Few nucleon systems without angular momentum decomposition

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24-28 VI 2018

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PW CALCULATIONS - IN A NUTSHELL

- Work in a finite sized basis of - angular momentum, spin, isospin eigenstates
- QM operators can be represented using matrices with finite size . . .

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- $\check{X}$ some 2N operator ($\check{}$ denotes operators) eg.: 2N potential ($\check{V}$), 2N transition operator (for given energy $\check{t}(E)$), free propagator (for a given energy $G_0(E)$), ...
- Each ■ in subspace with given orbital angular momentum l, spin s and total angular momentum j
- Different momentum magnitude states, and different projections of total angular momentum $\langle |\boldsymbol{p}'|(l's')j'm_{j'} \mid \ldots \mid |\boldsymbol{p}|(ls)jm_j\rangle$
- Impose pairity, time reversal and rotational symmetry ...

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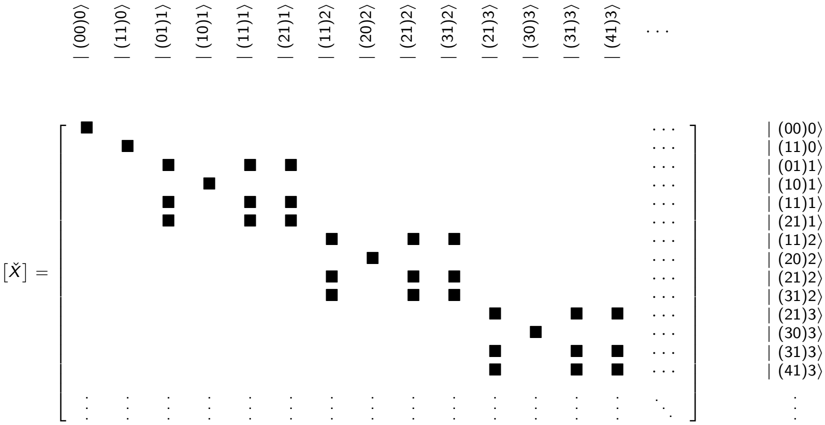

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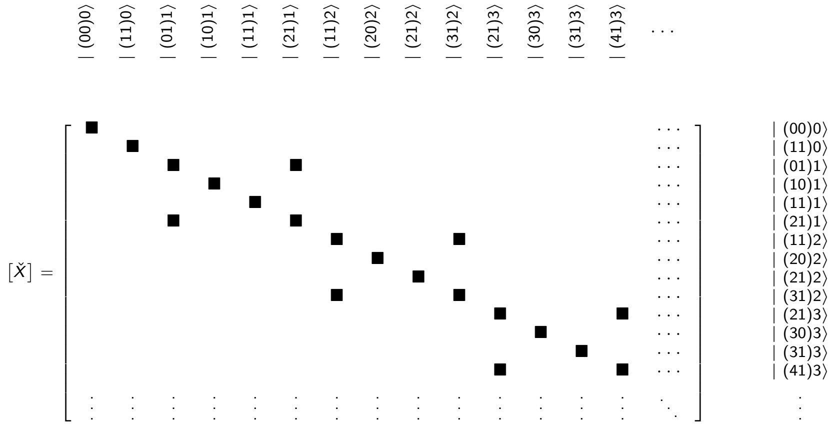

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- Solve equations (eg. LAPACK)

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PW CALCULATIONS - CONS

- Different coupling schemes for three, four, . . . particles
- Implementation requires heavily oscilating functions

$$Y_l^m(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

- Calculations need to include many partial waves in order to acheve convergence
  - High energies
  - Long range interactions (Coulomb!)

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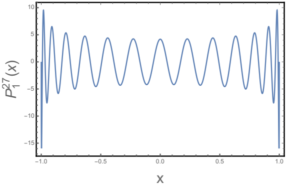

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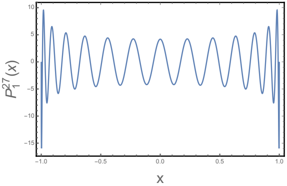

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3D CALCULATIONS - IN A NUTSHELL

- Example: the 2N transition operator.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.
- Solve the LSE: $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$

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- Ignoring isospin:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p}\rangle\right] =$$

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- $\check{V}$, $\check{G}_0$ in a similar form ($\check{G}_0$ - diagonal)
- Very wastefull - we need to calculate 16 functions $t_{ij}(p'_x, p'_y, p'_z, p_x, p_y, p_z)$ of 6 real parameters that satisfy the LSE

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- The matrix element in momentum space that satisfied appropriate symmetries (parity inversion, time reversal, particle exchange) can be written [Phys. Rev. 96 1654 (1954)] as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle\right] = \sum_{i=1}^{6} t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}}) \left[\check{w}_i(\boldsymbol{p}, \boldsymbol{p})\right]$$

- Instead of calculating 16 functions of 6 real variables we now only need to calculate 6 functions of 3 variables $t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}})$
- Less numerical work $\rightarrow$ a practical numerical implementation

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- The matrix element in momentum space that satisfied appropriate symmetries (parity inversion, time reversal, particle exchange) can be written [Phys. Rev. 96 1654 (1954)] as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p}\rangle\right] = \sum_{i=1}^{6} t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}}) \left[\check{w}_i(\boldsymbol{p}, \boldsymbol{p})\right]$$

- Instead of calculating 16 functions of 6 real variables we now only need to calculate 6 functions of 3 variables $t_i(|\boldsymbol{p}'|, |\boldsymbol{p}|, \hat{\boldsymbol{p}}' \cdot \hat{\boldsymbol{p}})$
- Less numerical work $\rightarrow$ a practical numerical implementation

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- No need to perform the PWD of operators
- No need to implement Legendre polynomials
  - Calculations at higher energies
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  - Two nucleon transition operator [Golak J. et al. *Phys. Rev. C 81, 034006 (2010)*] [Elster Ch. et al. *Few Body Syst. 24, 55 (1998)*]
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^{3}He - FADDEEV EQUATION

- Ingredients: bound state energy E ; free propagator $\check{G}_0(E)$; two nucleon potential between particles j, k (such that $i \neq j \neq j \neq k$) $\check{V}_i^{2N}$; three nucleon potential symmetric with respect to the exchange of particles j, k (such that $i \neq j \neq k \neq i$) $\check{V}_i^{3N}$; particle exchange operator P_{ij}.
- Faddeev component of the 3N bound state:

$$| \psi\rangle = \check{G}_0(E) \left(\check{V}_1^{2N} + \check{V}_1^{3N} \right) \left(1 + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}\right) | \psi\rangle$$

- Full bound state wave function:

$$| \Psi\rangle = \left(1 + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}\right) | \psi\rangle$$

OPERATOR FORM OF 3N STATE

- Ingredients: Jacobi momenta $\boldsymbol{p}$, $\boldsymbol{q}$; two particle subsystem isospin t ; total isospin T ; total isospin projection M_T ; given operators in the spin space of the 3N system $\check{O}_i$; spin state $\chi_m =| (1\frac{1}{2})\frac{1}{2}m\rangle$
- The operator form of the three nucleon state [Fachruddin I. et al. *Phys. Rev. C 69:064002 (2004)*]:

$$\langle \boldsymbol{pq}; (t\frac{1}{2})TM_T \mid \psi\rangle = \sum_{tT}\sum_{i=1}^{8} \phi_i^{tT}(p, q, \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})\, \check{O}_i(\boldsymbol{p}, \boldsymbol{q}) \mid \chi_m\rangle$$

can be used to transform the Faddeev equation into a set of coupled linear equations for the scalar functions $\phi_i^{tT}(p, q, \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})$

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PRACTICAL REALIZATION

- Ingredients: scalar functions that define the 3N state $\phi \in span(\{\phi_i^{tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})\})$; energy dependent operator $\check{A}(E)$
- The transformation:

$$|\,\psi\rangle = \check{G}_0(E)\left(\check{V}_1^{2N} + \check{V}_1^{3N}\right)\left(1 + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}\right)|\,\psi\rangle$$

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$$\phi = \check{A}(E)\phi$$

- Actually solve (try various energies, search for $\lambda = 1$):

$$\check{A}(E)\phi = \lambda\phi$$

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PROBLEMS

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  - Use Krylov subspace methods (Arnoldi algorithm) $span(\{\phi_0, \check{A}\phi_0, \check{A}^2\phi_0, \ldots, \check{A}^{N-1}\phi_0\})$
  - $N = 40$ results in a reduction $\approx 10^6 \times 10^6 \to 40 \times 40$
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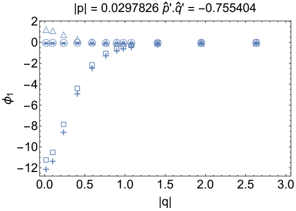

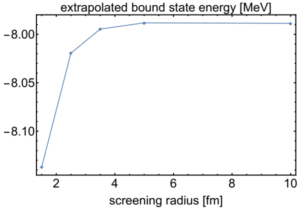

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- ^{3}He, bound state energy $-7.989[MeV]$, isospin:
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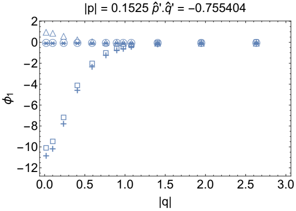

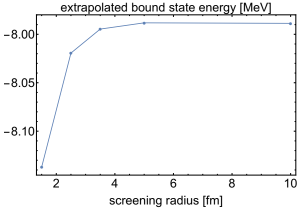

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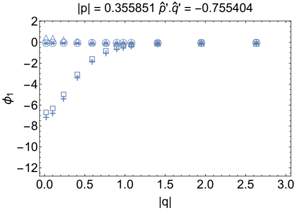

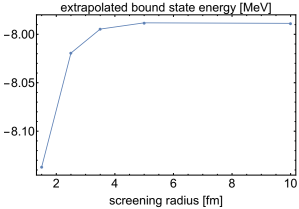

- ^{3}H, bound state energy $-8.074[MeV]$, isospin:
  - $\times \mid (0\tfrac{1}{2})\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$
  - $+ \mid (1\tfrac{1}{2})\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$
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  - $\square \mid (1\tfrac{1}{2})\tfrac{1}{2}\ \tfrac{1}{2}\rangle$
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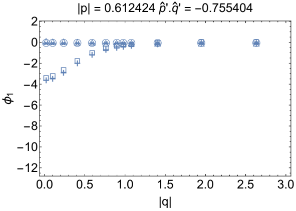

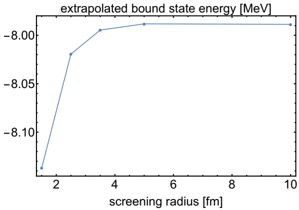

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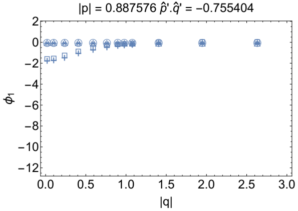

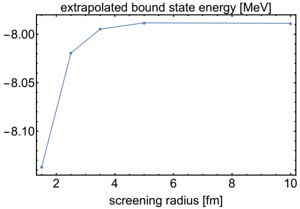

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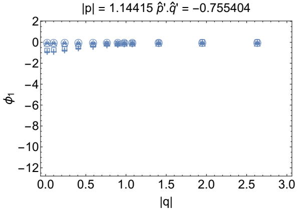

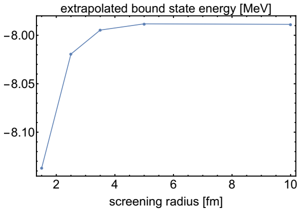

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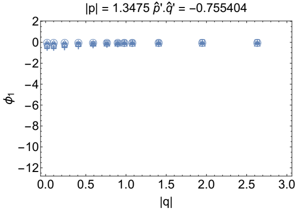

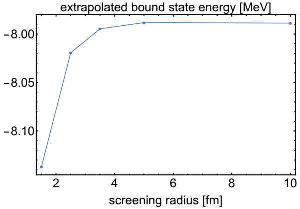

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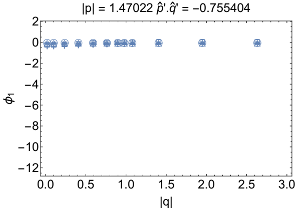

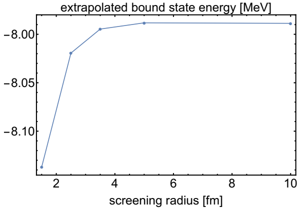

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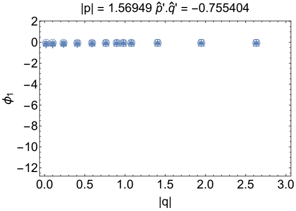

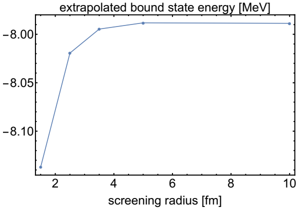

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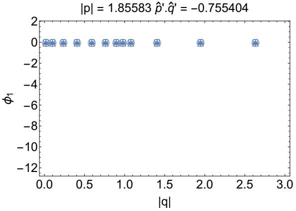

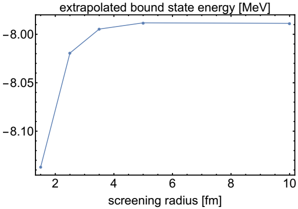

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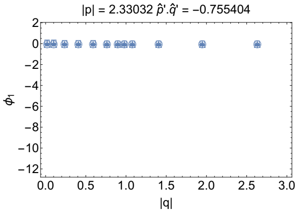

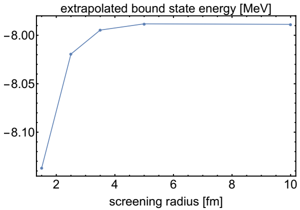

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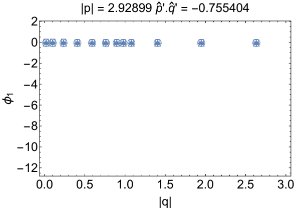

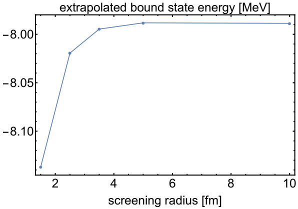

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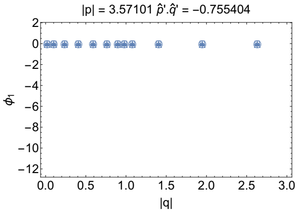

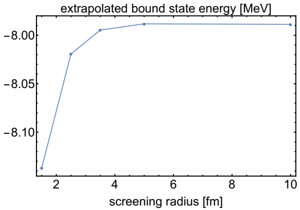

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- 3D calculations for ^{3}He are possible !
- Preliminary results were presented - new results coming soon
  - n-p + p-p 2N force
  - More grid points
  - Easy to calculate matrix elements between ^{3}H and ^{3}He - beta decay
- 3N scattering using the operator form of the 3N scattering amplitude

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THANK YOU

The project was financed from the resources of the National Science Center (Poland) under grants 2016/22/M/ST2/00173 and 2016/21/D/ST2/01120. Numerical work was performed on the supercomputers of JSC Jülich.

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