Neutron Decay – Standard Model and Beyond

Humboldt Kolleg
Discoveries and Open Puzzles in Particle Physics and Gravitation
Kitzbühel, Austria (June 23 – 28 2019)

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Outline

- Why study the neutron?
- Neutron β-decay in SM
- Disputes about neutron lifetime
- Search for BSM physics with neutron β-decay
- Neutron β-decay correlations in low magnetic field with electron tracking
- Summary

Why study the free neutron?

- **Main goal of Particle Physics**:
  Establish consistent picture of Nature's fundamental interactions

  - **High Energy PP**:
    - Operates at TeV scale (10^{12} eV)
      $\Rightarrow$ study of 2^{nd} (s, c, μ, ν_μ) and 3^{rd} (b, t, τ, ν_τ) particle families
  - **Low Energy PP (e.g. with neutrons)**:
    - Operates at neV scale (10^{-9} eV)
      $\Rightarrow$ study of 1^{st} (u, d, e, ν_e) particle family
    - Reveals respectable sensitivity:
      - Energy: $\Delta E/E \sim 10^{-11} \div 10^{-13}$ ($\Delta E \sim 10^{-23}$ eV)
      - Momentum: $\Delta p/p \sim 10^{-10} \div 10^{-11}$
      - Spin polarization: $\Delta s/s \sim 10^{-7}$
  - *Fundamental neutron physics provides more than **20** observables reach in information which is difficult to achieve (or not achievable at all) in other fields of Particle Physics*

"ENERGY frontier"

"PRECISION frontier"

Why study the free neutron?

Neutron in astrophysics:

- **Primordial Nucleosynthesis**:
  - ^{4}He abundance Y_p is sensitive to:

$$\Delta Y_p / Y_p = +0.72\ \Delta\tau_n / \tau_n \qquad \Delta Y_p / Y_p = +0.17\ \Delta N_\nu / N_\nu$$

$$\Delta Y_p / Y_p = +0.039\ \Delta\eta / \eta$$

- **Stellar nucleosynthesis**:
  - Neutrino flux (CNO cycle) depends on g_A most accurately extracted from n-decay

$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu_e$$

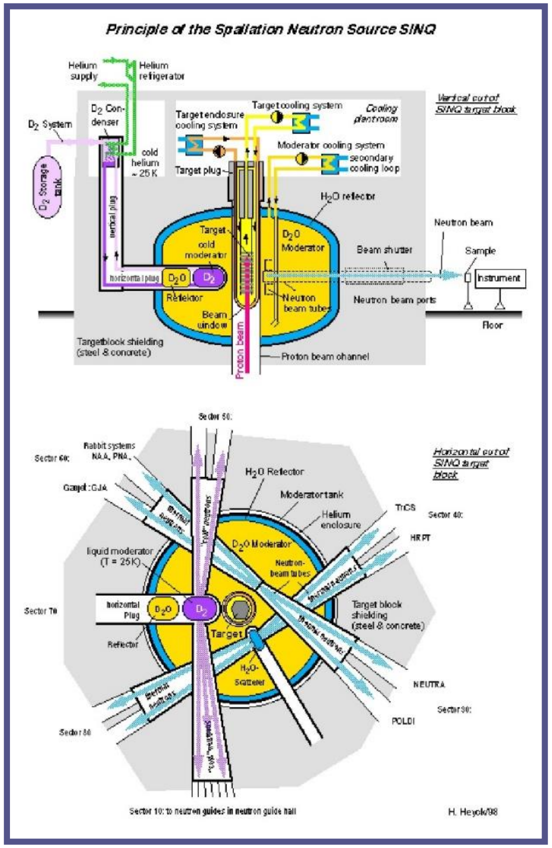
  - s-process (production of nuclides heavier than ^{56}Fe need n-capture cross sections

$$\Delta\Phi_8 / \Phi_8 = -5.2\ \Delta g_A / g_A$$

- *Interesting neutron physics can be extensively tested in the variety of nuclear processes involved in nucleosynthesis*

Cold neutrons (CN)

- **Cold neutrons**: $E_{\mathrm{kin}}^{\mathrm{CN}} \sim 5\ \mathrm{meV}, \qquad \upsilon^{\mathrm{CN}} \sim 1\ \mathrm{km/s}$
- **CN production via moderation of thermal neurons**:
  - Cold sources: moderators made of liquid hydrogen or deuterium operated at 20 K
  - Cold moderators are typically inserted close to reactor core or to spallation target



Facility	Pulsed	Time-averaged neutron capture flux [10^9 n/cm^2/s]	
ANNI (ESS)	Yes	40	(calculated, see Appendix 3)
PF1B (ILL)	No	20	
MEPHISTO (FRM II)	No	18	(calculated [Kle14], under construction)
NG-6 (NIST)	No	2	(decommissioned and replaced by NG-C)
NG-C (NIST)	No	8.3	[Wie14]
FunSpin (PSI)	No	1	(polarized, replaced by BOA)
FnPB (SNS)	Yes	1.4	(at 1.4 MW)
FP12 (LANSCE)	Yes	0.1	
NOP (J-PARC)	Yes	1.2/MW	(calculated [Ari12])

Ultra-cold neutrons (UCN)

- **Ultra-cold neutrons** – can be stored in material or magnetic traps

$$E_{\mathrm{kin}} < V_{\mathrm{F}} - \boldsymbol{\mu}_{\mathrm{n}} \cdot \mathbf{B} + mgh$$

$$V_{\mathrm{F}} = \frac{2\pi\hbar}{m} bN$$

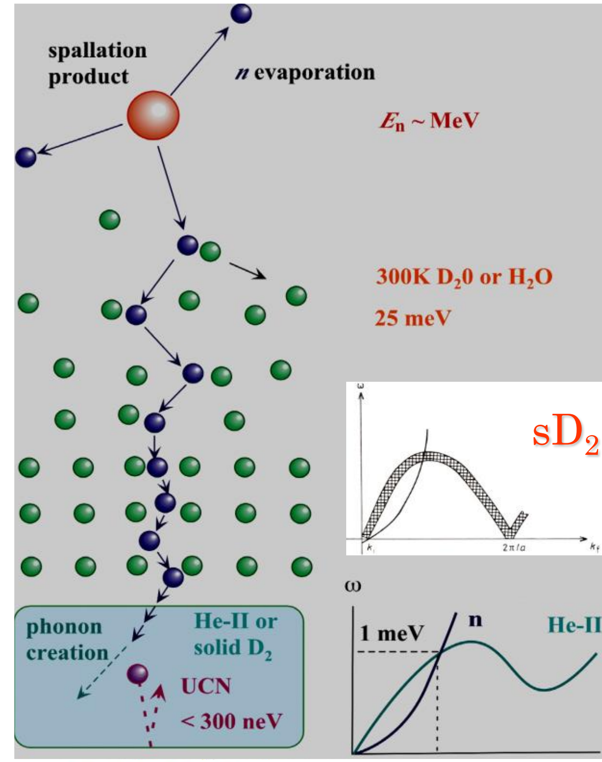
V_{F} – Fermi pseudo-potential,
b – scattering length,
N – number density

- $V_{\mathrm{F}}(\mathrm{Be}) \leftrightarrow E_{\mathrm{kin}} = 252$ neV,
- $\mu_{\mathrm{n}} B(1\ \mathrm{T}) \leftrightarrow E_{\mathrm{kin}} = 60$ neV,
- $mgh(1\ \mathrm{m}) \leftrightarrow E_{\mathrm{kin}} = 100$ neV

- $v^{UCN} < 8$ m/s,
- $T^{UCN} < 4$ mK,
- $\lambda^{UCN} > 50$ nm

- **UCN production via moderation of CN**:
  - Earth gravitational field and/or scattering from turbine blades (ILL)
  - Super-thermal process e.g. in solid D_2 (PSI, LANL, GUM) or super-fluid He (ILL; in development)

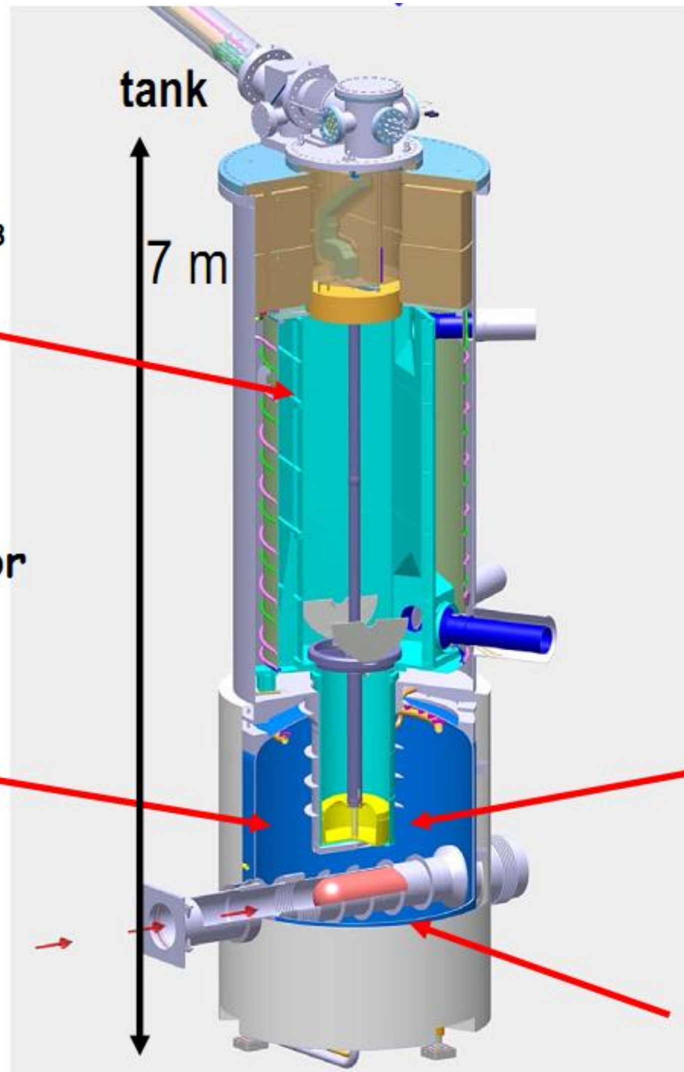
UCN spallation source at PSI



DLC coated
UCN storage volume
height 2.5 m, ~ 2 m^3
ρ_{UCN} ~ 2000 cm^{-3}

heavy water moderator
→ thermal neutrons
3.6m^3 D_2O

pulsed
1.3 MW p-beam
600 MeV, 2.4 mA,
1% duty cycle



cold UCN-converter
30 dm^3 solid D_2 at 5 K

spallation target (Pb/Zr)
(~ 8 neutrons/proton)

UCN sources – performance comparison

- From G. Bison, et al., PHYSICAL REVIEW C 95, 045503 (2017)

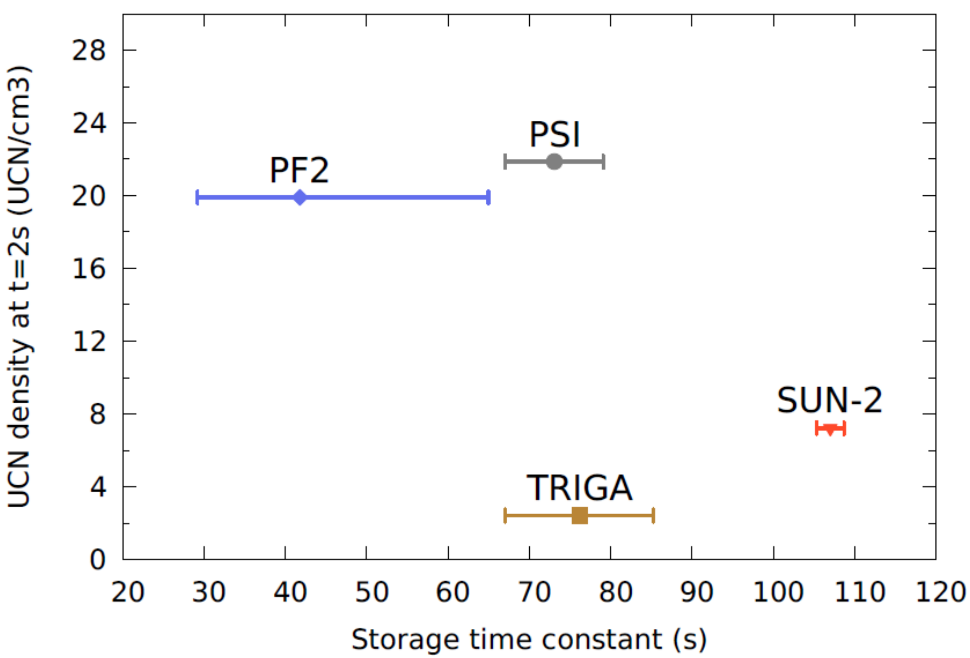


FIG. 29. Measured largest UCN density in the standard storage bottle at a given UCN source plotted versus the measured storage time constant. The measurement conditions are explained in the text. The PF2 value is without safety foil, which is not a standard user configuration. Errors on the UCN density are smaller than symbol size.

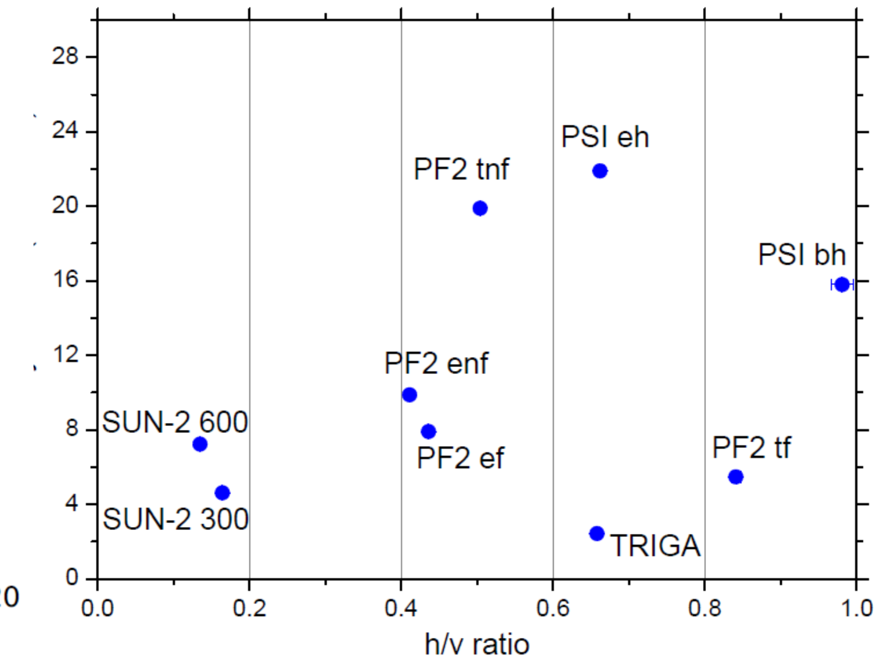


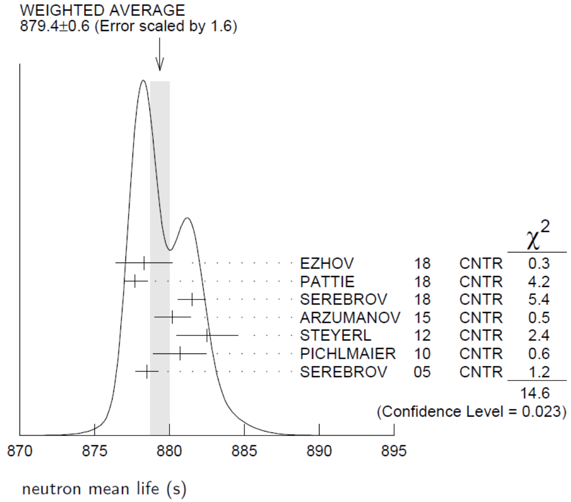
FIG. 30. Calculated ratio of measured UCN density in horizontal and vertical extraction, "h/v ratio", versus UCN density measured in vertical extraction for the given source in a 2 s storage measurement. Labels are explained in the text. Errors are smaller than symbol size.

Neutron β-decay

Neutron decay from PDG2019

n MEAN LIFE

VALUE (s)	*DOCUMENT ID*		*TECN*	*COMMENT*
879.4± 0.6 OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.			
878.3± 1.6± 1.0	EZHOV	18	CNTR	UCN magneto-gravit. trap
$877.7 \pm 0.7^{+0.4}_{-0.2}$	[1] PATTIE	18	CNTR	UCN asym. magnetic trap
881.5± 0.7± 0.6	SEREBROV	18	CNTR	UCN gravitational trap
880.2± 1.2	[2] ARZUMANOV	15	CNTR	UCN double bottle
882.5± 1.4± 1.5	[3] STEYERL	12	CNTR	UCN material bottle
880.7± 1.3± 1.2	PICHLMAIER	10	CNTR	UCN material bottle
878.5± 0.7± 0.3	SEREBROV	05	CNTR	UCN gravitational trap



n DECAY MODES

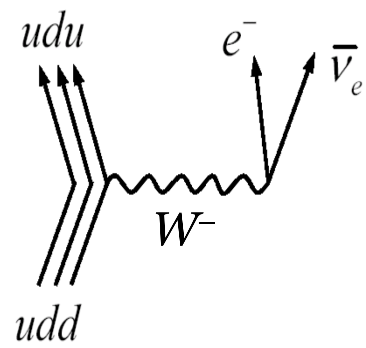
	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$p e^- \overline{\nu}_e$		100 %	
Γ_2	$p e^- \overline{\nu}_e \gamma$		[a] (9.2±0.7) $\times 10^{-3}$	
Γ_3	hydrogen-atom $\overline{\nu}_e$		< 2.7 $\times 10^{-3}$	95%
Charge conservation (*Q*) violating mode				
Γ_4	$p \nu_e \overline{\nu}_e$	*Q*	< 8 $\times 10^{-27}$	68%

[a] This limit is for γ energies between 0.4 and 782 keV.

Neutron β-decay in Standard Model

- In the Standard Model, at tree level, only two parameters survive:

$$H = \frac{G_F}{\sqrt{2}} V_{ud} \quad \bar{p}\left\{\gamma_\mu\left(1+\lambda\gamma_5\right)+\frac{\mu_p-\mu_n}{2m_p}\sigma_{\mu\nu}q^\nu\right\}n \quad \bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu_e$$



V_{ud} – CKM matrix element

$\lambda \equiv \frac{g_A}{g_V}$ – axial-to-vector coupling constant ratio

- **V_{ud}, λ can be extracted from:**
  - Neutron lifetime

f – phase space factor
δ_R – radiative correction (model independent)
Δ_R – radiative correction (model dependent)

$$\tau_n^{-1} = \frac{G_F^2 m_e^2}{2\pi^3}\left|V_{ud}\right|^2 f\left(1+\delta_R\right)\left(1+\Delta_R\right)\left(1+3\lambda^2\right) \Rightarrow \tau_n = \frac{\left(4908.7 \pm 1.9\right)\text{ s}}{\left|V_{ud}\right|^2\left(1+3\lambda^2\right)}$$

  - Differential decay rates: angular distribution of decay products (correlation coefficients)

Neutron β-decay correlations

- Depending on the initial state and measured quantities for the decay products, one can define various differential rates and split them into terms depending on momenta and spins (in lowest order) [J.D. Jackson et al.: Phys. Rev. 106 (1957) 517]

$$\omega(\langle \mathbf{J}_n \rangle | E_e \Omega_e \Omega_\nu) \cdot dE_e d\Omega_e d\Omega_\nu \quad \propto$$

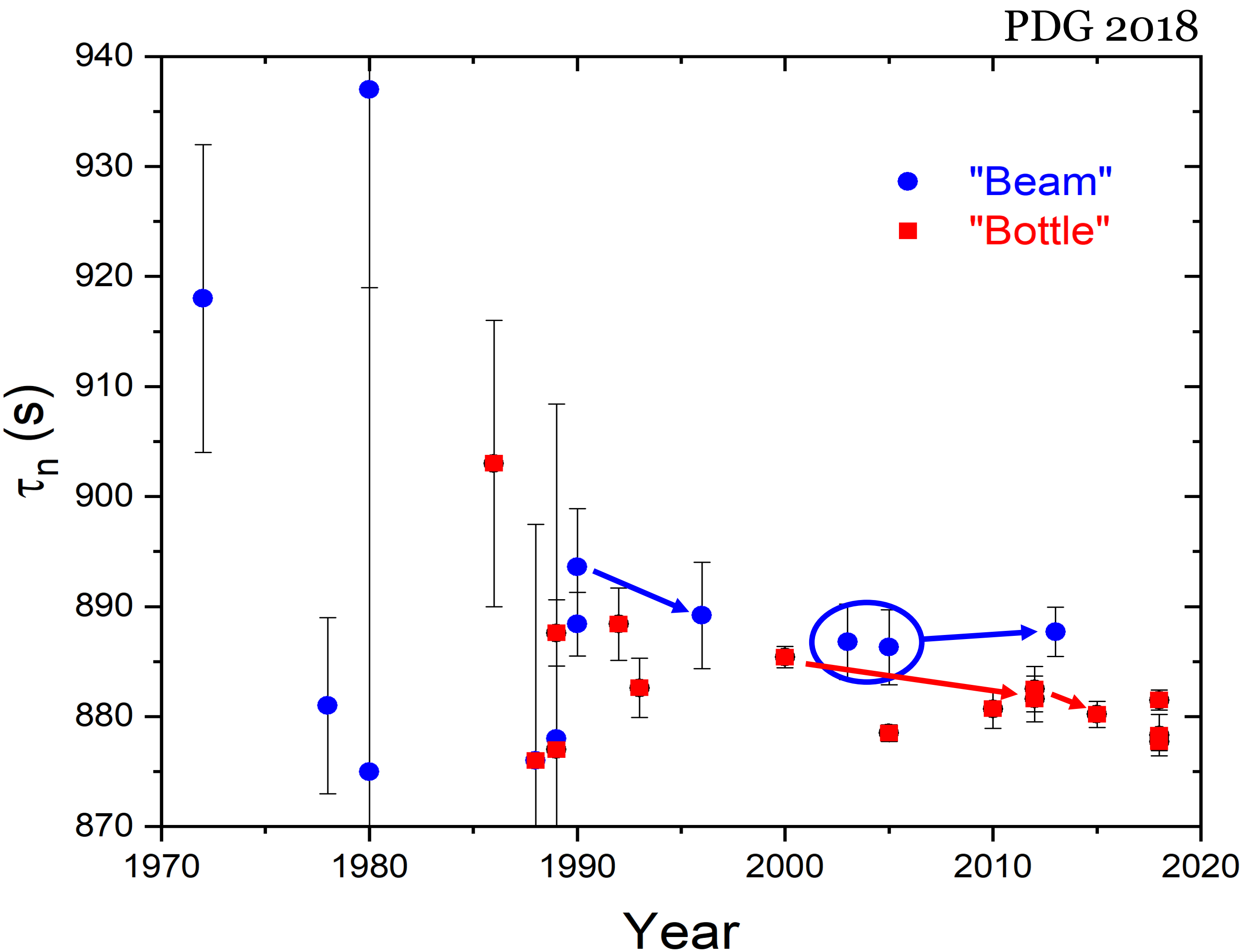
$$\left[1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} + \langle \mathbf{J}_n \rangle \left(A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{(\mathbf{p}_e \times \mathbf{p}_\nu)}{E_e E_\nu} \right) \right] \cdot dE_e d\Omega_e d\Omega_\nu$$

$$\omega(\langle \mathbf{J}_n \rangle \boldsymbol{\sigma} | E_e \Omega_e) \cdot dE_e d\Omega_e \quad \propto \quad \left[1 + \ldots + R \frac{(\mathbf{p}_e \times \boldsymbol{\sigma}) \cdot \langle \mathbf{J}_n \rangle}{E_e} + \ldots \right] \cdot dE_e d\Omega_e$$

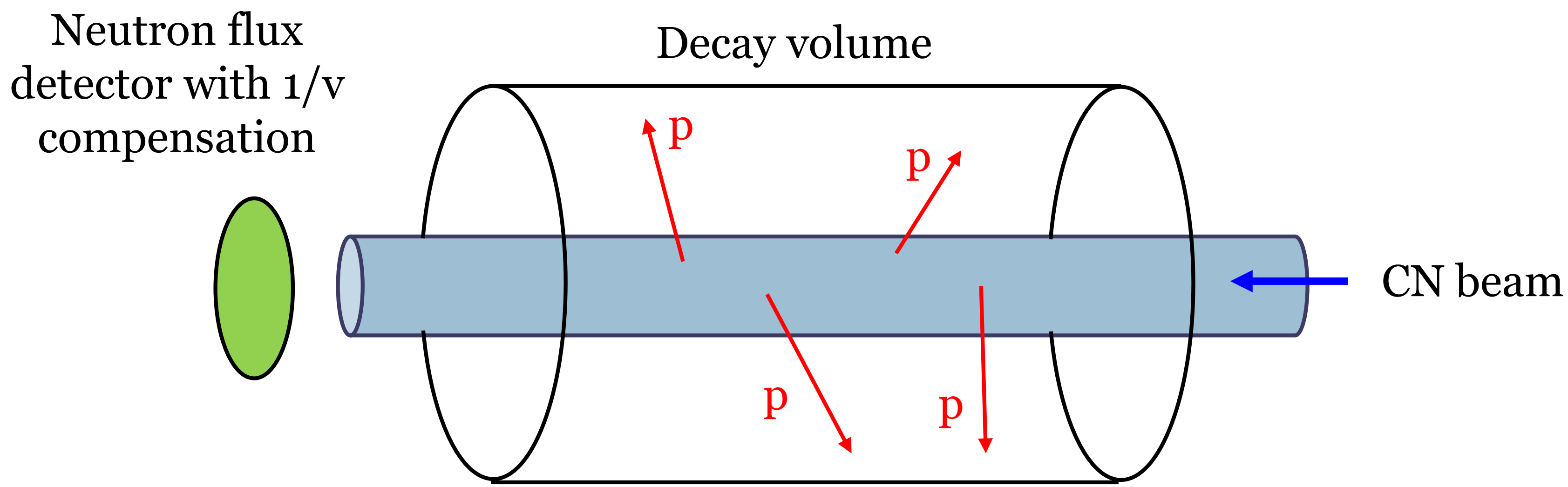
$$a = \frac{1-\lambda^2}{1+3\lambda^2}, \quad b = 0, \quad A = -2\frac{\lambda(1+\lambda)}{1+3\lambda^2}, \quad B = -2\frac{\lambda(1-\lambda)}{1+3\lambda^2}, \quad D = 0, \quad R = 0$$

- Overdetermination is advatageous in suppressing systematic errors

Neutron lifetime experiments



“**Beam**” experiment



$$-\frac{dN}{dt} = \frac{1}{\tau_n} N$$

$$\Gamma = N/\tau_n \quad \Rightarrow \quad \tau_n = \frac{\Phi V_{decay}}{v\,\Gamma}$$

- To be measured:
  - Decay rate Γ
  - Neutron flux Φ weighted by flight time in the decay volume ($\propto 1/v$)
  - Effective decay volume

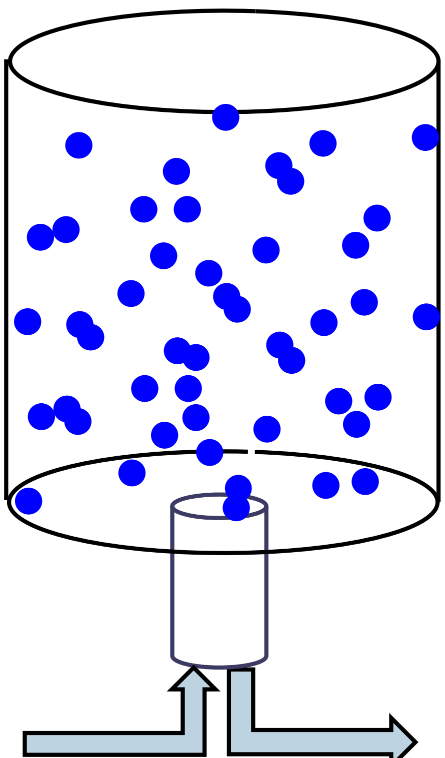
“Bottle” experiment

- Stored UCN in:
  - Material vessels
  - Magnetic bottles/traps

$$\frac{1}{\tau} = \frac{1}{\tau_n} + \frac{1}{\tau_{\mathrm{LOSS}}}$$

$$N_1/N_2 = \exp\left[-(t_1 - t_2)/\tau\right]$$

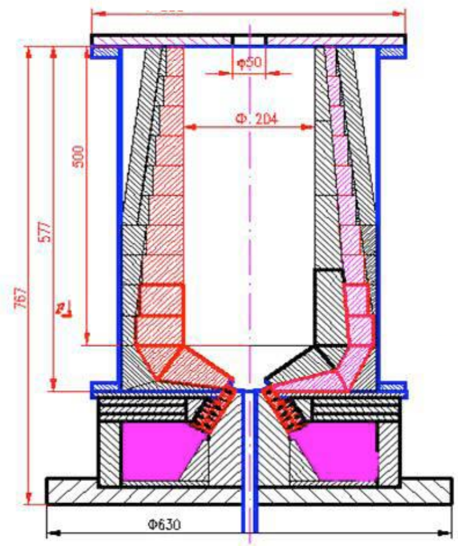
$$t/\tau = \ln\left(N_0/N\right)$$

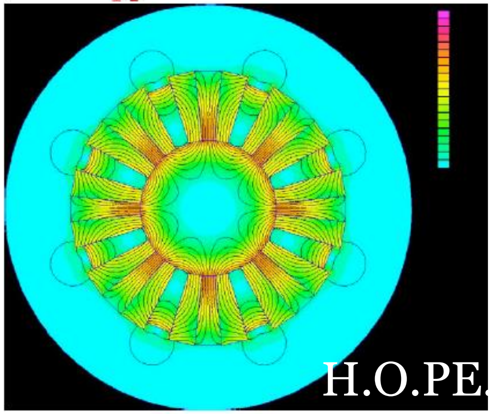


UCN filling

UCN empting & counting

V.F. Ezhov





Counting charged decay products

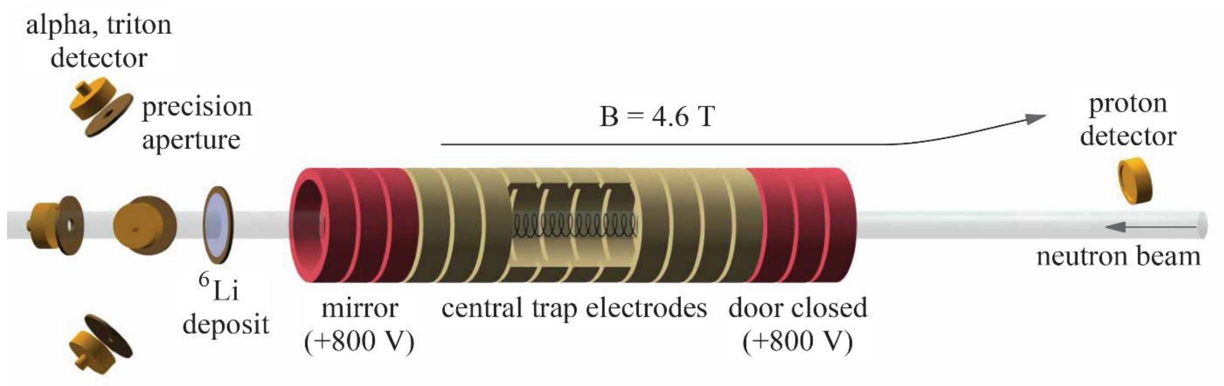
Neutron decay experiments – problems

- “**Beam**”
  - Absolute determination of neutron fluence
  - Accurate determination of fiducial volume
- “**Material bottle**”
  - Energy dependent wall collision loss mechanism (UCN energy spectrum evolution)
- “**Magnetic bottle/trap**”
  - Marginally trapped UCN – complicated particle orbits
  - Spin-flip losses

"**Beam**" experiment at NIST

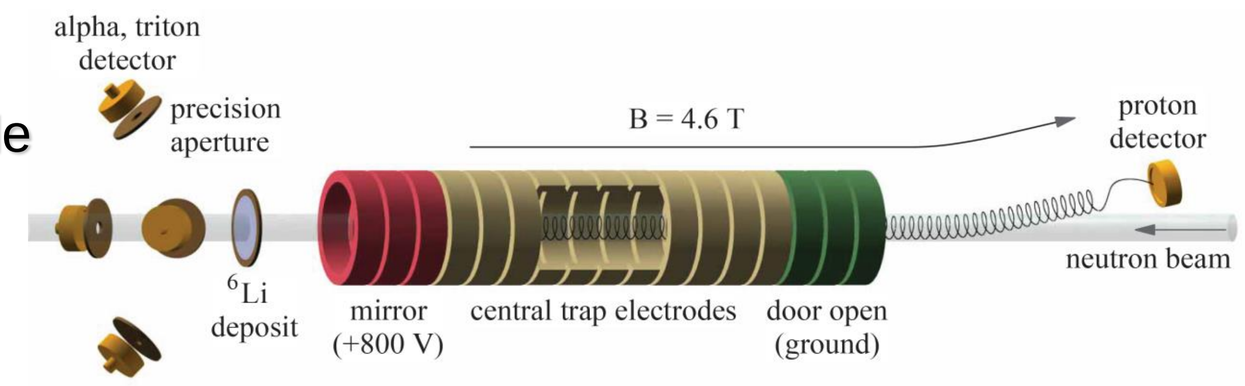
Trapping cycle

10 ms



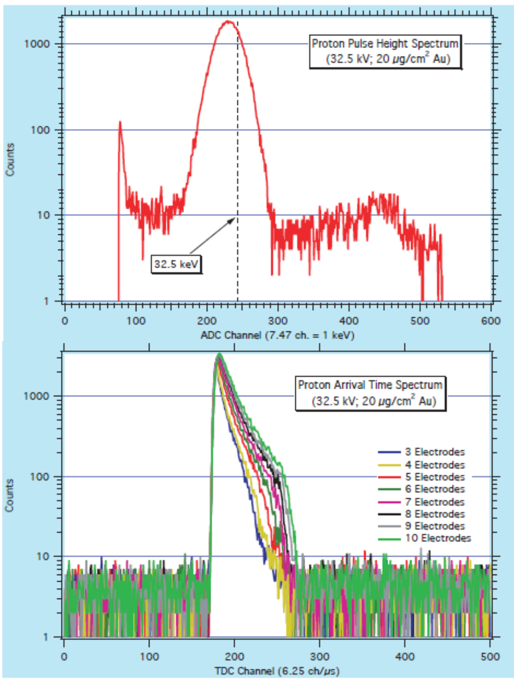
Counting cycle

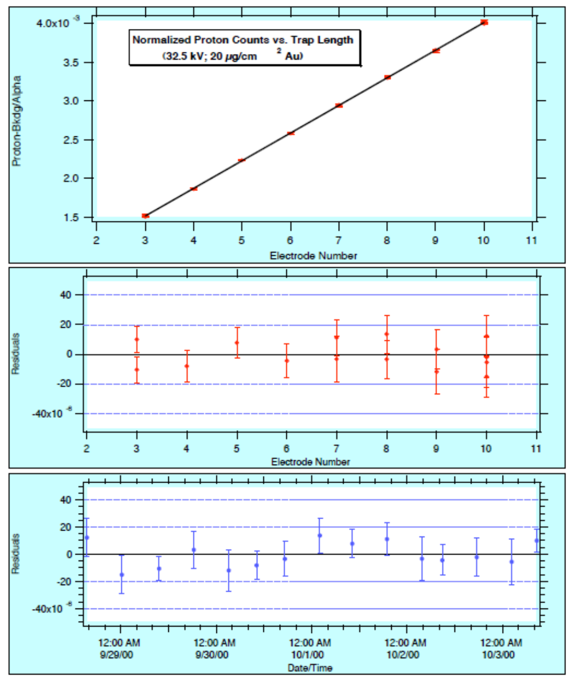
100 μs

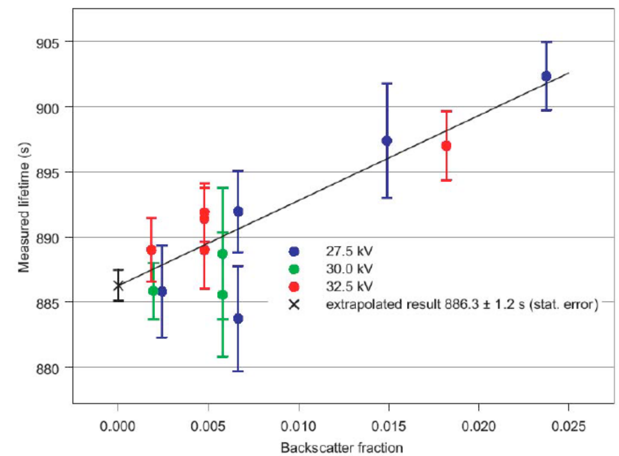


- Modular trap structure allows for changing trap lengths (control of decay volume)

"**Beam**" experiment at NIST







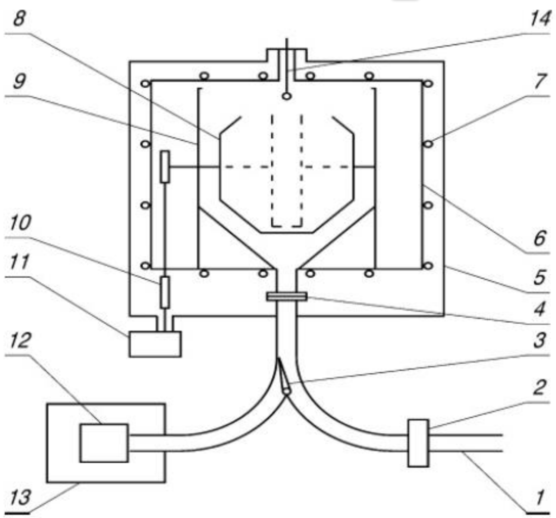
Precision efficiency of ^{6}Li flux monitor: 0.058 % (A.T. Yue, et al., Metrologia 55, 460 (2018))

$$\tau_n = (887.7 \pm 2.3)\ \mathrm{s}$$

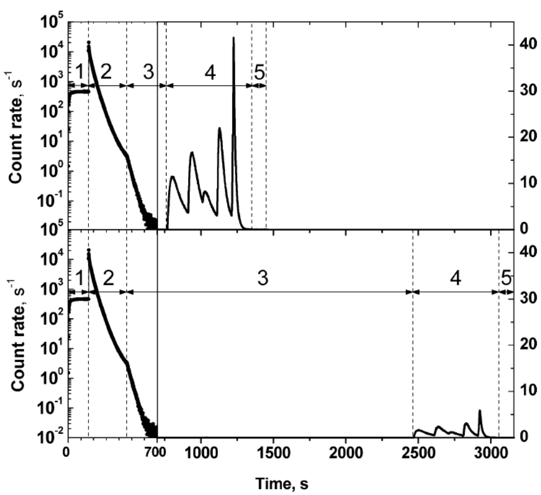
A.T. Yue, et al., PTL 111, 222501 (2013)

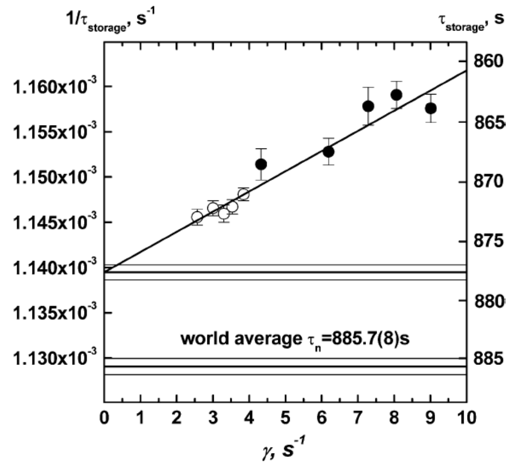
“**Bottle**” experiment

Gravitrap I
PNPI/ILL



$$\tau_{\mathrm{LOSS}}^{-1} = \eta(T)\gamma(E)$$



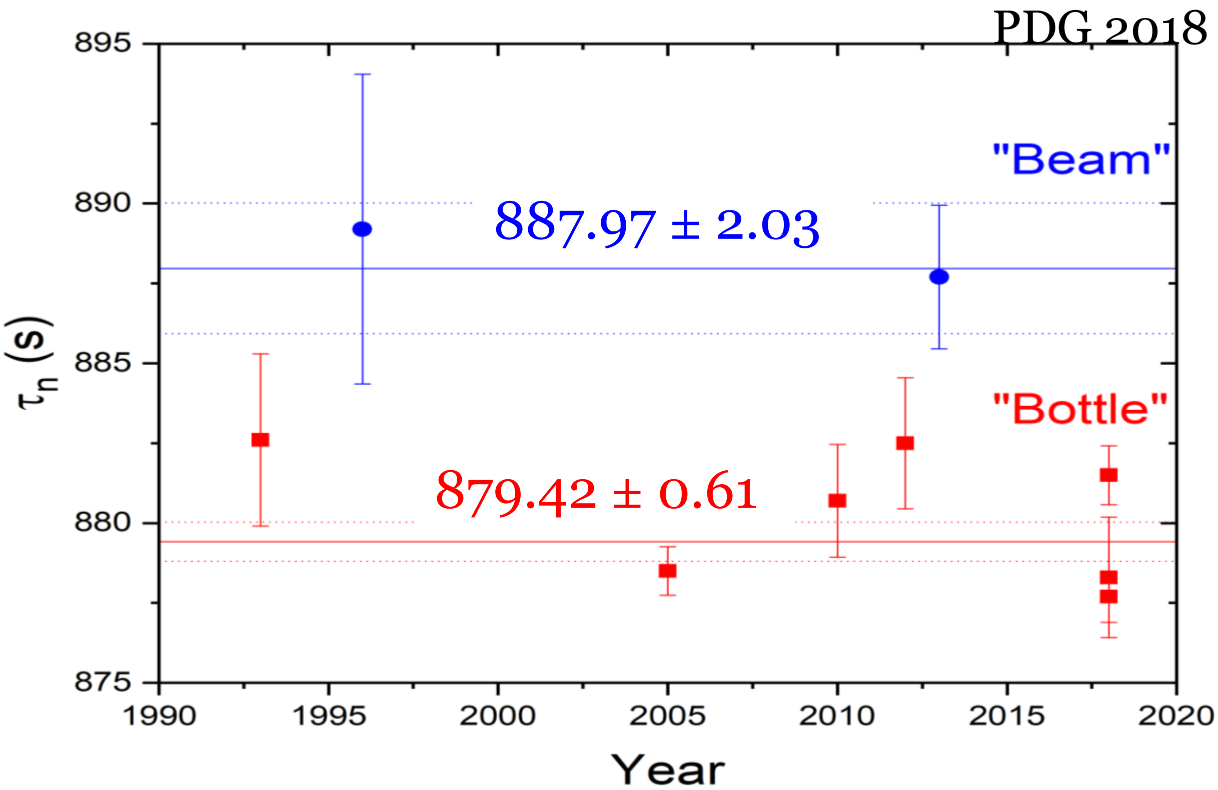


Gravitrap experiment

A.Serebrov et al. ,
Phys Lett B 605,
(2005) 72-78

878.5 ± 0.8 s

Neutron lifetime experiments: “Beam” vs. “Bottle”

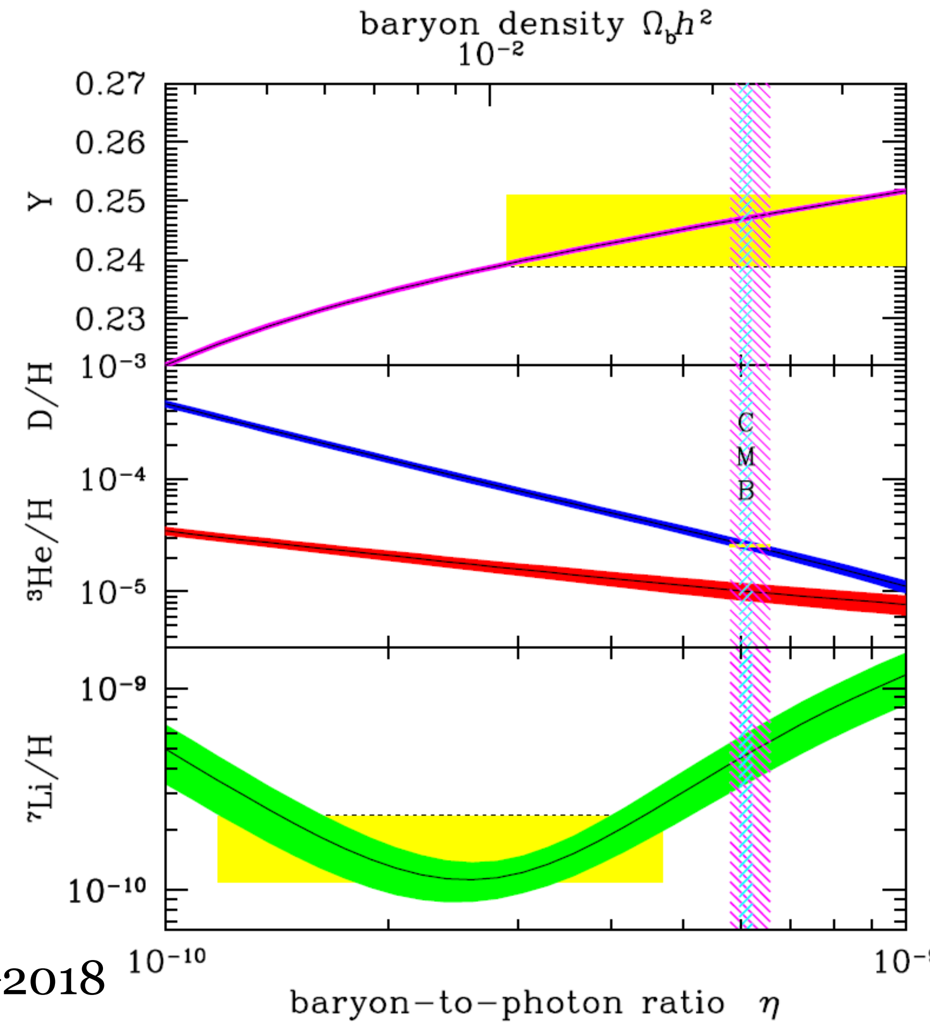


- Beam vs. bottle τ_n tension

Consequences for light element abundances in early Universe

- Primordial mass fraction of ^{4}He
  $Y_p = \rho(^4\mathrm{He})/\rho(\mathrm{H})$

$$Y_\mathrm{p} = \frac{2(n/p)}{1+n/p} \simeq 0.25$$



Following G.J. Mathews, et al., PRD 71 (2005) 021302

PDG2018

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- CMB measurements clearly prefer primordial light element production calculations assuming "Bottle" neutron lifetime

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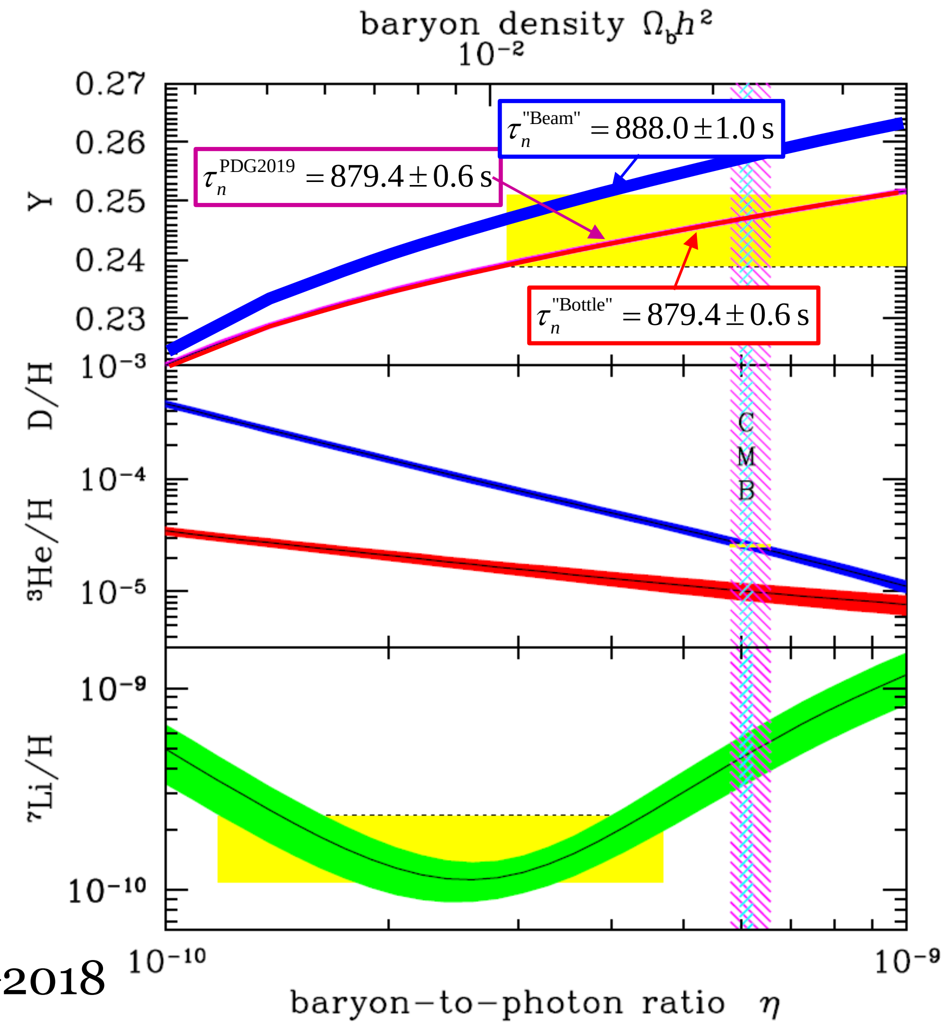


PDG2018

CKM unitarity – testing SM

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

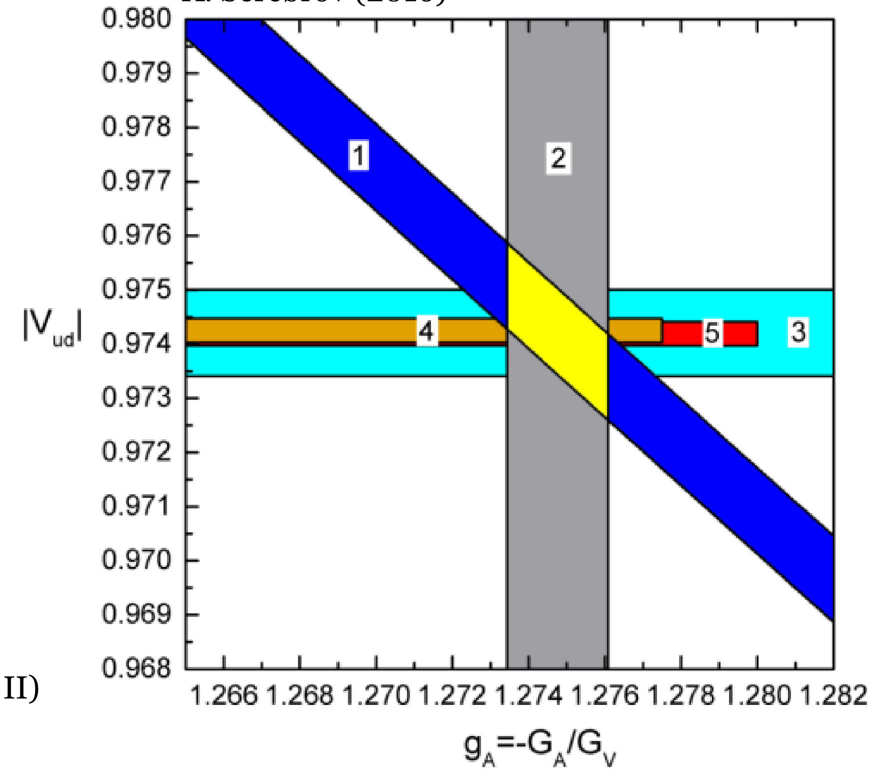
- Unitarity condition requires:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- V_{ub} is small ($V_{ub} = 3.6(7)\times10^{-3}$) so the unitarity test involves essentially only V_{ud} and V_{us}

- V_{ud} from:
  - **Nuclear super-allowed β-decays**: sophisticated nuclear structure calculations, some problems with *Q*-values
  - **From pion β-decay**: theoretically cleanest, statistically not competitive
  - **From neutron β-decay**: theoretically clean

A. Serebrov (2016)



1. Neutron decay lifetime
2. Neutron β-asymmetry *A* (PERKEO II)
3. Neutron β-decay (PDG 2015 + PERKEO II)
4. CKM Unitarity
5. $0^+ \rightarrow 0^+$ nuclear transitions

CKM unitarity – testing SM

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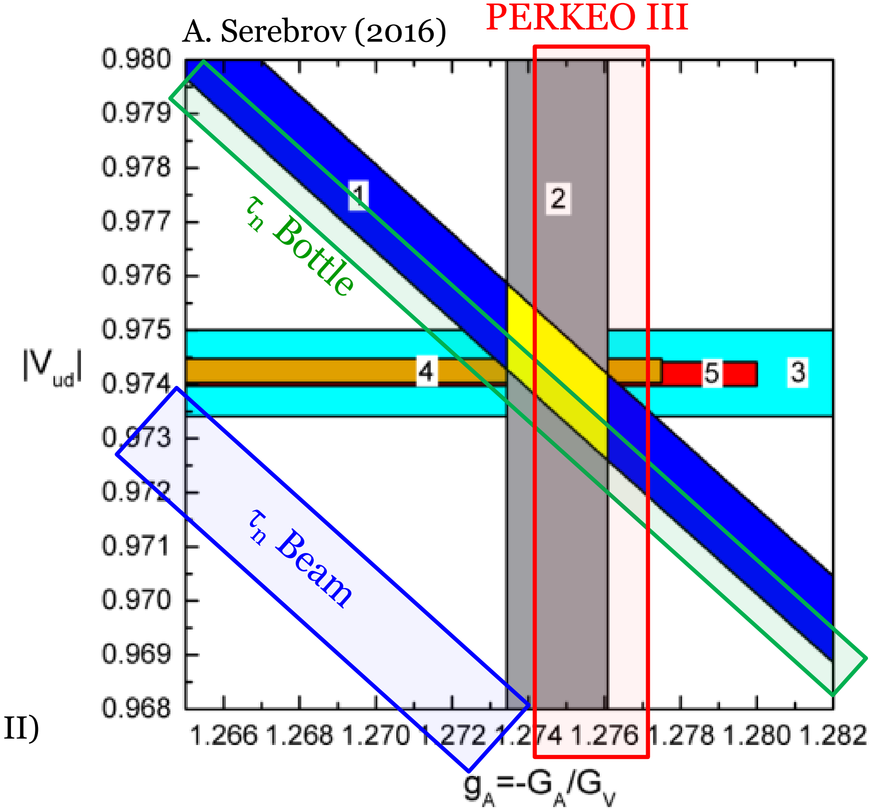

1. Neutron decay lifetime
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Neutron dark decay ?

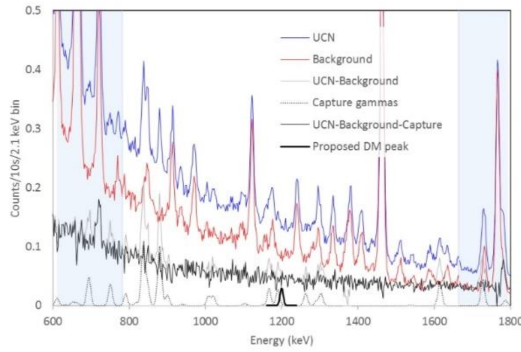
Fornal & Grinstein, PRL 120 (2018)
Dark Matter Interpretation of the Neutron Decay Anomaly

$$\mathrm{Br}(n \to p + \text{anything}) \approx 99\%$$

- From strong bounds on bound proton stability and nuclear masses:

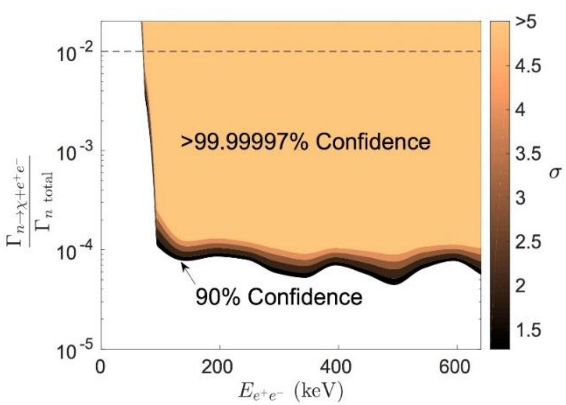
^{9}Be is stable if: 937.900 MeV < m_χ < 939.565 MeV

$$n \to \chi + \gamma$$
$$n \to \chi + e^+e^-$$
$$n \to \chi + \phi$$
$$n \to \ldots$$



UCN: Sun *et al.*, arXiv:1803.10890 [nucl-ex]

UCN: Tang *et al.*, arXiv:1802.01595 [nucl-ex]



Neutron dark decay – constraints from Astrophysics and Nuclear Physics

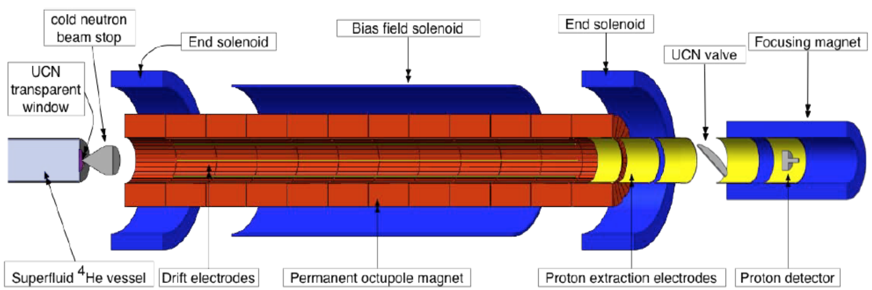
- McKeen, Nelson, Reddy & Zhou, *Neutron stars exclude light dark baryons*, arXiv:1802.08244 [hep-ph]
- Baym, Beck, Geltenbort & Shelton, *Coupling neutrons to dark fermions to explain the neutron lifetime anomaly is incompatible with observed neutron stars*, arXiv:1802.08282 [hep-ph]
- Motta, Guichon & Thomas, *Implications of neutron star properties for the existence of light dark matter*, J. Phys. G 45 05LT01 (2018)
- Cline & Cornell, *Dark decay of the neutron*, arXiv:1803.04961 [hep-ph] [$n \rightarrow \chi + A'$]
- Karananas & Kassiteridis, *Small-scale structure from neutron dark decay*, arXiv:1805.03656 [hep-ph] [*may resolve small-scale problems in ΛCDM*]
- Pfutzner & Riisager, *Examining the possibility to observe neutron dark decay in nuclei*, PRC 97, 042501(R) (2018)
- Riisager et al., *^{11}Be(βp), a quasi-free neutron decay?*, PLB 732, 305 (2014)

$$^{11}\mathrm{Be} \rightarrow {}^{10}\mathrm{Be} + \chi + \phi \qquad \mathrm{Br}\left({}^{11}\mathrm{Be} \rightarrow {}^{10}\mathrm{Be} + ?\right) \approx 8 \times 10^{-6}$$

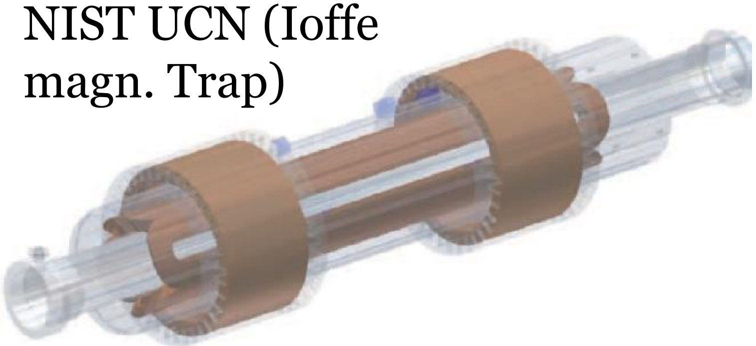
Ongoing and planned τ_n experiments

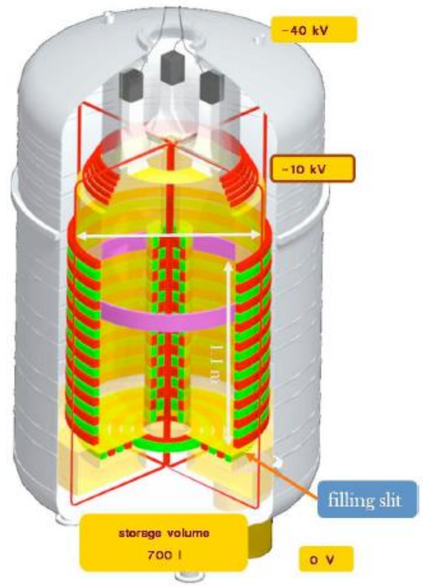
Gravitrap II

H.O.PE



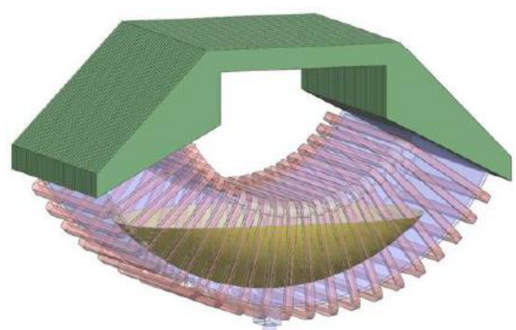
NIST UCN (Ioffe magn. Trap)



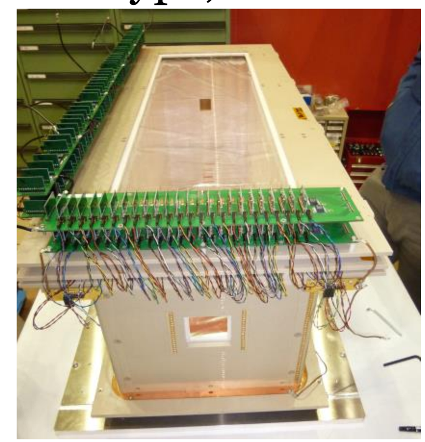


PENeLOPE

Halbach grav. trap

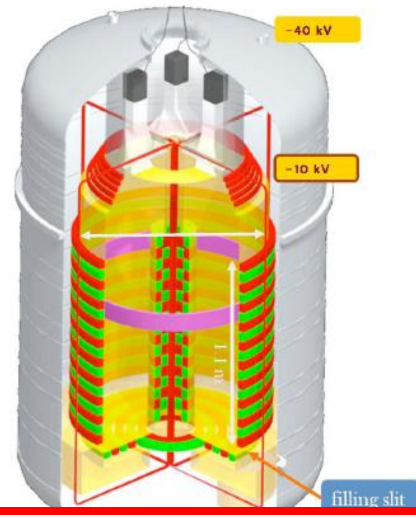


J-PARC, "Beam"-type, TPC

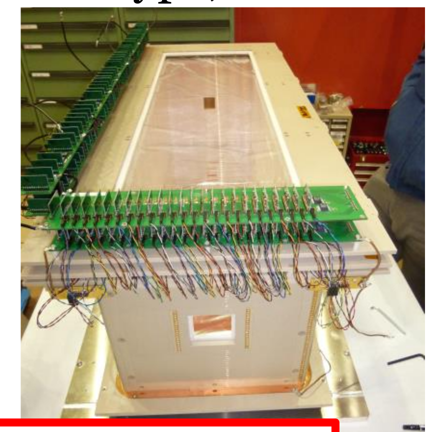


Ongoing and planned τ_n experiments

Gravitrap II



J-PARC, "Beam"-type, TPC

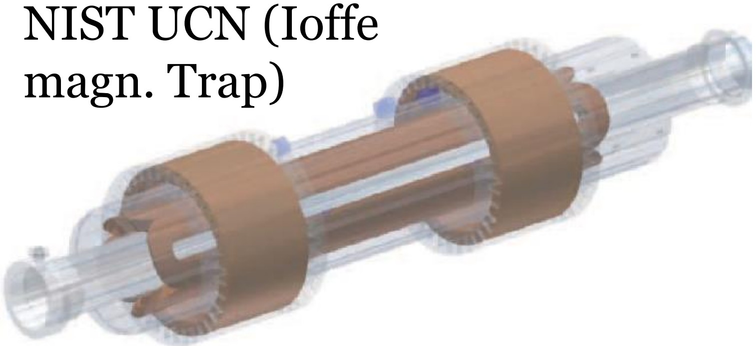


H.O.PE

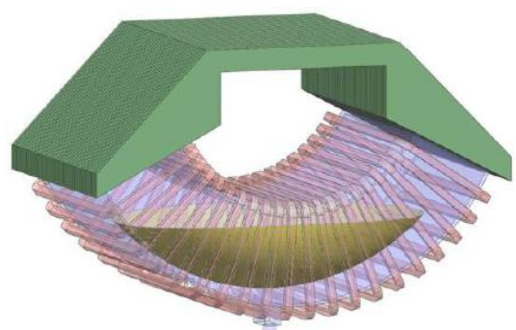
New projects aim at absolute accuracy

$\Delta\tau_n \sim 0.1 - 0.2$ s

NIST UCN (Ioffe magn. Trap)



Halbach grav. trap



Neutron β-decay correlations

- **For decay of polarized neutrons of (polarization ⟨J⟩/*J*):**

$$\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} \sim 1 + a\frac{\mathbf{p}}{E_e}\cdot\frac{\mathbf{q}}{E_\nu} + b\frac{m_e}{E_e} + \frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}}{E_e} + B\frac{\mathbf{q}}{E_\nu} + D\frac{\mathbf{p}}{E_e}\times\frac{\mathbf{q}}{E_\nu}\right] + \ldots$$

p – electron momentum **q** – neutrino momentum

σ – electron spin sensing direction

- **Coefficients *a*, *b*, …, *W* are functions of λ**

J.D. Jackson et al., Phys. Rev. 106, 517 (1957); J.D. Jackson et al., Nucl. Phys. 4, 206 (1957); M.E. Ebel et al., Nucl. Phys. 4, 213 (1957)

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$$+\boldsymbol{\sigma}\cdot\left[G\frac{\mathbf{p}}{E_e} + H\frac{\mathbf{q}}{E_\nu} + K\frac{\mathbf{p}}{E_e+m_e}\frac{\mathbf{p}}{E_e}\cdot\frac{\mathbf{q}}{E_\nu} + L\frac{\mathbf{p}}{E_e}\times\frac{\mathbf{q}}{E_\nu} + N\frac{\langle \mathbf{J}\rangle}{J}\right]$$

$$+\boldsymbol{\sigma}\cdot\left[Q\frac{\mathbf{p}}{E_e+m_e}\frac{\langle \mathbf{J}\rangle}{J}\cdot\frac{\mathbf{p}}{E_e} + R\frac{\langle \mathbf{J}\rangle}{J}\times\frac{\mathbf{p}}{E_e} + S\frac{\langle \mathbf{J}\rangle}{J}\frac{\mathbf{p}}{E_e}\cdot\frac{\mathbf{q}}{E_\nu} + T\frac{\mathbf{p}}{E_e}\frac{\langle \mathbf{J}\rangle}{J}\cdot\frac{\mathbf{q}}{E_\nu}\right]$$

$$+\boldsymbol{\sigma}\cdot\left[U\frac{\mathbf{q}}{E_\nu}\frac{\langle \mathbf{J}\rangle}{J}\cdot\frac{\mathbf{p}}{E_e} + V\frac{\mathbf{q}}{E_\nu}\times\frac{\langle \mathbf{J}\rangle}{J} + W\frac{\mathbf{p}}{E_e+m_e}\frac{\langle \mathbf{J}\rangle}{J}\frac{\mathbf{p}}{E_e}\times\frac{\mathbf{q}}{E_\nu}\right]$$

p – electron momentum **q** – neutrino momentum
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Neutron β-decay correlations worldwide

Experiment	Correlation and anticipated precision	Location and status
aSPECT	a (3×10^{-4})	FRM-2 (ongoing)
aCORN	a (5×10^{-4})	NIST (ongoing)
Nab/aBBa/PANDA	a ($\sim10^{-4}$), b (3×10^{-4}), A, B, C ($\sim10^{-4}$)	SNS (planned)
emiT	D ($\sim10^{-4}$) – measured	NIST (completed)
PERC	a, b, A (3×10^{-5}), B, C, D (?)	FRM-2 (ongoing)
PERKEO	A (2×10^{-4}), B, C (2×10^{-3}) – measured	ILL (ongoing)
UCNA	A (2.5×10^{-3})	LANL (ongoing)
UCNB	B ($<10^{-3}$)	LANL (ongoing)
nTRV	N, R ($\sim10^{-2}$) - measured	PSI (completed)
BRAND	a, A, B, D, H, L, N, R, S, U, V ($\sim5\times10^{-4}$)	ILL (in preparation), ESS (planned)

- Review on the interrelations between decay coefficients can be found e.g. in D. Dubbers and M.G. Schmidt, Rev. Mod. Phys. 83 (2011) 1111-1171.
- Up to date sensitivity analysis of new experiments given by S. Baessler at NORDITA Workshop, 10-14 Dec 2018 "*Particle Physics with Neutrons at the ESS*" – available on-line.

Transverse electron polarization in neutron decay – quest for BSM

- If only transverse electron polarization can be observed:

$$\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} \sim 1 + a\frac{\mathbf{p}}{E_e}\cdot\frac{\mathbf{q}}{E_\nu} + b\frac{m_e}{E_e} + \frac{\langle \mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}}{E_e} + B\frac{\mathbf{q}}{E_\nu} + D\frac{\mathbf{p}}{E_e}\times\frac{\mathbf{q}}{E_\nu}\right]$$

$$+\boldsymbol{\sigma}_\perp\cdot\left[H\frac{\mathbf{q}}{E_\nu} + L\frac{\mathbf{p}}{E_e}\times\frac{\mathbf{q}}{E_\nu} + N\frac{\langle \mathbf{J}\rangle}{J} + R\frac{\langle \mathbf{J}\rangle}{J}\times\frac{\mathbf{p}}{E_e}\right]$$

$$+\boldsymbol{\sigma}_\perp\cdot\left[S\frac{\langle \mathbf{J}\rangle}{J}\frac{\mathbf{p}}{E_e}\cdot\frac{\mathbf{q}}{E_\nu} + U\frac{\mathbf{q}}{E_\nu}\frac{\langle \mathbf{J}\rangle}{J}\cdot\frac{\mathbf{p}}{E_e} + V\frac{\mathbf{q}}{E_\nu}\times\frac{\langle \mathbf{J}\rangle}{J}\right]$$

- All correlation coefficients can be expressed as **combinations** of real and imaginary parts of exotic (**scalar** and **tensor**) couplings:

$$X = X_{\mathrm{SM}} + X_{\mathrm{FSI}} + c_{\mathrm{Re}S}\,\mathrm{Re}\,S + c_{\mathrm{Re}T}\,\mathrm{Re}\,T + c_{\mathrm{Im}S}\,\mathrm{Im}\,S + c_{\mathrm{Im}T}\,\mathrm{Im}\,T$$

$S = \frac{C_S + C_S'}{C_V}$, $T = \frac{C_T + C_T'}{C_A}$, $c_{\mathrm{Re}S}, c_{\mathrm{Re}T}, c_{\mathrm{Im}S}, c_{\mathrm{Im}T}$ – functions of $\lambda = C_A/C_V$ and kinematical quantities

Sensitivity factors for scalar and tensor couplings

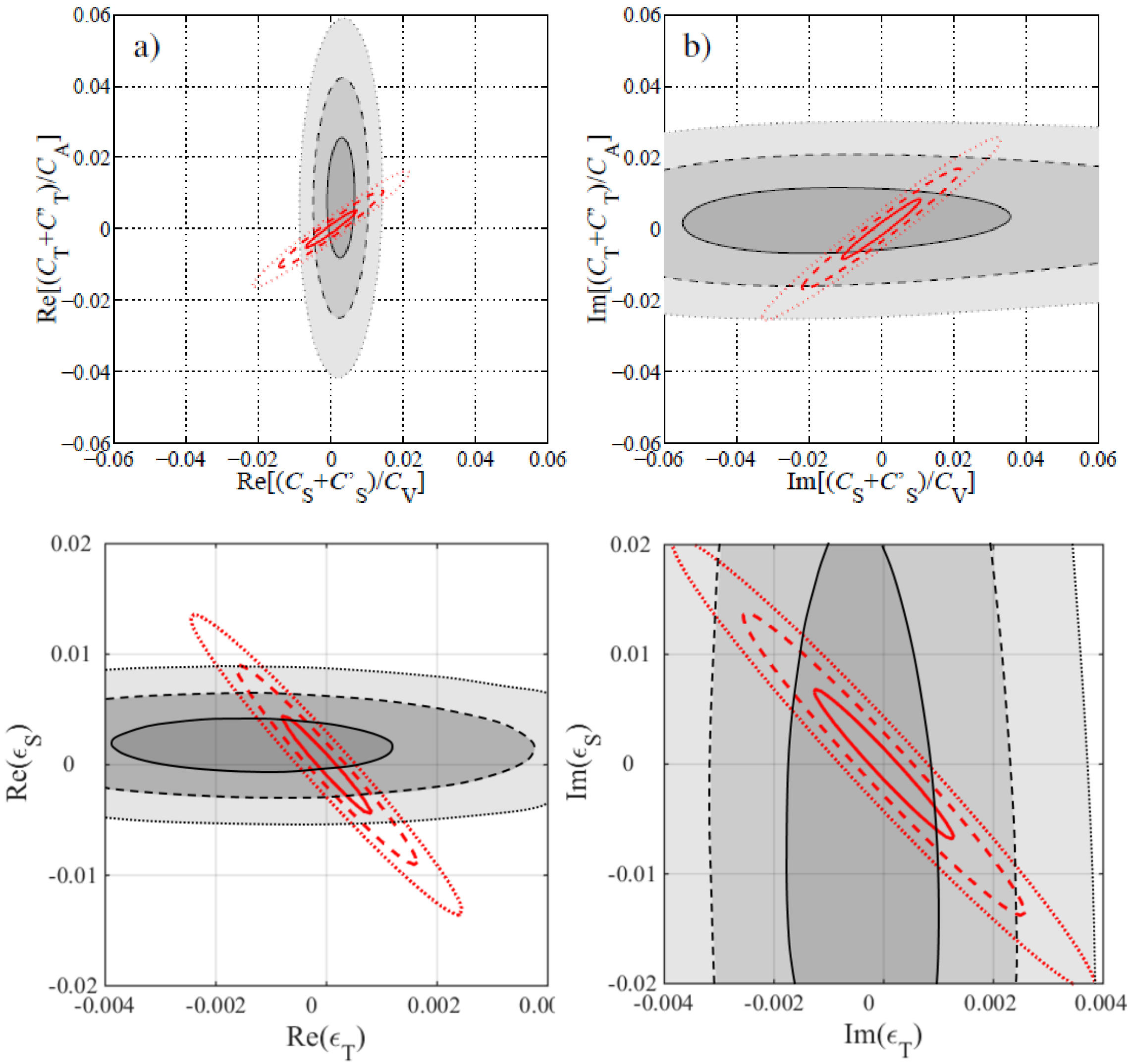
(leading order, no recoil, point charge)

	SM (λ)	FSI (λ)	c(Re$\mathcal{S}$)	c(Re$\mathcal{T}$)	c(Im$\mathcal{S}$)	c(Im$\mathcal{T}$)
a	-0.1048	0	-0.1714†	0.1714†	-0.0007	+0.0012
b	0	0	+0.1714	+0.8286	0	0
A	-0.1172	0	0	0	-0.0009	+0.0014
B	+0.9876	0	-0.1264	+0.1945	0	0
D	0	0	+0.0009	-0.0009	0	0
H	+0.0609	0	-0.1714	+0.2762	0	0
L	0	-0.0004	0	0	+0.1714	-0.2762
N	+0.0681	0	-0.2176	+0.3348	0	0
R	0	+0.0005	0	0	-0.2176	+0.3348
S	0	-0.0018	+0.2176	-0.2176	0	0
U	0	0	-0.2176	+0.2176	0	0
V	0	0	0	0	-0.2176	+0.2172

* Kinematical factor averaged over electron kinetic energy E_k = (200,783) keV

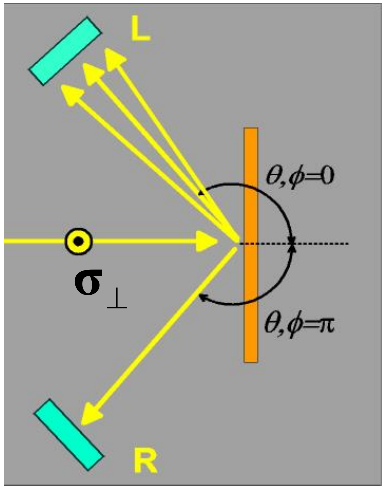
† $(|C_S|^2+|C'_S|^2)/2$ instead of ReS and $(|C_T|^2+|C'_T|^2)/2$ instead of ReT, respectively

Impact of *H*, *L*, *N*, *R*, *S*, *U* and *V* measurement with anticipated accuracy of 5×10^{-4}



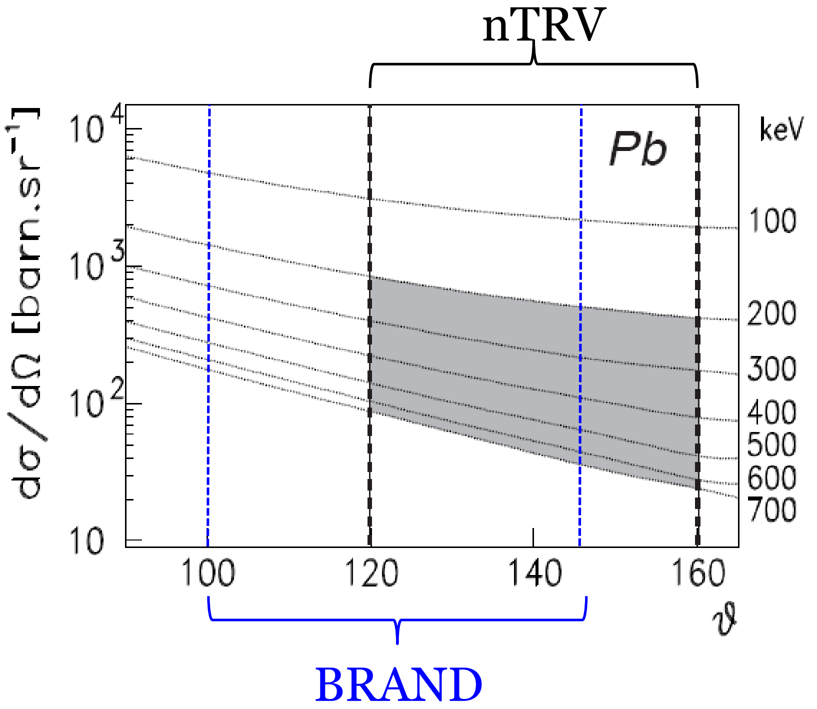
- Constraints on ReS and ReT contributions will be comparable to that obtained from Fierz term (spectrum shape) with $\Delta b < 10^{-3}$ but with completely different systematics
- Similar accuracy is expected for ImS and ImT contribution (TRV effects)
- **In order to be competitive with HE searches, new n-decay correlation coefficient measurements should improve by 1 order of magnitude**

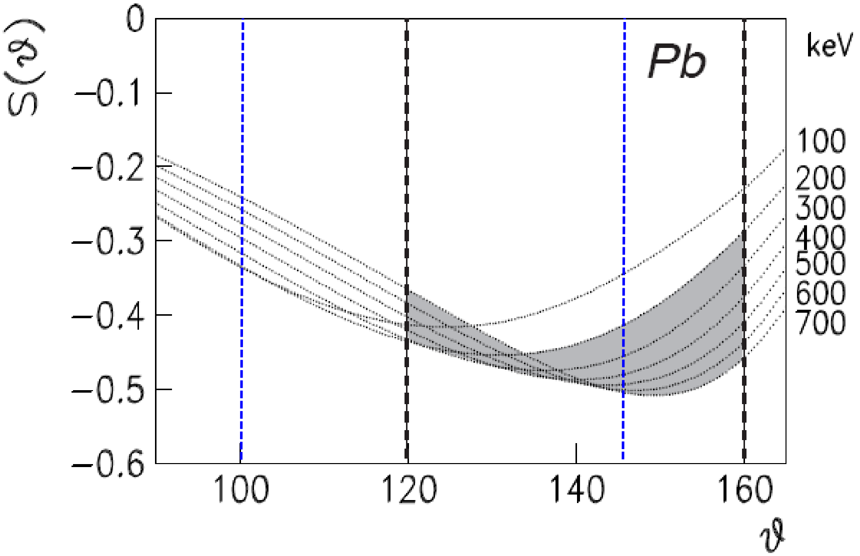
Electron spin analysis



- **Mott scattering:**
  - Analyzing power caused by spin-orbit force
  - Parity and time reversal conserving (electromagnetic process)
  - Sensitive **exclusively** to the transverse polarization

$$\frac{N_L - N_R}{N_L + N_R} = \frac{\langle \boldsymbol{\sigma}_\perp \rangle}{\sigma} S(\theta)$$





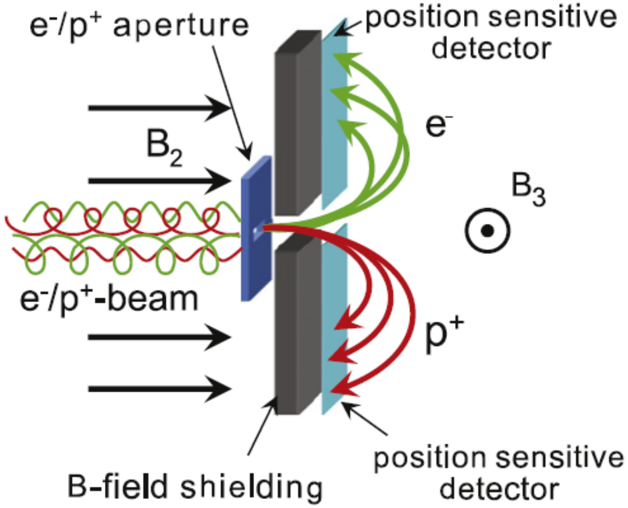
Obstacles for electron polarization measurements

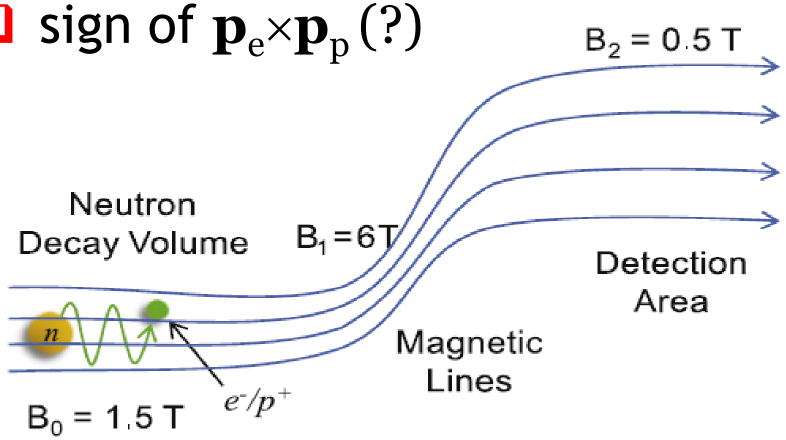
- **Momentum rotation in external electric field**
  - In uniform field step of 30 kV, incident energy of 100 keV and angle of 45^o, momentum vector rotates by about 12^o
  - Effect decreases with increasing energy and decreasing angle of incidence
  - Effect cancels in symmetric barrier or if symmetrically sampled (left-right)
- **„*g*-2 effect"**
  - 7 mrad per revolution de-synchronization between spin and momentum
- **Electron polarization can be determined only in well controlled electric field and in low magnetic field**

ep/n separators coupled to magnetic spectrometers

G. Konrad et al., J. Phys.: Conf. Series **340** (2012) 012048

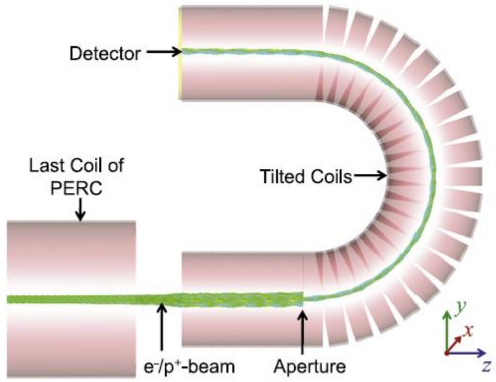
- ep/n-spectrometers adiabatically coupled to n-decay channel (PERC, ANNI)
- Conserve particle energy, angular (polar) distributions and spin helicity
- Reconstruct $|\mathbf{p}_e\times\mathbf{p}_p|$ using decay kinematics
- sign of $\mathbf{p}_e\times\mathbf{p}_p$ (?)





X. Wang et al., NIMA (2012) 254

R×B
NoMoS

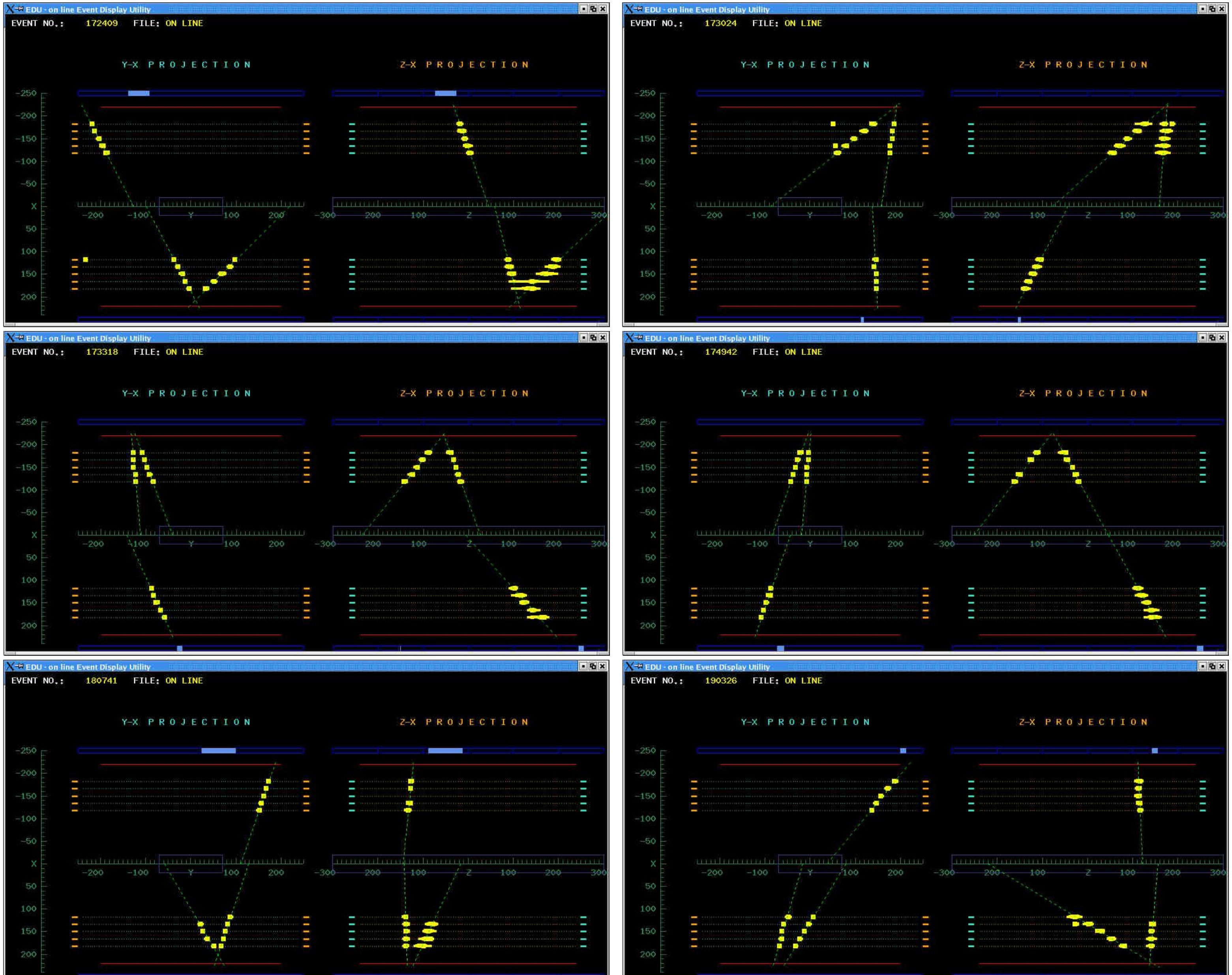


Electron tracking, vertex reconstruction (HE like approach)

- Unavoidable for electron spin analysis in Mott scattering for diffused and weak decay sources like e.g. cold neutron beam
- Allows for direct measurement of geometry factors
- Reduces gamma background in electron energy detector
- Allows for implementation of corrections based on parameter maps (e.g. effective Sherman function corrected for target thickness variation and for angle of incidence)
- Allows for accurate gain balance of large plastic scintillators
- Improves diagnostics of beam in fiducial volume

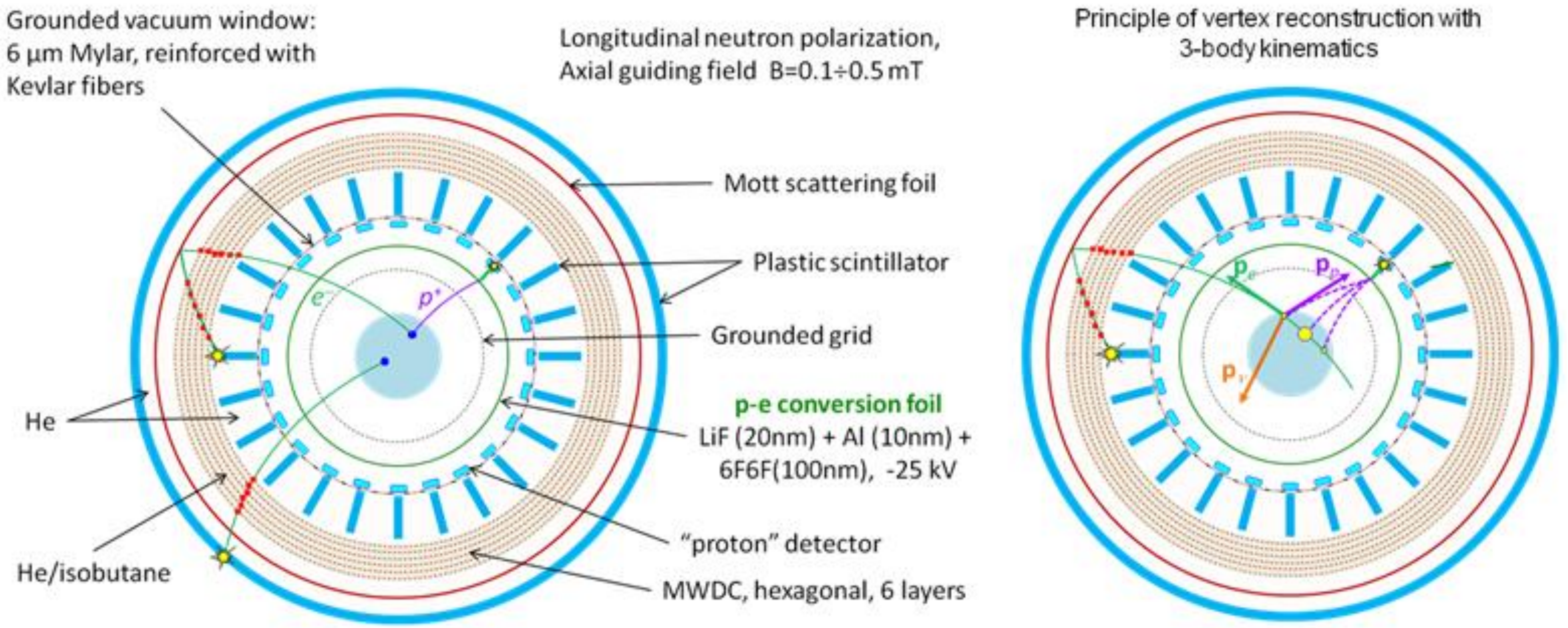
nTRV@PSI

“V-track” events - on-line display



BRAND project

- **Systematic exploration of electron spin dependent correlations:** ***H*, *L*, *N*, *R*, *S*, *U*, *V***



a) b)

Timeline

	BRAND I	BRAND II	BRAND III
Site	ILL Grenoble	ILL Grenoble	ESS Lund
Time	3 years	3 - 4 years	5-6 years
Pressure	Ambient	<300 mbar	<300 mbar
Mott target	Pb (Au)	Pb (Au)	Depleted U
Coverage of azimuthal angle	1/6	Full	Full
Statistical precision (goal)			
A	0.0008	0.00008	0.000016
a, B, D	0.005	0.0005	0.0001
R, N	0.01	0.001	0.0002
H, L, S, U, V	0.02	0.002	0.0003
Systematic errors (goal)			
R, N, H, L, S, U, V	0.002	0.001	0.0004

Summary and outlook

- **Neutron observables**:
  - *Test directly SM and search for TeV scale physics beyond SM*
- **The dream scenario**:
  - *LHC finds BSM particle(s) on-shell and β-decay has to confirm it in observables (off-shell corrections)*
- **Fundamental neutron research is**:
  - *Important for Particle Physics*
  - *Addressed in several labs worldwide*
  - *Promising as new installations (CN-beams, UCN) are under construction*
- **New results**:
  - *Expected soon from variety of ongoing and planned projects*

Backup slides

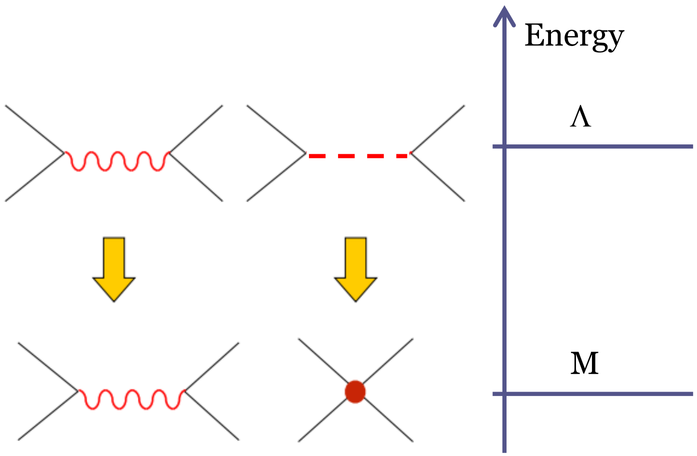
BRAND Collaboration

- **Presently BRAND collaboration consists of:**
  - JU Krakow: K. Bodek[1)], D. Rozpedzik, J. Zejma[1)], K. Lojek, M. Perkowski, M. Kolodziej:
    e- and *p*-detectors, front end electronics and DAQ, simulations
  - INP PAS Krakow: A. Kozela[1)], K. Pysz & Co.:
    mechanical structure, vacuum window, MWDC tracker, Mott target, Slow Control
  - ILL Grenoble: T. Soldner:
    polarized CN beam and infrastructure, vacuum
  - KU Leuven: N. Severijns[1)] , L. De Keukeleere.:
    guiding magnetic field
  - NCSU Raleigh: A. Young:
    pe-converter film
  - ...

[1)] Members of nTRV@PSI

BRAND phase-1 (ILL) funded by NCN (OPUS-15)

EFT approach in β-decay



- For experiments at energy significantly lower than BSM scale (Λ_i):

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum \frac{1}{\Lambda_i^2}\mathcal{L}_i \approx \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum \alpha_i \mathcal{O}_i^{(6)}$$

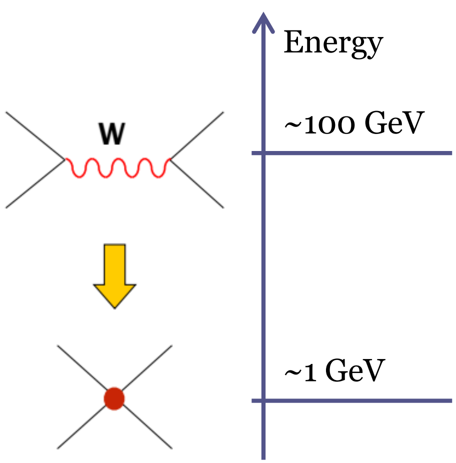
$\mathcal{O}_i^{(6)}$ – dimension-6 operators

α_i – *Wilson coefficients* $\alpha_i = \Lambda^2 f_i\left(g_{\text{BSM}}, M_{\text{BSM}}\right)$

Observables for $E << \Lambda$: $\mathcal{R} = \mathcal{R}_0\left(1 + \frac{\mathcal{O}(M)}{\Lambda} + \frac{\mathcal{O}(M^2, E^2, ME)}{\Lambda^2} + \ldots\right)$

- Semi-leptonic processes, partonic level, exchanged W-boson is heavy – SM interaction Lagrangian takes the contact (V-A)×(V-A) form

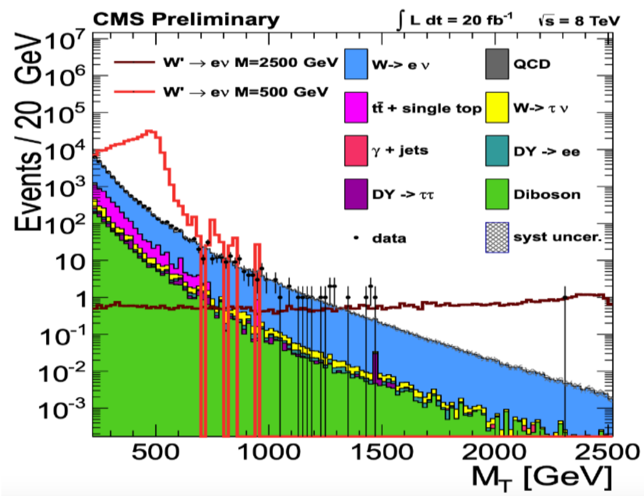
$$\mathcal{L}_{\text{SM}} = -\frac{G_F V_{ud}}{\sqrt{2}}\, \bar{e}\gamma_\mu(1-\gamma_5)\nu_e \cdot \bar{u}\gamma^\mu(1-\gamma_5)d$$

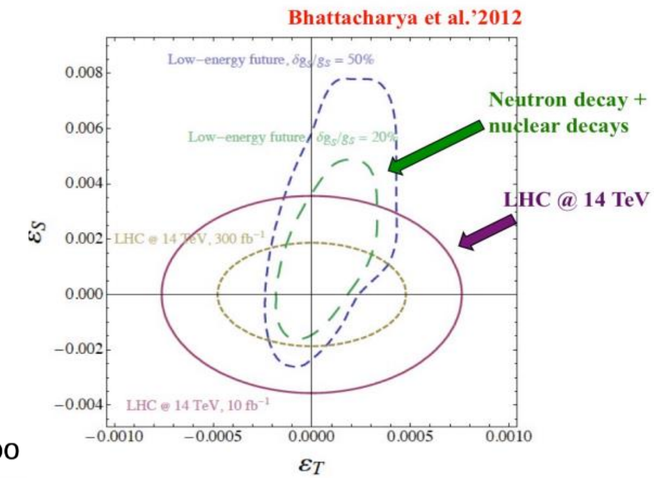


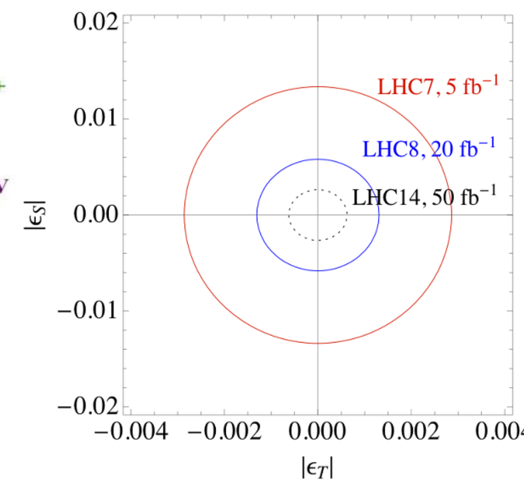
LE-HE competition: CMS results

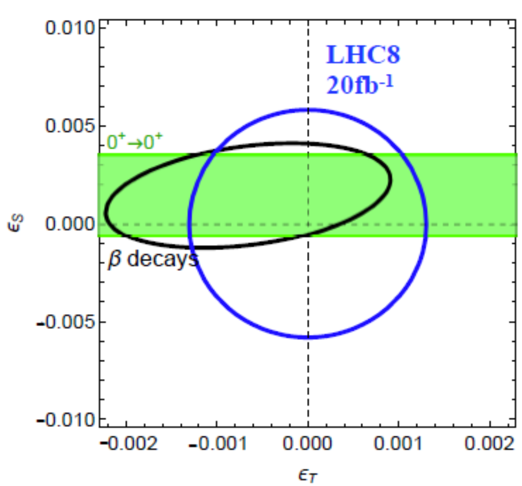
- Electrons and missing transverse energy (MET) channel

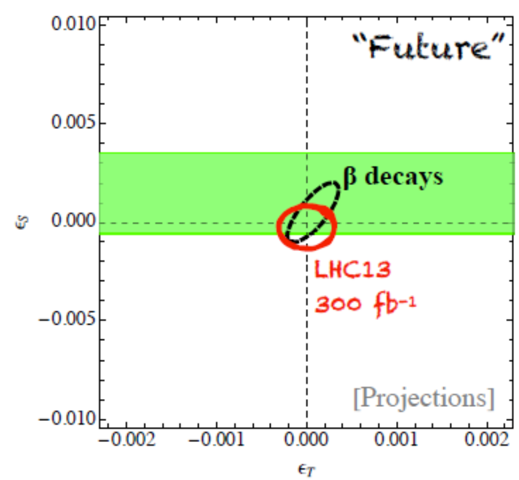
$$\sigma(pp \to e + \mathrm{MET} + X)$$











M. Gonzalez-Alonso et al., Ann. Phys. 525 (2013), 180308732

Gupta et al., 1806.09006

EFT approach in β-decay (cont.)

Model independent EFT parameters

V. Cirigliano et al., Nucl. Phys. B 830 (2010)
T. Bhattacharya et al., Phys. Rev. D 85 (2012)
V. Cirigliano et al., JHEP 1302 (2013)
M. Gonzalez-Alonso et al., Ann. Phys. 525 (2013)
M. Gonzalez-Alonso et al., Phys. Rev. Lett. 112 (2014)

$$\begin{aligned}\mathcal{L}_{\rm eff} = & -\frac{G_F V_{ud}}{\sqrt{2}}\,[\,(1+\epsilon_L)\;\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\cdot\bar{u}\gamma^\mu(1-\gamma_5)d \\ & + \tilde{\epsilon}_L\;\bar{e}\gamma_\mu(1+\gamma_5)\nu_e\cdot\bar{u}\gamma^\mu(1-\gamma_5)d \\ & + \epsilon_R\;\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\cdot\bar{u}\gamma^\mu(1+\gamma_5)d \\ & + \tilde{\epsilon}_R\;\bar{e}\gamma_\mu(1+\gamma_5)\nu_e\cdot\bar{u}\gamma^\mu(1+\gamma_5)d \\ & + \epsilon_S\;\bar{e}(1-\gamma_5)\nu_e\cdot\bar{u}d + \tilde{\epsilon}_S\;\bar{e}(1+\gamma_5)\nu_e\cdot\bar{u}d \\ & - \epsilon_P\;\bar{e}(1-\gamma_5)\nu_e\cdot\bar{u}\gamma_5 d - \tilde{\epsilon}_P\;\bar{e}(1+\gamma_5)\nu_e\cdot\bar{u}\gamma_5 d \\ & + \epsilon_T\;\bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)d \\ & + \tilde{\epsilon}_T\;\bar{e}\sigma_{\mu\nu}(1+\gamma_5)\nu_e\cdot\bar{u}\sigma^{\mu\nu}(1+\gamma_5)d] + \text{h.c.}\,.\end{aligned}$$

Valid also for $\pi^\pm \to \pi^0 e^\pm \nu$

Low-energy simplifications:

- Neglect RH neutrinos – $\tilde{\epsilon}_{L,R,S,P,T} = 0$
- Pseudo-scalar contribution (non-relativistic limit) – $\epsilon_P = 0$

$$\begin{aligned}\mathcal{L}_{\rm eff} = & -\frac{G_F V_{ud}}{\sqrt{2}}\,[1+\text{Re}\,(\epsilon_L+\epsilon_R)]\,\times \\ & \times\{\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\cdot\bar{u}\gamma^\mu\,[1-(1-2\epsilon_R)\,\gamma_5]\,d \\ & + \epsilon_S\;\bar{e}(1-\gamma_5)\nu_e\cdot\bar{u}d \\ & + \epsilon_T\;\bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)d\} + \text{h.c.}\end{aligned}$$

Nucleon-level effective couplings

- Lee-Yang effective Lagrangian (leading order, low momentum transfer):

$$
\begin{aligned}
-\mathcal{L}_{n\to pe^-\bar{\nu}_e} = \; & \bar{p}\, n \; (C_S \bar{e}\nu_e - C_S' \bar{e}\gamma_5\nu_e) \\
& + \bar{p}\gamma^\mu n \,(C_V \bar{e}\gamma_\mu\nu_e - C_V' \bar{e}\gamma_\mu\gamma_5\nu_e) \\
& + \bar{p}\sigma^{\mu\nu} n \,(C_T \bar{e}\sigma_{\mu\nu_e}\nu_e - C_T' \bar{e}\sigma_{\mu\nu}\gamma_5\nu_e) \\
& - \bar{p}\gamma^\mu\gamma_5 n \,(C_A \bar{e}\gamma_\mu\gamma_5\nu_e - C_A' \bar{e}\gamma_\mu\nu_e) \\
& + \bar{p}\gamma_5 n \; (C_P \bar{e}\gamma_5\nu_e - C_P' \bar{e}\nu_e) + \text{h.c.}\;.
\end{aligned}
$$

$$C_i,\; C_i' \;\; (i \in \{V, A, S, T\})$$

$$C_i = \frac{G_F}{\sqrt{2}} V_{ud}\, \overline{C}_i$$

$$\langle p|\bar{u}\Gamma d|n\rangle = g_\Gamma\, \bar{\psi}_p \Gamma \psi_n$$

- Effective nucleon-level couplings can be expressed in parton-level parameters:

$$
\begin{aligned}
\overline{C}_V &= g_V\,(1 + \epsilon_L + \epsilon_R + \tilde{\epsilon}_L + \tilde{\epsilon}_R) \\
\overline{C}_V' &= g_V\,(1 + \epsilon_L + \epsilon_R - \tilde{\epsilon}_L - \tilde{\epsilon}_R) \\
\overline{C}_A &= -g_A\,(1 + \epsilon_L - \epsilon_R - \tilde{\epsilon}_L + \tilde{\epsilon}_R) \\
\overline{C}_A' &= -g_A\,(1 + \epsilon_L - \epsilon_R + \tilde{\epsilon}_L - \tilde{\epsilon}_R)
\end{aligned}
$$

$$
\begin{aligned}
\overline{C}_S &= g_S\;(\epsilon_S + \tilde{\epsilon}_S) \\
\overline{C}_S' &= g_S\;(\epsilon_S - \tilde{\epsilon}_S) \\
\overline{C}_P &= g_P\;(\epsilon_P - \tilde{\epsilon}_P) \\
\overline{C}_P' &= g_P\;(\epsilon_P + \tilde{\epsilon}_P) \\
\overline{C}_T &= 4\, g_T\;(\epsilon_T + \tilde{\epsilon}_T) \\
\overline{C}_T' &= 4\, g_T\;(\epsilon_T - \tilde{\epsilon}_T)
\end{aligned}
$$

- *Form factors are the key ingredients for translation of hadron-level coupling constants to parton-level parameters*

EFT approach in β-decay (cont.)

- g_A from experiment (Lattice QCD still not accurate):

$$g_A \to g_A \,\mathrm{Re}\left[\frac{1+\epsilon_L-\epsilon_R}{1+\epsilon_L+\epsilon_R}\right] \approx g_A\left[1-2\mathrm{Re}(\epsilon_R)\right]+\mathcal{O}\left(\epsilon_i^2\right)$$

- 6 parameters left for probing:
  - $\epsilon_L+\epsilon_R$ – can be absorbed in V_{ud} (CKM unitarity tests)
  - Real parts of ϵ_S and ϵ_T
  - Imaginary parts of ϵ_R , ϵ_S and ϵ_T

- FF from Lattice QCD calculation
- Modest knowledge of g_S and g_T is still sufficient for present accuracy level of experimental observables

	g_S	g_T
Adler et al.'1975	0.60(40)	1.45(85)
PNDME 2011	0.80(40)	1.05(35)
LHPC 2012	1.08(32)	1.04(02)
PNDME 2013	0.66(24)	1.09(05)

Limits from high energy

- Electrons and missing transverse energy (MET) channel

$$\sigma(pp \to e + \mathrm{MET} + X)$$

- Underlying partonic process is the same as in β-decay $(\bar{u}d \to e\bar{\nu})$
- If BSM particles are too heavy to be produced on-shell → EFT analysis appropriate
- Express weak scale Lagrangian in terms of EFT parameters and calculate cross section

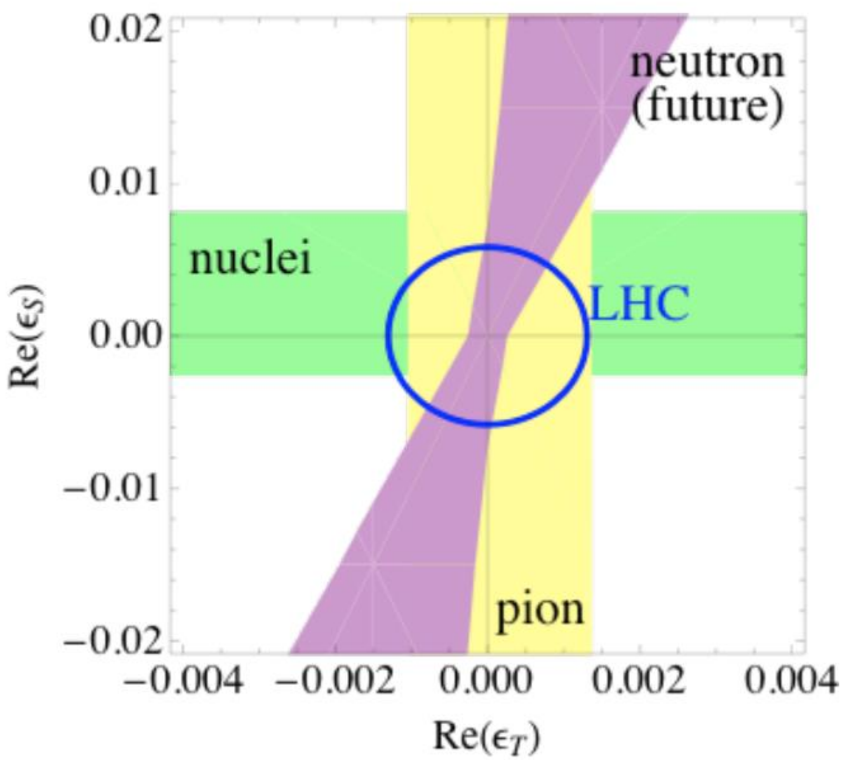
$$\begin{aligned}\sigma(m_T > \overline{m}_T) \quad = \quad & \sigma_W \left[\left| 1 + \epsilon_L^{(v)} \right|^2 + |\tilde{\epsilon}_L|^2 + |\epsilon_R|^2 \right] \\ & -2\,\sigma_{WL}\,\mathrm{Re}\left(\epsilon_L^{(c)} + \epsilon_L^{(c)} \epsilon_L^{(v)*} \right) + \sigma_R \left[|\tilde{\epsilon}_R|^2 + |\epsilon_L^{(c)}|^2 \right] \\ & + \sigma_S \left[|\epsilon_S|^2 + |\tilde{\epsilon}_S|^2 + |\epsilon_P|^2 + |\tilde{\epsilon}_P|^2 \right] + \sigma_T \left[|\epsilon_T|^2 + |\tilde{\epsilon}_T|^2 \right]\end{aligned}$$

LE-HE competition

- Benefits for β-decay analysis from better determination of g_S and g_T FF

	$\langle p\vert\bar{u}d\vert n\rangle$ g_S	$\langle p\vert\bar{u}\sigma_{\mu\nu}\gamma_5 d\vert n\rangle$ g_T
Adler et al.'1975	0.60(40)	1.45(85)
PNDME 2011	0.80(40)	1.05(35)
LHPC 2012	1.08(32)	1.04(02)
RQCD 2014	1.02(35)	1.01(02)
PNDME 2013/15	0.72(32)	1.02(08)
ETMC 2015/17	0.93(33)	1.00(03)
CVC	1.02(11)	-
PNDME 2016/18	1.02(10)	0.99(03)
JLQCD'18	0.88(11)	1.08(10)

M. Gonzalez-Alonso (2016)



TRV tests

- True TRV tests require (i) reversal of motion and (ii) exchange of initial and final states

(a) $t \to -t$ (b)

Initial Final Initial Final

Scattering, binary reaction:

- In particle decay exchange of initial and final states is impossible

Particle decay:

- If interaction violates TR symmetry: $\Gamma(a) \neq \Gamma(b)$

D- and *R*-correlations

- In ordinary neutron decay, two observables are particularly interesting

$D \sim \langle \mathbf{J} \rangle / J \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$ $\quad\quad$ $R \sim \langle \mathbf{J} \rangle / J \cdot (\mathbf{p}_e \times \boldsymbol{\sigma})$

- *D*-correlation (C-odd, P-even, T-odd)

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_{\bar\nu}} = S(E_e)\left[1 + a\frac{\mathbf{p}_e \cdot \mathbf{p}_{\bar\nu}}{E_e E_{\bar\nu}} + b\frac{m_e}{E_e} + \frac{\langle \mathbf{J} \rangle}{J} \cdot \left(A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_{\bar\nu}}{E_{\bar\nu}} + D\frac{\mathbf{p}_e \times \mathbf{p}_{\bar\nu}}{E_e E_{\bar\nu}}\right)\right]$$

- *R*-correlation (C-even, P-odd, T-odd)

$$\frac{d\Gamma}{dE_e d\Omega_e} = S(E_e)\left[1 + b\frac{m_e}{E_e} + A\frac{\langle \mathbf{J} \rangle}{J} \cdot \frac{\mathbf{p}_e}{E_e} + G\frac{\mathbf{p}_e \cdot \boldsymbol{\sigma}}{E_e} + \frac{\langle \mathbf{J} \rangle}{J} \cdot \left(Q\frac{\mathbf{p}_e}{E_e}\frac{\mathbf{p}_e \cdot \boldsymbol{\sigma}}{E_e + m_e} + N\boldsymbol{\sigma} + R\frac{\mathbf{p}_e \times \boldsymbol{\sigma}}{E_e}\right)\right]$$

$$\frac{d\Gamma}{dE_e d\Omega_e} = S(E_e)\left[1 + b\frac{m_e}{E_e} + \frac{\langle \mathbf{J} \rangle}{J} \cdot \left(A\frac{\mathbf{p}_e}{E_e} + N\boldsymbol{\sigma}_\perp + R\frac{\mathbf{p}_e \times \boldsymbol{\sigma}_\perp}{E_e}\right)\right]; \qquad \boldsymbol{\sigma}_\perp \perp \mathbf{p}_e$$

D-correlation

J.D. Jackson et al., Phys. Rev. 106, 517 (1957); J.D. Jackson et al., Nucl. Phys. 4, 206 (1957); M.E. Ebel et al., Nucl. Phys. 4, 213 (1957)

- For left-handed *V-A* interactions ($C_V = C'_V,\ C_A = C'_V$), defining $\lambda = C_A/C_V$, neglecting terms quadratic in C_S and C_T, point charge, no recoil:

$$D = D_{\not T} + D_{FSI} \approx D_{\not T} + 1.2\times 10^{-5}$$

$$D_{\not T} \approx \frac{1}{1+3|\lambda|^2}\left\{-2\frac{\mathrm{Im}\left(C_V C_A^*\right)}{|C_V|^2} + \frac{\mathrm{Im}\left(C_S C_T^* + C'_S C'^*_T\right)}{|C_V|^2}\right\}$$

$$+\frac{\alpha m}{p_e}\frac{1}{1+3|\lambda|^2}\mathrm{Re}\left(\lambda^*\frac{C_T^* + C'^*_T}{C_A^*} - \lambda^*\frac{C_S + C'_S}{C_V}\right)$$

$$D_{\not T} \approx \frac{1}{1+3|\lambda|^2}\left\{2\sin\phi_{AV} + \mathrm{Im}\left(S^+T^{+*} + S^-T^{-*}\right)\right\};\quad S^\pm = \frac{C_S \pm C'_S}{C_V},\quad T^\pm = \frac{C_T \pm C'_T}{C_A}$$

$$D_{\not T}^{VA} \approx 0.435\,\sin\phi_{AV}$$

V-correlation

$$\frac{d\Gamma}{dE_{\bar{\nu}}d\Omega_{\bar{\nu}}} \sim 1+\ldots+\boldsymbol{\sigma}_{\perp}\cdot\left[V\frac{\mathbf{p}_{\bar{\nu}}}{E_{\bar{\nu}}}\times\frac{\langle\mathbf{J}\rangle}{J}\right]$$

$$V = V_{\not{T}} + V_{FSI} \approx V_{\not{T}} + 10^{-5}\ (?)$$

$$V_{\not{T}} \approx \frac{1}{1+3|\lambda|^2}\frac{m}{E_e}\left\{2\frac{\mathrm{Im}\left(C_V C_A^*\right)}{|C_V|^2}+\frac{\mathrm{Im}\left(C_S C_T^* + C'_S C'^*_T\right)}{|C_V|^2}\right\}$$

$$+\frac{1}{1+3|\lambda|^2}\mathrm{Im}\left(\lambda^*\frac{C_S+C'_S}{C_V}-\lambda^*\frac{C_T+C'_T}{C_A}\right)$$

$$V_{\not{T}} \approx \frac{1}{1+3|\lambda|^2}\left\{\frac{m}{E_e}\left[2\sin\phi_{AV}+\mathrm{Im}\left(S^+T^{+*}-S^-T^{-*}\right)\right]-|\lambda|\left[\mathrm{Im}\,S^+-\mathrm{Im}\,T^+\right]\right\}$$

$$\boxed{V_{\not{T}}^{VA} \approx 0.261\sin\phi_{AV}} \qquad \left(T_e \geq 200\ \mathrm{keV}\right)$$