CLARIANT

LORENTZ COVARIANT NEURAL NETWORK

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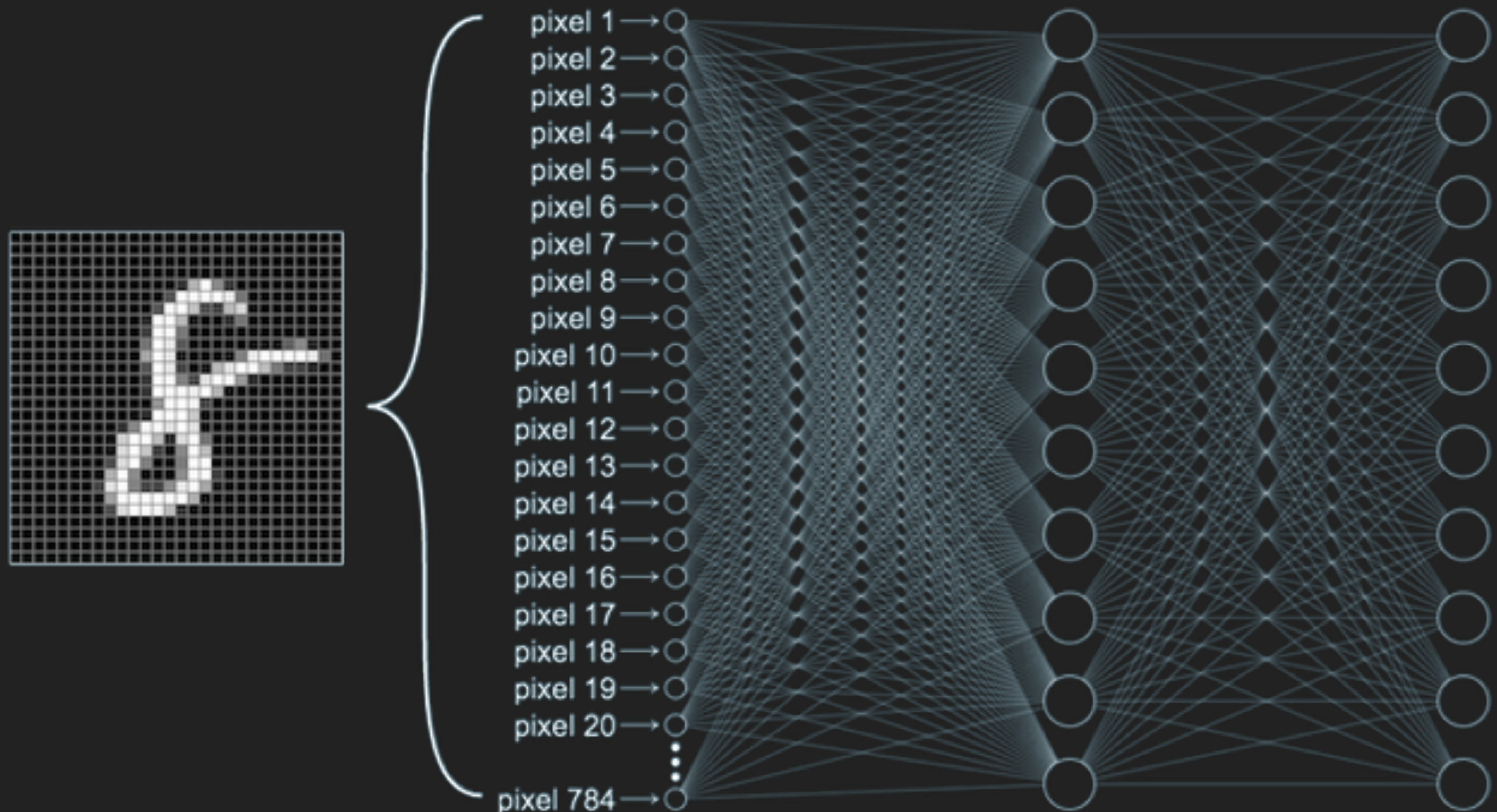
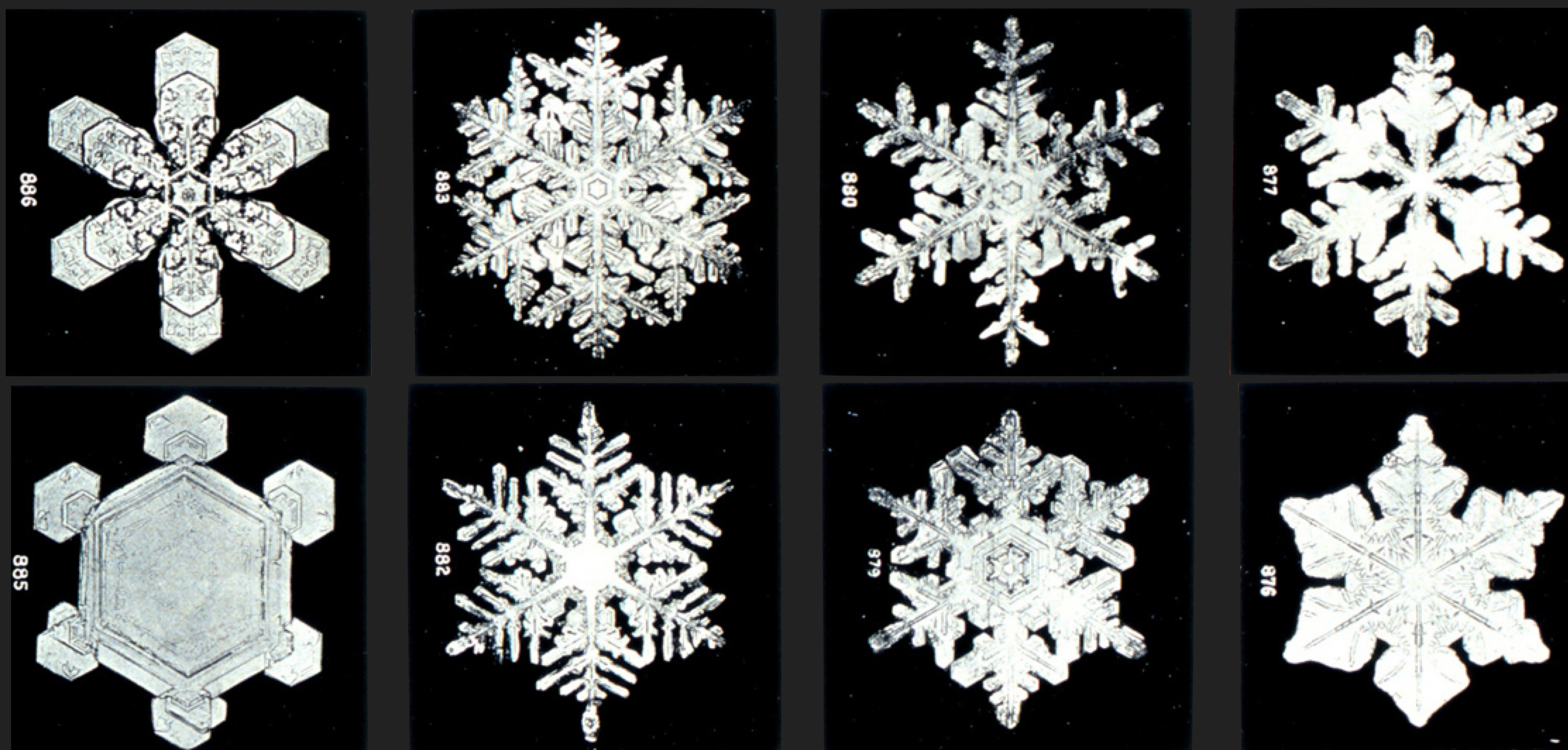


[ml4a.github.io]



[NOAA]

- Solution: CNN’s

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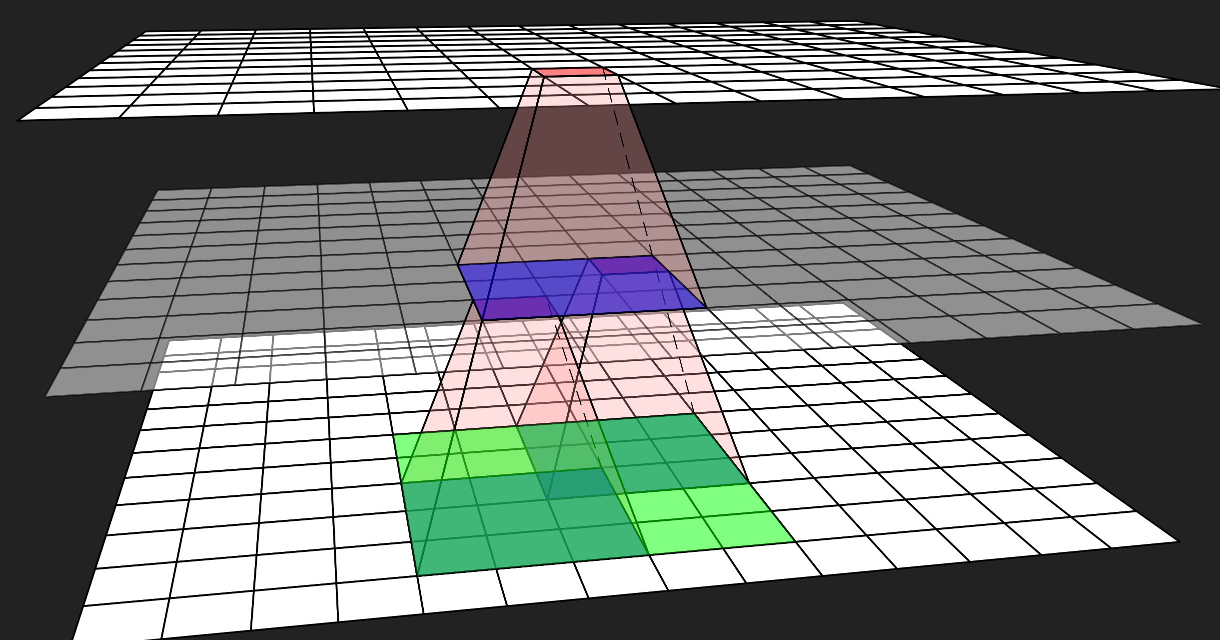

[Cohen & Welling: Group equivariant CNNs (ICML 2016)]

- Solution: randomly rotated training samples?

  preprocessing?

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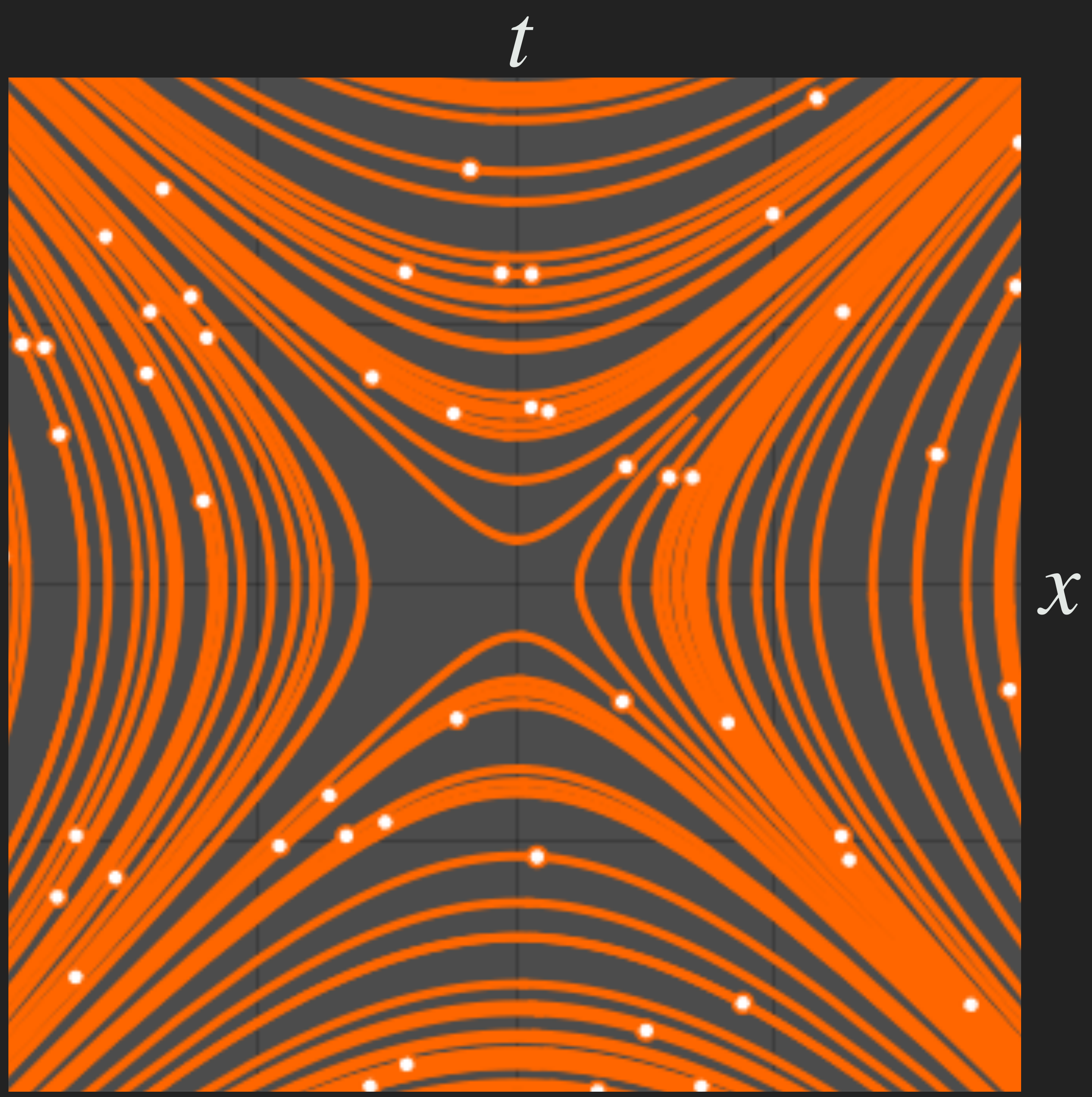
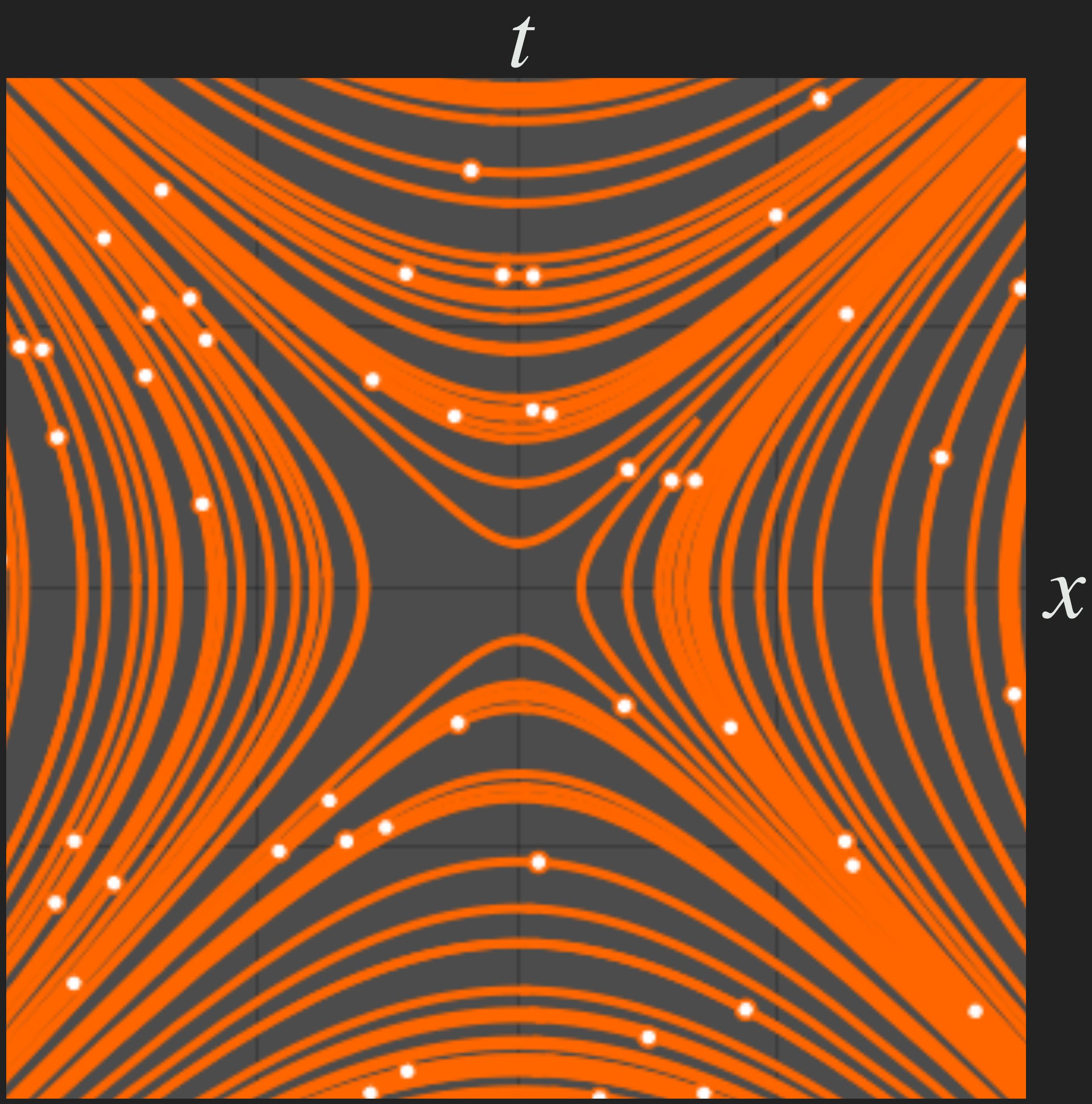
  ~~preprocessing?~~

- Solution: convolutions on Lie groups

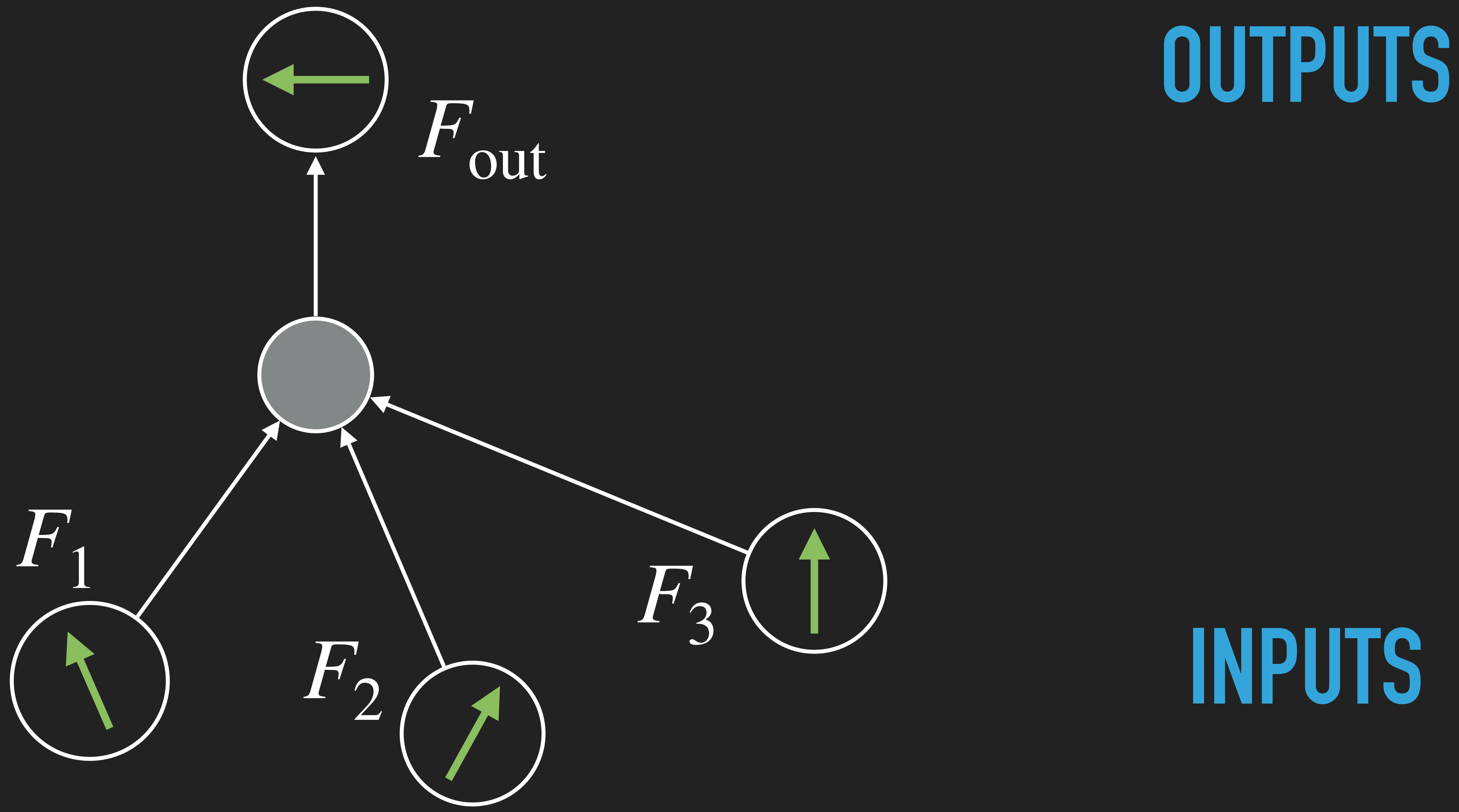
[Cohen & Welling: Group equivariant CNNs (ICML 2016)]
[Cohen & Welling: Steerable CNNs (ICLR 2017)]

- Greatly reduce the number of learnable parameters
- Reduce the number of training samples
- Improve generalization and regression ability
- Provide physically interpretable models
- Bring elegance and mathematical transparency
- Build on physical principles of symmetries and constraints


t
x


t
x

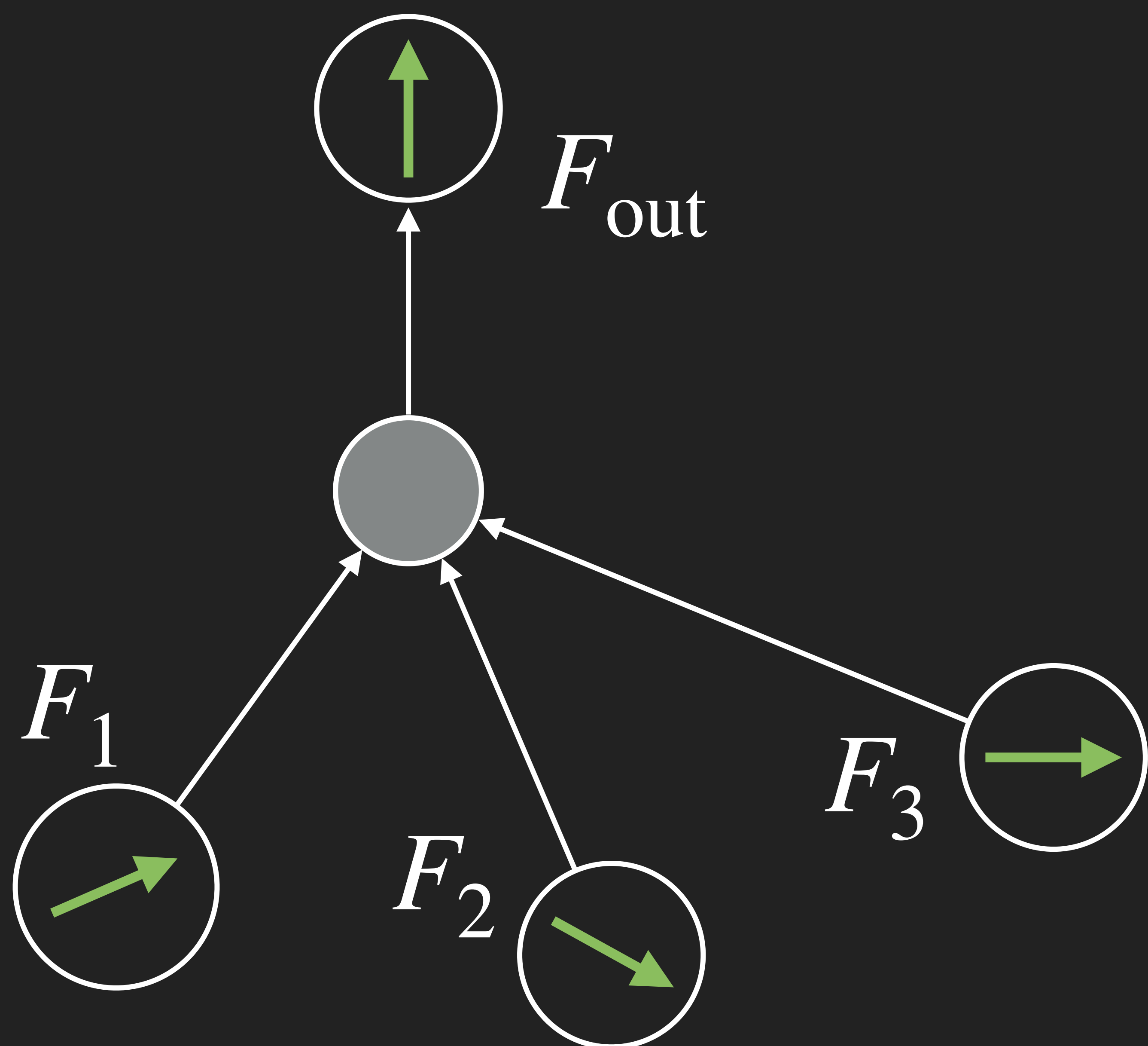
$$F_{\text{out}} \to \rho_{\text{out}}(g) F_{\text{out}}$$



$F_1 \to \rho_1(g) F_1$ $\quad$ $F_2 \to \rho_2(g) F_2$ $\quad$ $F_3 \to \rho_3(g) F_3$

$$F_{\text{out}} \to \rho_{\text{out}}(g)F_{\text{out}}$$

OUTPUTS



INPUTS

$$F_1 \to \rho_1(g)F_1 \qquad F_2 \to \rho_2(g)F_2 \qquad F_3 \to \rho_3(g)F_3$$

Any continuous function can be represented:

$$F(x_1, \ldots, x_n) = \sum_{k=0}^{2n \cdot d} F_k \left(\sum_{i=1}^{n} f_{ki}(x_i) \right)$$

[Kolmogorov-Arnold representation theorem]

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[Kolmogorov-Arnold representation theorem]

Symmetric functions:

$$F(x_1, \ldots, x_n) = \hat{F} \left(\sum_{i} f(x_i) \right)$$

[Zaheer et al. (Deep Sets, 2017)]

Group equivariant neural networks (Cohen & Welling, 2016)

Harmonic networks: deep translation and rotation equivariance (Worrall, Garbin, Turmukhanbetov & Brostow, 2016)

Steerable CNNs (Cohen & Welling, 2017)

On the generalization of convolution and equivariance (Kondor & Trivedi, 2018)

Intertwiners between induced representations (Cohen, Geiger & Weiler, 2018)

3D steerable neural networks (Weiler, Geiger, Wellig, Boomsma & Cohen, 2018)

Gauge equivariant neural networks (Cohen, Weiler, Kicanaoglu, Welling, 2019)

Tensor field networks (Thomas, Smidt, Kearns, Yang, Li Kohlhoff & Riley, 2018)

Relativistic Harmonic Networks (Chase Shimmin @ Boost 2019)

Cormorant: Covariant Molecular Neural Networks (Anderson, Hy & Kondor, 2019)

INGREDIENTS

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- Activations valued in a vector space

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- learnable linear operation

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- Activations valued in a vector space
- learnable linear operation
- composable nonlinear operation

INGREDIENTS

- Activations valued in vector representations of a group
- Equivariant learnable linear operation
- Equivariant composable nonlinear operation

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  ("commutes" with the action of the group)
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  (maps representations to representations)

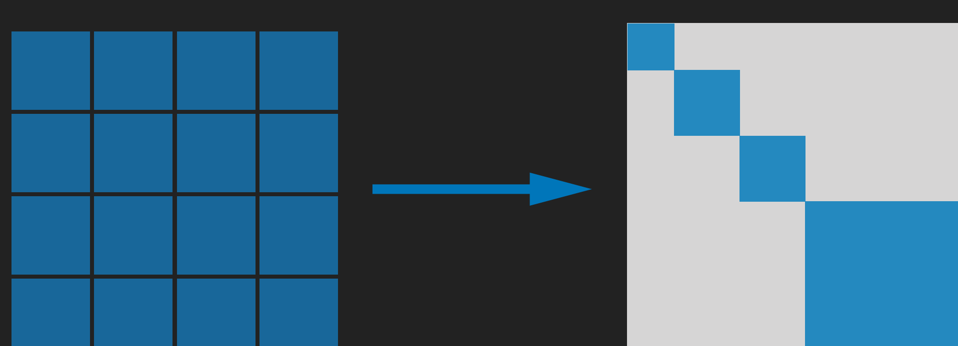
I. MIXING IRREDUCIBLES

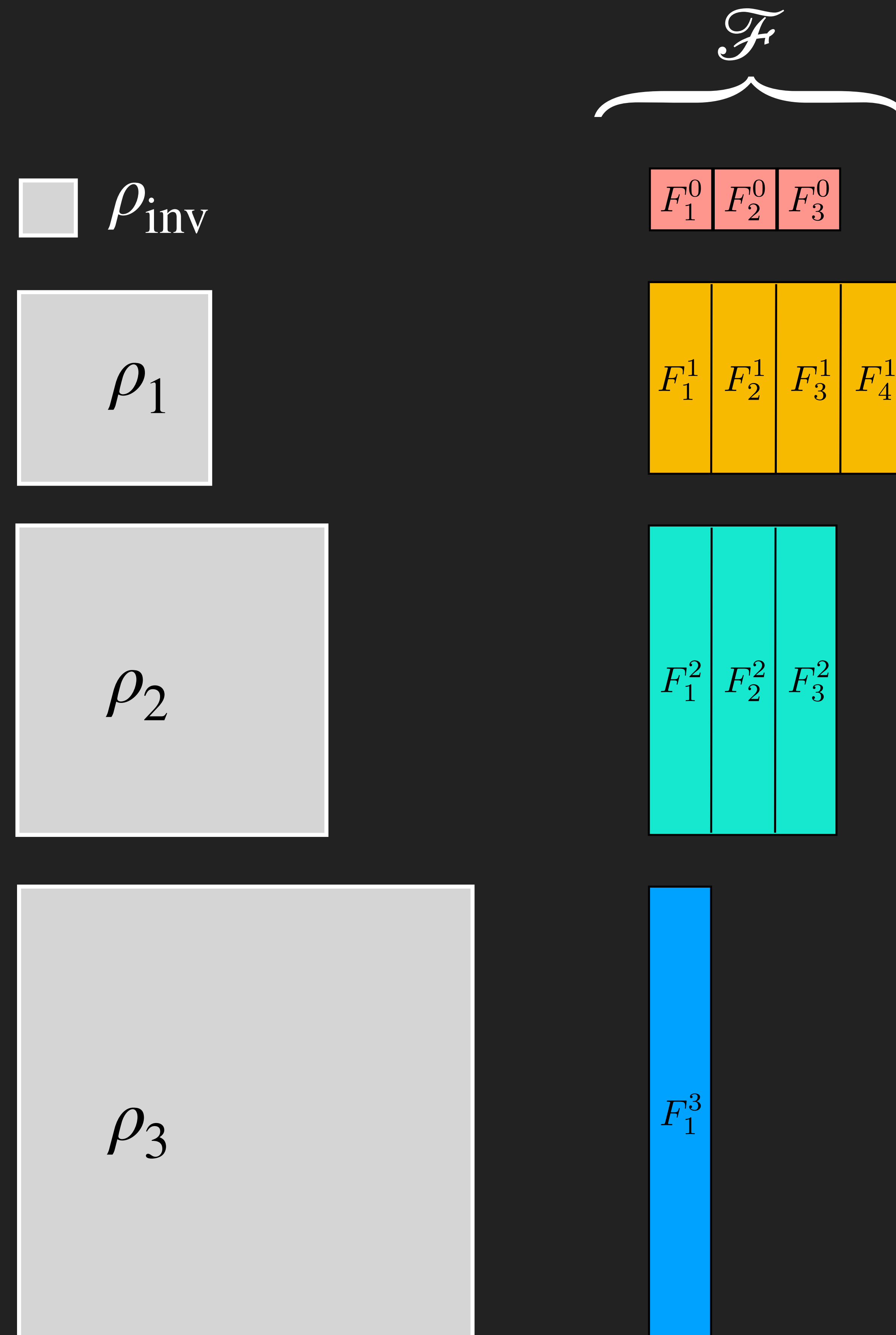
Finite-dimensional representations of the Lorentz group are **decomposable**, i.e. are direct sums of irreps.

$$W : R \to R', \quad W \cdot \rho(g) = \rho'(g) \cdot W$$

Schur's lemma implies that W acts as scalar multiplication on each irrep, and only linearly combines vectors of the same weight.

Activations need to be stored as collections of irreducible components.





II. CLEBSCH-GORDAN PRODUCT

The "only" bilinear equivariant operation mapping two representations to another one is the **tensor product**

$$\otimes : R_1 \times R_2 \to R_3$$

$$\rho_1(g) \otimes \rho_2(g) = C_{\rho_1,\rho_2} \left[\bigoplus_{\rho} \bigoplus_{1}^{\mu(\rho)} \rho(g) \right] C^{\dagger}_{\rho_1,\rho_2}$$

Since our linear operation requires knowledge of irreducible components, each product must be followed by a **Clebsch-Gordan decomposition**.

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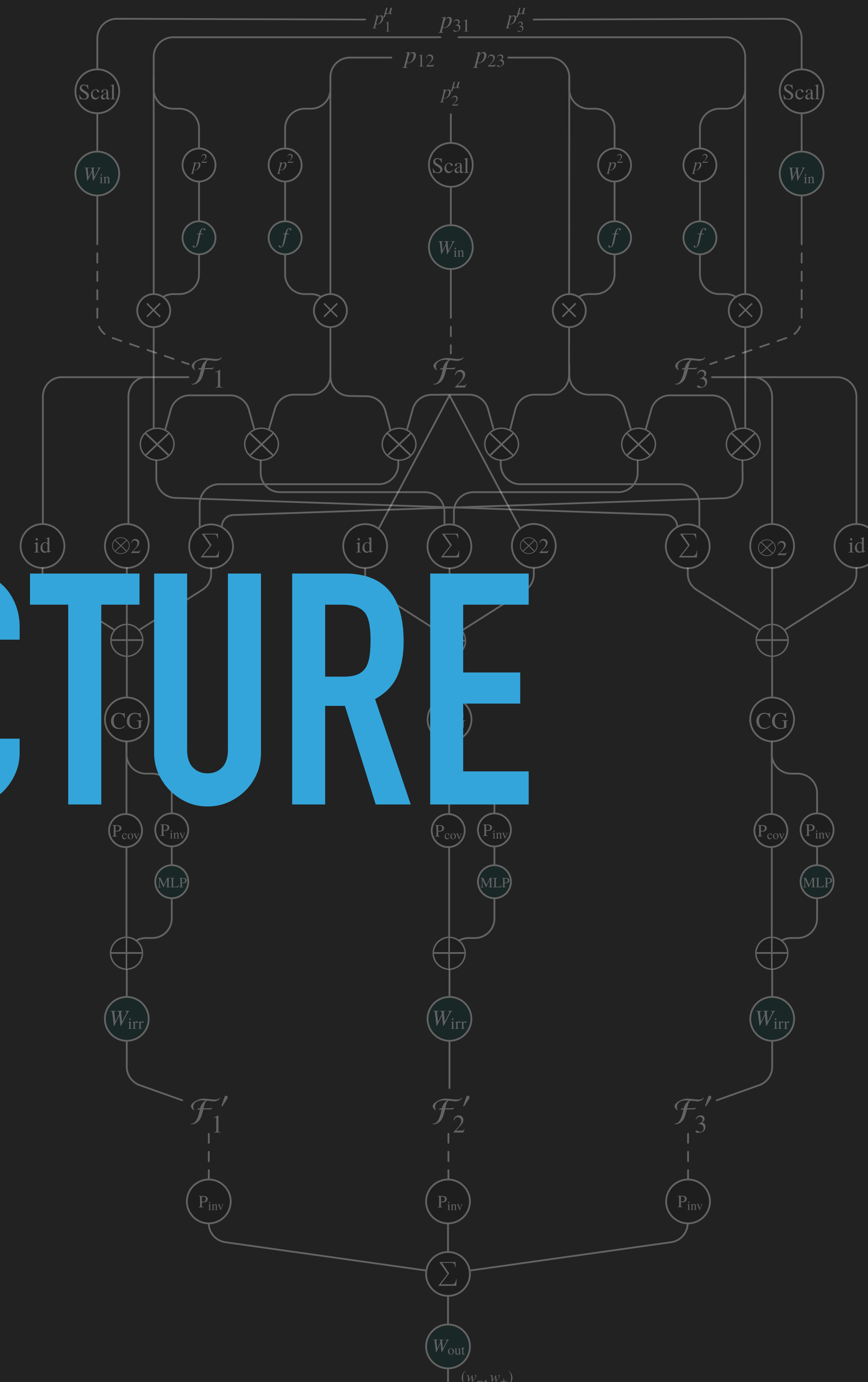
ARCHITECTURE



- N particles with input 4-momenta p_i^μ
- N activations $\mathscr{F}_i$ at each level live in representations of the Lorentz group
- The update rule involves pair interactions

$$\mathscr{F}_i \mapsto W \cdot \left(\mathscr{F}_i \oplus \mathscr{F}_i^{\otimes 2} \oplus \sum_j f\left(p_{ij}^2\right) \cdot p_{ij} \otimes \mathscr{F}_j \right) \quad *$$

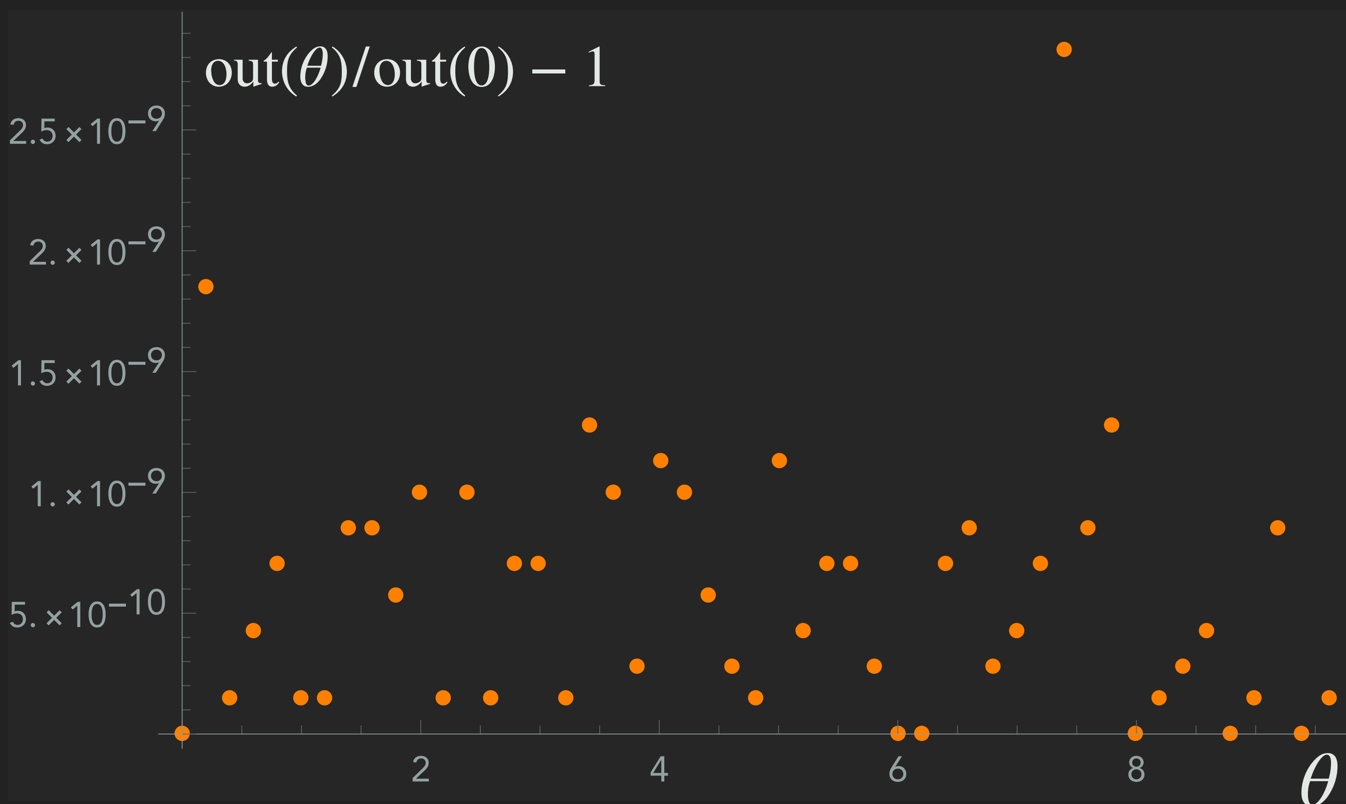
- Arbitrary traditional sub-networks can be applied to Lorentz invariants
- Output layer sums over i and projects onto invariants (or other irrep)

$$\mathscr{M}^2 = \frac{1}{4} \frac{(2g_c)^4}{((p_1 - p_3)^2 - m_\gamma c^2)^2} \left[p_1 \cdot p_3 + m_e c^2\right] \left[p_2 \cdot p_4 + m_\mu c^2\right]$$

IRC Safety to be addressed separately

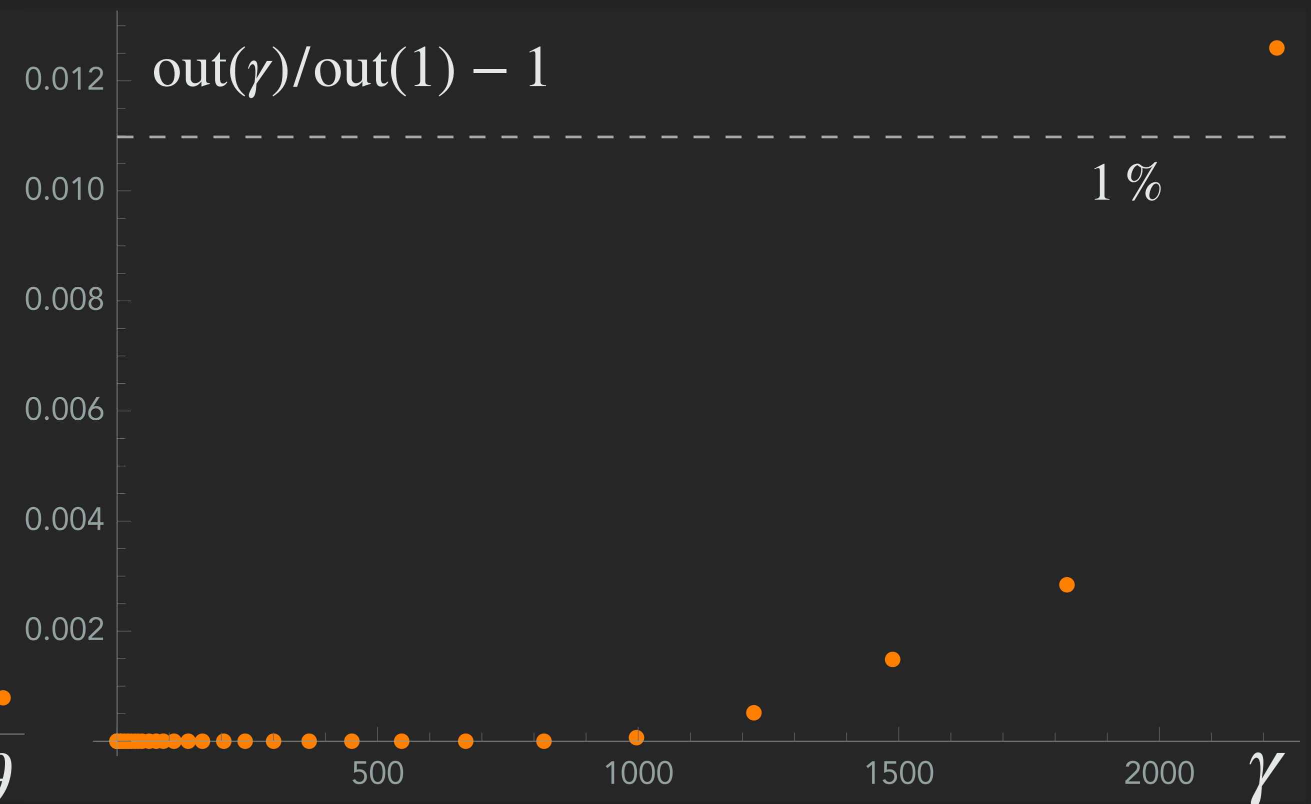
PERFORMANCE

Rotational invariance

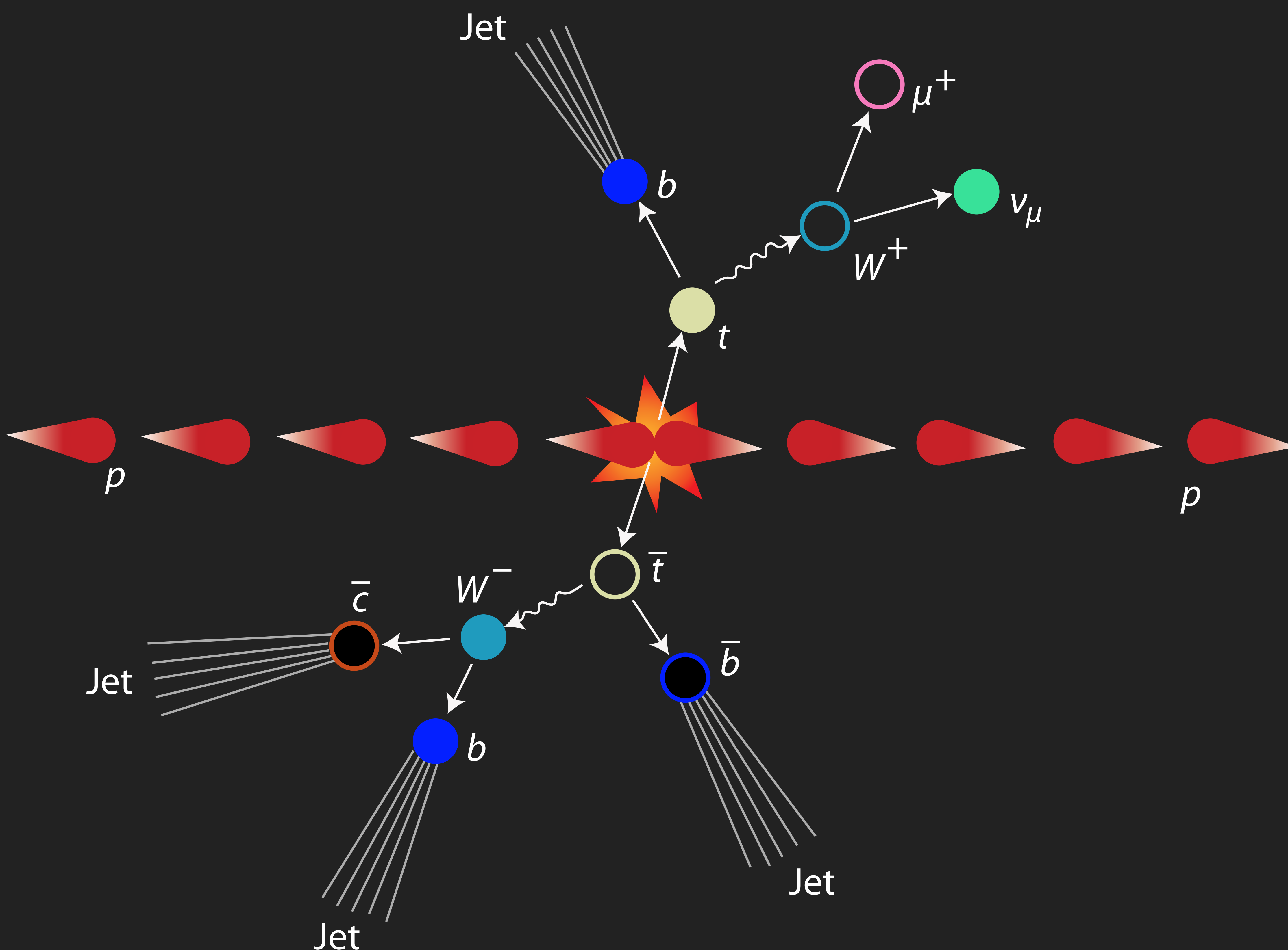


Relative invariance within 10^{-9} using double precision

Boost invariance

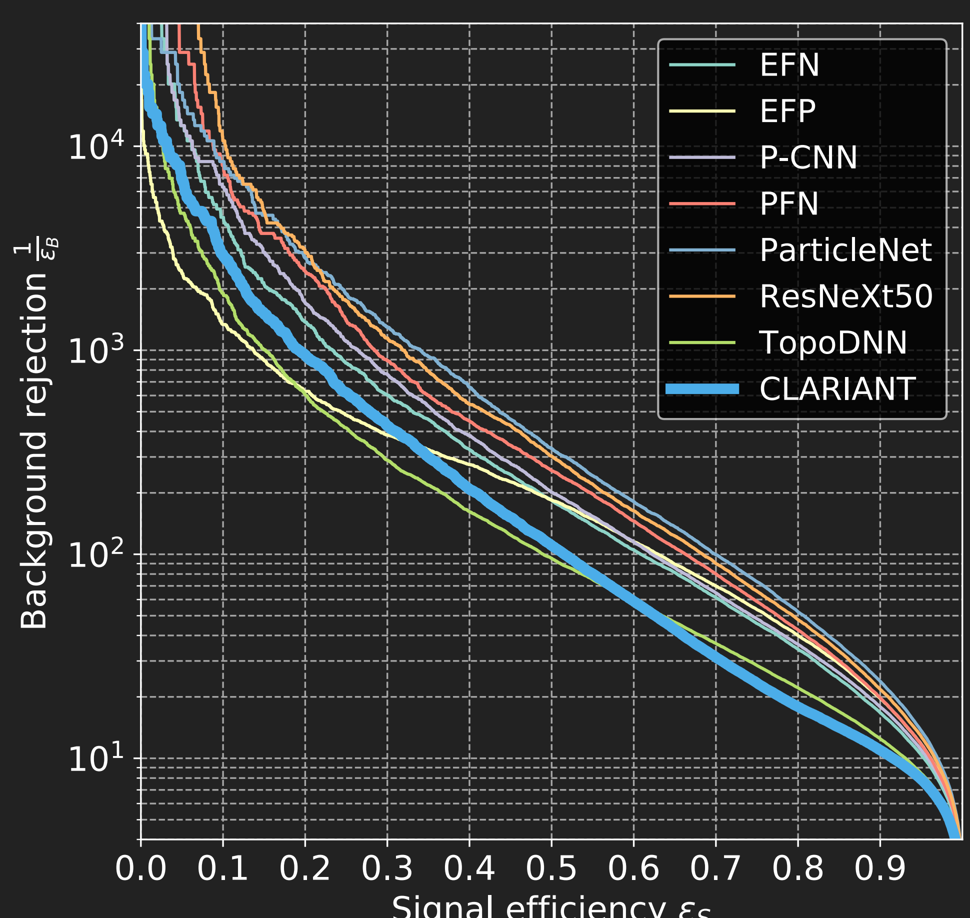


Relative invariance within 10^{-2} limited by floating point precision



Dataset: [Butter, Kasieczka, Russell & Russell (*Deep-learned Top Tagging with a Lorentz Layer*) arXiv: 1707.08966]

	AUC	Acc	$1/\epsilon_B$ ($\epsilon_S = 0.3$) single	mean	median	#Param
CNN	0.981	0.930	914±14	995±15	975±18	610k
ResNeXt	0.984	0.936	1122±47	1270±28	1286±31	1.46M
TopoDNN	0.972	0.916	295±5	382± 5	378 ± 8	59k
Multi-body N-subjettiness 6	0.979	0.922	792±18	798±12	808±13	57k
Multi-body N-subjettiness 8	0.981	0.929	867±15	918±20	926±18	58k
TreeNiN	0.982	0.933	1025±11	1202±23	1188±24	34k
P-CNN	0.980	0.930	732±24	845±13	834±14	348k
ParticleNet	0.985	0.938	1298±46	1412±45	1393±41	498k
LBN	0.981	0.931	836±17	859±67	966±20	705k
LoLa	0.980	0.929	722±17	768±11	765±11	127k
LDA	0.955	0.892	151±0.4	151.5±0.5	151.7±0.4	184k
Energy Flow Polynomials	0.980	0.932	384			1k
Energy Flow Network	0.979	0.927	633±31	729±13	726±11	82k
Particle Flow Network	0.982	0.932	891±18	1063±21	1052±29	82k
CLARIANT	0.970	0.922	426			3k



[Kasieczka et. al. (ML Landscape of top taggers, 2019)]

[Xia et. al. (ResNeXt, 2016)]
[Moore at. al. (Multi-body N-subjettiness, 2018)]
[Qu & Gouskos (ParticleNet, 2019)]
[Thaler et. al. (Energy/Particle Flow Polynomials, 2018/19)]
[Butter et. al. (LoLa – Lorentz Layer, 2018)]

FUTURE WORK AND POTENTIAL EXTENSIONS

- Complete hyperparameter optimization;
- Include particle information (label, charge, spin, …);
- Regression tasks and measurements:
  - invariant mass detection,
  - covariant 4-momentum measurements;
- Detection of hidden symmetries;
- Multiple symmetries combined (Standard Model?);

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Computer Science Department

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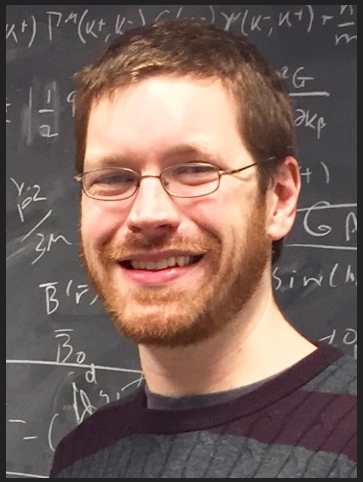
Center for Data and Computing
AT THE UNIVERSITY OF CHICAGO

KADANOFF CENTER
FOR THEORETICAL PHYSICS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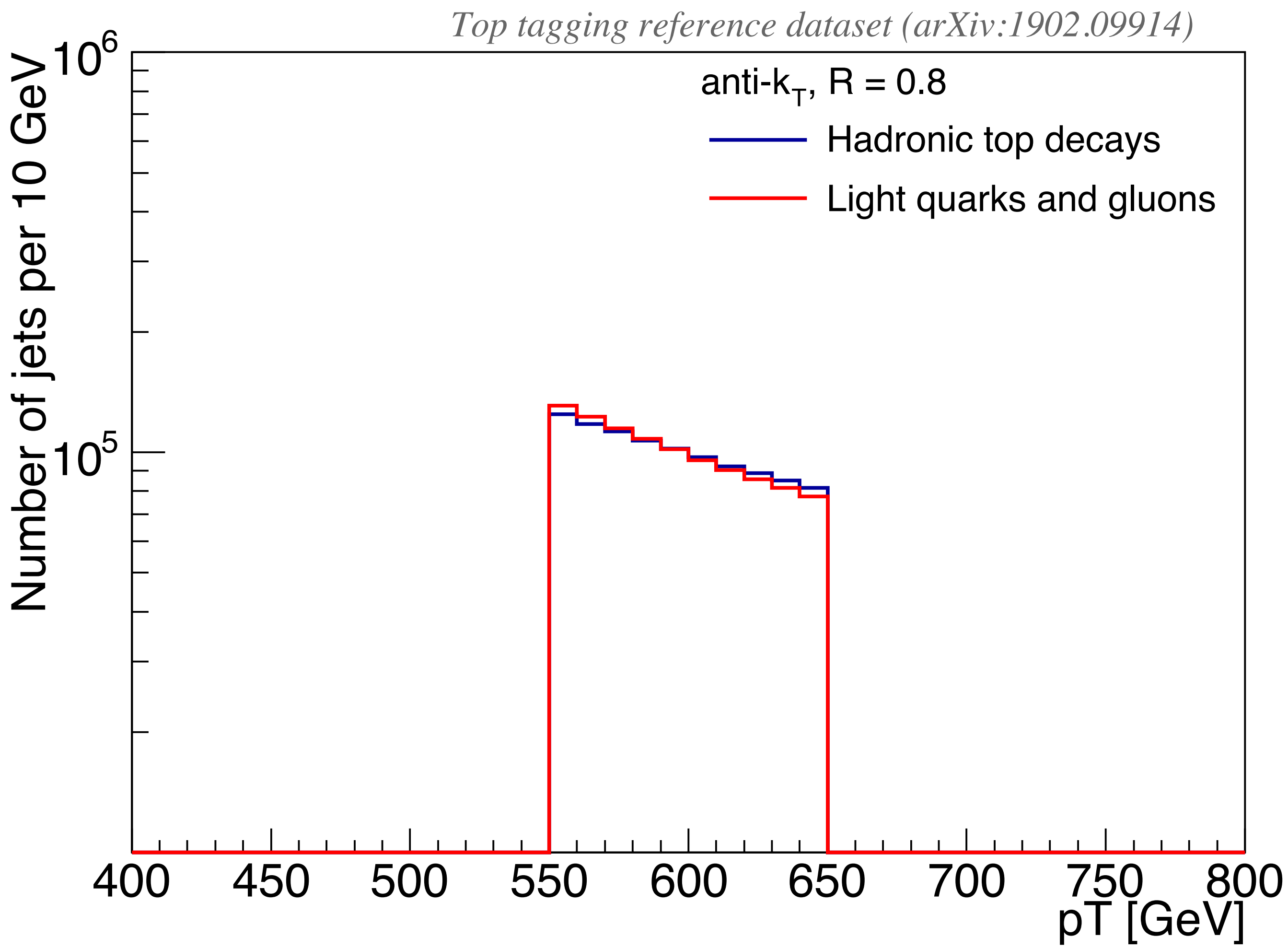
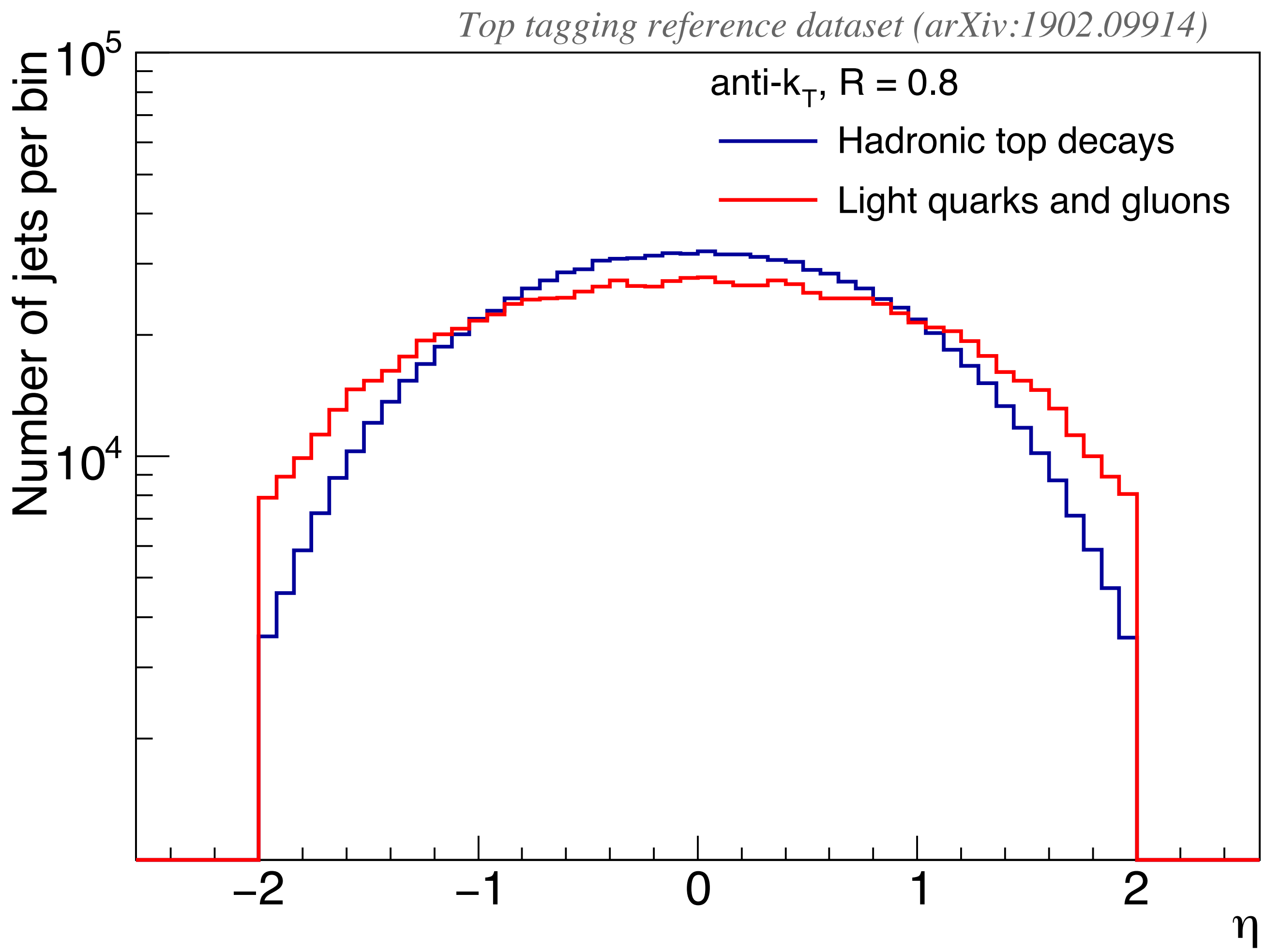
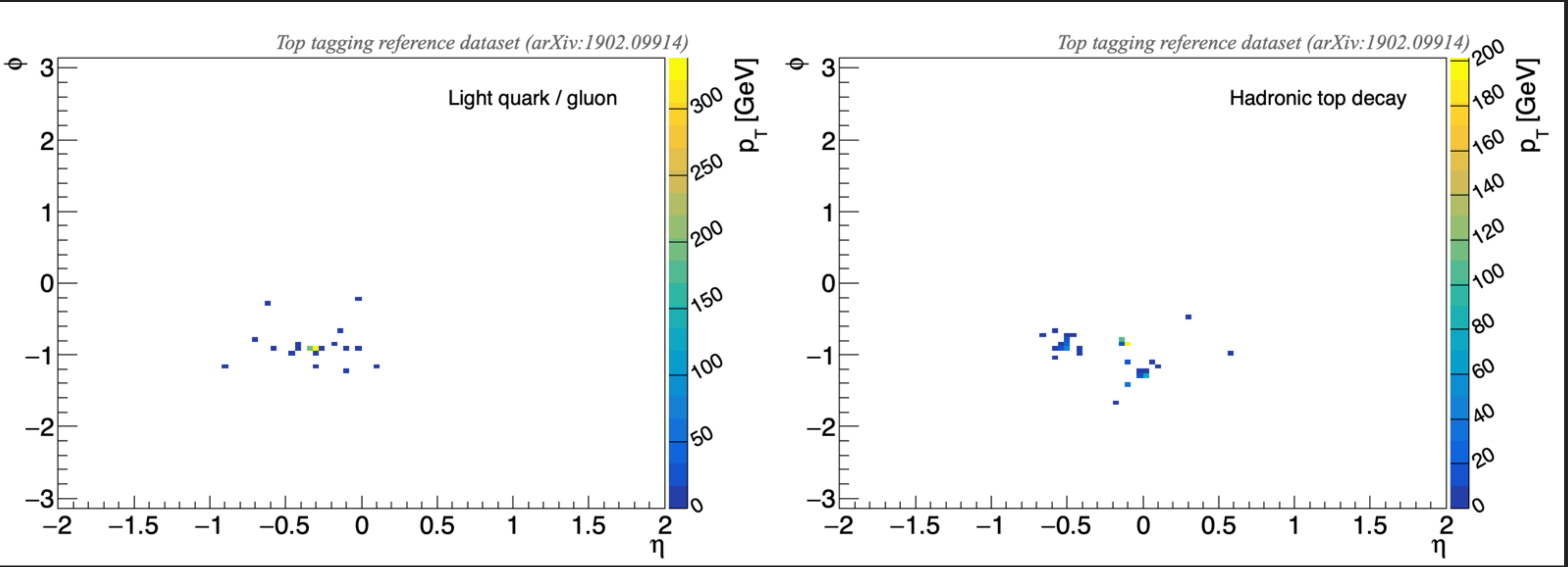
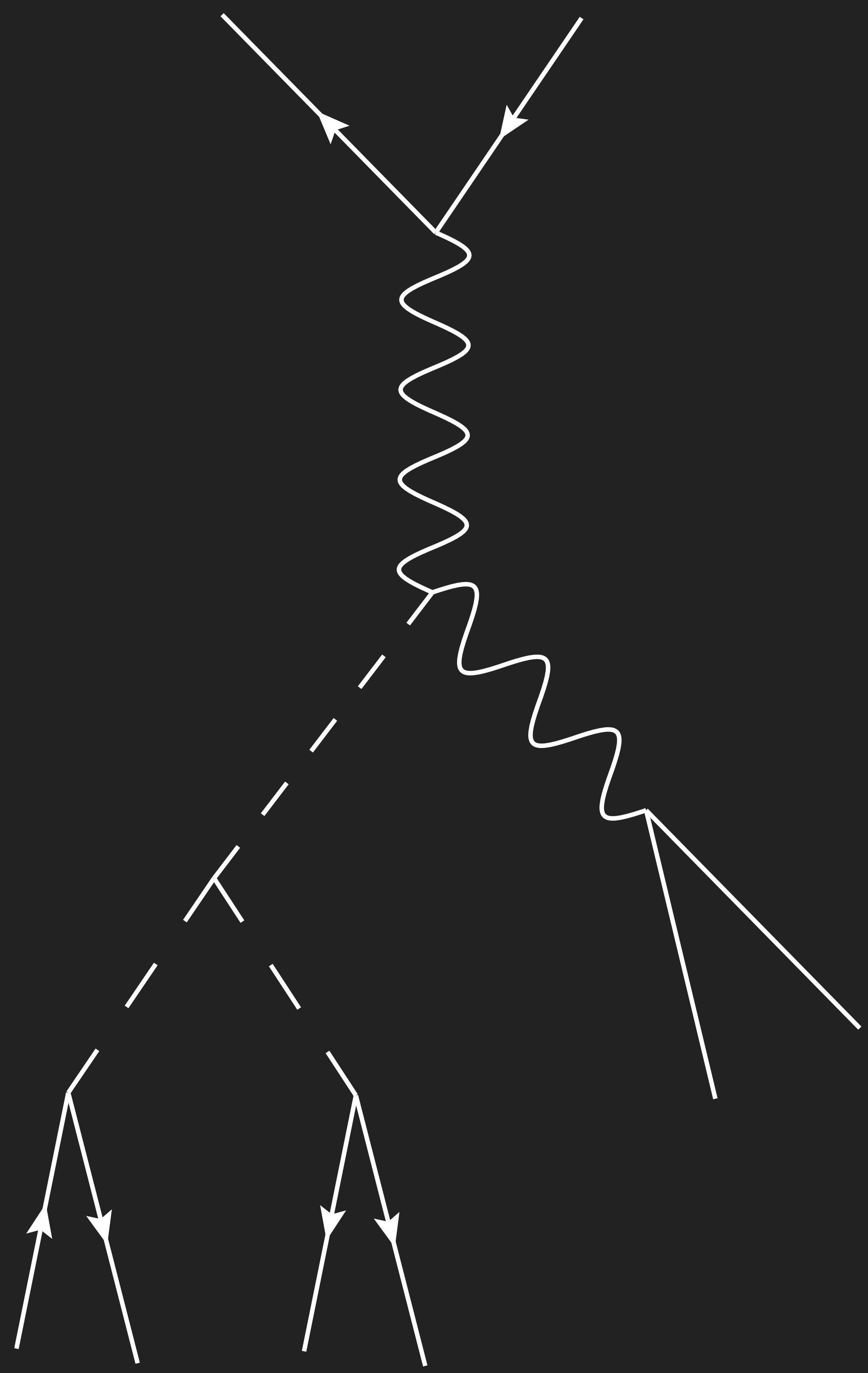
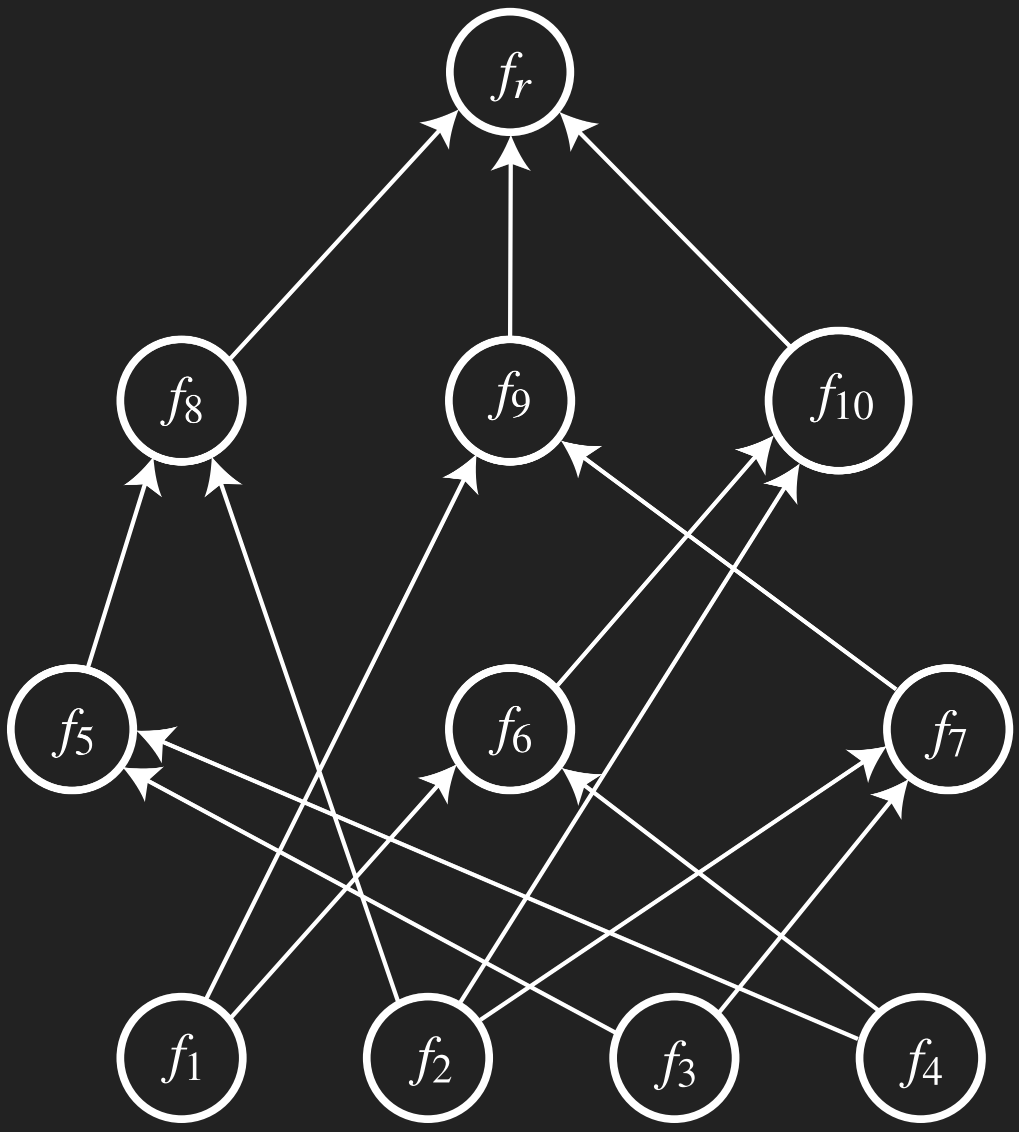
DATASET ENERGY DISTRIBUTIONS






Top tagging reference dataset (arXiv:1902.09914)
Light quark / gluon
φ
η
p_T [GeV]
Top tagging reference dataset (arXiv:1902.09914)
Hadronic top decay
φ
η
p_T [GeV]




f_r
f_8
f_9
f_{10}
f_5
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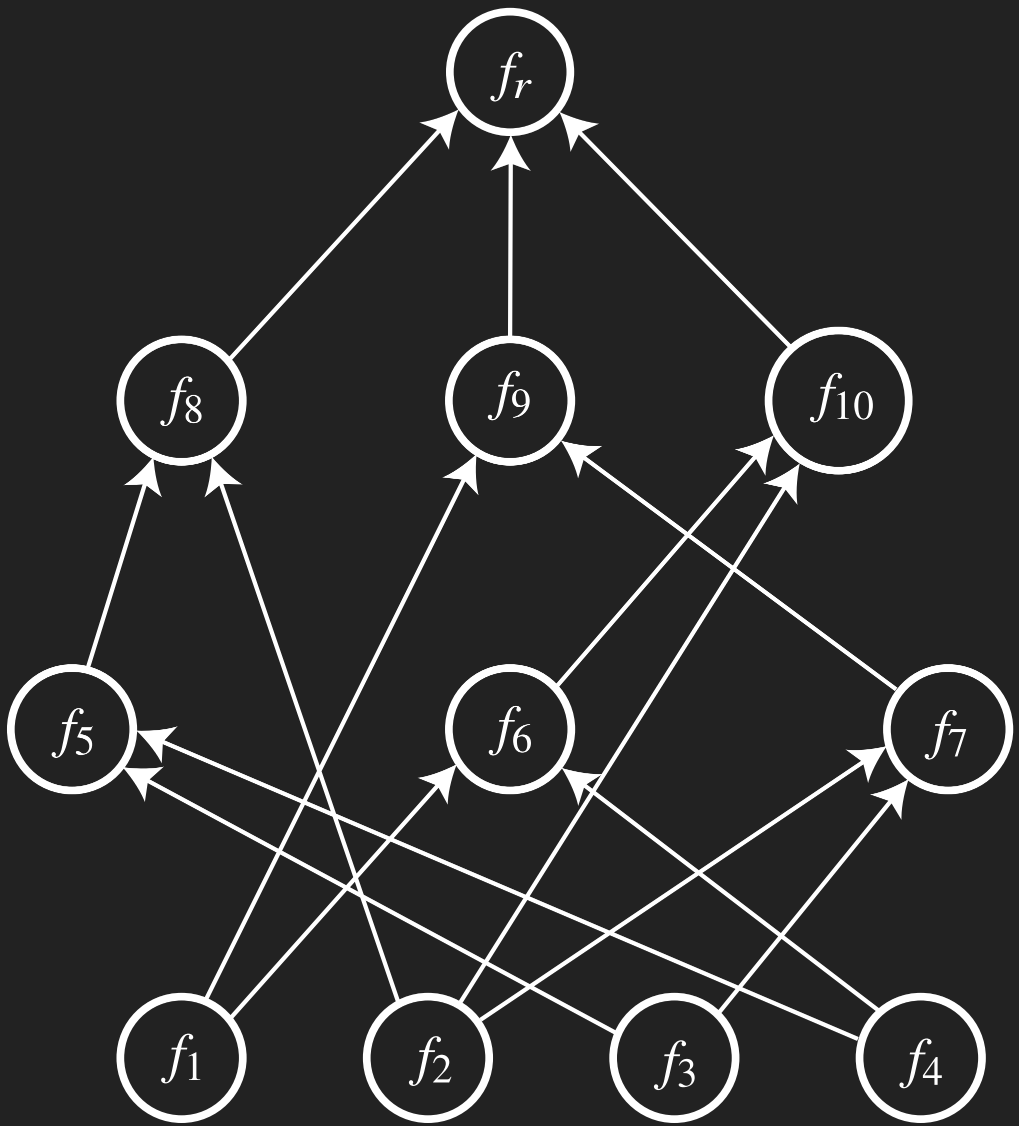
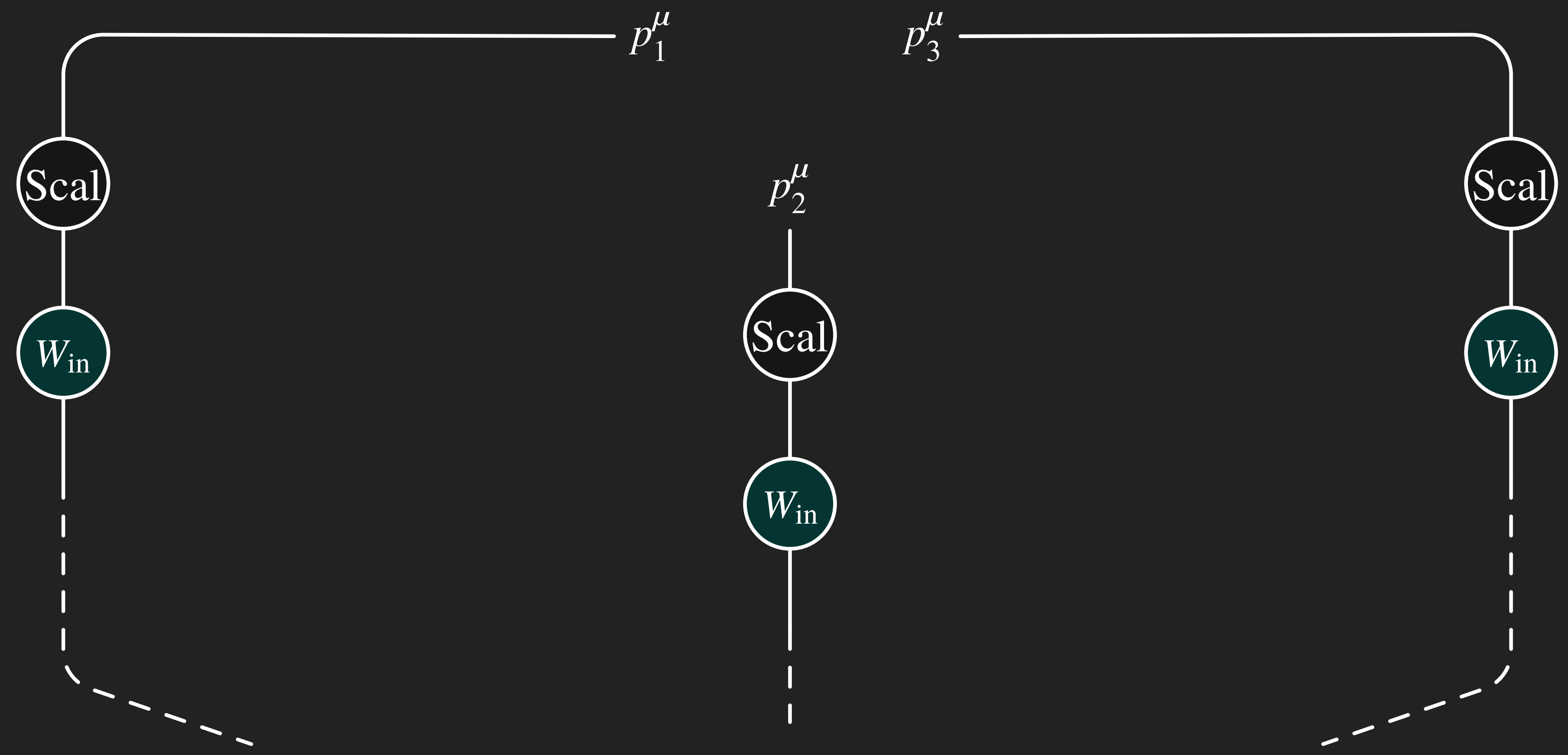
f_1
f_2
f_3
f_4

ATLAS



INPUT LAYER



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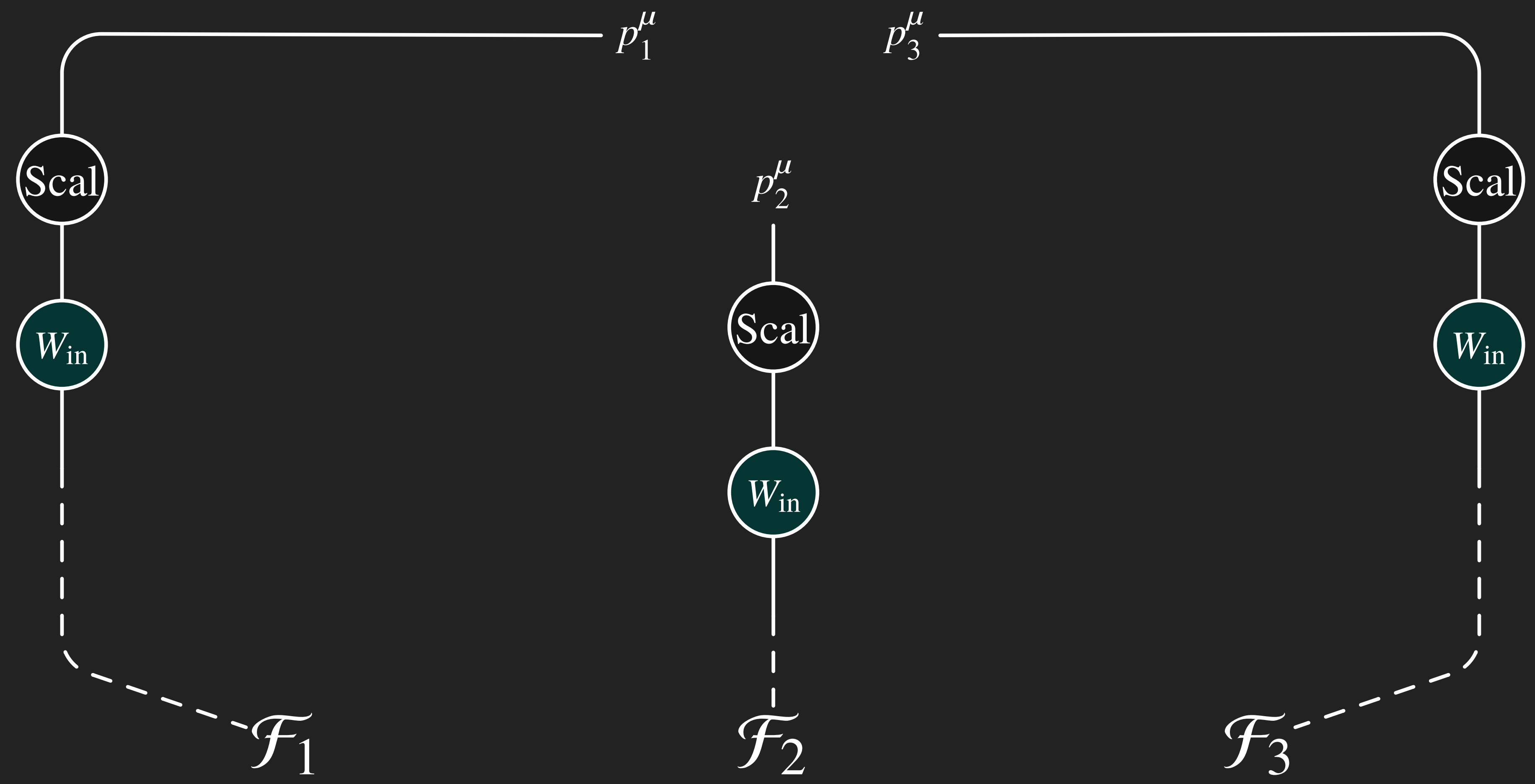
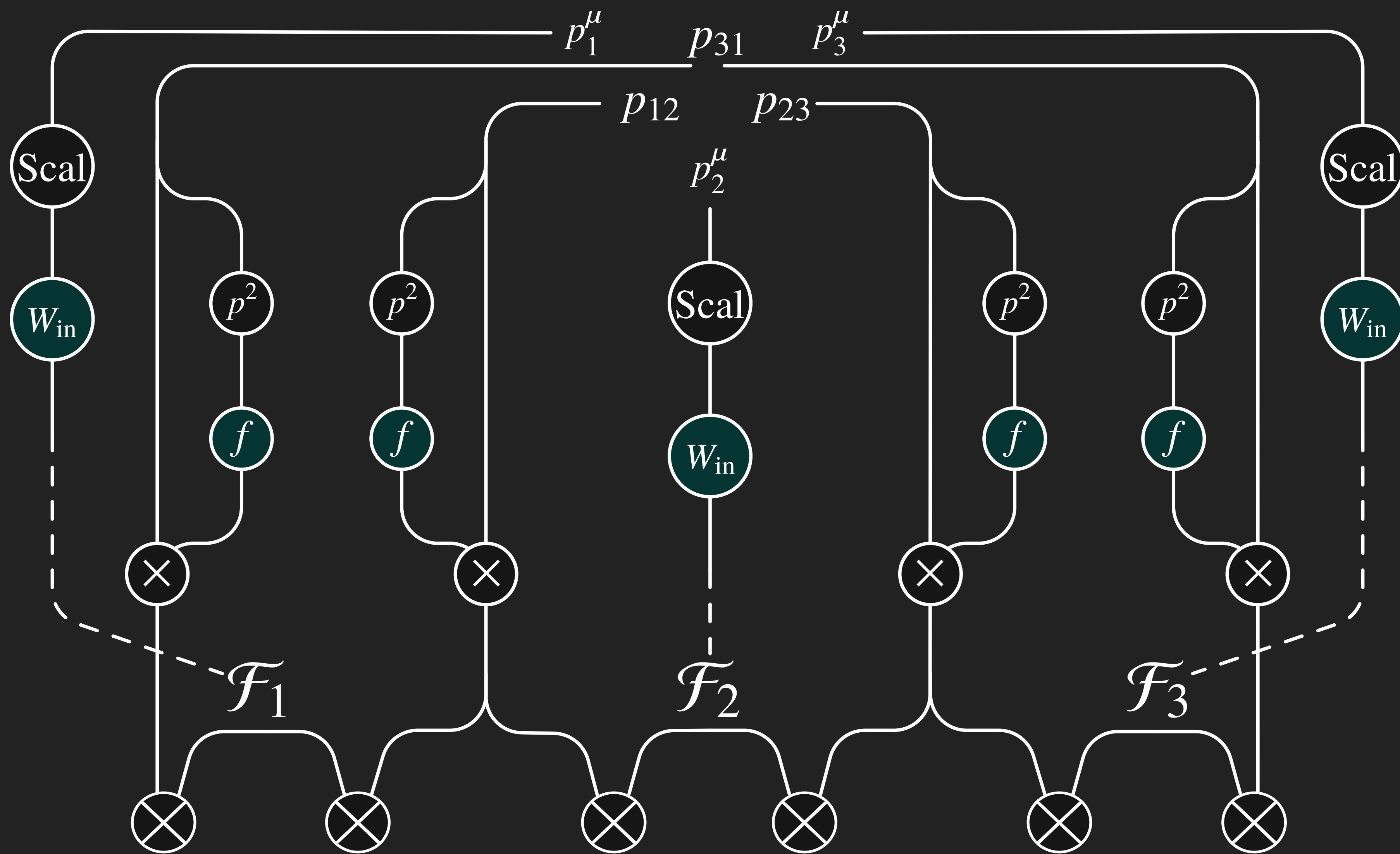


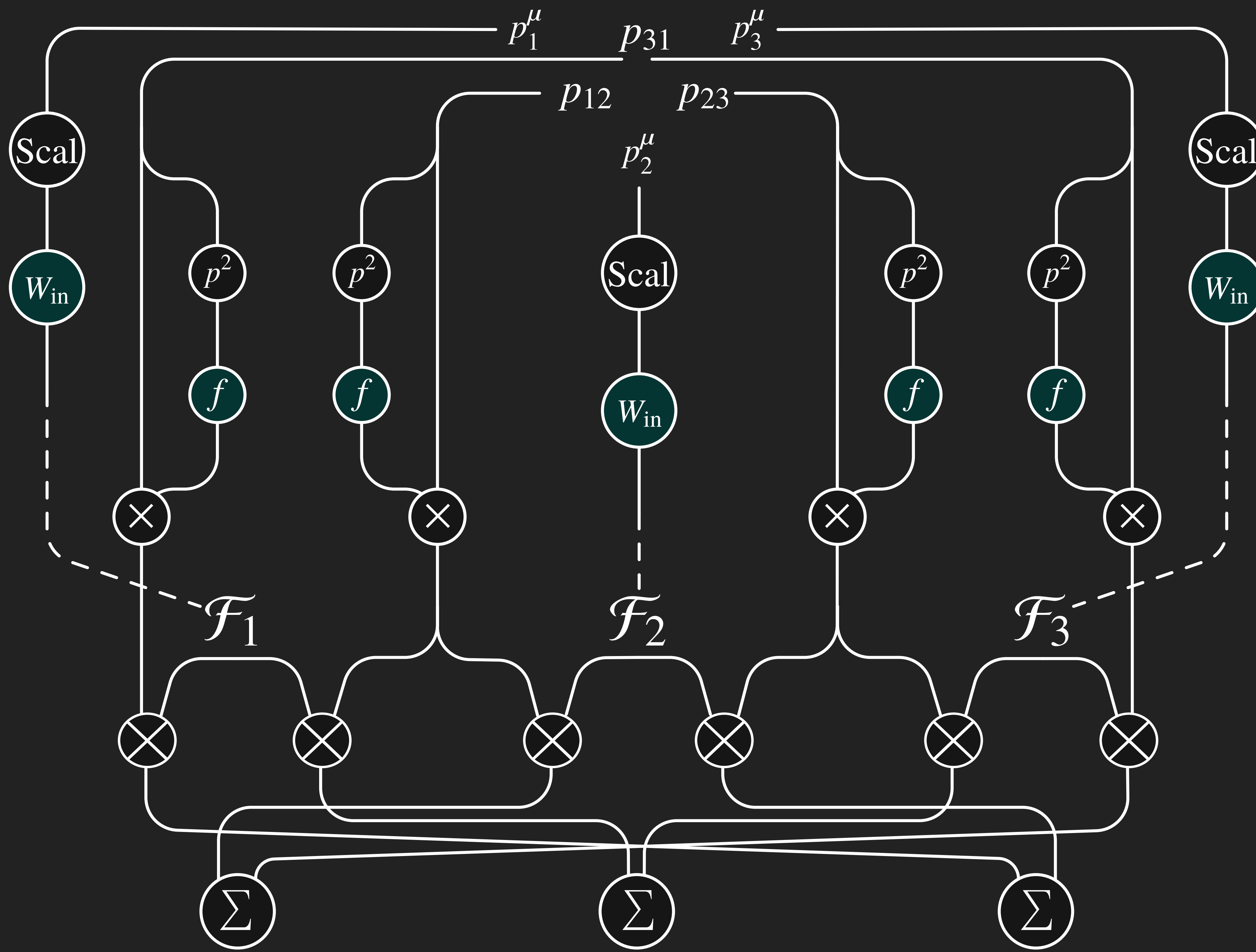
p_1^μ
p_3^μ
p_2^μ
Scal
Scal
Scal
W_{in}
W_{in}
W_{in}
$\mathcal{F}_1$
$\mathcal{F}_2$
$\mathcal{F}_3$

INPUT LAYER



ITERATED CG LAYER

INPUT LAYER



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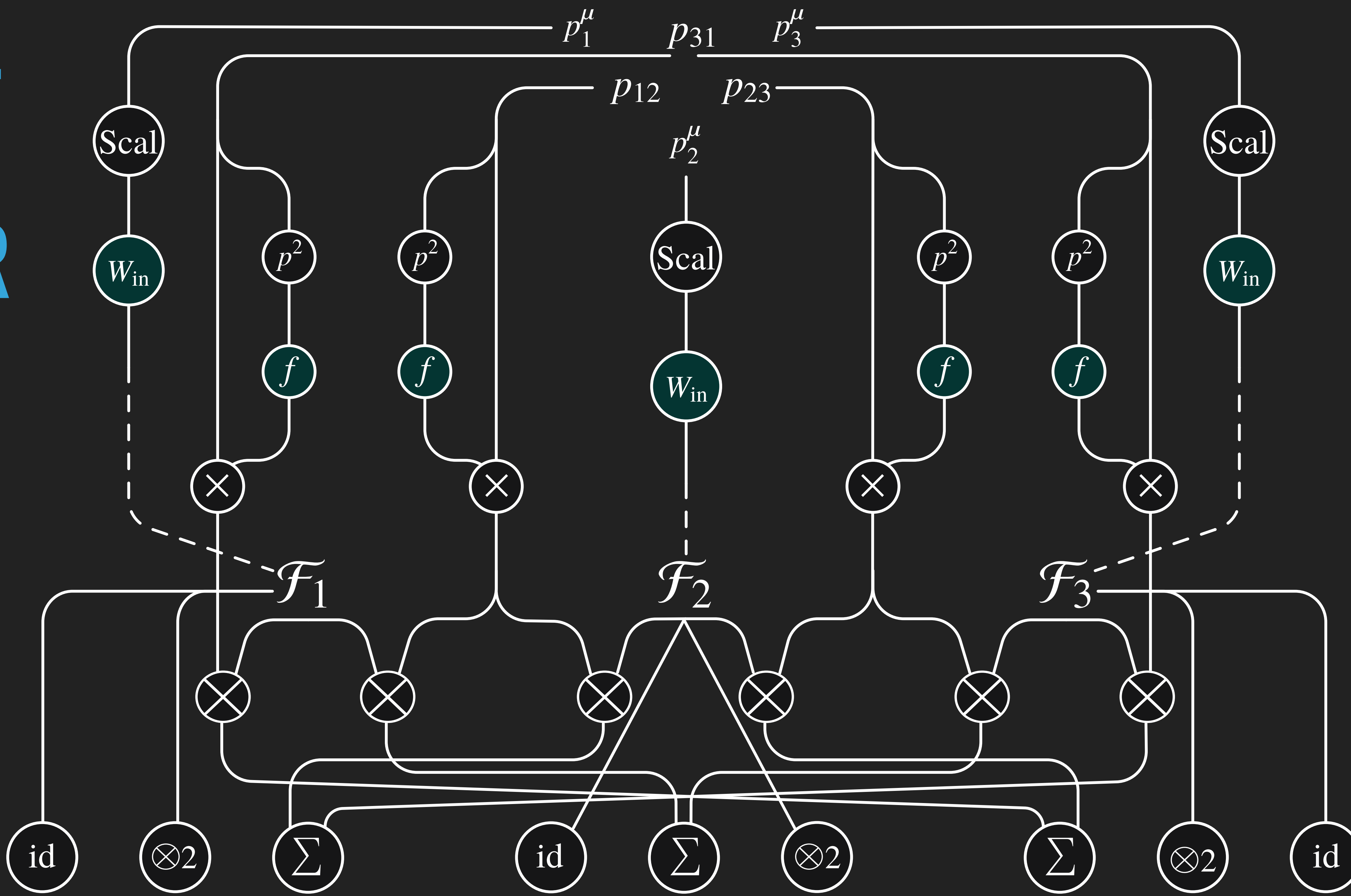

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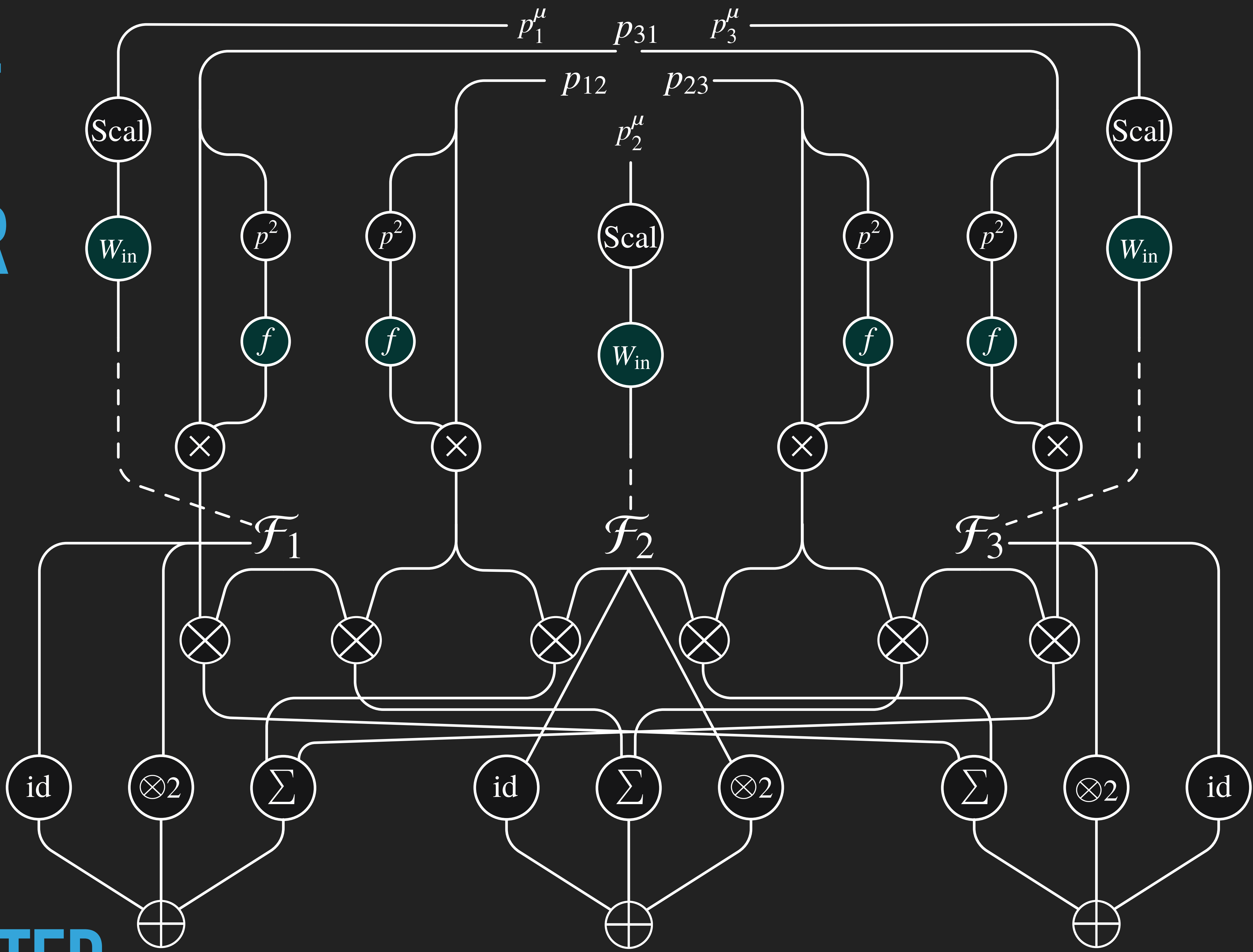

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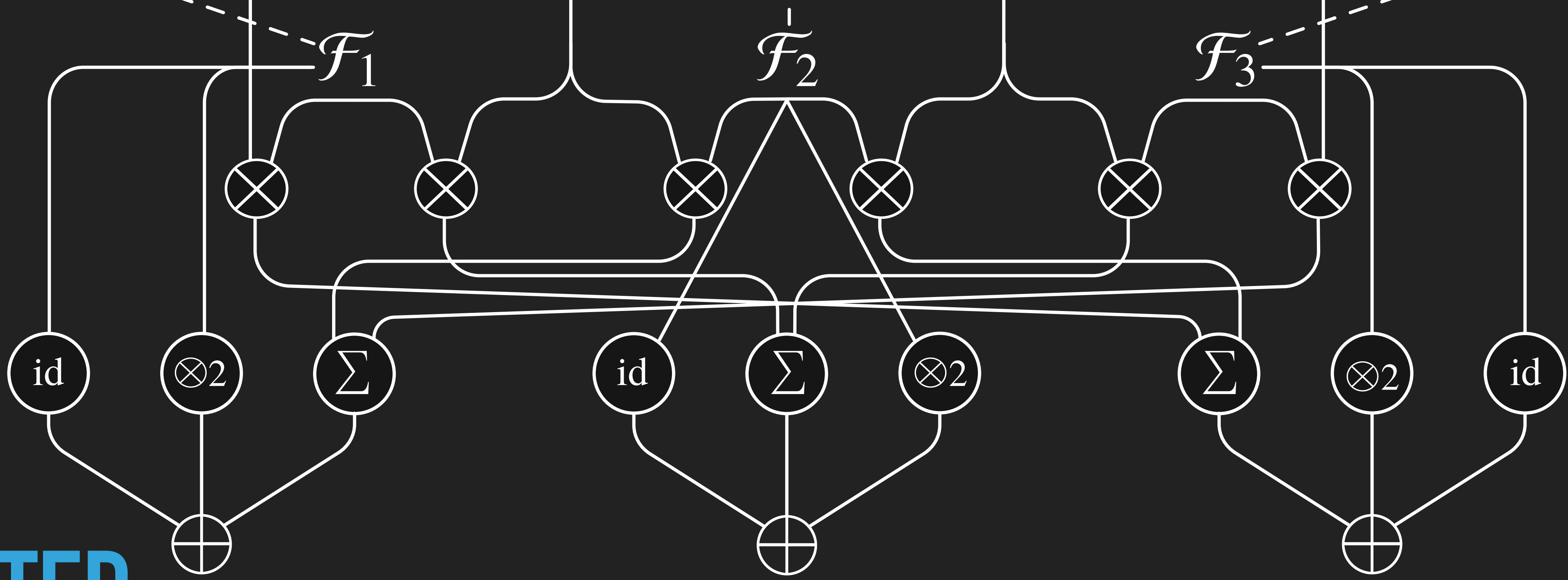

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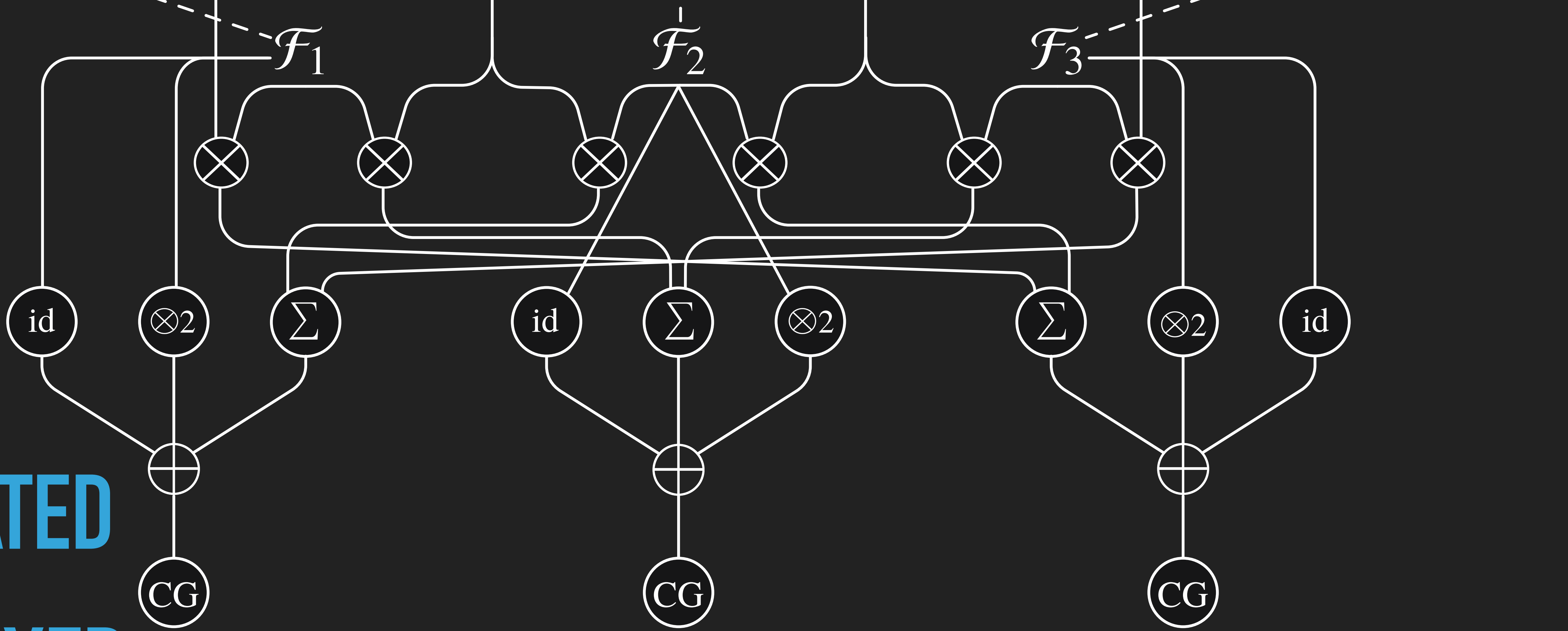

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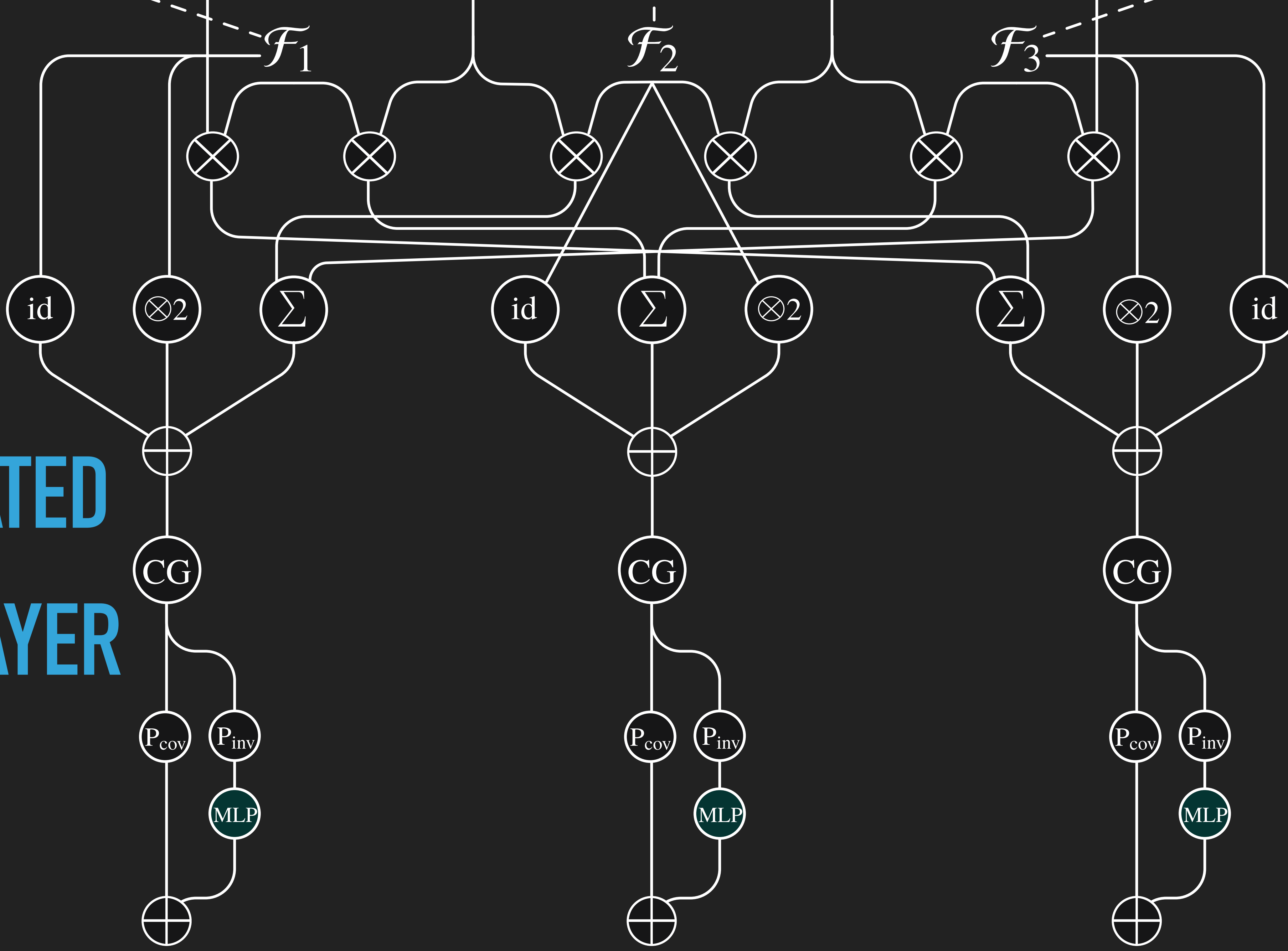

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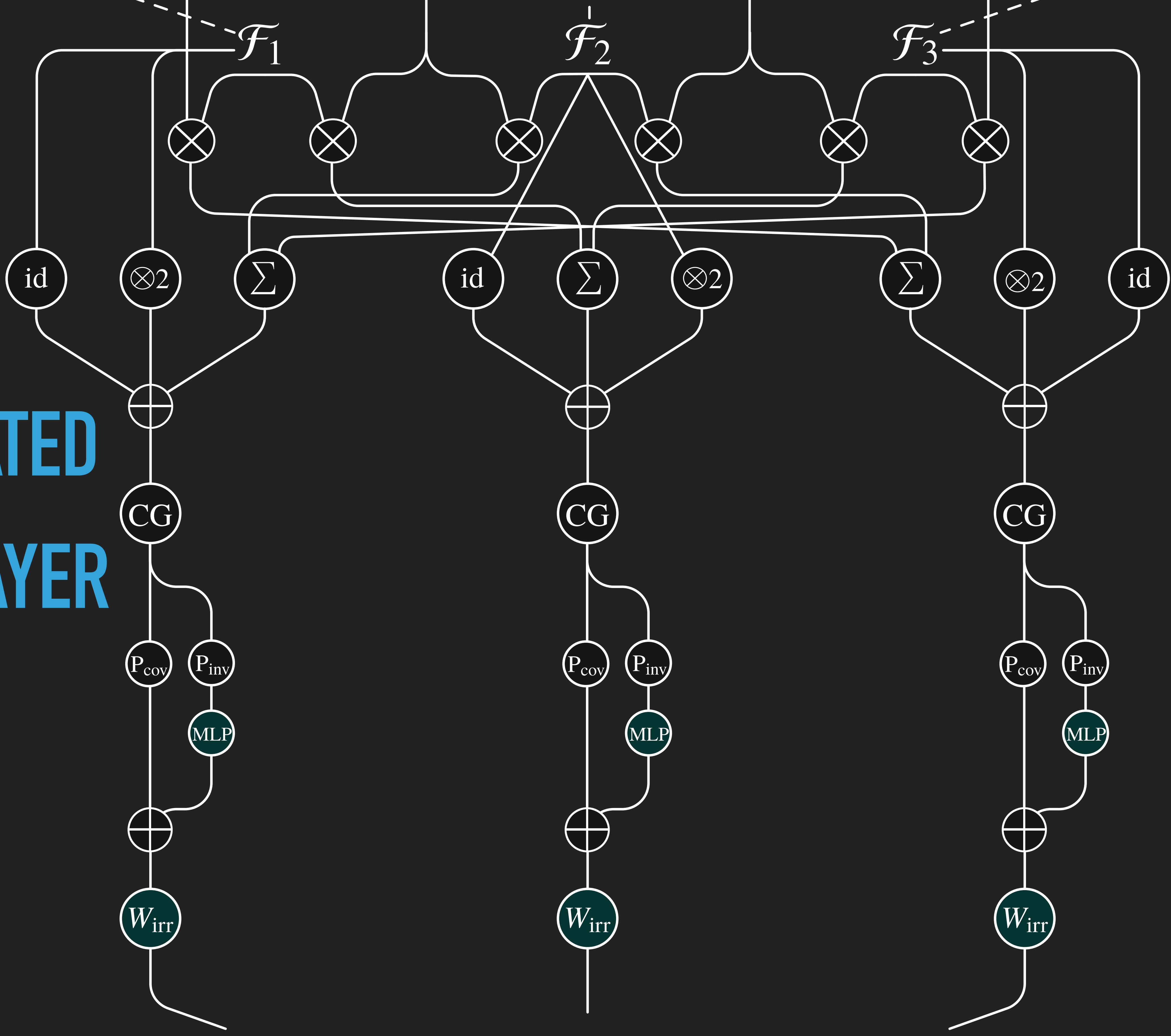

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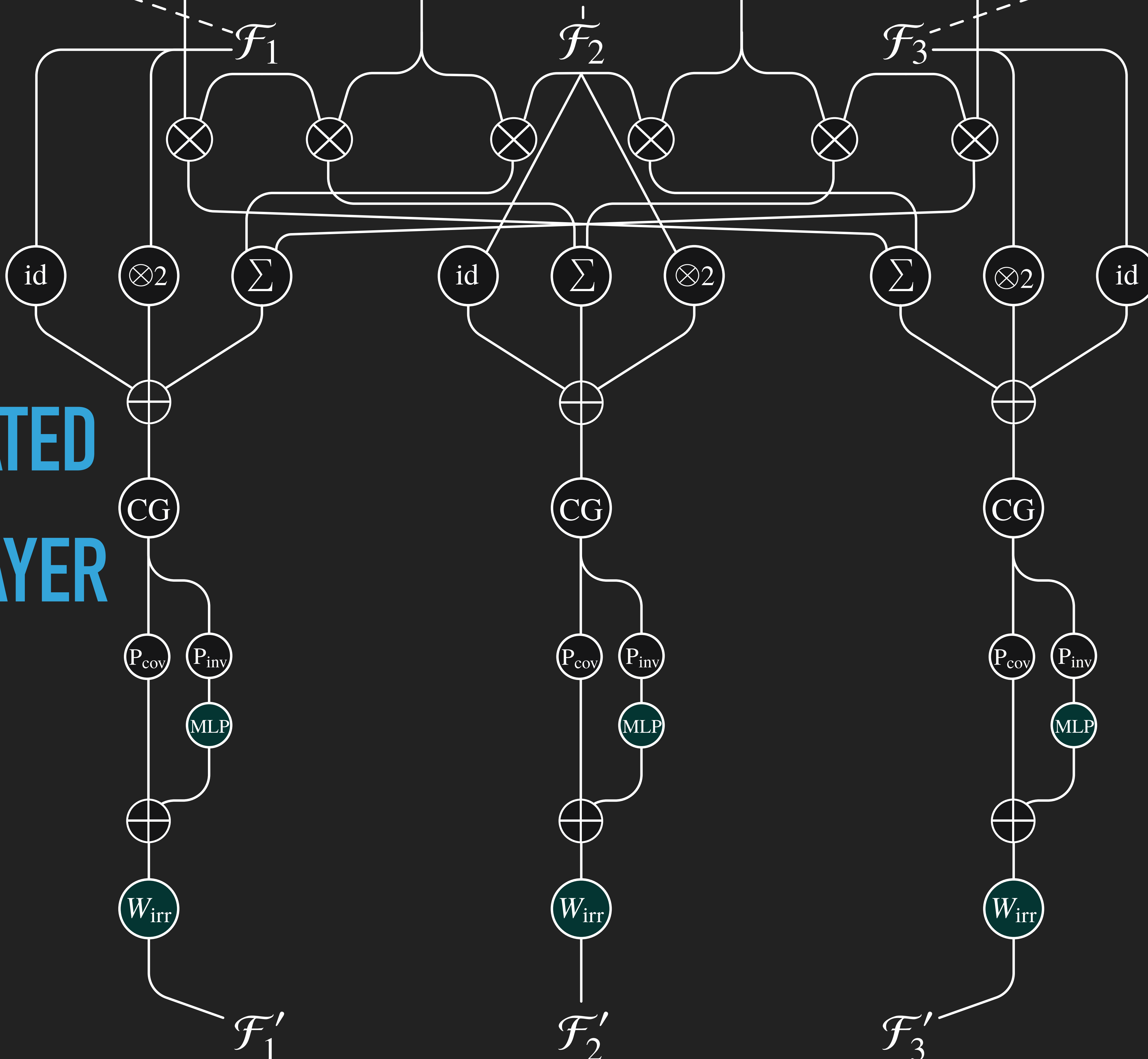

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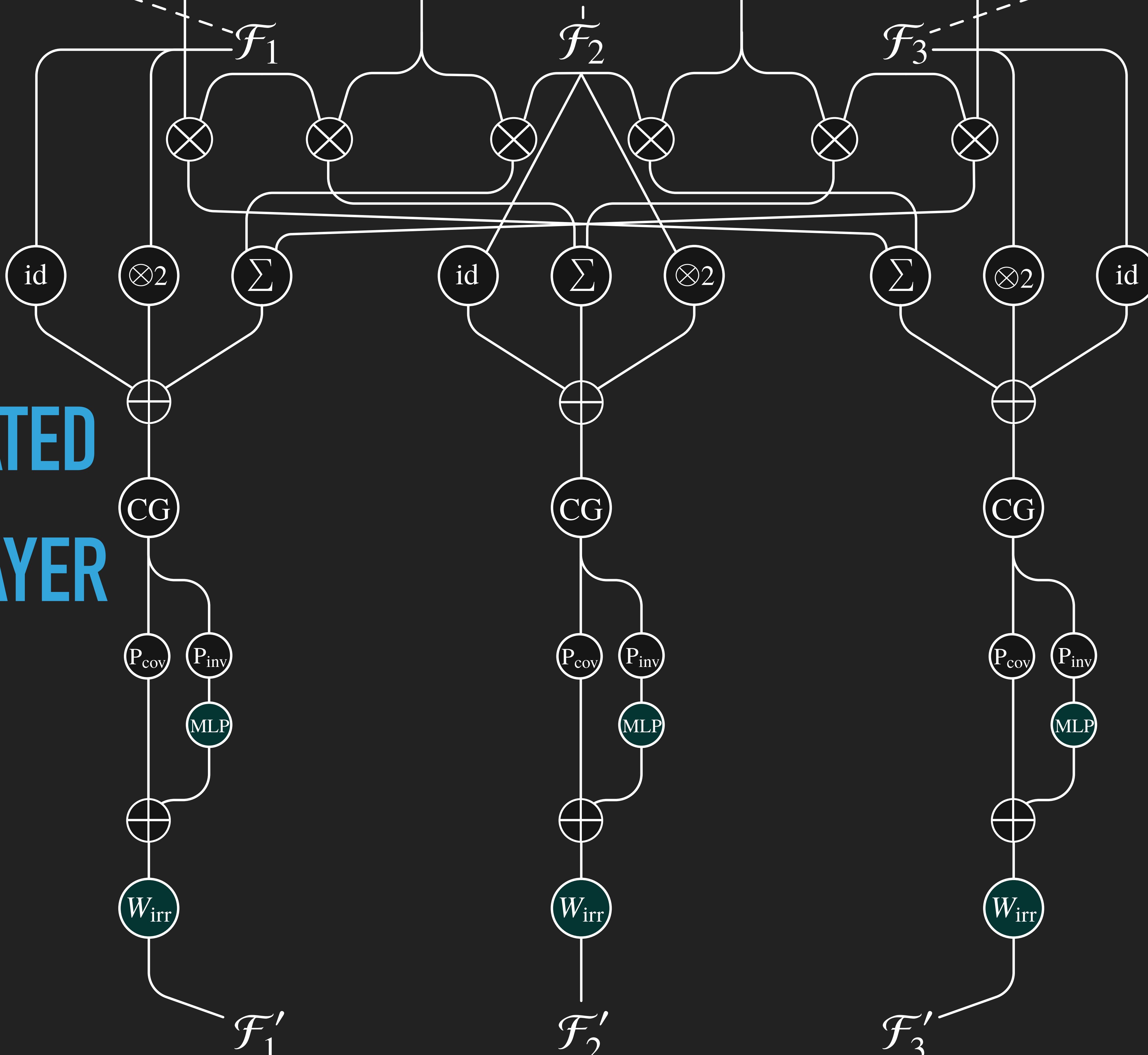
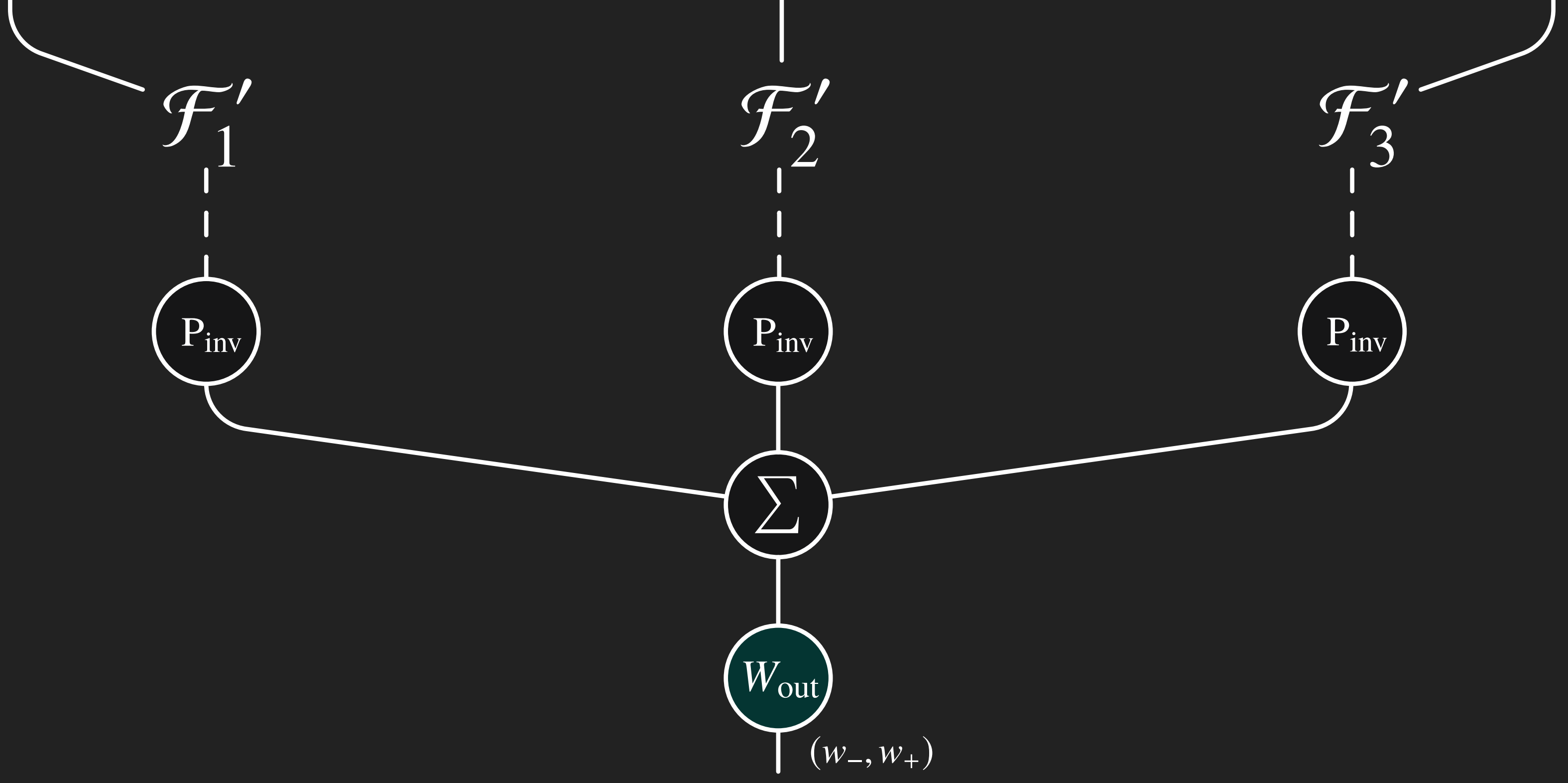



$\mathcal{F}_1'$
$\mathcal{F}_2'$
$\mathcal{F}_3'$

OUTPUT
LAYER



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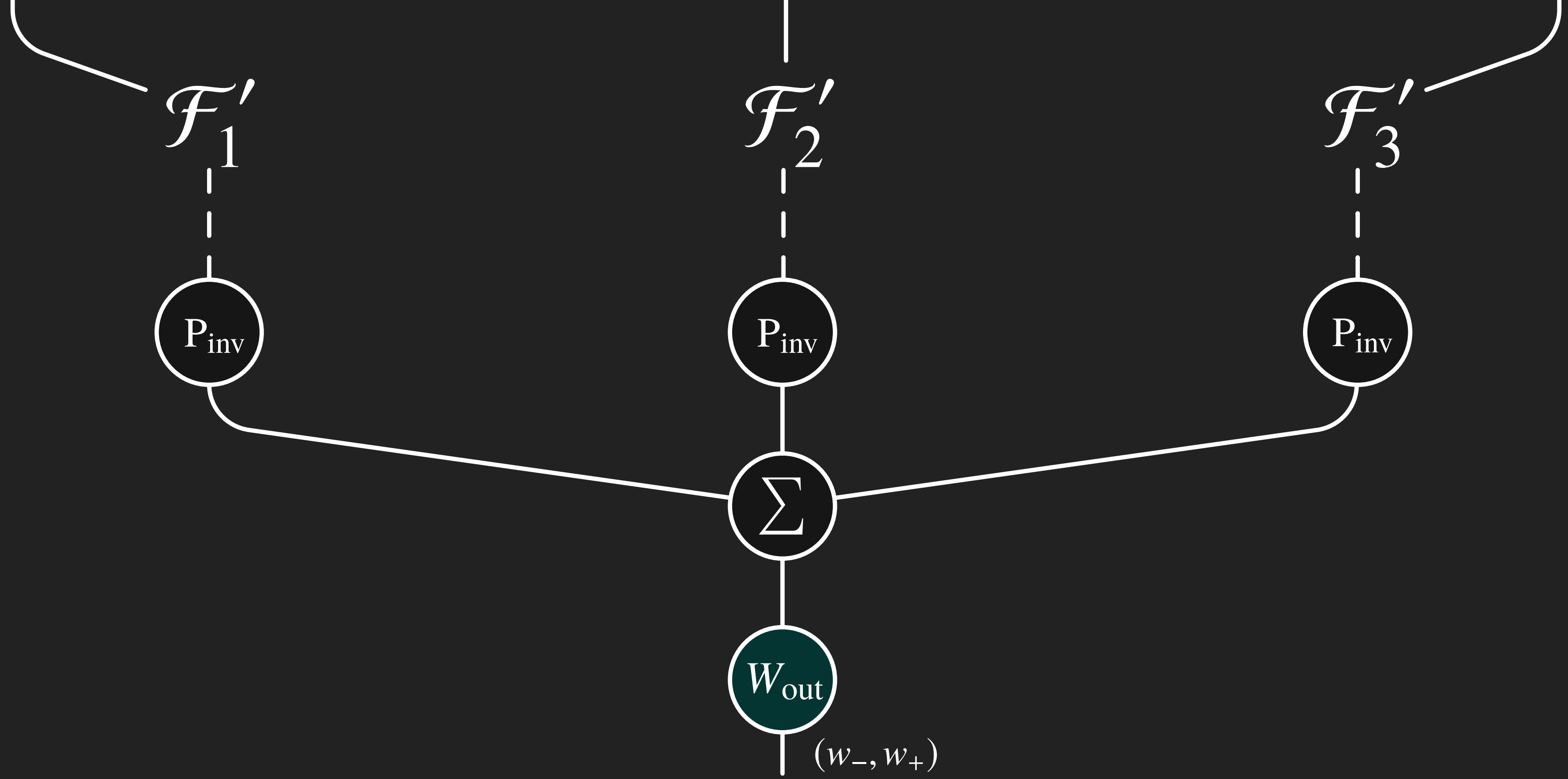
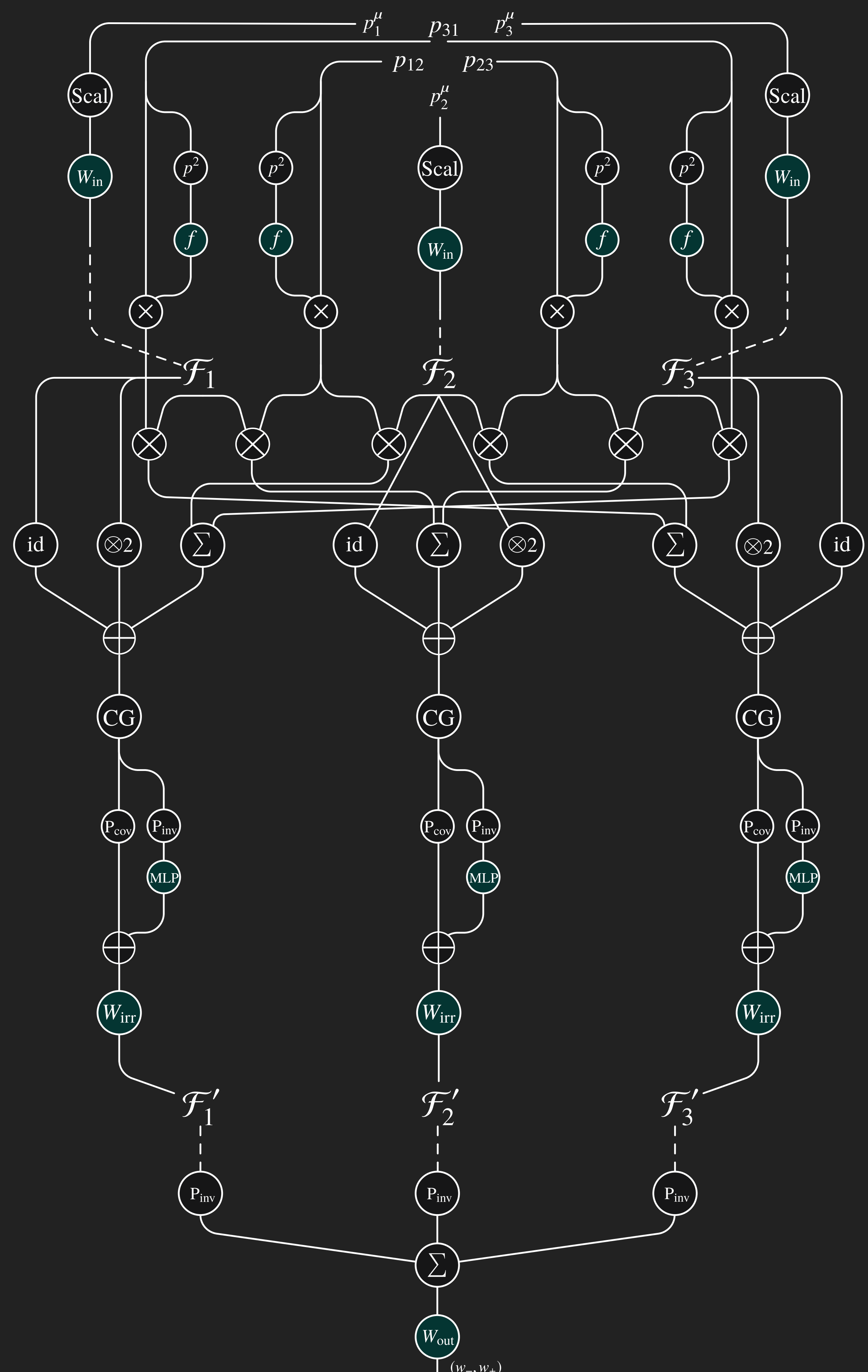


CLARIANT



p_1^μ
p_{31}
p_3^μ
p_{12}
p_{23}
p_2^μ
Scal
W_{in}
p^2
f
$\mathcal{F}_1$
$\mathcal{F}_2$
$\mathcal{F}_3$
id
$\otimes 2$
$\sum$
CG
P_{cov}
P_{inv}
MLP
W_{irr}
$\mathcal{F}_1'$
$\mathcal{F}_2'$
$\mathcal{F}_3'$
W_{out}
(w_-, w_+)