LORENTZ FLOWS FOR BOOSTED JETS

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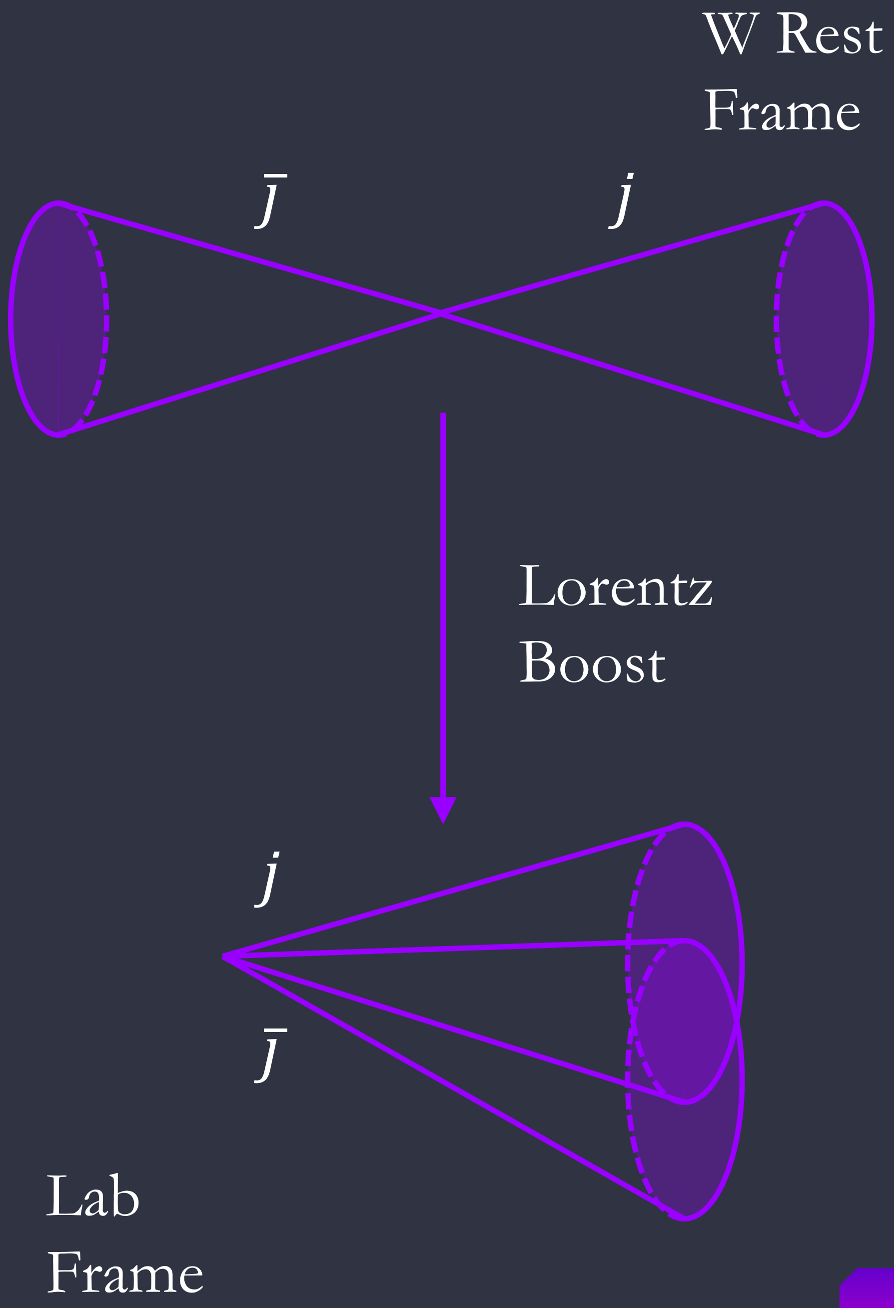
PROJECT GOAL

We model the momenta of particles in a boosted fat jet from $W \to jj$

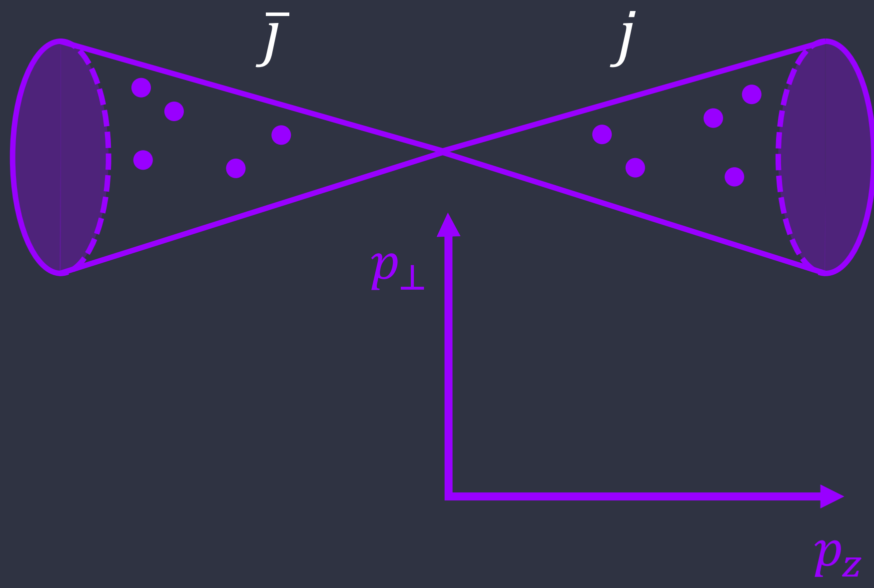
- Begin with a simple probability distribution for the momenta in the rest frame of the jet's parent
- Use the Lorentz transformation to map the rest frame momentum distribution into a model of the boosted momenta parameterized by the velocity of the jet's parent



A SIMPLE MODEL FOR A JET

We want to see how well we can do with a simple model

- Think of a jet as a point cloud of 4-momenta where the constituents are sampled i.i.d. from some distribution
  - Neglect possible correlation among particles
- Momentum along the jet-axis (p_z) and the transverse momentum $(p_\perp)$ are independent $f_P(p) = f_\perp(p_\perp) f_z(p_z)$
  - Transverse momenta coordinates are not independent



$$f_{jet}(\mathbf{p}_1, \ldots, \mathbf{p}_N) = \prod_{i=1}^{N} f_P(\mathbf{p}_i)$$

WHY?

These simplifications allow our model to be:

- Interpretable: the model is low dimensional, can be visualized and checked with data
- Data driven: we can build the model from data using low-pT W->jj and 1-d distributions p_z and $|p_\perp|$
  - initially we neglect how high-pT W jets differ from low-pT W jets in Lorentz invariant quantities
- Simple: should know how well such a simple baseline works
- Can use gradient descent to fit boost and orientation of W
- Generative model: It's a generative model with a tractable likelihood

NORMALIZING FLOWS

Normalizing flows allow you to model complicated distributions through finding a bijection, or sequence of bijections, that transform the distribution into a simpler one.

This simpler distribution is often chosen to be a Gaussian, while the bijections are arbitrary and often parameterized by neural networks.

$$\mathbf{z}_K = g_K \circ \cdots \circ g_1(\mathbf{z}_0)$$

$$\mathbf{z}_0 \sim f_0(\mathbf{z}_0)$$

$$\mathbf{z}_K \sim f_K(\mathbf{z}_K) = f_0(\mathbf{z}_0) \prod_{k=1}^{K} \left| \det \frac{\partial g_k}{\partial \mathbf{z}_{k-1}} \right|^{-1}$$

http://akosiorek.github.io/ml/2018/04/03/norm_flows.html

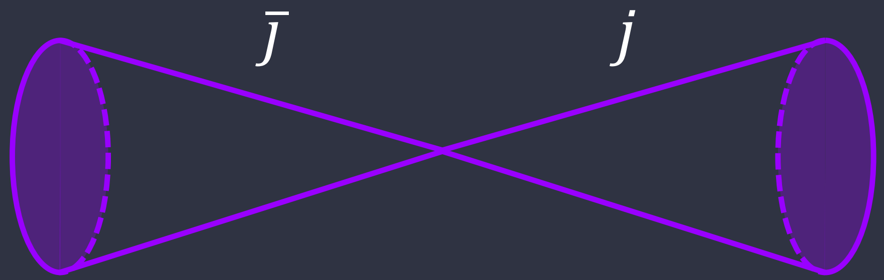
THE LORENTZ FLOW

We will use one bijection, the Lorentz transformation, to transform the rest frame distribution into a boosted frame distribution.

$$\mathbf{p}_{\text{rest frame}} \sim f_{\text{rest frame}}(\mathbf{p}_{\text{rest frame}})$$

$$\mathbf{p}'_{\text{lab frame}} \sim f_{\text{rest frame}}(\mathbf{p}_{\text{rest frame}}) \left| \det \frac{\partial \text{Boost}}{\partial \mathbf{p}_{\text{rest frame}}} \right|^{-1}$$

REST FRAME MODEL



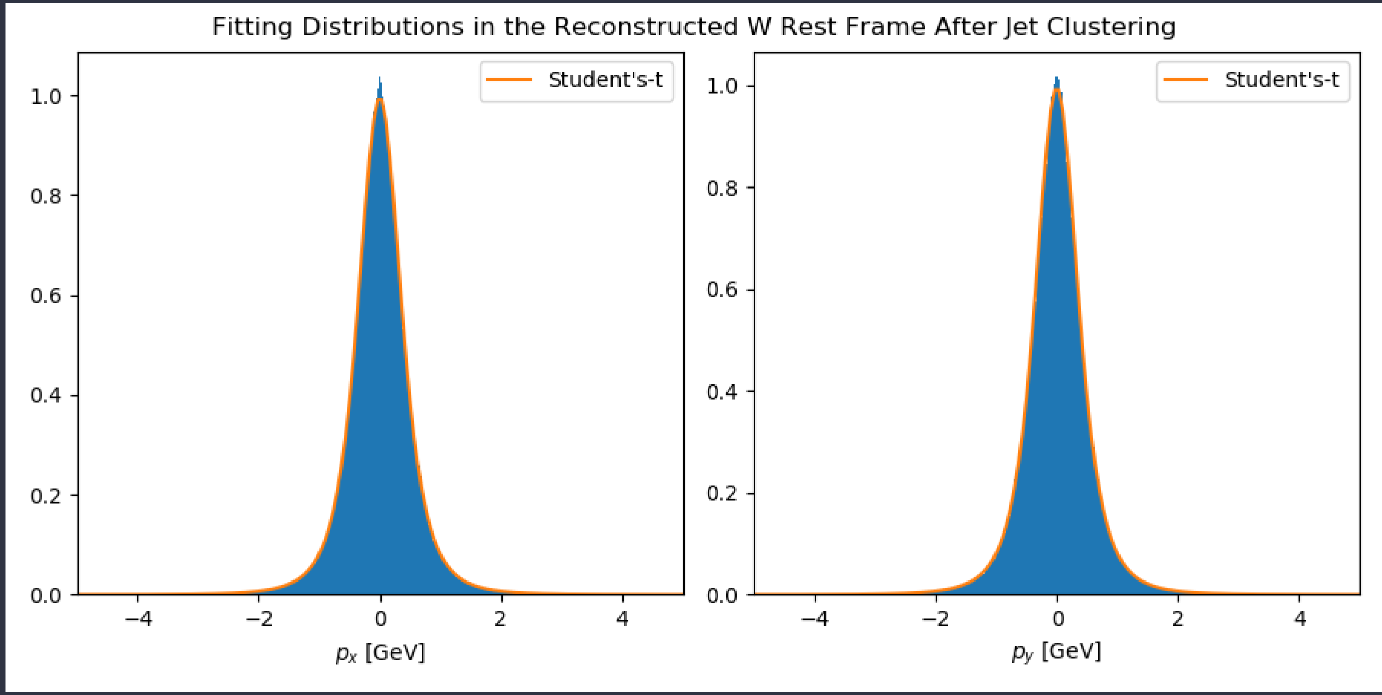
DATA GENERATION

- Used Pythia to simulate the events
- Keep the two highest pT Anti-kt R=1 jets
- Reconstruct W boson 4-momenta by summing over the two jets
- Boost each constituent of the jets into the reco W rest frame
- Rotate each particle such that the z-axis corresponds to the jet axis and the two transverse axes are arbitrary
- Set mass of hadrons to be 0.1 GeV

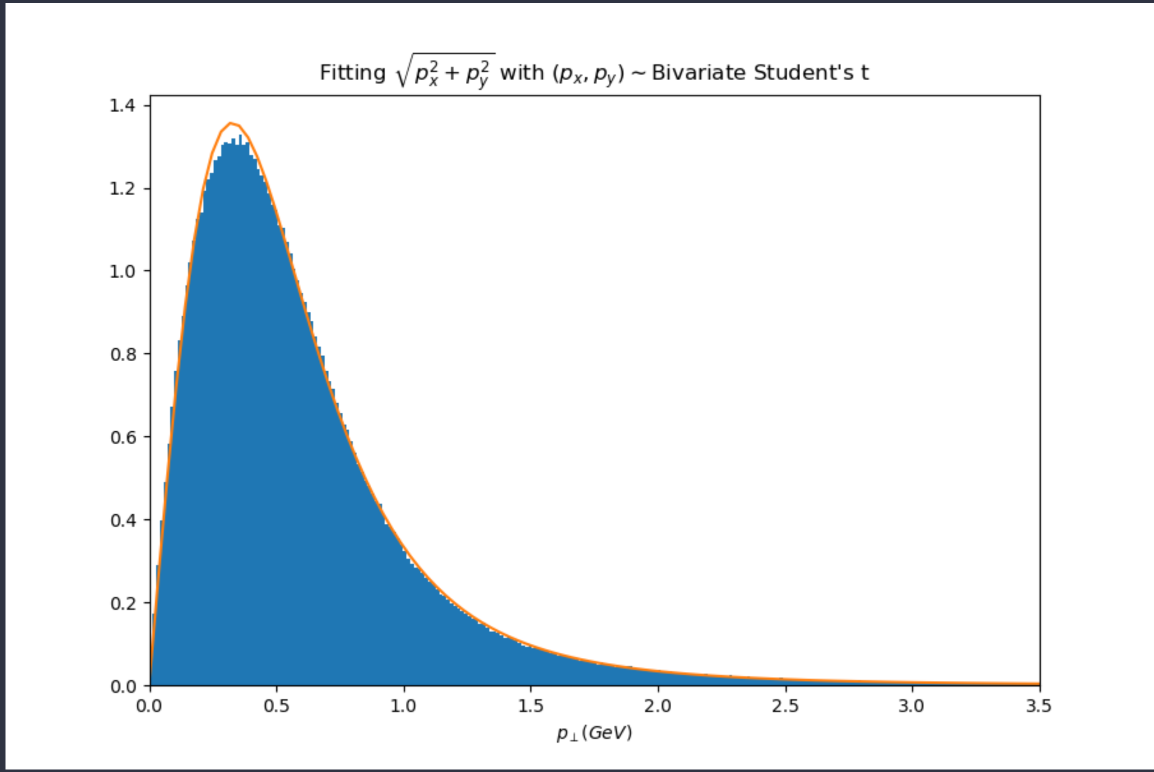
TRANSVERSE MOMENTUM DISTRIBUTION

The marginal distributions of the x and y components of the momenta closely followed identical Student's-t distributions, which motivated a Bivariate Student's-t distribution for the joint momentum (P_x, P_y) and a simple form for the transverse momentum $P_\perp = \sqrt{P_x^2 + P_y^2}$

$$f_{P_\perp}(x; \nu, \sigma) = \frac{x}{\sigma^2}\left(1 + \frac{x^2}{\nu\sigma^2}\right)^{-(\nu+2)/2}$$



Model Fits: Transverse Momentum

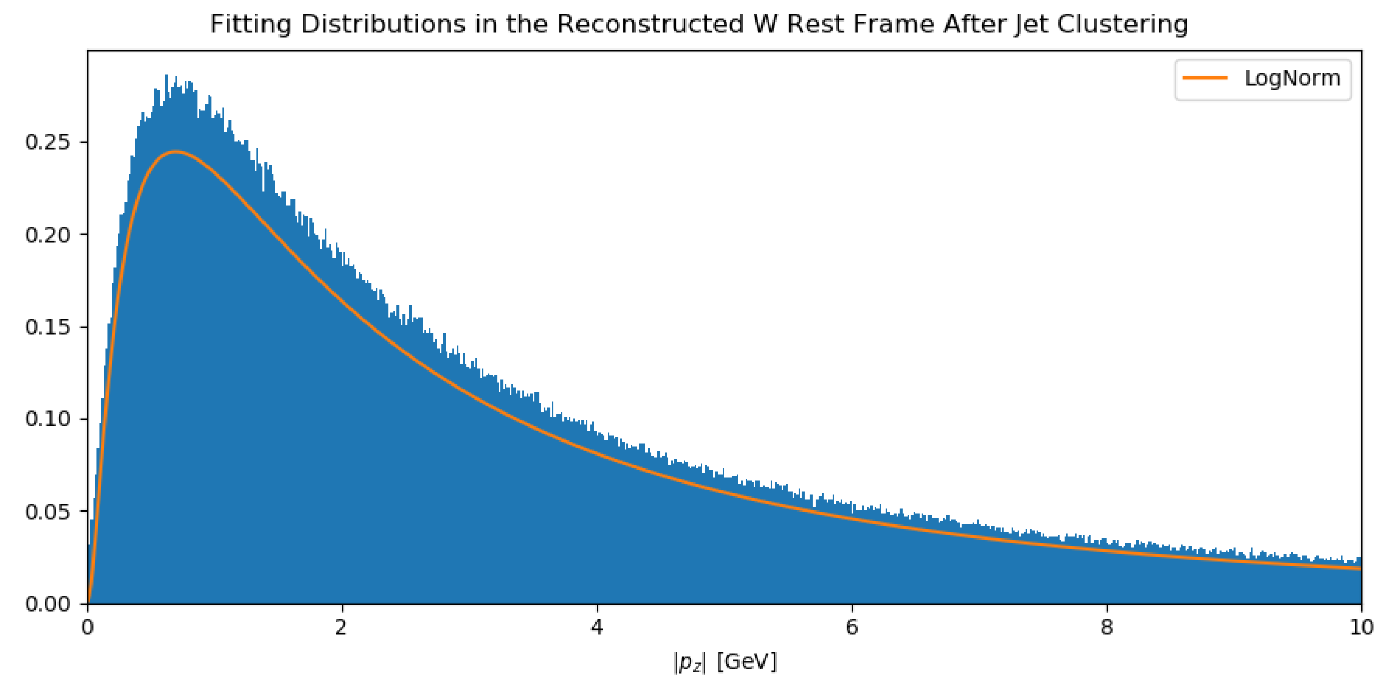


Model Fits: Transverse Momentum

JET-AXIS MOMENTUM DISTRIBUTION

Many simple, known distributions were fitted to the distribution, and Log Normal provided the best fit

$$f_{P_z}(x; s, \sigma) = \frac{\sigma}{sx\sqrt{2\pi}} \exp\left(-\frac{\log^2(x/\sigma)}{2s^2}\right)$$



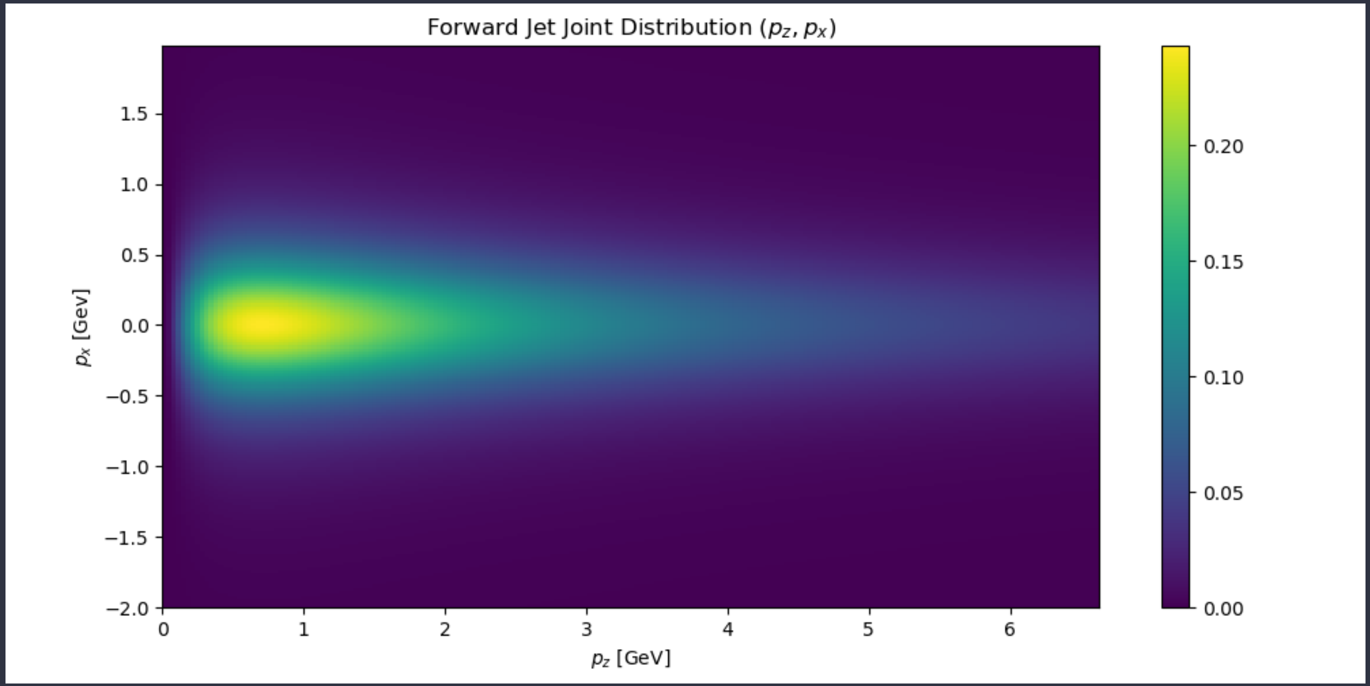
Model Fits: Jet-Axis Momentum

$$f_{P_z}(x; s, \sigma) = \frac{\sigma}{sx\sqrt{2\pi}} \exp\left(-\frac{\log^2(x/\sigma)}{2s^2}\right)$$

PUTTING THEM TOGETHER

The overall distribution density is then simply the product of the three independent densities.

$$f(\mathbf{p}) = \frac{f_{P_\perp}\left(\sqrt{p_x^2 + p_y^2}\right) f_{P_z}(p_z)}{2\pi\sqrt{p_x^2 + p_y^2}}$$



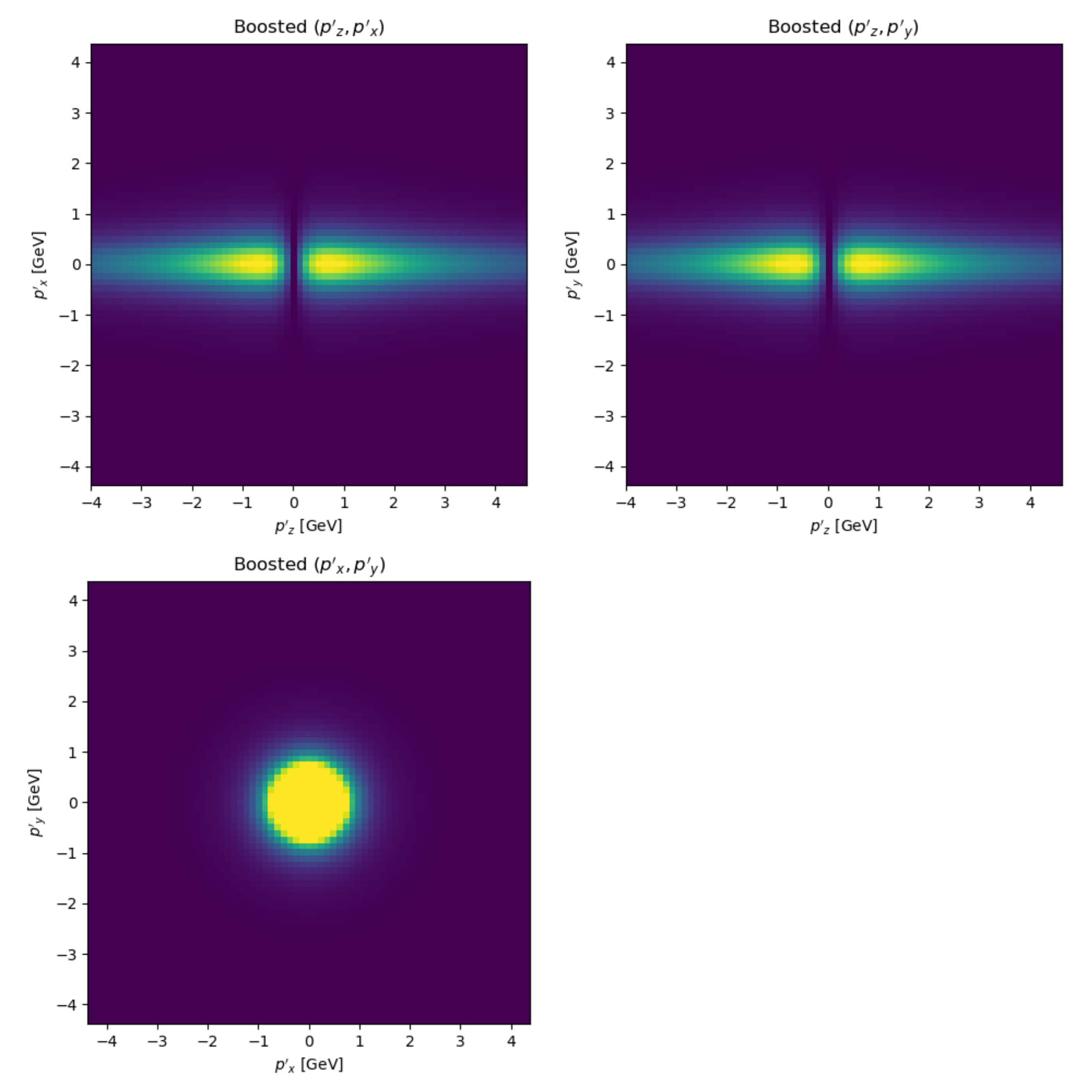
Single Jet Model Distribution

DIJET MODEL

To extend this to a two-jet model, we just use a simple mixture model that represents two single jet models back-to-back

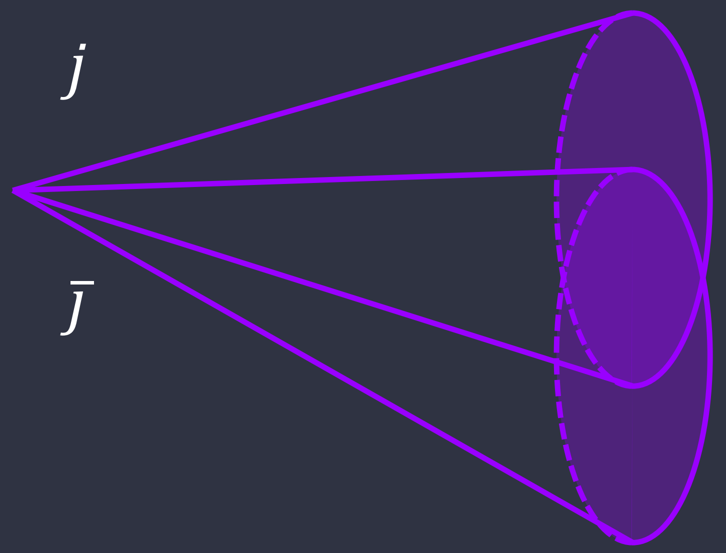
$$f_{dijet}(\mathbf{p}) = 0.5\left[f_P(\mathbf{p}) + f_P(-\mathbf{p})\right]$$

When sampling, we must introduce some rules to ensure the jets are balanced and give $m_{jj} \cong m_W$. Although not shown here, we can also easily account for rotations of the jet axis.



Dijet Model Distributions

LAB FRAME MODEL



THE LORENTZ TRANSFORMATION

To boost the momentum from the rest frame into a frame with relative velocity **v** we have

$$\mathbf{p}' = \mathbf{p} + \left[\frac{(\gamma_v - 1)(\mathbf{p}\cdot\mathbf{v})}{v^2} - \gamma_v\sqrt{m^2 + p^2}\right]\mathbf{v}$$

with Jacobian

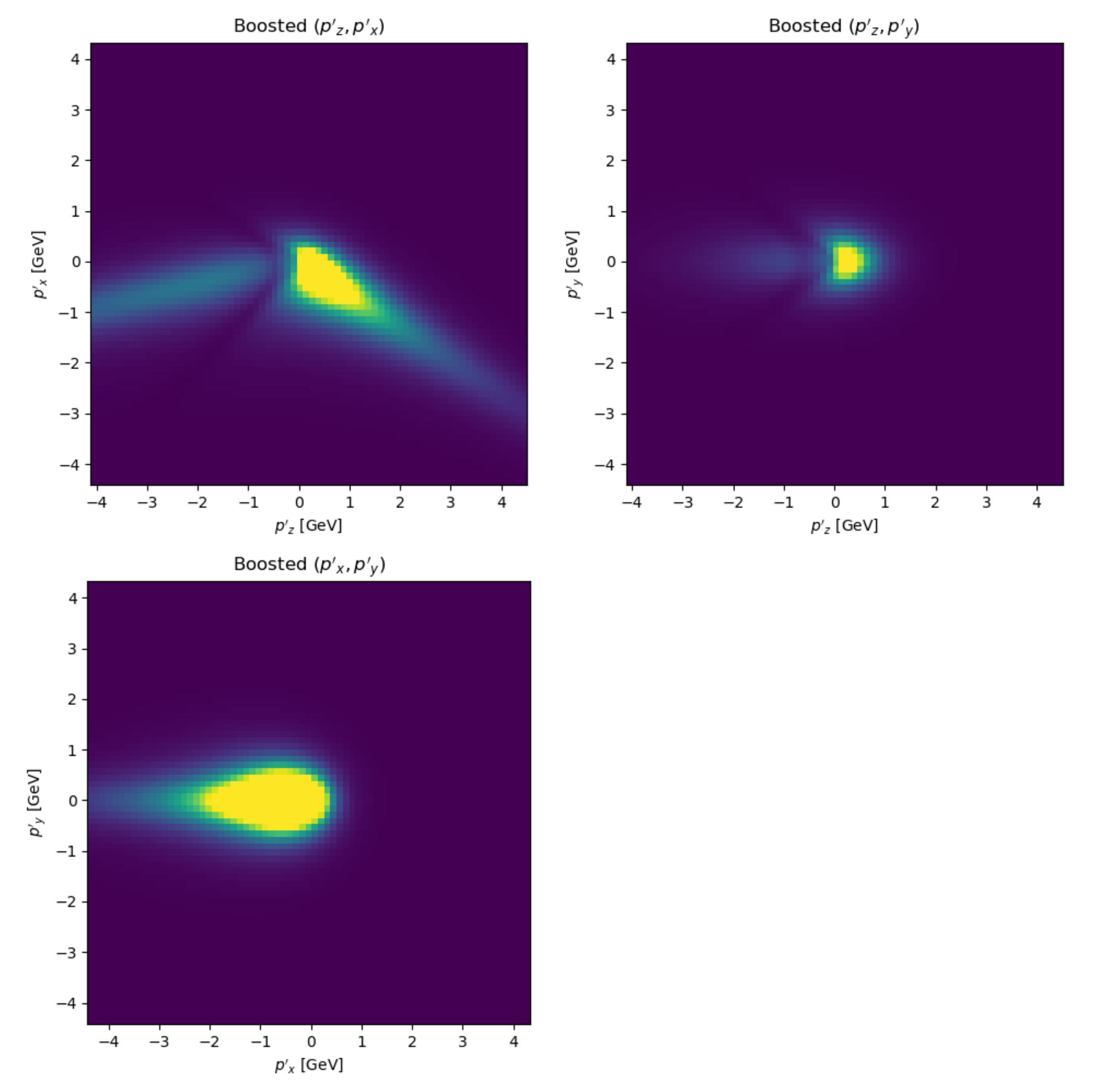
$$\frac{\partial p_i'}{\partial p_j} = \delta_{ij} + \left[\frac{(\gamma_v - 1)v_j}{v^2} - \frac{\gamma_v p_j}{\sqrt{m^2 + p^2}}\right]v_i$$

THE LORENTZ FLOW

Since the Lorentz transformation is a diffeomorphism, we can use the idea of a normalizing flow to get a conditional probability density in the lab frame

$$f_{P'}(\mathbf{p}'|\mathbf{v}) = f_P(\mathrm{Boost}(\mathbf{p}', -\mathbf{v})) \left| \det \left[\frac{\partial \mathrm{Boost}_i}{\partial p'_j}(\mathbf{p}', -\mathbf{v}) \right] \right|$$

$$= f_P(\mathrm{Boost}(\mathbf{p}', -\mathbf{v}))\gamma_v \left(1 + \frac{\mathbf{p}' \cdot \mathbf{v}}{\sqrt{m^2 + p'^2}} \right)$$



We have used this model to fit the boost vector v and orientation of the W in $W \to jj$ events generated from Pythia

Lab Frame Distribution Example

TAKEAWAYS

We can build a simple, data-driven model for boosted jets.

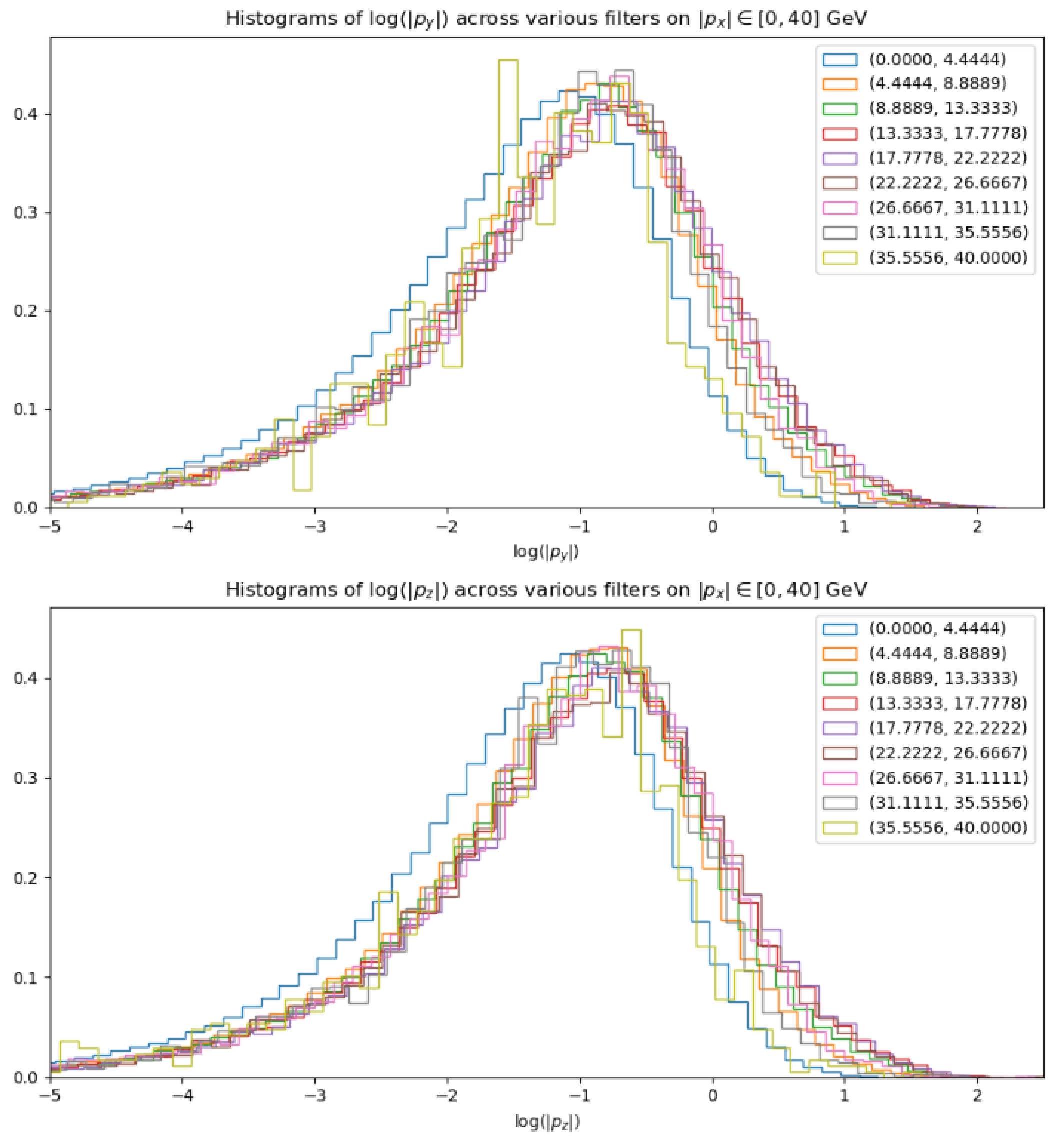
We can then apply this to

- Inference on kinematics of the boosted system
  - We have some results for this
- As a tagger
- As a generative model
- As a transfer function for matrix element method in boosted jet context

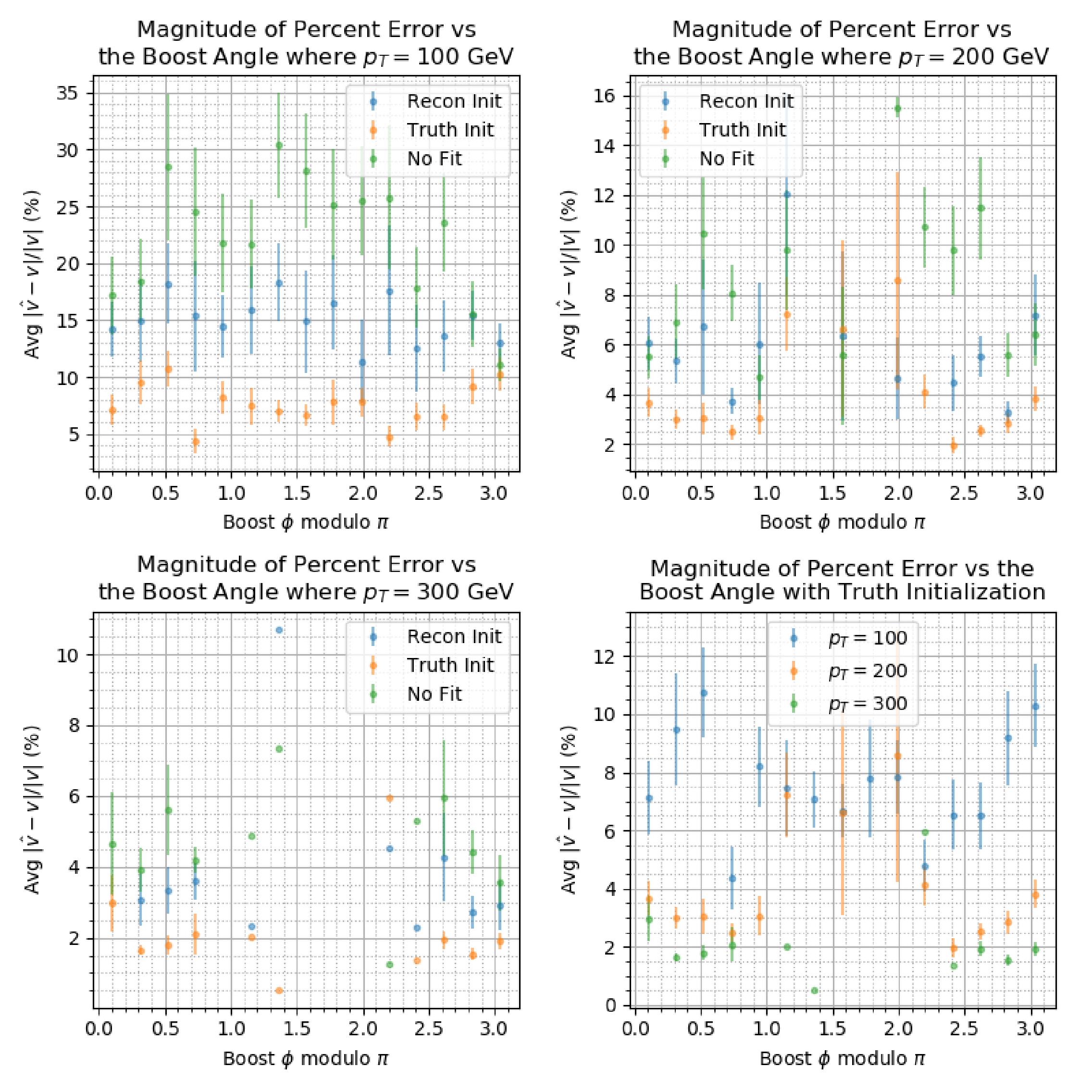
THANK YOU

Questions?

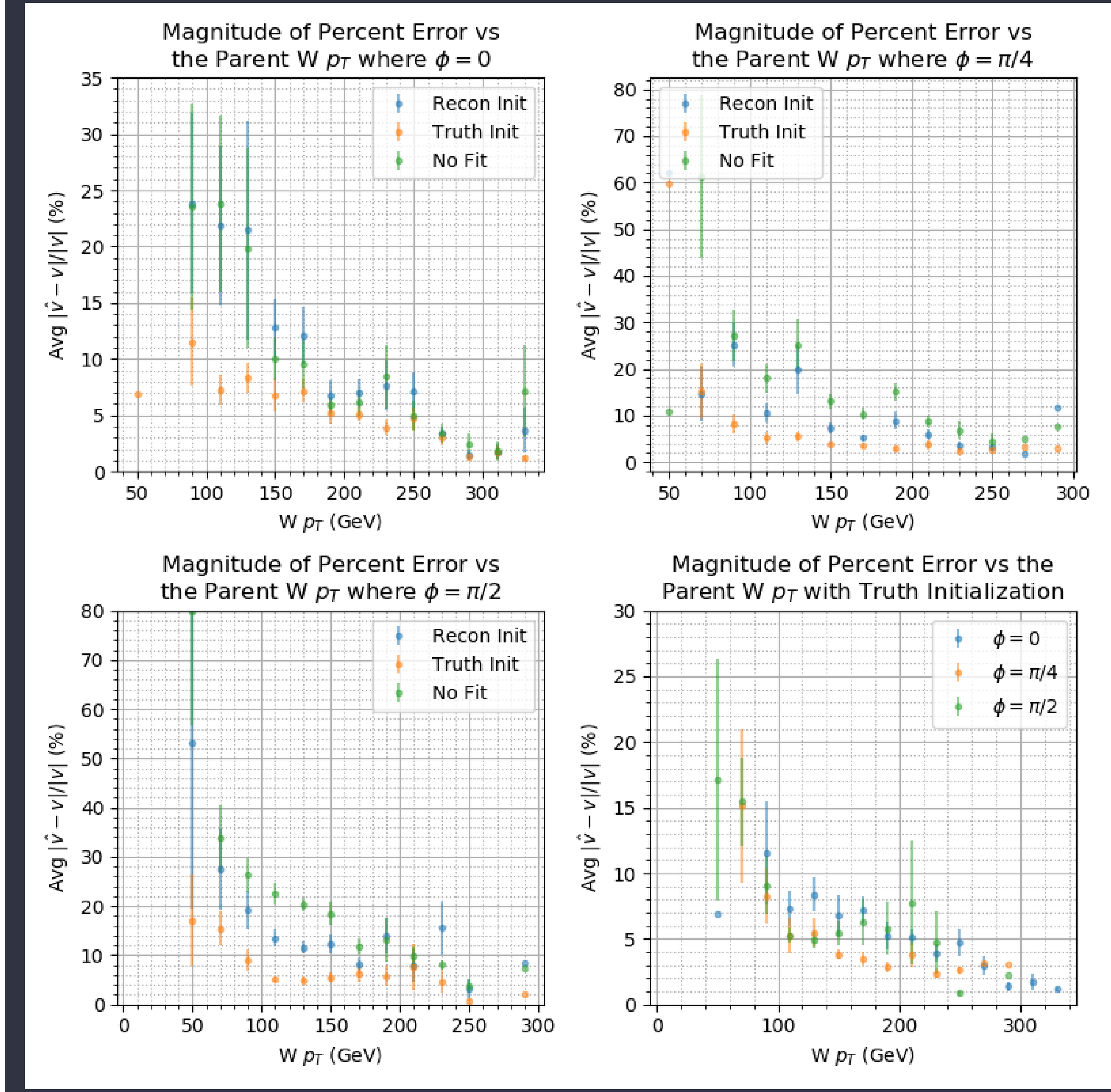
APPENDIX



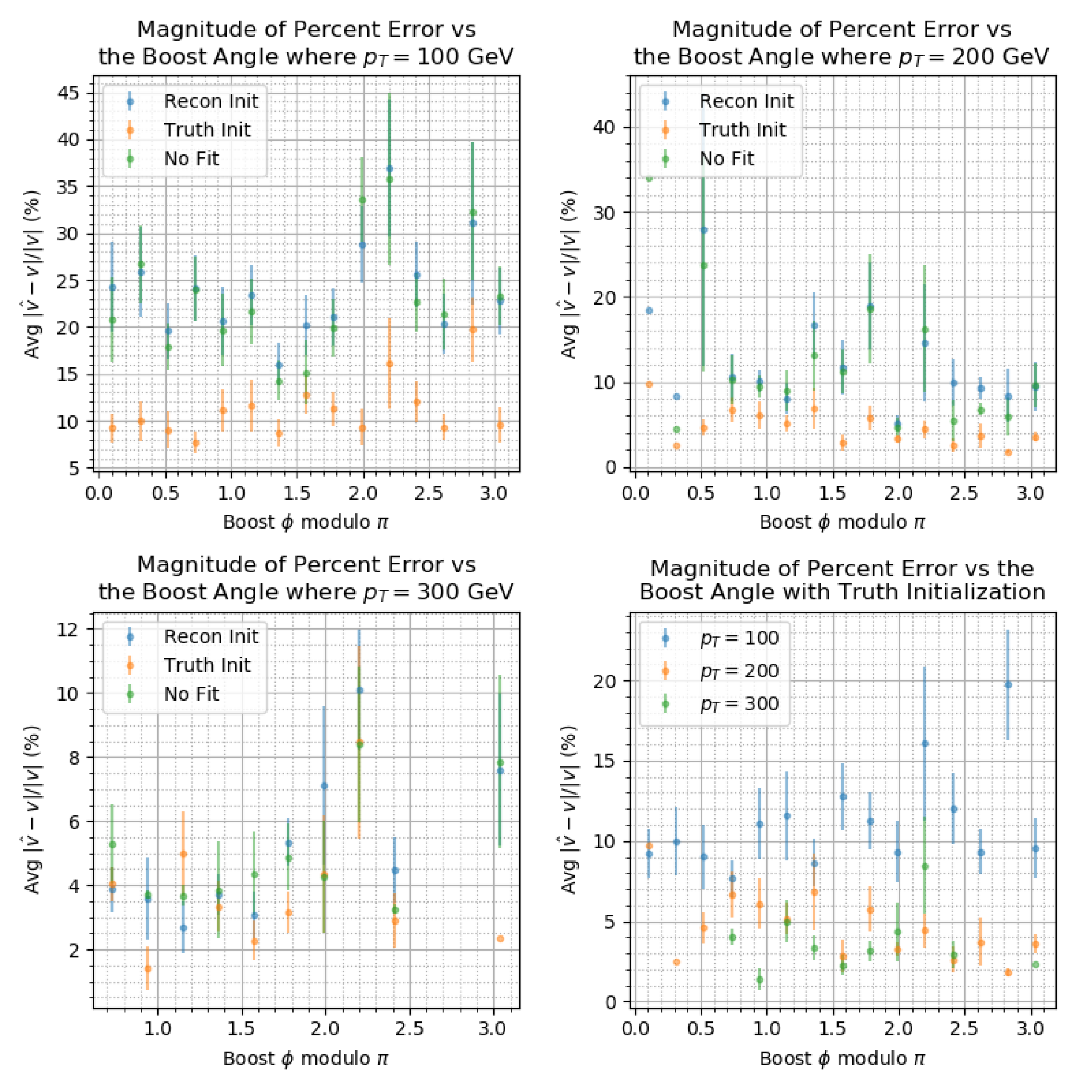
Jet-Axis and Transverse Momentum Independence



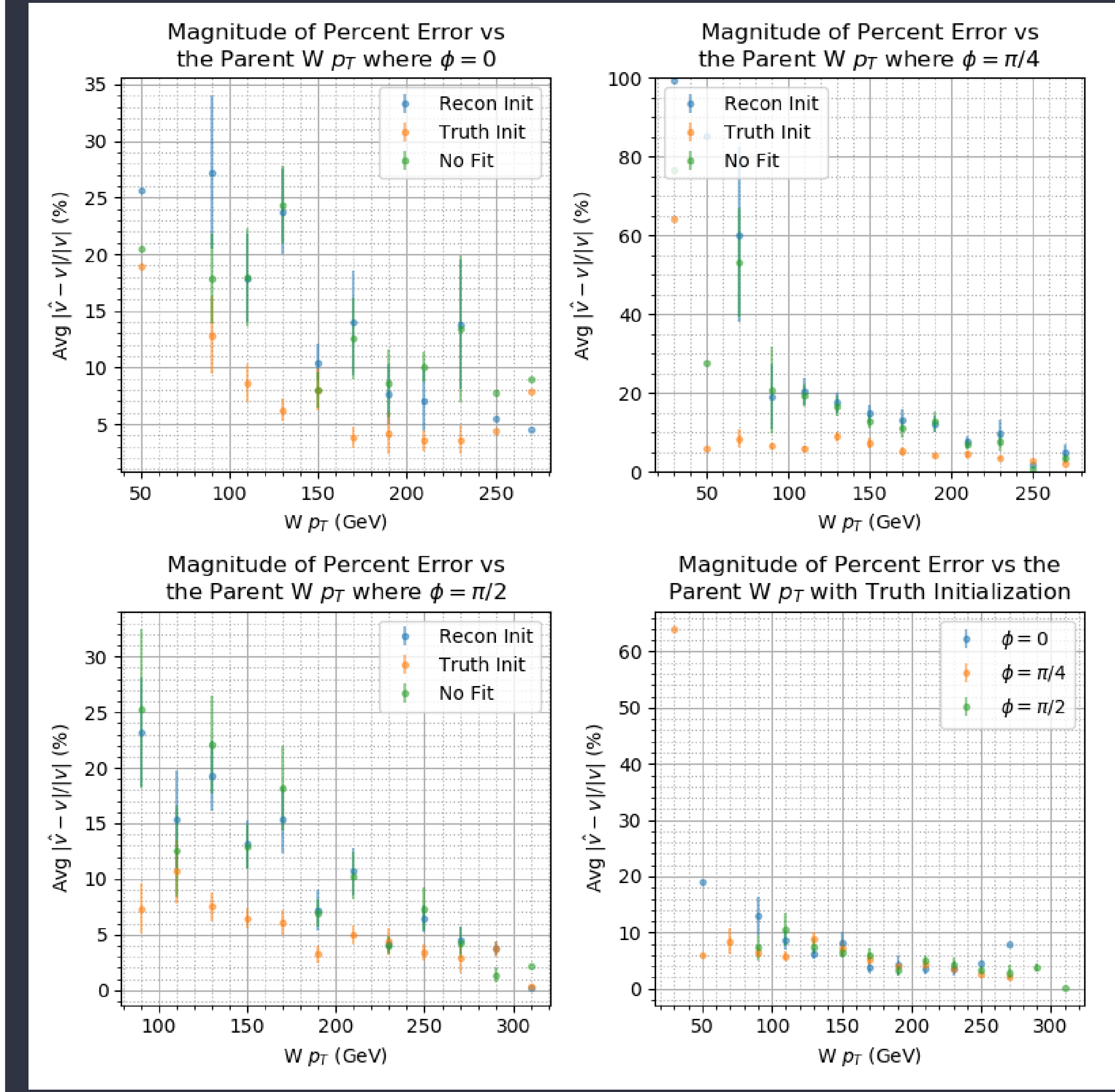
Inference on Boost Velocity (no rotations)



Inference on Boost Velocity (no rotations)



Inference on Boost Velocity (with rotations)



Inference on Boost Velocity (with rotations)