Theory anatomy & uncertainties in rare semileptonic B-decays

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Amplitude analyses for Flavour Anomalies, Bristol, 05/07/2019

Content

Structure of rare decay amplitudes

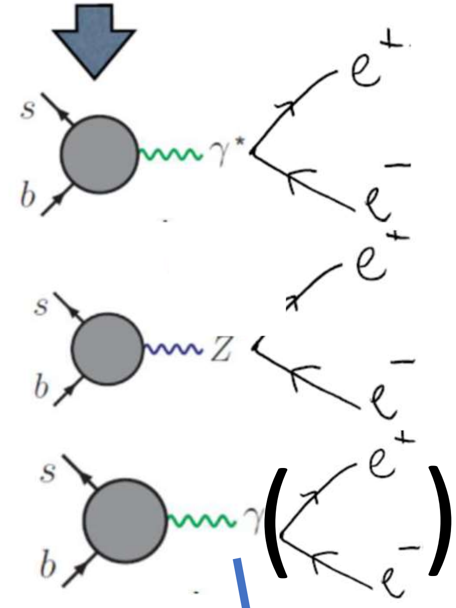
Anatomy of decay amplitudes and observables

+ a few implications & suggestions

Rare B-decay: short-distance

BSM (and SM weak interactions) enter flavour physics through effective contact interactions (SMEFT/H_{weak})

in SM mainly



C_9: dilepton from vector current

$$(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l)$$

C_{10}: dilepton from axial current

$$(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma^5 l)$$

C_7: dilepton from dipole

$$(\bar{s}\sigma^{\mu\nu} P_R b)F_{\mu\nu}$$

Can also have real photon

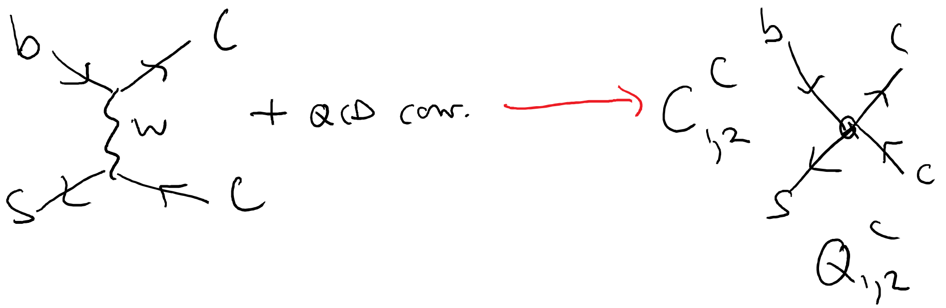
+parity conjugate "right-handed currents" - C_7', C_9', C_{10}'
suppressed by m_s/m_b in SM

Alternative basis with chiral leptons l_L, l_R

$C_L = (C_9 - C_{10})/2$ $\quad$ $C_R = (C_9 + C_{10})/2$

Importance of virtual charm

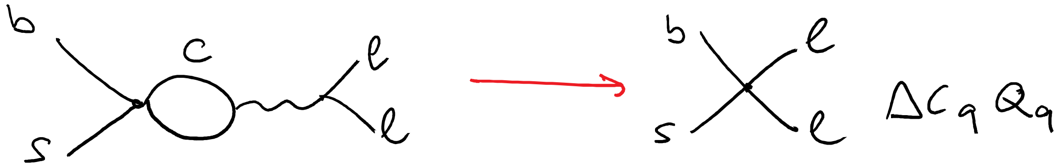
Also **purely hadronic** operators enter, in SM primarily:



$$Q_1^c = (\bar{c}_L^i \gamma_\mu b_L^j)(\bar{s}_L^j \gamma^\mu c_L^i)$$

$$Q_2^c = (\bar{c}_L^i \gamma_\mu b_L^i)(\bar{s}_L^j \gamma^\mu c_L^j)$$

RG mixes these into C_9 and C_7



+ dipole

$$C_7^{\rm eff}(4.6{\rm GeV}) = 0.02\, C_1(M_W) - 0.19\, C_2(M_W)$$

$$C_9(4.6{\rm GeV}) = 8.48\, C_1(M_W) + 1.96\, C_2(M_W)$$

SM: O(50%) of total in both cases!

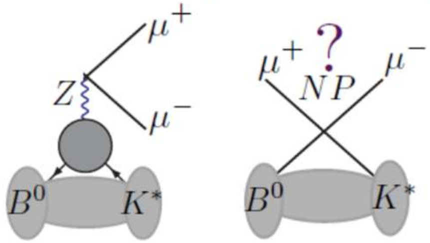
At μ=m_b: C_7^{eff} ~ -0.3 , C_L ~ 4 , $C_R \approx 0$

- SM: accidentally almost left-chiral muon interactions
- Long-distance virtual charm important theory uncertainty

Decay amplitude structure

Two mechanisms to produce dilepton in & beyond SM

- via axial lepton current (in SM: Z, boxes) C10

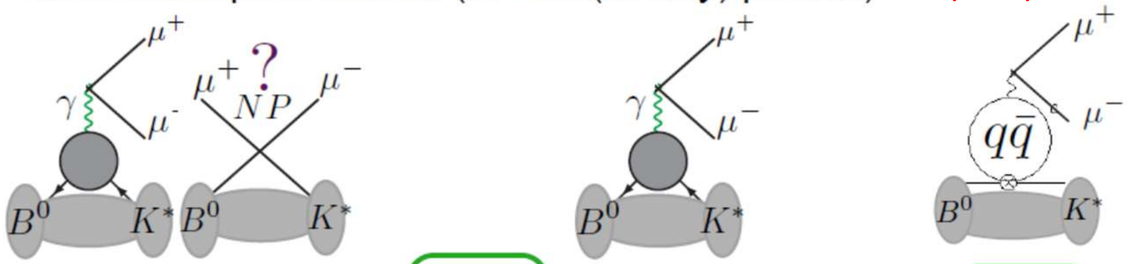


K* helicity

$$H_A(\lambda) \propto \boxed{\tilde{V}_\lambda(q^2)} C_{10} - V_{-\lambda}(q^2) C'_{10}$$

one form factor (nonperturbative) per helicity
amplitudes factorize naively
[nb - one more amplitude if not neglecting lepton mass]

- via vector lepton current (in SM: (mainly) photon) C7, C9, hadronic hamiltonian



$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C'_9 + \boxed{\frac{2\, m_b m_B}{q^2}} \left(\tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C'_7\right) - \boxed{\frac{16\,\pi^2 m_B^2}{q^2}} h_\lambda(q^2)$$

photon pole at $q^2=0$

two form factors interfere for each helicity

nonlocal "quark loops"
do **not** factorize naively

Natural, systematic discussion in terms of helicity amplitudes SJ, Martin Camalich 2012, 2014

Photon pole absent for helicity-0 (form factor rescaling)

Form factors

In helicity basis (makes for simple expressions in HQ limit):

$$-im_B\tilde{V}_{L(R)\lambda}(q^2) = \langle M(\lambda)|\bar{s}\not\epsilon^*(\lambda)P_{L(R)}b|\bar{B}\rangle,$$
$$m_B^2\tilde{T}_{L(R)\lambda}(q^2) = \epsilon^{*\mu}(\lambda)q^\nu\langle M(\lambda)|\bar{s}\sigma_{\mu\nu}P_{R(L)}b|\bar{B}\rangle$$
$$im_B\tilde{S}_{L(R)}(q^2) = \langle M(\lambda=0)|\bar{s}P_{R(L)}b|\bar{B}\rangle.$$

SJ, Martin Camalich 2012
(Bharucha, Feldmann, Wick 2010)

Close to q^2_{max}: determinations from lattice QCD (B -> pi; K; stable V)

Low q^2: no first-principles determinations

- heavy-quark limit: calculable relations, eg 7 FF -> 2 FF for B->V
  uncontrolled systematic: power corrections (Λ/m_b = 10% ? 20% ?)

- light-cone sum rules (LCSR)

  correlation function $$G_{F\lambda}(q^2;p^2) = i\int d^4y e^{-ip\cdot y}\langle K^*(k)|T\{\epsilon^*_\mu(q;\lambda)(\bar{s}\Gamma^\mu_F b)[0]\, j^\dagger_B(y)\}|0\rangle$$

hadronic representation

Form factor (output)

$$G_{F\lambda} = \frac{\boxed{\langle K^*(k,\lambda)|\epsilon^*_\mu(q;\lambda)\bar{s}\Gamma^\mu_F b|B\rangle}}{p^2-m_B^2}\frac{f_B m_B^2}{m_b} + \ldots,$$

collinear factorisation

Kernel (calc. PT) LCDA (input)

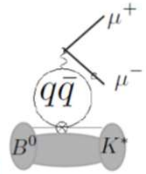
$$G_{F\lambda} = \sum_i \boxed{t_i(\alpha_s)} \star \boxed{\phi_i}$$

model omitted higher states: Borel transform & continuum threshold ("semilocal parton-hadron duality")

Main uncontrolled systematic: continuum threshold (not parametrically suppressed)

Nonlocal term and scale dependence

$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2)C_9 - V_{-\lambda}(q^2)C_9' + \frac{2\,m_b m_B}{q^2}\left(\tilde{T}_\lambda(q^2)C_7 - \tilde{T}_{-\lambda}(q^2)C_7'\right) - \boxed{\frac{16\,\pi^2 m_B^2}{q^2}\,h_\lambda(q^2)}$$



+ strong interactions!

more properly:

$$\frac{e^2}{q^2} L_V^\mu a_\mu^{\mathrm{had}} = -i\frac{e^2}{q^2}\int d^4x e^{-iq\cdot x}\langle \ell^+\ell^-|j_\mu^{\mathrm{em,lept}}(x)|0\rangle \boxed{\int d^4y\, e^{iq\cdot y}\langle M|j^{\mathrm{em,had},\mu}(y)\mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0)|\bar{B}\rangle}$$

$$h_\lambda \equiv \frac{i}{m_B^2}\epsilon^{\mu *}(\lambda) a_\mu^{\mathrm{had}}$$

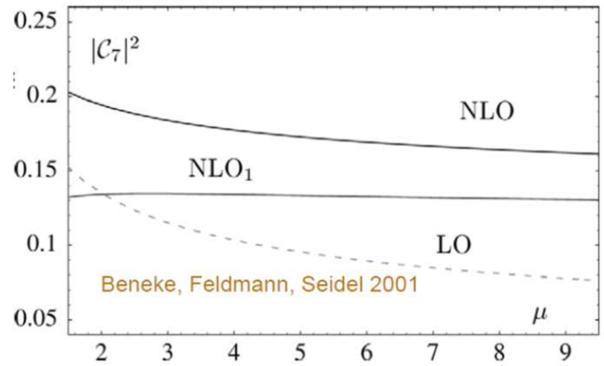
nonlocal, nonperturbative, large normalisation ($V_{cb}^* V_{cs}$ C_2)

traditional “ad hoc fix” : C_9 -> $C_9 + Y(q^2) = C_9^{\mathrm{eff}}(q^2)$

C_7 -> C_7^{eff}

Systematic justification in QCD factorisation
(low q2, heavy quark limit)
scale dependence cancels order by order in PT

Beneke, Feldmann, Seidel 2001,2004



power corrections ?
But subdominant to FF (SJ, Martin Camalich 2012, 2014)

High q2: OPE in 1/q2

duality violation ? Grinstein,Pirjol 2004; Beylich/Buchalla/Feldmann 2008, Lyon & Zwicky 2014

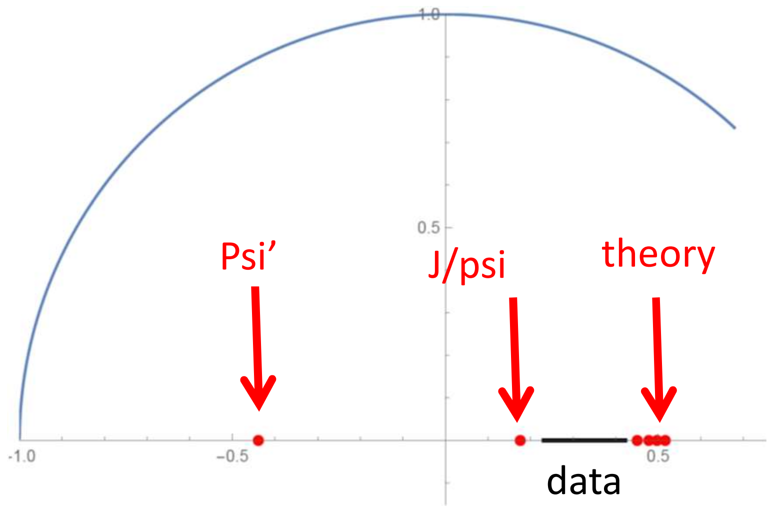
Data-driven determination?

Bobeth, Chrzaszcz, Van Dyk, Virto 2017
Chrzaszcz, Mauri, Serra, Coutinho, Van Dyk 2018

Basic idea: reduce theory dependence of h_λ by using data & assuming analyticity

- Ignoring CKM-suppressed terms, assume h_λ is analytic in q2 except for a cut from $4\ m_D^2$ to infinity, and poles at the J/psi and psi'

- Use QCDF (+LCSR pc estimate) only at q2 <~ 0
- And experimental data to fix/constrain the residues at the poles (and/or use rare decay data)
- Conformal mapping to increase separation of the input data from the cut in hope of a fast-converging Taylor series (truncate after 3 terms)



k	0	1	2
$\mathrm{Re}[\alpha_k^{(\perp)}]$	-0.06 ± 0.21	-6.77 ± 0.27	18.96 ± 0.59
$\mathrm{Re}[\alpha_k^{(\parallel)}]$	-0.35 ± 0.62	-3.13 ± 0.41	12.20 ± 1.34
$\mathrm{Re}[\alpha_k^{(0)}]$	0.05 ± 1.52	17.26 ± 1.64	–
$\mathrm{Im}[\alpha_k^{(\perp)}]$	-0.21 ± 2.25	1.17 ± 3.58	-0.08 ± 2.24
$\mathrm{Im}[\alpha_k^{(\parallel)}]$	-0.04 ± 3.67	-2.14 ± 2.46	6.03 ± 2.50
$\mathrm{Im}[\alpha_k^{(0)}]$	-0.05 ± 4.99	4.29 ± 3.14	–

Used with LCSR form factors gives BSM C9 consistent with when HQE computation is used with LCSR FF form factors

If the convergence/stability of this method can be established, it may eliminate the charm loop as a source of concern for interpreting low-q2 data.

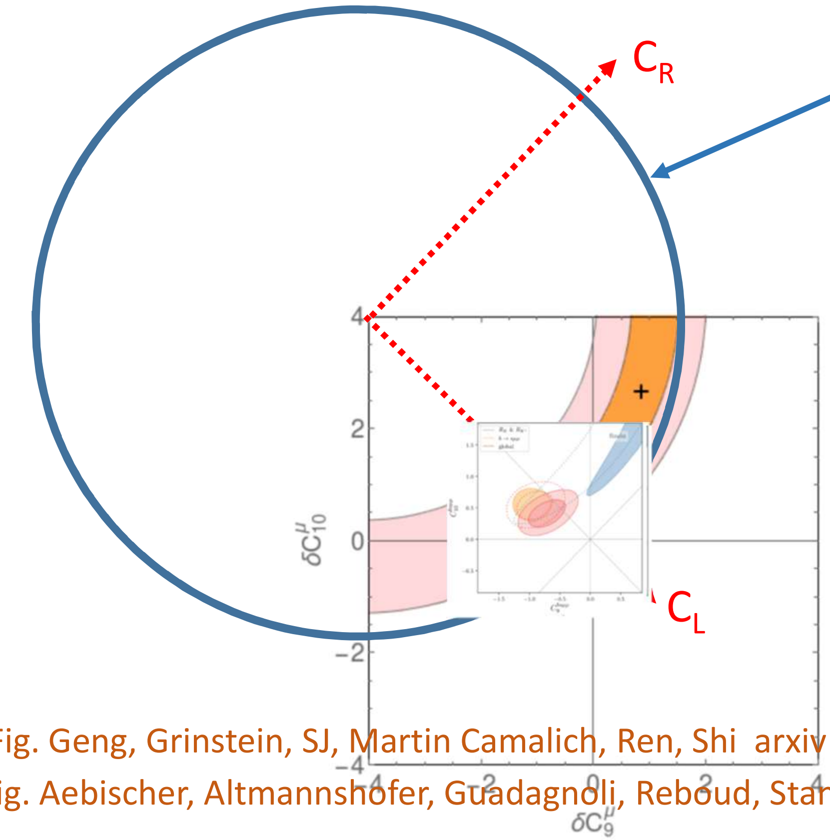
No new information on form factors – **could they similarly be fit to data?**

Scalar BR / $R_K^{(*)}$ and the chirality of NP

Assume here that the BSM effect is in the muonic mode, and no right-handed currents.

Because in the SM, $|C_R|$, $|C_7| << |C_L|$,

$$BR \approx \text{const}\,(|C_L^{SM} + C_L^{BSM}|^2 + |C_R^{BSM}|^2) \approx \text{const}\,(|4 + C_L^{BSM}|^2 + |C_R^{BSM}|^2)$$



BR(B->K(*)μμ) = SM value

Only C_L^{BSM} can interfere destructively: $R_K^{(*)}$ point to purely left-handed coupling

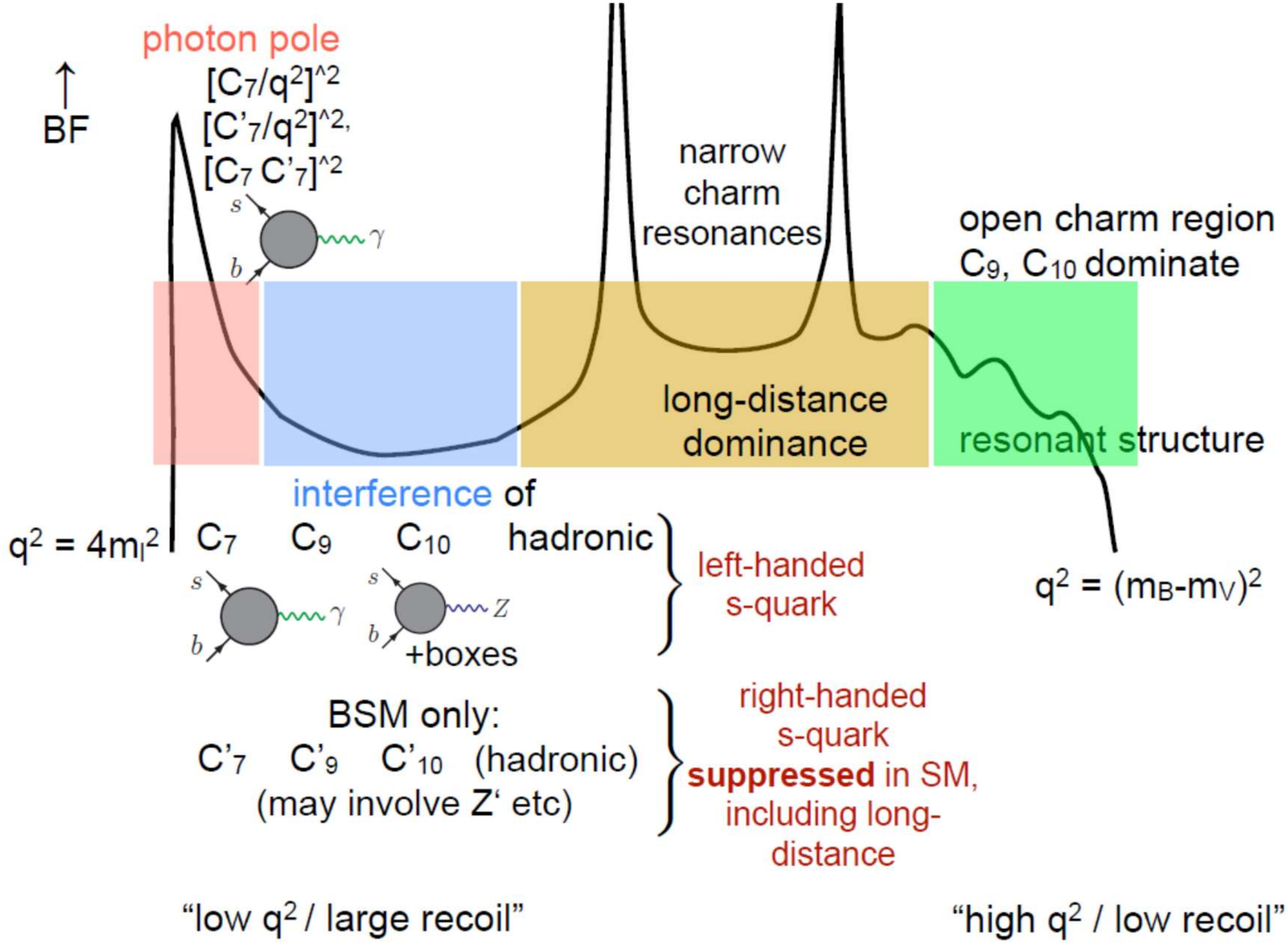
$$(\bar{s}_L \gamma^\mu b_L)\,(\bar{\mu}_L \gamma_\mu \mu_L)$$

with ~ -10% of SM value

Fig. Geng, Grinstein, SJ, Martin Camalich, Ren, Shi arxiv:1704.05446

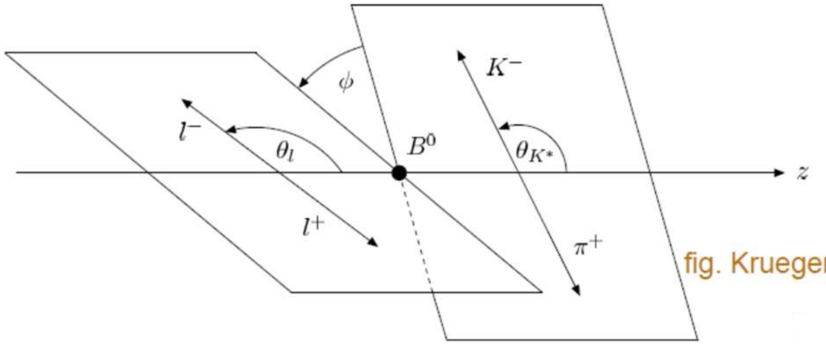
Fig. Aebischer, Altmannshofer, Guadagnoli, Reboud, Stangl, Straub 1903.10434

B->V l l: rate (schematic)



B->V l l: angular distribution

Vector observed as two-particle spin-1 resonance. Six helicity amplitudes. Many angular observables



θ_K in K* rest frame

θ_l in dilepton cm frame

ϕ boost-invariant (w.r.t. z axis)

fig. Krueger, Matias 2002

$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d(\cos\theta_k) d\phi} = \frac{9}{32\,\pi}$$

$$\times\Big(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l$$
$$+I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi$$
$$+I_5 \sin 2\theta_k \sin\theta_l \cos\phi + (I_6^s \sin^2\theta_k + I_6^c \cos^2\theta_K)\cos\theta_l$$
$$+I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi\Big)$$

Angular observables

For zero mass there are the following independent observables:

$$I_2^c = -F\,\frac{\beta^2}{2}\left(|H_V^0|^2 + |H_A^0|^2\right),$$

$$I_2^s = F\,\frac{\beta^2}{8}\left(|H_V^+|^2 + |H_V^-|^2\right) + (V \to A)$$

"longitudinal" rate (sim. to scalar BR)

"transverse" rate

Usually reported as BR and FL

$$I_6^s = F\beta\,\mathrm{Re}\left[H_V^-(H_A^-)^* - H_V^+(H_A^+)^*\right]$$

Lepton forward-backward rate asymmetry

Usually reported as AFB or P2

$$I_4 = F\,\frac{\beta^2}{4}\mathrm{Re}\left[\left(H_V^- + H_V^+\right)\left(H_V^0\right)^*\right] + (V \to A).$$

$$I_5 = F\left\{\frac{\beta}{2}\mathrm{Re}\left[\left(H_V^- - H_V^+\right)\left(H_A^0\right)^*\right] + (V \leftrightarrow A)\right.$$

Often discuss P4' and P5' instead

$$I_3 = -\frac{F}{2}\mathrm{Re}\left[H_V^+(H_V^-)^*\right] + (V \to A)$$

$$I_9 = F\,\frac{\beta^2}{2}\mathrm{Im}\left[H_V^+(H_V^-)^*\right] + (V \to A)$$

Require presence of "wrong-helicity" amplitudes (suppressed in SM)

Probe right-handed currents

Flavour: the dogs that did not bark

From AC Doyle, “The Adventure of Silver Blaze”
[with thanks to J Ellis]

> Gregory (Scotland Yard detective): "Is there any other point to which you would wish to draw my attention?"
> Holmes: "To the curious incident of the dog in the night-time."
> Gregory: "The dog did nothing in the night-time."
> Holmes: "That was the curious incident."

Quote and S Paget's illustration via Wikipedia

Absence of an effect in a BSM-sensitive observable can be as important a clue as an anomaly.

Eg Meson-antimeson mixing → constrain NP scales up to 10^5 TeV (for maximally flavor-violating BP)

Null results

Clean null tests of SM from (mainly) B → K*ɣ and B→ K*μμ

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 0$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 0$$

(Melikhov 1998)
Krueger, Matias 2002
Lunghi, Matias 2006
Becirevic, Schneider 2011
Becirevic, Kou, et al 2012
SJ, Martin Camalich 2012

Generated in the present of right-handed currents. No effect seen in data.
'Pseudo-observables:' Wilson coefficients from global fit

$$C'_{7\gamma} = 0.018 \pm 0.037$$ Aebischer et al arXiv:1903.10434

$$C'_{9V} = 0.09 \pm 0.15$$ Paul & Straub arXiv:1608.02556

ΔF=2: Neutral meson mixing also stringent constraints

Null results: Implications

SJ, Kirk, Lenz, Leslie, to appear – PRELIMINARY!

Delta C<0

Delta C>0

Coeff.	$\Delta\chi^2 \leq 1$	$\Lambda_-(\mathrm{TeV})$	$\Lambda_+(\mathrm{TeV})$
ΔC_5	$[-0.01, 0.01]$	9.7	10.5
ΔC_6	$[-0.02, 0.02]$	5.6	5.8
ΔC_7	$[-0.01, 0.01]$	8.8	9.7
ΔC_8	$[-0.02, 0.02]$	6.2	6.9
ΔC_9	$[-0.001, 0.005]$	22.3	12.6
ΔC_{10}	$[0.01, 0.05]$	-	3.8
$\Delta C'_1$	$[-0.01, 0.02]$	11.9	5.5
$\Delta C'_2$	$[-0.04, 0.09]$	4.5	2.8
$\Delta C'_3$	$[-0.04, 0.02]$	4.5	7.0
$\Delta C'_4$	$[-0.07, 0.03]$	3.2	5.1
$\Delta C'_5$	$[-0.02, 0.03]$	5.9	4.8
$\Delta C'_6$	$[-0.07, 0.10]$	3.3	2.8
$\Delta C'_7$	$[-0.03, 0.02]$	5.2	6.6
$\Delta C'_8$	$[-0.05, 0.04]$	3.7	4.3
$\Delta C'_9$	$[0.002, 0.010]$	-	8.6
$\Delta C'_{10}$	$[-0.08, -0.06], [0.02, 0.05]$	7.1	3.5

$(\bar{s}_R \Gamma c)(\bar{c}\Gamma' b)$

stringently constrained

Forward-backward asymmetry / P2

The zero-crossing of $I_6^s = F\beta\,\mathrm{Re}\left[H_V^-(H_A^-)^* - H_V^+(H_A^+)^*\right]$ (or of AFB, or P2)

approximately coincides with that of HV-, because HV+ HA+ is doubly suppressed in the heavy-quark limit (and constrained by non-signal in I3, I9).

Have

$$H_V^- \propto \frac{2m_b^2}{q^2} C_7 T_- + C_9 V_- + h_-$$

Zero depends on form factor ratio T-/V- (besides on nonlocal term h-).
This ratio is calculable in the heavy-quark limit (in terms of meson LCDA's).

Charles et al 1999
Beneke, Feldmann 2000
...

Forms the basis for the 'optimised observables' (P2, P5', etc) Descotes-Genon, Hofer, Matias, Virto

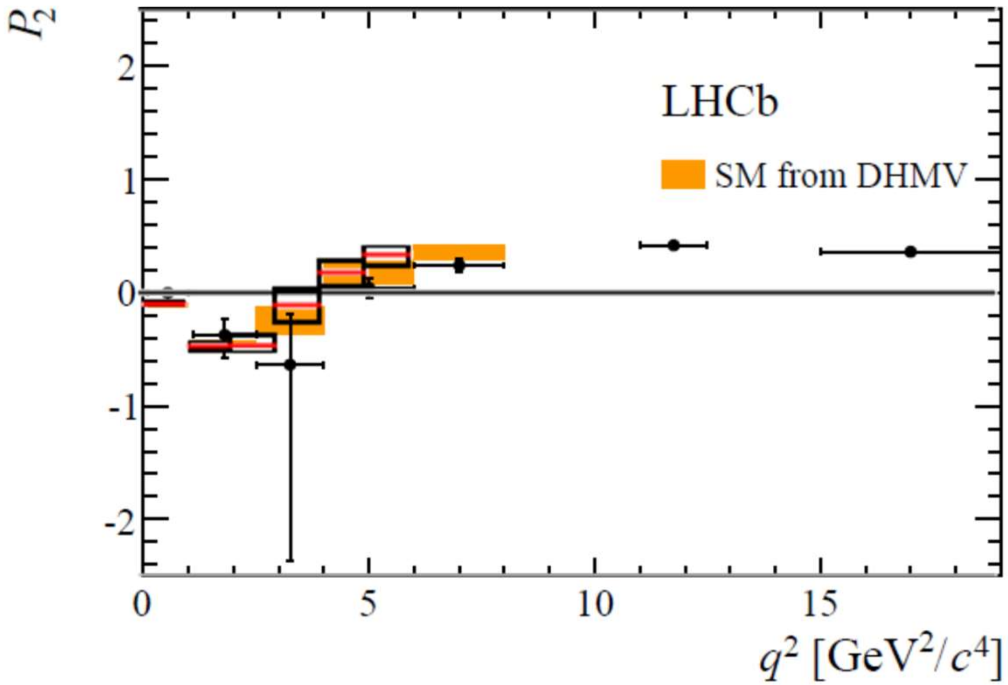
HQ limit: $T_-(0)/V_-(0) \sim 1.05 > 1$

compare to: $T_-(0)/V_-(0) = 0.94 \pm 0.04$ [D Straub, priv comm based on Bharucha, Straub, Zwicky 1503.05534]

LCSR computation with correlated parameter variations.

Size consistent with a power correction; 5% uncertainty estimate.

P2 – theory vs data



arXiv:1512.04442 (LHCb)

Boxes – predictions from SJ, Martin Camalich 2014

(pure heavy-quark limit, general power correction parameterisation, varying in 10% range, Gaussian error combination)

Good agreement with data, even for pure heavy-quark limit with no power corrections (red lines)

P5'

Defined through $P_5' = \frac{I_5}{\sqrt{-I_{2s} I_{2c}}}$ Descotes-Genon, Hofer, Matias, Virto

$$I_5 = F\left\{\frac{\beta}{2}\mathrm{Re}\left[(H_V^- - H_V^+)\,(H_A^0)^*\right] + (V \leftrightarrow A)\right.$$

Approximately:

suspressed at 3-6 GeV2 (AFB zero)

proportional to C10

proportional to C9 x C10

$$I_2^c = -F\,\frac{\beta^2}{2}\left(|H_V^0|^2 + |H_A^0|^2\right),$$

Proportional to CL^2

$$I_2^s = F\,\frac{\beta^2}{8}\left(|H_V^+|^2 + |H_V^-|^2\right) + (V \to A)$$

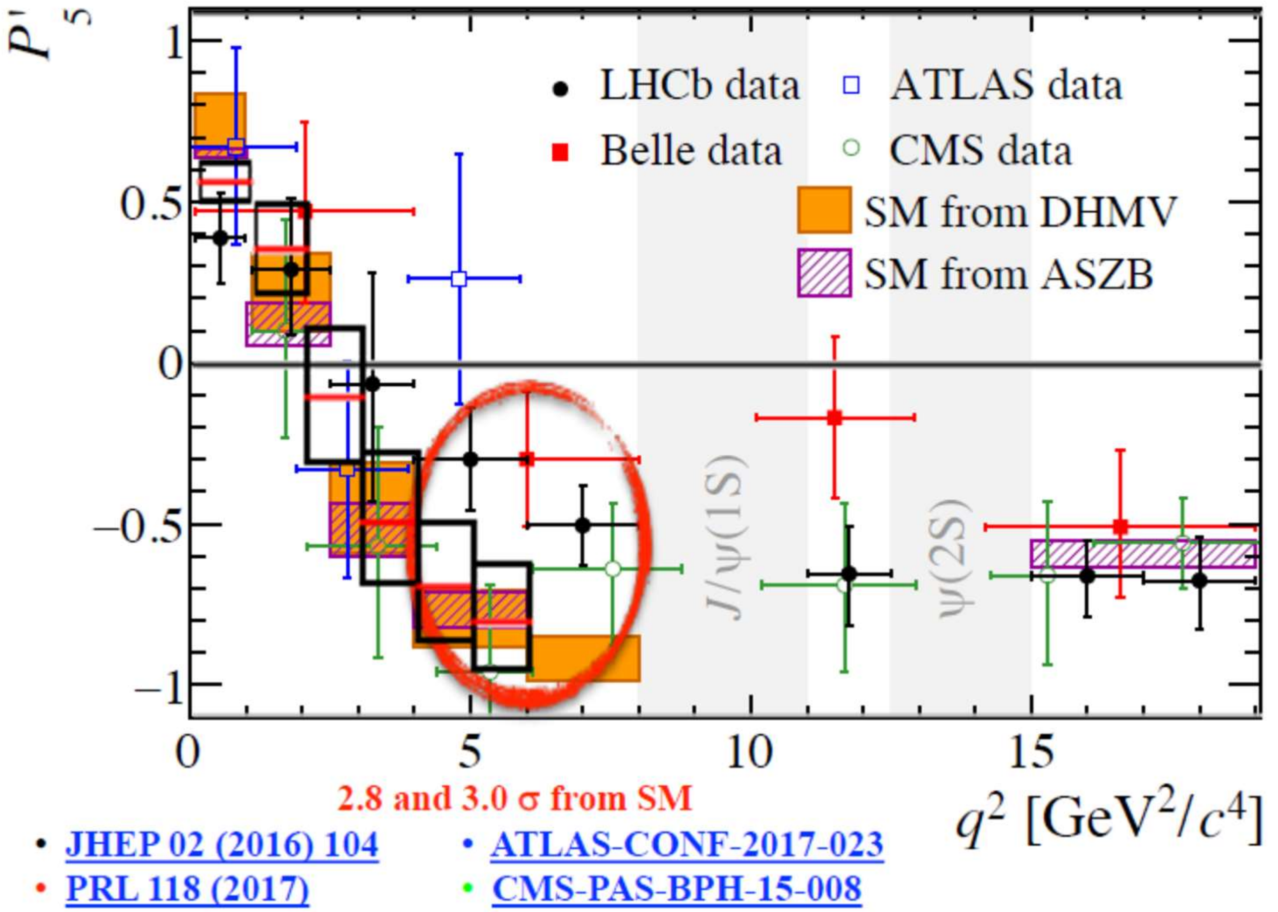
Dominated by axial amplitude

As a result, the C10 (as well as form factor) dependence largely cancels, and the observable is strongly dependent on C9 (very roughly proportional)

However, the number of independent hadronic inputs (for which power corrections must be estimated, LCSRs used, etc) is larger, because both transverse and longitudinal helicities enter.

Emphatic claims in literature that this does not matter Descotes-Genon et al; Capdevila et al

P5’



Simone Bifani, seminar at CERN (overlaid predictions from SJ&Martin Camalich 2014)

Modest discrepancy around 4-6 GeV, consistent with reduced C9

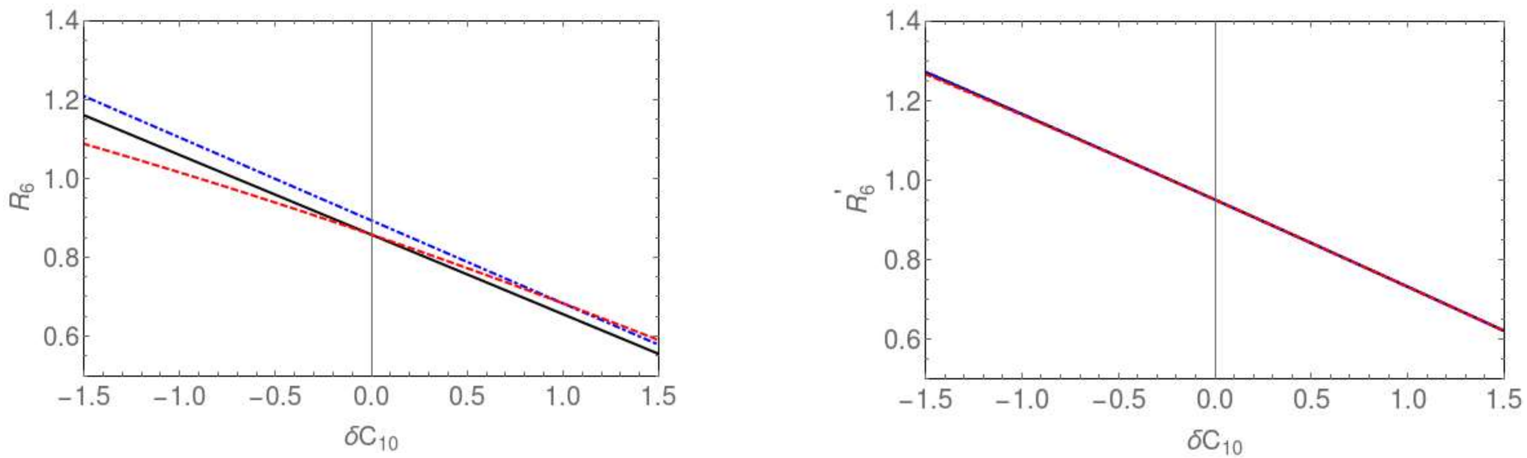
Determining CR (break C9/C10 degeneracy)

Geng, Grinstein, SJ, Martin Camalich, Ren, Shi arxiv:1704.05446

Propose to measure observable

$$R_6[a,b] = \frac{\int_a^b \Sigma_6^\mu dq^2}{\int_a^b \Sigma_6^e dq^2} \approx \frac{C_{10}^\mu}{C_{10}^e} \times \frac{\int_a^b |\vec{k}| q^2 \beta_\mu^2 \,\mathrm{Re}[H_{V-}^{(\mu)}(q^2)] V_-(q^2)}{\int_a^b |\vec{k}| q^2 \,\mathrm{Re}[H_{V-}^{(e)}(q^2)] V_-(q^2)}$$

and/or $R_6' = \langle P_2^{(\mu)} \rangle / \langle P_2^{(e)} \rangle$

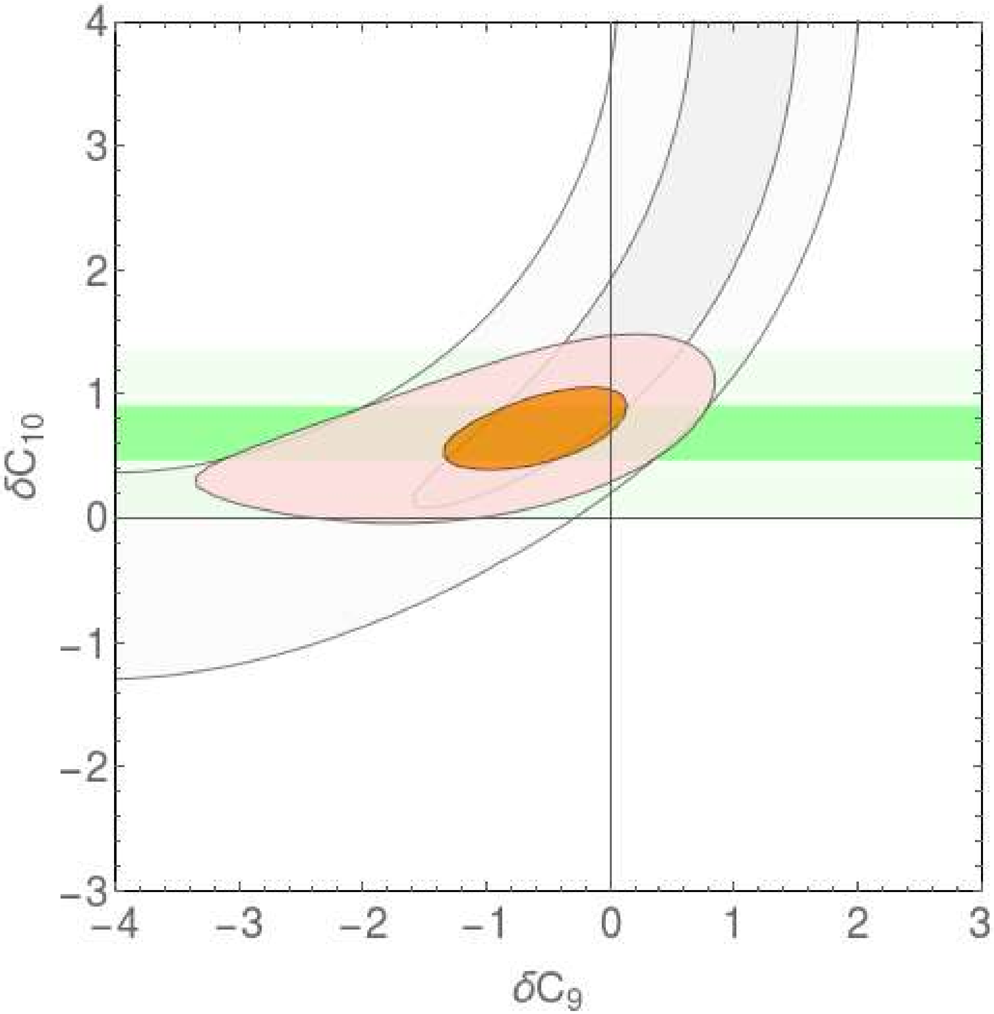


Remains very clean in presence of new physics.
Probes a LUV C10 precisely, irrespective of values of C9e, C9mu

Prospective fit with LUV obs. only

Geng, Grinstein, SJ, Martin Camalich, Ren, Shi arxiv:1704.05446

Consider a hypothetical experimental result R6’ = 0.80(5)



Conclusions

Rare B-decays show intriguing anomalies – and also null tests which provide powerful constraints on Wilson coefficients and new physics!

Main theory uncertainties remain from form factors (in my opinion more important than long-distance virtual-charm effects which can be computed up to power corrections, at least if one stays well below the virtual charm resonances)

- require few-percent accuracy on FF for a BR
- require few-percent accuracy of FF ratios for P5'

-> use analyticity-based FF expansion (z polynomial or Taylor series in q2) to fit form factors to data, rather than taking them from calculations with unclear systematics

Backup

RG evolution - numerical

SJ, Kirk, Lenz, Leslie arxiv:1701.09183 and to appear – PRELIMINARY!

Some elements first arise at two loops – still give important constraints.

$$\begin{pmatrix} C_1(\mu_b) \\ C_2(\mu_b) \\ C_3(\mu_b) \\ C_4(\mu_b) \\ C_5(\mu_b) \\ C_6(\mu_b) \\ C_7(\mu_b) \\ C_8(\mu_b) \\ C_9(\mu_b) \\ C_{10}(\mu_b) \\ C_{7\gamma}^{\rm eff}(\mu_b) \\ C_{9V}(\mu_b) \end{pmatrix} = \begin{pmatrix} 1.1 & -0.27 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.27 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & 1.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.9 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0.05 & 2.70 & 1.70 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.37 & 2.0 & 2.30 & -0.55 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.07 & 0.07 & 1.80 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & -0.02 & -0.29 & 0.82 \\ 0.02 & -0.19 & -0.015 & -0.13 & 0.56 & 0.17 & -1.0 & -0.47 & 4.00 & 0.70 \\ 8.50 & 2.10 & -4.30 & -2.00 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} C_1(M_W) \\ C_2(M_W) \\ C_3(M_W) \\ C_4(M_W) \\ C_5(M_W) \\ C_6(M_W) \\ C_7(M_W) \\ C_8(M_W) \\ C_9(M_W) \\ C_{10}(M_W) \\ C_{7\gamma}^{\rm eff}(M_W) \\ C_{9V}(M_W) \end{pmatrix}$$

Enormous RG effects - can accommodate P_5'. **But lepton-universal**

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RH(primed) 4-quark ops constrained by both C_7' and C_9'

Form factor relations

At most 1-2% over entire 0..6 GeV^2 range -> ignore

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit — Power corrections - parameterise

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al) (Beneke, Feldmann)

q^2 dependence in heavy-quark limit not known (model by a power p, and/or a pole model)

$V_+^\infty(0) = 0$ $T_+^\infty(0)=0$

$V_-^\infty(0) = T_-^\infty(0)$

$V_0^\infty(0) = T_0^\infty(0)$

from heavy-quark/ large energy symmetry

hence

$$\begin{aligned} T_+(q^2) &= \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b) \\ V_+(q^2) &= \mathcal{O}(\Lambda/m_b). \end{aligned}$$

Corrections are calculable in terms of perturbation theory, decay constants, light cone distribution amplitudes

$V_+^\infty(q^2) = 0$ $T_+^\infty(q^2)=0$

- **"naively factorizing" part of the helicity amplitudes $H_{V,A}^+$ strongly suppressed** as a consequence of chiral SM weak interactions
- We see the suppression is **particularly strong** near low-q^2 endpoint
- Form factor relations imply reduced uncertainties in suitable observables

Burdman, Hiller 1999 (quark picture)

Beneke, Feldmann, Seidel 2001 (QCDF)

Power corrections

SJ, Martin Camalich 1412.3183

Compare

$$P_5' = P_5'|_\infty \left(1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} \right.$$
$$+ \frac{a_{V_0} - a_{T_0}}{\xi_\|} 2\, C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\|}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})}$$
$$\left. + 8\pi^2 \frac{\tilde{h}_-}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{C_{9,\perp} + C_{9,\|}} + \text{further terms} \right) + \mathcal{O}(\Lambda^2/m_B^2)$$

(truncated after 3 out of 11 independent power-correction terms!)
also, dependence on soft form factors reappears at PC level

and

$$P_1 = \frac{1}{C_{9,\perp}^2 + C_{10}^2} \frac{m_B}{|\vec{k}|} \left(-\frac{a_{T_+}}{\xi_\perp} \frac{2\, m_B^2}{q^2} C_7^{\rm eff} C_{9,\perp} - \frac{a_{V_+}}{\xi_\perp} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) - \frac{b_{T_+}}{\xi_\perp} 2 C_7^{\rm eff} C_{9,\perp} \right.$$
$$\left. - \frac{b_{V_+}}{\xi_\perp} \frac{q^2}{m_B^2} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) + 16\pi^2 \frac{h_+}{\xi_\perp} \frac{m_B^2}{q^2} C_{9,\perp} \right) + \mathcal{O}(\Lambda^2/m_B^2).$$

(complete expression)

Further notice that a_{T+} vanishes as $q^2 \to 0$, h_+ helicity suppressed [will show], and the other three terms lacks the photon pole.

Hence P_1 **much** cleaner than P_5', especially at very low q^2