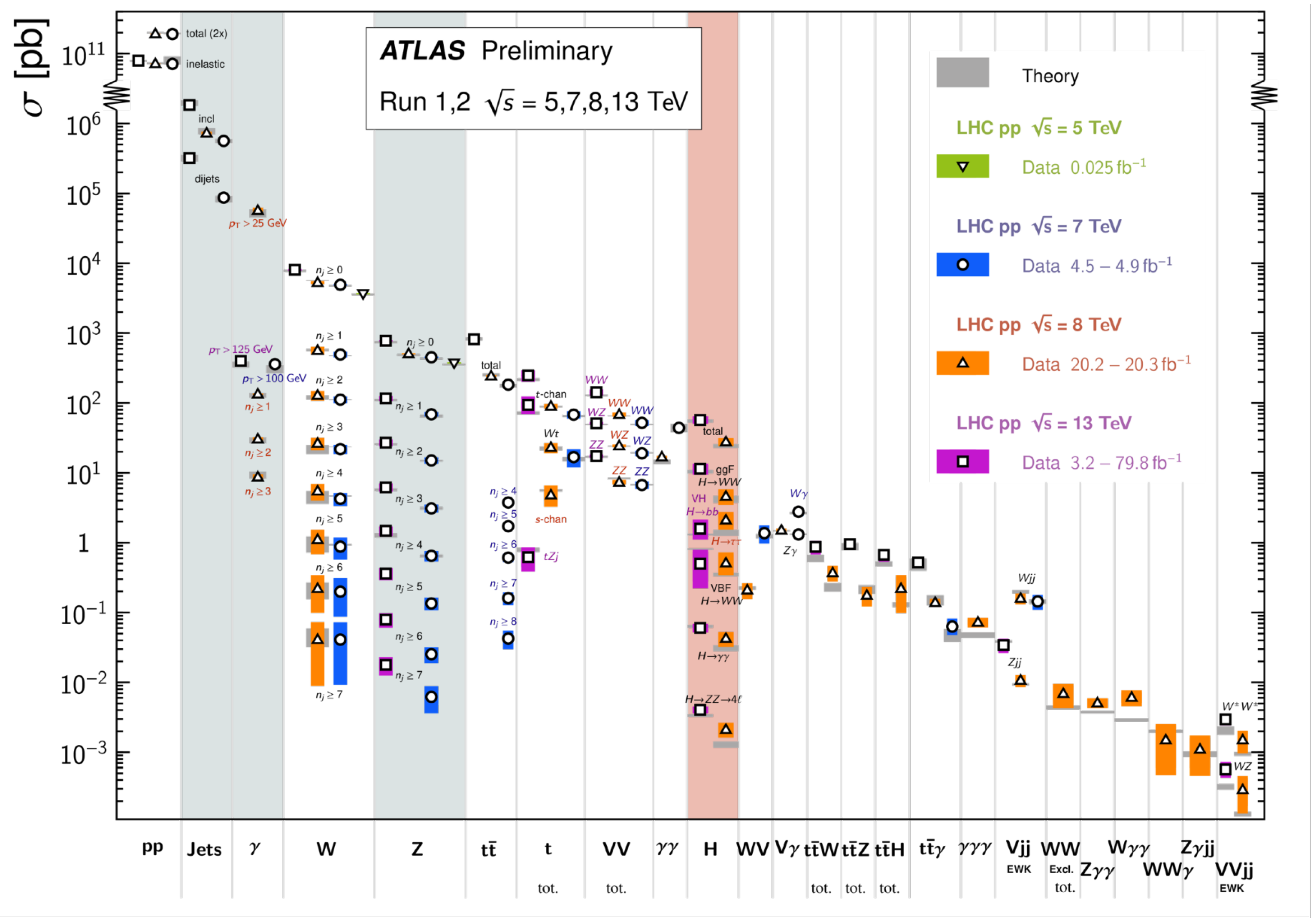
- $\mathcal{O}(\alpha_s)$ prediction:
  - ↪ vanishes (Lam–Tung)
- $\mathcal{O}(\alpha_s^2)$ prediction:
  - ↪ $\simeq$ DYNNLO (NNLO)
  - ↪ tension with data
  - ↪ $\chi^2/N_{\mathrm{dat.}} \sim 4.89$
- $\mathcal{O}(\alpha_s^3)$ prediction:
  - ↪ large positive corrections
  - ↪ $\chi^2/N_{\mathrm{dat}} \sim 1.75$
- ▶ data: [ATLAS arXiv:1606.00689]
  - ↪ applies "regularization"

No significant data* vs. theory disagreement between
(un-regularized) ATLAS & **theory @ $\mathcal{O}(\alpha_s^3)$**

$$^* \chi^2 = \sum_{i,j}^{N_{\mathrm{dat.}}} (O^i_{\mathrm{exp}} - O^i_{\mathrm{th.}})\sigma^{-1}_{ij}(O^j_{\mathrm{exp}} - O^j_{\mathrm{th.}})$$

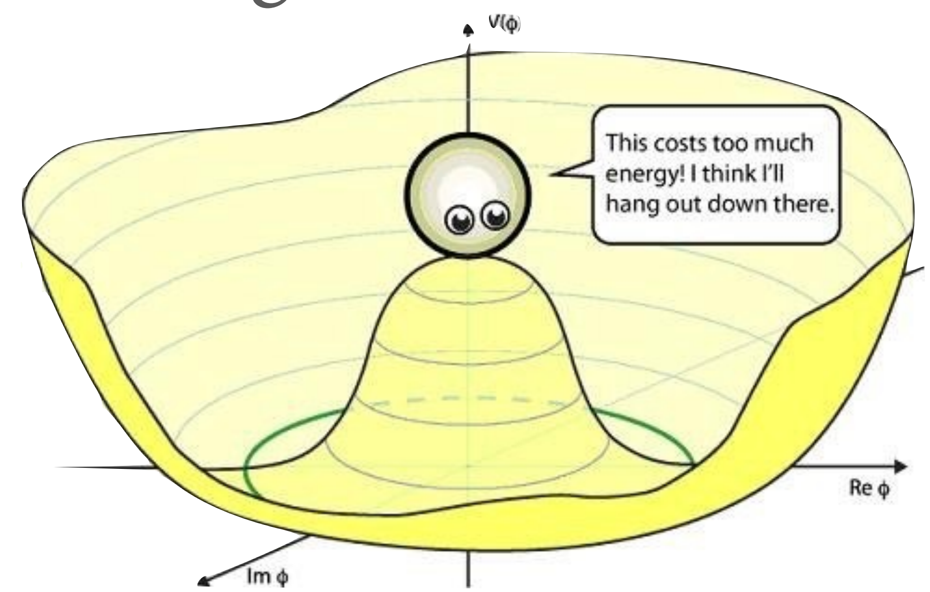
THE PLAN.

1. *Predictions for the LHC*
2. *Phenomenological Applications*
   - *Jets — Photon — Z-boson — Higgs*



The Higgs Boson

- experimental era of Higgs physics just starting
  - scrutinise all properties
  - couplings/interactions
  - probe the potential



- notoriously bad perturbative convergence (N³LO needed)

$$\sigma_{\text{tot}}^{\text{N}^3\text{LO}} = 48.68\,\text{pb}^{+2.07\,\text{pb}}_{-3.16\,\text{pb}}$$

[Anastasiou et al. '15] [Mistlberger '18]

✓ analytic integration of full phase space
✗ no information on final state

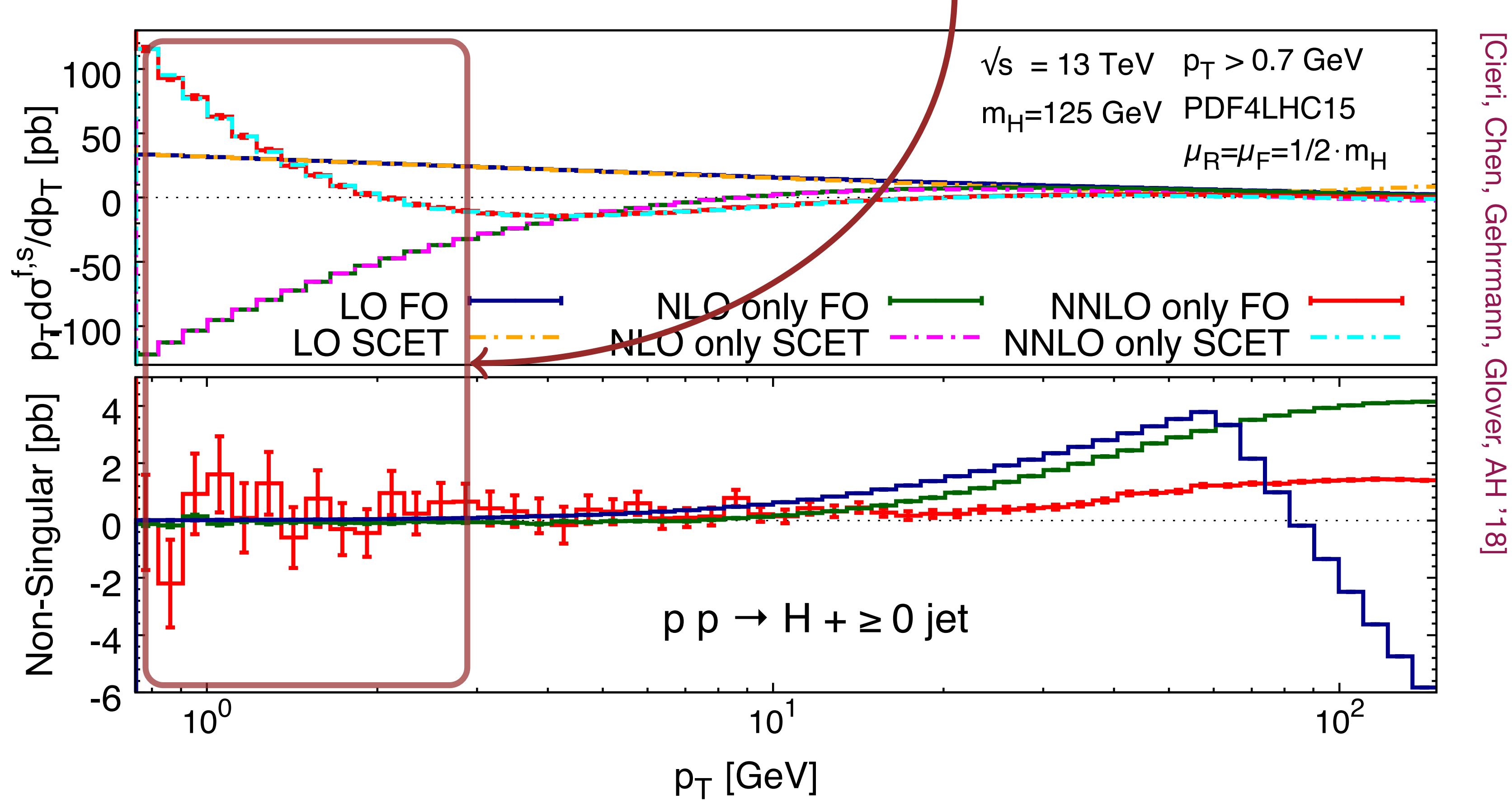
Fully Differential?

q_T Subtraction @ N³LO

[Catani, Grazzini '07]

$$\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}} = \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_T<q_T^{\mathrm{cut}}}}_{q_T\ \text{resummation}} + \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_T>q_T^{\mathrm{cut}}}}_{\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}}}$$

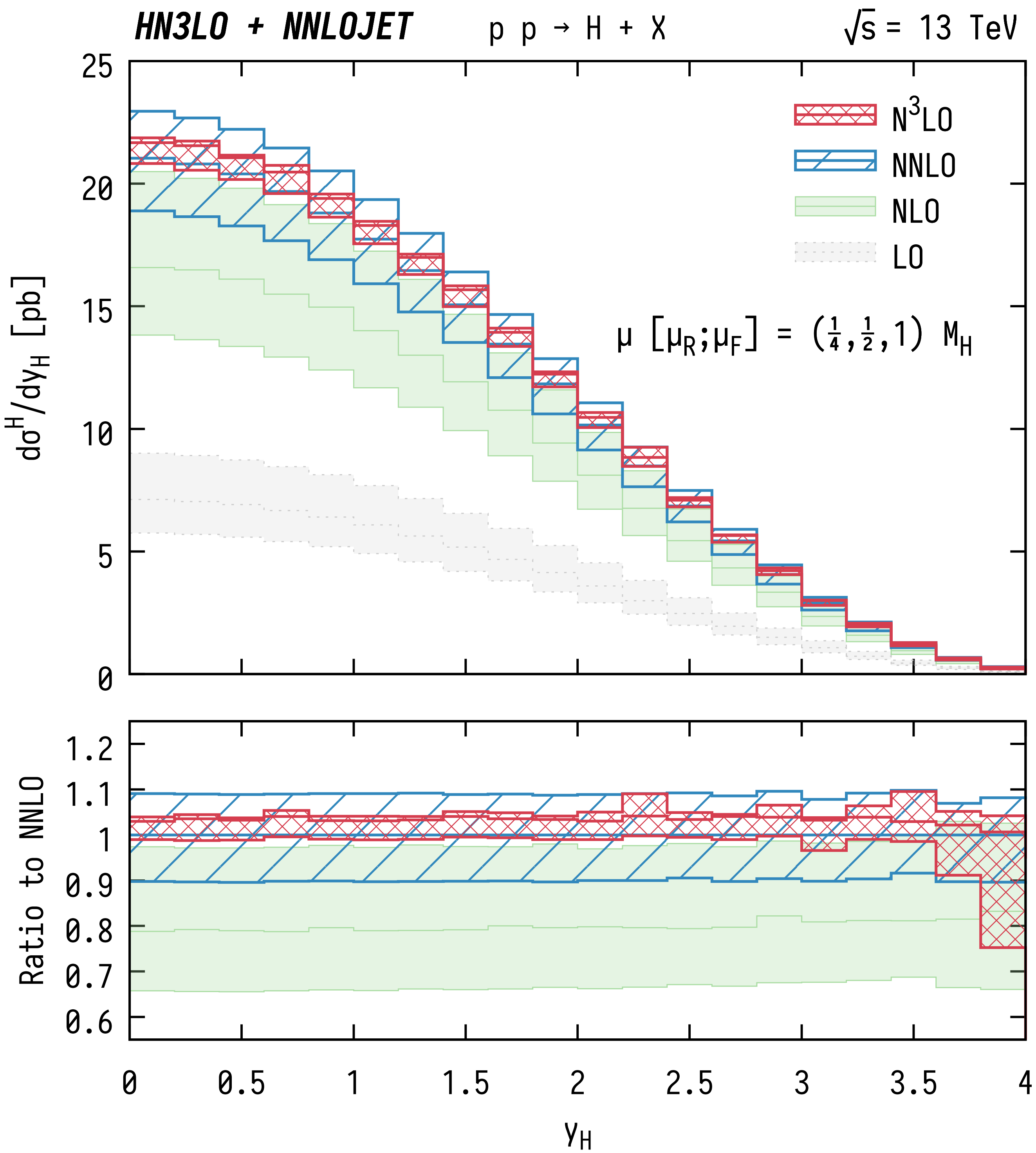
$$\simeq \mathcal{H}^{\mathrm{H}}_{\mathrm{N^3LO}} \otimes \mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{LO}} + \underbrace{\left[\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{H,\,CT}}_{\mathrm{N^3LO}}\right]}_{}{}_{q_T>q_T^{\mathrm{cut}}}$$



** numerical approx. for unknown coefficients*

HIGGS RAPIDITY @ N^3LO

[Cieri, Chen, Gehrmann, Glover, AH '18]



- *four* successive orders

N^3LO

- few %
- fully contained within **NNLO** band

↔ convergence!

- analytical result for y_{H}:
  [Dulat, Mistlberger, Pelloni '18]
  in good agreement!

Conclusions & Outlook

- **LHC** — remarkable opportunity to study high-energy physics
  - search for new physics & probe the Higgs sector
  - precision measurements using "standard candles"

⇒ high-precision predictions essential!

(reduced uncertainties & often resolves tension to data)

- **Remarkable progress** in precision calculations:
  - $2 \to 2$ @ NNLO, $2 \to 1$ @ N^{3}LO, NLO EW $2 \to 6$

➡ **precision phenomenology** using these calculations has only started!

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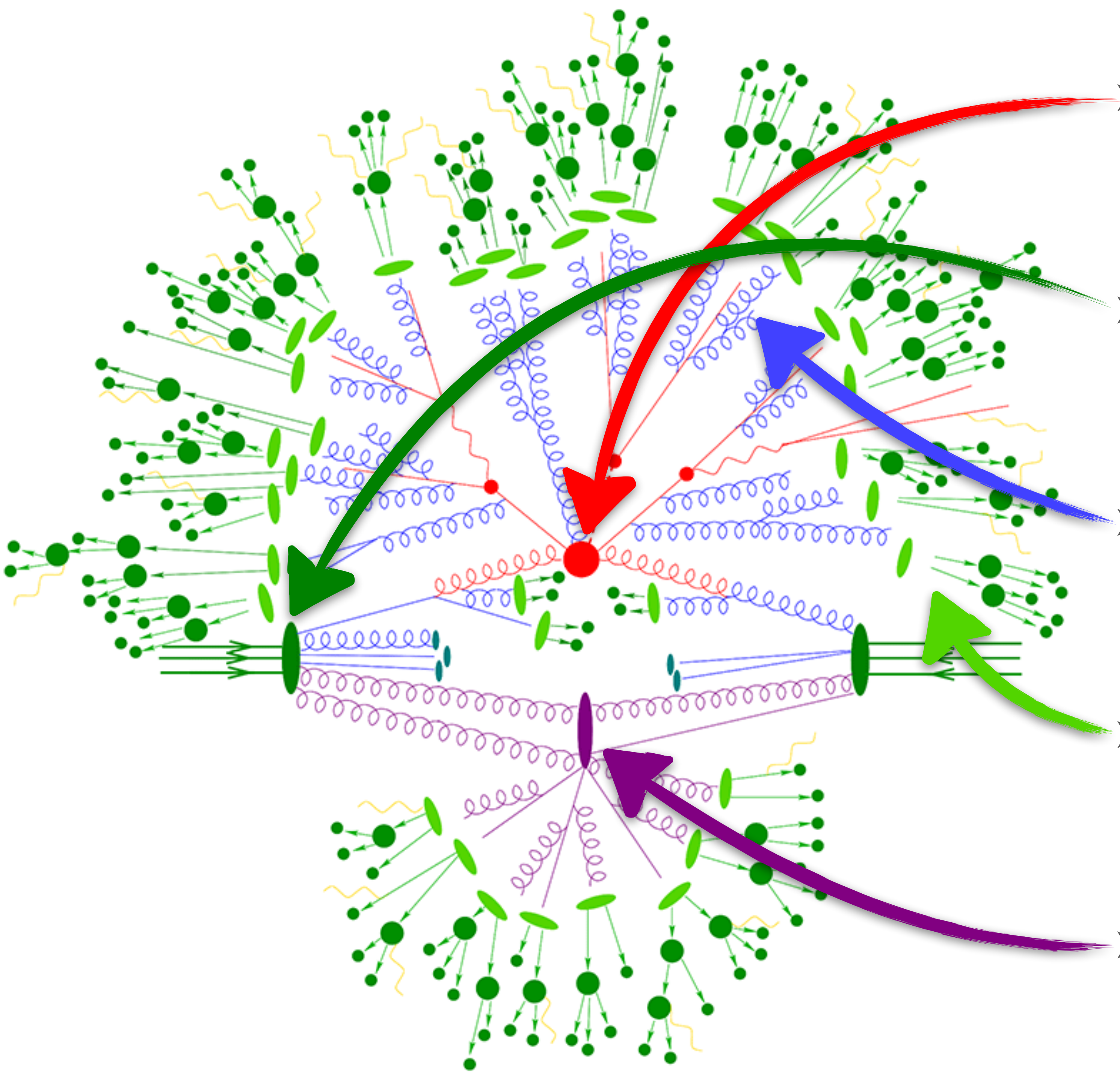
THANK YOU!

Backup.

PREDICTIONS @ LHC



- Hard subprocess
- PDFs
- Parton shower
- Hadronization + decays
- Underlying event
- Jets, substructure, res.

SUBTRACTION METHODS

- Remarkable progress in the development of methods to perform NNLO computations!

(not an exhaustive list)	local subtraction	analytic	pp collisions	final-state jet(s)
Antenna	✗ (local after rot^{n})	✓	✓	✓
CoLorFul	✓	✓	✗	✓
q_T-Subtr.	✗	✓	✓	✗ (only t)
STRIPPER / nested soft–coll.	✓	✗ / ✓	✓	✓
N-jettiness	✗	✓	✓	✓ ($\leq$ 1 jet so far)

- Projection-to-Born, Local Analytic Sectors, Geometric, …

** more painful with massless particles*

q_T Subtraction @ N^{3}LO

[Catani, Grazzini '07]

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$$\simeq \mathcal{H}^{\mathrm{H}}_{\mathsf{N^3LO}} \otimes \mathrm{d}\sigma^{\mathrm{H}}_{\mathsf{LO}} + \Big[\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathsf{NNLO}} - \mathrm{d}\sigma^{\mathrm{H,\,CT}}_{\mathsf{N^3LO}}\Big]_{q_T>q_T^{\mathrm{cut}}}$$

Unknown pieces in $\mathcal{H}^{\mathrm{H}}_{N^3LO}$: $H_{\mathrm{g}}^{\mathrm{H};(3)}$, $C_{ga}^{(3)}(z)$, $G_{ga}^{(2)}(z)$

But we know $\sigma^{tot}_{N^3LO}$! [Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15] [Mistlberger '18]

$$\sigma^{\mathsf{tot}}_{\mathsf{N^3LO}} = \int_0^\infty \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\mathrm{H}}_{\mathsf{N^3LO}}}{\mathrm{d}q_T} = \frac{M^2}{\hat{s}}\mathcal{H}^{\mathrm{H}}_{\mathsf{N^3LO}} + \int_0^\infty \mathrm{d}q_T \left[\frac{\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathsf{NNLO}}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\mathrm{H,\,CT}}_{\mathsf{N^3LO}}}{\mathrm{d}q_T}\right]$$

$\Rightarrow$ *numerical* extraction of unknown pieces: $\to C_{N3}$

- approximation: $f(z) \to$ const. $\delta(1-z)$

$\leadsto$ how well does this work?! $\leadsto$ *per-mille level @ NNLO*