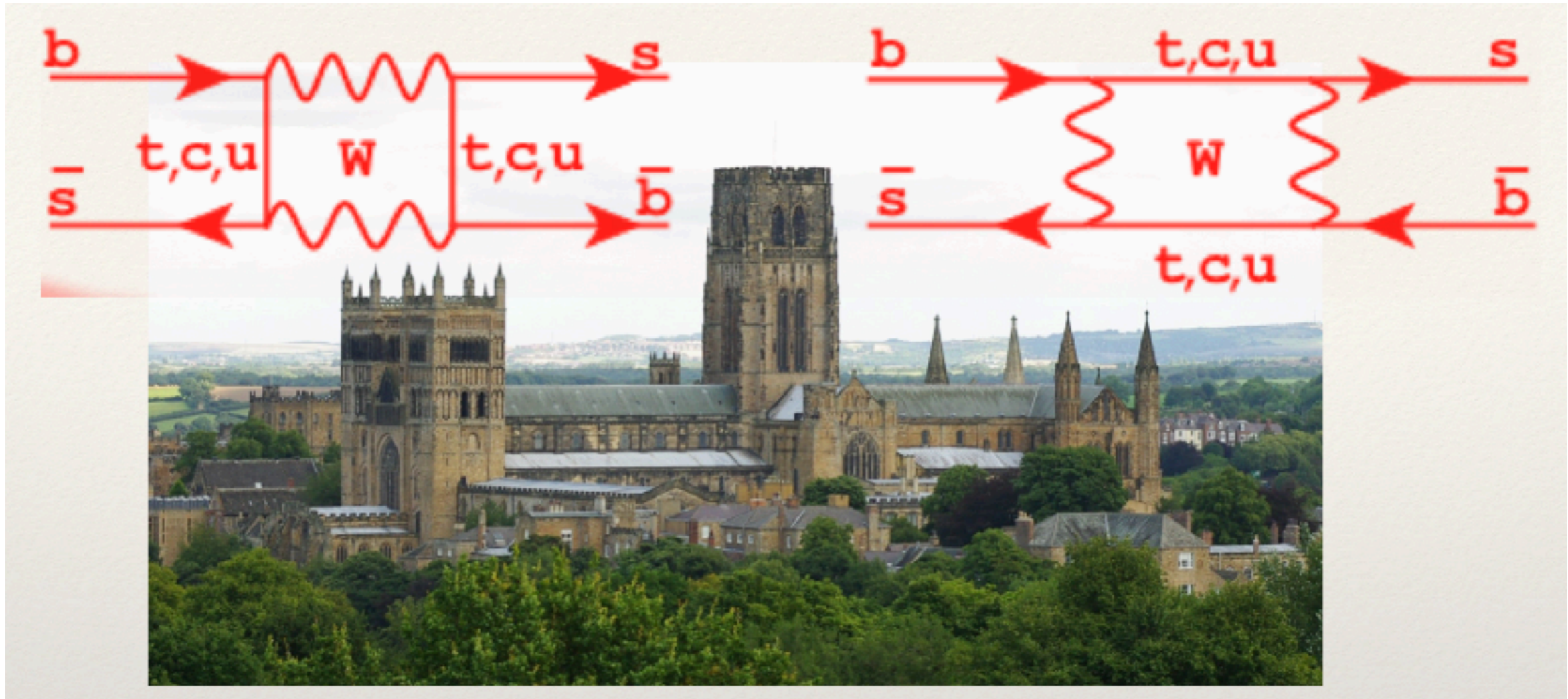


Interplay of Bs mixing and Flavour Anomalies



4.7.2019, Bristol, Amplitude Analysis
Alexander Lenz, IPPP Durham

Messages from the machine room to the top deck



4.7.2019, Bristol, Amplitude Analysis Taken from Thomas Mannel
Alexander Lenz, IPPP Durham

Outline

- **Flavour Anomalies and BSM explanations**
- **B-mixing: Anomalie vs. Bound on BSM explanations**
- **Status of theory predictions for B-mixing**

Fresh from the press!!!!

FLAVOUR ANOMALIES

➤ **Message 3:** First deviations start to show up **and they stay**

σ

- 3-6: Semi-leptonic loop-level decays (small BSM)
- 3.9: Semi-leptonic tree-level decays (large BSM)
- 3.6: B-mixing phase (dimuon asymmetry)
- 3.5: Muon $g-2$
- 2.8: K-mixing/ ϵ' (huge lattice progress)
- 2.6: Zbb coupling (LEP FB asym)
- 2.x: K-pi puzzle
- 2.x: tau to mu nu nu/tau to e nu nu
- 2.x: V_{us} : K vs. tau
- 2.0: B-mixing modulus (mass difference)

4 σ in neutron lifetime? Proton radius seems to be solved by Hänsch et al

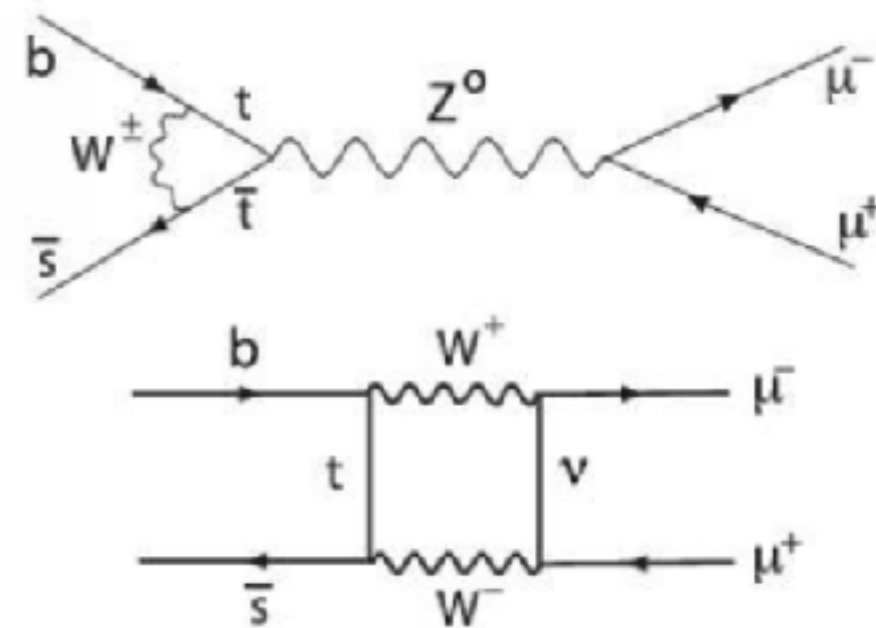
SEMI-LEPTONIC LOOP LEVEL

$$b \rightarrow s\mu\mu$$

“relatively” simple hadronic structure

$B_{d,s} \rightarrow \mu\mu$: decay constant

$H_b \rightarrow H_q\mu\mu$: form factor



Can be determined with lattice, sum rules,...

Observables:

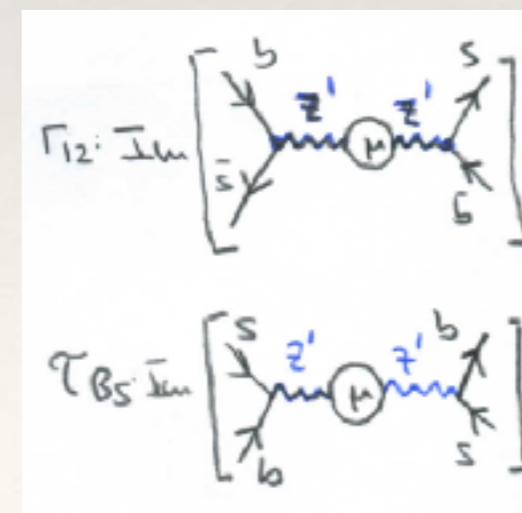
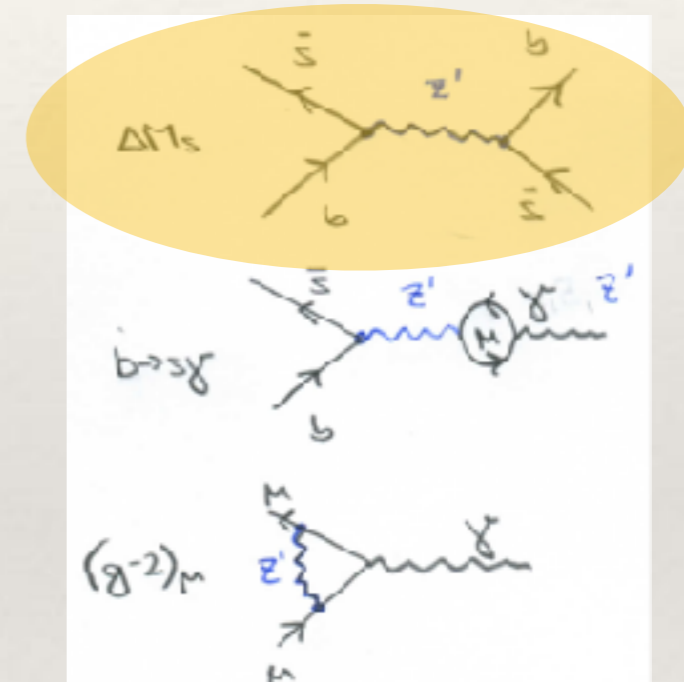
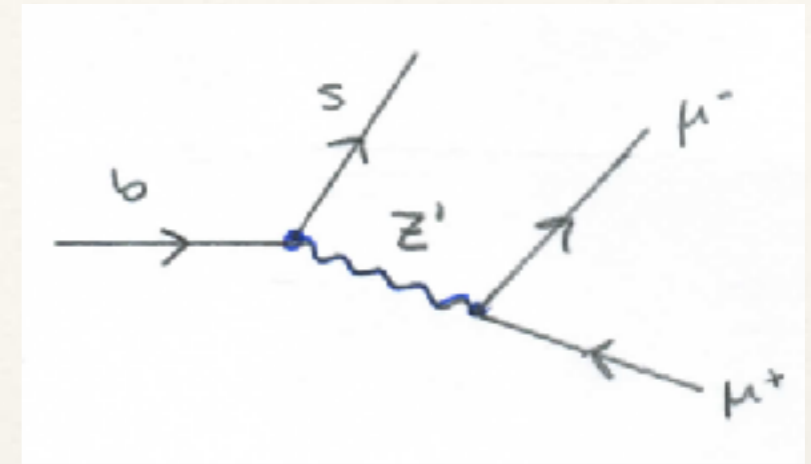
- Branching ratios $Br(B_s \rightarrow \phi\mu\mu), Br(B \rightarrow K^*\mu\mu),$
- Angular observables, e.g. P'_5 hadronic uncertainties cancel partially
- Ratios $R_K = \frac{Br(B^+ \rightarrow K^+\mu^-\mu^+)}{Br(B^+ \rightarrow K^+e^-e^+)}$ hadronic uncertainties cancel completely

BSM physics is on the Horizon?

A popular BSM model for solving the anomalies related to loop-level (semi) leptonic decays are Z' models:

Such a new tree-level transition will also affect many other observables, most notably **B-mixing at tree-level**, but also many loop processes.

Make sure all relevant bounds are included, e.g. electro-weak precision bounds



NP IN TREE-LEVEL DECAYS



Do a systematic study of tree-level observables that are both well known in experiment and theory

Main Chapters

Introduction

Why Time Dilation must be impossible

$$C_{1,2}^{SM} \rightarrow C_{1,2}^{SM} + \Delta C_{1,2}$$

4.3 Constraints from $b \rightarrow u\bar{u}d$ transitions

4.3.1 R_{KK}

4.3.2 $S_{\pi\pi}$ and $S_{\rho\pi}$

4.3.3 $R_{\rho\rho}$

4.4 Constraints from $b \rightarrow c\bar{u}d$ transitions

4.4.1 $\bar{B}^0 \rightarrow D^{*+}\pi^-$

4.4.2 S_{D^*h}

4.5 Observables constraining $b \rightarrow c\bar{c}d$ transitions

4.5.1 M_{12}^d

4.5.2 $B \rightarrow X_d\gamma$

4.6 Constraints from $b \rightarrow c\bar{c}s$ transitions

4.6.1 $\bar{B} \rightarrow X_s\gamma$

4.6.2 $\text{Sin}(2\beta_d)$

4.7 Constraints using multiple channels observables: α_{at}^d , α_{at}^d and $\Delta\Gamma$

$$\hat{\mathcal{H}}_{eff} = \frac{V_{cb}V_{ud}^*}{\sqrt{2}} (C_1\hat{Q}_1 + C_2\hat{Q}_2)$$

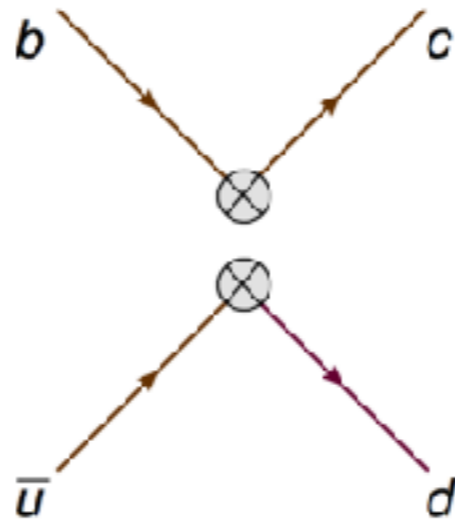
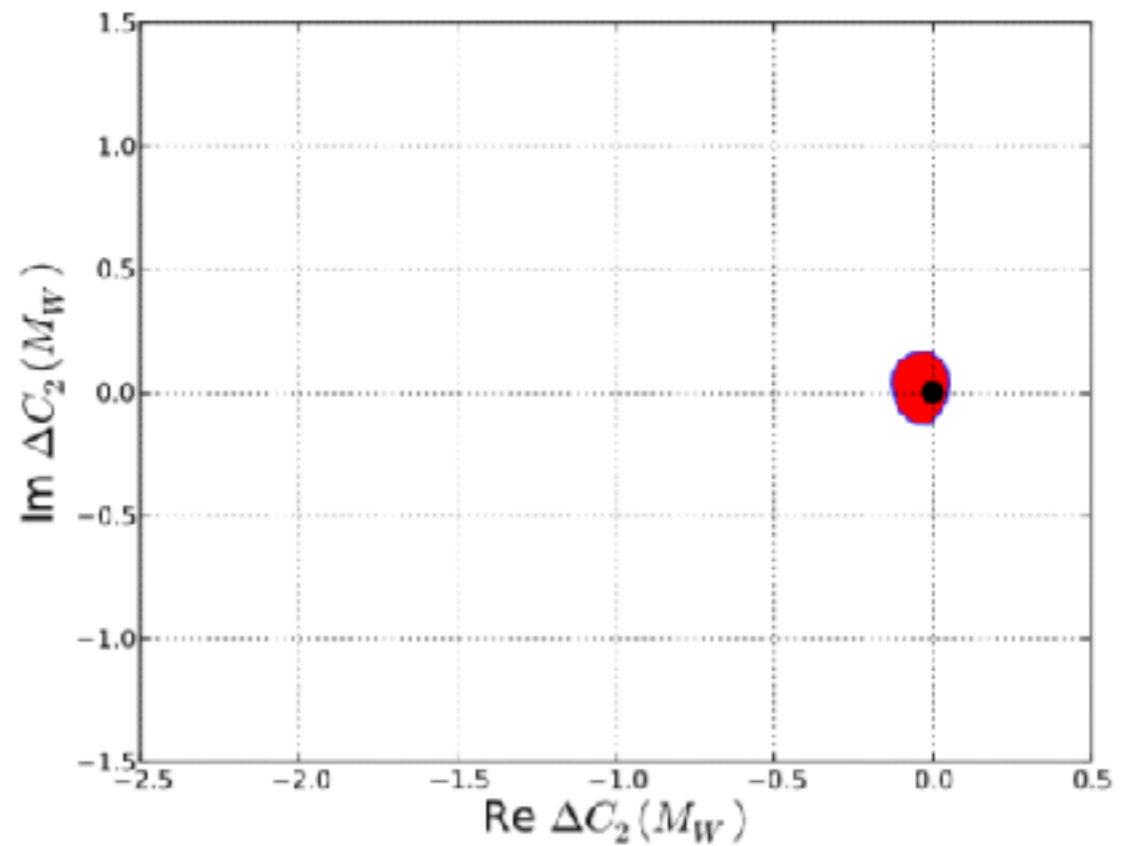
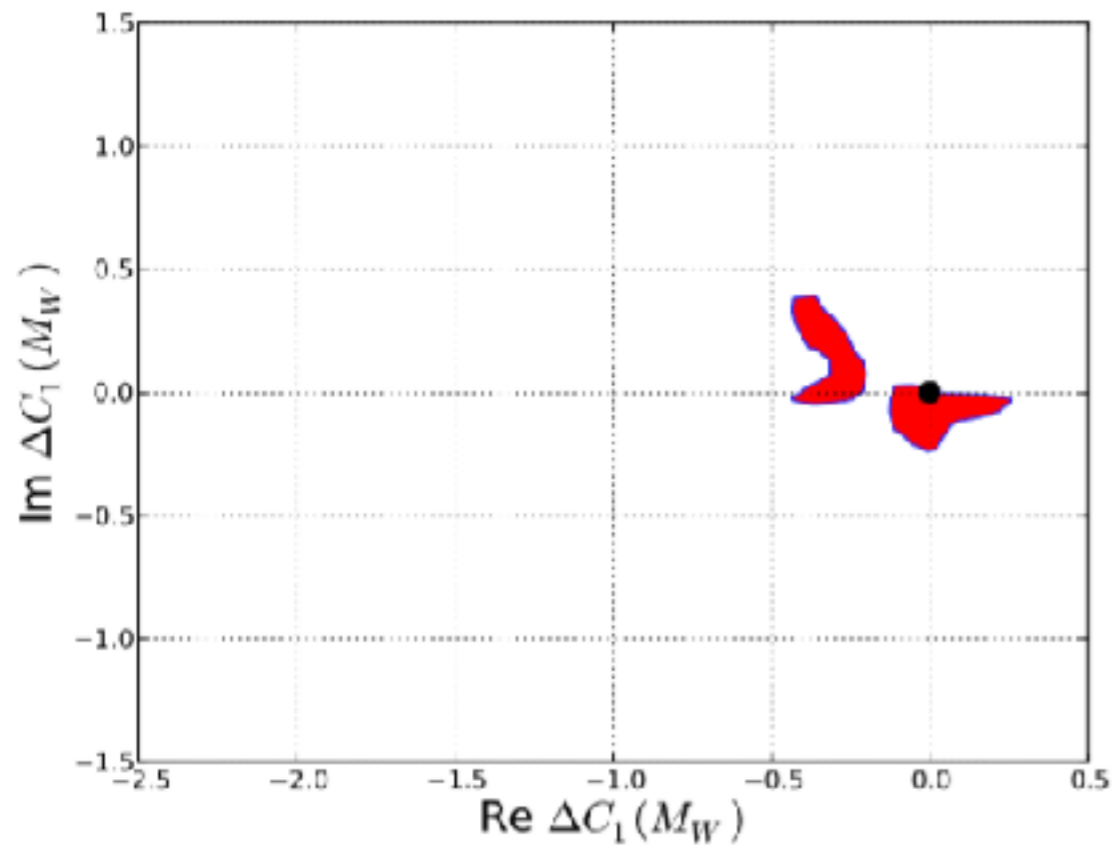


IMAGE Anyone else get these weird emails from Gabor Fekete? (1.png)
submitted 2 years ago by Astrekhw (Astrophysics)
72 comments share

$t_a = \frac{1}{\omega}$
 $\omega = 2\pi \cdot f = \frac{1}{\sqrt{L \cdot C}}$
 $s_a = \frac{c^2}{a} = \frac{\lambda}{2\pi}$
 $r \cdot \omega = c$
 $m \cdot r \cdot \omega = m \cdot c$
 $m \cdot r^2 \cdot \omega^2 = m \cdot c^2$
 $F_{cf} = m \cdot r \cdot \omega^2$
 $r = A = \frac{\lambda}{2\pi}$
 $F_a = E_{em} \cdot \frac{2\pi}{\lambda}$
 a, F_a, c, P, E_k
 the direction of movement
 $f = \frac{c}{\lambda}$
 $m = 7.3724191 \cdot 10^{-51} \text{ kgs} \cdot \Delta f = \dots \text{ kg}$
 $E_{em} = h \cdot f = \frac{1}{2} C \cdot U^2 + \frac{1}{2} L \cdot I^2$ the effect cross-section
 $A_e = 2r \cdot d$

NP IN TREE-LEVEL DECAYS

Result:



What does this mean?

Is this an important effect?

NP IN TREE-LEVEL DECAYS

- Decay rate difference of neutral Bd mesons, $\Delta\Gamma_d$, can be enhanced by several 100%

work triggered by D0 di-muon asymmetry - **Borissov**
work triggered ATLAS measurement of $\Delta\Gamma_d$ - **Borissov**

On new physics in $\Delta\Gamma_d$
Bobeth, Haisch, Lenz, Pecjak, Tetlalmatzi-Xolocotzi
JHEP 1406 (2014) 040

- Extraction of CKM angle γ can be modified by several degrees

SM precision: 1 ppm

Experimental precision: now 6deg, future 1 deg

NP effects in tree-level decay and the precision of γ
Brod, Lenz, Tetlalmatzi-Xolocotzi Alexander Lenz
Rev.Mod.Phys. 88 (2016) no.4,045002

- More profound analysis in progress

AL, Tetlalmatzi-Xolocotzi

till now only SM Dirac structures

BSM PHYSICS IS ON THE HORIZON?

**Look for non-standard BSM models*

Is there a connection between mixing and rare decays?

Charming new physics in rare B-decays and mixing

Jaeger, Kirk, Lenz, Leslie

arXiv: 1701.09183

New paper 1901.xxxxx

Consider NP in tree-level $b \rightarrow ccs$ traditions with general Dirac structures

$$\mathcal{H}_{\text{eff}}^{c\bar{c}} = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_{i=1}^{10} (C_i^c Q_i^c + C_i^{c'} Q_i^{c'})$$

$$\begin{aligned} Q_1^c &= (\bar{c}_L^i \gamma_\mu b_L^j)(\bar{s}_L^j \gamma^\mu c_L^i), & Q_2^c &= (\bar{c}_L^i \gamma_\mu b_L^i)(\bar{s}_L^j \gamma^\mu c_L^j), \\ Q_3^c &= (\bar{c}_R^i b_L^j)(\bar{s}_L^j c_R^i), & Q_4^c &= (\bar{c}_R^i b_L^i)(\bar{s}_L^j c_R^j). \end{aligned} \quad (2)$$

This affects rare decays and mixing/lifetimes:

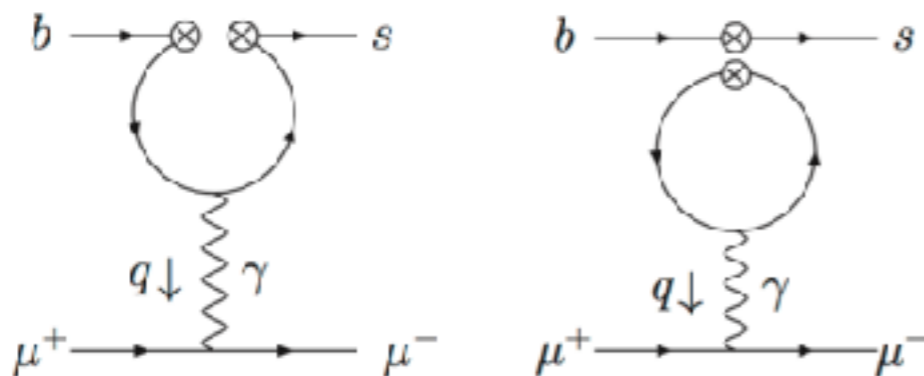


FIG. 1. Leading Feynman diagrams for CBSM contributions to rare and semileptonic decays. With our choice of Fierz-ordering, only the diagram on the left is relevant.

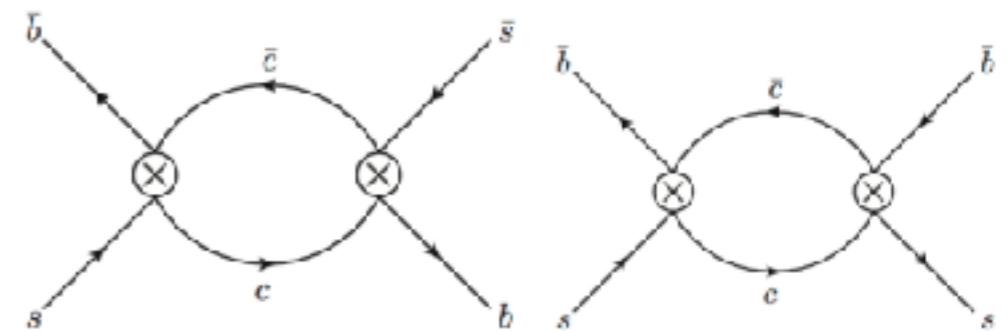


FIG. 2. Leading Feynman diagrams for CBSM contributions to the width difference $\Delta\Gamma_s$ (left) and the lifetime ratio $\tau(B_s)/\tau(B_d)$ (right).

q^2 dependent BSM contributions possible

This was considered to be a smoking gun for a hadronic origin of the anomalies

Outline

- **Flavour Anomalies and BSM explanations**
- **B-mixing: Anomalie vs. Bound on BSM explanations**
- **Status of theory predictions for B-mixing**

Fresh from the press!!!!

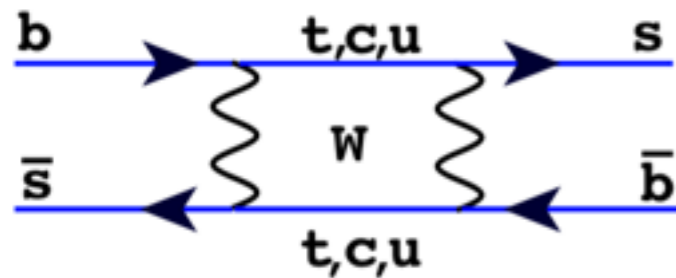
Mass difference ΔM_q

Experiment.: HFLAV 2019

$$\Delta m_s = 17.757 \pm 0.021 \text{ ps}^{-1}$$

$$|\Delta m_d = 0.5064 \pm 0.0019 \text{ ps}^{-1}$$

Theory



$$M_{12}^s = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s} \hat{\eta}_B$$

CKM
Inami-Lim
Buras Jamin Weisz

In the SM one operator:

$$Q = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1 - \gamma_5) b^\beta$$

$$\langle Q \rangle \equiv \langle B_s^0 | Q | \bar{B}_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu)$$

Non-perturbative theory input:

- 1) Lattice: ETM, FNAL-MILC, RBC-UKQCD
- 2) Sum rules: Siegen, Durham

Mass difference ΔM_q



Why is this interesting?

1. Interesting SM test per se
2. Many BSM models predict large effects in ΔM_q

Source	$f_{B_s} \sqrt{\hat{B}}$	ΔM_s^{SM}
HPQCD14 [132]	$(247 \pm 12) \text{ MeV}$	$(16.2 \pm 1.7) \text{ ps}^{-1}$
ETMC13 [133]	$(262 \pm 10) \text{ MeV}$	$(18.3 \pm 1.5) \text{ ps}^{-1}$
HPQCD09 [134] = FLAG13 [135]	$(266 \pm 18) \text{ MeV}$	$(18.9 \pm 2.6) \text{ ps}^{-1}$
FLAG17 [70]	$(274 \pm 8) \text{ MeV}$	$(20.01 \pm 1.25) \text{ ps}^{-1}$
Fermilab16 [72]	$(274.6 \pm 8.8) \text{ MeV}$	$(20.1 \pm 1.5) \text{ ps}^{-1}$
HQET-SR [77, 136]	$(278_{-24}^{+28}) \text{ MeV}$	$(20.6_{-3.4}^{+4.4}) \text{ ps}^{-1}$
HPQCD06 [137]	$(281 \pm 20) \text{ MeV}$	$(21.0 \pm 3.0) \text{ ps}^{-1}$
RBC/UKQCD14 [138]	$(290 \pm 20) \text{ MeV}$	$(22.4 \pm 3.4) \text{ ps}^{-1}$
Fermilab11 [139]	$(291 \pm 18) \text{ MeV}$	$(22.6 \pm 2.8) \text{ ps}^{-1}$

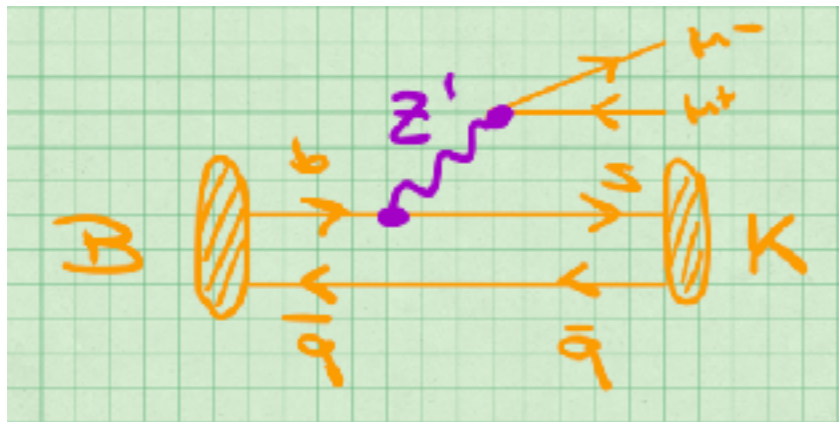
2 sigma away from experiment
Independent confirmation desirable

*In agreement with SM
*uses most recent 2+1+1 lattice results for decay constants

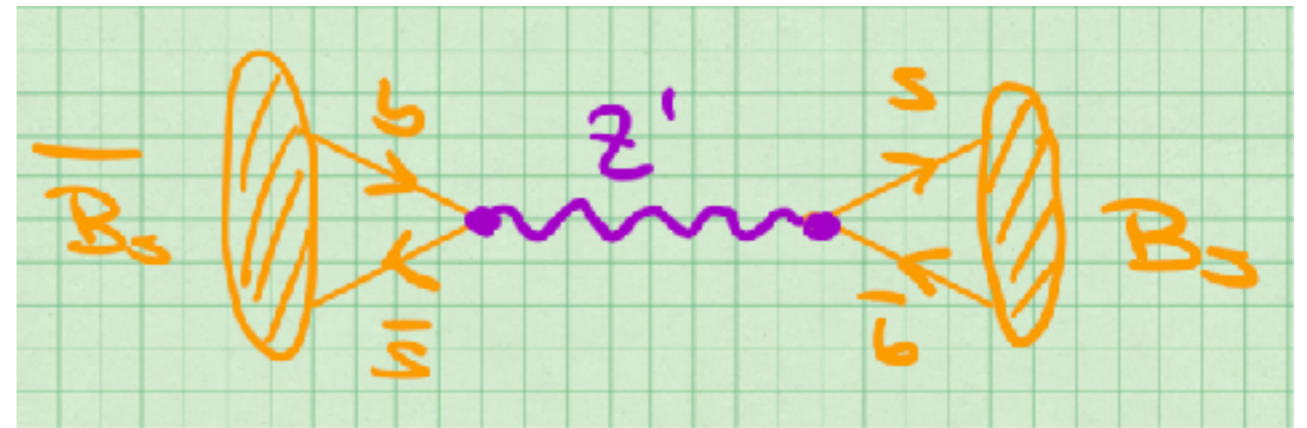
King, AL, Rauh 2019 $(263.00_{-8.1}^{+7.7}) \text{ MeV}$ $(18.5_{-1.5}^{+1.2}) \text{ ps}^{-1}$

Mass difference ΔM_q

Flavour anomalies could e.g. be explained by Z' models



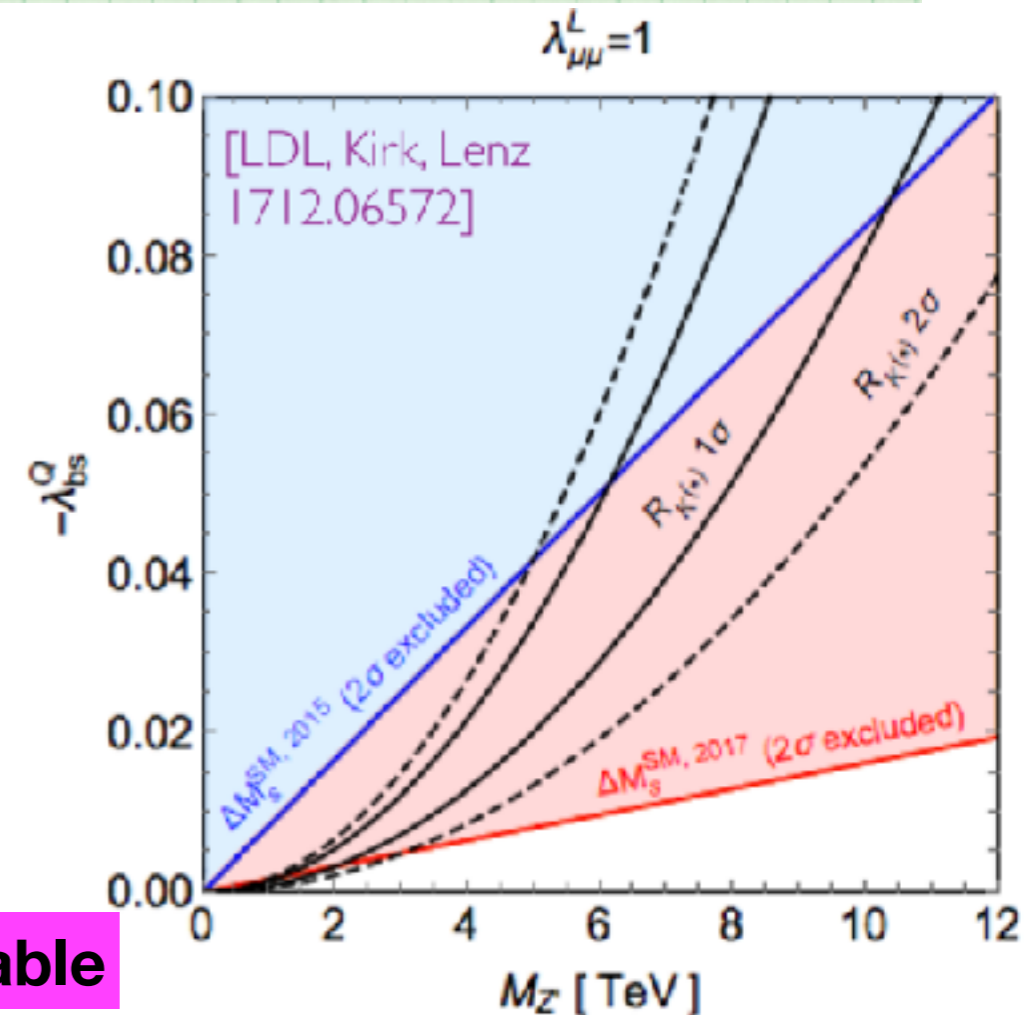
Such a model also modifies the mass difference of neutral mesons



Many times the BSM contribution to ΔM_q is positive

Using the large FNAL-MILC value:

One constraint to kill them all! Di Luzio, Kirk, AL

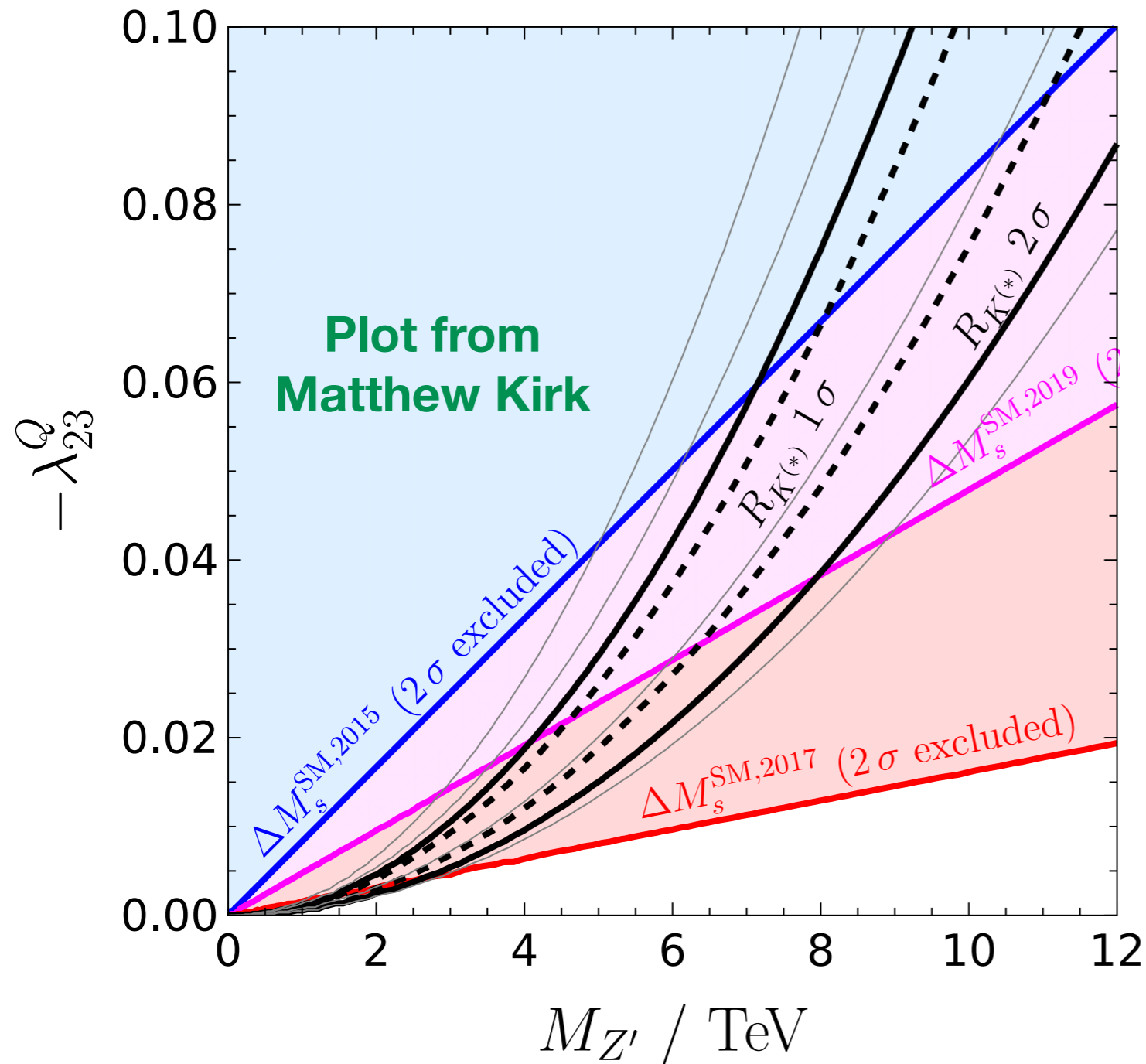


Independent determination of B_s mixing inputs desirable

Mass difference ΔM_q

Using the new HQET sum rule results from 2019

$$\lambda_{22}^L = 1$$



The party can go on

Outline

- **Flavour Anomalies and BSM explanations**
- **B-mixing: Anomalie vs. Bound on BSM explanations**
- **Status of theory predictions for B-mixing**

Fresh from the press!!!!

Mass difference ΔM_q

Mixing
Operators
Delta B = 2

$$\begin{array}{l}
 Q_1 = \bar{b}_i \gamma_\mu (1-\gamma_5) s_i \times \bar{b}_j \gamma^\mu (1-\gamma_5) s_j \\
 Q_2 = \bar{b}_i (1-\gamma_5) s_i \times \bar{b}_j (1-\gamma_5) s_j \\
 Q_3 = \bar{b}_i (1-\gamma_5) s_j \times \bar{b}_j (1-\gamma_5) s_i \\
 Q_4 = \bar{b}_i (1-\gamma_5) s_i \times \bar{b}_j (1+\gamma_5) s_j \\
 Q_5 = \bar{b}_i (1-\gamma_5) s_j \times \bar{b}_j (1+\gamma_5) s_i
 \end{array}
 \left. \vphantom{\begin{array}{l} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{array}} \right\} \Delta \Gamma_s^{\text{SM}}$$

$\Delta \Gamma_s^{\text{SM}}$
 $\&$
 $\Delta \Gamma_s^{\text{BSM}}$

$\uparrow 3$
 $\downarrow 1$

Parameterisation
in terms of
decay constants
and
Bag parameter

$$\begin{array}{l}
 \langle B_s | Q_1 | \bar{B}_s \rangle = \frac{8}{3} \Gamma_{B_s}^2 f_{B_s}^2 B_1 \\
 \langle B_s | Q_2 | \bar{B}_s \rangle = -\frac{5}{3} \left[\frac{\Gamma_{B_s}}{m_b + m_s} \right]^2 \Gamma_{B_s}^2 f_{B_s}^2 B_2 \\
 \langle B_s | Q_3 | \bar{B}_s \rangle = \frac{1}{3} \left[\frac{\Gamma_{B_s}}{m_b + m_s} \right]^2 \Gamma_{B_s}^2 f_{B_s}^2 B_3 \\
 \langle B_s | Q_4 | \bar{B}_s \rangle = \left(2 \left[\frac{\Gamma_{B_s}}{m_b + m_s} \right]^2 + \frac{1}{3} \right) \Gamma_{B_s}^2 f_{B_s}^2 B_4 \\
 \langle B_s | Q_5 | \bar{B}_s \rangle = \left(\frac{2}{3} \left[\frac{\Gamma_{B_s}}{m_b + m_s} \right]^2 + 1 \right) \Gamma_{B_s}^2 f_{B_s}^2 B_5
 \end{array}$$

• lattice • HQET-SR
• lattice

Non-perturbative input for ΔM_q

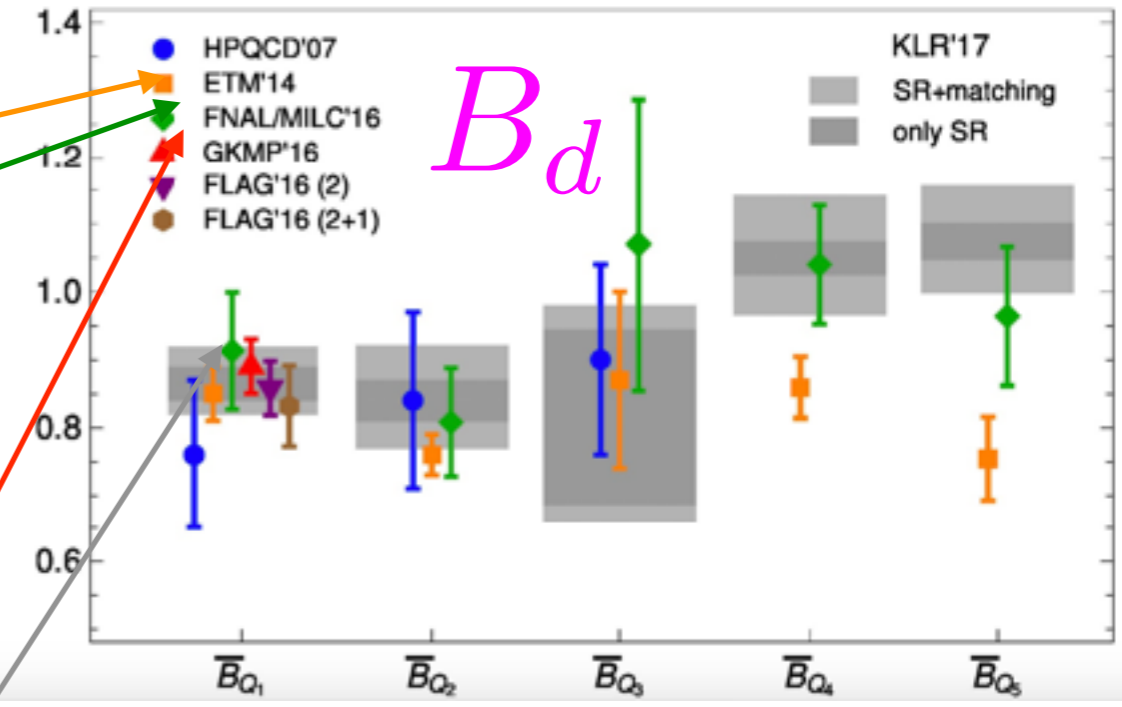
B_d-mixing

1. Lattice

- * ETM 1308.1851
- * FNAL-MILC 1602.03560

2. HQET-sum rules: 3-loop + NNLO matching:

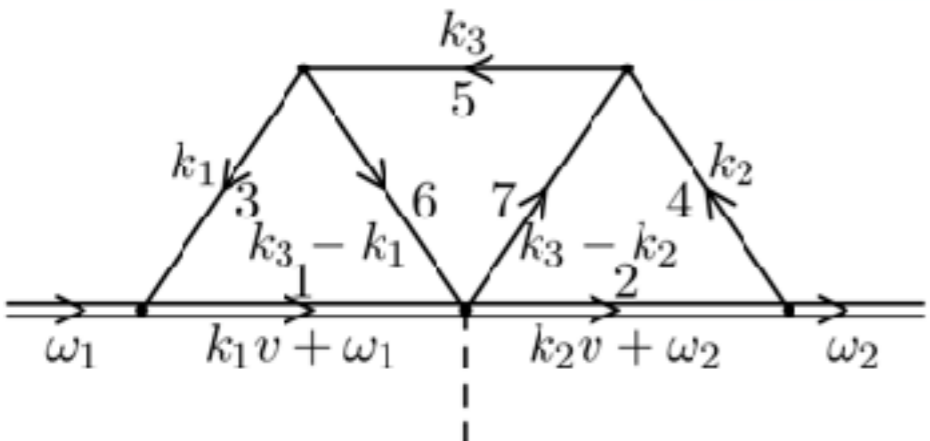
- * Siegen: Grozin, Klein, Mannel, Pivovarov 1606.06054, 1706.05910, 1806.00253
- * Durham: Kirk (Rome), AL, Rauh (Bern) 1711.02100



Three-loop HQET vertex diagrams for $B^0-\bar{B}^0$ mixing

Andrey G. Grozin and Roman N. Lee

arXiv:0812.4522v2



The various NLO contributions:

- ▶ Perturbative contribution (3-loop)
 $\Delta B_{PT} = -0.10 \pm 0.02 \pm 0.03$
A.Grozin,R.Klein,ThM,AAP, Phys.Rev. D94, 034024 (2016)
- ▶ Quark condensate contribution (2-loop)
 $\Delta B_q = -0.002 \pm 0.001$
A.Grozin,R.Klein,ThM,AAP, Phys.Rev. D94, 034024 (2016)
- ▶ Other condensates (tree-level+2-loop gluon cond)
 $\Delta B_{ncnPT} = -0.006 \pm 0.005$
ThM, B.D. Pecjak, AAP, Eur.Phys.J. C71 (2011) 1607

Total $\Delta B = -0.11 \pm 0.04 \pm 0.03$

Non-perturbative input for ΔM_q

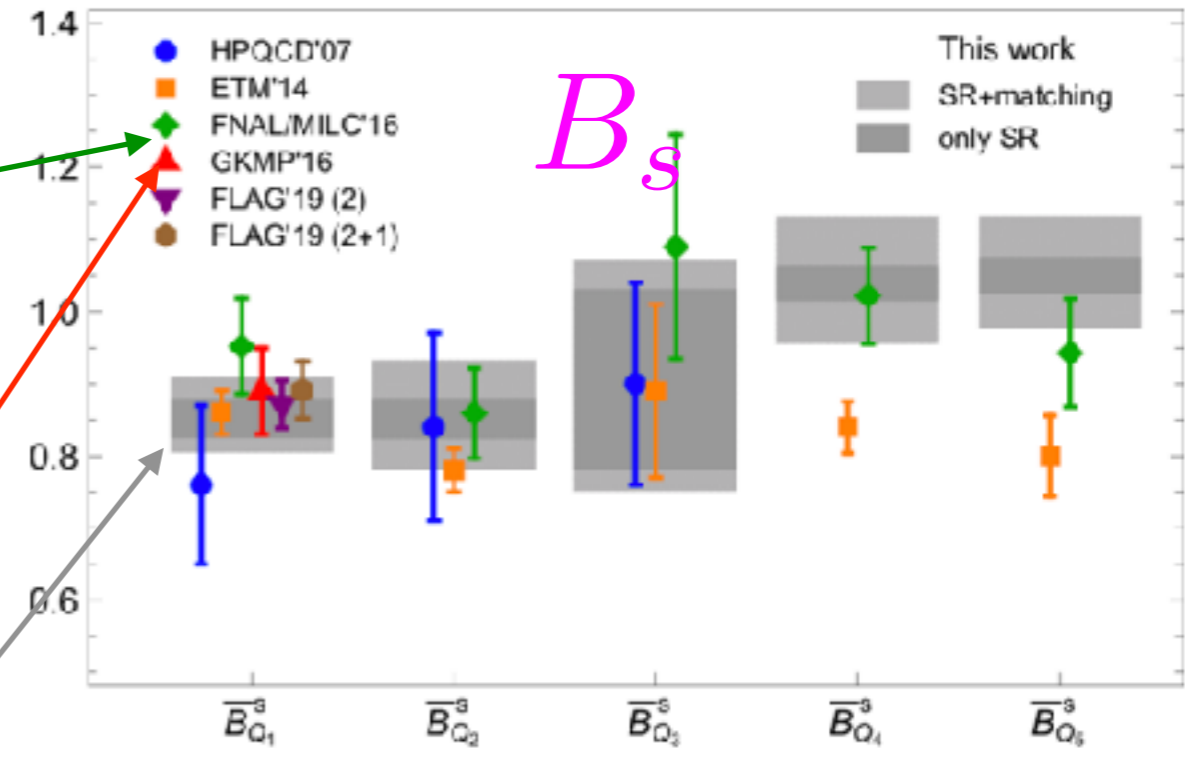
B_s-mixing

1. Lattice

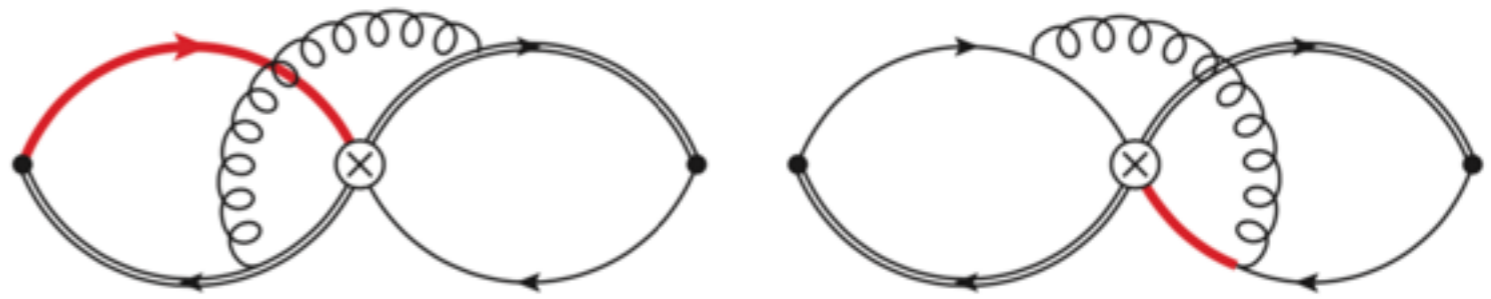
- * **ETM 1308.1851**
- * **FNAL-MILC 1602.03560**

2. HQET-sum rules: 3-loop + NNLO matching:

- *Durham: **King, AL, Rauh (Bern) 1904.00940**



$$r_{\tilde{Q}_1}^{(0)} = 8 - \frac{a_2}{2} - \frac{8\pi^2}{3}$$



$$r_{\tilde{Q}_1}^{(2)} = \frac{1}{1+x^2} \left[\frac{(1-x)^2 a_2}{4} + \frac{2\pi^2(1-4x+x^2)}{3} + 2x\psi(x) \left(2 + \frac{1+x}{1-x} \ln(x) \right) \right]$$

$$+ \left\{ \begin{array}{ll} -\frac{2(6+6x-x^2+2x^3)}{3} + 2(2-4x+x^2)\ln(x) - 4(1-x^2)\text{Li}_2(1-1/x), & x \leq 1, \\ -\frac{2(2-x+6x^2+6x^3)}{3x} - 2(1-4x+2x^2)\ln(x) + 4(1-x^2)\text{Li}_2(1-x), & x > 1, \end{array} \right.$$

Non-perturbative input for ΔM_q

Comparison with experiment

$$\Delta M_d^{\text{exp}} = (0.5064 \pm 0.0019) \text{ ps}^{-1}, \quad \mathbf{4 \text{ per mille}}$$

$$\Delta M_d^{\text{SR}} = (0.547_{-0.046}^{+0.035}) \text{ ps}^{-1}$$

$$= (0.547_{-0.032}^{+0.033} (\text{had.})_{-0.002}^{+0.004} (\text{scale})_{-0.032}^{+0.011} (\text{param.})) \text{ ps}^{-1},$$

$$\Delta M_s^{\text{exp}} = (17.757 \pm 0.021) \text{ ps}^{-1}, \quad \mathbf{1 \text{ per mille}}$$

$$\Delta M_s^{\text{SR}} = (18.5_{-1.5}^{+1.2}) \text{ ps}^{-1}$$

$$= (18.5 \pm 1.1 (\text{had.}) \pm 0.1 (\text{scale})_{-1.0}^{+0.3} (\text{param.})) \text{ ps}^{-1}$$

$$\Delta M_s^{\text{Lat.}} = (20.3_{-1.7}^{+1.3}) \text{ ps}^{-1}$$

Good agreement with experiment - lattice 1.5 sigma above experiment
Experiment is 21/64 times more precise than theory

Assuming validity of the SM: Extraction of CKM parameter

$$|V_{td}|_{\text{SR}} = (8.36_{-0.24}^{+0.26}) \cdot 10^{-3}$$

$$= (8.36_{-0.24}^{+0.26} (\text{had.})_{-0.03}^{+0.02} (\mu) \pm 0.02 (\text{param.})) \cdot 10^{-3}.$$

$$|V_{ts}|_{\text{SR}} = (40.74_{-1.21}^{+1.30}) \cdot 10^{-3}$$

$$= (40.74_{-1.20}^{+1.29} (\text{had.})_{-0.14}^{+0.09} (\mu) \pm 0.05 (\text{param.})) \cdot 10^{-3}$$

$$|V_{ts}|_{\text{CKMfitter}} = (41.69_{-1.08}^{+0.28}) \cdot 10^{-3}$$

$$|V_{td}|_{\text{CKMfitter}} = (8.710_{-0.246}^{+0.086}) \cdot 10^{-3}.$$

$$|V_{ts}|_{\text{CKMfitter, tree}} = (41.63_{-1.45}^{+0.39}) \cdot 10^{-3}$$

$$|V_{td}|_{\text{CKMfitter, tree}} = (9.08_{-0.45}^{+0.23}) \cdot 10^{-3}.$$

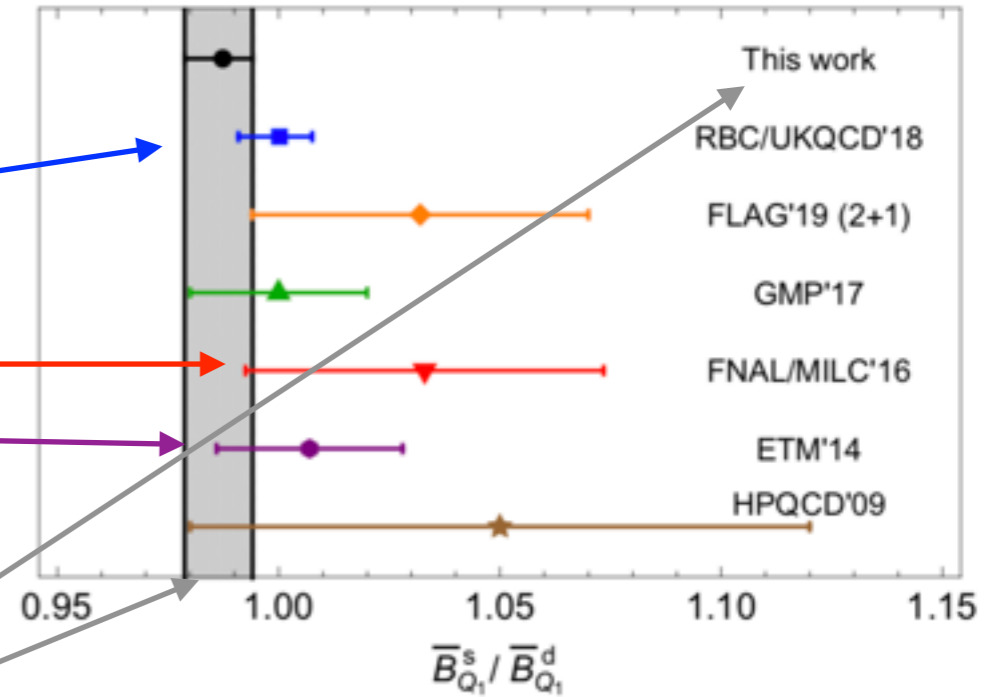
Good agreement with CKMfits - competitive precision - “deviation” from tree level fit

Non-perturbative input for ΔM_q

Uncertainties cancel in B_s/B_d

1. Lattice

- * RBC/UKQCD 1812.09791
- * FNAL-MILC 1602.03560
- * ETM 1308.1851

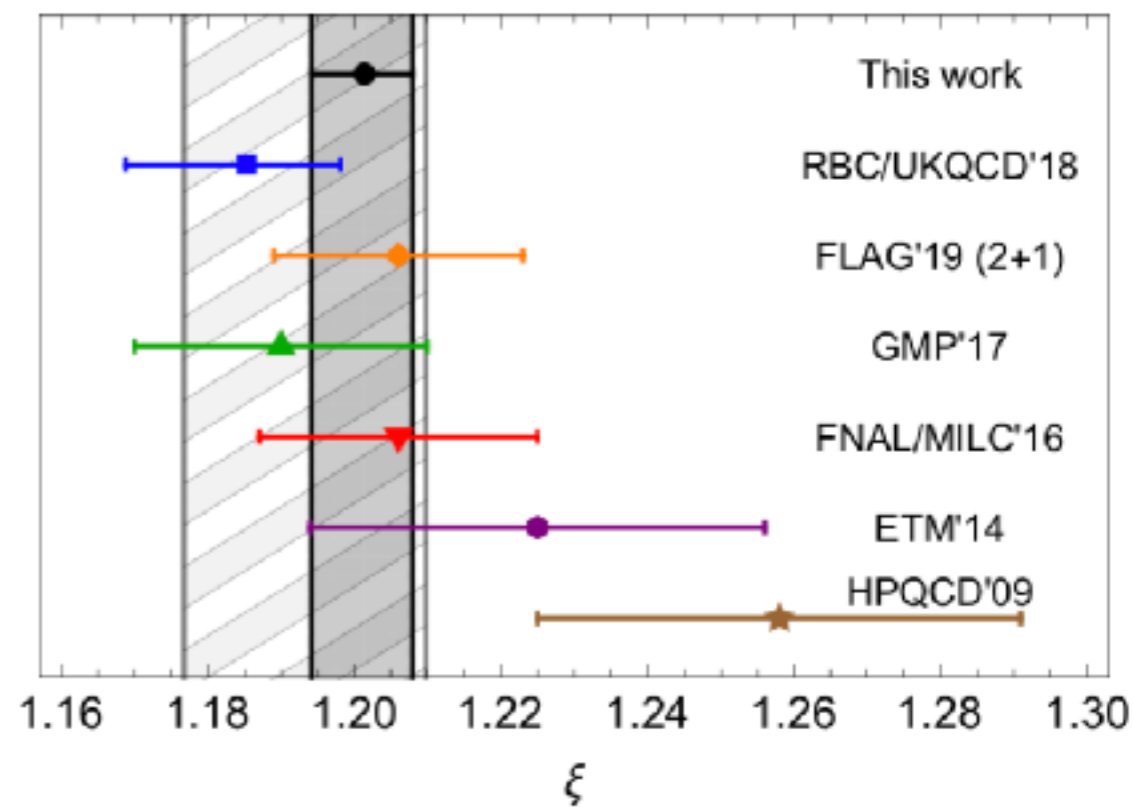


2. HQET-sum rules: 3-loop + NNLO matching:

- * Durham: King, AL, Rauh (Bern) 1904.00940

Take decay constants from most recent 2+1+1 lattice evaluation =>

$$\xi \equiv \frac{f_{B_s}}{f_B} \sqrt{\overline{B}_{Q_1}^{s/d}} = 1.2014^{+0.0065}_{-0.0072}$$



Non-perturbative input for ΔM_q

Very precise prediction of the ratios of mass differences

$$\frac{\Delta M_d}{\Delta M_s} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}}.$$

$$\left(\frac{\Delta M_d}{\Delta M_s} \right)_{\text{exp}} = 0.0285 \pm 0.0001,$$
$$\left(\frac{\Delta M_d}{\Delta M_s} \right)_{\text{SR}} = 0.0297_{-0.0009}^{+0.0006} = 0.0297_{-0.0003}^{+0.0004} (\text{had.})_{-0.0008}^{+0.0005} (\text{exp.}).$$

1.3 sigma above experiment

Experiment is 7-8 times more precise than theory

Assuming validity of the SM: Extraction of CKM parameter

$$|V_{td}/V_{ts}|_{\text{SR}} = 0.2045_{-0.0013}^{+0.0012} \text{ vs. } \begin{array}{l} |V_{td}/V_{ts}| = 0.2088_{-0.0030}^{+0.0016} \quad [\text{CKMfitter}], \\ |V_{td}/V_{ts}| = 0.211 \pm 0.003 \quad [\text{UTfit}], \end{array} \text{ vs. } |V_{td}/V_{ts}| = 0.2186_{-0.0059}^{+0.0049} \quad [\text{CKMfitter, tree}]$$

Slightly below CKMfits - slightly higher precision - 2.3 sigma below tree-level fits

Assuming validity of the SM: Independent determination of m_{top}

$$\bar{m}_t(\bar{m}_t) = (157_{-6}^{+8}) \text{ GeV} \text{ vs. } \bar{m}_t(\bar{m}_t) = (160_{-4}^{+5}) \text{ GeV, (PDG)}$$

Very good agreement - almost comparable precision

Non-perturbative input for ΔM_q

Very precise prediction of the ratios of mass differences

$$\frac{\Delta M_d}{\Delta M_s} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}}$$

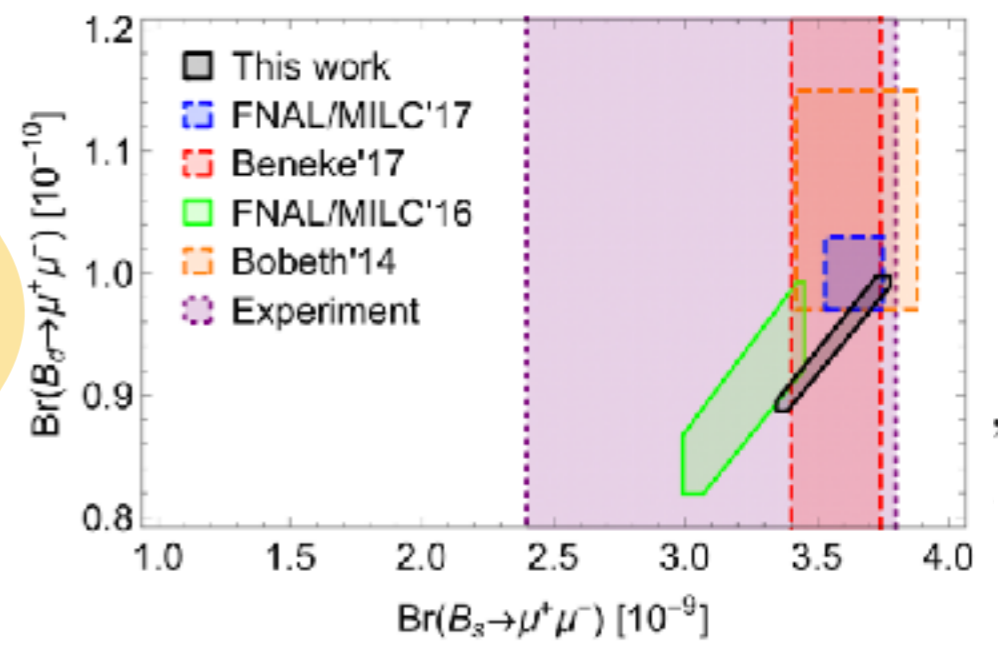
Consequences for BSM??

Further cancellations (no CKM, no decay constant - See AJ Buras) in

$$\frac{\text{Br}(B_q \rightarrow l^+ l^-)}{\Delta M_q} = \frac{3G_F^2 M_W^2 m_l^2 \tau_{B_q^H}}{\pi^3} \sqrt{1 - \frac{4m_l^2}{M_{B_q}^2}} \frac{|C_A(\mu)|^2}{S_0(x_t) \hat{\eta}_B \overline{B}_{Q_1}^q(\mu)}$$

Assuming validity of the SM: precise prediction of rare branching ratios

$$\begin{aligned} \text{Br}(B_s^0 \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (3.55^{+0.23}_{-0.20}) \cdot 10^{-9}, \\ \text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (9.40^{+0.58}_{-0.53}) \cdot 10^{-11}, \\ \left(\frac{\text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)}{\text{Br}(B_s^0 \rightarrow \mu^+ \mu^-)} \right)_{\text{SM}} &= 0.0265 \pm 0.0003 = \end{aligned}$$



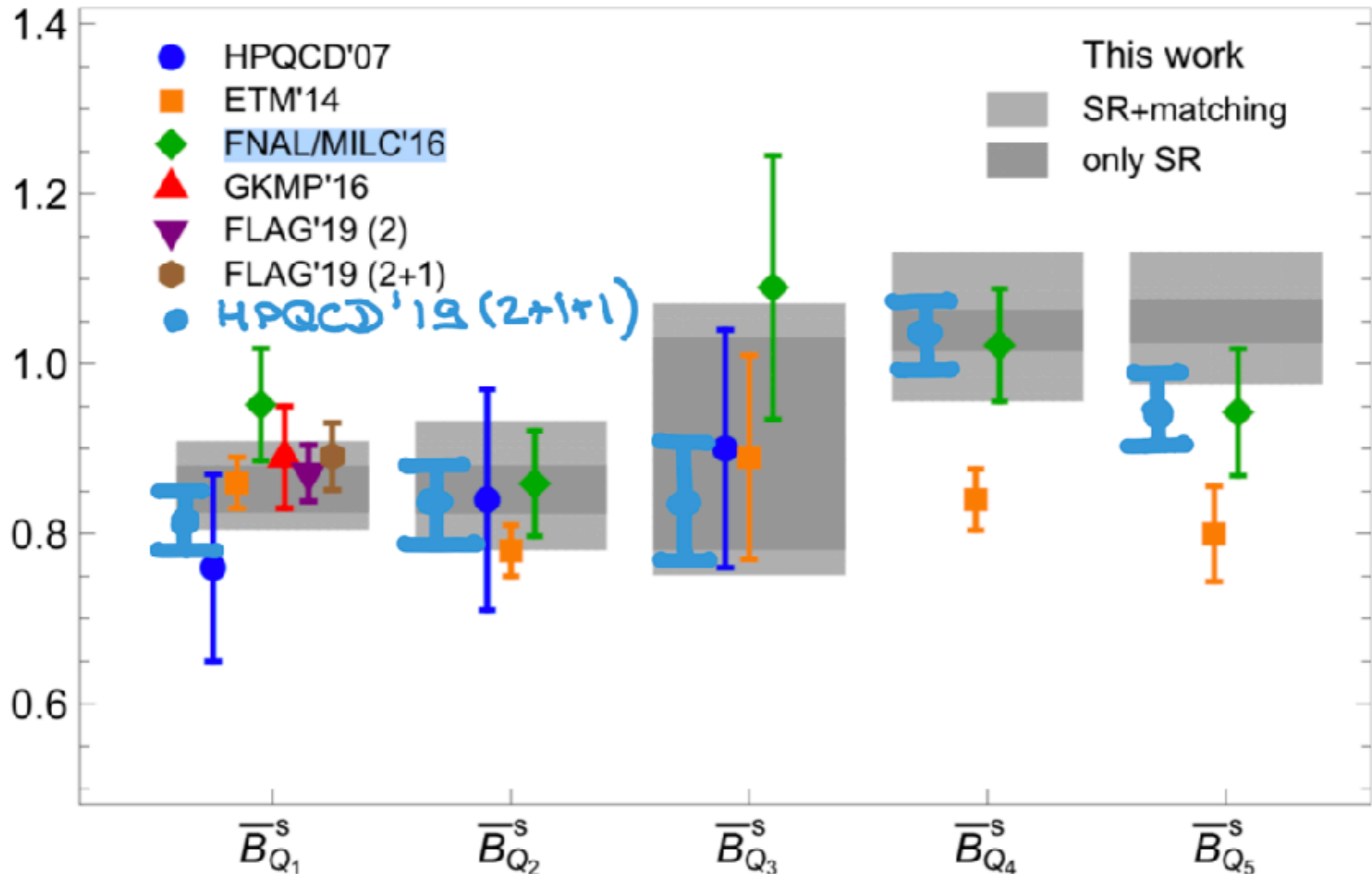
Non-perturbative input for ΔM_q

- Most recent lattice values give large values for ΔM_q
- HQET sum rules give smaller values
- **Are sum rules competitive at all with lattice???**

Non-perturbative input for ΔM_q

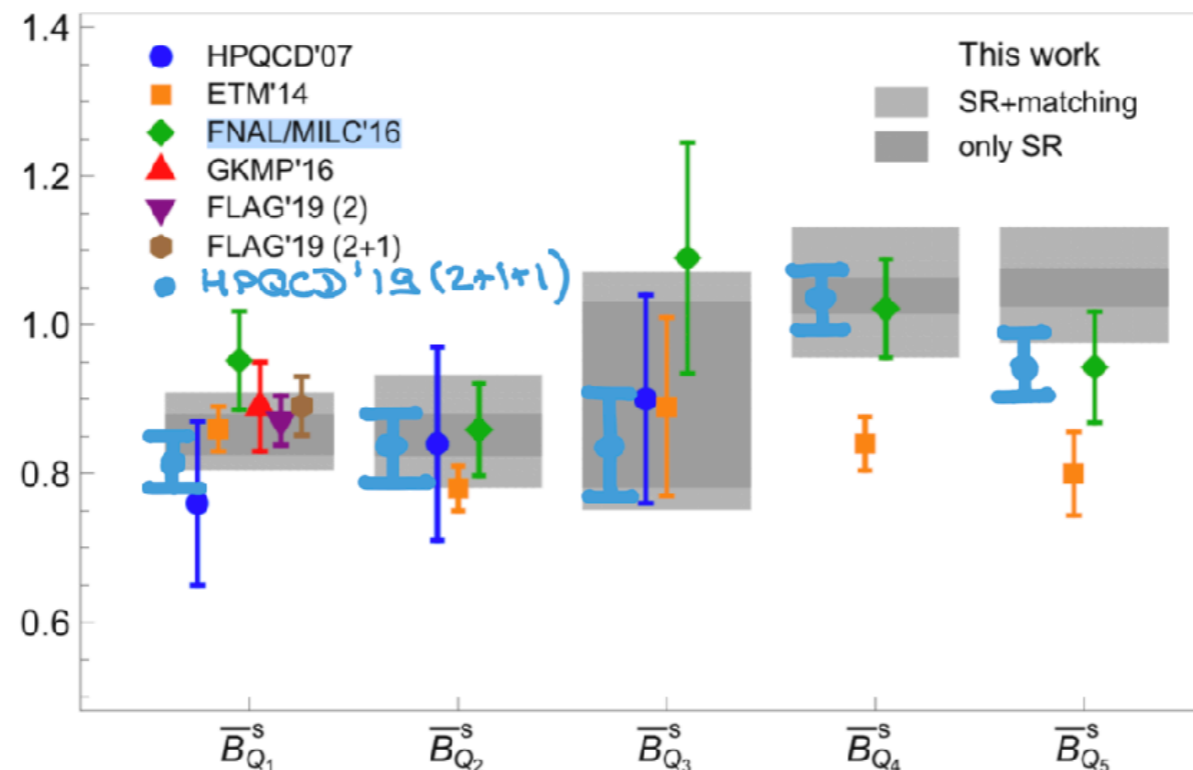
- Most recent lattice values give large values for ΔM_q
- HQET sum rules give smaller values
- **Are sum rules competitive at all with lattice???**

Yesterday:
HPQCD 1907.01025



Non-perturbative input for ΔM_q

- Theory prediction for mass differences agree with experiment
- Weaker bounds on BSM models
- For some quantities sum rules are highly competitive to lattice, e.g. bag parameter for mixing
- For some quantities sum rules are not competitive to lattice, e.g. decay constants
- For some quantities sum rules are the only available, reliable method, e.g. bag parameter for lifetime predictions



Decay rate difference $\Delta\Gamma_s$

Calculation is more difficult than mass difference - use Heavy Quark Expansion

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \Gamma_3 + \frac{\Lambda^4}{m_b^4} \Gamma_4 + \dots$$

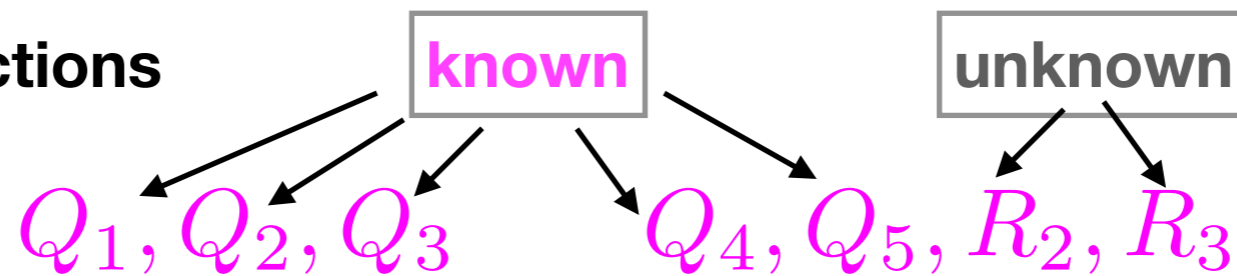
Each term can be split up into a perturbative part and non-perturbative matrix elements

$$\Gamma_i = \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi} \Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2} \Gamma_i^{(2)} + \dots \right] \langle O^{d=i+3} \rangle$$

$$R_2 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1 - \gamma^5) D^\rho s^\alpha) (\bar{b}^\beta \gamma_\mu (1 - \gamma^5) s^\beta)$$

$$R_3 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho (1 - \gamma^5) D^\rho s^\alpha) (\bar{b}^\beta (1 - \gamma^5) s^\beta)$$

Status of theory predictions



Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ
Γ_{12}^s	++	++	$\frac{\pm}{2}$	+ ++	++	0	0	9.5 + . (***)
Γ_{12}^d	++	++	0	+ + +	++	0	0	9 + (***)

Decay rate difference $\Delta\Gamma_s$

Calculation is more difficult than mass difference - use Heavy Quark Expansion

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \dots$$

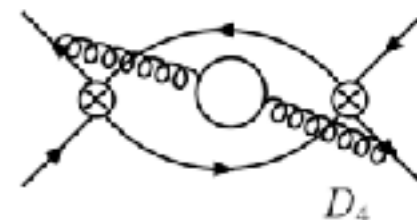
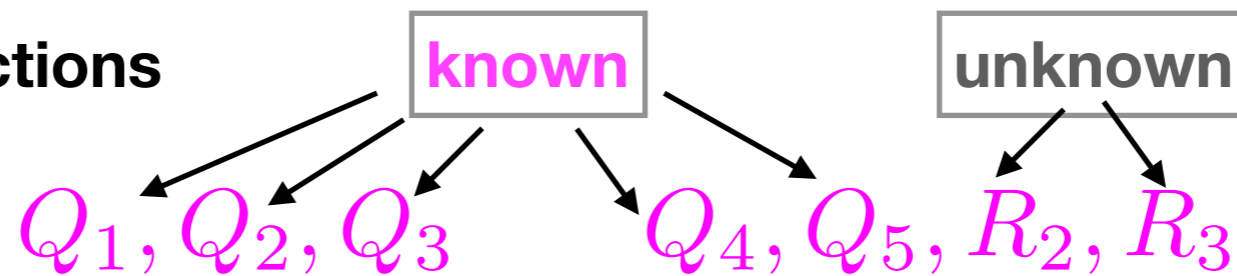
Each term can be split up into a perturbative part and non-perturbative matrix elements

$$\Gamma_i = \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi}\Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2}\Gamma_i^{(2)} + \dots \right] \langle O^{d=i+3} \rangle$$

$$R_2 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1 - \gamma^5) D^\rho s^\alpha) (\bar{b}^\beta \gamma_\mu (1 - \gamma^5) s^\beta)$$

$$R_3 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho (1 - \gamma^5) D^\rho s^\alpha) (\bar{b}^\beta (1 - \gamma^5) s^\beta)$$

Status of theory predictions



Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ
Γ_{12}^s	++	++	$\frac{\pm}{2}$					*
Γ_{12}^d	++	++	0					

CKM 2020: α_s/m_b corrections to $\Delta\Gamma$
 CKM 2022: NNLO corrections to $\Delta\Gamma$
 CKM 2024: NNLO corrections to semileptonic CP asymmetry in $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$
$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- Decay constants cancel completely
- Bag parameter cancel largely

SM predictions

$$\Delta\Gamma_s^{\text{exp}} = (0.088 \pm 0.006) \text{ ps}^{-1},$$
$$\Delta\Gamma_s^{\text{SR}} = (0.091_{-0.030}^{+0.022}) \text{ ps}^{-1}$$
$$= (0.091 \pm 0.020 \text{ (had.)}_{-0.021}^{+0.008} \text{ (scale)}_{-0.005}^{+0.002} \text{ (param.)}) \text{ ps}^{-1}.$$

- Good agreement
- Experiment about 4 times more precise

$$\Delta\Gamma_d^{\text{exp}} = (-1.3 \pm 6.6) \cdot 10^{-3} \text{ ps}^{-1},$$
$$\Delta\Gamma_d^{\text{SR}} = (2.6_{-0.9}^{+0.6}) \cdot 10^{-3} \text{ ps}^{-1}$$
$$= (2.6 \pm 0.6 \text{ (had.)}_{-0.6}^{+0.2} \text{ (scale)}_{-0.2}^{+0.1} \text{ (param.)}) \cdot 10^{-3} \text{ ps}^{-1},$$

- Strong test of HQE
- Violation of Quark hadron duality must be small
- Dim 7 operator have to be determined
- NNLO-QCD corrections have to be determined

- Might be a solution to the D0 di-muon asymmetry
- Experimental number needed

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- Decay constants cancel completely
- Bag parameter cancel largely

SM predictions

$$\Delta\Gamma_s^{\text{exp}} = (0.088 \pm 0.006) \text{ ps}^{-1},$$

$$\Delta\Gamma_s^{\text{SR}} = (0.091_{-0.030}^{+0.022}) \text{ ps}^{-1}$$

$$= (0.091 \pm 0.020 \text{ (had.)}_{-0.021}^{+0.008} \text{ (scale)}_{-0.005}^{+0.002} \text{ (para.))} \text{ ps}^{-1}$$

- Good agreement
- Experiment about

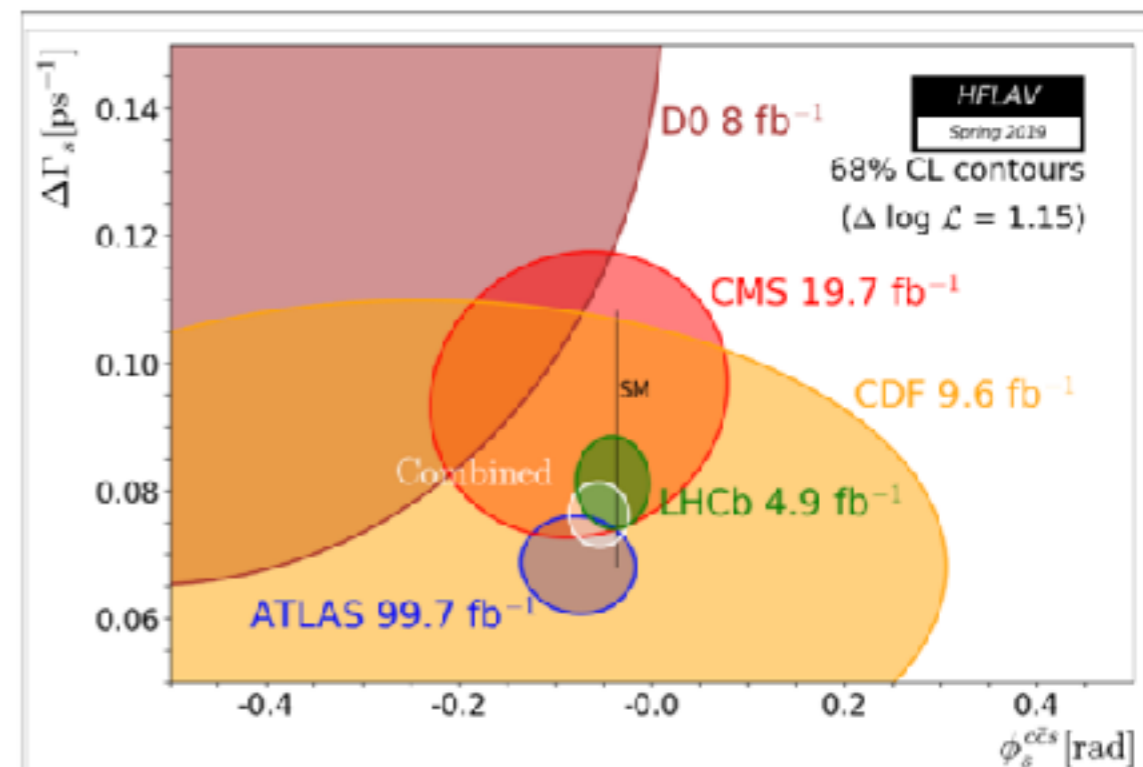
$$\Delta\Gamma_d^{\text{exp}} = (-1.3 \pm 6.6) \cdot 10^{-3} \text{ ps}^{-1},$$

$$\Delta\Gamma_d^{\text{SR}} = (2.6_{-0.9}^{+0.6}) \cdot 10^{-3} \text{ ps}^{-1}$$

$$= (2.6 \pm 0.6 \text{ (had.)}_{-0.6}^{+0.2} \text{ (scale)}_{-0.2}^{+0.1} \text{ (param.)}) \cdot 10^{-3} \text{ ps}^{-1}$$

- Strong test
- Violation of
- Dim 7 operators
- NNLO-QCD

- Might be a solution
- Experimental number



Very preliminary HFLAV combination

$$\varphi_s = -0.054 \pm 0.021 \text{ rad}$$

$$\Delta\Gamma_s = 0.0762 \pm 0.0034 \text{ ps}^{-1}$$

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$
$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

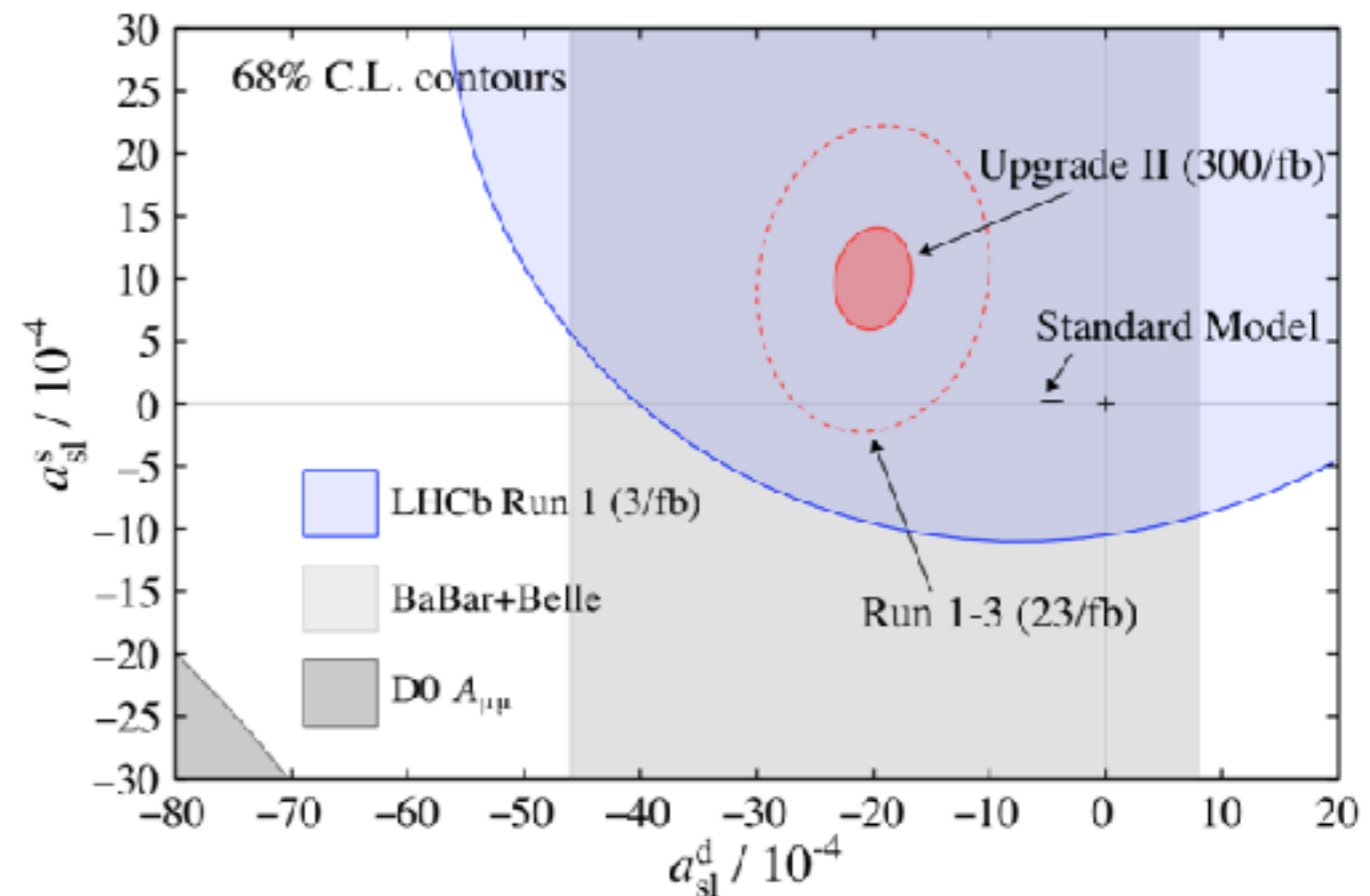
- Decay constants cancel completely
- Bag parameter cancel largely

SM predictions

$$a_{fs}^{d,SM,2015} = (-4.7 \pm 0.6) \cdot 10^{-5}$$

$$a_{fs}^{s,SM,2015} = (2.22 \pm 0.27) \cdot 10^{-5}$$

- Very sensitive to BSM effects!
- Experimental number needed



Conclusion

- 1) Severe theoretical progress in mixing and lifetime predictions in recent years
- 2) Theory prediction for mass differences agree with experiment - weaker bounds on BSM models
- 3) For some quantities sum rules are highly competitive to lattice, e.g. Bag parameter for mixing
- 4) Theory uncertainties can still be reduced significantly with current technology
 - BSM effects in box diagrams
 - $b \rightarrow s \mu \mu$ anomalies
 - Convergence of the HQE in charm $\rightarrow \Delta A_{CP}$
- 4) A lot of work has still to be done - but it can be done!

