FINITE ELECTROWEAK MONOPOLE FROM BRANEWORLD

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in collaboration with
Masato Arai, Minoru Eto and *Norisuke Sakai*

PART I:

WHAT HAVE THE EXTRA DIMENSIONS EVER DONE FOR US?

TWO WAYS OF HIDING EXTRA DIMENSIONS

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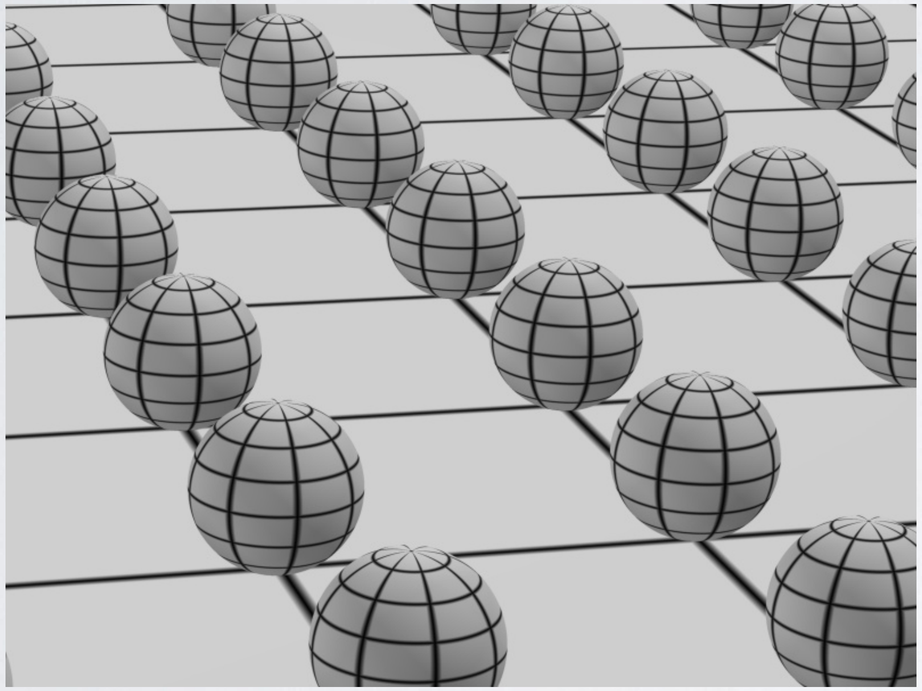

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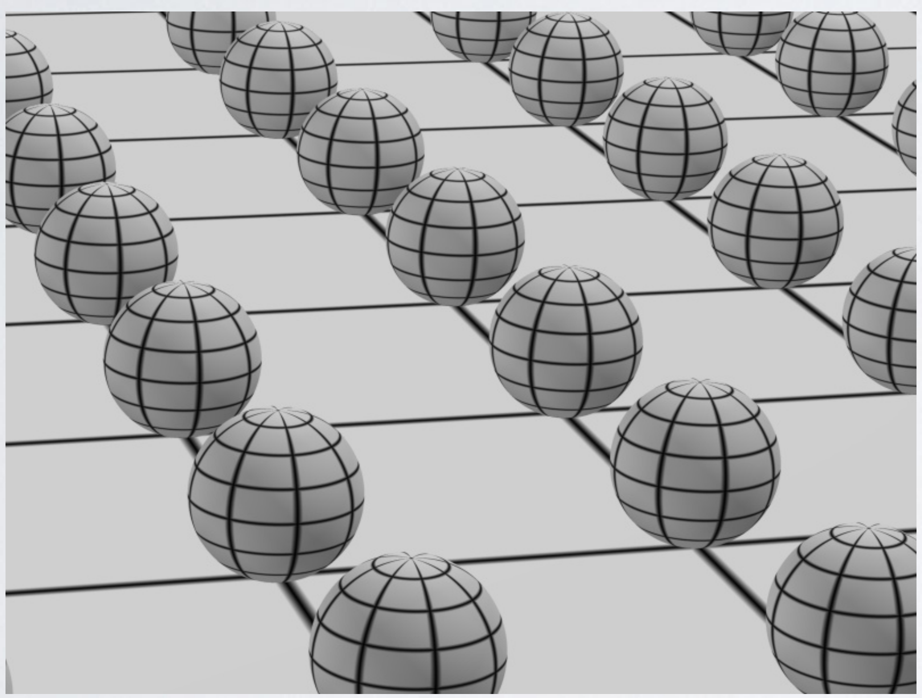

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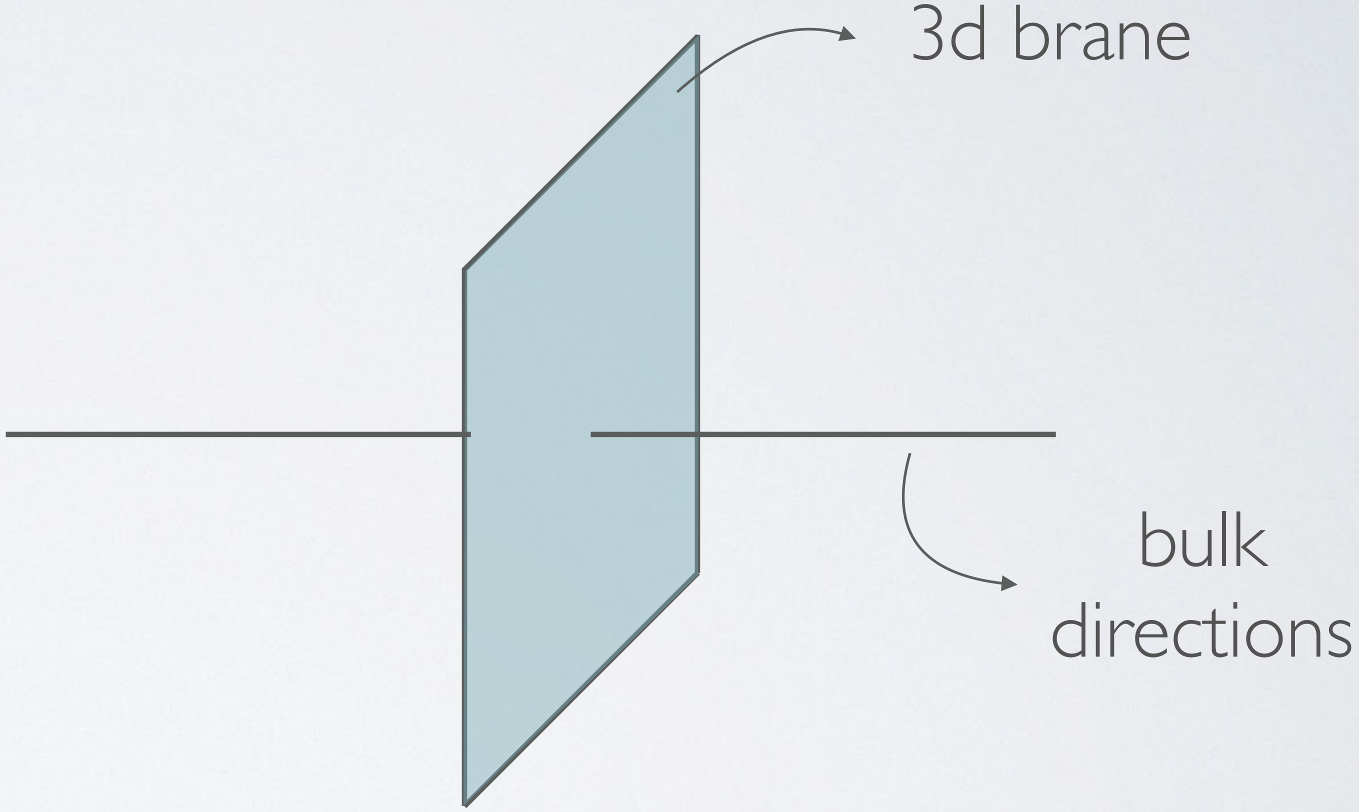
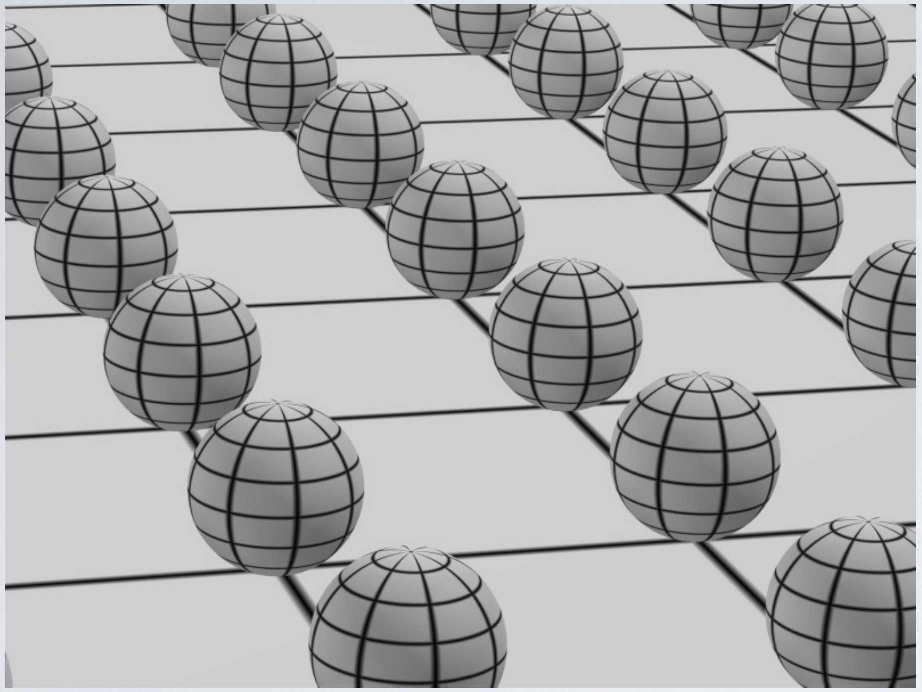
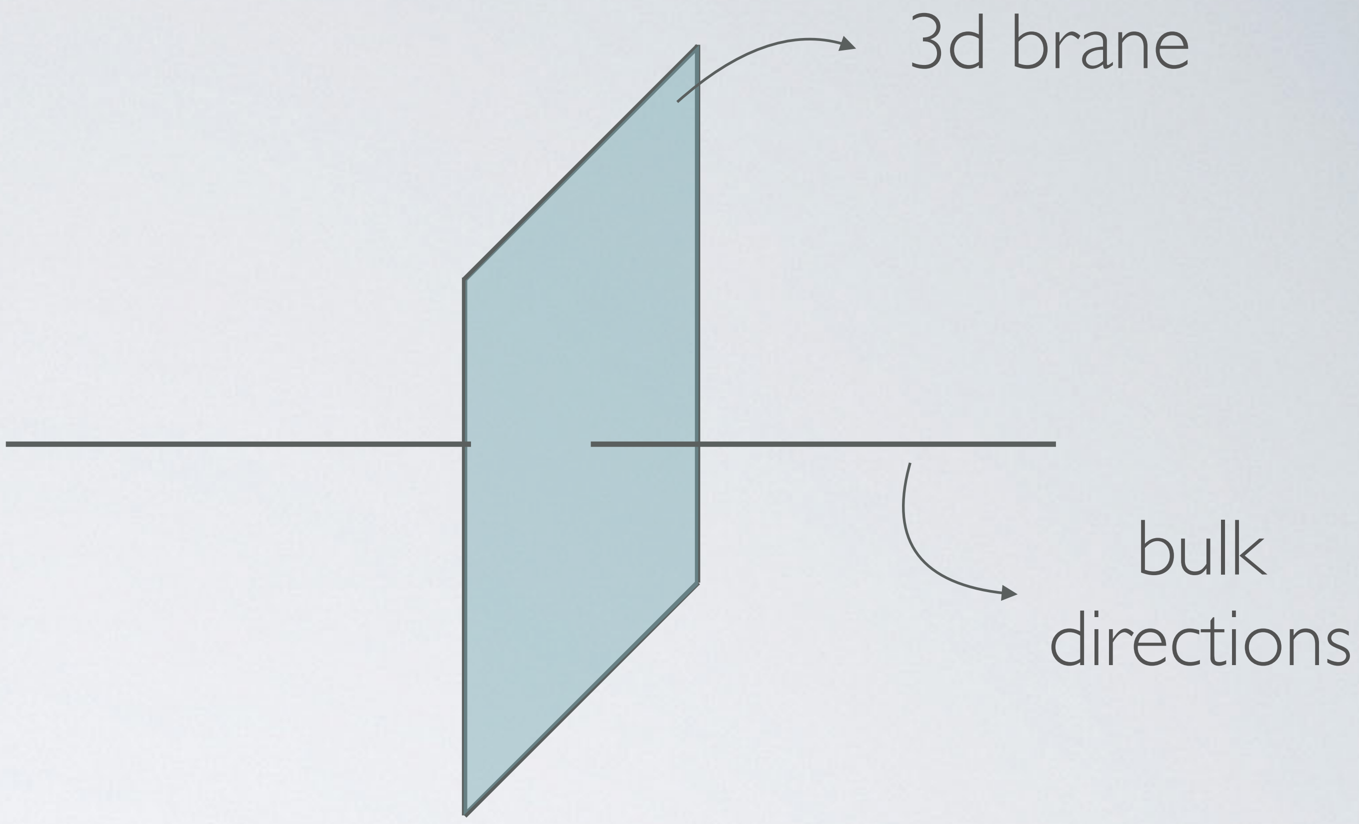
2) In plain sight



In the **brane-world** scenario matter is confined on 3-dimensional brane floating inside multi-dimensional **bulk**.




3d brane
bulk
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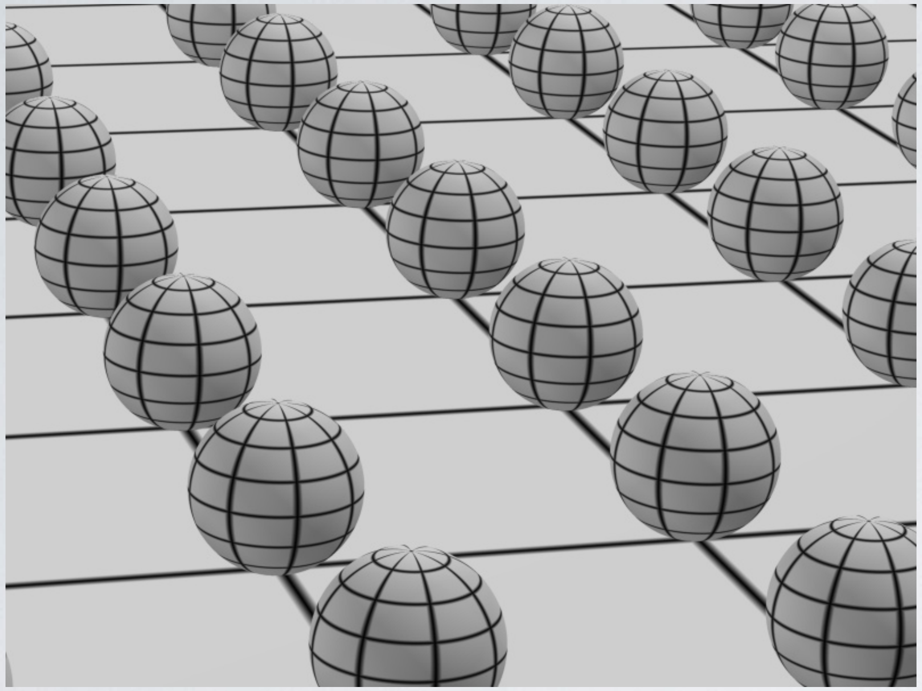

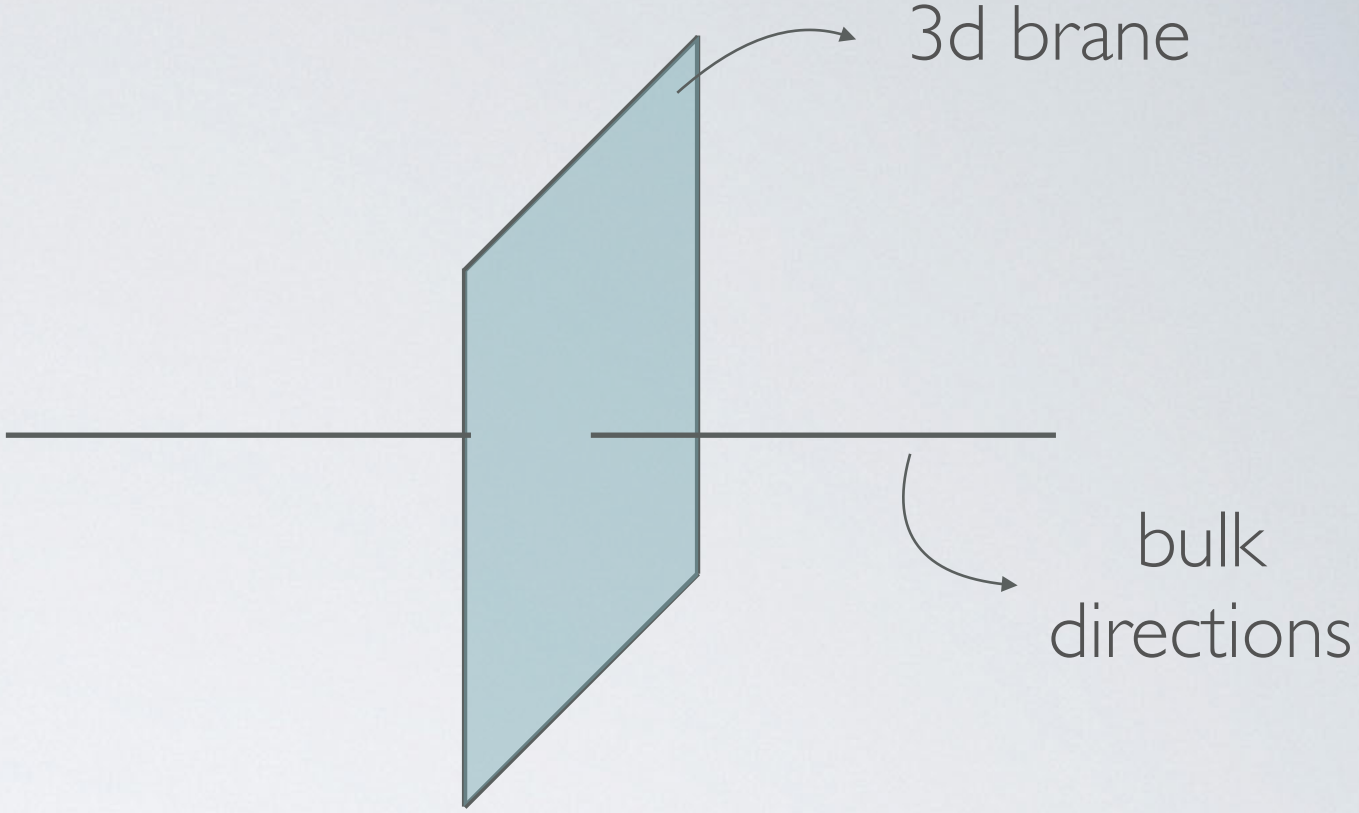
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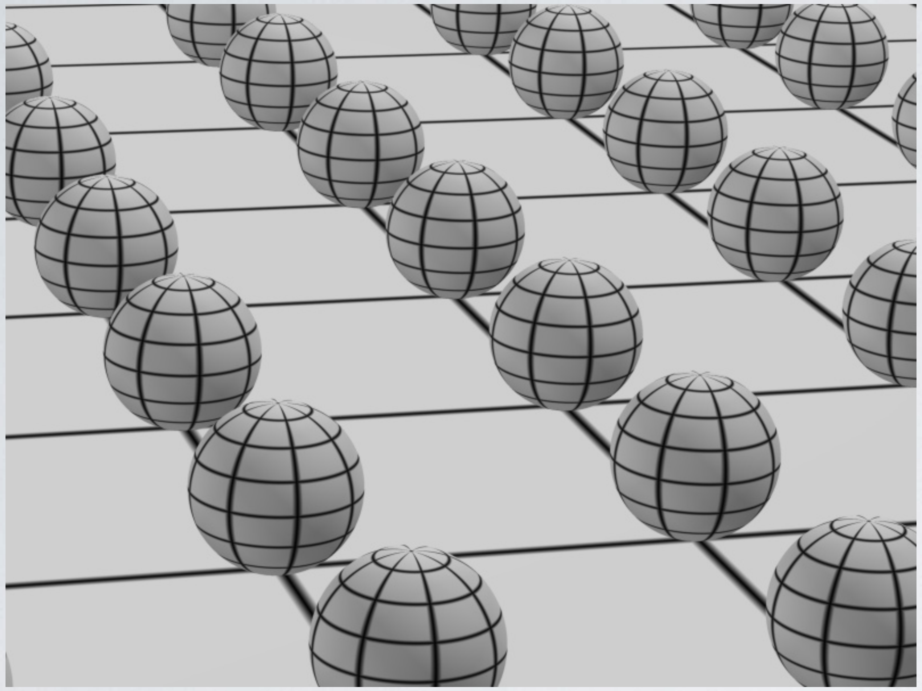
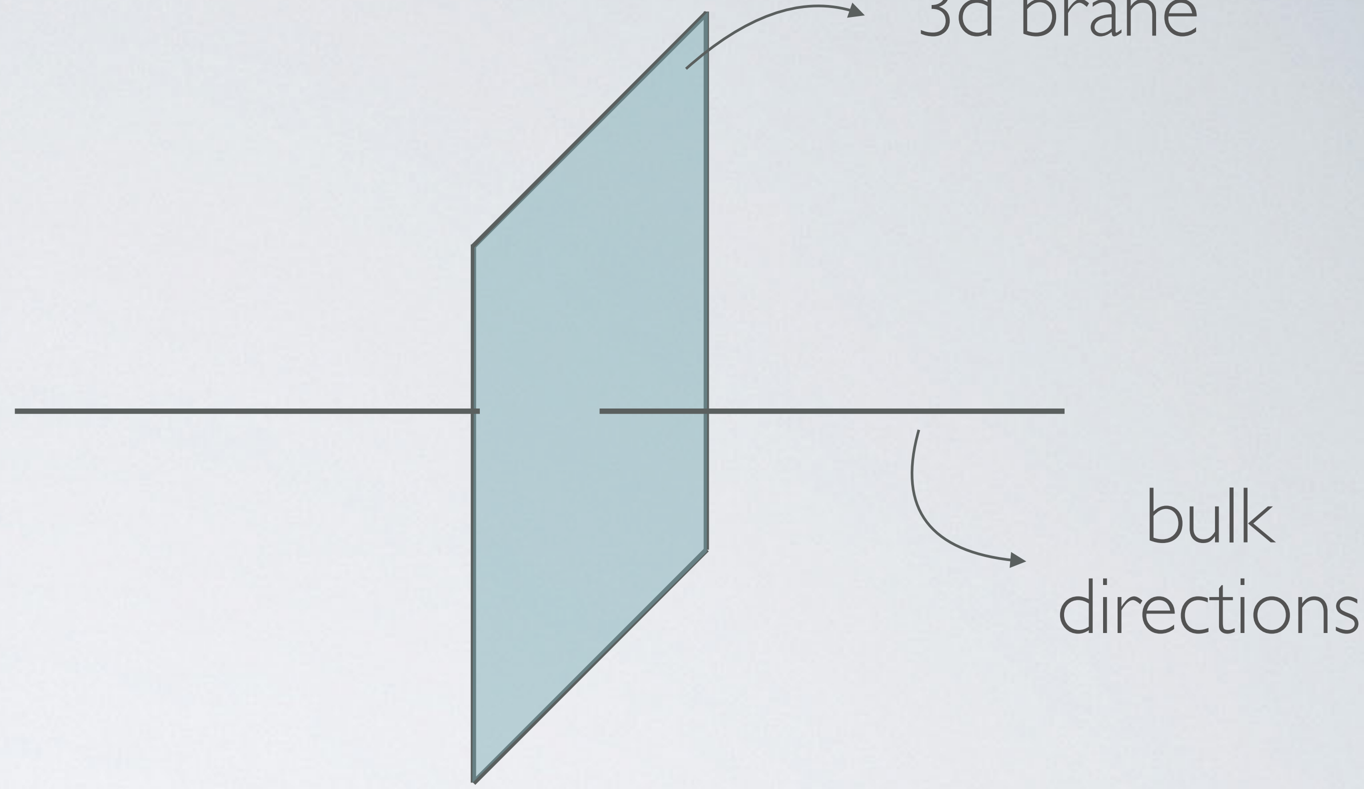
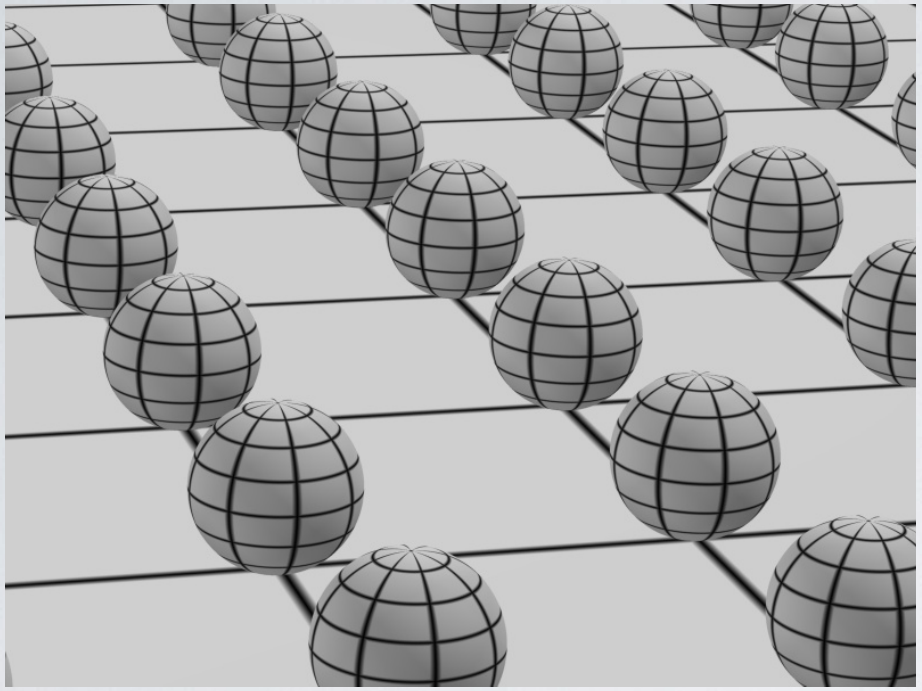


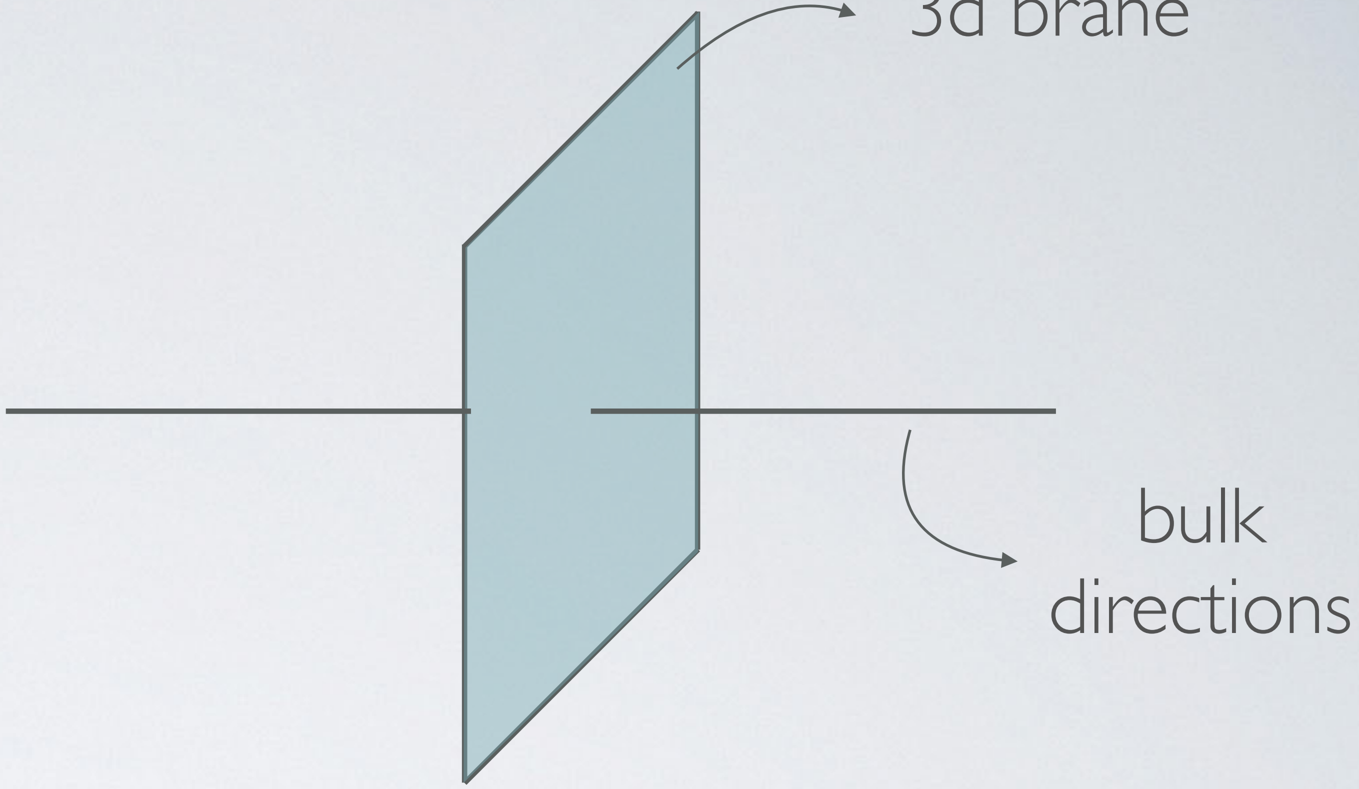


A signature of extra dimensions is an infinite tower of KK modes

$$\phi(x,y) = \sum_{n=-\infty}^{\infty} \phi^{(n)}(x) e^{2i\pi n y/R}$$

$$m_n^2 = \frac{4\pi^2 n^2}{R^2}$$





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- Quantization of charge

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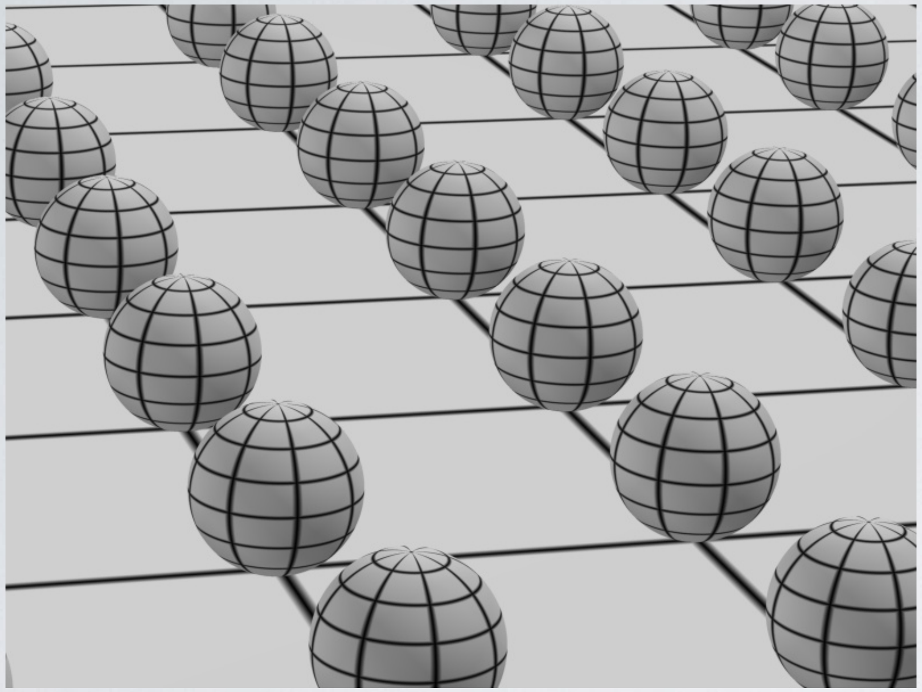

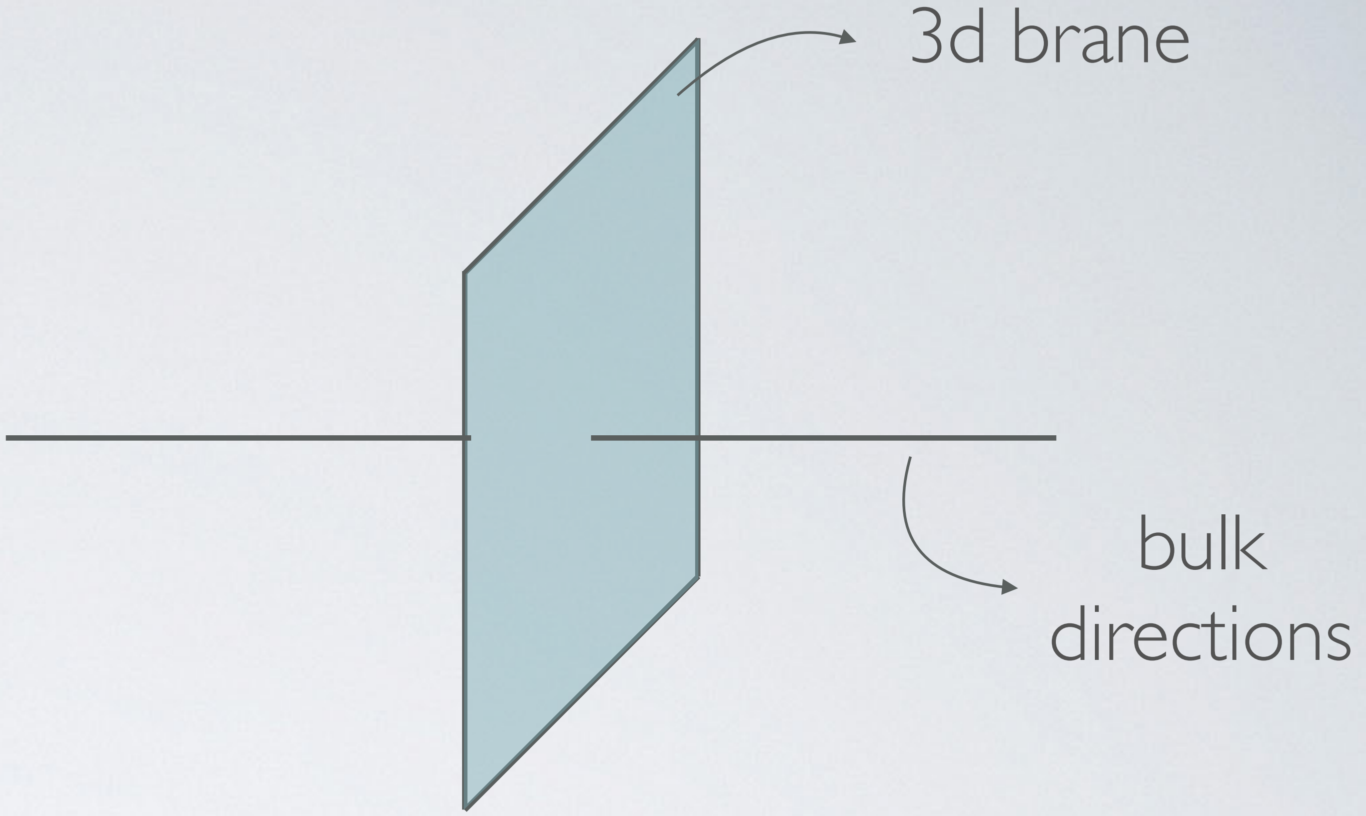

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In field theory, a brane is an extended object (background solution) made of some scalar fields. It creates a potential well which traps low-energy modes of fields.

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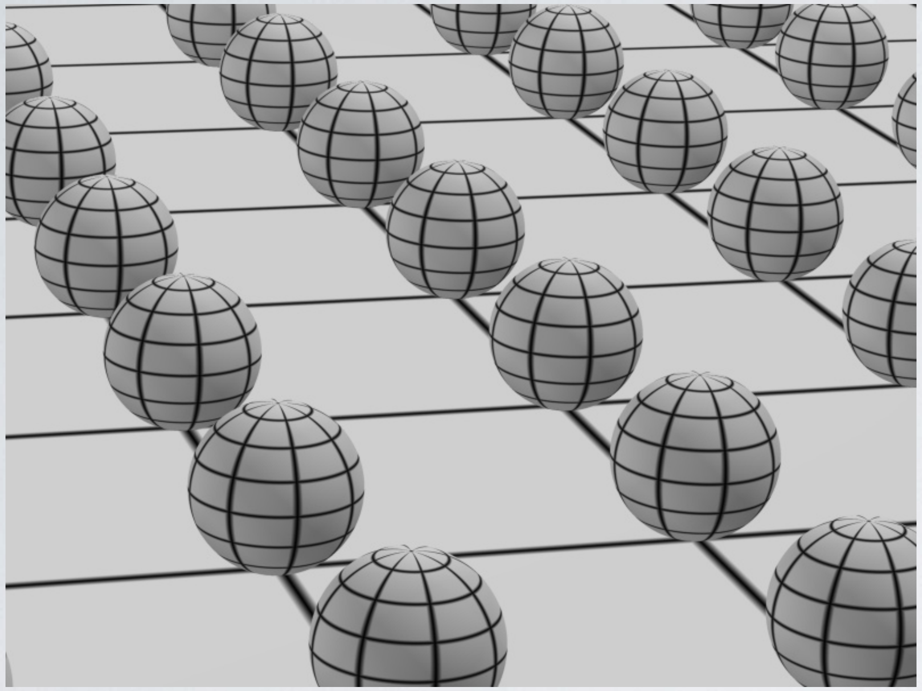

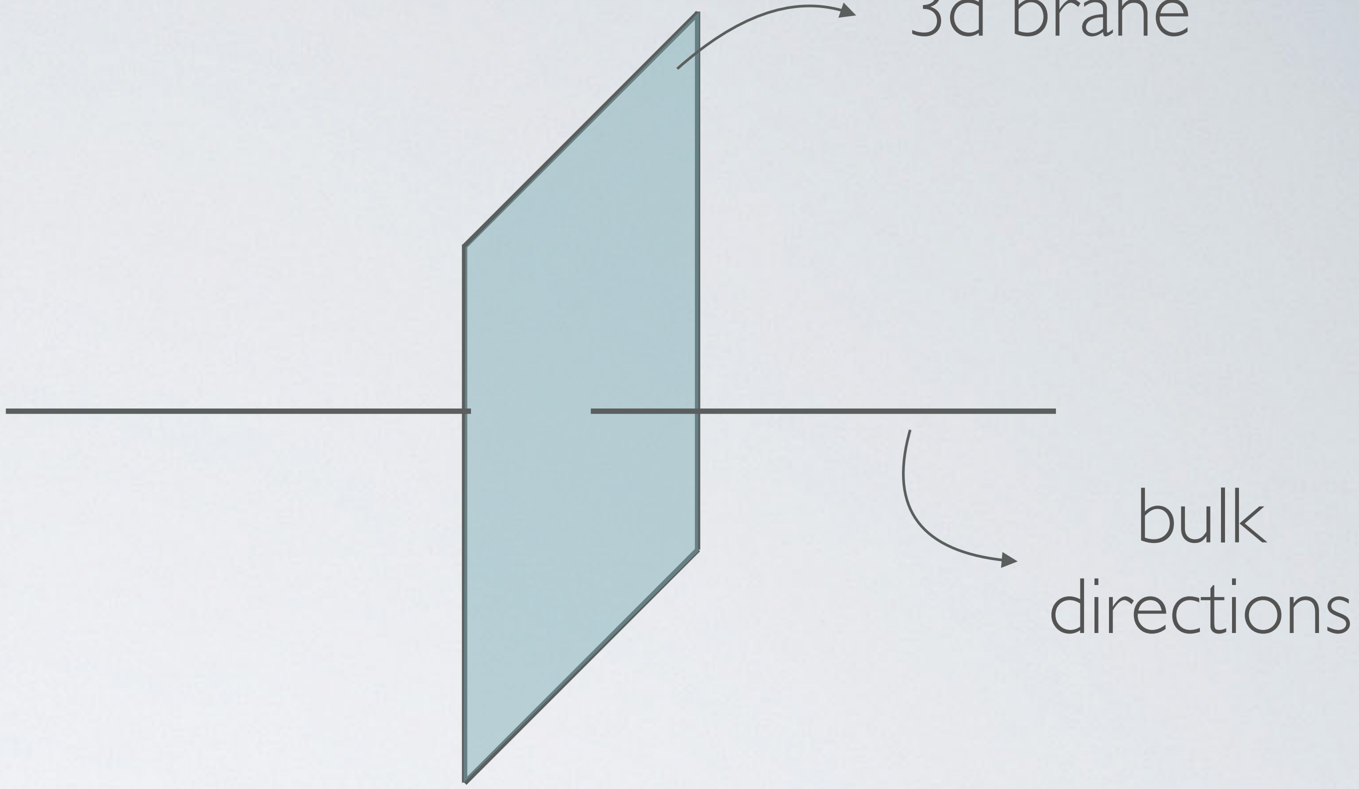

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KK compactification is, in fact, a special case of infinitely deep (square) potential well.

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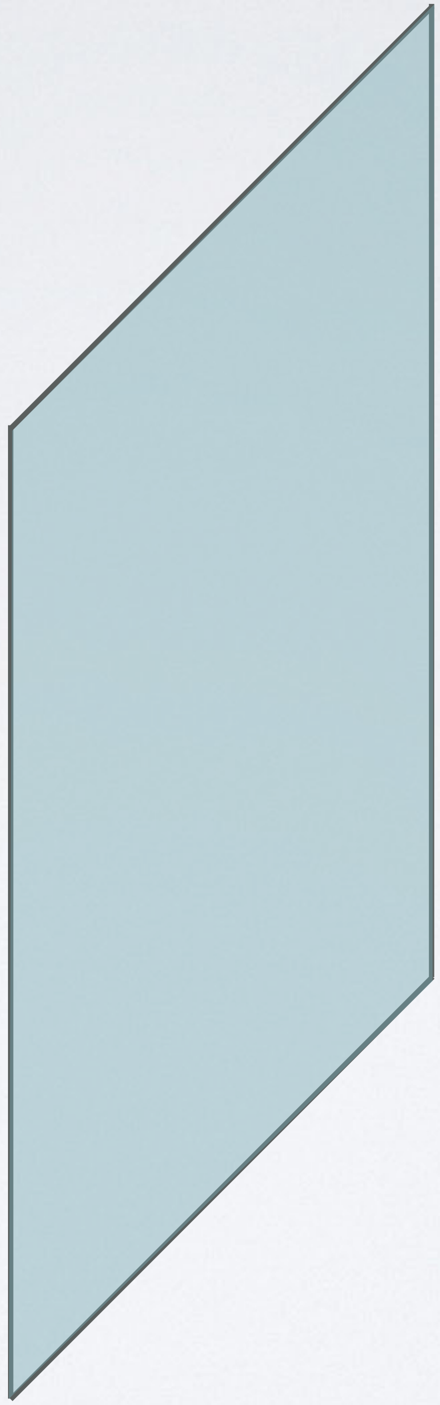

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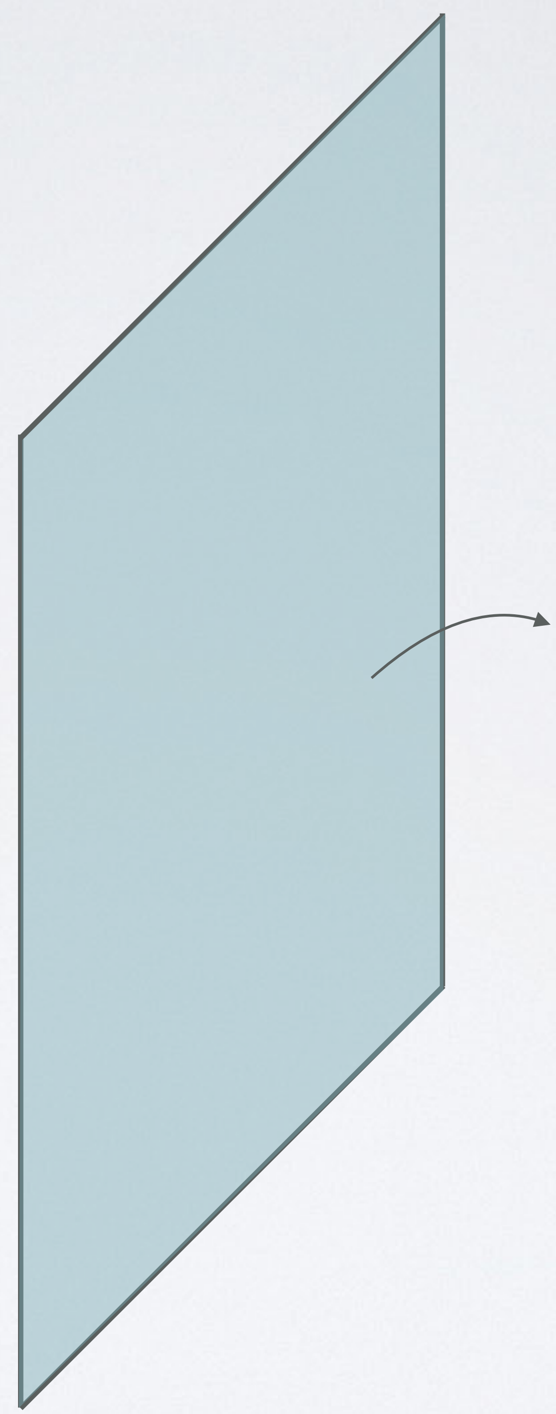

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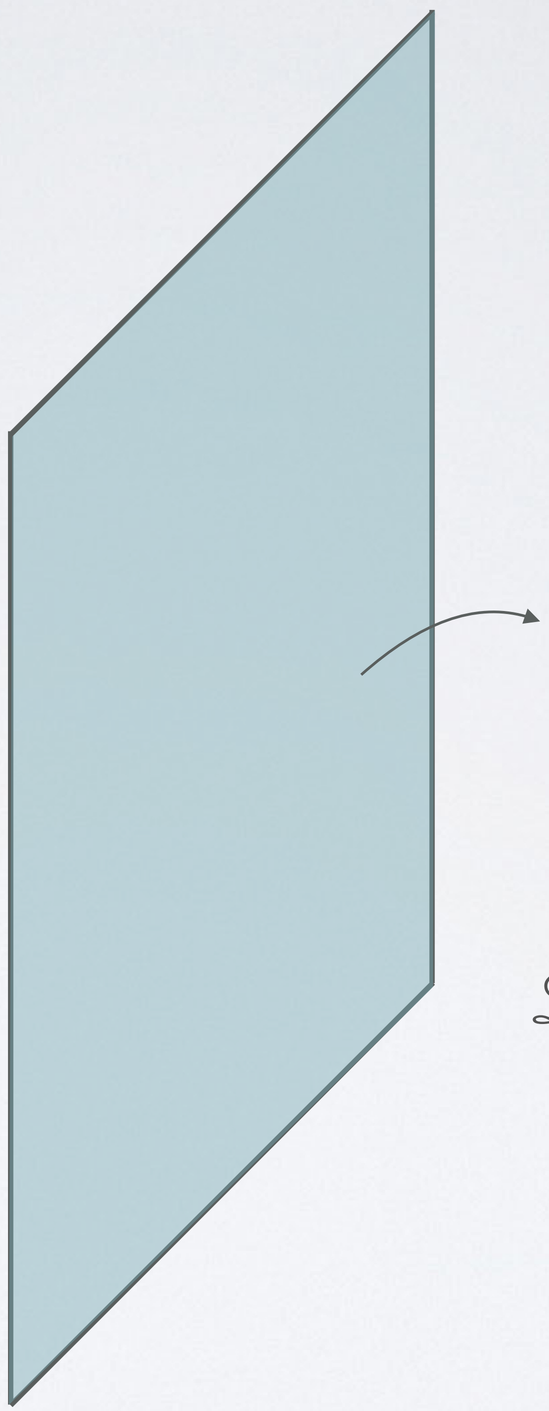

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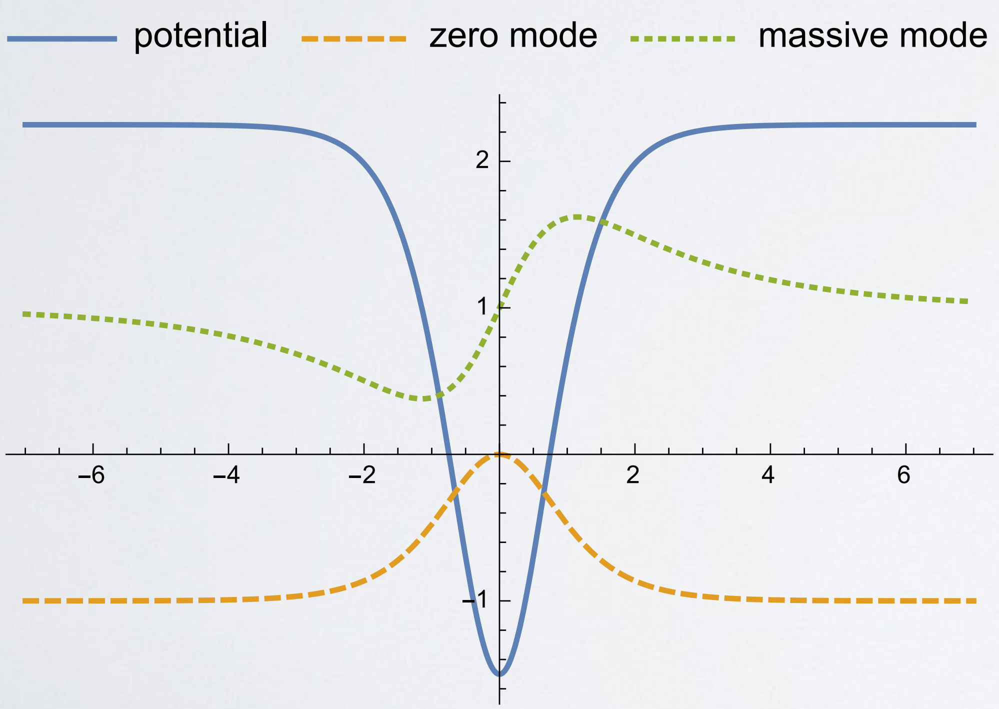
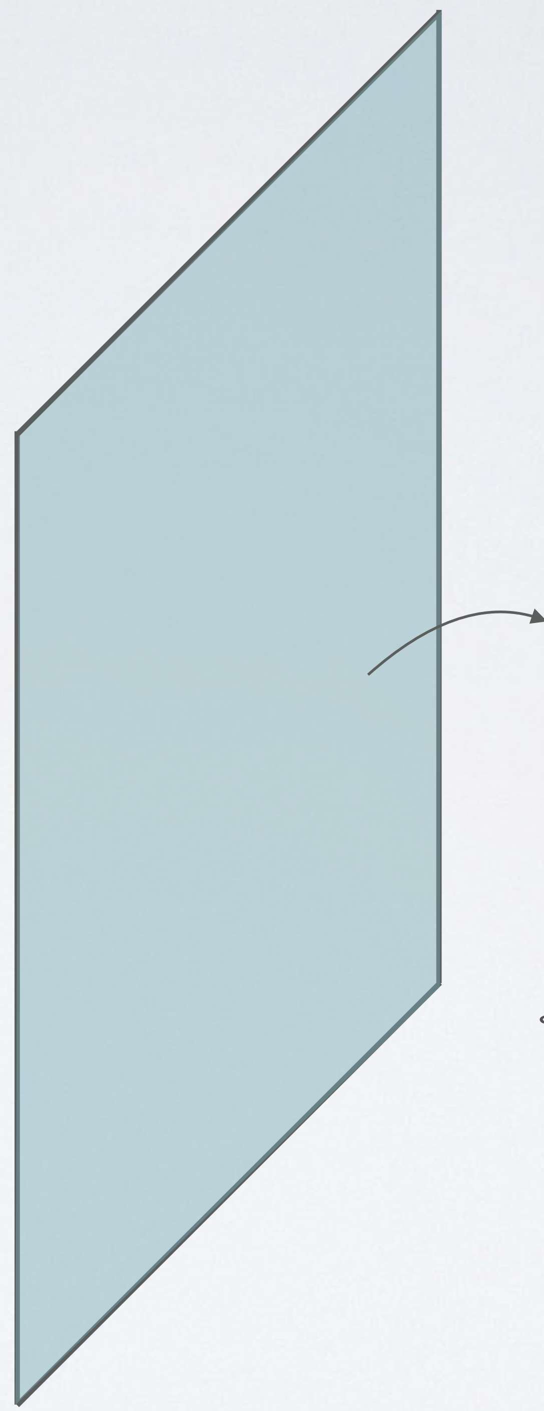
$$\mathscr{L} = \frac{1}{2}\left(\partial\Phi\right)^2 - \frac{\lambda^2}{2}\left(v^2 - \Phi^2\right)^2$$

$$\Phi_{\mathrm{DW}} = v\tanh\left(v\lambda y\right)$$

LOCALIZATION MECHANISMS

The idea: domain wall = position dependent vacuum.
Fields naturally condensate in the middle (false) vacuum





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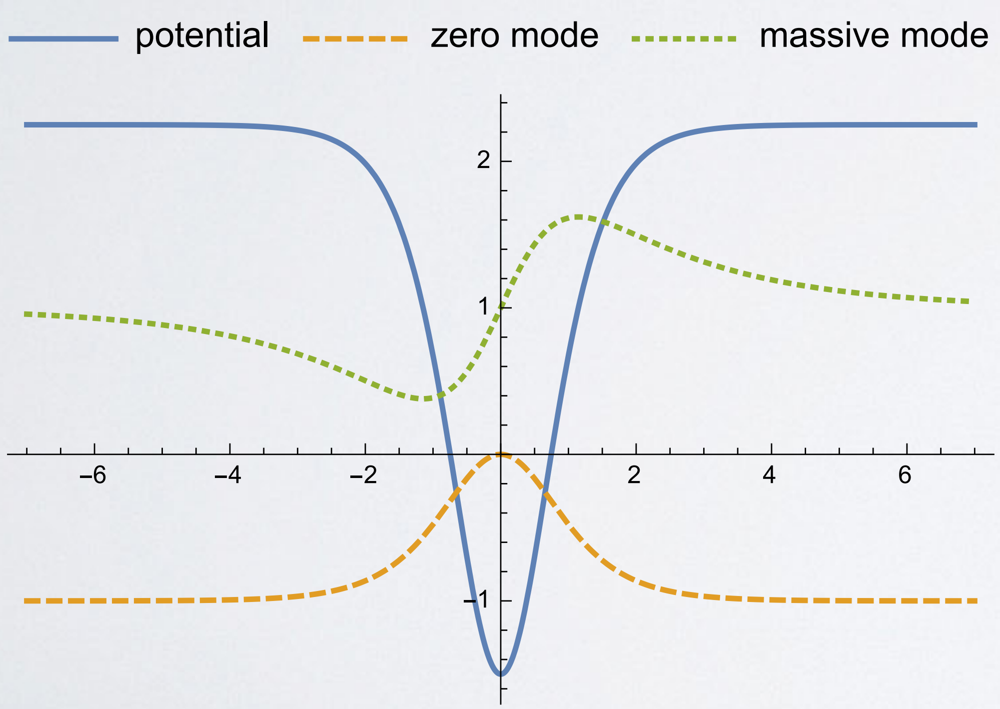

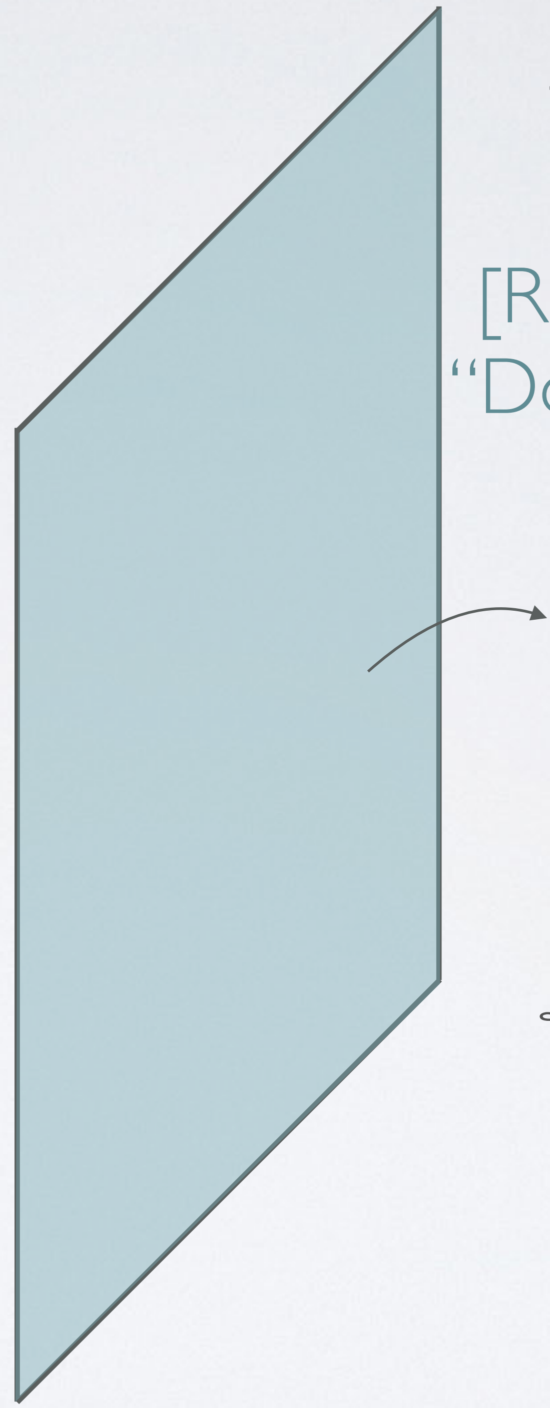

This works far better than it should
[Rubakov & Shaposhnikov, 1983]
"Do we live inside a domain wall?"

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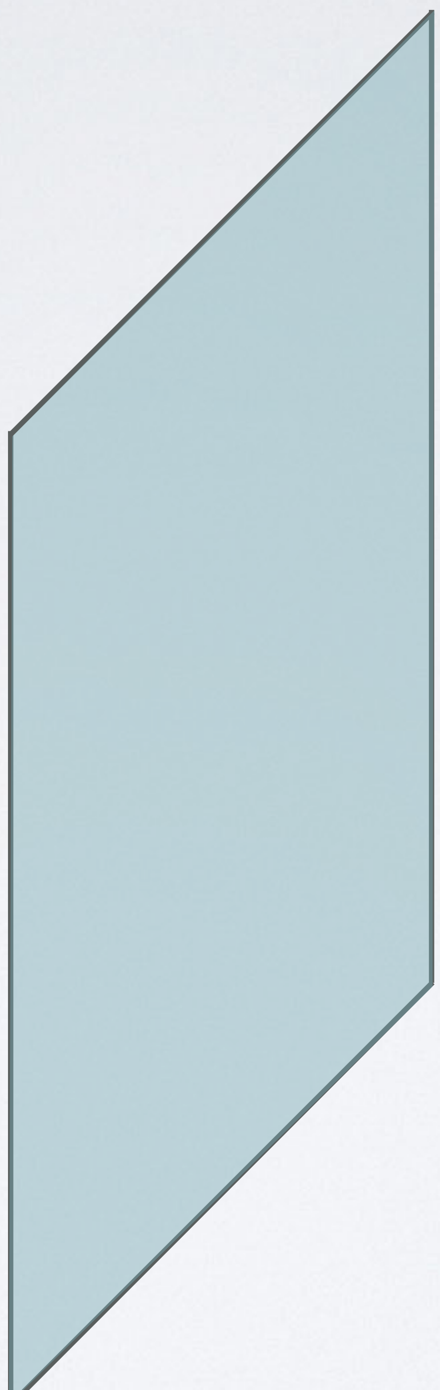
Typical domain wall profile

$$m(y) \sim \tanh(y)$$



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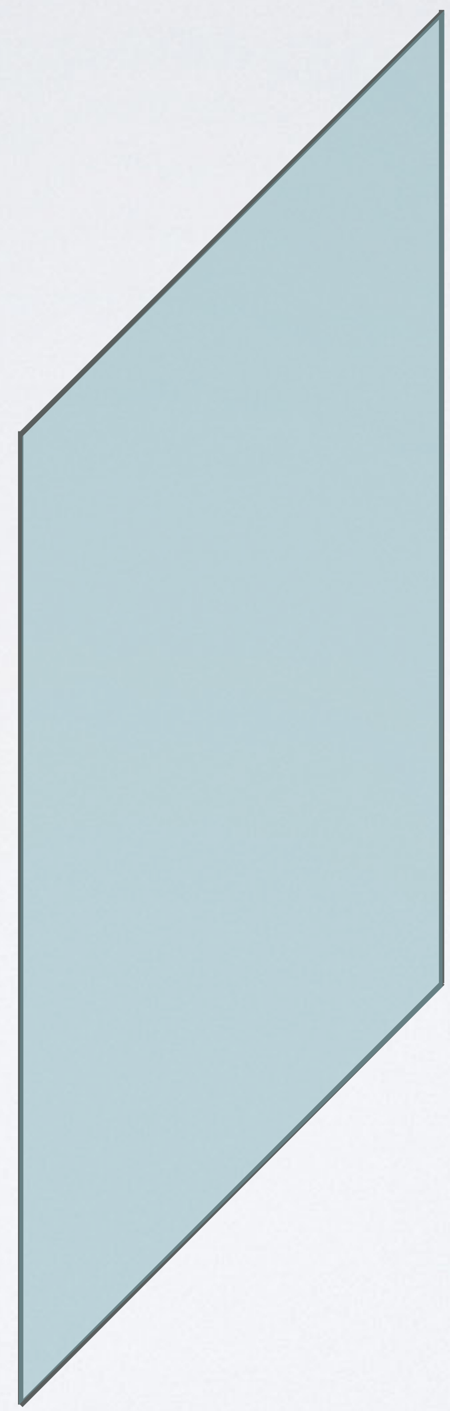

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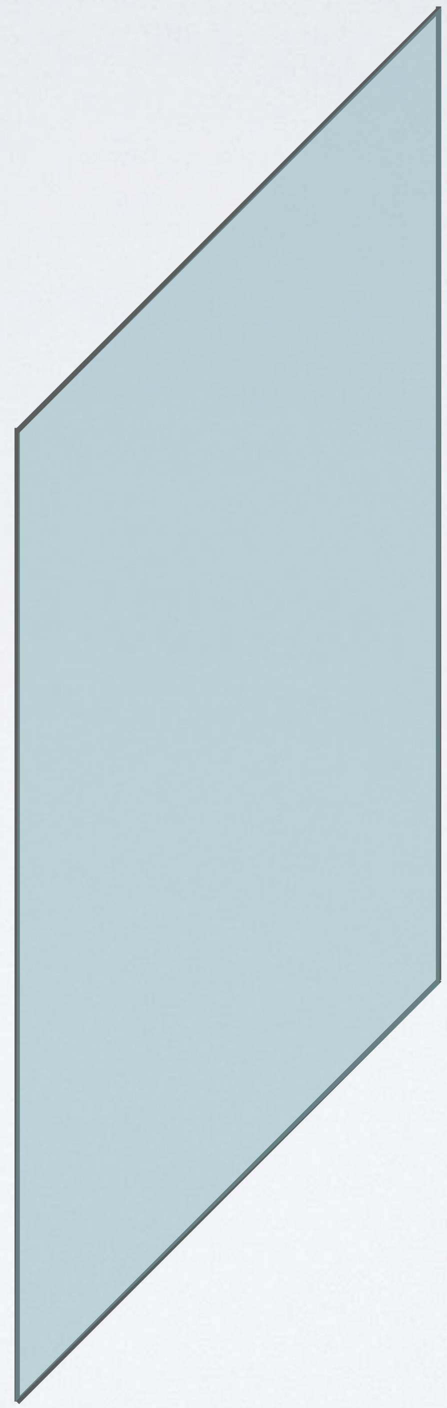

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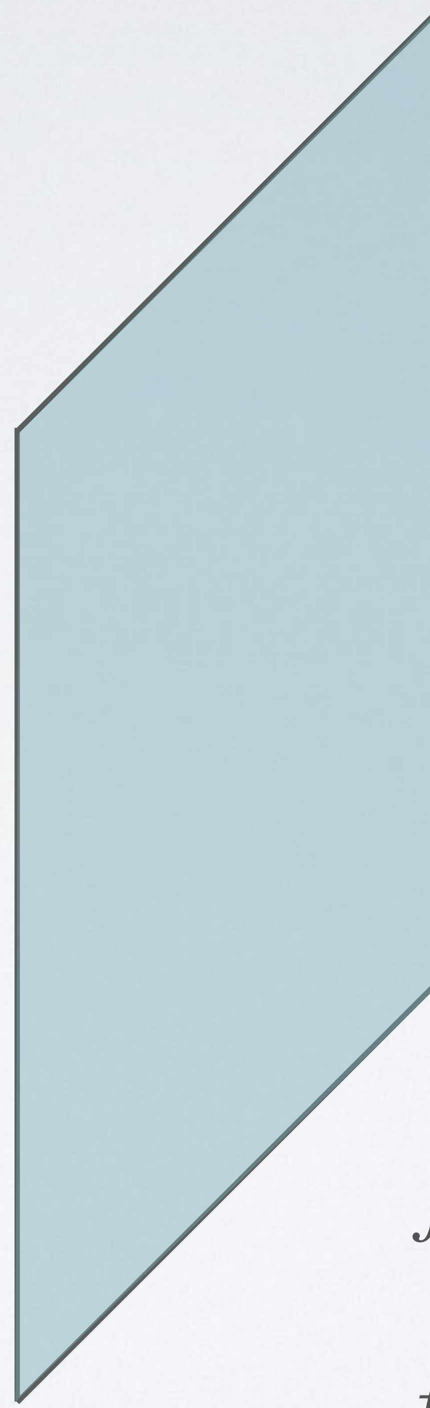

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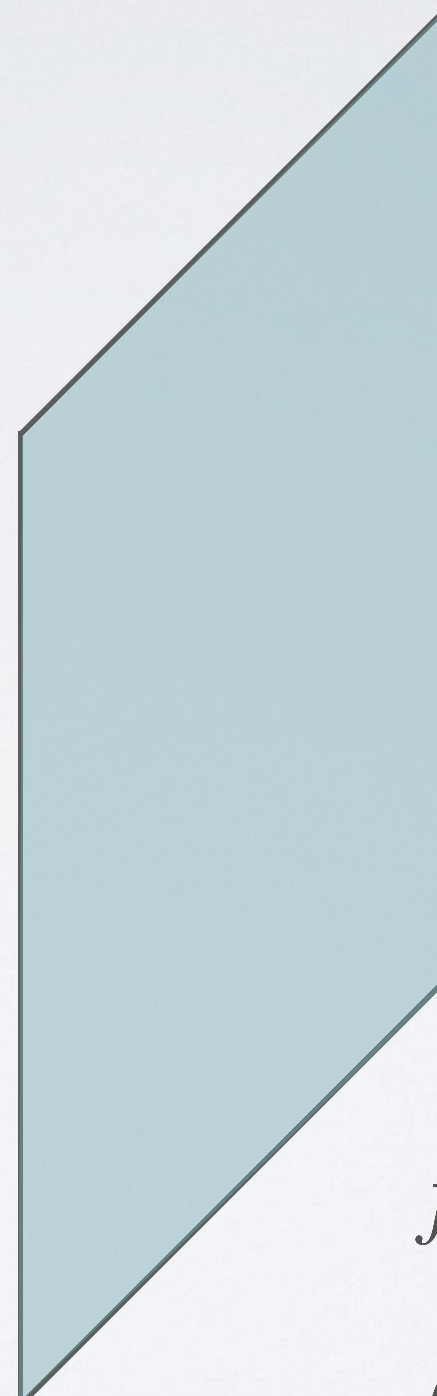

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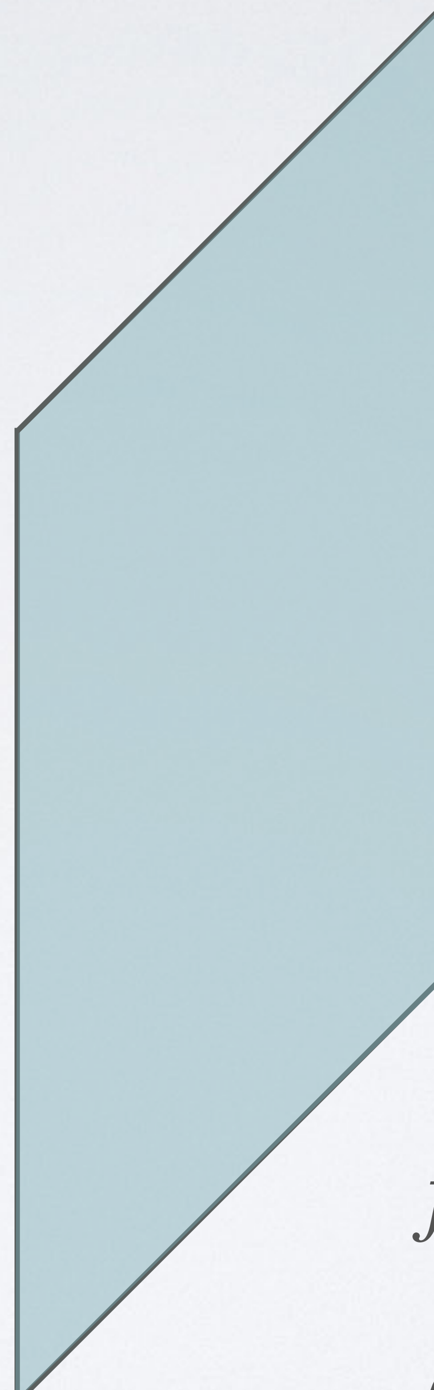

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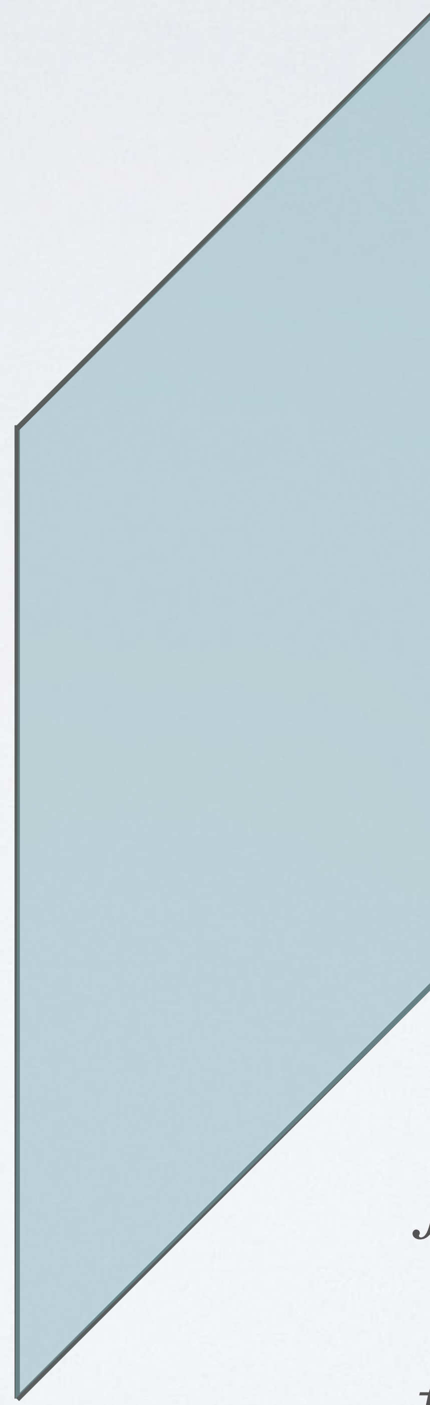
This is known as Jackiw-Rebbi mechanism in condense matter physics.



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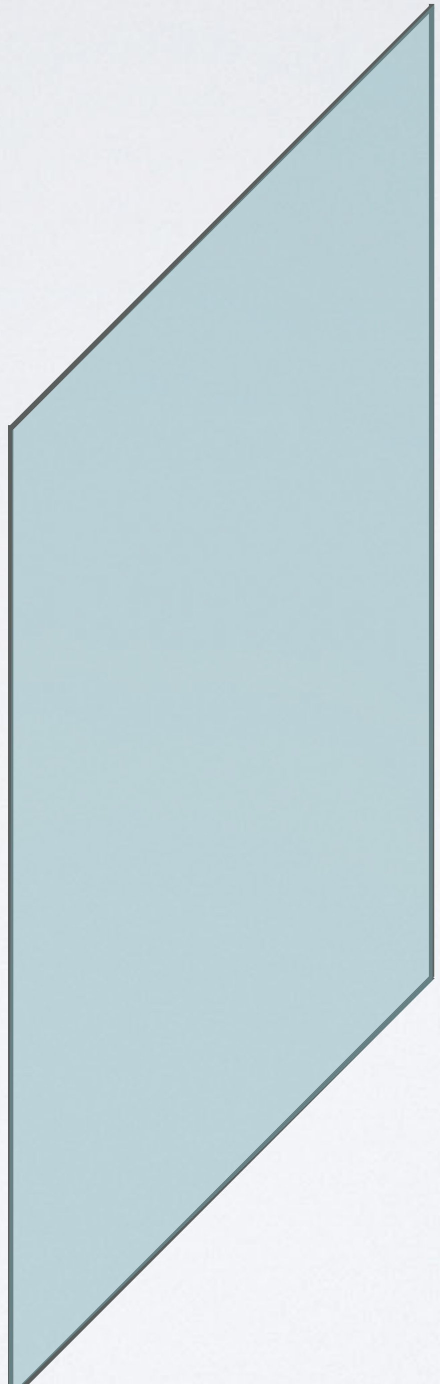

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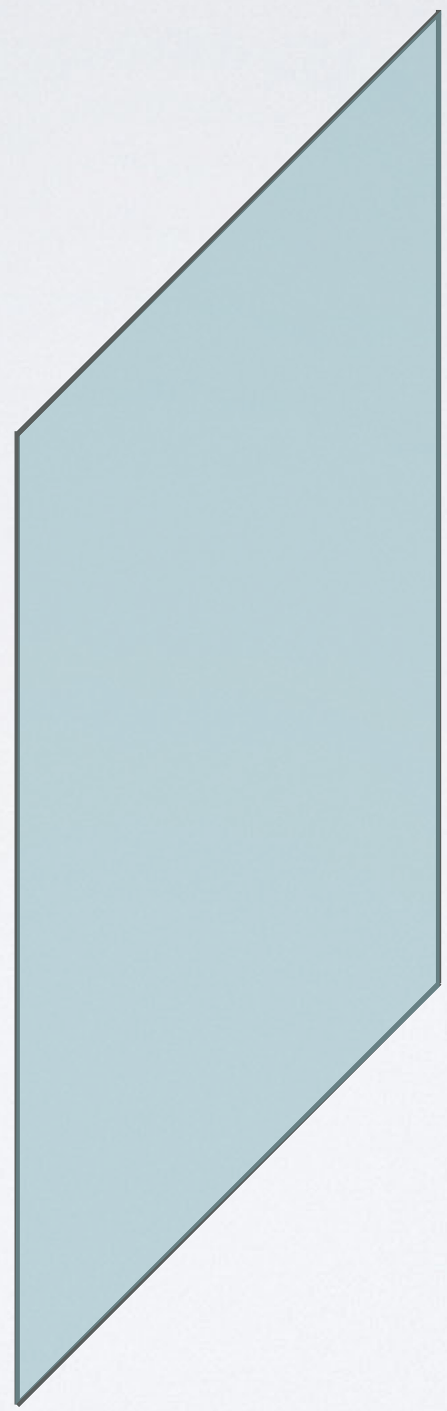

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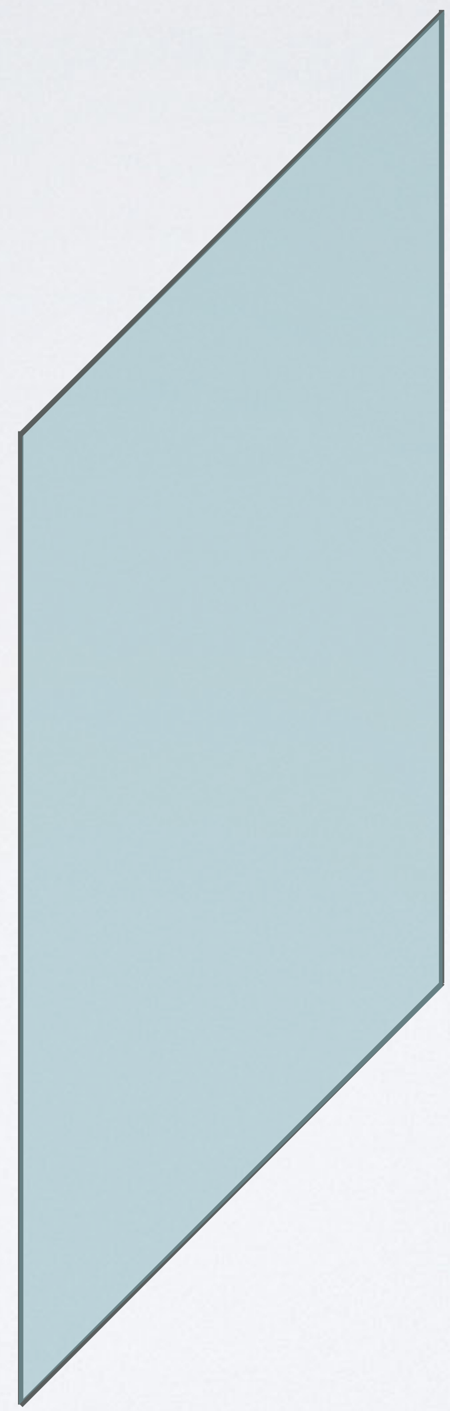

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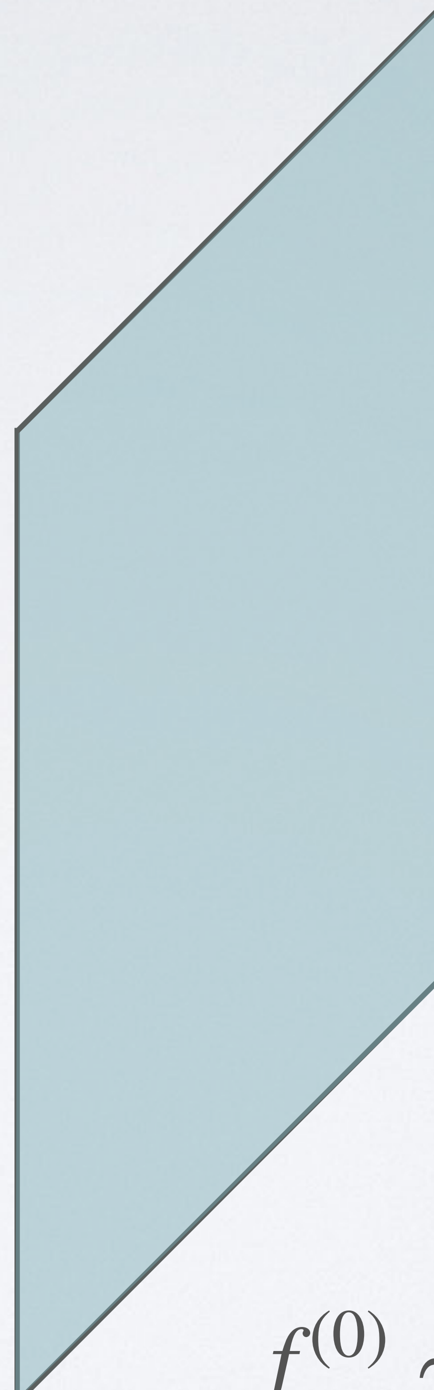

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hep-th:1801.02498

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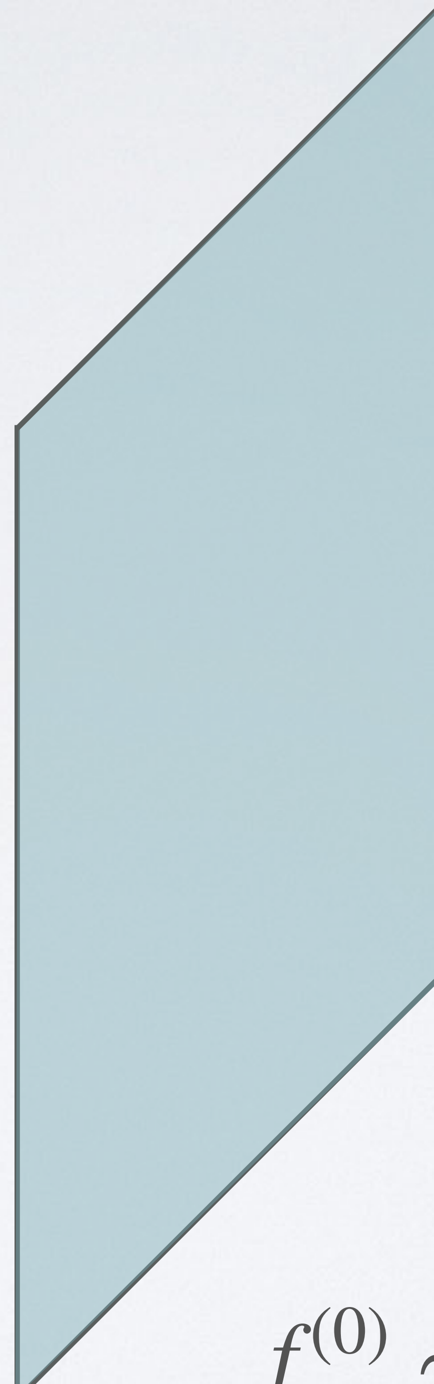

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hep-th:1811.08708

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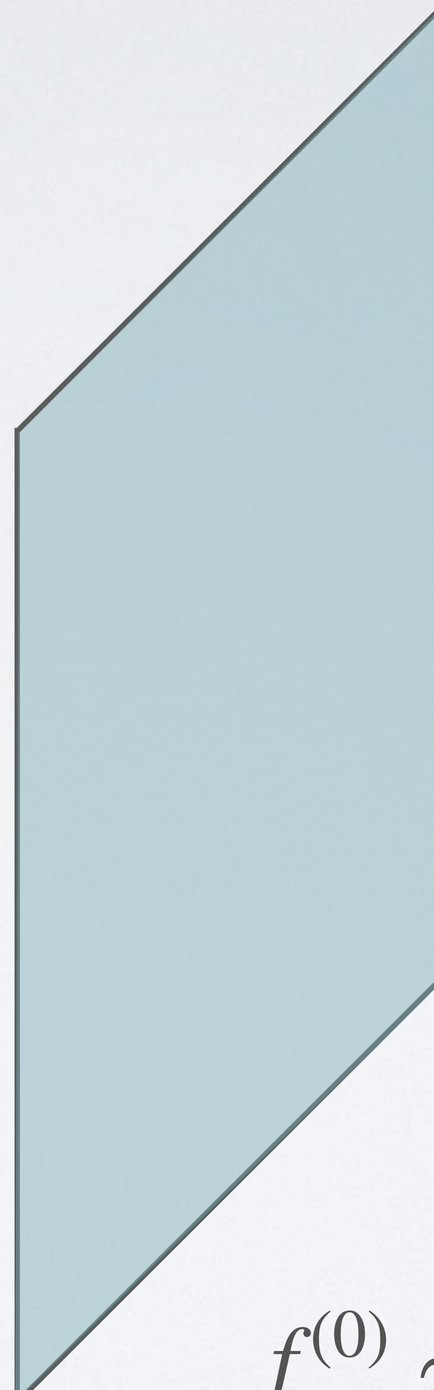

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DOMAIN WALL PARADIGM

- Successful localization of chiral fermions and gauge bosons with charge universality.
- Scalar fields (i.e. **Higgs**) can be localized using both position-dependent mass or coupling ideas (the latter is more robust, see hep-th\1811.08708).
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SM bugfixing/enhancements:

- gauge hierarchy problem
- fermion generations problem
- grand unification+geometric Higgs mechanism
- SUSY breaking
- seesaw, ….

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hep-th:1703.00351

PART II:

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THE GOAL OF OUR WORK

is to find **minimal but realistic** model with a single extra-dimension and a domain wall on which **SM arise as an effective four-dimensional low-energy theory**.

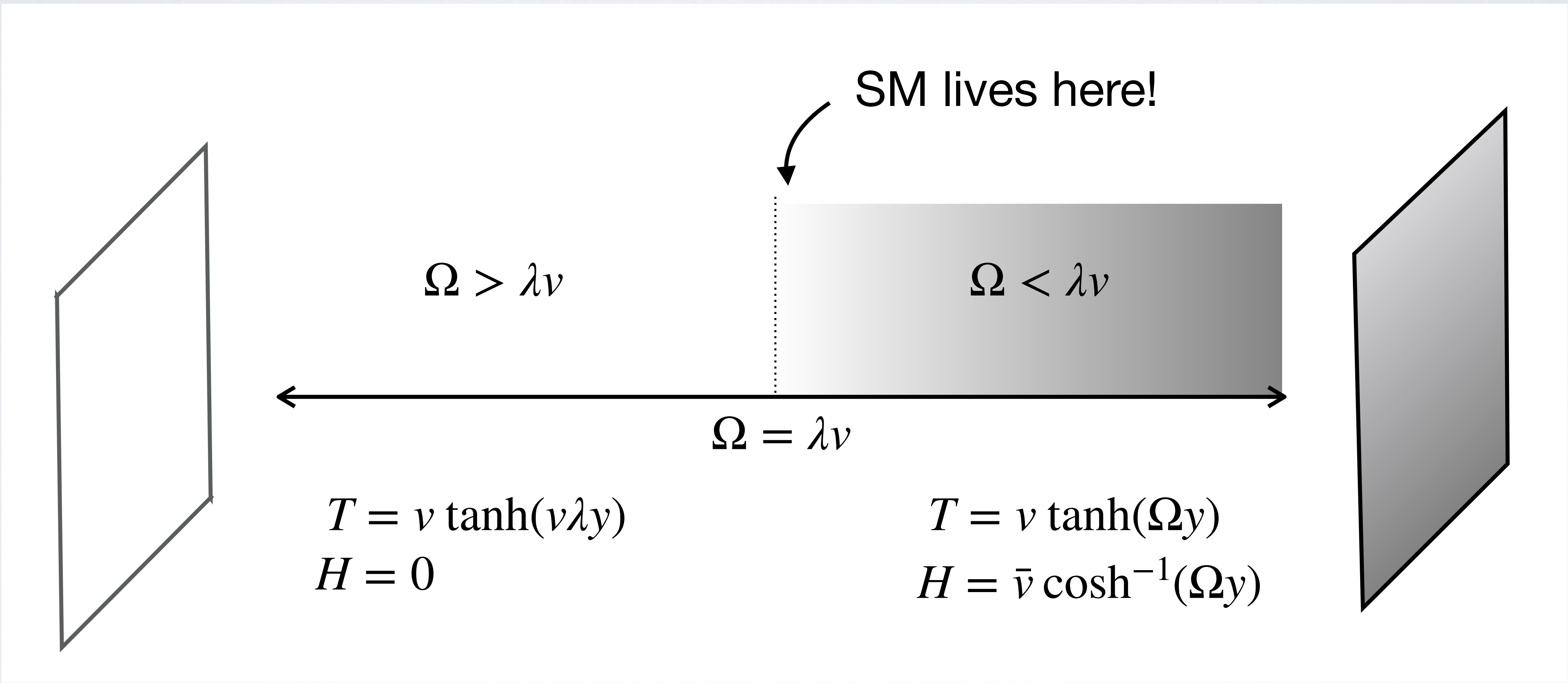
In our model, the Higgs:

- spontaneously breaks SM gauge group from *SU(2)xU(1)$_Y$* to *U(1)$_{em}$*
- gives mass to W and Z bosons and fermions via Yukawa magic
- **provides a localization mechanism for gauge fields.**

At the same time, a large gap between EW and 5D energy scales emerges naturally and protects low-energy physics from phenomenologically self-terminatory effects coming from extra dimensions.

SM AT THE CRITICAL POINT

In our model, the domain wall has **two phases** which are separated by **a critical point** in the parameter space, where the Higgs doublet obtains a non-trivial background.



This facilitates spontaneous symmetry breaking of SM gauge group but it also provides **position dependent gauge coupling** for gauge fields and renders the mass of the electroweak monopole finite.

A TOY MODEL

Let us illustrate how it works on a simple U(1) theory:

$$\mathcal{L} = -\beta(\mathcal{H})^2 \mathcal{F}_{MN}^2 + |\mathcal{D}_M \mathcal{H}|^2 + (\partial_M \mathcal{T})^2 - V(\mathcal{T})$$
$$+ i\bar{\Psi}\Gamma_M \mathcal{D}^M \Psi + i\bar{\tilde{\Psi}}\Gamma_M \partial^M \tilde{\Psi} + \left(\eta \mathcal{T}\bar{\Psi}\Psi - \tilde{\eta}\mathcal{T}\bar{\tilde{\Psi}}\tilde{\Psi} + \chi \mathcal{H}\bar{\Psi}\tilde{\Psi} + \text{h.c.}\right)$$

$$\beta(\mathcal{H})^2 = \frac{1}{4\mu^2}|\mathcal{H}|^2 \qquad V = \Omega^2|\mathcal{H}|^2 + \lambda^2\left(|\mathcal{H}|^2 + \mathcal{T}^2 - v^2\right)^2,$$

The domain wall has a Higgs condensation above the threshold:

$$\mathcal{T}_{\text{bkg}} = v\tanh\lambda v y, \qquad \mathcal{H}_{\text{bkg}} = 0, \qquad (\lambda v \le \Omega)$$
$$\mathcal{T}_{\text{bkg}} = v\tanh\Omega y, \qquad \mathcal{H}_{\text{bkg}} = \bar{v}\,\text{sech}\,\Omega y, \qquad (\lambda v > \Omega)$$

Slightly above the critical point, the Higgs has a nearly massless mode:

$$\mathcal{H}(x,y) = \sqrt{\frac{\Omega}{2}}\,H(x)\,\text{sech}\,\Omega y$$

$$\mathcal{L}_{\text{Higgs}}(H) = |D_\mu H|^2 - V_H, \quad V_H = \lambda_2^2|H|^2 + \frac{\lambda_4^2}{2}|H|^4,$$
$$\lambda_2^2 = -\frac{4\lambda^2\bar{v}^2}{3}, \quad \lambda_4^2 = \frac{2\lambda^2\Omega}{3},$$

ARBITRARY LARGE MASS GAP

5D parameters

$[\Omega] = 1$	$[v\lambda] = 1$	$[\mu] = 1$	$[\eta, \tilde{\eta}, \chi] = -1/2$
Wall thickness phase II	wall thickness phase I	5D gauge coupling	Yukawas

effective 4D parameters

$m_A = \sqrt{2}\mu$	$v_h = \frac{2\sqrt{\Omega}}{\lambda}\varepsilon$	$m_h = \sqrt{\frac{8}{3}\Omega\,\varepsilon}$	$e = \frac{\sqrt{2}\mu}{v_h}$
gauge bosons mass	Higgs VEV	Mass of the Higgs	4D gauge coupling

Mild fine-tuning gives an arbitrary large mass gap

$$\Omega\,\varepsilon \equiv \sqrt{v^2\lambda^2 - \Omega^2} \sim \mu \sim 10^2 \text{ GeV}$$

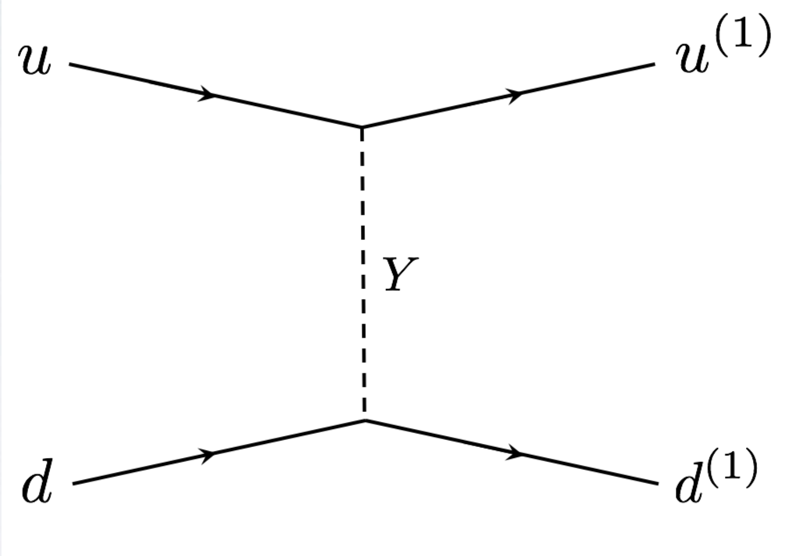
$$\Omega \sim \lambda^{-2} \sim v^{\frac{2}{3}} \sim \eta^{-2}, \tilde{\eta}^{-2}, \chi^{-2} \geq 10^3 \text{ GeV}$$

SMOKING GUNS OF OUR MODEL

Production channel for KK quarks via NG boson *Y*

$$T = v \tanh\left(\Omega y - \frac{1}{f_Y} Y(x)\right) \quad H = \sqrt{\frac{\Omega}{2}} H(x)\,\mathrm{sech}\left(\Omega y - \frac{1}{f_Y} Y(x)\right)$$

$$\int_{-\infty}^{\infty} dy\, i\bar{\Psi}\Gamma_M D^M \Psi \supset i\alpha \frac{\sqrt{\Omega}}{v} \partial_\mu Y \left(\bar{\psi}_L^{(1)} \gamma^\mu \psi_L^{(0)} - \bar{\psi}_L^{(0)} \gamma^\mu \psi_L^{(1)}\right)$$



New tree-level diagram for $h \to \gamma\gamma$

$$H = \bar{v}\left(1 + \frac{\sqrt{2}h(x)}{v_h}\right) \mathrm{sech}\,\Omega y\,.$$

$$-\int_{-\infty}^{\infty} dy\, |\beta|^2 (\mathscr{F}_{MN})^2 = -\frac{1}{4}\left(1 + 2\frac{\sqrt{2}h}{v_h} + \frac{2h^2}{v_h^2}\right)(F_{\mu\nu}^{(0)})^2\,.$$

MONOPOLE MASS

In SM the mass of the Cho-Maison monopole is divergent.
We can regularize it by assuming non-canonical kinetic term for $U(1)_Y$.
In our model, this regularization is a byproduct of localization of SM gauge fields on the domain wall.

EMY proposal: $$\epsilon_1 = 5\left(\frac{H}{v_h}\right)^8 - 4\left(\frac{H}{v_h}\right)^{10}$$

Our model: $$\beta^2 = \frac{|H|^2}{\mu^2}\left(10\frac{|H|^6}{\bar{v}^6} - 9\frac{|H|^8}{\bar{v}^8}\right)$$

$$-\int_{-\infty}^{\infty} dy\, \beta^2 (\mathscr{B}_{\mu\nu})^2 = -\frac{\epsilon_1}{4}(B^{(0)}_{\mu\nu})^2$$

THANK YOU!