Electron dynamics with Synchrotron Radiation

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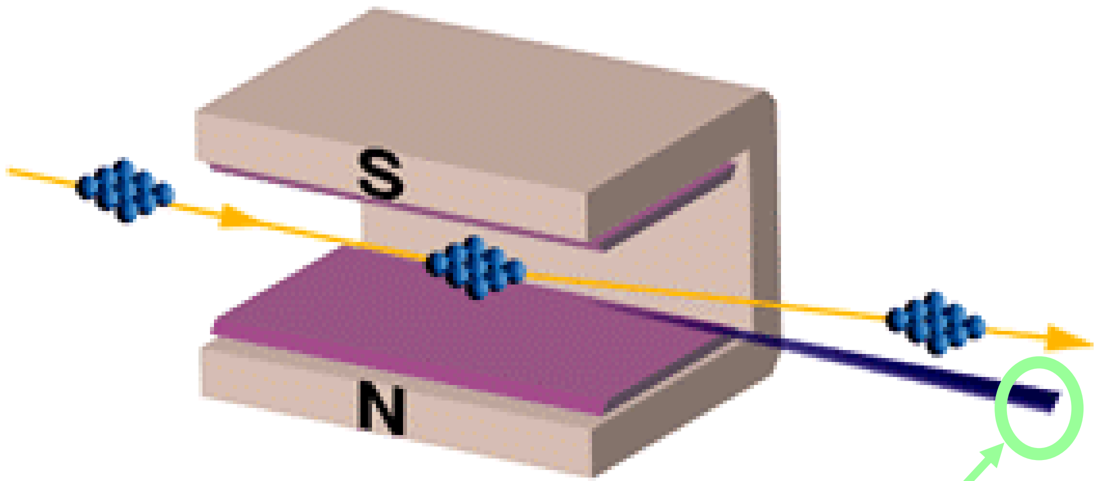
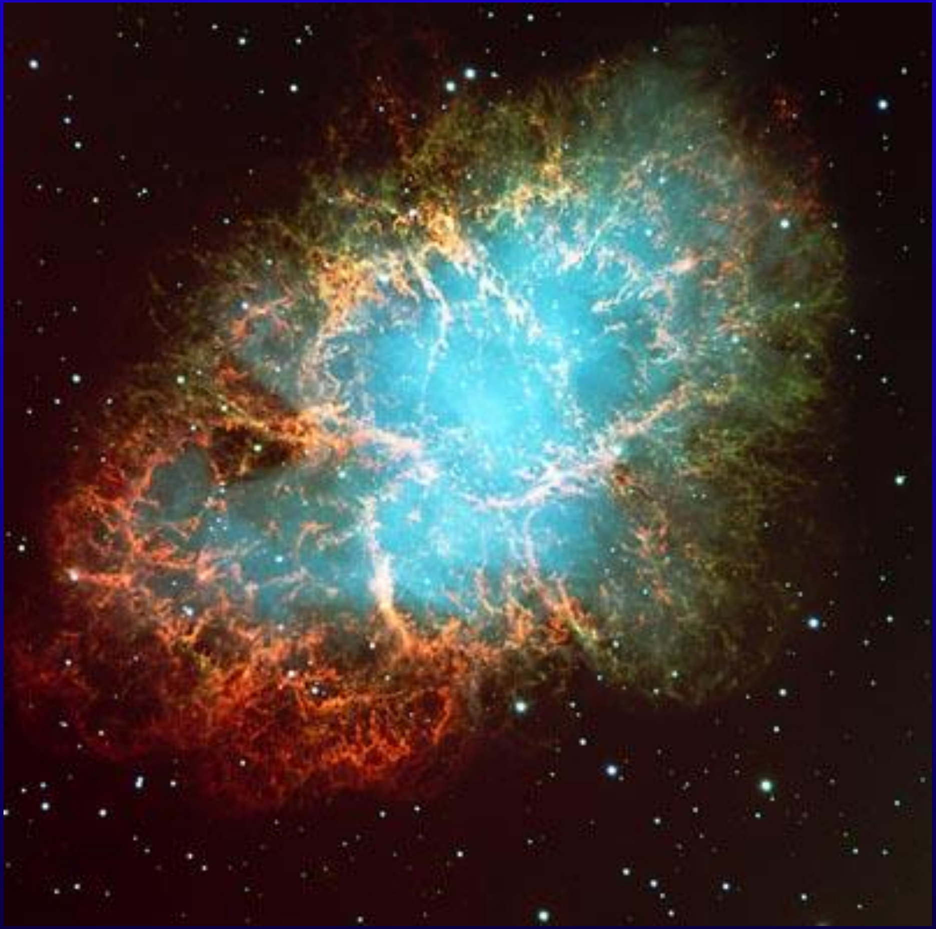
Curved orbit of electrons in magnet field



Accelerated charge → Electromagnetic radiation

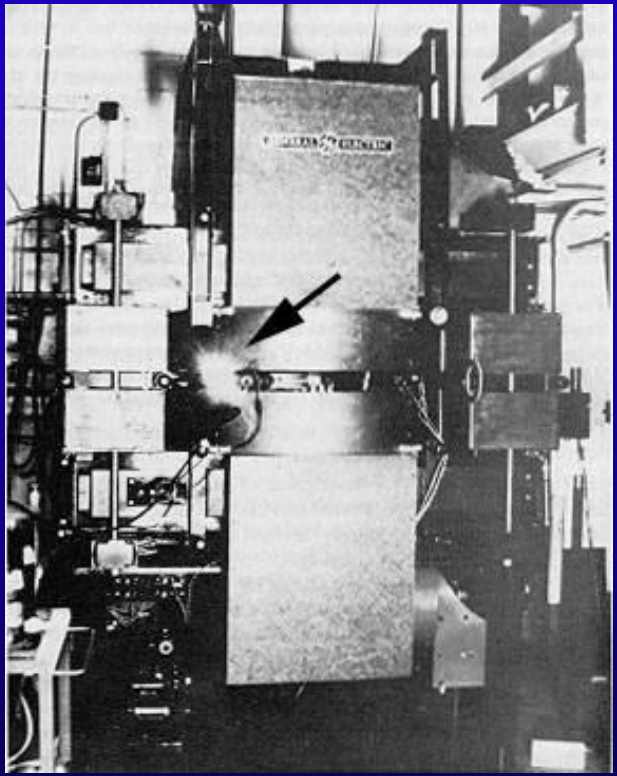
Electromagnetic waves

Crab Nebula
6000 light years away



First light observed
1054 AD

GE Synchrotron
New York State



First light observed
1947

Synchrotron radiation: some dates

- 1873 Maxwell's equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
- 1900 Wiechert: retarded potentials
- 1908 Schott: Adams Prize Essay

... waiting for accelerators …
1940: 2.3 MeV betatron,Kerst, Serber

Maxwell equations (poetry)

War es ein Gott, der diese Zeichen schrieb
Die mit geheimnisvoll verborg'nem Trieb
Die Kräfte der Natur um mich enthüllen
Und mir das Herz mit stiller Freude füllen.
Ludwig Boltzman

Was it a God whose inspiration
Led him to write these fine equations
Nature's fields to me he shows
And so my heart with pleasure glows.
translated by John P. Blewett

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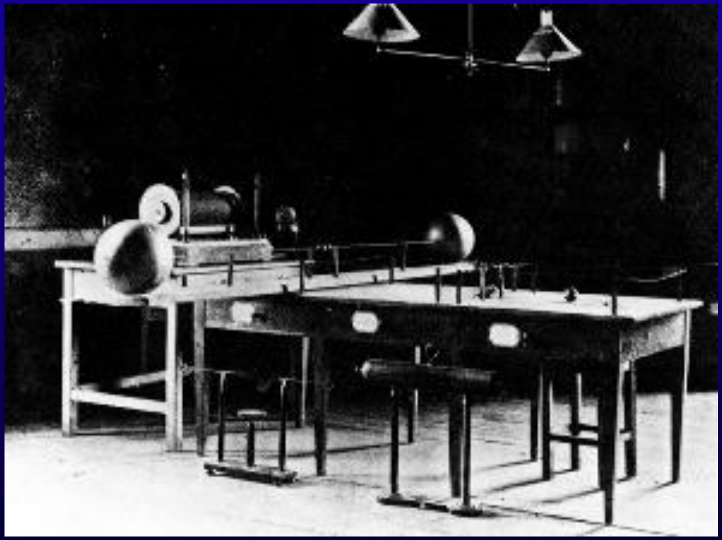
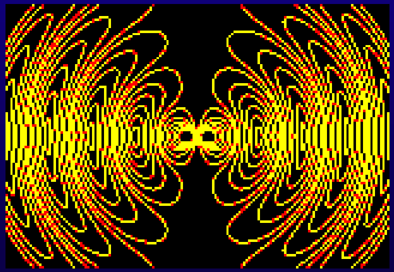
THEORETICAL UNDERSTANDING →

1873 **Maxwell's equations**

→ **made evident that changing charge densities would result in electric fields that would radiate outward**

1887 Heinrich Hertz demonstrated such waves:





It's of no use whatsoever[...] *this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.*


Radiowaves für www.senderfotos.de

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Donald Kerst: first betatron (1940)

"Ausserordentlichhochgeschwindigkeitelektronenent wickelndenschwerarbeitsbeigollitron"

Synchrotron radiation: some dates

- 1946 Blewett observes energy loss due to synchrotron radiation
  100 MeV betatron
- 1947 First visual observation of SR
  70 MeV **synchrotron**, GE Lab

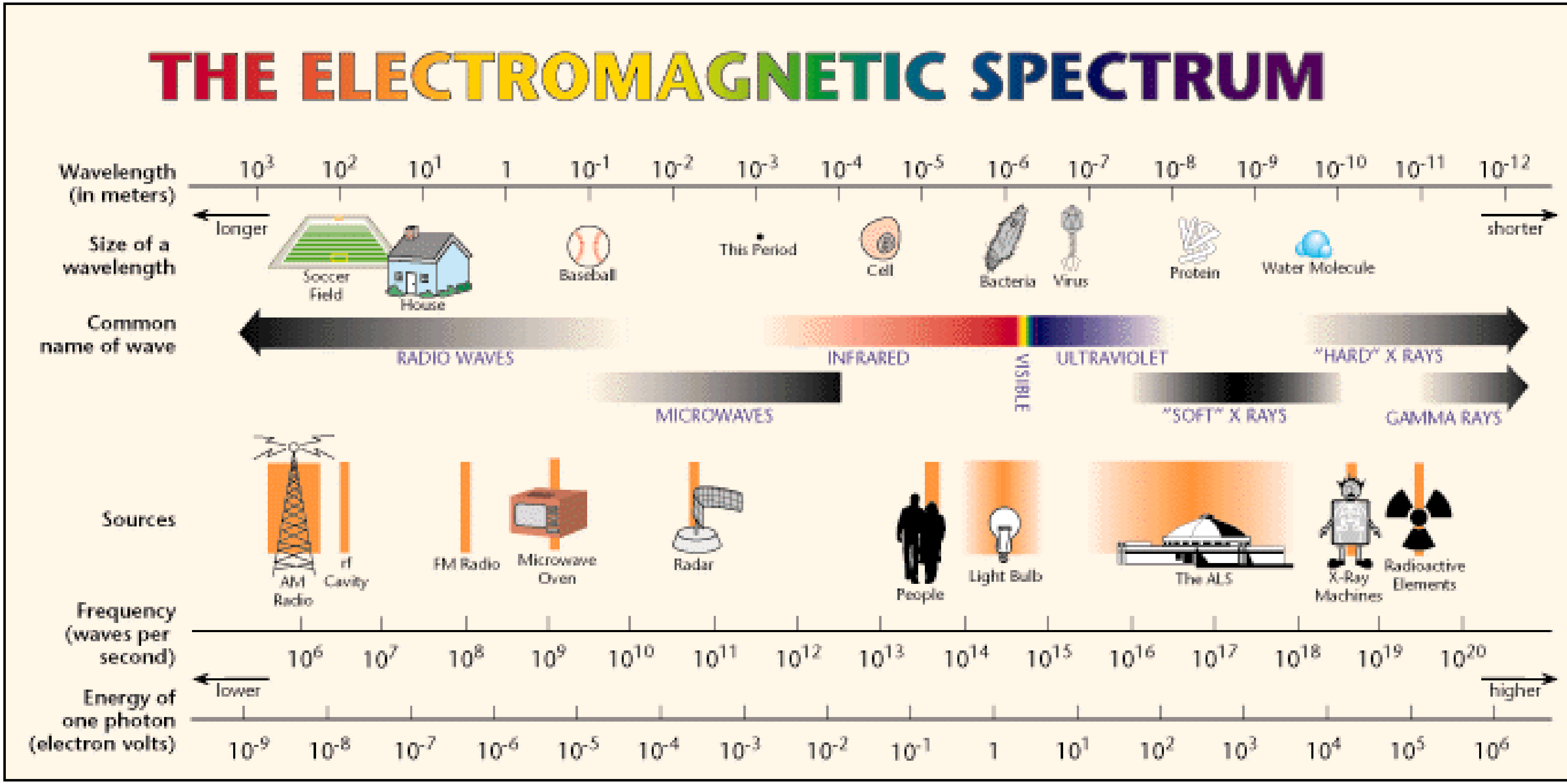
  NAME!

- 1949 Schwinger PhysRev paper

…

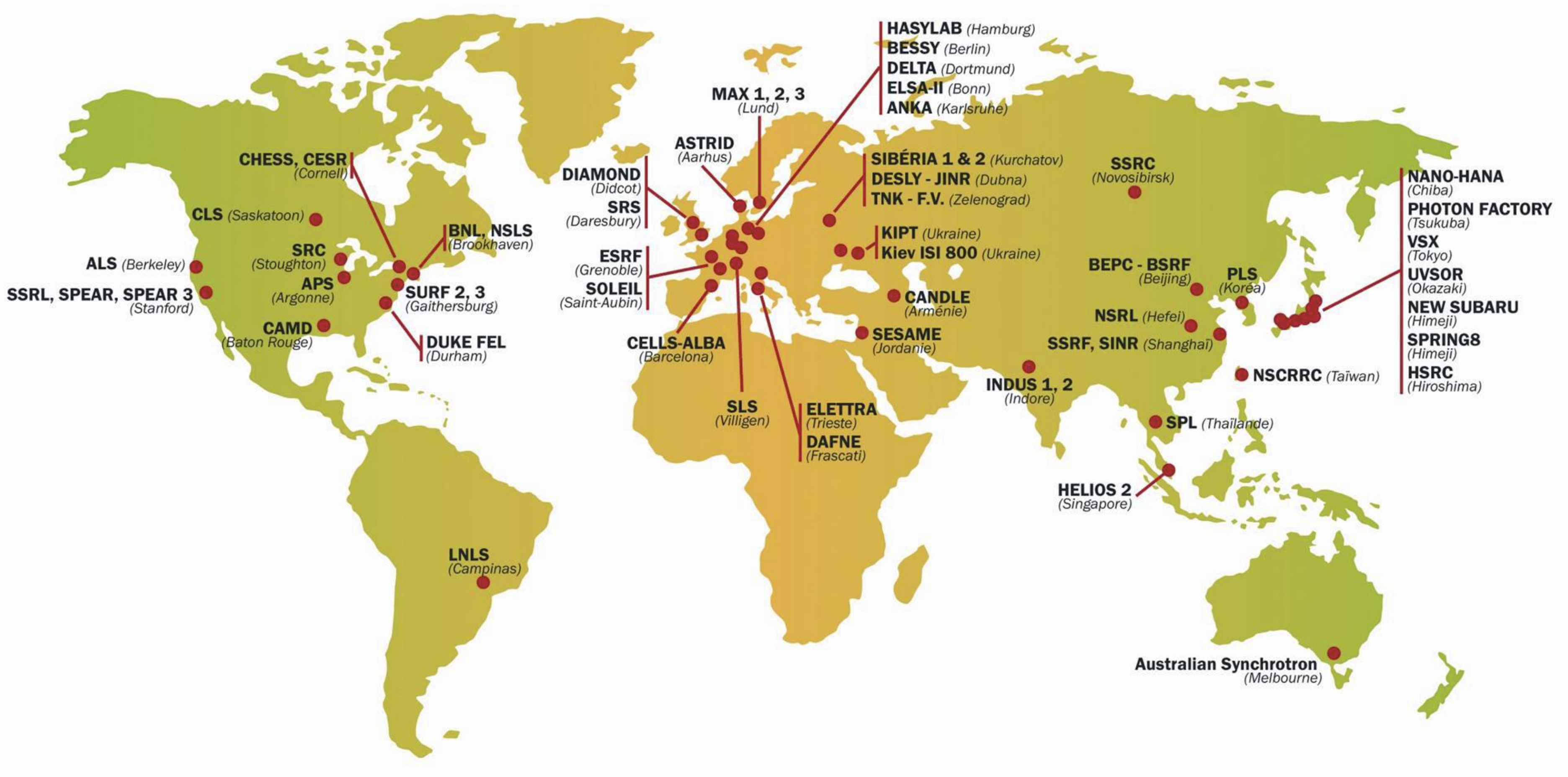
- 1976 Madey: first demonstration of Free Electron laser

Paul Scherrer Institute, Switzerland

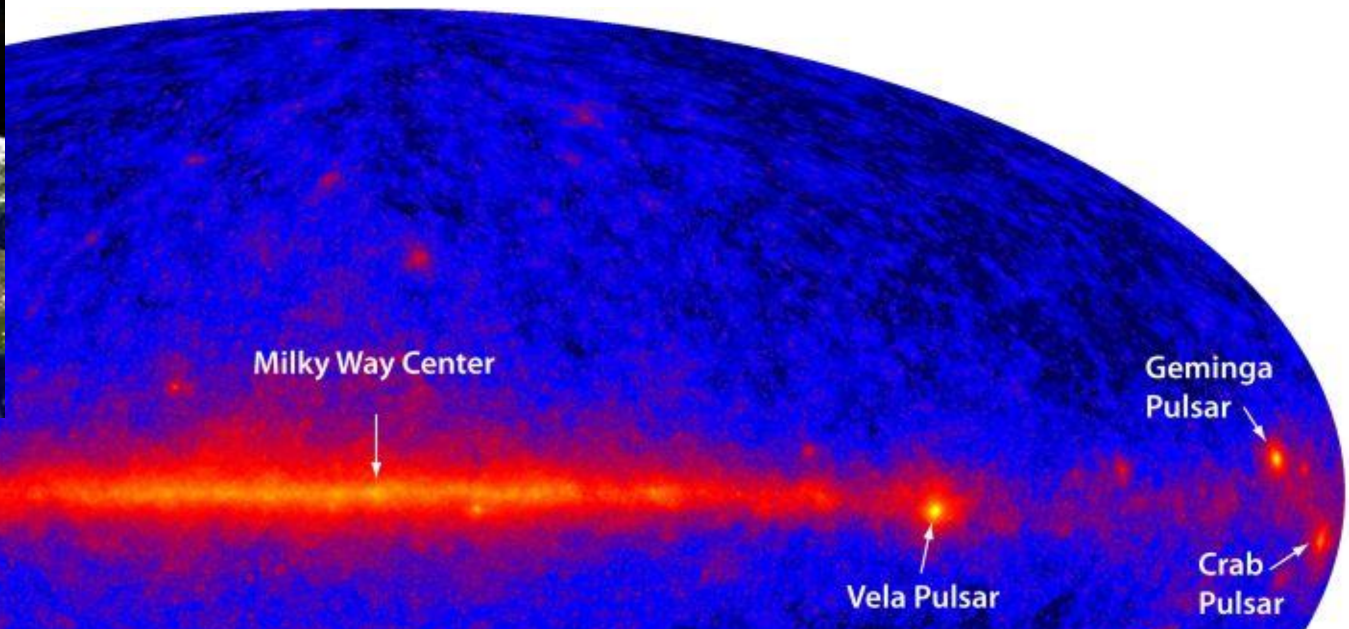


Wavelength continuously tunable !

60‘000 SR users world-wide



A larger view



Crab Nebula sent a 450 TeV photon

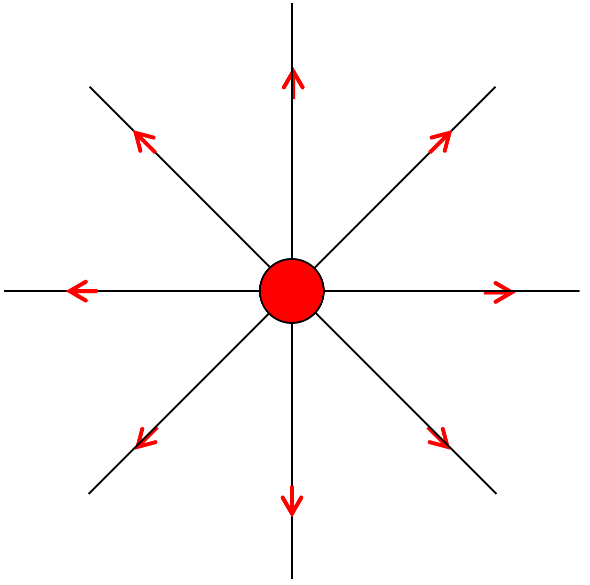
Tibet
ASγ
Collaboration

Why do they radiate?

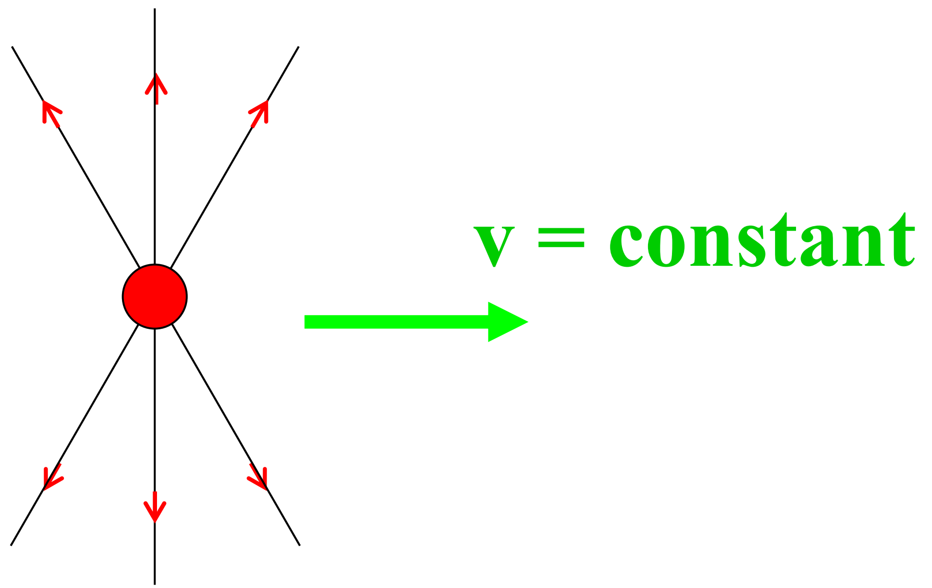
Synchrotron Radiation is not as simple as it seems

… I will try to show that it is much simpler

Charge at rest
Coulomb field, no radiation



Uniformly moving charge does not radiate



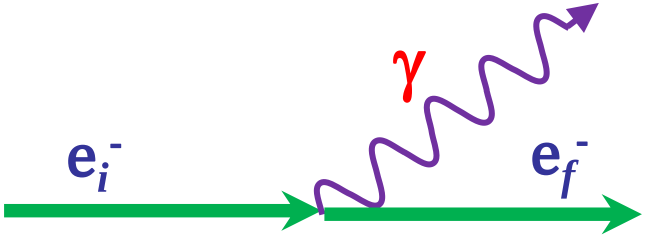
But! Cerenkov!

Free isolated electron cannot emit a photon

Easy proof using 4–vectors and relativity

- momentum conservation if a photon is emitted

$$\boldsymbol{P}_i = \boldsymbol{P}_f + \boldsymbol{P}_\gamma$$



- square both sides

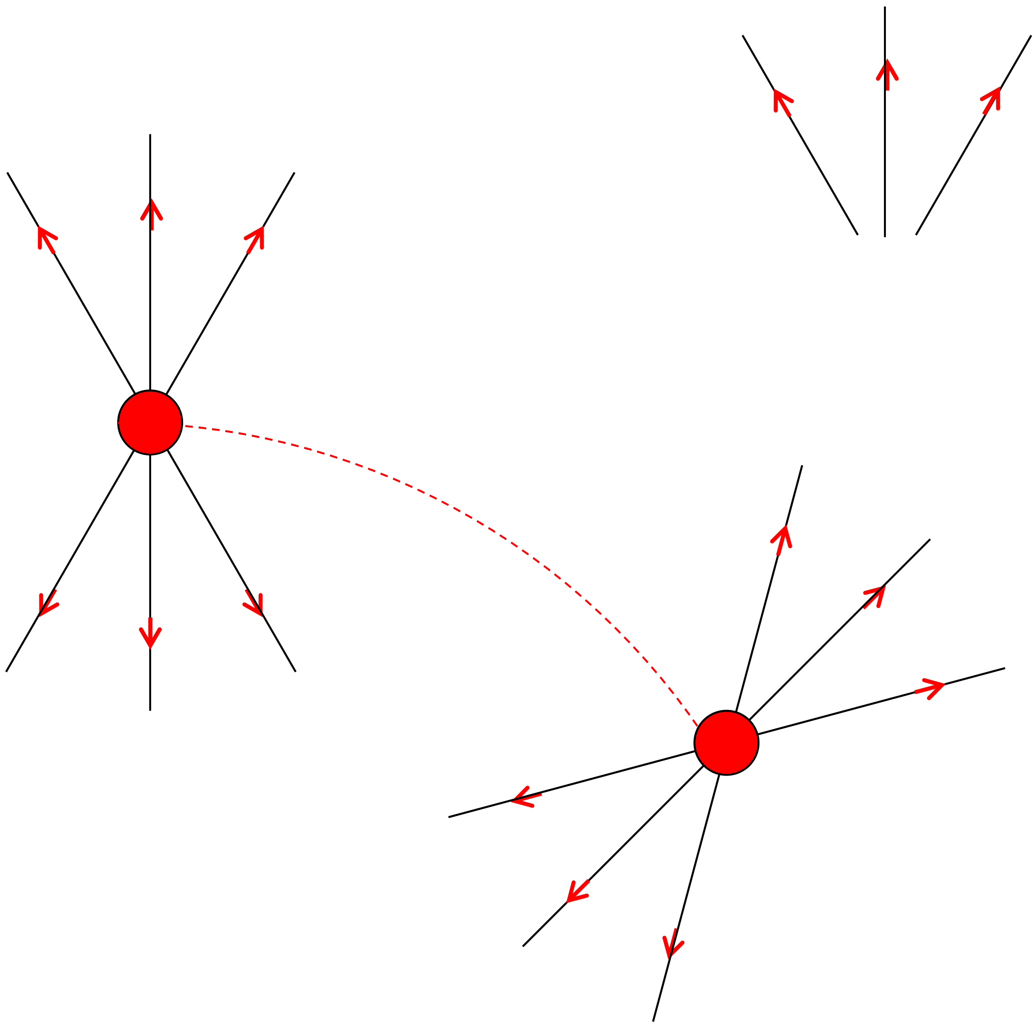
$$m^2 = m^2 + 2\boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma + 0 \Rightarrow \boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma = 0$$

- in the rest frame of the electron

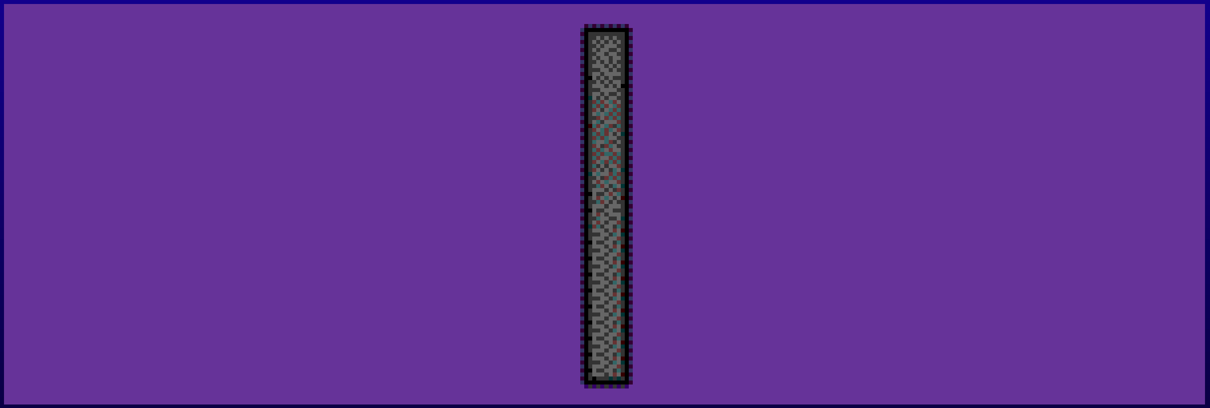
$$\boldsymbol{P}_f = (m, 0) \qquad \boldsymbol{P}_\gamma = (E_\gamma, p_\gamma)$$

this means that the photon energy must be zero.

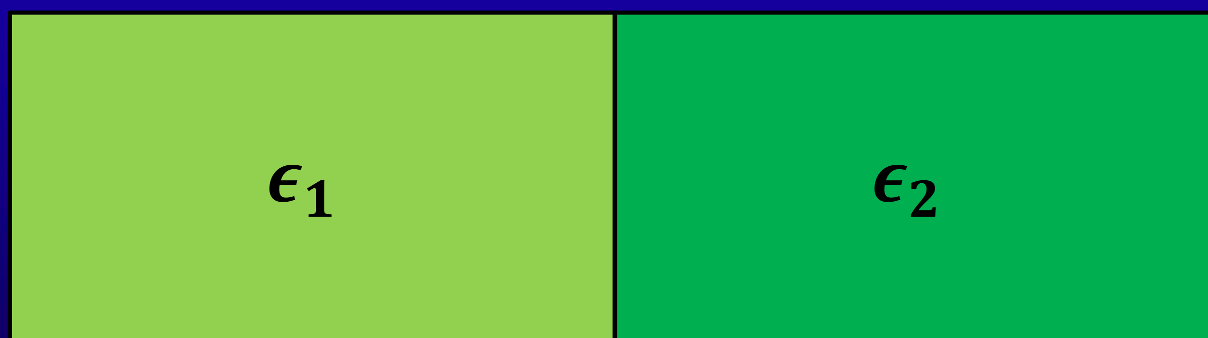
We need to separate the field from charge



Bremsstrahlung
or
“braking” radiation



Transition Radiation



$$c_1 = \frac{1}{\sqrt{\epsilon_1 \mu_1}} \qquad c_2 = \frac{1}{\sqrt{\epsilon_2 \mu_2}}$$

Liénard–Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left[r\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)\right]_{ret}} \qquad \vec{\mathbf{A}}(t) = \frac{q}{4\pi\varepsilon_0 c^2}\left[\frac{\vec{\mathbf{v}}}{r\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)}\right]_{ret}$$

and the electromagnetic fields:

$$\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0$$

(Lorentz gauge)

$$\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\nabla \varphi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

Fields of a moving charge

$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \frac{\mathbf{1}}{\mathbf{r}^2}\right]_{ret} +$$

"near field"

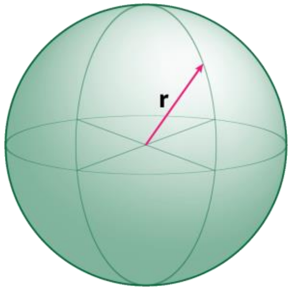
$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}} \times \left[\left(\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}\right) \times \dot{\vec{\boldsymbol{\beta}}}\right]}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \frac{\mathbf{1}}{\mathbf{r}}\right]_{ret}$$

"far field"

$$\vec{\mathbf{B}}(t) = \frac{1}{c}\left[\vec{\mathbf{n}} \times \vec{\mathbf{E}}\right]$$

Energy flow integrated over a sphere

$$Power \sim E^2 \cdot \text{Area}$$



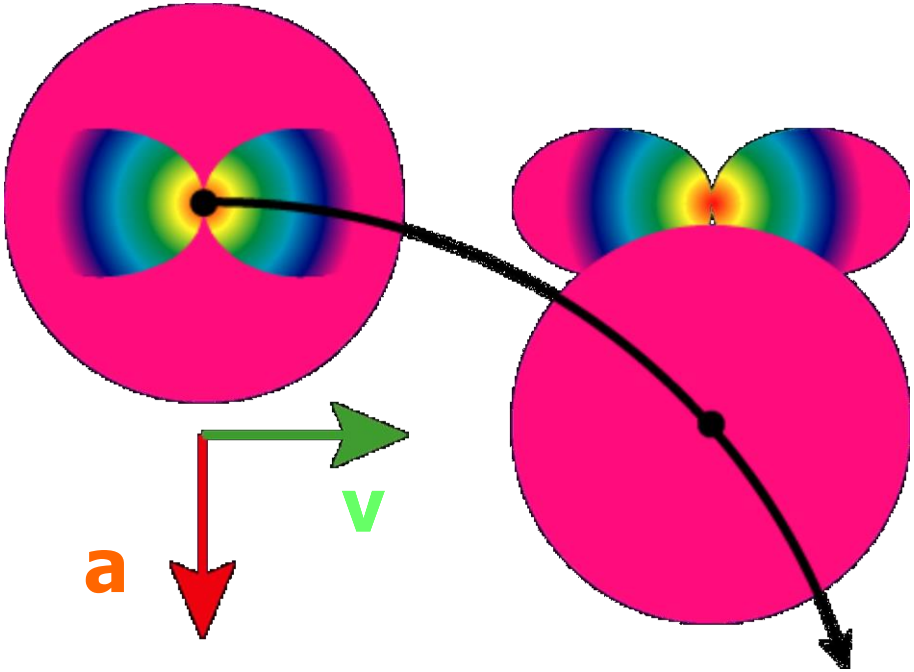
$$A = 4\pi r^2$$

Near field $$P \propto \frac{1}{r^4} r^2 \propto \frac{1}{r^2}$$

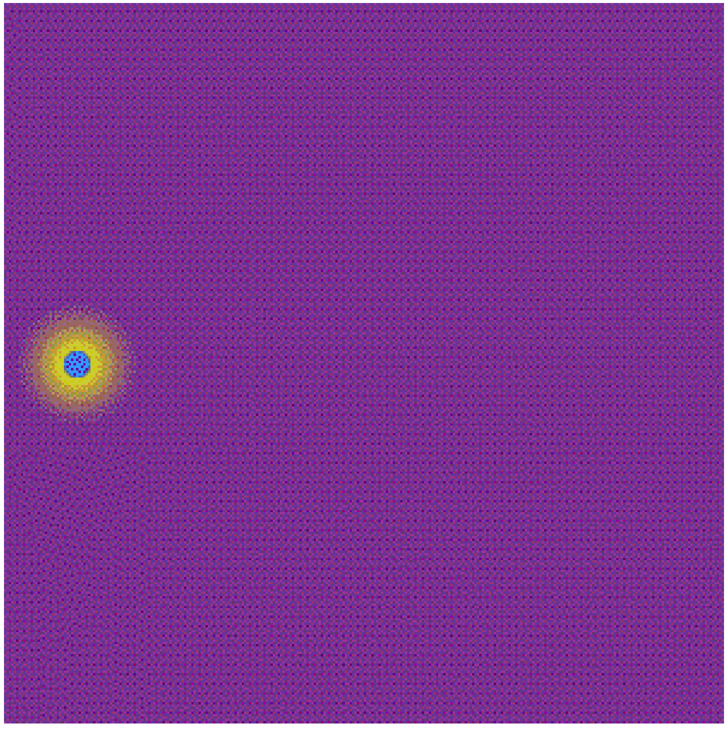
Far field $$P \propto \frac{1}{r^2} r^2 \propto const$$

Radiation = constant flow of energy to infinity

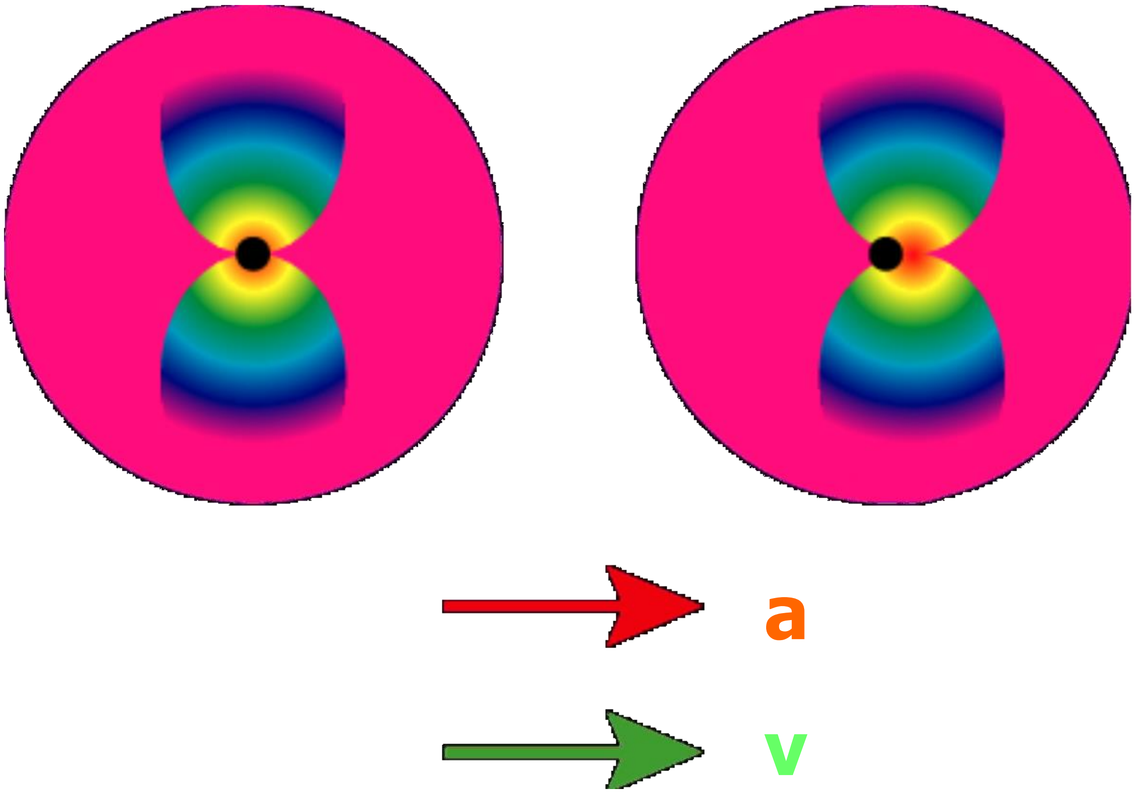
Transverse acceleration



Radiation field quickly separates itself from the Coulomb field



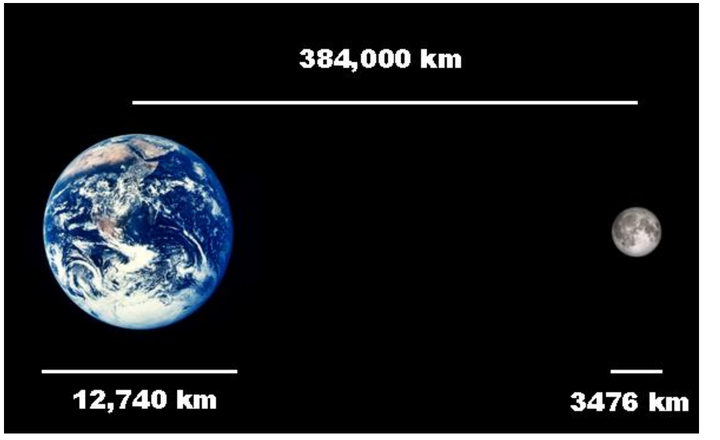
Longitudinal acceleration



Radiation field cannot separate itself from the Coulomb field

Synchrotron Radiation Basic Properties

Beams of ultra-relativistic particles: e.g. a race to the Moon



An electron with energy of a few GeV emits a photon… a race to the Moon!

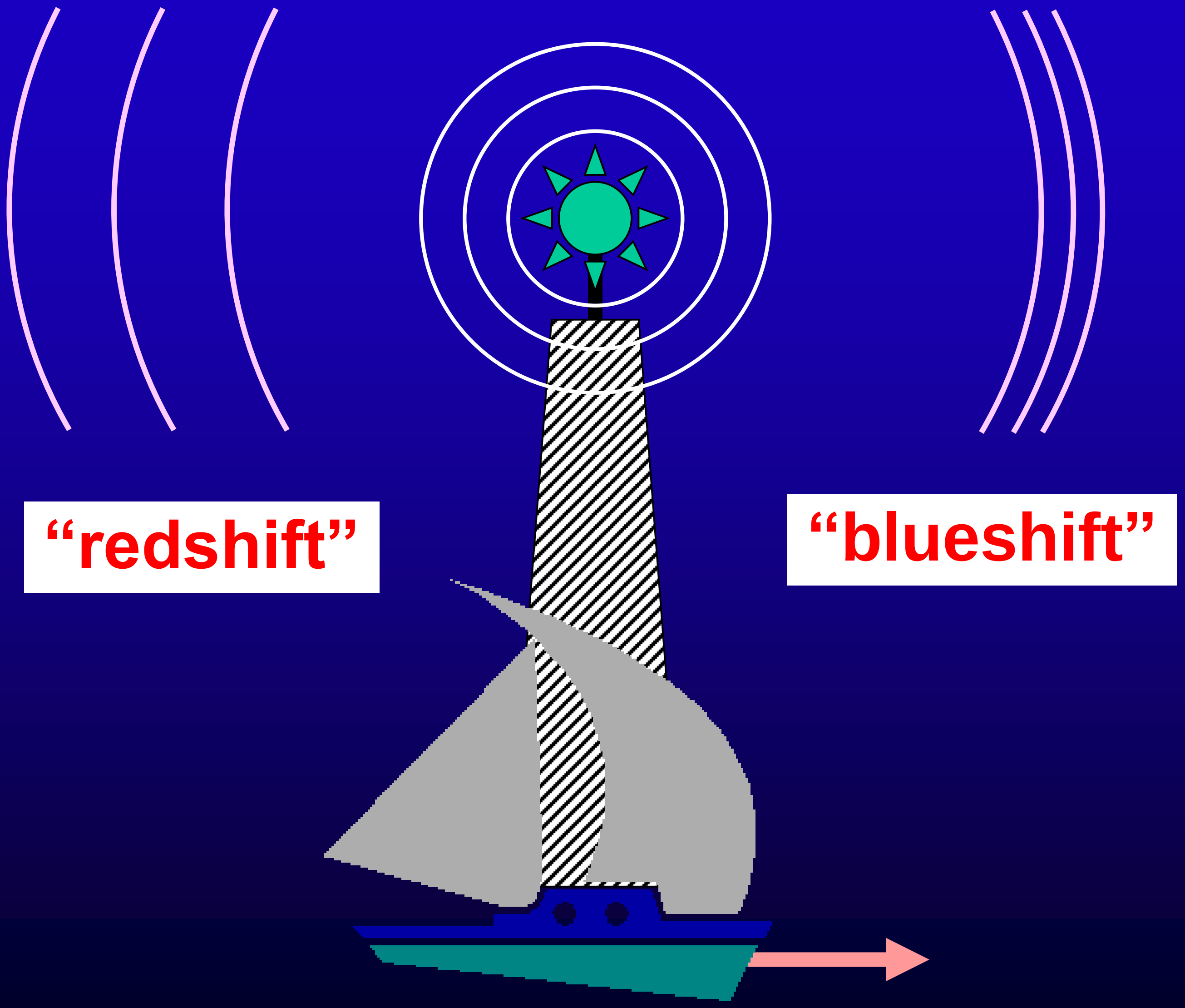
$$\Delta t = \frac{L}{\beta c} - \frac{L}{c} = \frac{L}{\beta c}(1-\beta) \sim \frac{L}{\beta c} \cdot \frac{1}{2\gamma^2}$$

Electron will lose

- by only 8 meters
- the race will last only 1.3 seconds

$$\Delta L = L(1-\beta) \cong \frac{L}{2\gamma^2}$$

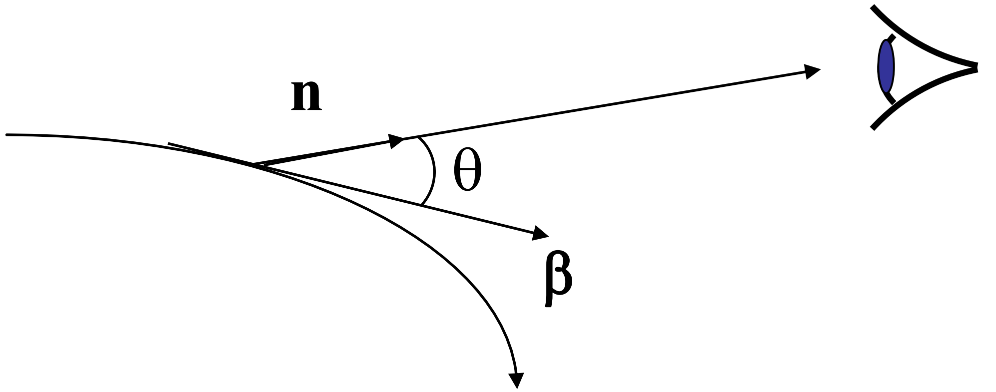
Moving Source of Waves: Doppler effect



Cape Hatteras, 1999

Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta}) T_{emit}$$

The wavelength is shortened by the same factor

$$\lambda_{obs} = (1 - \beta \cos\theta) \lambda_{emit}$$

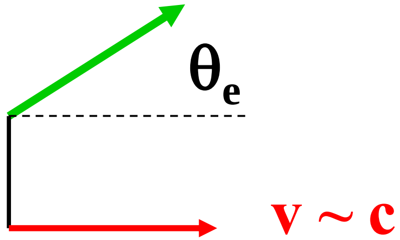
in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}$$

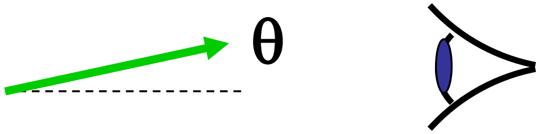
since

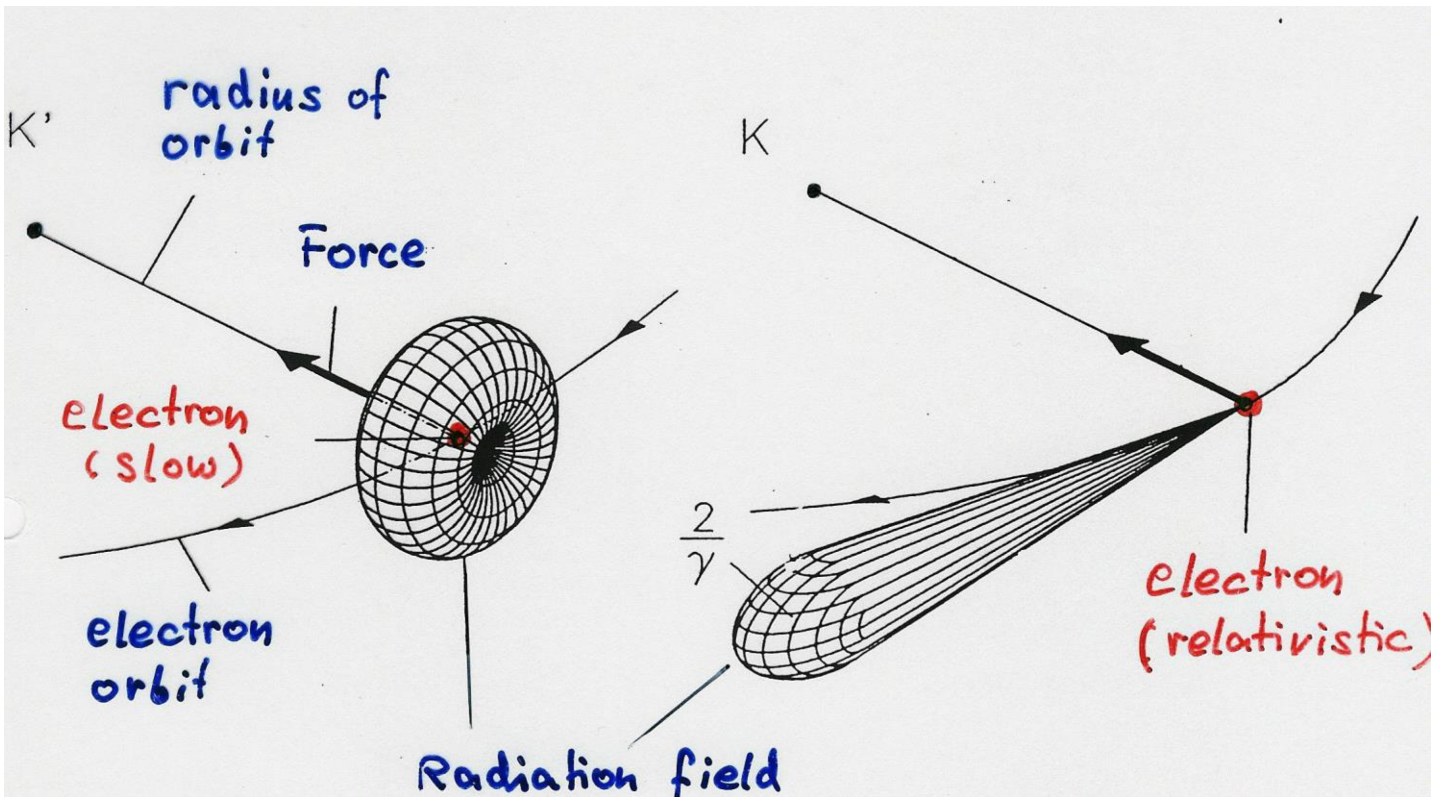
$$1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}$$

Radiation is emitted into a narrow cone



$$\theta = \frac{1}{\gamma} \cdot \theta_e$$



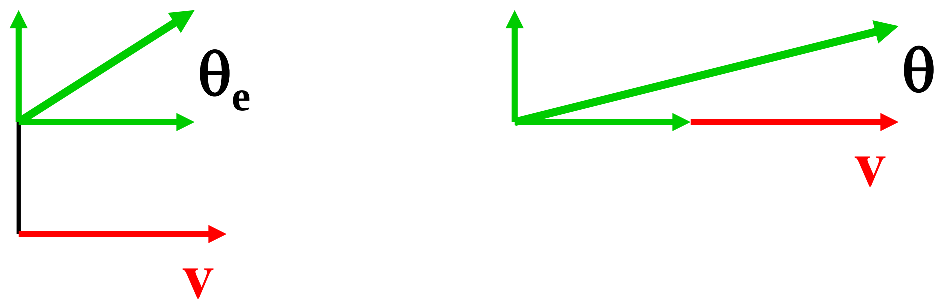


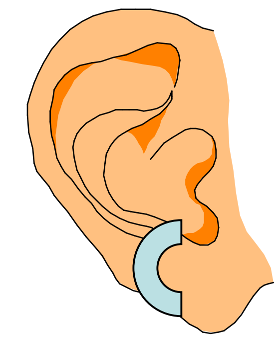
v << c

v ≈ c

Sound waves (non-relativistic)

Angular collimation





$$\theta = \frac{v_{s\perp}}{v_{s\|} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\|}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted}\left(1 - \frac{\mathbf{v}}{v_s}\right)$$

Synchrotron radiation power

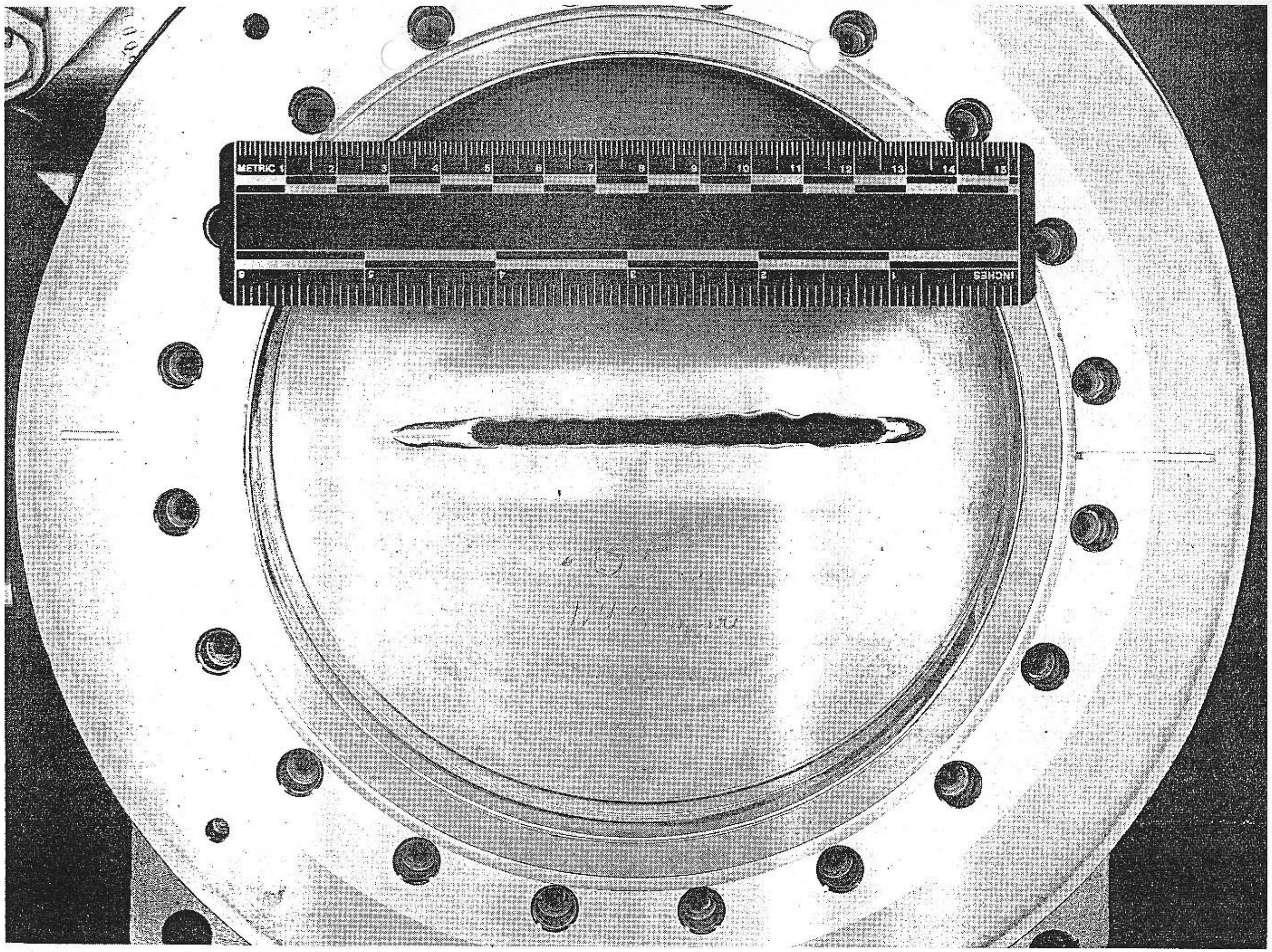
Power emitted is proportional to: $P \propto E^2 B^2$

$$P_\gamma = \frac{c C_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

E = Energy!

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{\left(m_e c^2\right)^3} = 8.858 \cdot 10^{-5} \left[\frac{\text{m}}{\text{GeV}^3}\right]$$

The power is all too real!



ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Synchrotron radiation power

Power emitted is proportional to:

$$\boldsymbol{P \propto E^2 B^2}$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3}\alpha\hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197\ \mathrm{Mev} \cdot \mathrm{fm}$$

Energy loss per turn:

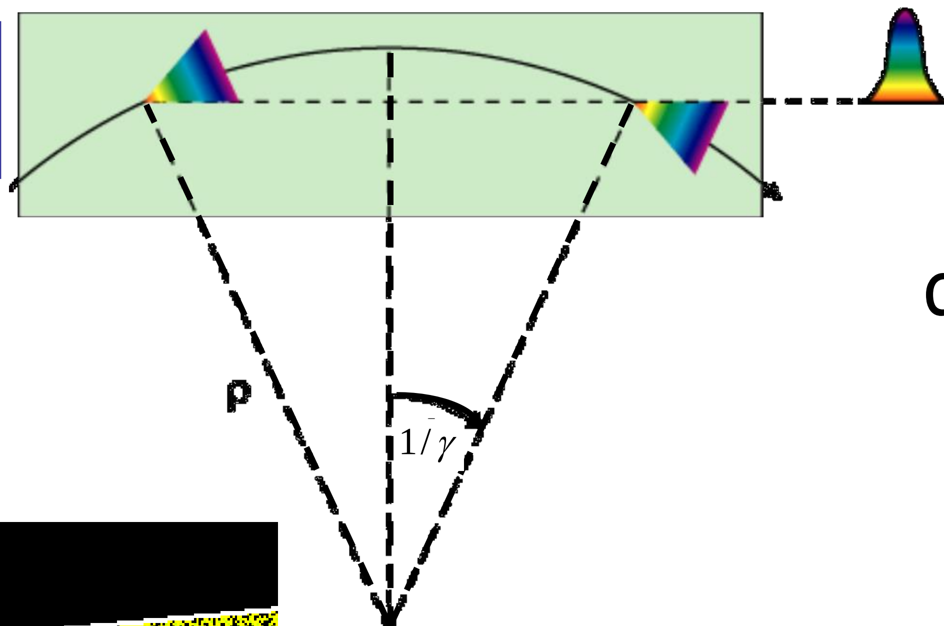
$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3}\alpha\hbar c\frac{\gamma^4}{\rho}$$

Typical frequency of synchrotron light

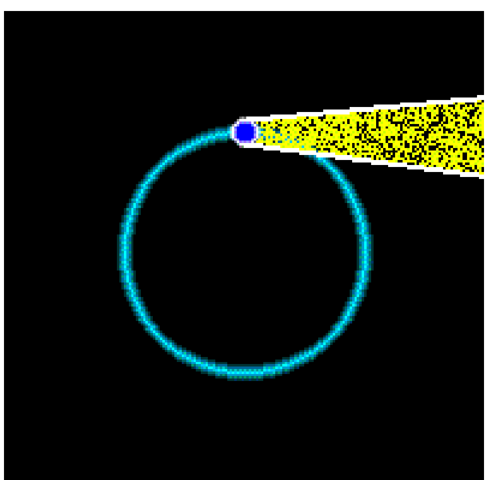
Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

$$l \sim \frac{2\rho}{\gamma}$$



Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$



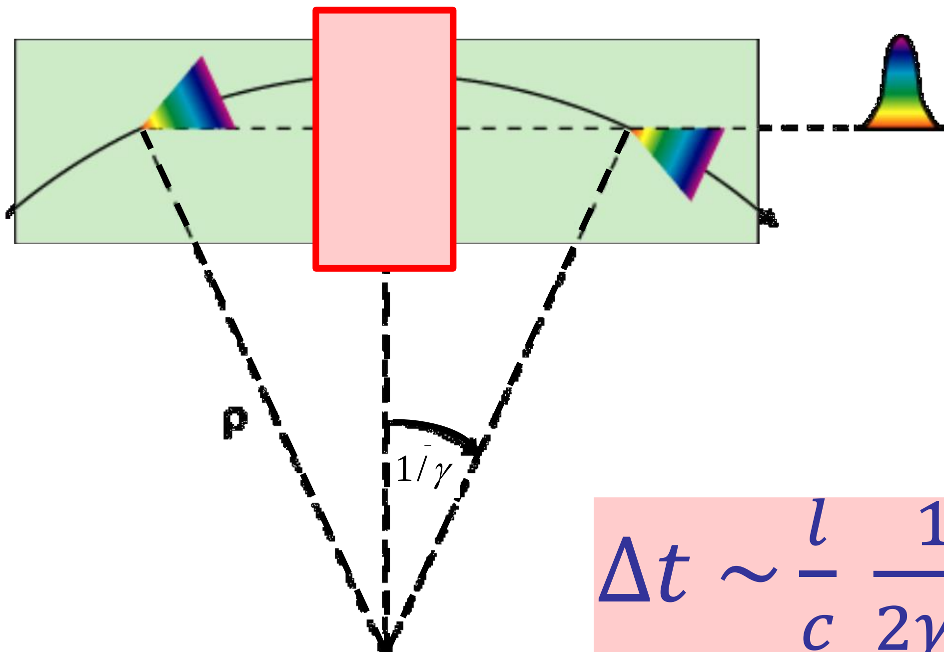
$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$

Short magnet: higher energy photons

When Lorentz factor is not very high (e.g. protons)…

$$l \ll \frac{2\rho}{\gamma}$$



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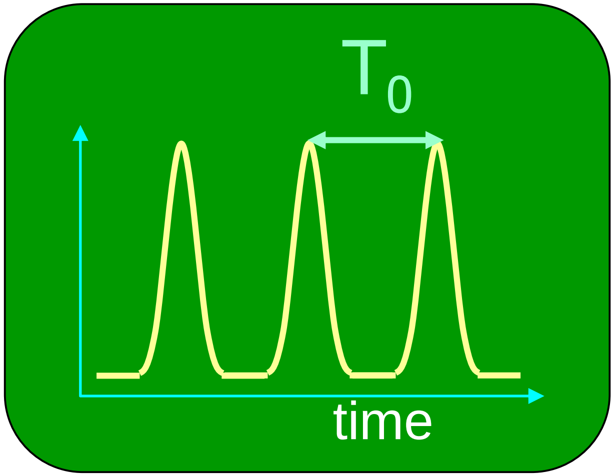
$$\Delta t \sim \frac{l}{c}\,\frac{1}{2\gamma^2}$$

Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every T_0 (revolution period)



- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

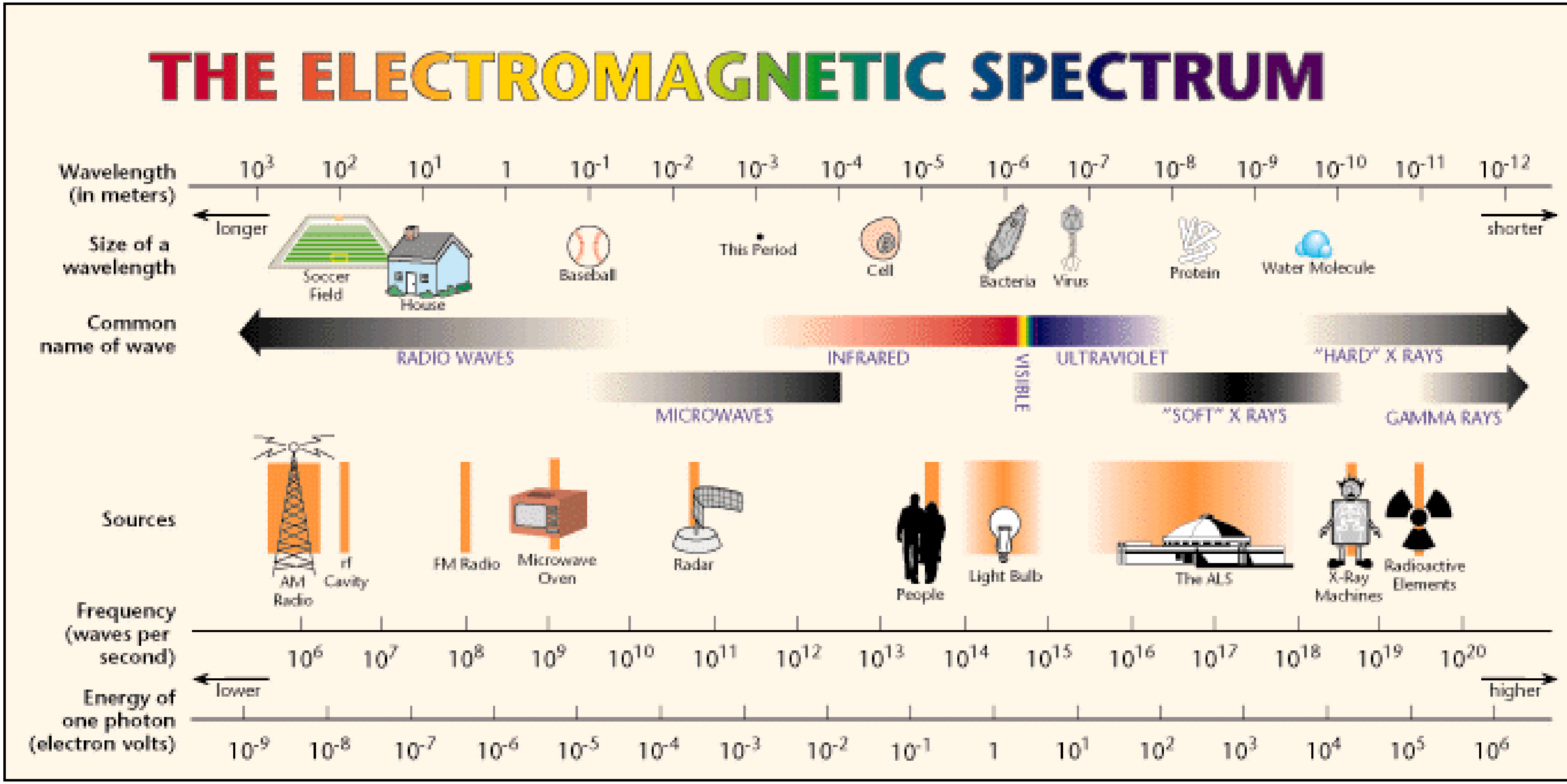
$$\omega_0 \sim 1 \text{ MHz}$$

$$\gamma \sim 4000$$

$$\omega_{\text{typ}} \sim 10^{16} \text{ Hz !}$$

- At high frequencies the individual harmonics overlap

continuous spectrum !



Wavelength continuously tunable !

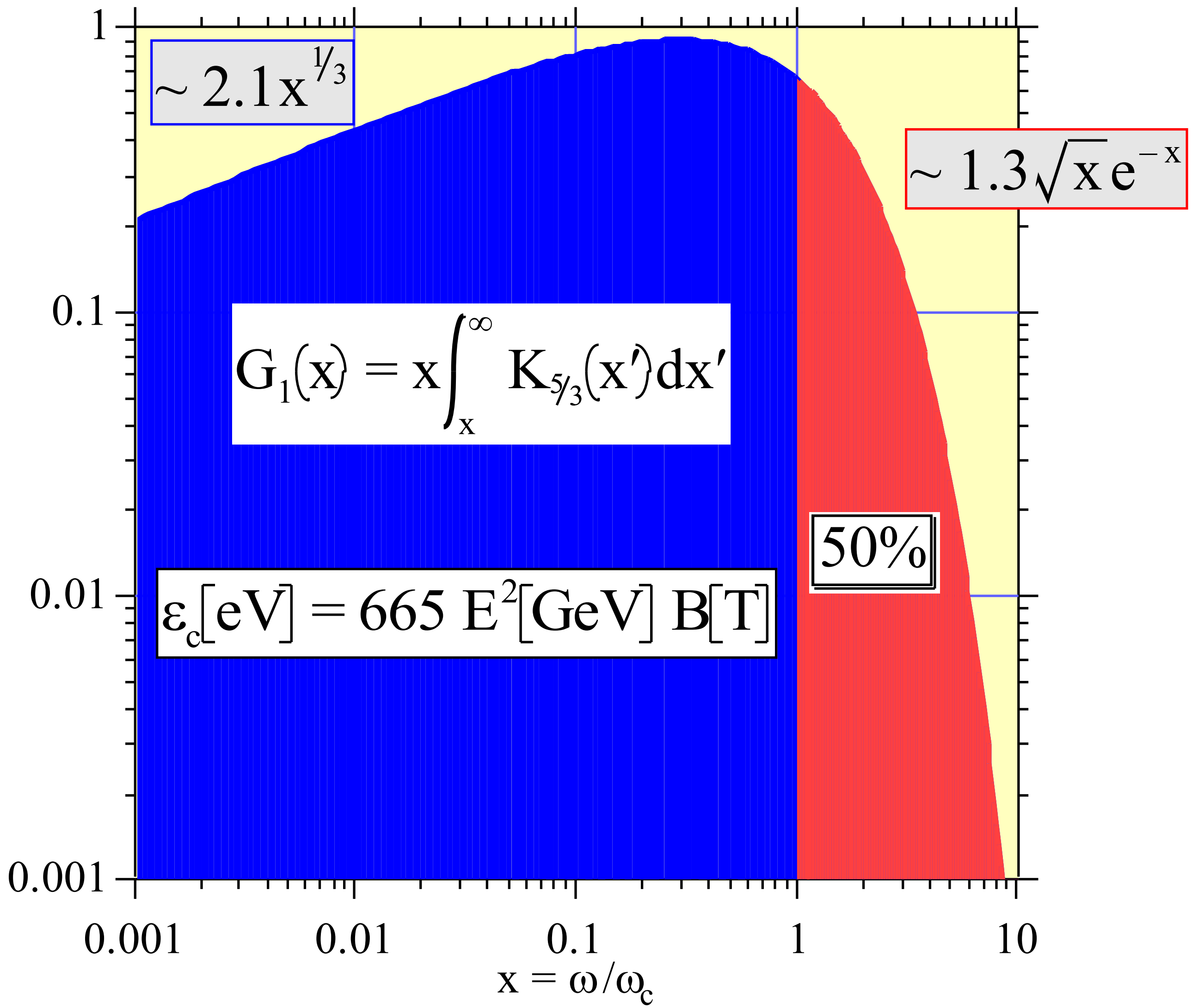
$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')dx'$$

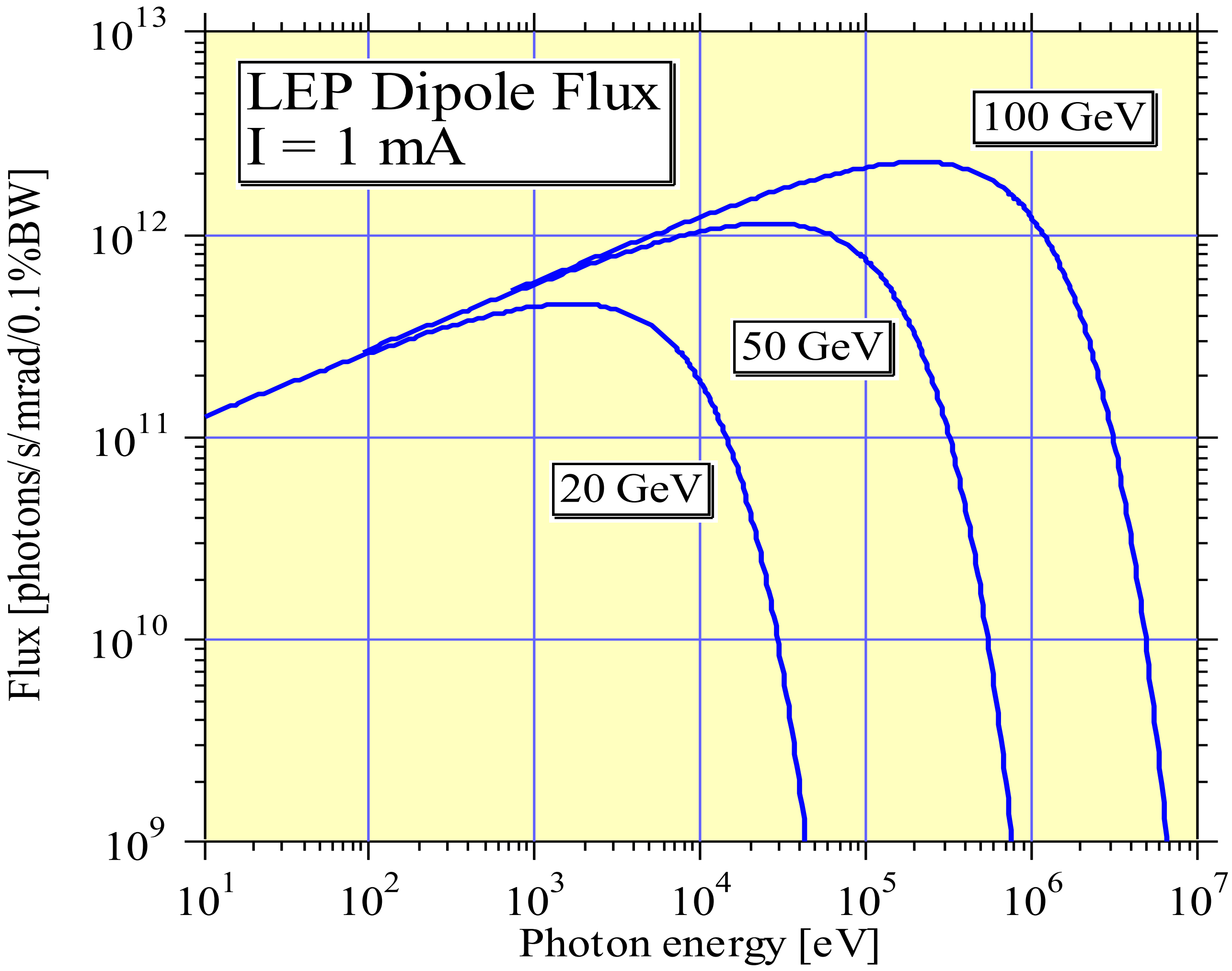
$$\int_0^\infty S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$



Synchrotron radiation flux for different electron energies



Angular divergence of radiation

The rms opening angle R'

- **at the critical frequency:**

$$\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$$

- **well below**

$$\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4\left(\frac{\lambda}{\rho}\right)^{1/3}$$

independent of γ !

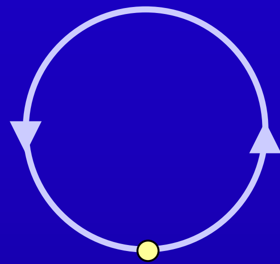
- **well above**

$$\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$$

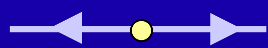
Synchrotron light polarization

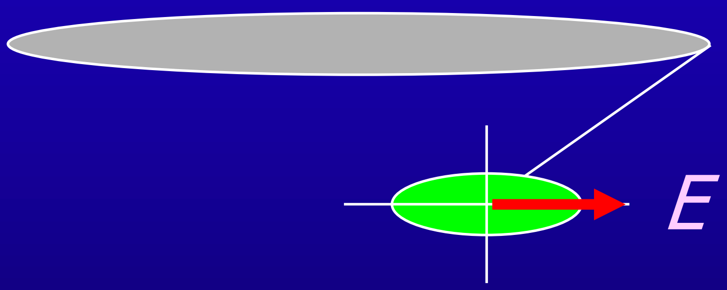
An electron in a storage ring

TOP VIEW



SIDE VIEW



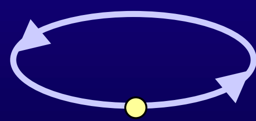


<u>Polarization:</u>

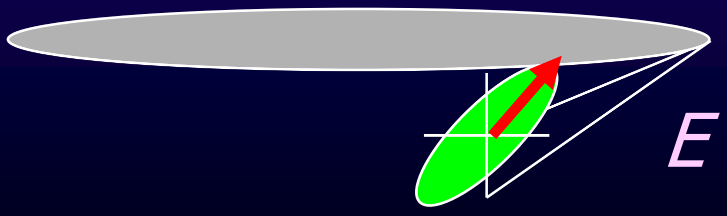
Linear in the plane of the ring

the electric field vector

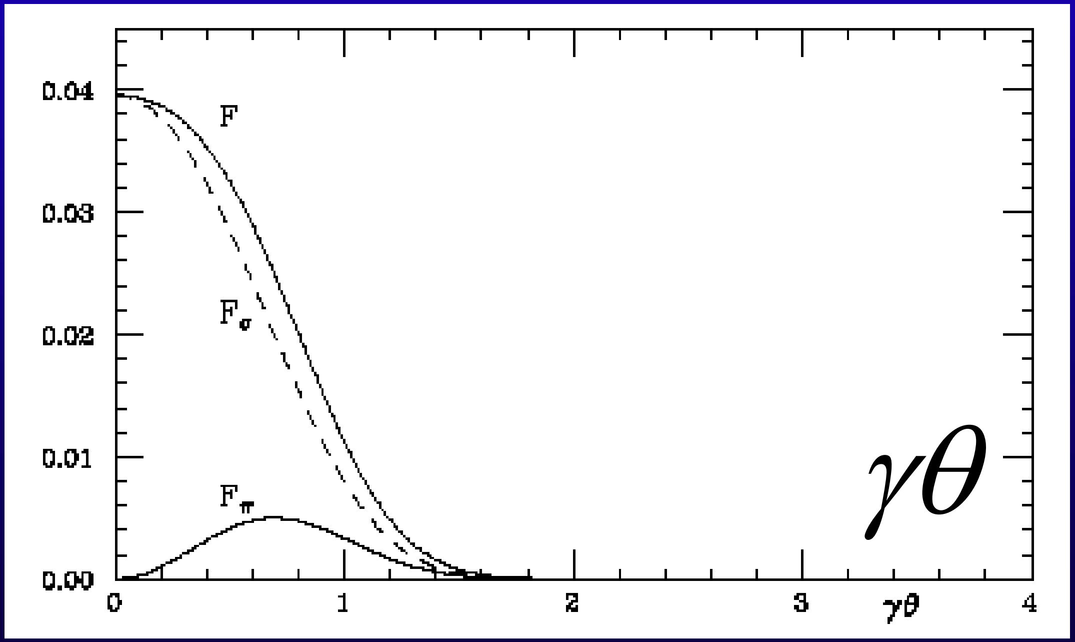
TILTED VIEW

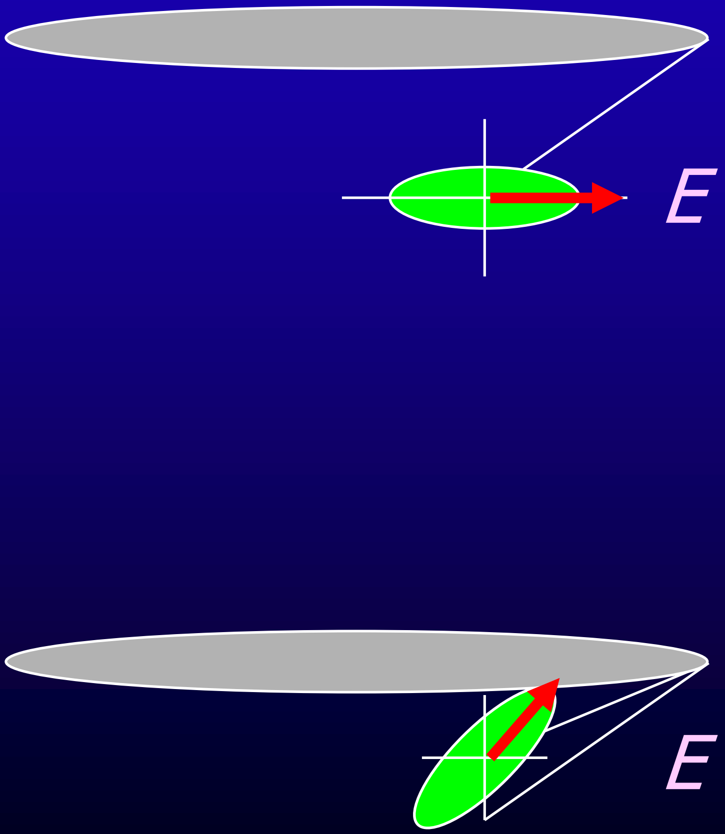


elliptical out of the plane



Angular distribution of SR

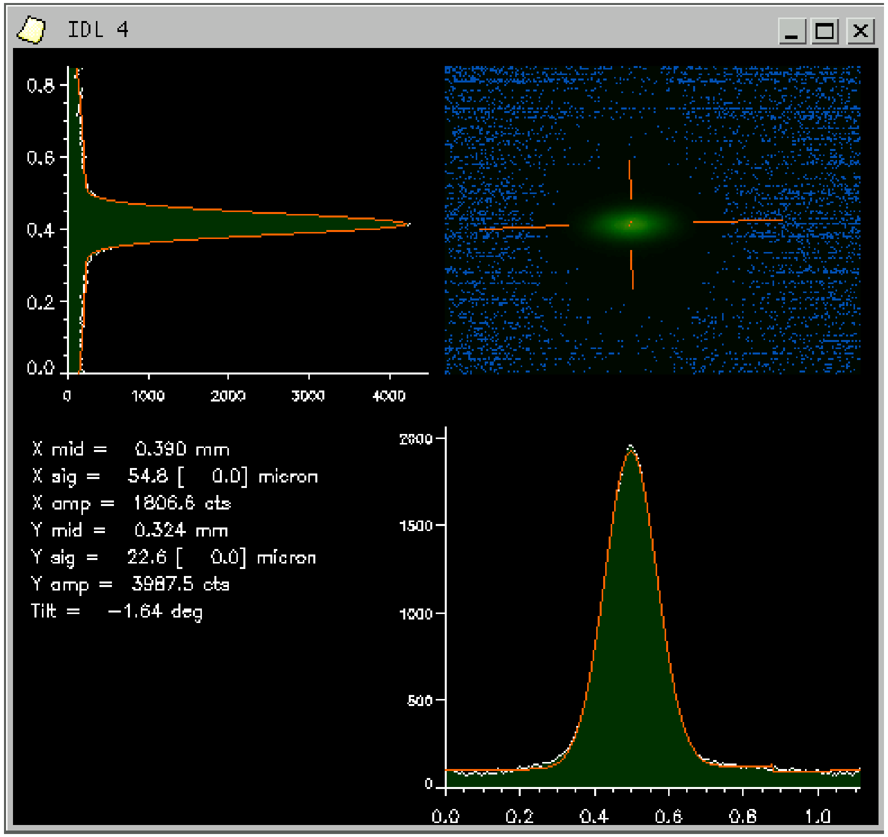


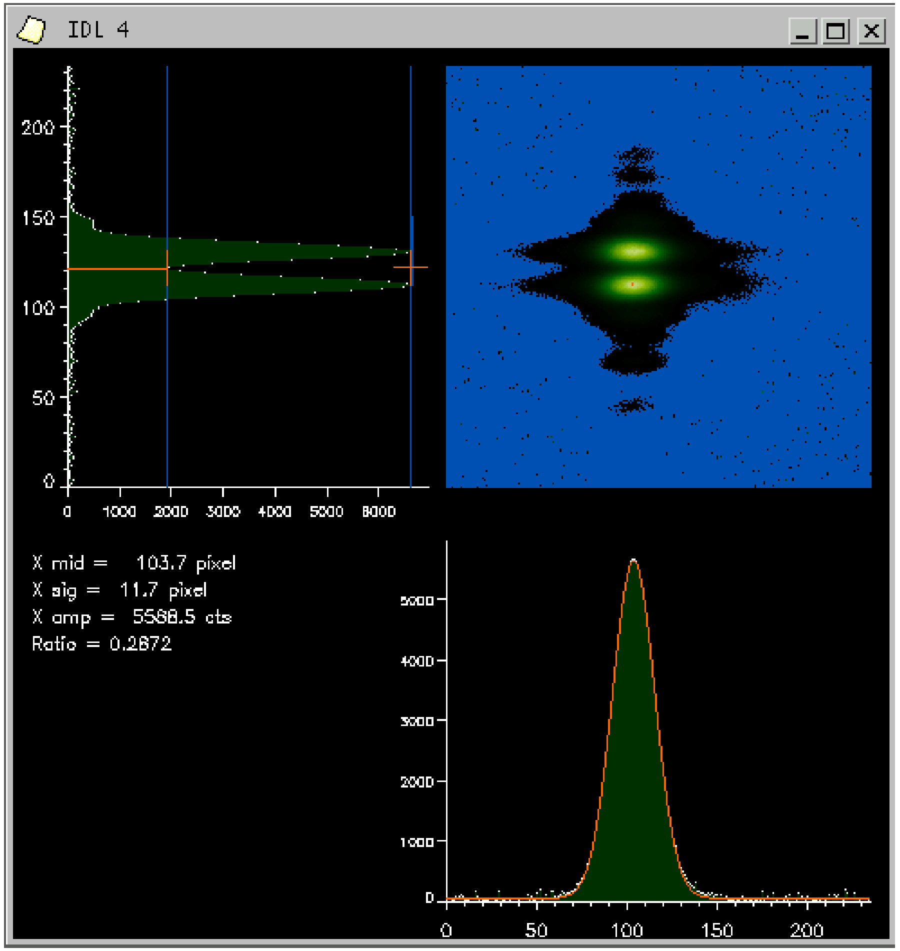


Synchrotron light based electron beam diagnostics

Seeing the electron beam (SLS)

X rays | visible light, vertically polarised

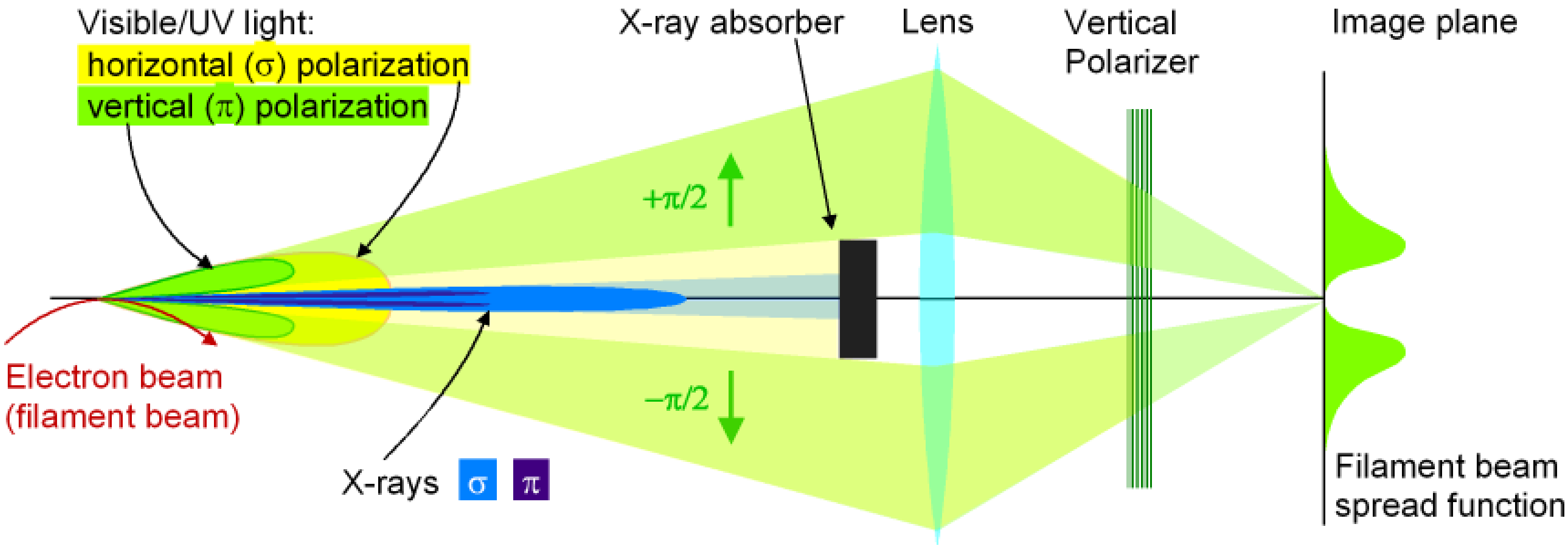




$\sigma_x \sim 55\mu m$

Seeing the electron beam (SLS)

Making an image of the electron beam using the vertically polarised synchrotron light



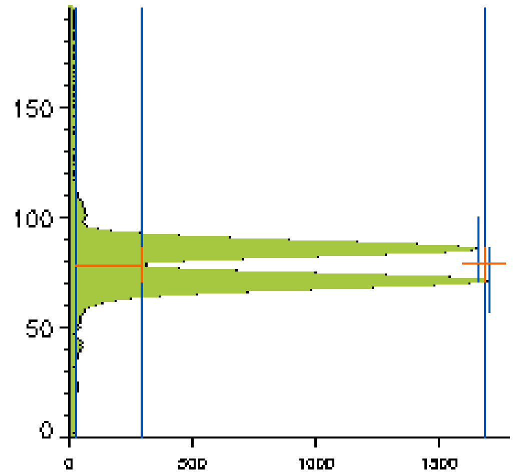
High resolution measurement

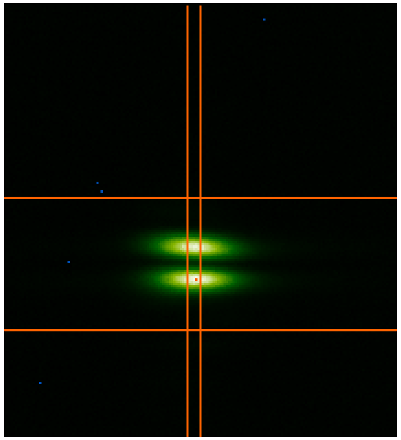
Wavelength used: 364 nm

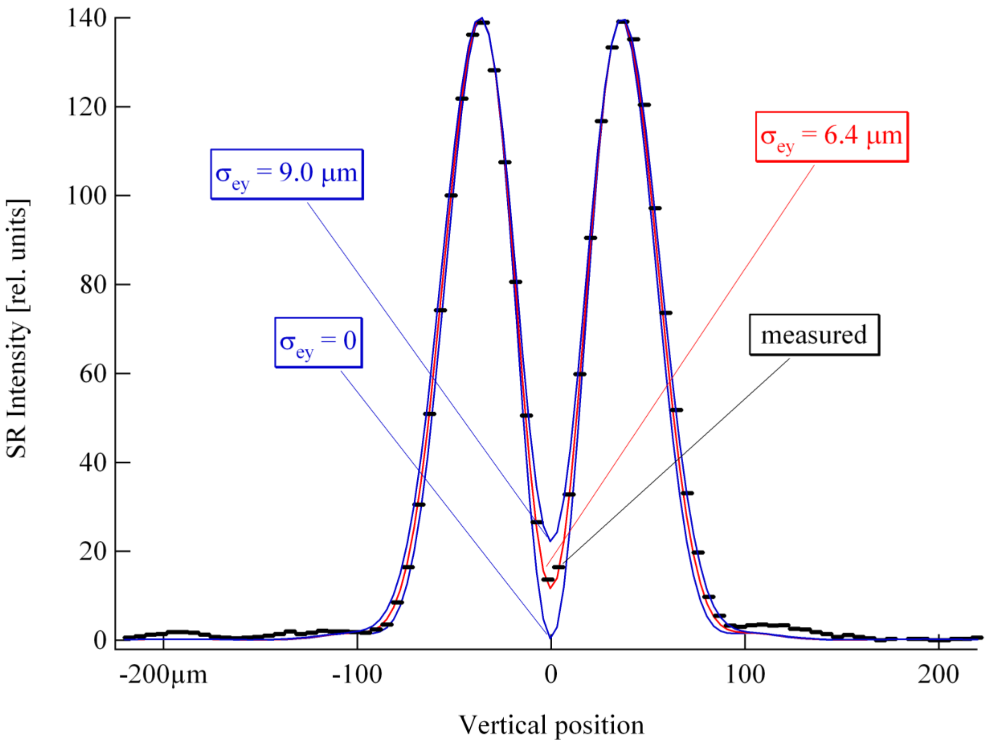
For point-like source the intensity on axis is zero

Peak-to-valley intensity ratio is determined by the beam height

Present resolution: **3.5 µm**







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