Introduction to Longitudinal Hands-on Calculations

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CERN

Introduction to Accelerator Physics

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Outline

- **Introduction**
- **Design of RF systems**
  - **Design flow and constraints**
  - **Examples of RF systems at CERN**
- **Longitudinal particle tracking**
  - **Basic tracking equations**
  - **Single and multiple particle tracking**
- **Summary**

Introduction

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical

- **Describe particle motion by differential equations**

→ **Continuous trajectories of particle motion**

→ **Deduce useful parameters for stable acceleration:**
  - → RF bucket
  - → Synchrotron frequency
  - → Stable phase
  - → …

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical	Numerical: tracking
• Describe particle motion by differential equations → Continuous trajectories of particle motion → Deduce useful parameters for stable acceleration: → RF bucket → Synchrotron frequency → Stable phase → ...	• Track particle parameters from turn to turn → Profit from discretization of motion: turn-by-turn, RF station-by-RF station → No notion of RF bucket, synchrotron, stable phase, etc. → Follow ensemble of particles to study evolution of bunch

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical	Numerical: tracking
• Describe particle motion by differential equations → Continuous trajectories of particle motion → Deduce useful parameters for stable acceleration: → RF bucket → Synchrotron frequency → Stable phase → …	• Track particle parameters from turn to turn → Profit from discretization of motion: turn-by-turn, RF station-by-RF station → No notion of RF bucket, synchrotron, stable phase, etc. → Follow ensemble of particles to study evolution of bunch
→ Classical introduction of longitudinal beam dynamics	→ Flexible brute-force approach

Objectives of longitudinal hands-on

1. **Design RF system (upgrade)**

   `LongitudinalHandsOnDraftRFSystemCalculations_empty.ipynb`

   - **Study boundary constraints**
   - **Derive requirements for RF system**
   - **Choose main components**
   - **Compare with existing facilities**

2. **Play with longitudinal beam dynamics**

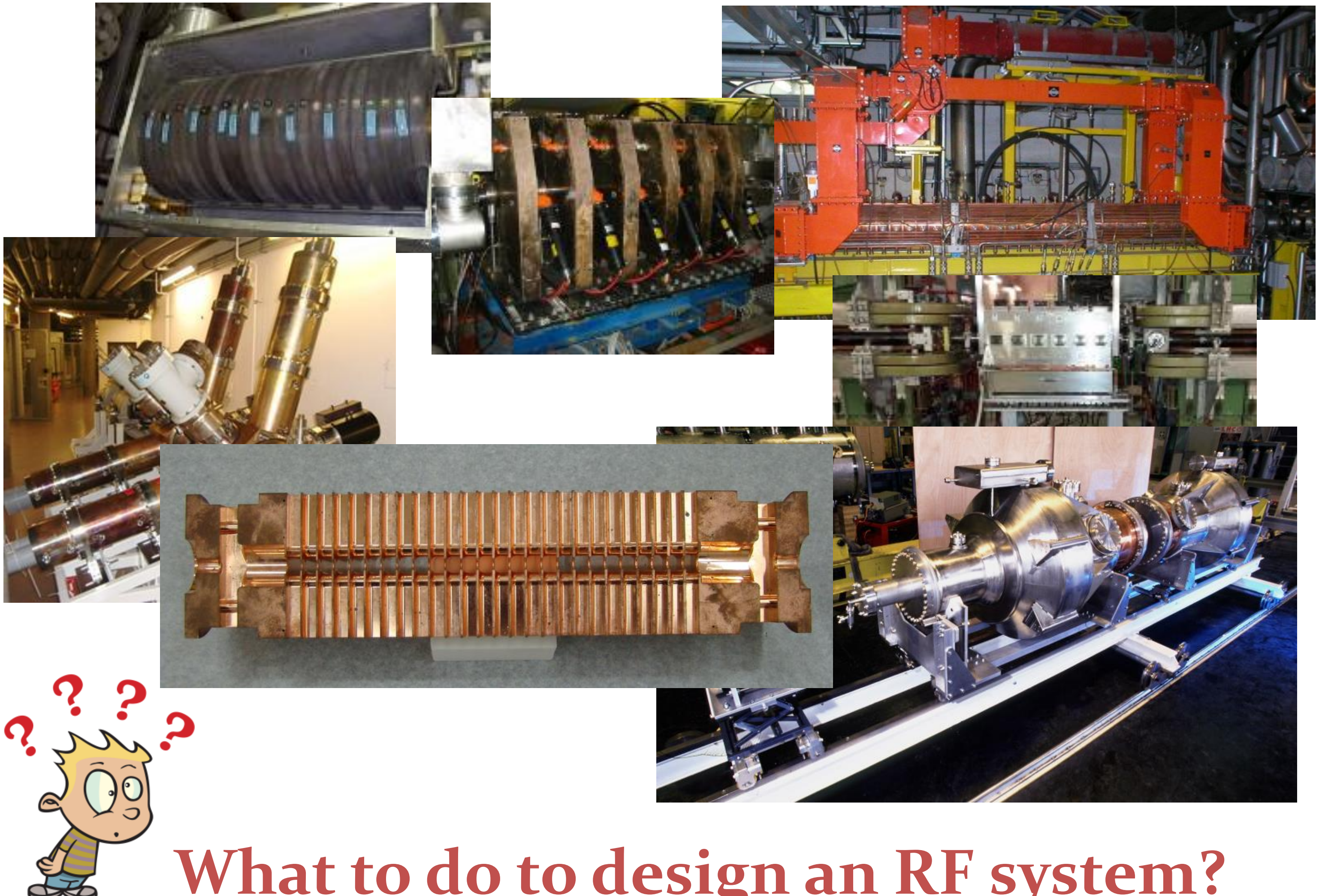
   `LongitudinalHandsOnDraftTracking_empty.ipynb`

   - **Build your own particle tracker**
   - **Understand motion of particles in longitudinal phase space**
   - **Transition from single particle motion to evolution of an entire bunch**

RF system design

Tomorrow afternoon

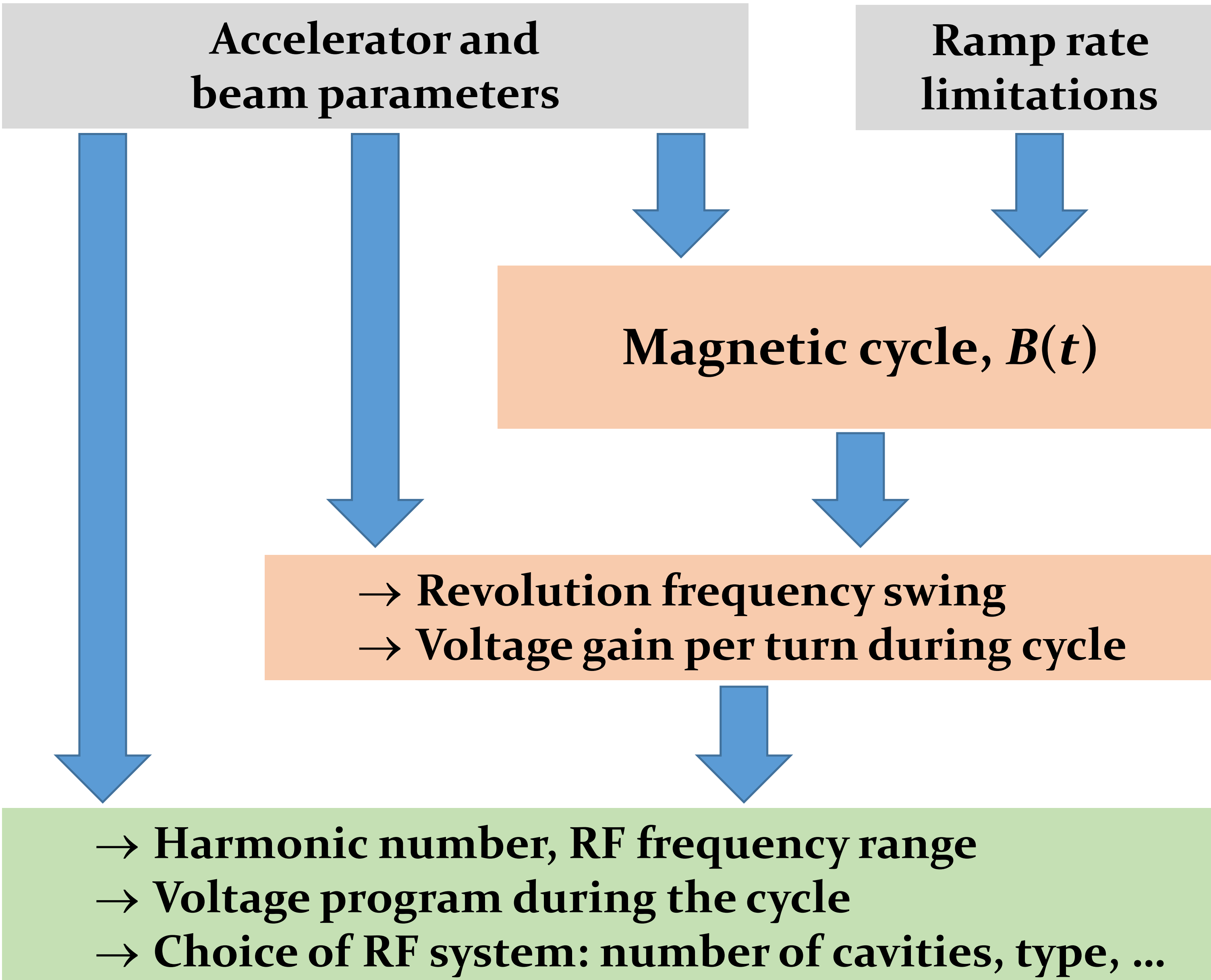
Introduction



What to do to design an RF system?

How to choose the right one?

Simplified design work flow

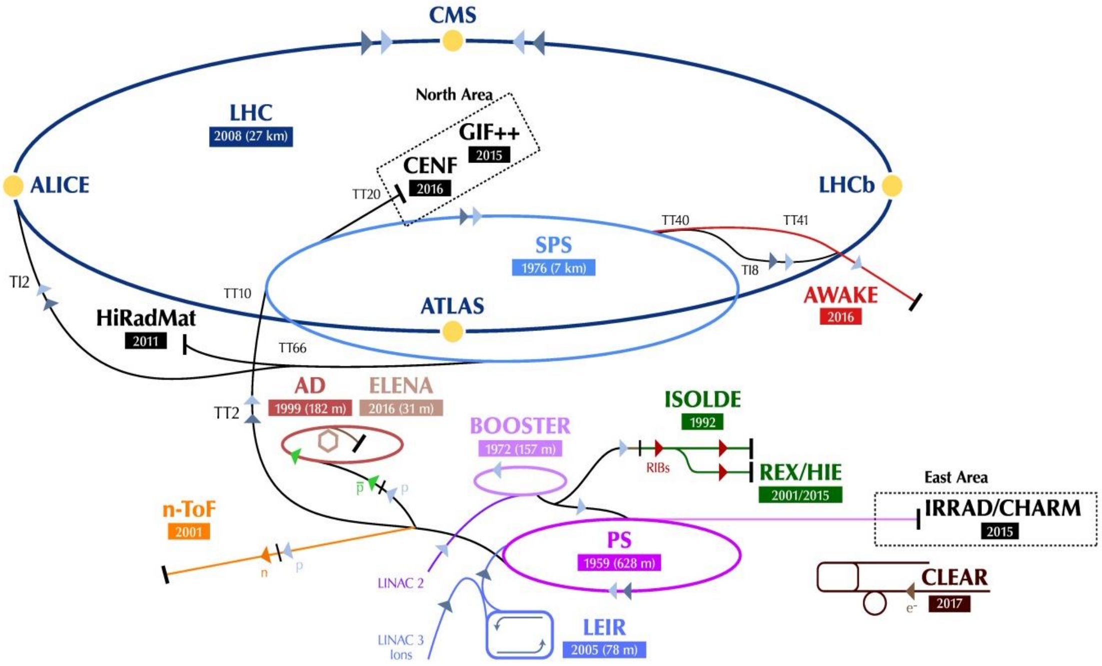


RF parameters of existing accelerators

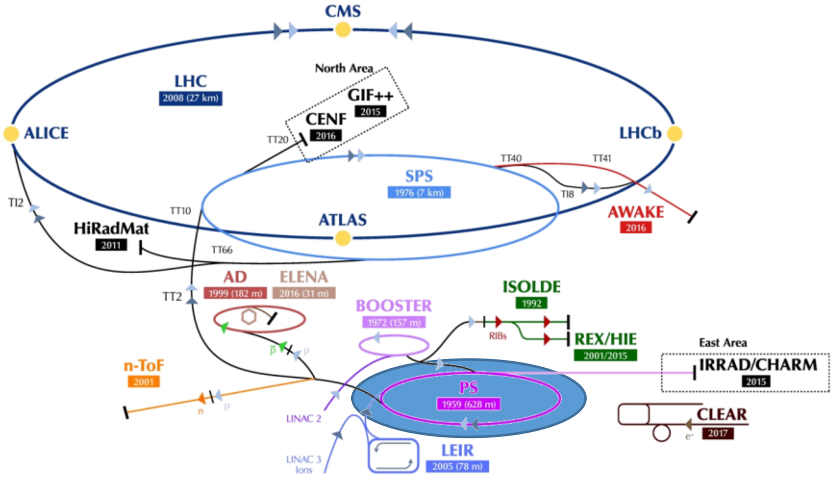
Try to follow design choices of existing accelerator

- **Can we understand the arguments?**
- **Are the choices reasonable?**

Good design?



Proton Synchrotron



Example: RF System for CERN PS

- **Attention**
  - → **Present RF system designed in ~1969**
  - → **Not the same energy range as today**

Parameter	**Value**
Circumference, $2\pi R$	$2\pi \cdot 100$ m = 628 m
Acceleration time, t_{cycle}	1 s
Maximum ramp rate, dB/dt	**2.3 T/s**
Injection energy, E_{kin}	**45 MeV**
Flat-top energy, E_{tot}	initially **28 GeV**

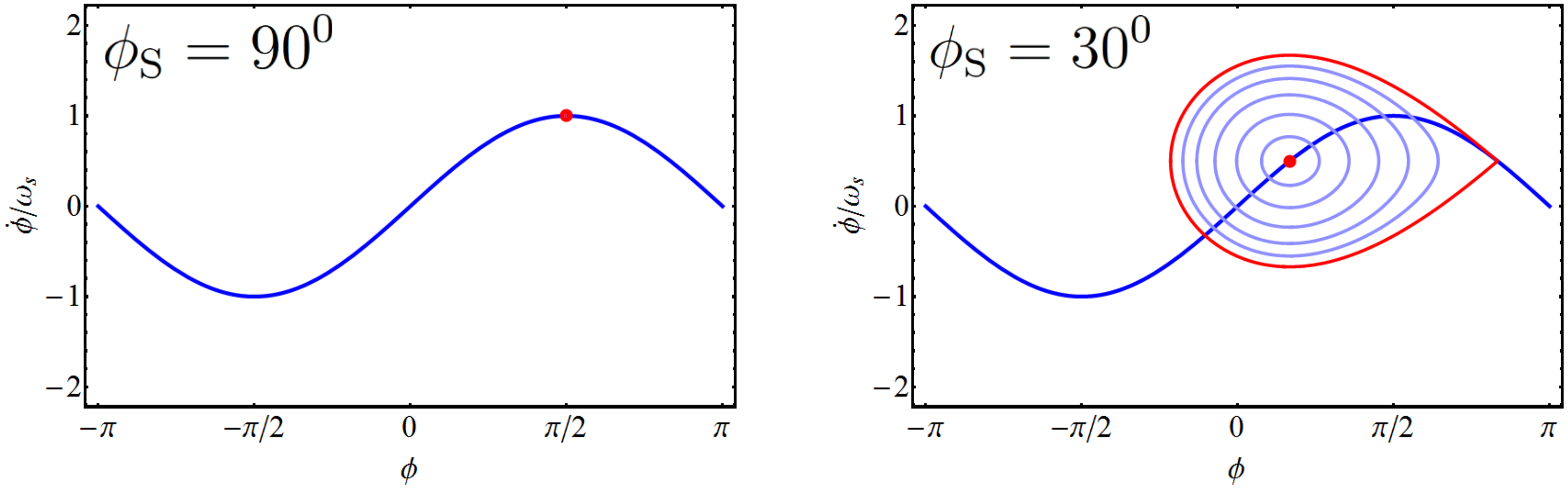
Example: CERN PS - choice of RF voltage

→ Energy gain per turn defined by size and ramp rate

$$\Delta E_{\text{turn}} = 2\pi q \rho R \dot{B}$$

→ At 2.3 T/s ramp rate: ~100 keV gain per turn

→ Just sufficient to accelerate synchronous particle

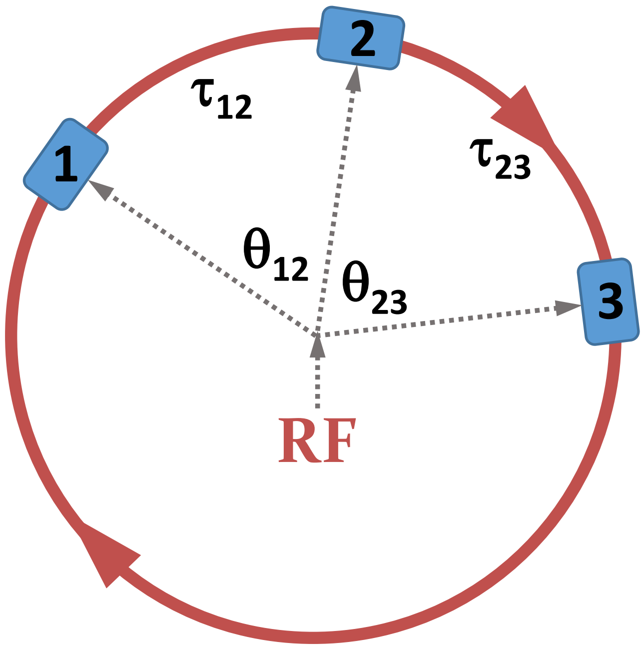


→ Over-voltage for bucket area: $V_{\text{RF}} = \dfrac{1}{\sin\phi_S}\dfrac{\Delta E}{q} \simeq 200\ \text{kV}$

Example: CERN PS - choice of RF harmonic

→ **Operate RF stations in phase with respect to beam**

→ **Use common RF signal**



- **Time of flight, τ_{nm} between RF cavities:**

→ **Multiple of RF period**

→ $\tau_{\mathrm{pq}} = n \cdot T_{\mathrm{RF}} = n/hT_{\mathrm{rev}}$

$$\theta_{\mathrm{pq}} = n \cdot 2\pi/h$$

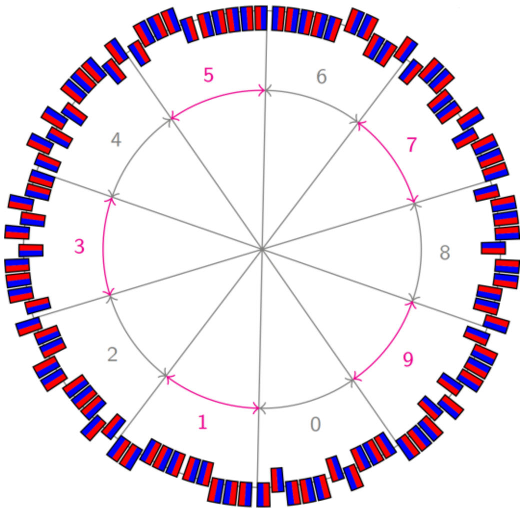
→ **RF stations must be located an multiples of $2\pi/h$**

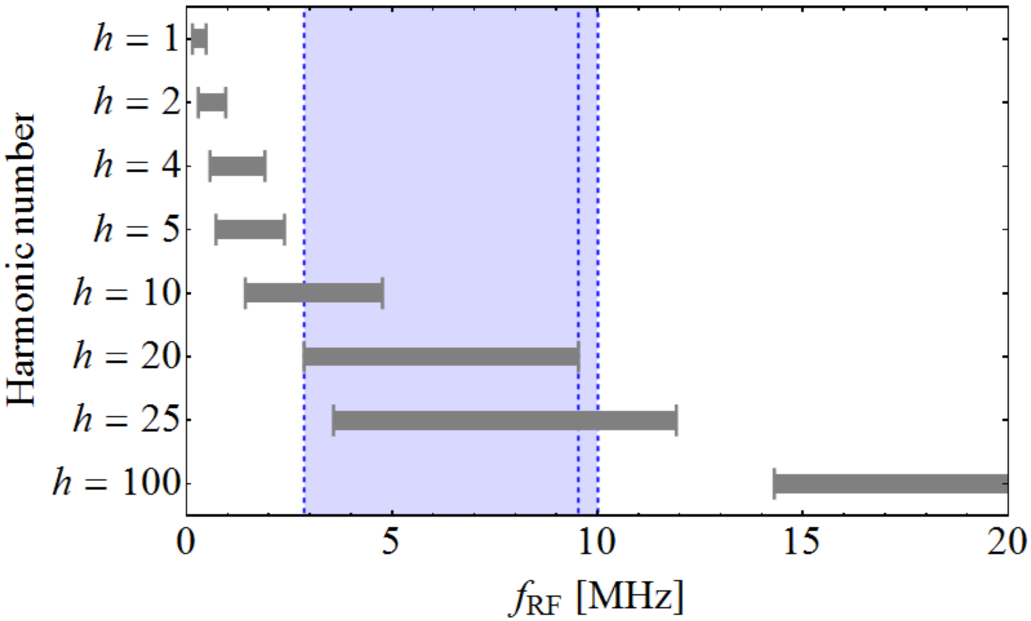
Example: CERN PS - choice of harmonic

→ **Main elements: 100 bending magnets**

→ **100 possible location for RF stations in-between**

→ **$100 = 2 \cdot 2 \cdot 5 \cdot 5$, hence divisible by 2, 4, 5, 10, 20, 25, 50**





→ **Distribute total RF voltage over many cavities**

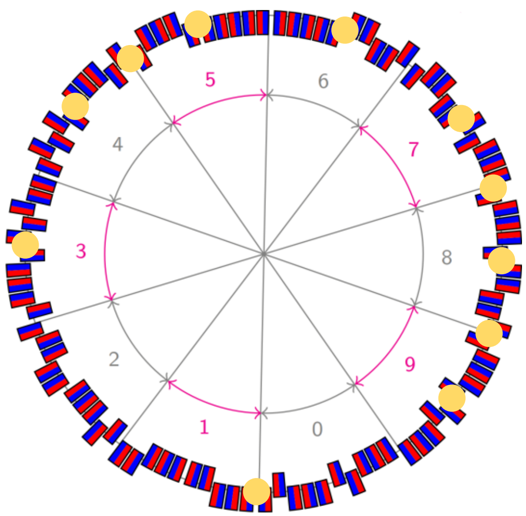
→ **Possible harmonic numbers 20 or 25 → *h* = 20 retained**

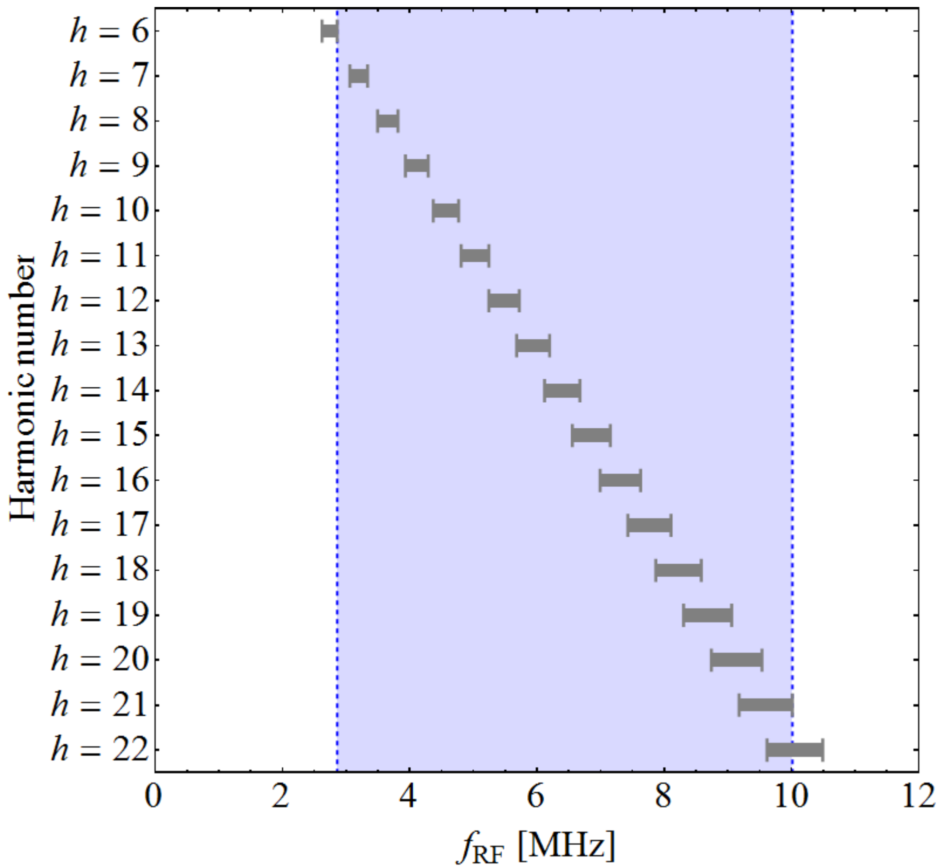
Example: CERN PS choice of harmonic

→ **Distance between RF stations: multiples of 2π/20**

→ **No need to use common RF with todays technology**

→ **Injection energy at 1.4 GeV (2 GeV) → 10% (5%) swing**





→ **Early design choices based on h = 20**

→ **Today's flexibility**

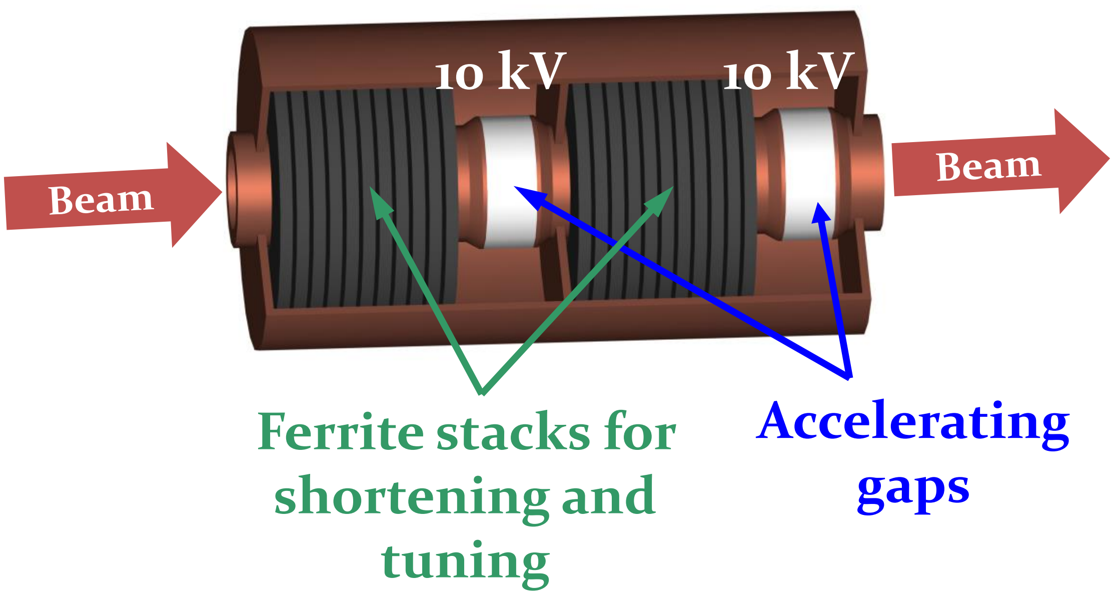
Example: CERN PS choice of cavity

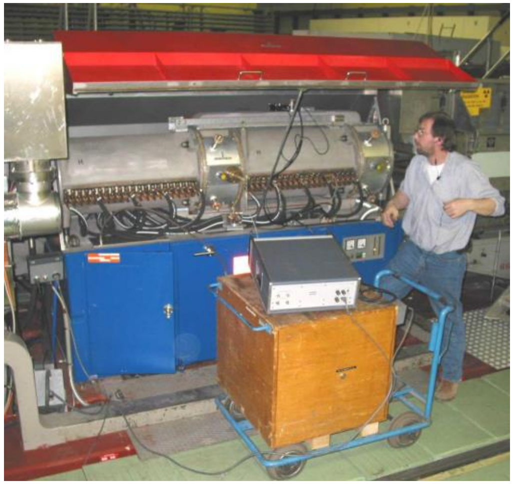
→ **RF system parameters:**

Parameter	
Harmonic, h	**7…, 20, 21**
Frequency, f_{RF}	**2.8-10 MHz**
Voltage, V_{RF}	**10 (+1) × 20 kV**

→ **Distribute voltage over 10 RF stations: 20 kV/cavity**

Shortened λ/4 coaxial resonators with ferrite tuning





Electrons in the PS

- **As an injector of LEP electrons were accelerated in the PS to E = 3.5 GeV**
- **Is the RF system for acceleration of protons usable?**

$$\Delta E_{\mathrm{turn}} = \frac{e^2}{3\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho} \simeq 190\ \mathrm{keV/turn}$$

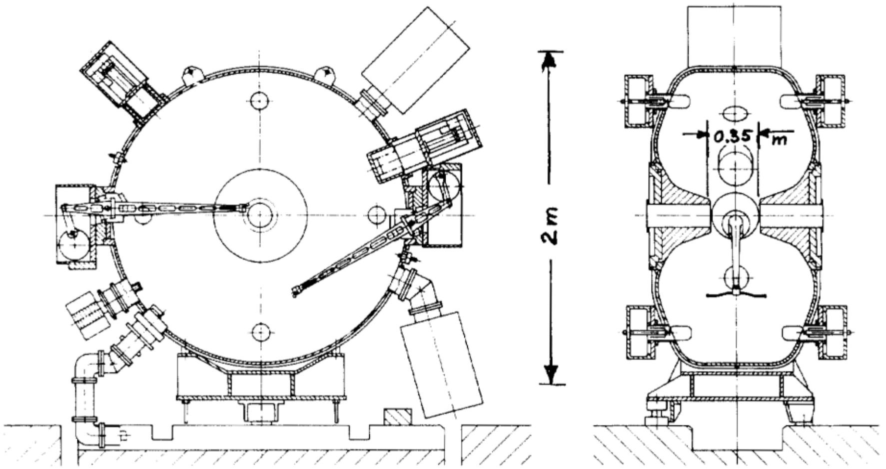
$$\epsilon_0 \simeq 8.85 \cdot 10^{-12}\ \mathrm{As/Vm}$$

→ **Bucket area too small and bunches too long at 3.5 GeV**

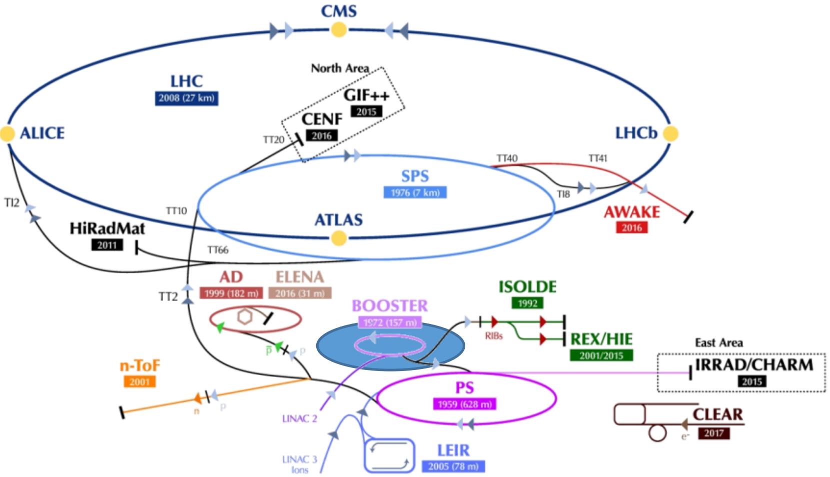
→ **Optimized RF system for electron acceleration**

Parameter	
Harmonic, h	240
Frequency, f_{RF}	**114 MHz**
Voltage, V_{RF}	**1 MV**

(5 × more than 10 MHz cavities)

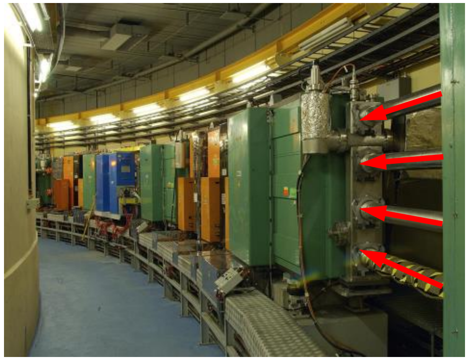


PS Booster


CMS
LHC
2008 (27 km)
North Area
GIF++
2015
CENF
2016
ALICE
LHCb
TT20
TT40
TT41
SPS
1976 (7 km)
TI8
TI2
TT10
ATLAS
AWAKE
2016
HiRadMat
2011
TT66
AD
1999 (182 m)
ELENA
2016 (31 m)
TT2
ISOLDE
1992
BOOSTER
1972 (157 m)
RIBs
REX/HIE
2001/2015
East Area
IRRAD/CHARM
2015
n-ToF
2001
PS
1959 (628 m)
CLEAR
2017
LINAC 2
LINAC 3
Ions
LEIR
2005 (78 m)

Example: RF System for CERN PS Booster

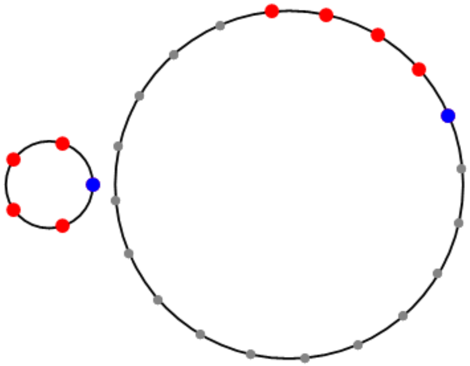
- **PS injector synchrotron**
  - → $2\pi R_{\mathrm{PSB}} = 2\pi R_{\mathrm{PS}}/4$
  - → **Sandwich of 4 rings**
  - → **Total length as PS circumference**



Parameter	Value
Circumference, $2\pi R$	$2\pi \cdot 25$ m = 157 m
Acceleration time, t_{cycle}	1 s
Maximum ramp rate, dB/dt	2.3 T/s
Injection energy, E_{kin}	**50 MeV**
Flat-top energy, E_{kin}	**0.8/1.0/1.4/2.0 GeV**

Example: CERN PS Booster (PSB)

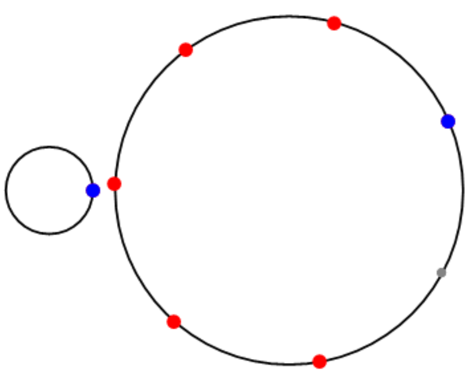
- **Circumference** $2\pi R_{PSB} = 2\pi R_{PS}/4 = 157$ m
- **Initial design as PS injector**



$$f_{RF,PSB} = f_{RF,PS}$$

$$h_{PSB} = h_{PS}/4 = 5$$

→ **Modifications as pre-injector to LHC:**

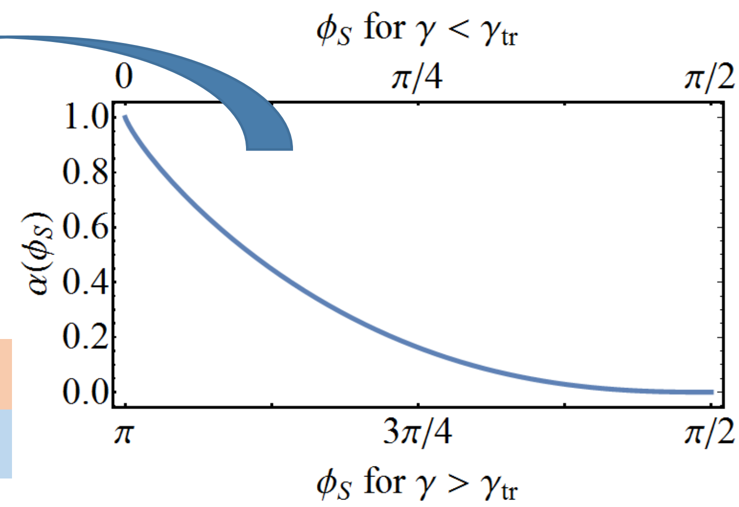


Parameter	
Harmonic, h	1 or/and 2
Frequency, f_{RF}	0.6...1.8 MHz
Voltage, V_{RF}	8 kV

Example: CERN PSB (single harmonic, $h = 1$)

Bucket area:

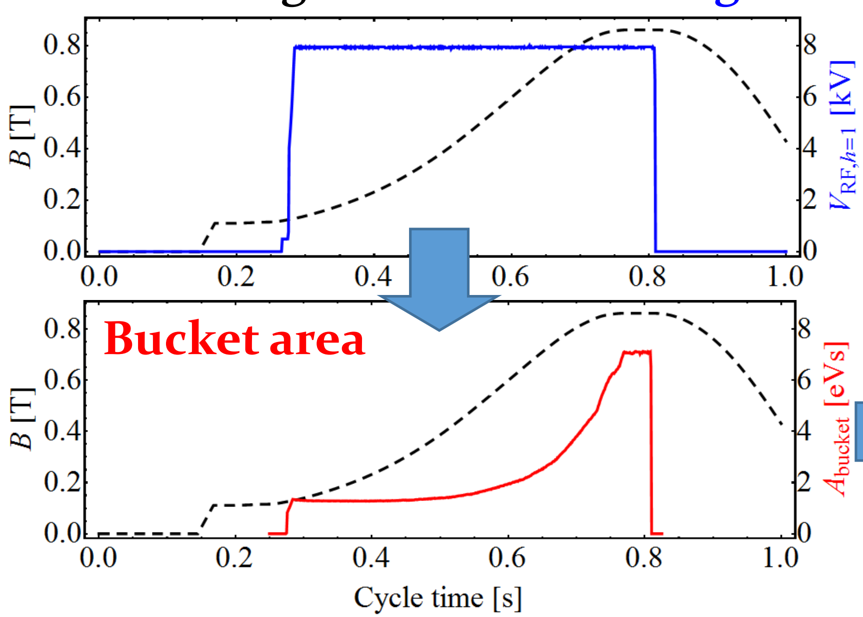
$$A_{\text{bucket}} = \frac{8\sqrt{2}}{h\omega_0}\sqrt{\frac{E\beta^2 qV}{\pi h|\eta|}}\cdot\alpha(\phi_S)$$



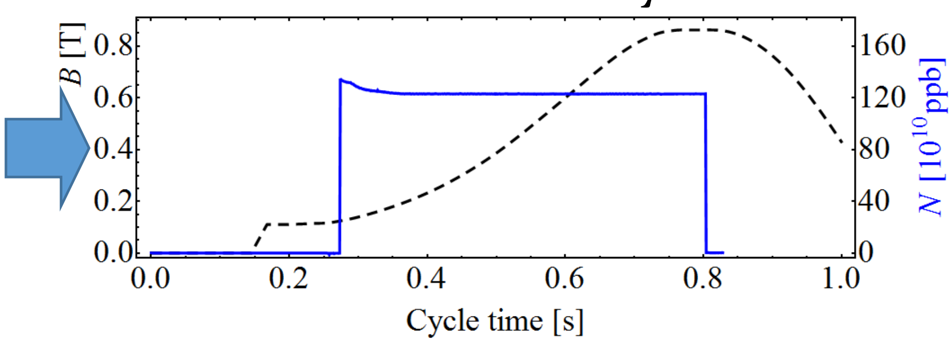
Depends on:

- **Bending field, *B* and ramp rate *dB/dt***
- **RF voltage, *V***

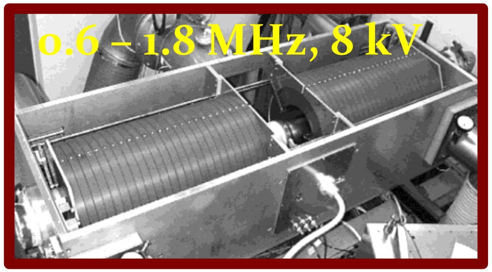
Bending field and RF voltage



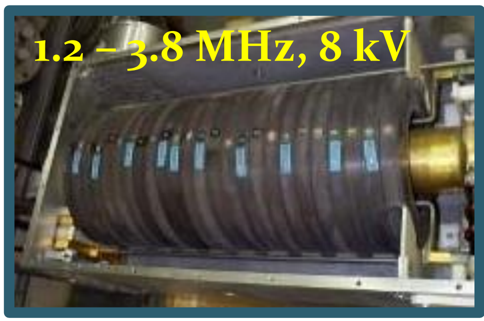
Beam intensity



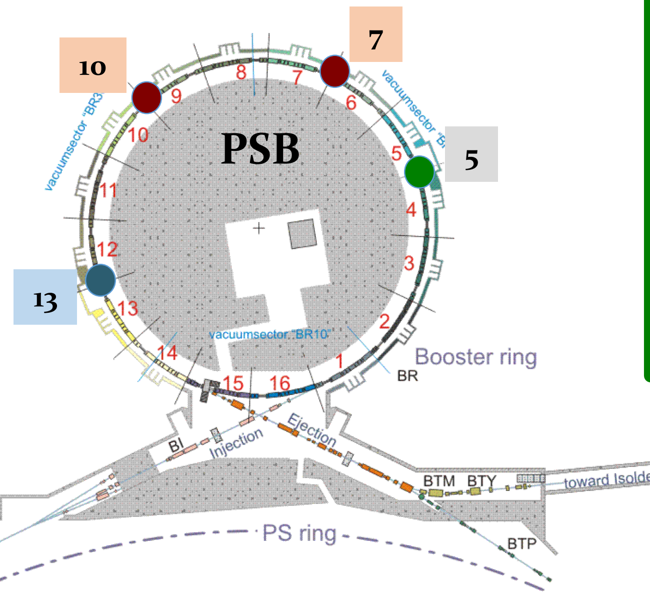
RF systems in the PS Booster



Acceleration and splitting



Acceleration and splitting

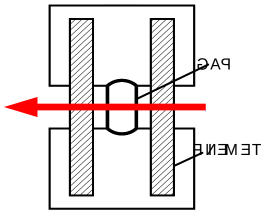


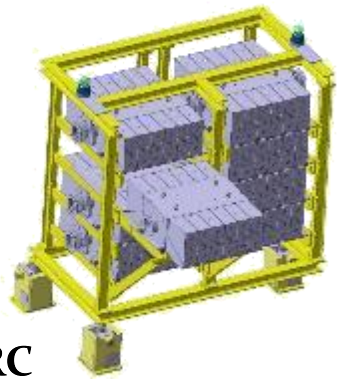
Controlled longitu-
dinal blow-up

→ **4 rings** with **3 cavities**

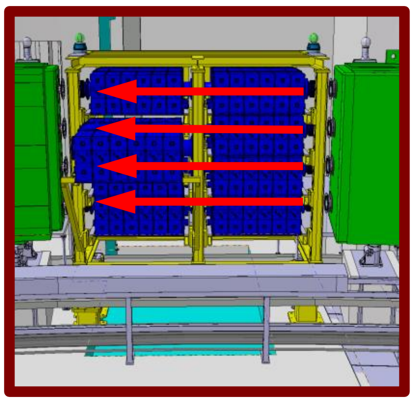
→ **PS Booster RF systems based on tuned ferrite cavities**

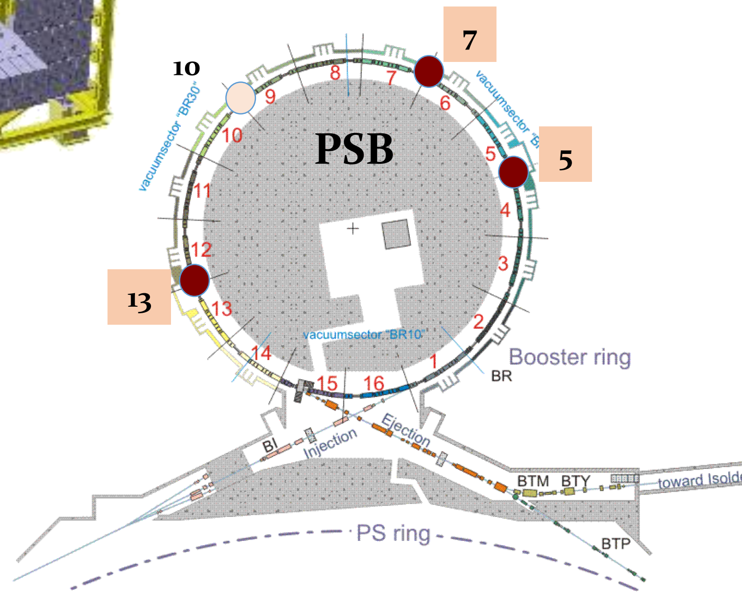
RF systems in the PS Booster after upgrade

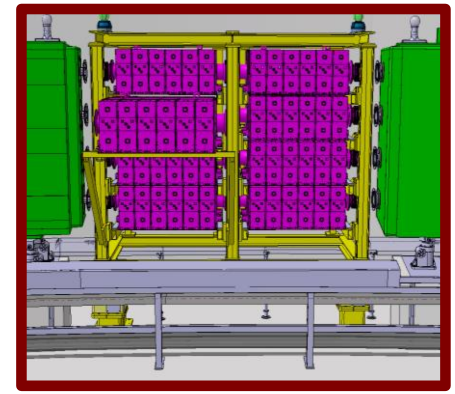


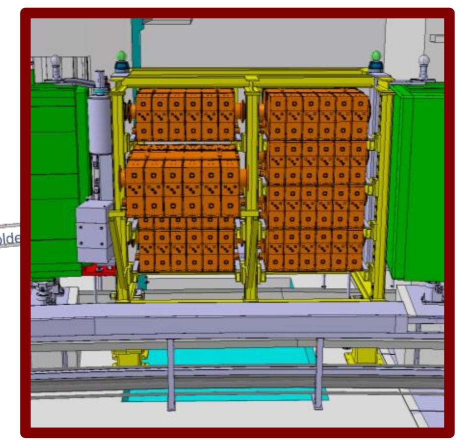


Collaboration with KEK/JPARC







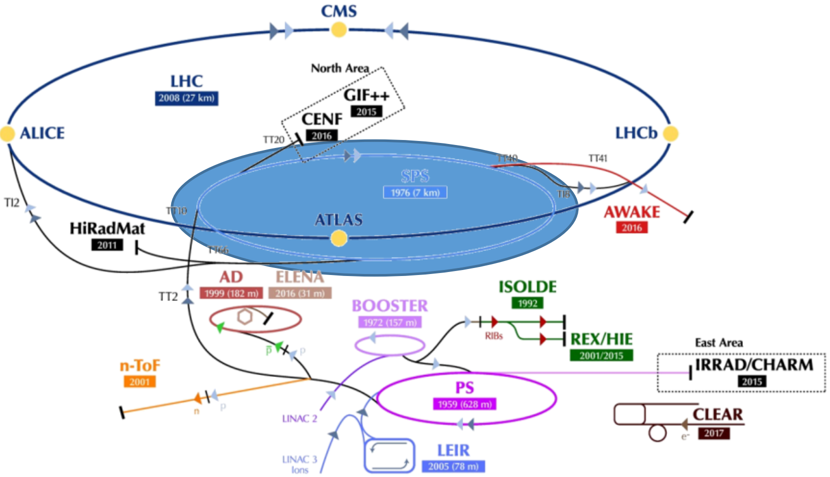


→ New **wide-band cavities** covering ***h*** **= 1, 2, and higher**

→ **Based on innovative Finemet material**

→ **Much increased flexibility**

Super Proton Synchrotron



Example: RF System for CERN SPS

$$\Delta E_{\text{turn}} = 2\pi q \rho R \dot{B}$$

→ **Needs significantly more RF voltage: several MV**

Parameter	Value
Circumference, $2\pi R$	$2\pi \cdot$ **1.1 km** = 6.91 km
Acceleration time, t_{cycle}	1 s
Maximum ramp rate, dB/dt	~0.74 T/s
Injection Energy, E_{tot}	initially **10 GeV**
Flat-top energy, E_{tot}	**450 GeV**

Example: SPS - choice of RF harmonic

Harmonic number should be multiple of	
Revolution frequency ratio of PS and SPS	11
Acceleration harmonic in the PS	20
Super-periodicity of SPS	6

→ **Looking for multiples of 660**

h	660	1320	1980	2640	3300	3960	4620	5280	5940
f_{RF} [MHz]	29	57	86	115	143	172	200	229	258

Lower RF frequency	Higher RF frequency

Example: SPS choice of RF cavities

- **Requirements:**

Parameter	
Harmonic, h	4620
Frequency, f_{RF}	**200 MHz**
Bandwidth, Δf_{RF}	**0.44%**
Voltage, V_{RF}	**Few MV**

How to build such an RF system?

→ **Cavity resonator would need tuning or low** $Q < 1/0.44\% \approx 230$

Mechanically tuned cavity

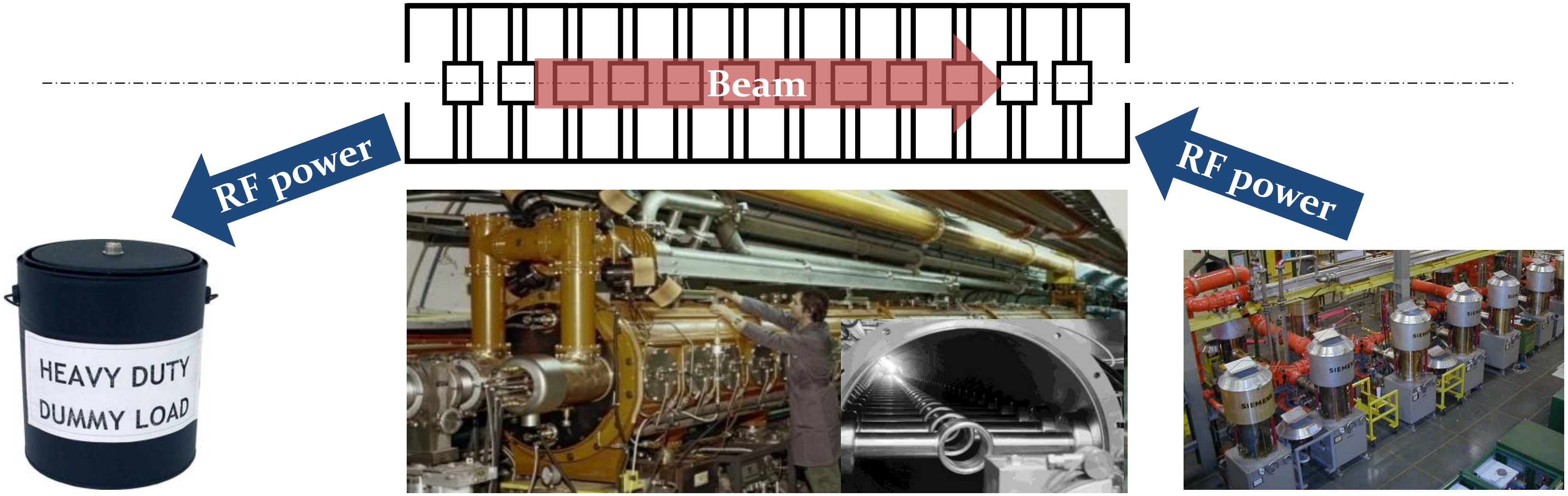
Avoid cycling mechanics in vacuum

Travelling wave structure

High voltage in moderate bandwidth

Example: SPS travelling wave cavities

→ **Multi-cell structure operated as a waveguide**



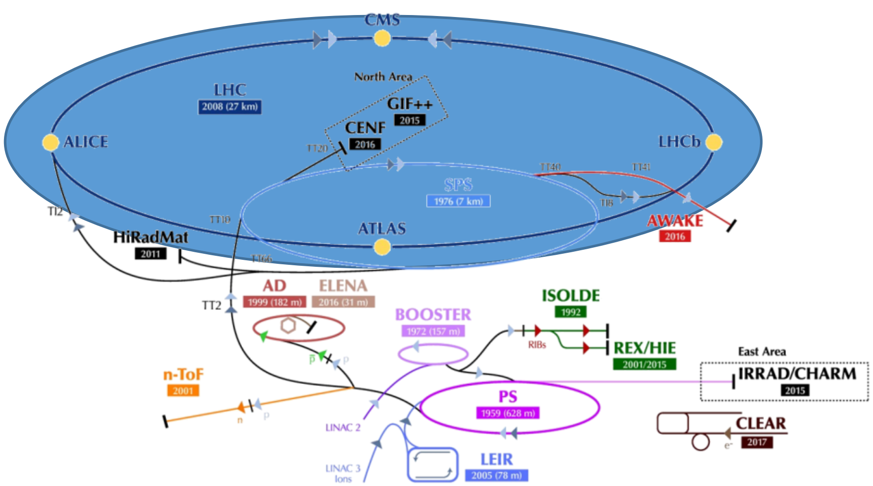
→ **Sufficient bandwidth without mechanically moving parts**

→ **Travelling wave structure always matched to amplifier**

→ **Beam takes power it needs from the waveguide**

$$P_{\mathrm{load}} = P_{\mathrm{in}} - P_{\mathrm{beam}} - P_{\mathrm{loss}}$$

Large Electron Positron and Hadron Colliders



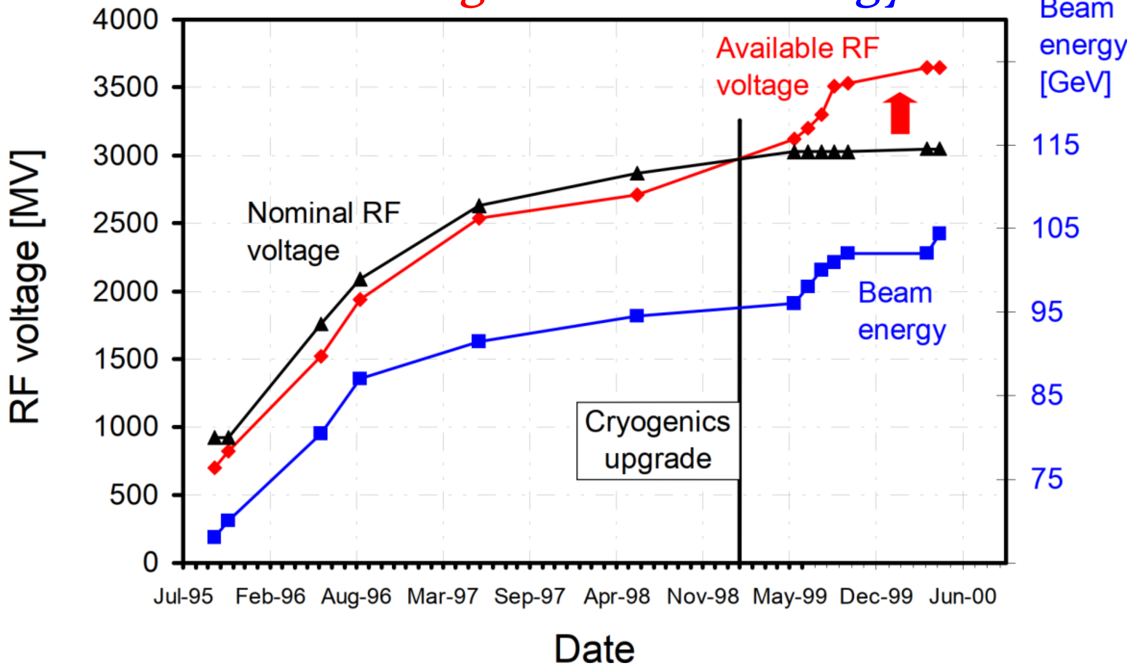
Ex.: RF against synchrotron radiation in LEP

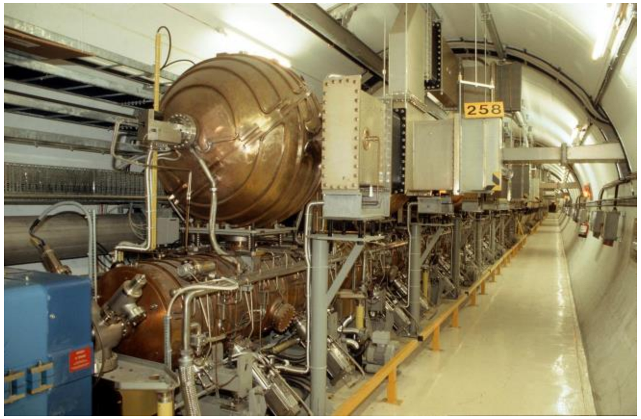
- **LEP energy was entirely dominated by synchrotron radiation**
- **At *E* = 100 GeV:** $$\Delta E_{\text{turn}} = \frac{e^2}{3\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho} \simeq 3\ \text{GeV/turn}$$

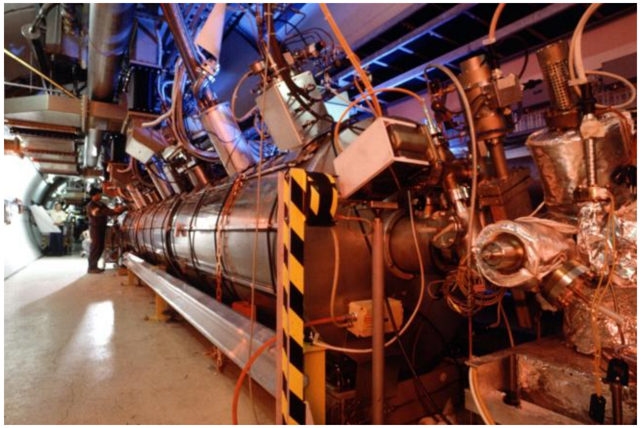
$\epsilon_0 \simeq 8.85 \cdot 10^{-12}\ \text{As/Vm}$

→ About 3 % of beam energy lost each turn

RF voltage and beam energy







Example: LHC

- **LHC maximum energy and ramp rate limited by super-conducting bending magnets: 20 minutes ramp time**

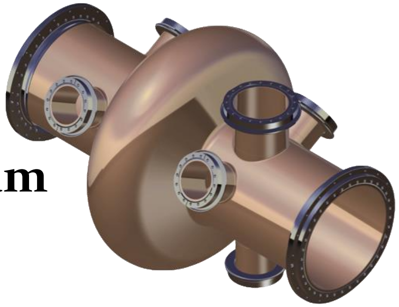
→ Average energy gain per turn only $\Delta E_{\mathrm{turn}} \approx 500$ keV/turn

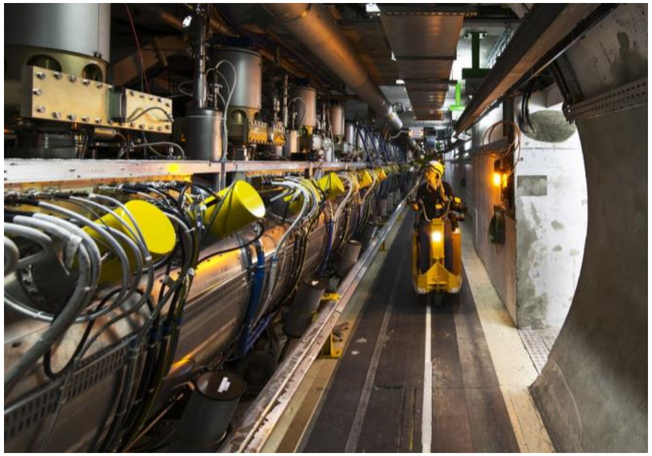
→ Revolution frequency stays almost constant

→ RF voltage required to keep bunches short

→ Superconducting cavities chosen to reduce beam induced voltage (small R/Q)

8× per beam





Parameter (per beam)	
Harmonic, h	35640
Frequency, f_{RF}	400.8 MHz
Voltage, V_{RF}	16 MV

(cf. LEP: 3.5 GeV)

You will design an RF system (upgrade)

→ **Protons accelerator:** **Special RF system for the CERN PS**

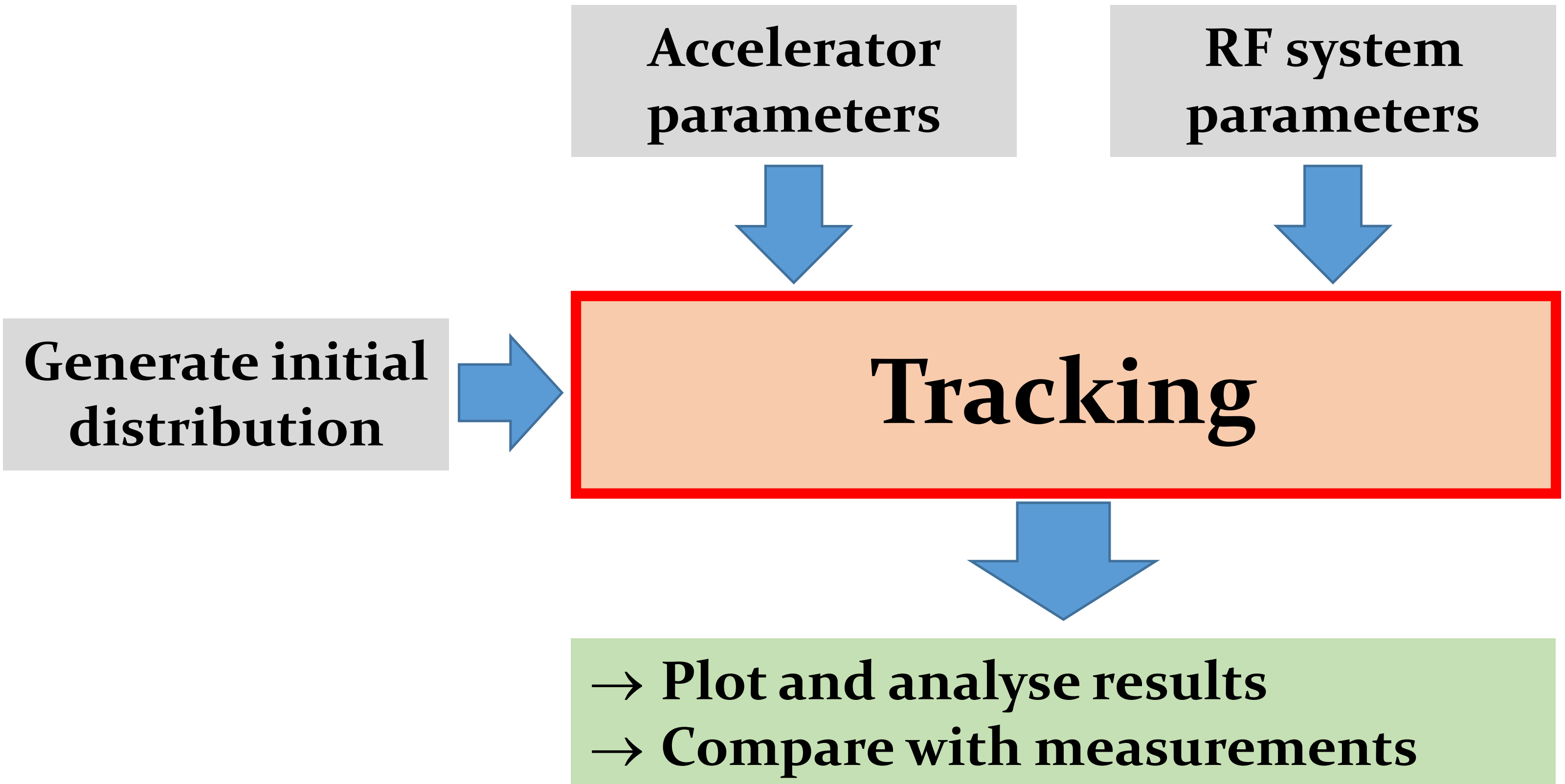
→ **Electron storage ring:** **Energy and current upgrade**

… tomorrow

Longitudinal tracking

this afternoon

Tracking simulation flow

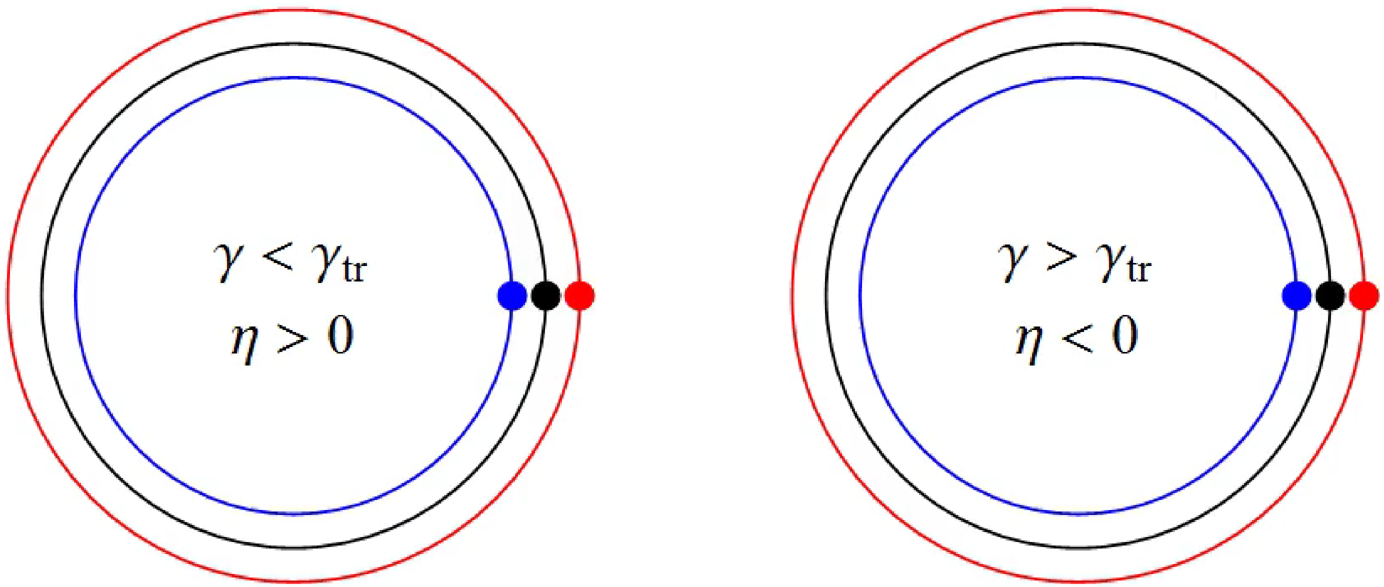


→ Follow the coordinates of one or more particles determine its behaviour

Circular ~~accelerator~~ without RF system
↳ ring

- **Particles with higher or lower momentum have a different orbit compared to a reference particle**

→ **Arrival time/phase depends on energy** $$\frac{\Delta L}{L} = \frac{1}{\gamma_{\mathrm{tr}}^2}\frac{\Delta p}{p}$$



$$\frac{\Delta f}{f} = \eta\frac{\Delta p}{p}$$ **, phase slip factor:** $$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\mathrm{tr}}^2}$$

Arrival phase of a particle at next turn

$$\frac{\Delta f}{f} = \eta \frac{\Delta p}{p}$$

$$\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta E}{E}$$

$$\frac{\Delta f}{f} = \frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

$$\frac{\Delta \theta}{\theta} = -\frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

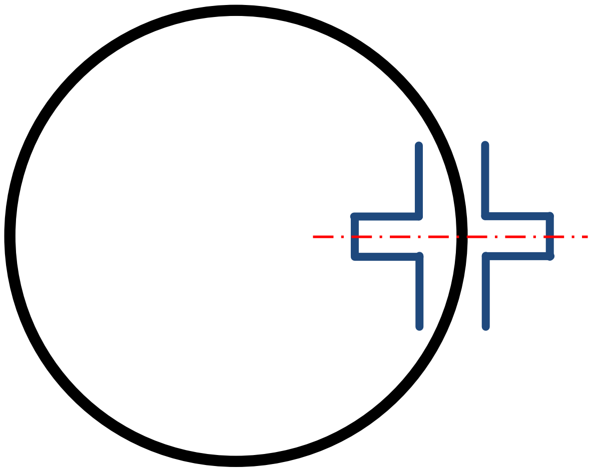
→ **Turn-by-turn drift equation**

$$\theta_{n+1} = \theta_n - 2\pi \frac{\eta}{\beta^2}\frac{\Delta E_{\mathrm{n}}}{E}$$

$$\phi_{n+1} = \phi_n - 2\pi h \frac{\eta}{\beta^2}\frac{\Delta E_{\mathrm{n}}}{E}$$

→ **Azimuth, θ or phase, ϕ a particles arrives next turn**

Circular accelerator with RF station



- **Particle energy changes at passage through cavity**

→ **For sinusoidal RF voltage:**

$$\Delta E_{n+1} = \Delta E_n + qV \sin \phi_{n+1}$$

→ **With acceleration:**

$$\Delta E_{n+1} = \Delta E_n + qV \left[\sin \phi_{n+1} - \sin \phi_{\mathrm{S}} \right]$$

Reference particle: $\phi = \phi_{\mathrm{S}}$

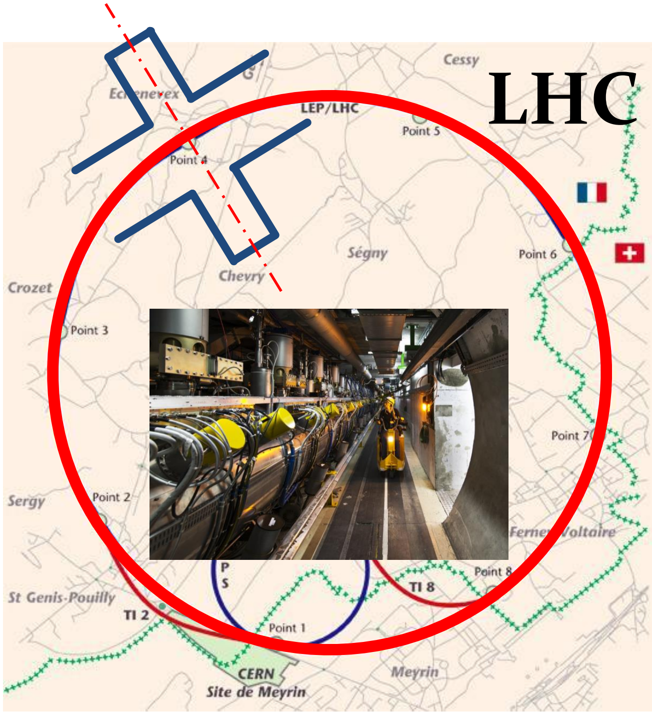
→ **General energy change:**

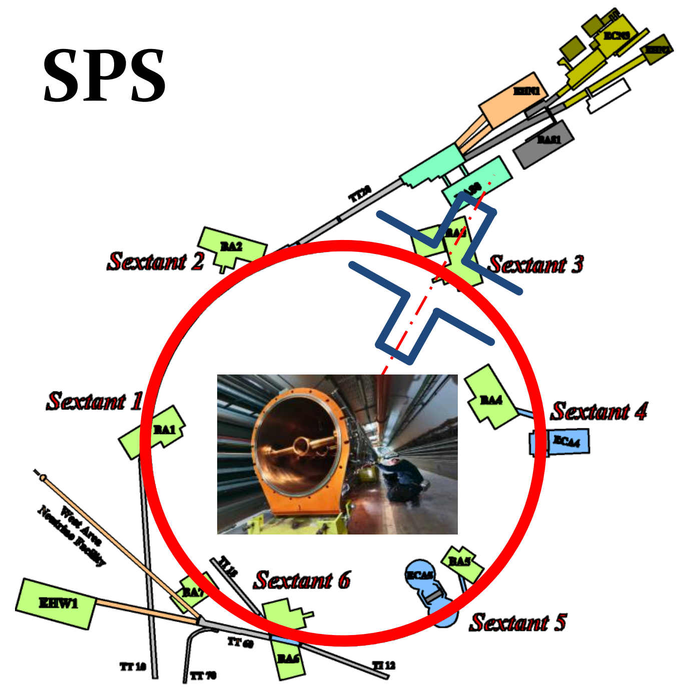
$$\Delta E_{n+1} = \Delta E_n + qV \left[g(\phi_{n+1}) - g(\phi_{\mathrm{S}}) \right] + \Delta E_{\mathrm{ext}} + \Delta E_{\mathrm{self}}$$

Multiple RF stations

- RF systems modelled point-like mostly valid approximation

→ Valid in most cases





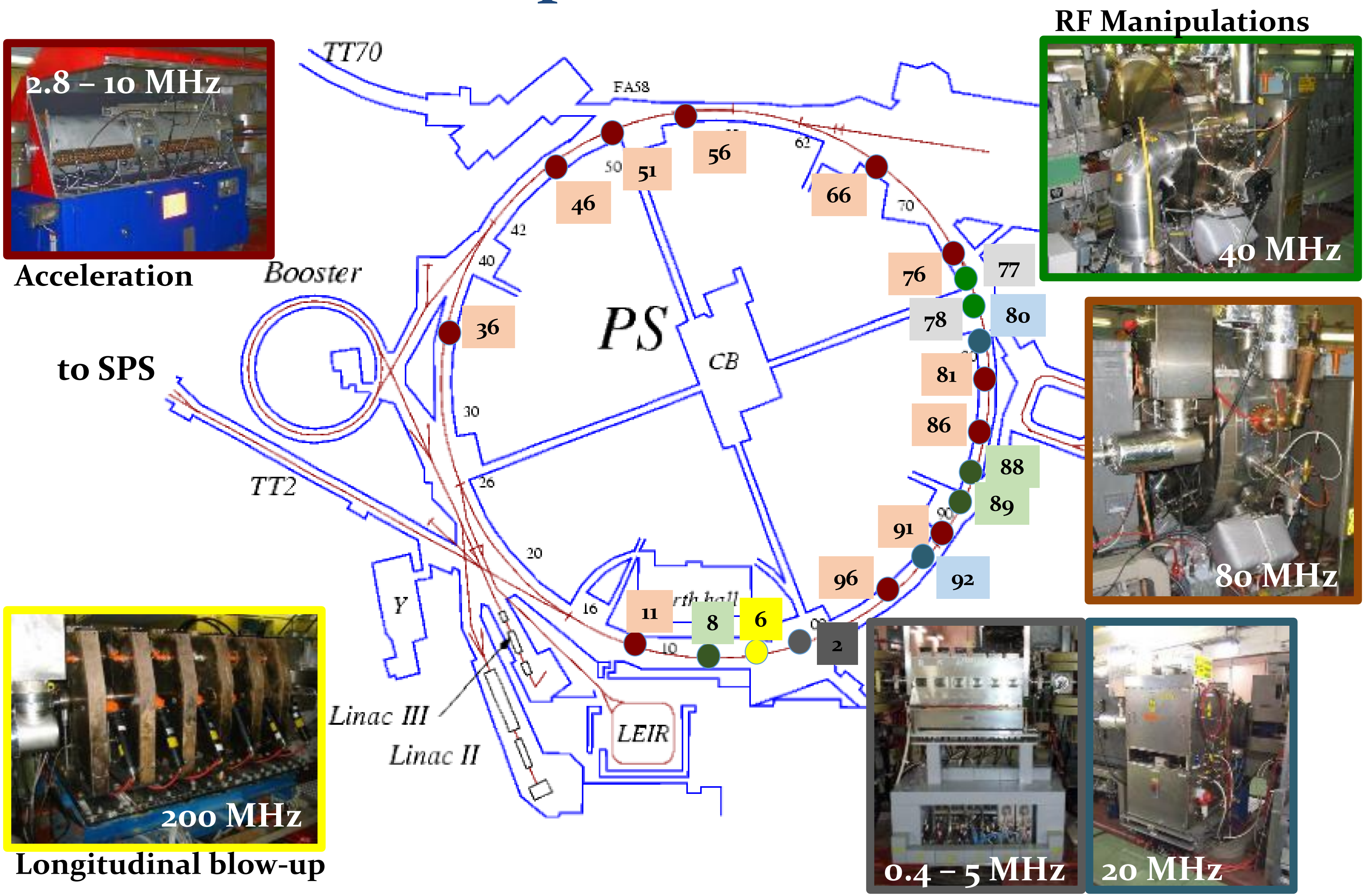
Exceptions:

→ Large synchrotron tune f_S/f_{rev}

→ Strong intensity effects: interaction within one turn

→ Beam energy changing during turn

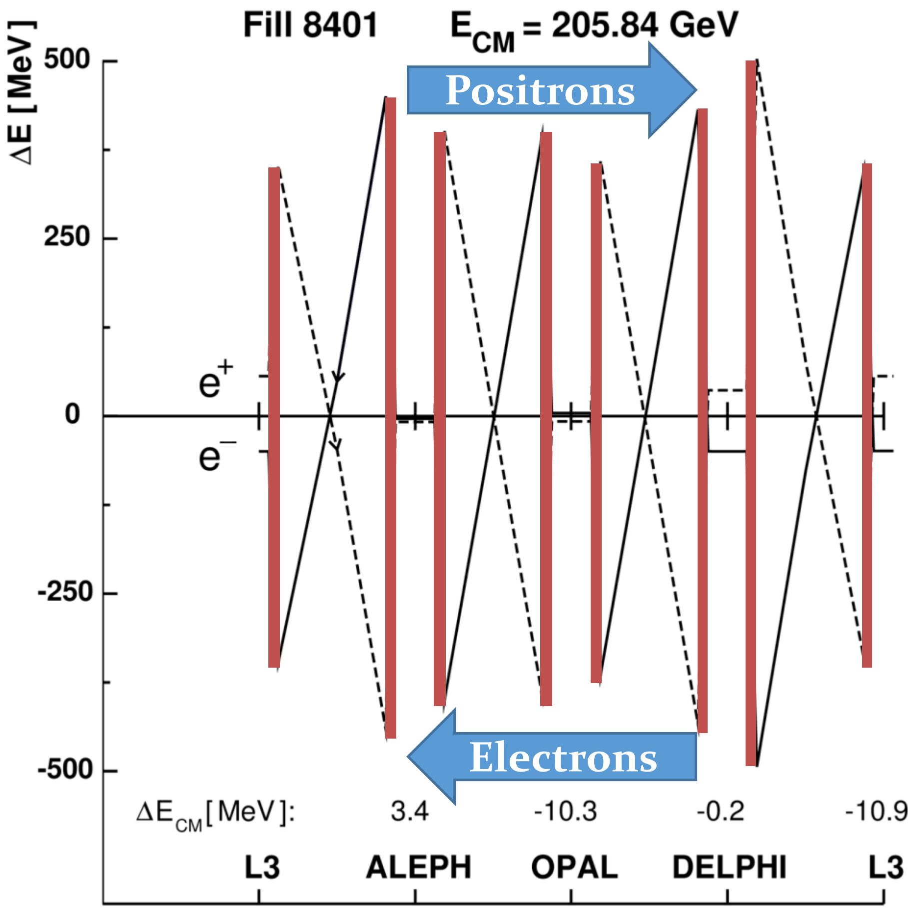
Multiple RF stations

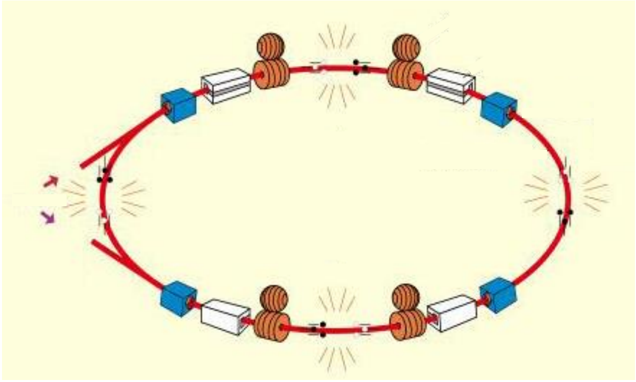


$\rightarrow$ **Small f_S/f_{rev}: Single kick per turn fully sufficient**

Example: Electrons and positrons in LEP

- **Beam energy changed in LEP along turn due to strong synchrotron radiation**





- **4 × 2 RF sections**

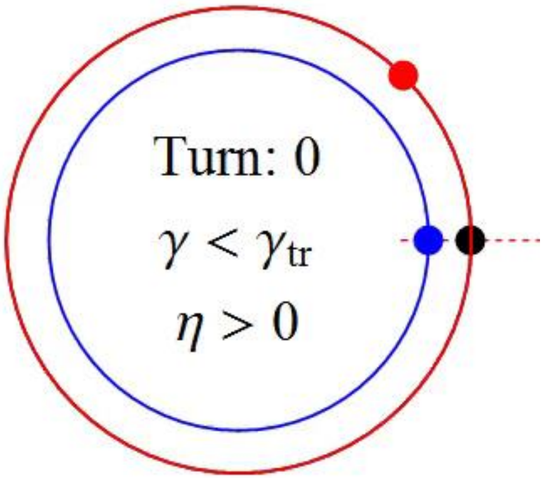
→ **Energy loss in bending magnets**

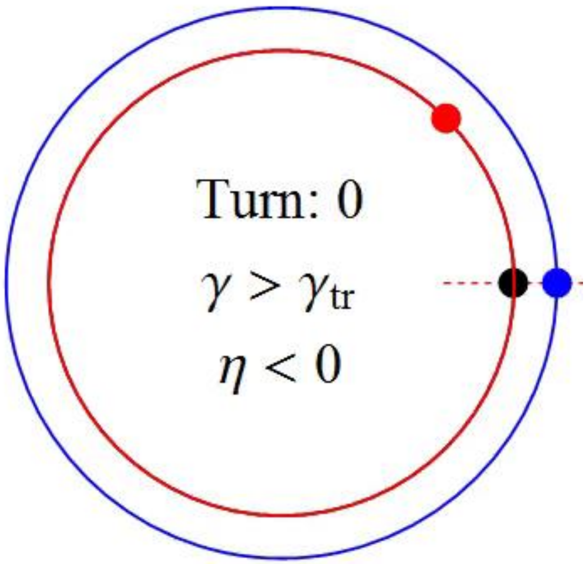
→ **Track from RF section to RF section**

Combining both tracking equations

- **Observe phase and energy error at each turn with respect to reference particle**
- **Test particles:**

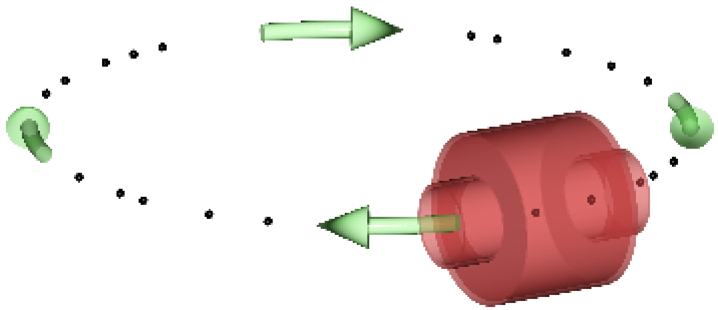
$\Delta\phi = \phi - \phi_S = 0$	$\Delta E = 0$
$\Delta\phi \neq 0$	$\Delta E = 0$
$\Delta\phi = 0$	$\Delta E \neq 0$

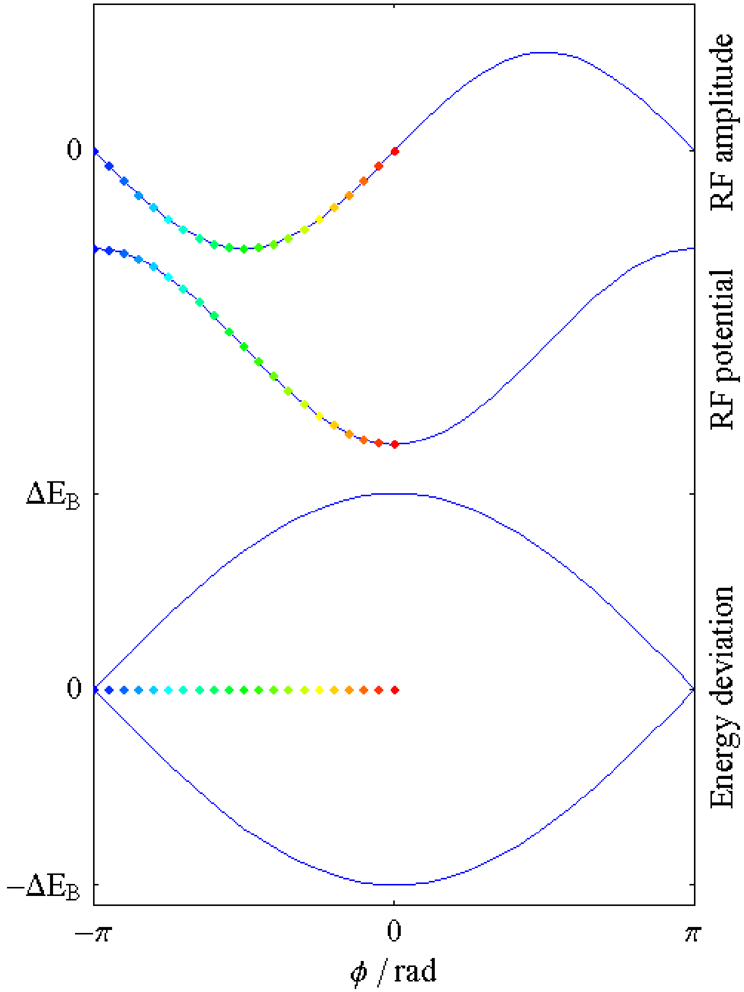




Longitudinal phase space

Simple accelerator model:





Energy dependent phase advance, ϕ:

$$\phi_{n+1} = \phi_n - 2\pi h\eta/\beta^2 \frac{\Delta E_n}{E_0},\ \eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\mathrm{tr}}^2}$$

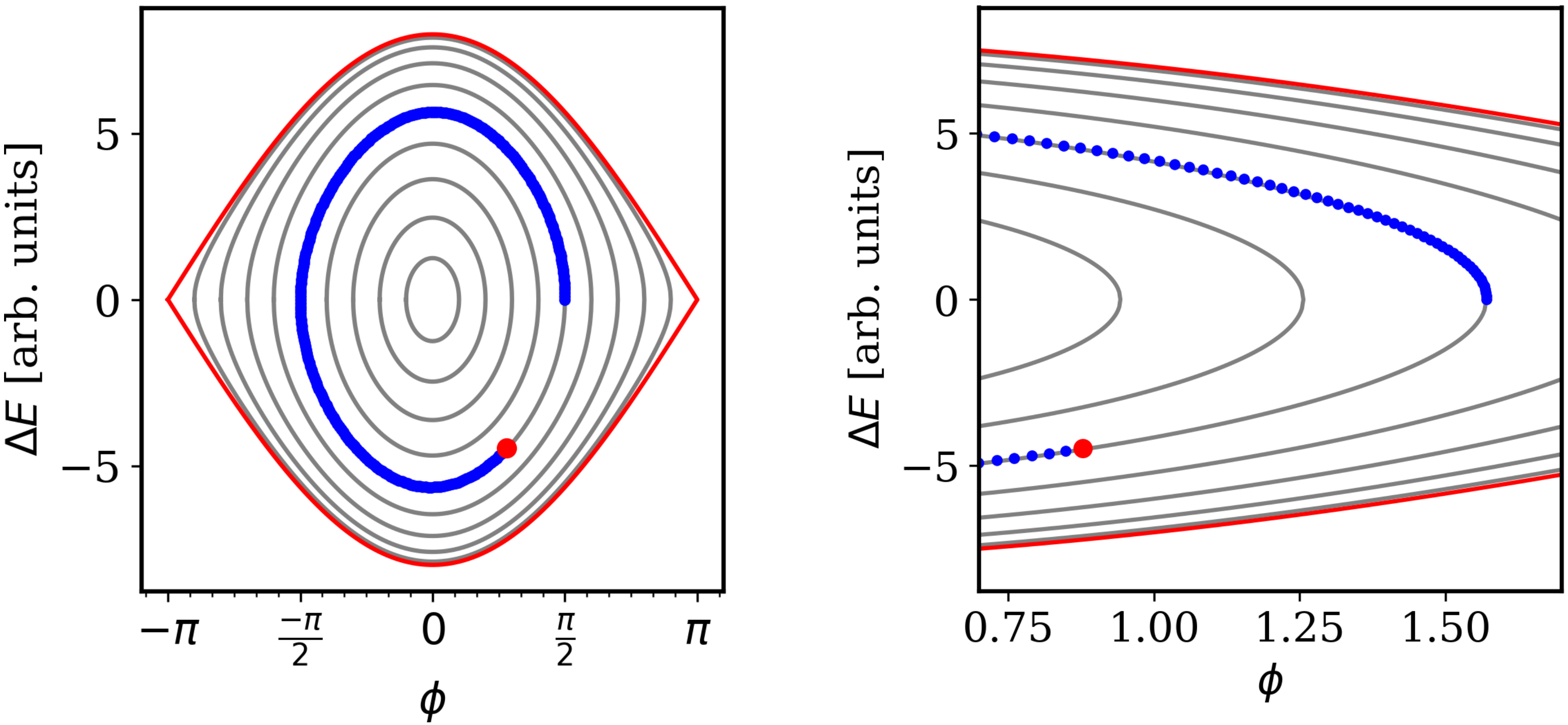
Phase dependent energy gain, Δ*E*:

$$\Delta E_{n+1} = \Delta E_n + qVg(\phi_{n+1})$$

Works for arbitrary shape of acceleration amplitude *g*(ϕ)

Continuous versus discrete

- **Analytical solution describes static condition**
- **No notion of turn-by-turn evolution**



→ **Same result with both approaches for f_S/f_{rev} << 1**

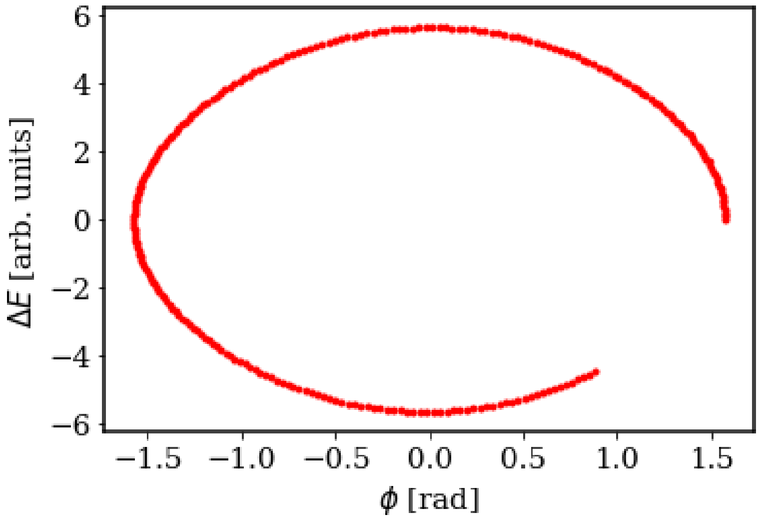
Example: simple tracking in Python

- **Follow the trajectory of a single particle**

```
def trackOneTurn(particles):
    particles[0] = particles[0] \
                 - 2*np.pi*eta/beta**2*particles[1]/energy
    particles[1] = particles [1] \
                 + charge*voltage*np.sin(harmonic*particles[0])
```

Turn $n \rightarrow n+1$

```
particle = [np.pi/2, 0]                      # Initial particle
numberOfTurns=250
trajectoryTrack = np.zeros((numberOfTurns,2)) # Empty 2D array
for turn in range(numberOfTurns):            #
    trajectoryTrack[turn] = particle         #
    trackOneTurn(particle)                   # Track
```

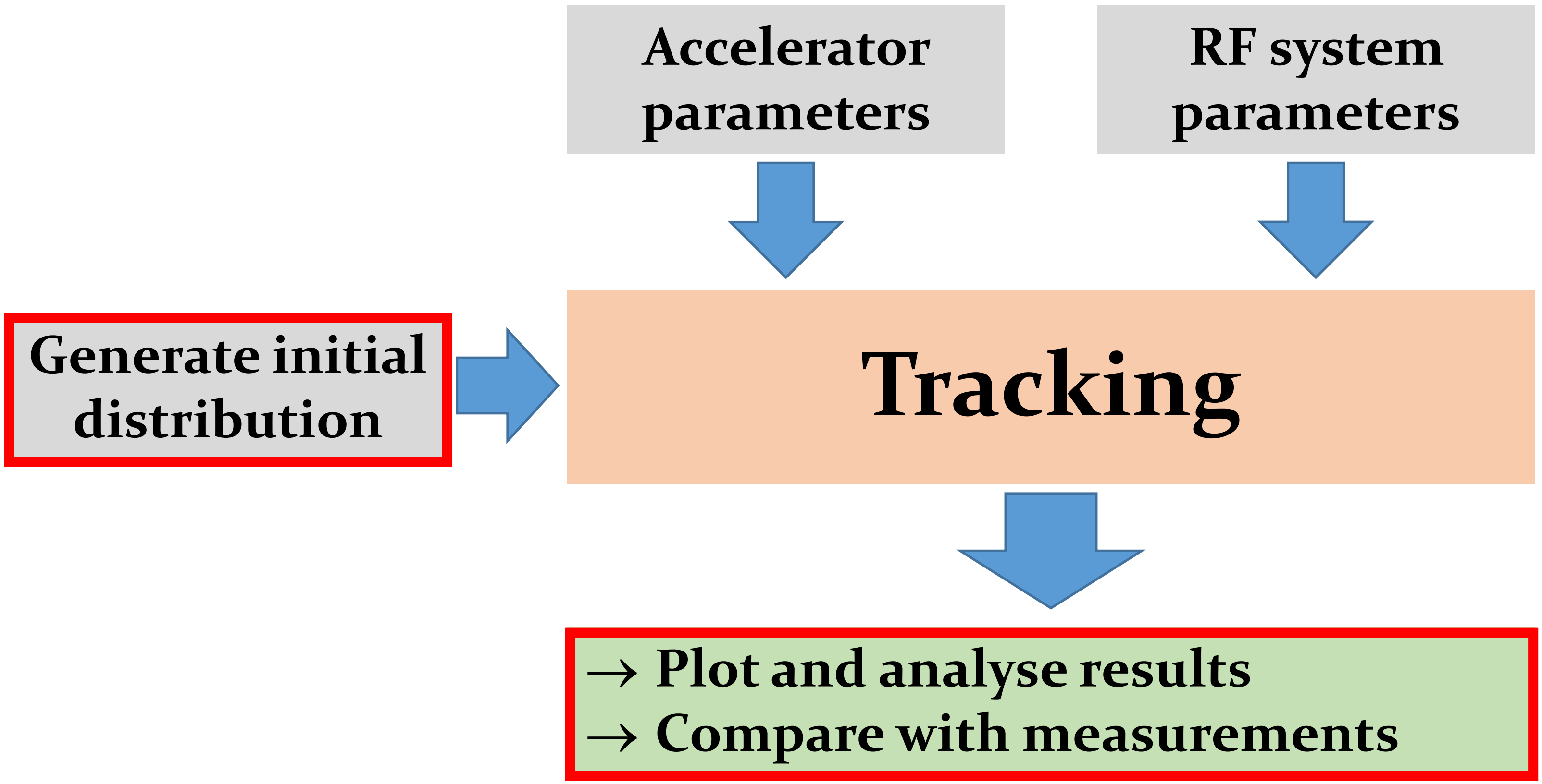


Choice of particle coordinates

- **Time or phase? Momentum or energy?**
- **Absolute or relative coordinates**

		Advantages	Disadvantages
t	E	• Most universal • Suitable for any tracking • Canonically conjugated	• Numerical precision: large absolute value • Relative bunch motion more difficult to follow
Δt	ΔE	• Relevant deviations only • Canonically conjugated • Most suited for multiple h	• Required synchronous particle as reference • Duration of turn may change
Φ	E, ΔE	• Turn length always 2π • Relevant deviations only	• Requires synchronous particle as reference • Not canonically conjugated
ϕ	E, ΔE	• RF bucket length always 2π • Relevant deviations only • **Most suited for single *h***	• Requires synchronous particle as reference • Not canonically conjugated

Tracking simulation flow



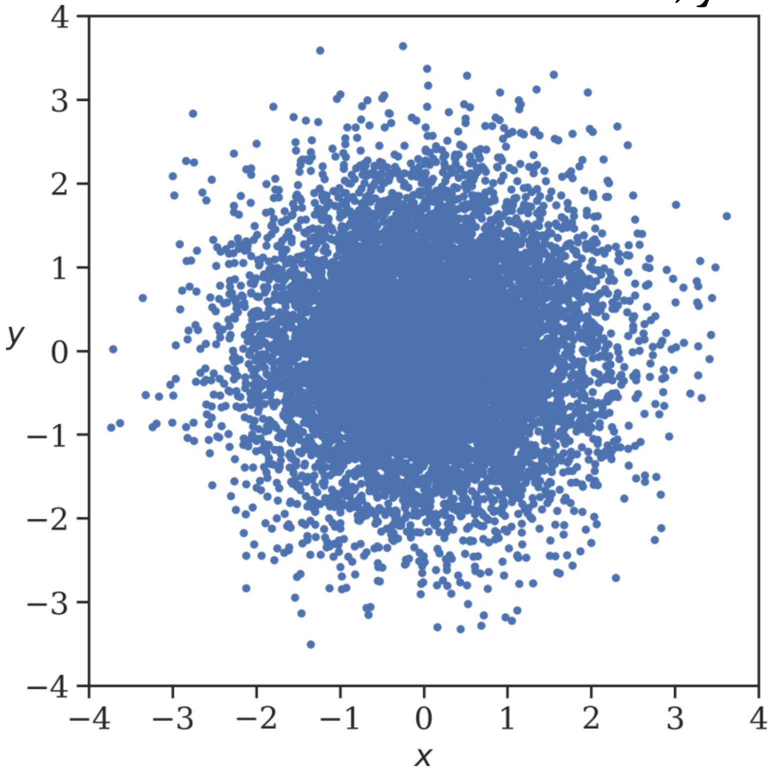
Distributions and projections

From single particle tracking to distribution

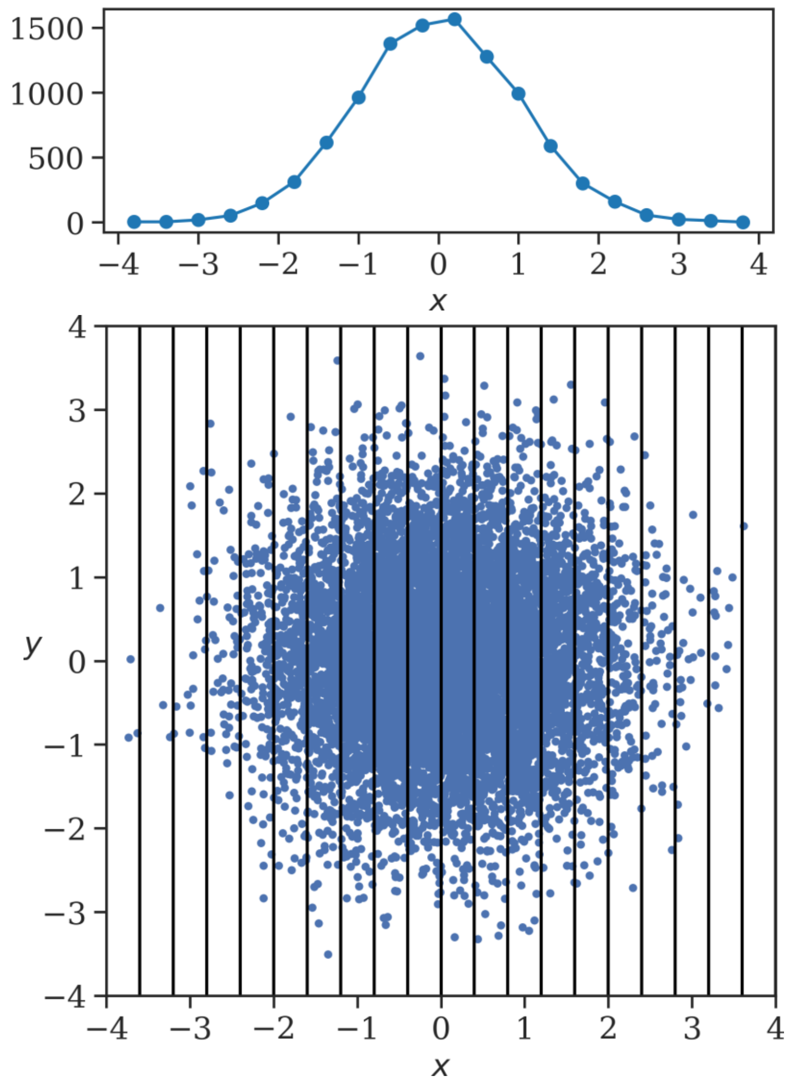
→ **10^{10}…10^{12} particles per bunch** → **too much computing power**

→ **Macro-particles to reduce** → **up to few 10^6 per bunch**

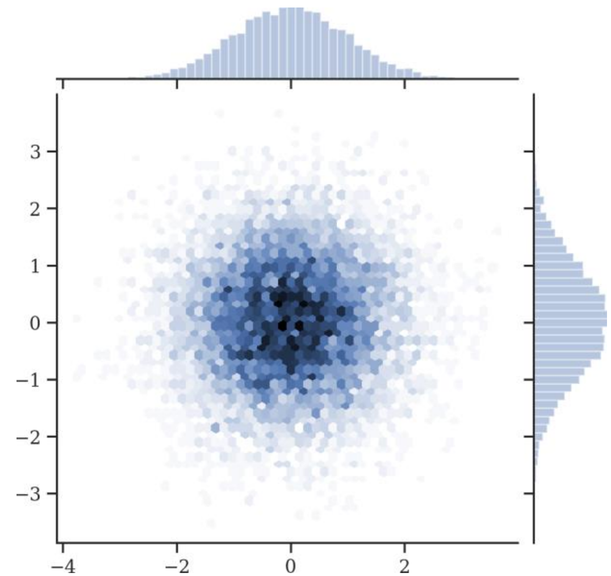
Normal distribution in *x*, *y*



Projections of distributions



- **Very common task:**
  - → **e.g. Python seaborn**
  - → **`plotPhaseSpace`**

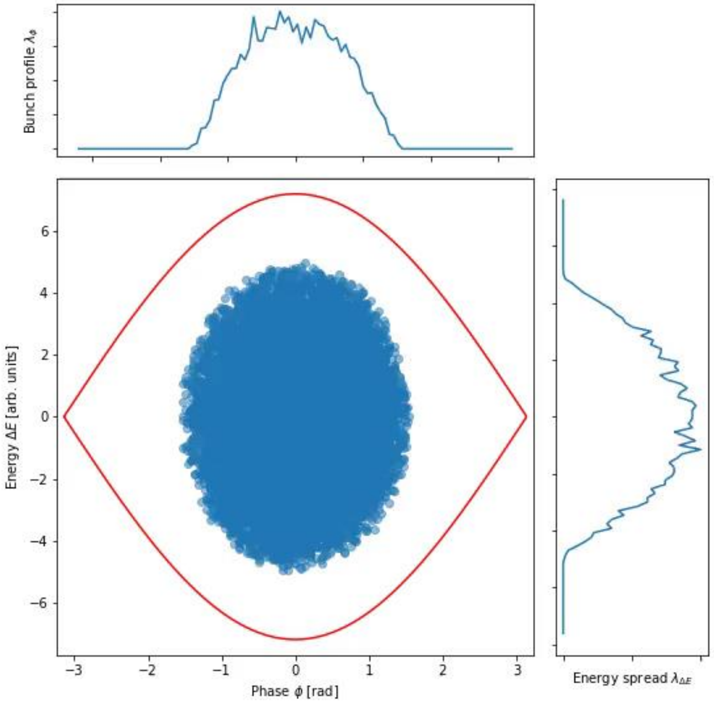


→ **Time projection directly observable: bunch profile**

Example: Tracking of a single bunch

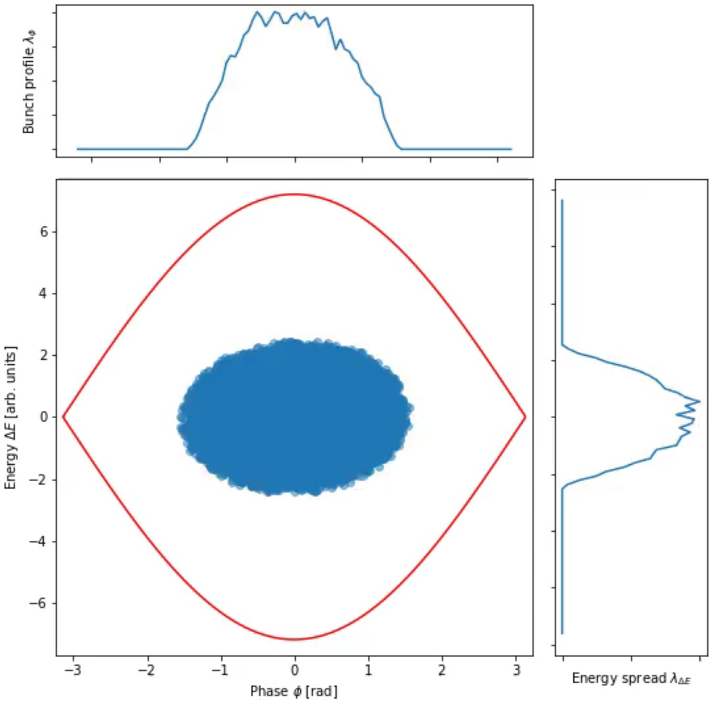
- Set-up bunch with parabolic distribution: `generateBunch`
- Most simple case: single harmonic RF without acceleration

Correct voltage at correct phase



→ Matched bunch

Wrong voltage at correct phase

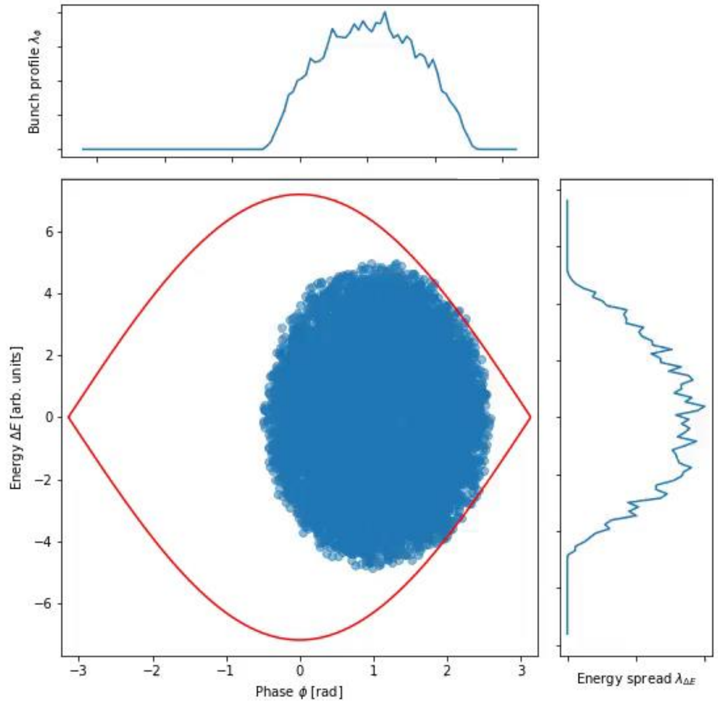


→ Breathing bunch (quadrupole)

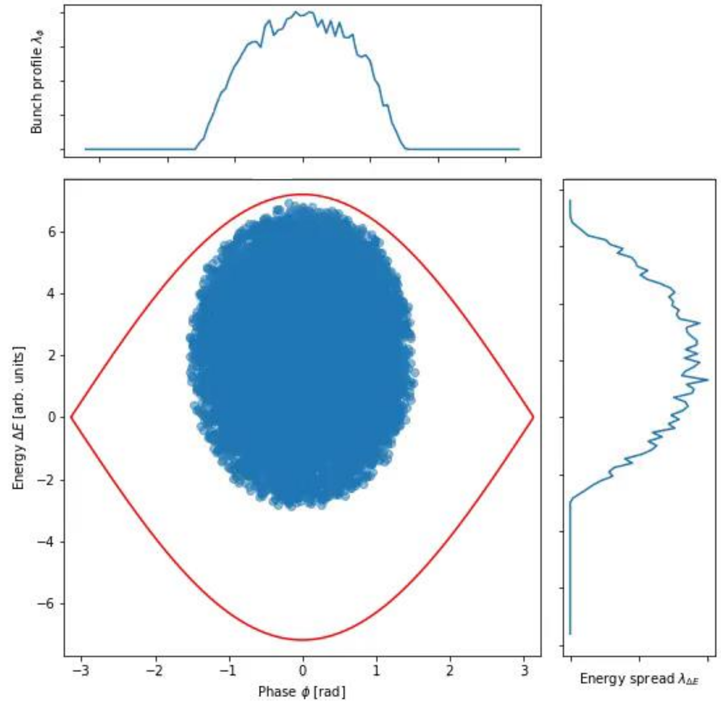
Example: Tracking of a single bunch

- **Set-up bunch with parabolic distribution: `generateBunch`**
- **Most simple case: single harmonic RF without acceleration**

Correct voltage at wrong phase



All correct, but wrong energy



→ Dipole oscillations
→ Phase and energy offset for example at injection

Getting closer to reality

→ State-of-the-art tracking may include much more

Non-linear phase slip factor, e.g. transition

$$\phi_{n+1} = \phi_n - 2\pi h \frac{\eta}{\beta^2} \frac{\Delta E_{\mathrm{n}}}{E}$$

Multiple RF systems with changing parameters

Energy loss: synchrotron radiation or impedances

Beam induced voltage

$$\Delta E_{n+1} = \Delta E_n + qV \left[g(\phi_{n+1}) - g(\phi_{\mathrm{S}}) \right] + \Delta E_{\mathrm{ext}} + \Delta E_{\mathrm{self}}$$

Global regulation loops for beam phase and radial position

Acceleration

Feedbacks around cavities

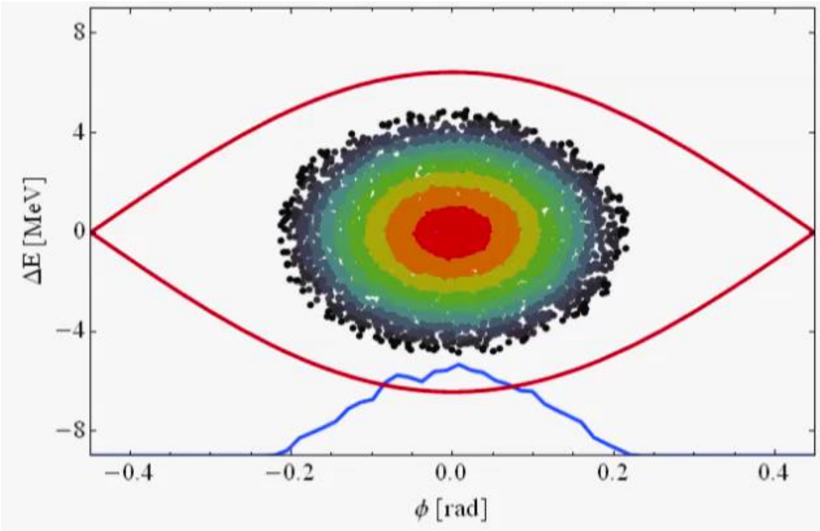
Longitudinal tracking codes

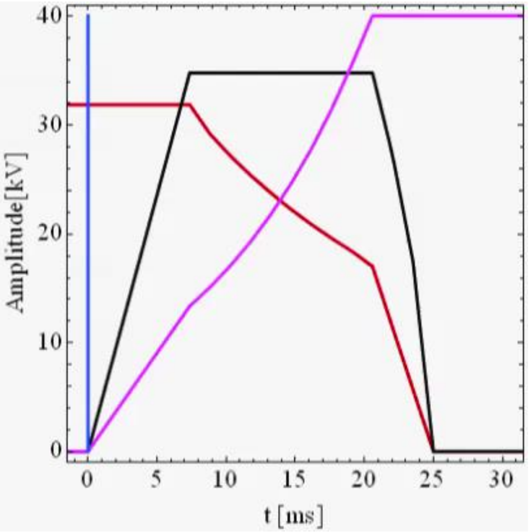
- **Dedicated to longitudinal dynamics: fast and focussed on RF aspects**
- **Combined transverse and longitudinal tracking**

Name	Remarks	
BLonD	• Widely used at CERN • Complex RF manipulations and feedbacks • Longitudinal intensity effects http://blond.web.cern.ch/	Longitudinal, 1D
ESME	• Longitudinal work horse code for many years • RF manipulations with multiple RF systems • Intensity effects esme.fnal.gov	
PyHeadTail	• Longitudinal and transverse combined simulation https://twiki.cern.ch/twiki/bin/view/ABPComputing/PyHEADTAIL	Combined, 3D
PyOrbit	• Longitudinal and transverse combined simulation https://twiki.cern.ch/twiki/bin/view/ABPComputing/PyORBIT	
elegant	• Longitudinal and transverse combined simulation • Mainly used for electron accelerators https://ops.aps.anl.gov/elegant.html	
…	…	

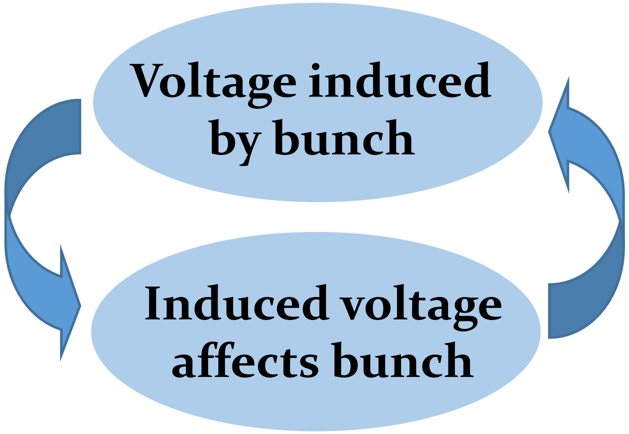
Examples of particle tracking

- **Multiple RF systems with changing RF voltages: RF manipulations**





- **Single bunch with intensity effects (example from SPS)**



You will build a (small) longitudinal tracking code

... after the coffee break

Summary

- Design of RF system for circular accelerator
    1. Start from accelerator parameters
    2. Define RF parameters based on beam requirements
    3. Chose RF system

  → Mostly several design options are possible

- Longitudinal simulations using particle tracking

  → Complementary approach to longitudinal beam dynamics

  → Flexibility to change parameters during tracking

  → Powerful technique to study
    - Multi-harmonic RF systems
    - Complicated intensity effects
    - Longitudinal dynamics with feedbacks and RF loops

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Thank you very much for your attention!

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