Minimal Realizations of Dirac Neutrino Mass from Generic One-loop and Two-loop Topologies

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Based on: arxiv: 1904.07407 (accepted for publication in EPJC), arxiv: 1910.xxxx

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Particle Physics on the Plains,
University of Kansas
13th Oct, 2019

❖ Outline

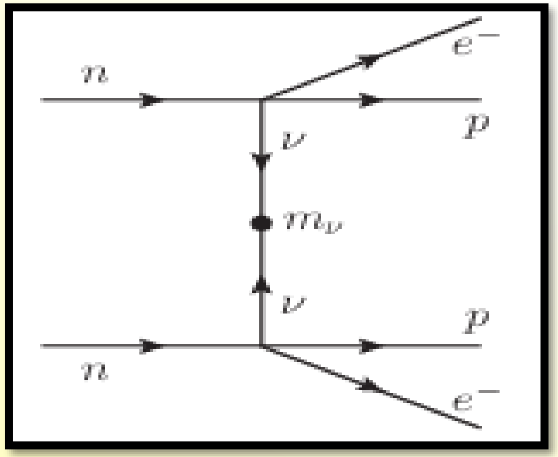
- Motivation
- Neutrino Mass Models
- Minimal Dirac Neutrino Mass Models: One-loop Topology
- Minimal Dirac Neutrino Mass Models: Two-loop Topology
- Phenomenology of Specific Models
- Summary

❖ Neutrino Mass and Nature of Neutrinos

- Neutrino Oscillation data suggests that neutrinos have tiny masses
  → Physics Beyond SM.

Origin of neutrino mass is still unknown

- Neutrinos can be either Dirac type or Majorana type in nature.
  Can be resolved by neutrinoless double beta decay.



No Conclusive Evidence

❖ Neutrino Mass Models

- Most of the popular models assumes that the neutrinos are Majorana type in nature.
  - Seesaw : Type I, Type II, Type III, etc…
  - Radiative Mechanism : 1-loop (Zee), 2-loop (Zee-Babu)
- Building Dirac neutrino mass models require RH neutrinos, $\boldsymbol{\nu_R}$ singlets under the SM

$$\mathcal{L}_4 = -y^{\nu}_{ij}\, \overline{L}_i \widetilde{H}\, \nu_{R_j} + h.c., \quad \Rightarrow \quad y_\nu \lesssim 10^{-11}$$

- Using additional symmetries one can **forbid the tree-level mass term** as well as **Majorana neutrino mass terms at all order**
  - Discrete Symmetries: D. Borah et al.
  - In Left-Right Symmetry Model: Babu-He, Branco-Senjanovic
  - SM with $\boldsymbol{U(1)}$ symmetry (global/local): $\boldsymbol{U(1)_{B-L}}$(Ma), $\boldsymbol{U(1)_R}$(S.Jana,**VPK**,S.Saad (arXiv:1904.07407))

❖ Dirac Neutrino Mass: SM x $U(1)_{B-L}$

- SM x $U(1)_{B-L}$ is anomaly free with three RH neutrinos.
- Anomaly free B-L charge assignment of RH neutrinos:

$$\nu_{Ri} = \{-1, -1, -1\} \text{ or } \{5, -4, -4\}$$

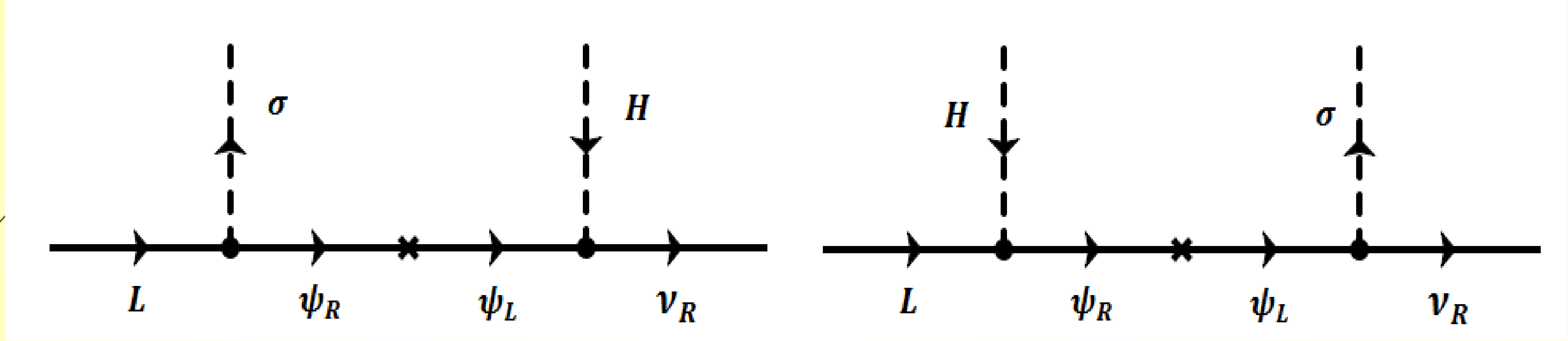
- The second possibility will naturally forbid *tree-level neutrino mass term* as well as *Majorana mass term* at all order.
- Neutrino mass generated by $d = 5$ effective operator of the form

$$\mathcal{L}_5 = -\frac{h_{ij}}{\Lambda}\,\overline{L}_i \widetilde{H}\,\nu_{Rj}\sigma + h.c.,$$

σ : SM singlet scalar charged +3 under $U(1)_{B-L}$

❖ Dirac Neutrino Mass: SM x $U(1)_{B-L}$

- Dirac Neutrino Mass via Seesaw mechanism



With $\psi_{L/R} = (1, 1, 0, -1)$ or $(1, 2, -1/2, -4)$

- Interested in NP at TeV Scale

Radiative Models

❖ Dirac Neutrino Mass: SM x $U(1)_{B-L}$

- Systematically search for the minimal Dirac neutrino mass models arise from this $d = 5$ effective operator.
- Strategy: Construct the generic one-loop and two-loop topologies and then build the associated minimal models .
- Minimality refers to

✓ Models with minimum number of BSM states are preferable.

✓ $SU(2)_L$ singlet BSM states are preferred. If BSM particles are required not to be iso-singlet, then we minimize the number of states that are charged under $SU(2)_L$.

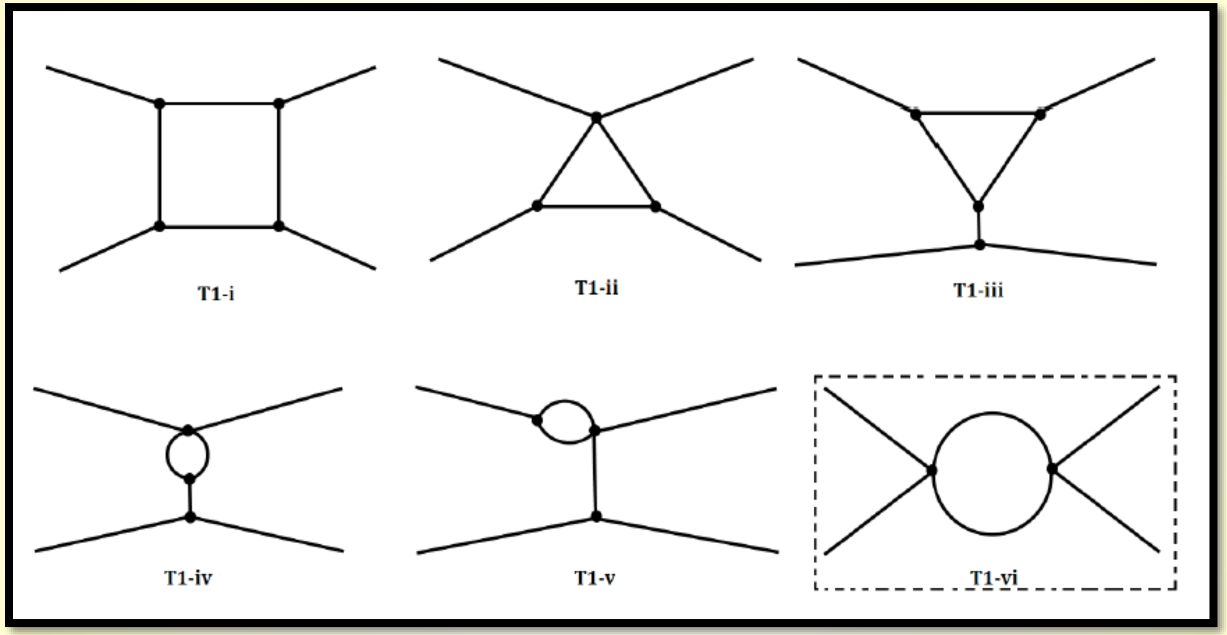
✓ If possible, introduction of any BSM fermion is prohibited. If the presence of BSM fermionic state is required, we assume it to be vector like under SM x $U(1)_{B-L}$.

✓ BSM states with lowest dimensional representation under $SU(2)_L$ are preferred.

❖ One-loop topologies: Viable topologies

$$\mathcal{L}_5 = -\frac{h_{ij}}{\Lambda}\,\overline{L}_i\widetilde{H}\,\nu_{Rj}\sigma + h.c.,$$

- *All possible one-loop topologies with four external legs*

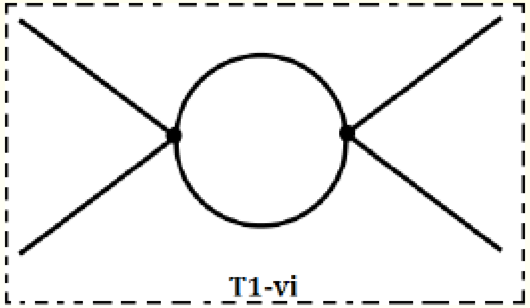


F.Bonnet, M.Hirsch, T. Ota, W. Winter **JHEP 1207 (2012)153**

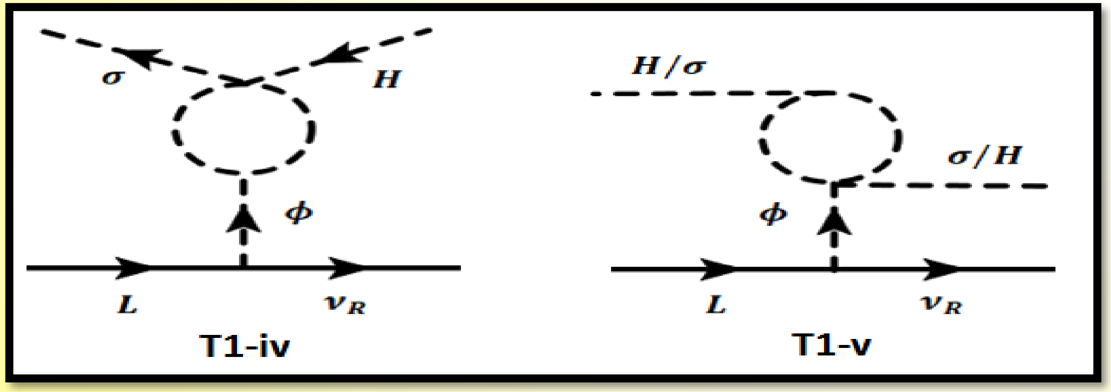
- *Not all these topologies can be lead to successful one-loop neutrino mass in our framework*

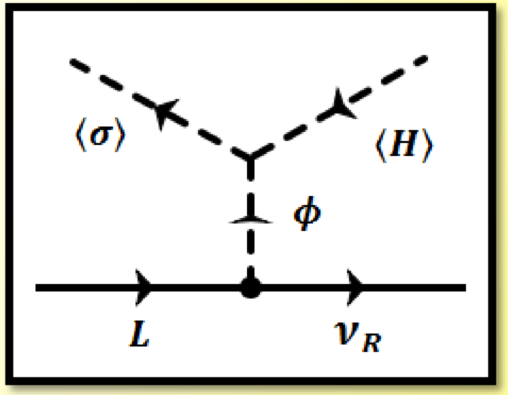
❖ One-loop topologies: Viable topologies

- $\boldsymbol{T1-vi}$ corresponds to non-renormalizable topologies



- $\boldsymbol{T1-iv\ /v}$ can't give viable one loop models in our framework.



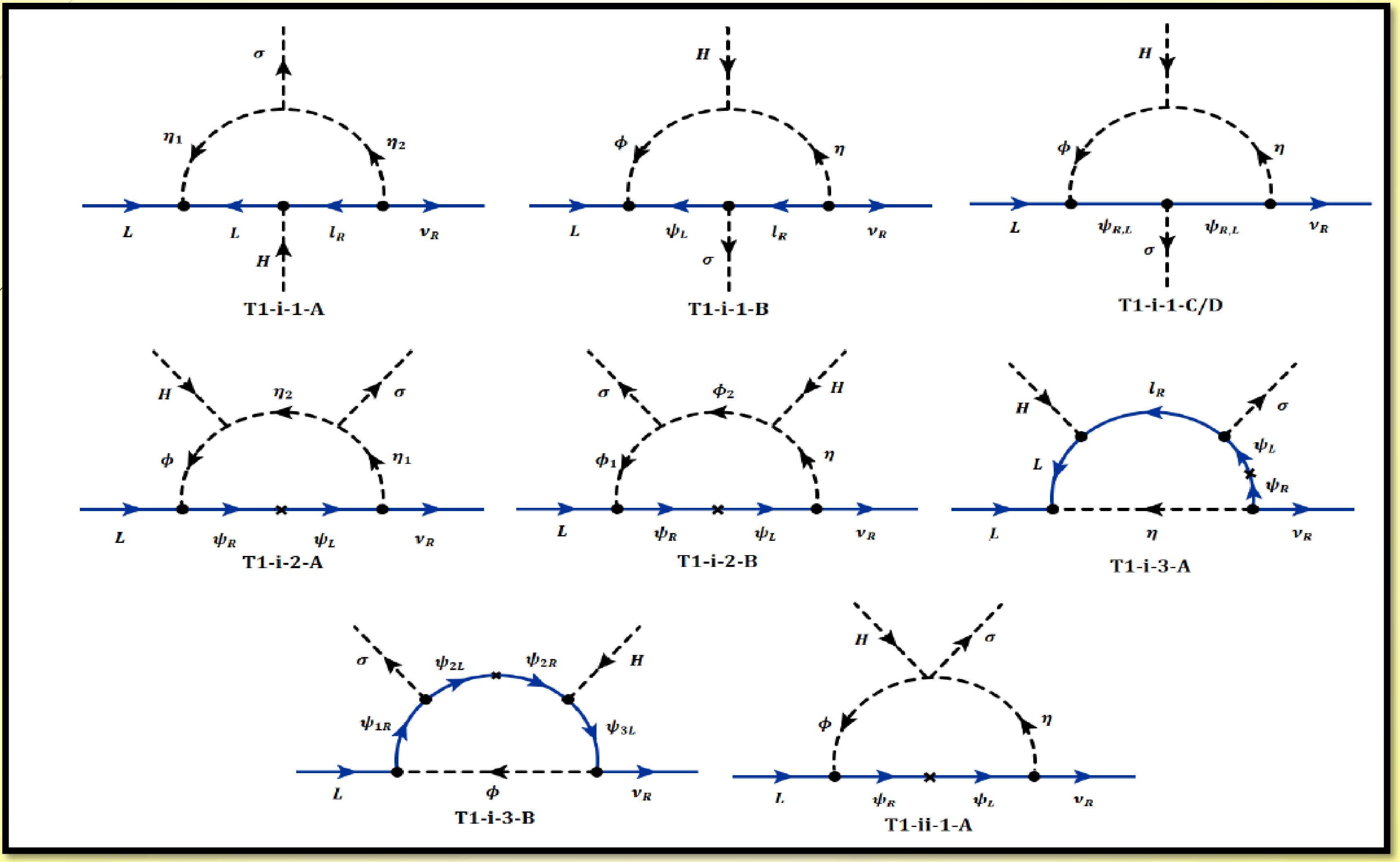


- $\boldsymbol{T1-iii}$ can't give viable one loop models in our framework.

❖ Minimal One-loop models

- Constructing minimal models arising from $T1-i, ii$

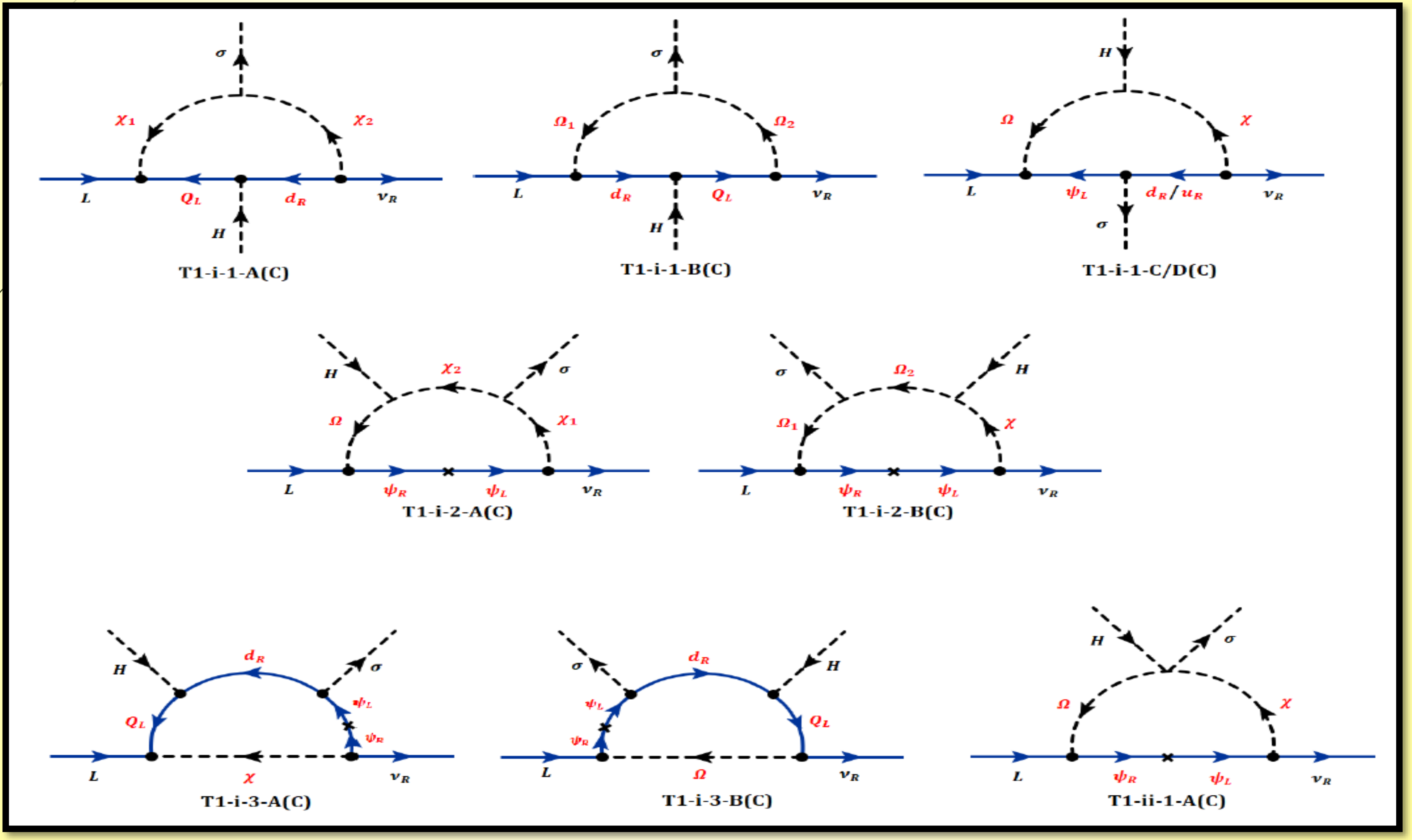
S.Jana,**VPK**,S.Saad (arXiv:1910.xxxx)



❖ Minimal One-loop models – Colored version

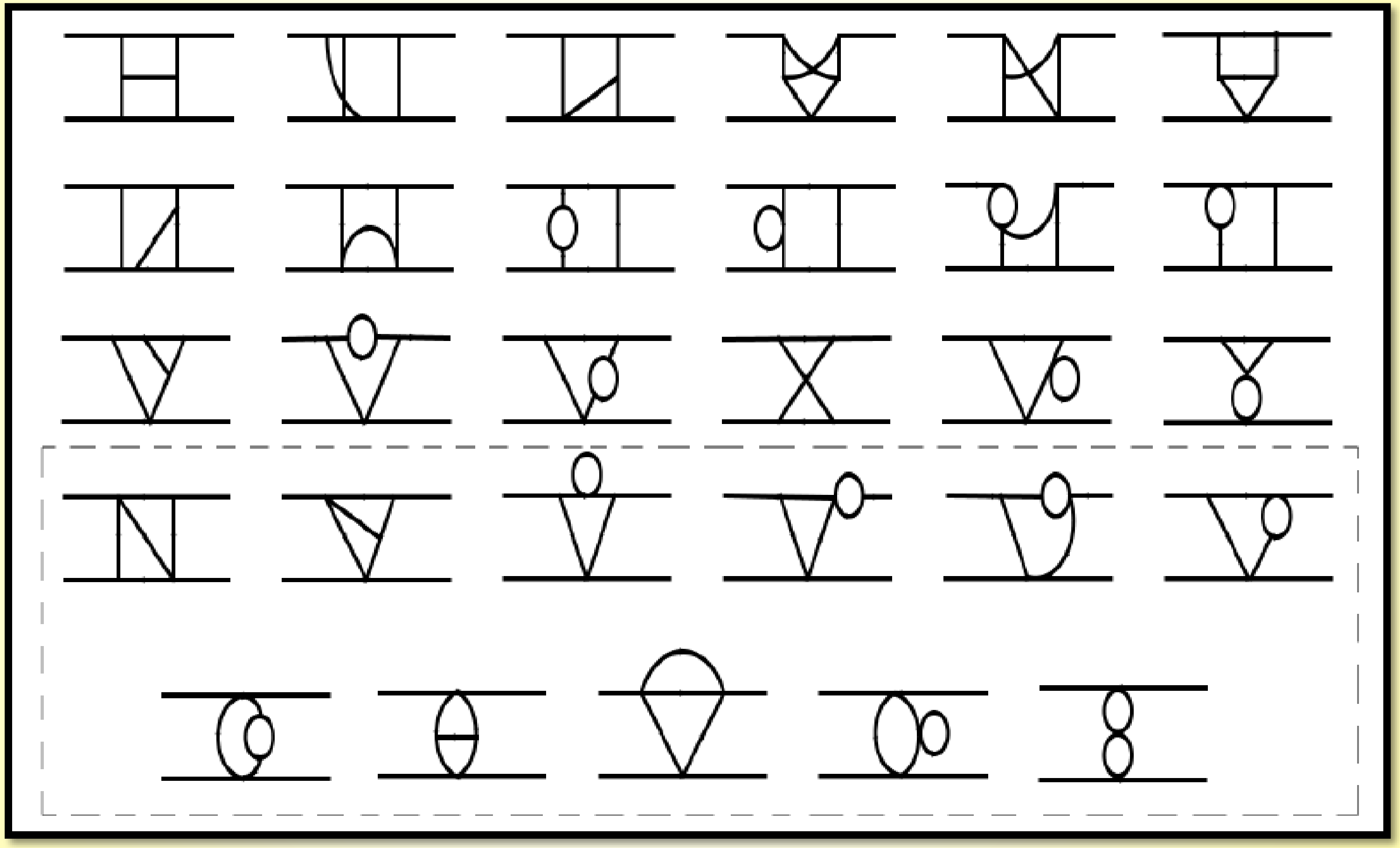
- Corresponding minimal models for the colored version

S.Jana,**VPK**,S.Saad (arXiv:1910.xxxx)



❖ Two-loop topologies

- ❑ All possible two-loop topologies with four external legs

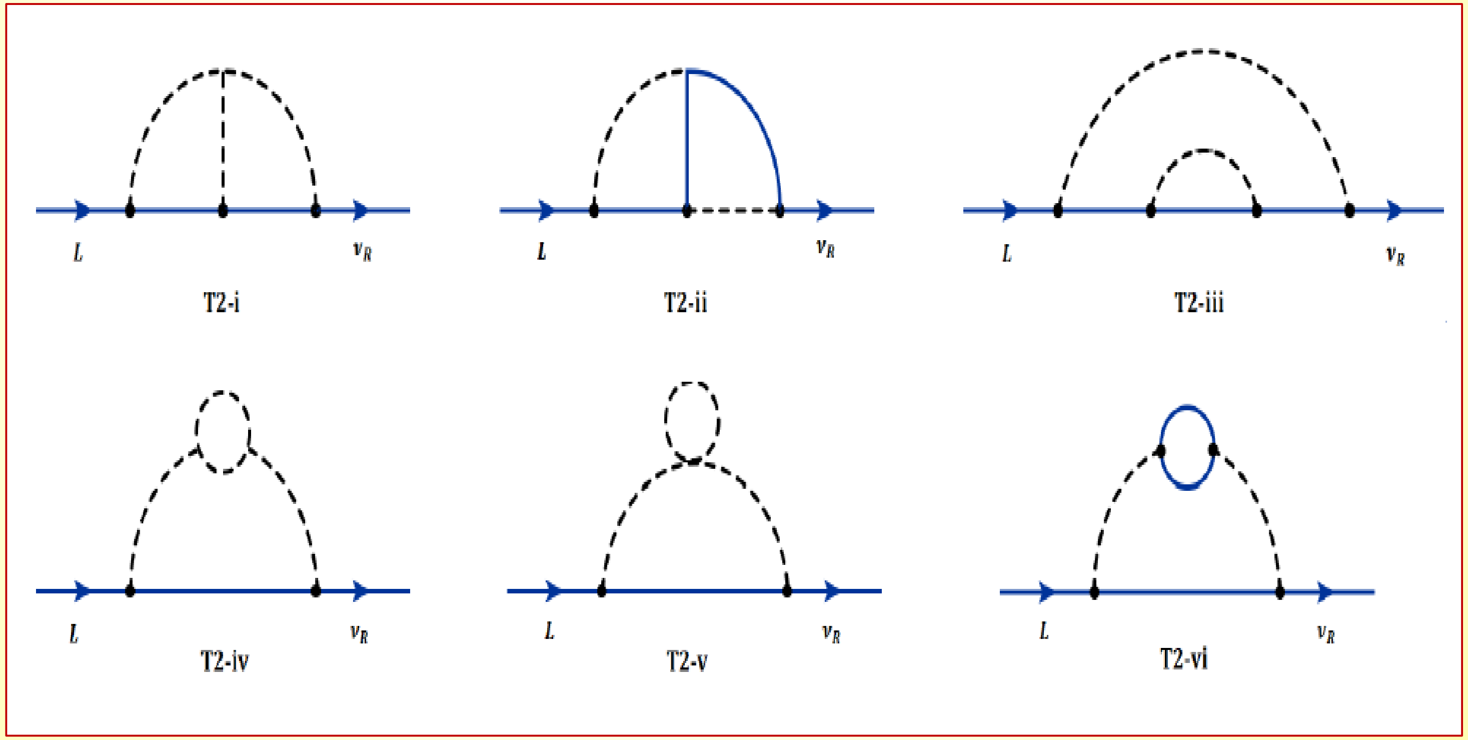


- ❑ Not all the topologies can be lead to successful two-loop neutrino mass in our framework. For example, last 11 topologies corresponds to non-renormalizable topologies

D.A.Sierra,A.Degree,L.Dorame,M.Hirsch **JHEP 1503 (2015) 040**

❖ Two-loop Skeleton Diagrams

- ❑ Remaining 18 topologies can give vast number of diagrams.
- ❑ By suppressing the external scalar legs, all these diagrams can be reduced into six basic diagrams → ***Skeleton diagrams***

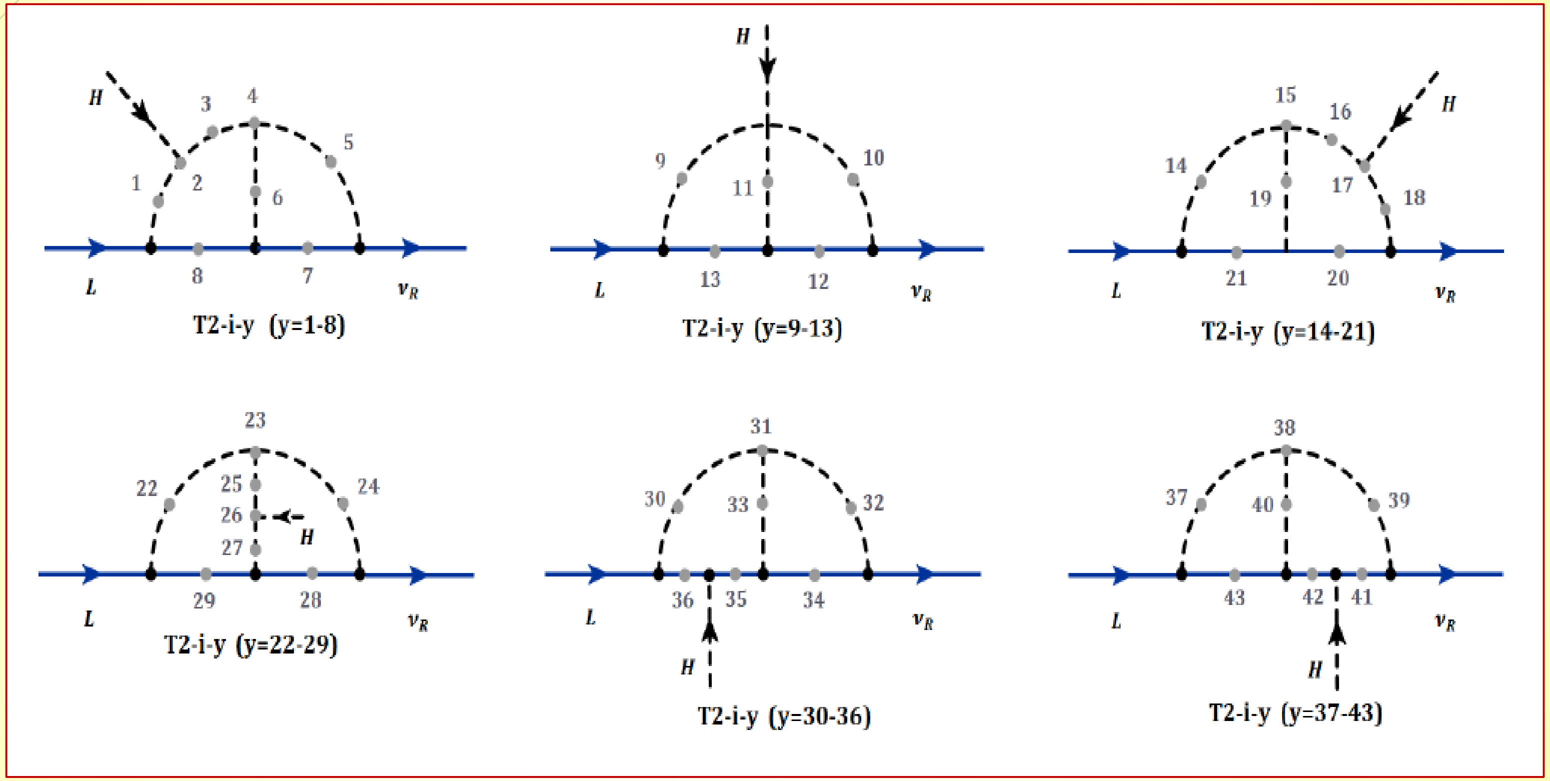


- ❑ Constructing minimal models arising from each of the skeleton diagrams.

D.A.Sierra,A.Degree,L.Dorame,M.Hirsch,**JHEP 1503 (2015) 040**

❖ Search for Minimal Two-loop Models

- All possible diagrams emerging from $T2-i$



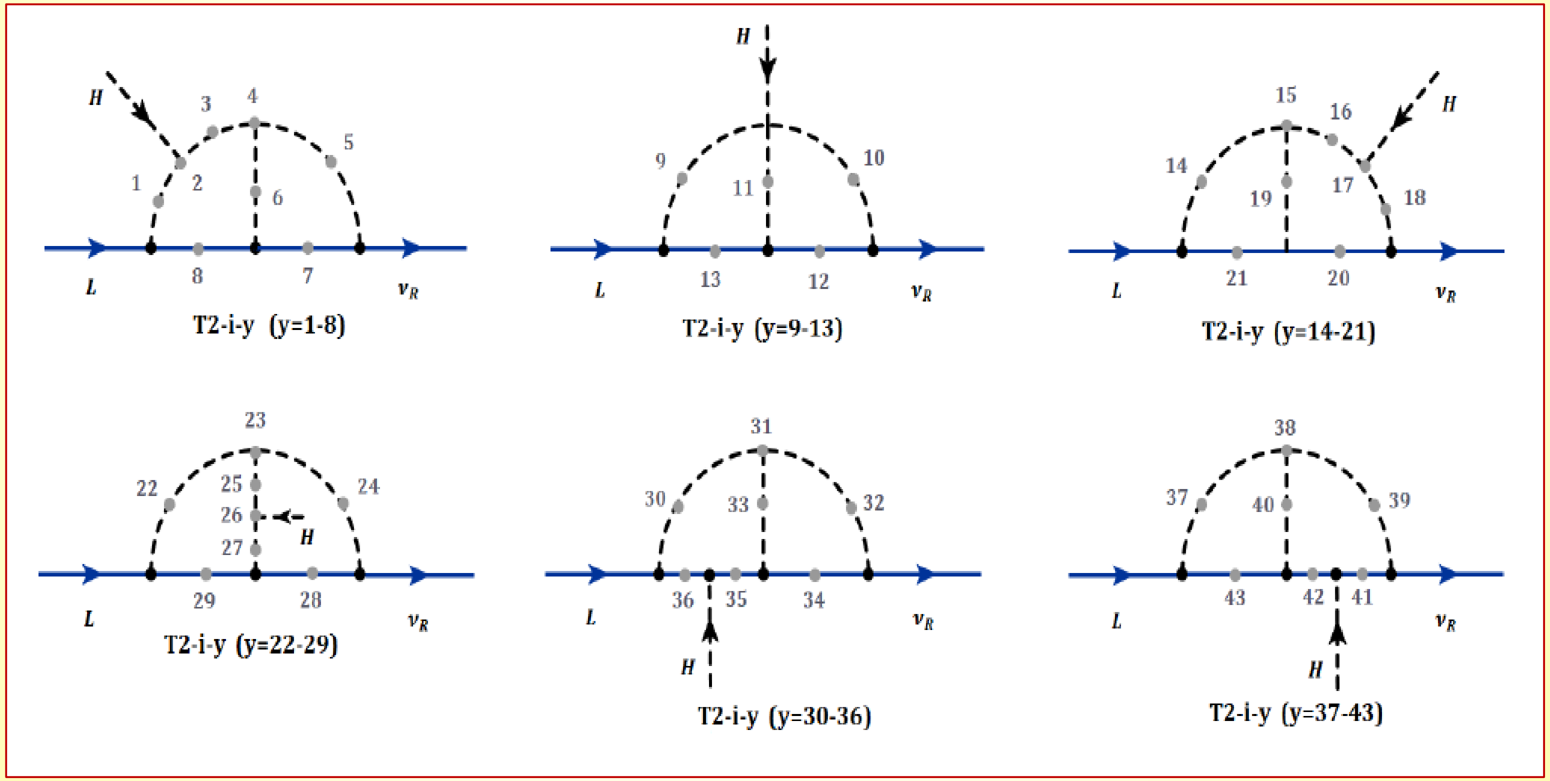
- Discarding 22 viable diagrams, $T2-i-y$, $y = \{7-8, 12-13, 20-21, 28-29, 34- 36, 41-43\}$

Required introduction of BSM fermions

❖ Search for Minimal Two-loop Models

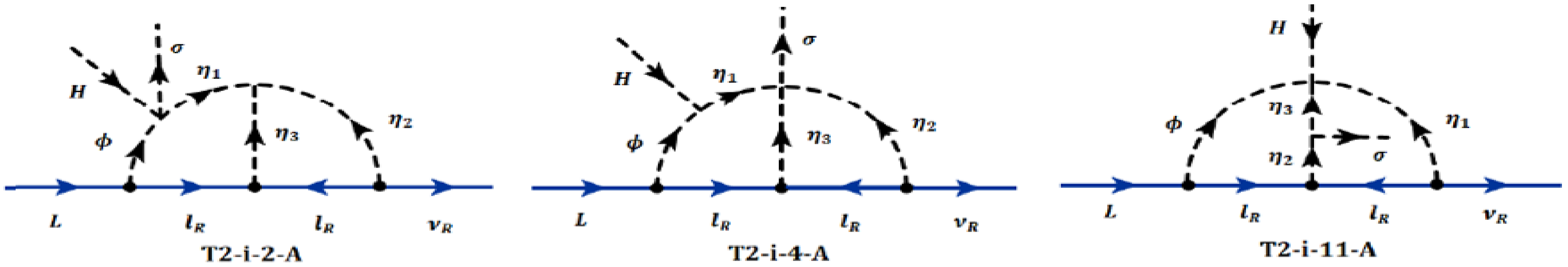
- ❑ Fixing internal fermion lines to be $\boldsymbol{l_R}$,

  excluding all diagrams other than $T2-i-y,\ y=\{2-6, 10-11\}$, required more than one isodoublet



- ❑ Among these 7 diagrams selecting diagrams with minimal number of BSM scalars.

❖ Minimal Two-loop Models

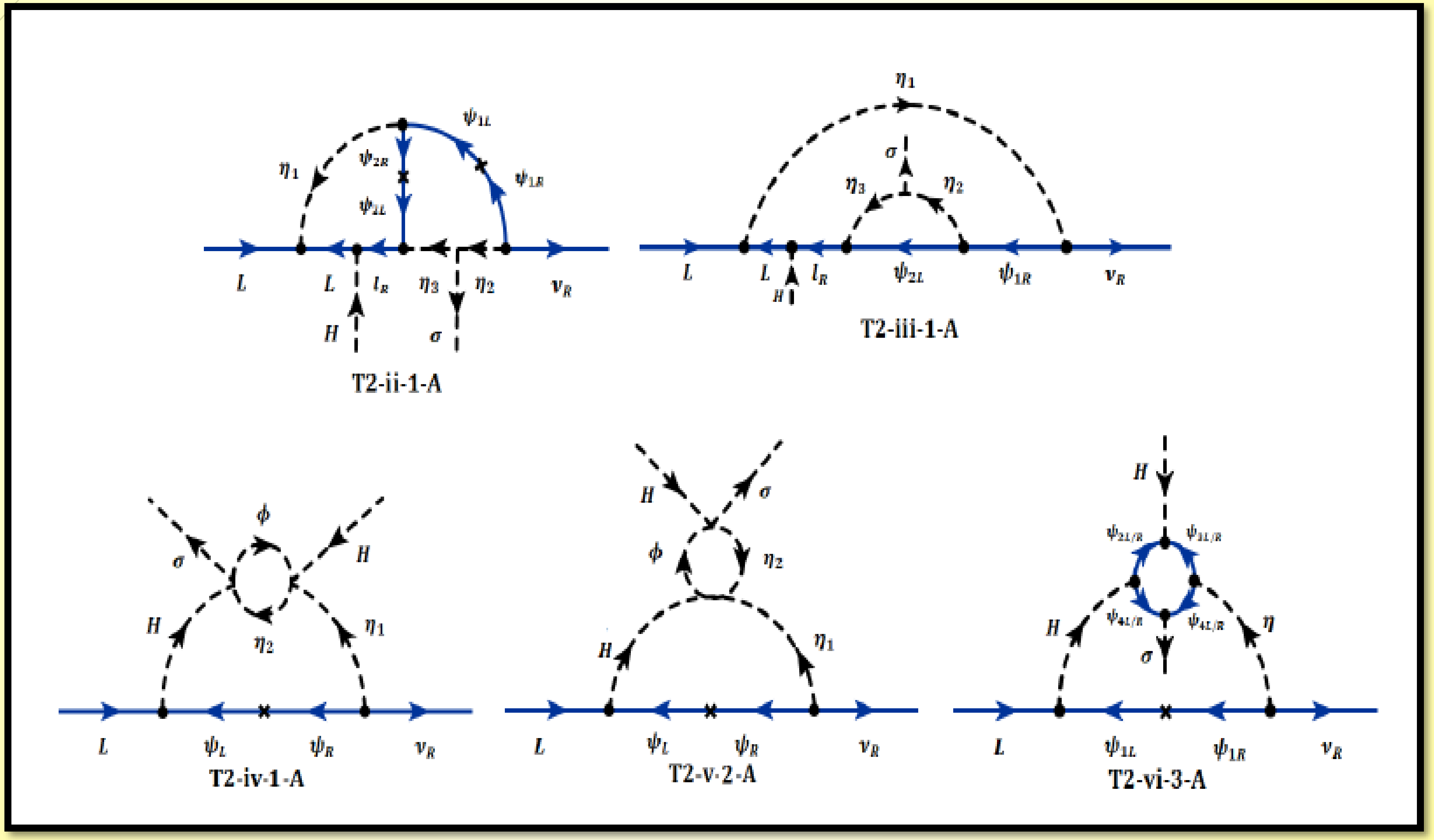


Skeleton	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T2-i	T2-i-2-A	$\eta_1(1,1,1,-3)$ $\eta_2(1,1,1,5)$ $\eta_3(1,1,-2,-2)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_3^* l_R$ $+y_3\overline{l^c_R}\eta_2\nu_R + \mu\eta_1\eta_2\eta_3$ $+\lambda\phi\epsilon H\sigma^*\eta_1^*$	(Saad)
	T2-i-4-A	$\eta_1(1,1,1,0)$ $\eta_2(1,1,1,5)$ $\eta_3(1,1,-2,-2)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_3^* l_R$ $+y_3\overline{l^c_R}\eta_2\nu_R + \mu\phi\epsilon H\eta_1^*$ $+\lambda\eta_1\eta_2\eta_3\sigma^*$	✔
	T2-i-11-A	$\eta_1(1,1,1,5)$ $\eta_2(1,1,-2,-2)$ $\eta_3(1,1,-2,-5)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_2^* l_R$ $+y_3\overline{l^c_R}\eta_1\nu_R + \mu\eta_2\eta_3^*\sigma^*$ $+\lambda\phi\epsilon H\eta_1\eta_3$	✔

S.Jana,**VPK**,S.Saad (arXiv:1910.xxxx)

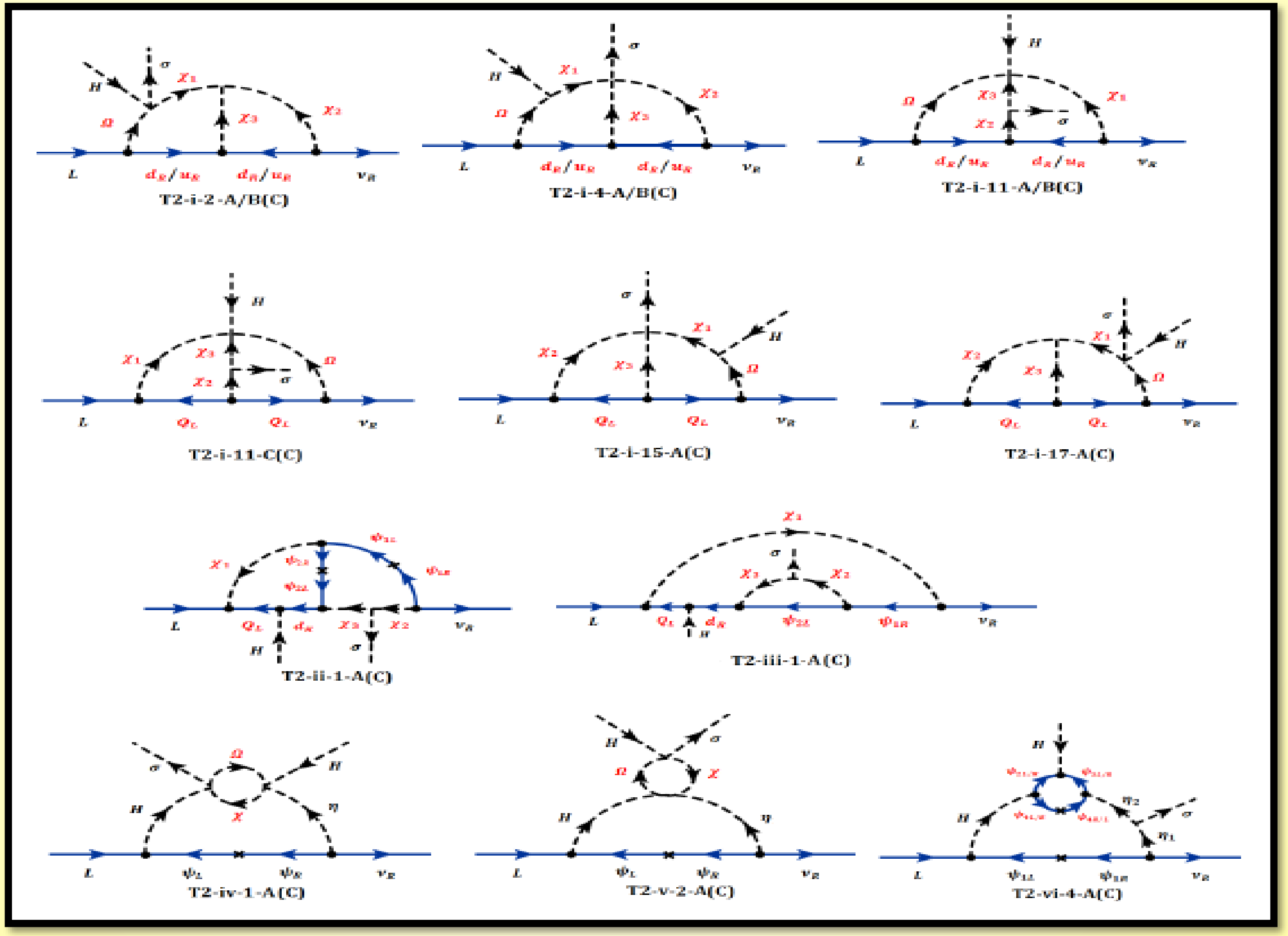
❖ Minimal Two-loop Models

❑ Repeating similar strategy for $T2 - ii\ to\ vi$



❖ Minimal Two-loop models – Colored version

- Corresponding minimal models for the colored version



❖ Possible DM Candidates

- ❑ Spontaneously broken $\boldsymbol{U(1)_{B-L}}$ symmetry, may leave a residual unbroken symmetry that can potentially stabilize the DM particle.

One-Loop Models

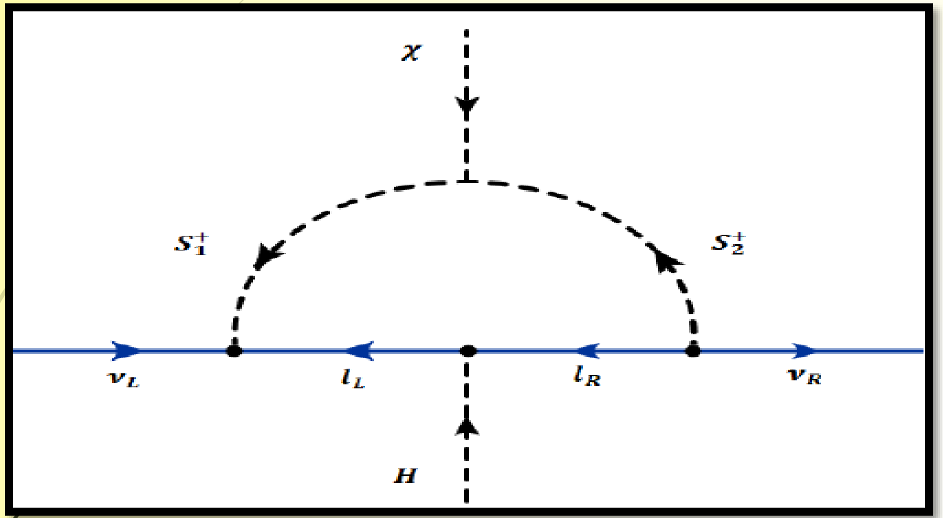
Models	Residual lepton symmetry	Residual dark symmetry	Choice of Y	Possible DM candidate
T1-i-1-A	Z_3	✗	—	✗
T1-i-1-B	Z_3	✗	—	✗
T1-i-1-C/D	Z_6	✓	—	$\psi_{L,R}, \eta, \phi$
T1-i-2-A	Z_6	✓	0	$\psi_{L,R}, \phi, \eta_1, \eta_2$
			-1	ϕ
T1-i-2-B	Z_6	✓	0	$\psi_{L,R}, \phi_1, \phi_2, \eta_1, \eta_2$
			-1	ϕ_1, ϕ_2
T1-i-3-A	Z_3	✗	—	✗
T1-i-3-B	Z_6	✓	0	$\psi_{1L,R}, \psi_{2L,R}, \psi_{3L,R}, \phi$
			-1	$\psi_{3L,R}, \phi$
T1-ii-1-A	Z_6	✓	0	$\psi_{L,R}, \eta, \phi$
			-1	ϕ

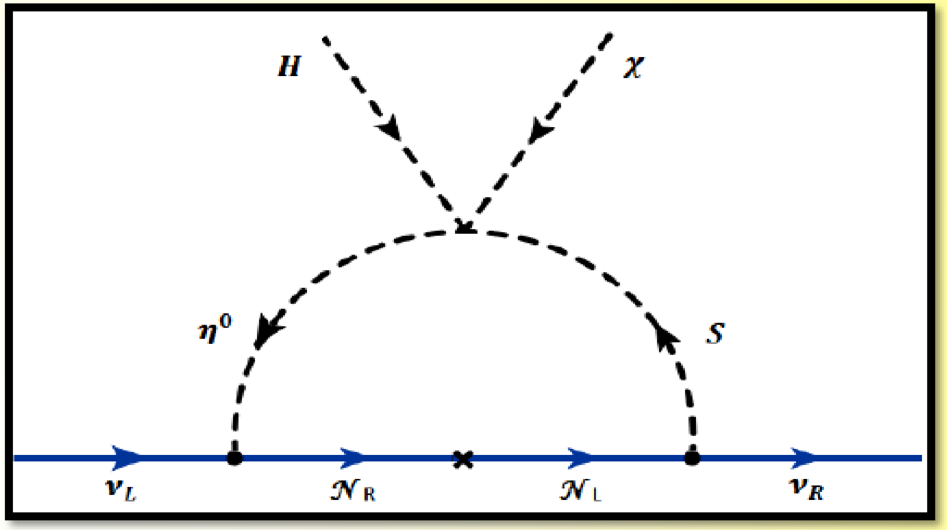
Two-Loop Models

Models	Residual lepton symmetry	Residual dark symmetry	Choice of Y	Possible DM candidate
T2-i-2-A	Z_3	✗	—	✗
T2-i-4-A	Z_3	✗	—	✗
T2-i-11-A	Z_3	✗	—	✗
T2-ii-1-A	Z_6	✓	0	$\psi_{2L,R}$
			-1	η_2, η_3
T2-iii-1-A	Z_6	✓	0	$\psi_{2L,R}$
			-1	η_2, η_3
T2-iv-1-A	Z_6	✓	$1/2$	ϕ, η
			$-1/2$	ϕ
T2-v-2-A	Z_6	✓	$1/2$	ϕ
			$-1/2$	ϕ, η_2
T2-vi-3-A	Z_6	✓	—	$\psi_{2L,R}, \psi_{4L,R}$

S.Jana,**VPK**,S.Saad (arXiv:1910.xxxx)

❖ Phenomenology of Specific Models





Multiplets	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$
Leptons	$L_{Li}(1,2,-\frac{1}{2},0)$ $\ell_{Ri}(1,1,-1,-1)$ $\nu_{Ri}(1,1,0,\{-5,4,4\})$
Scalars	$H(1,2,\frac{1}{2},1)$ $\chi(1,1,0,3)$ $S_1^+(1,1,1,0)$ $S_2^+(1,1,1,-3)$

Multiplets	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$
Leptons	$L_{Li}(1,2,-\frac{1}{2},0)$ $\ell_{Ri}(1,1,-1,-1)$ $\nu_{Ri}(1,1,0,\{-5,4,4\})$
Scalars	$H(1,2,\frac{1}{2},1)$ $\chi(1,1,0,3)$ $S(1,1,0,-\frac{7}{2})$ $\eta(1,2,\frac{1}{2},\frac{1}{2})$
Vector-like fermion	$\mathcal{N}_{L,R}(1,1,0,\frac{1}{2})$

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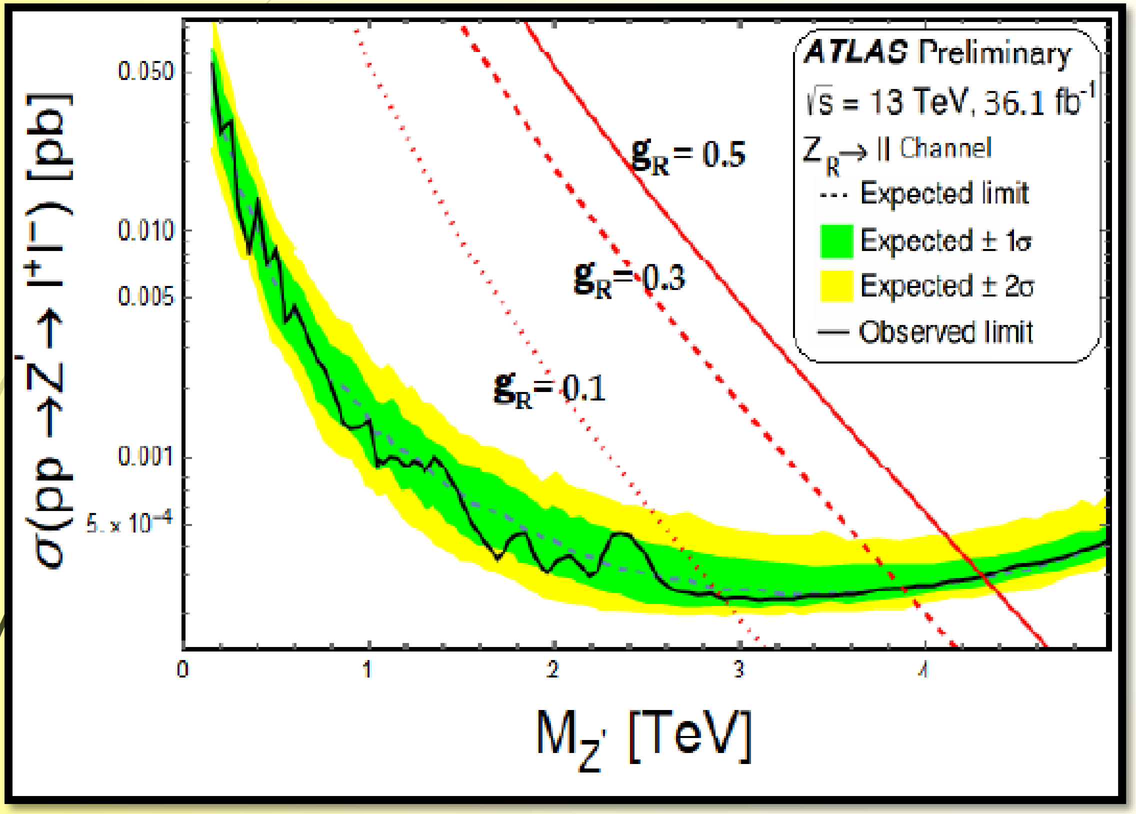
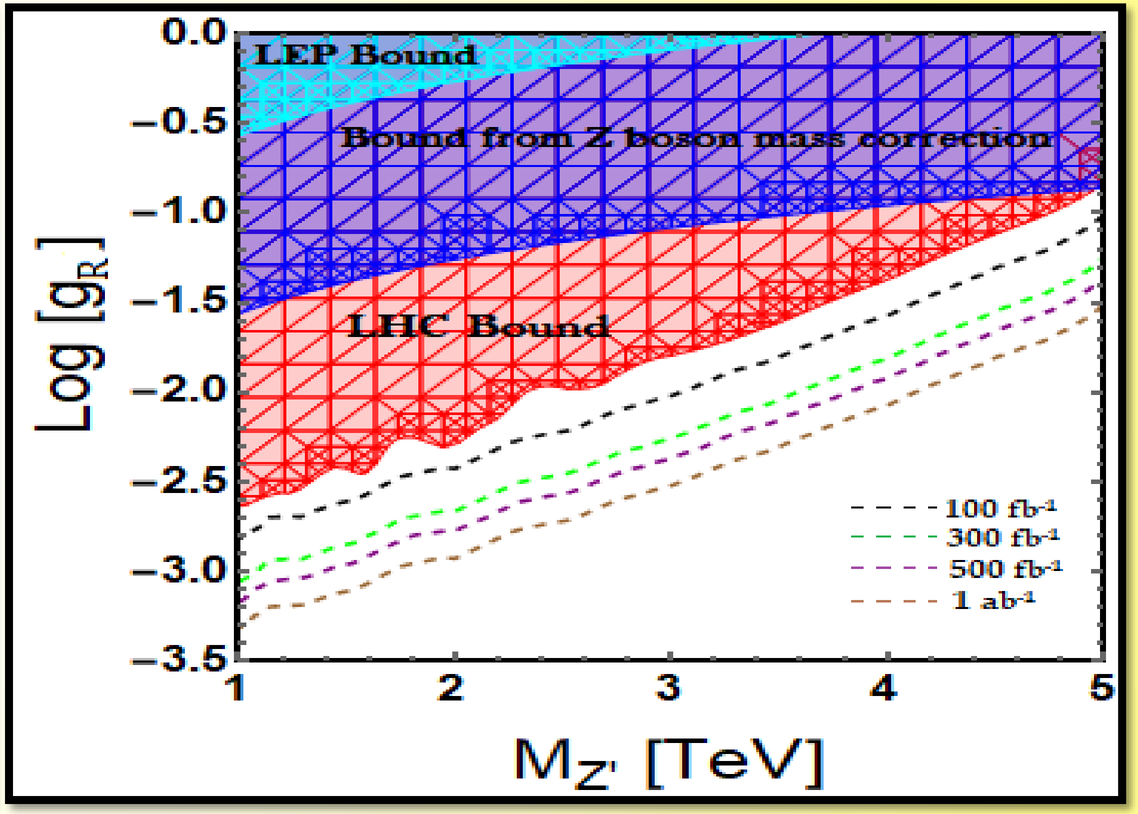
❖ Constraints from LEP and LHC

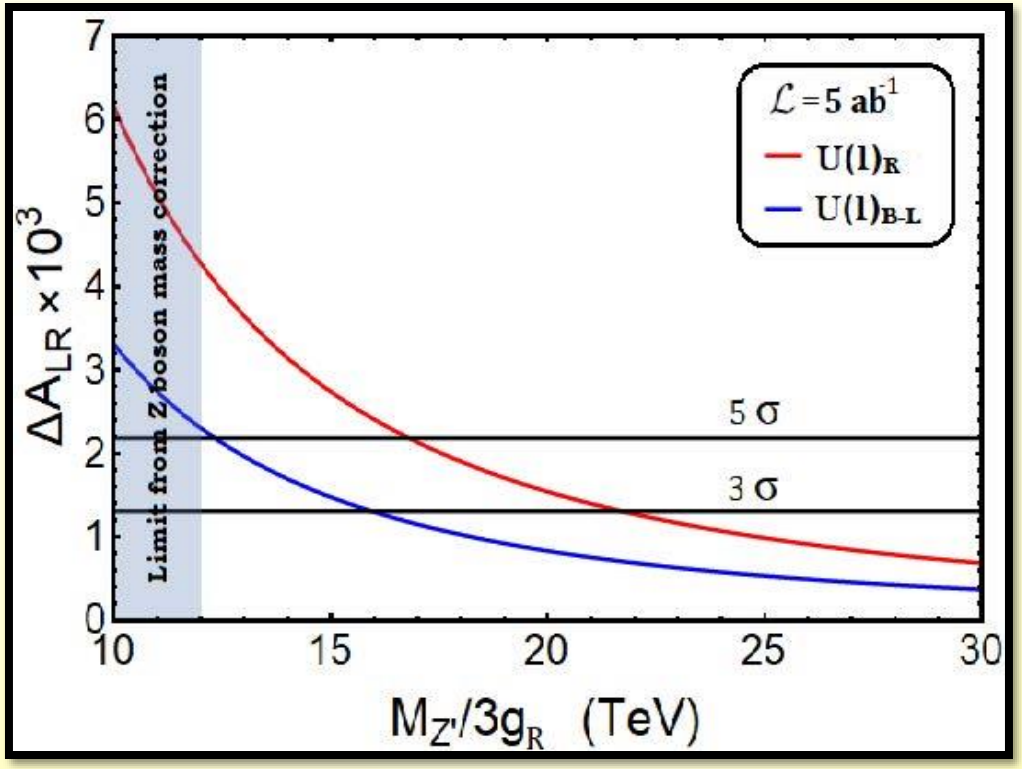




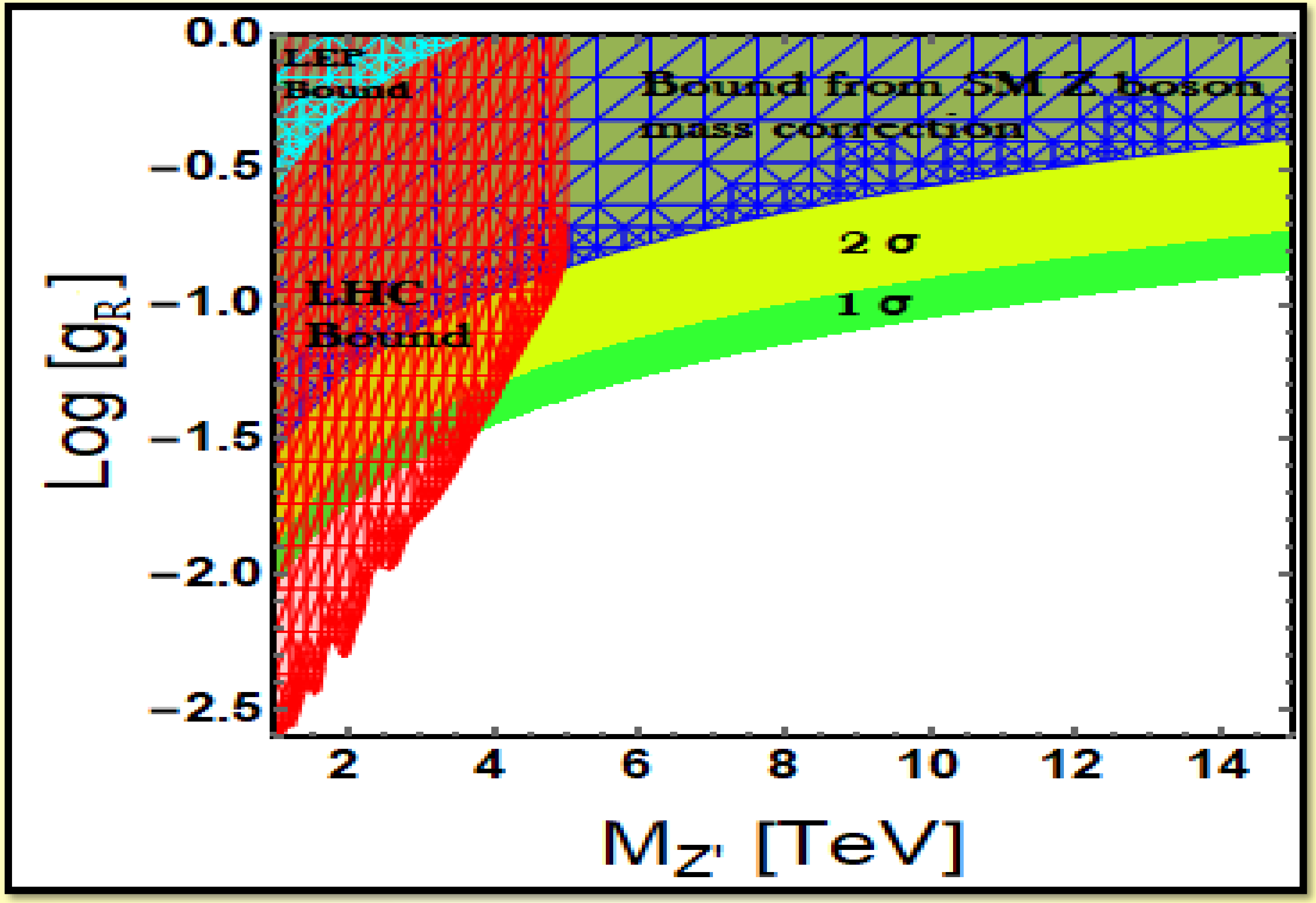
❖ Heavy Gauge Boson Z′ at ILC : Left-Right Asymmetry

$$\mathcal{L}_{eff} = \frac{1}{1+\delta_{ef}} \frac{g_R^2}{M_{Z'}^2} (\bar{e}\gamma^\mu \mathcal{P}_R e)(\bar{f}\gamma_\mu \mathcal{P}_R f)$$

- ❑ Analysis with the polarized initial states at ILC can be used to understand the chirality structure of the effective interaction



❖ Phenomenology of Z’

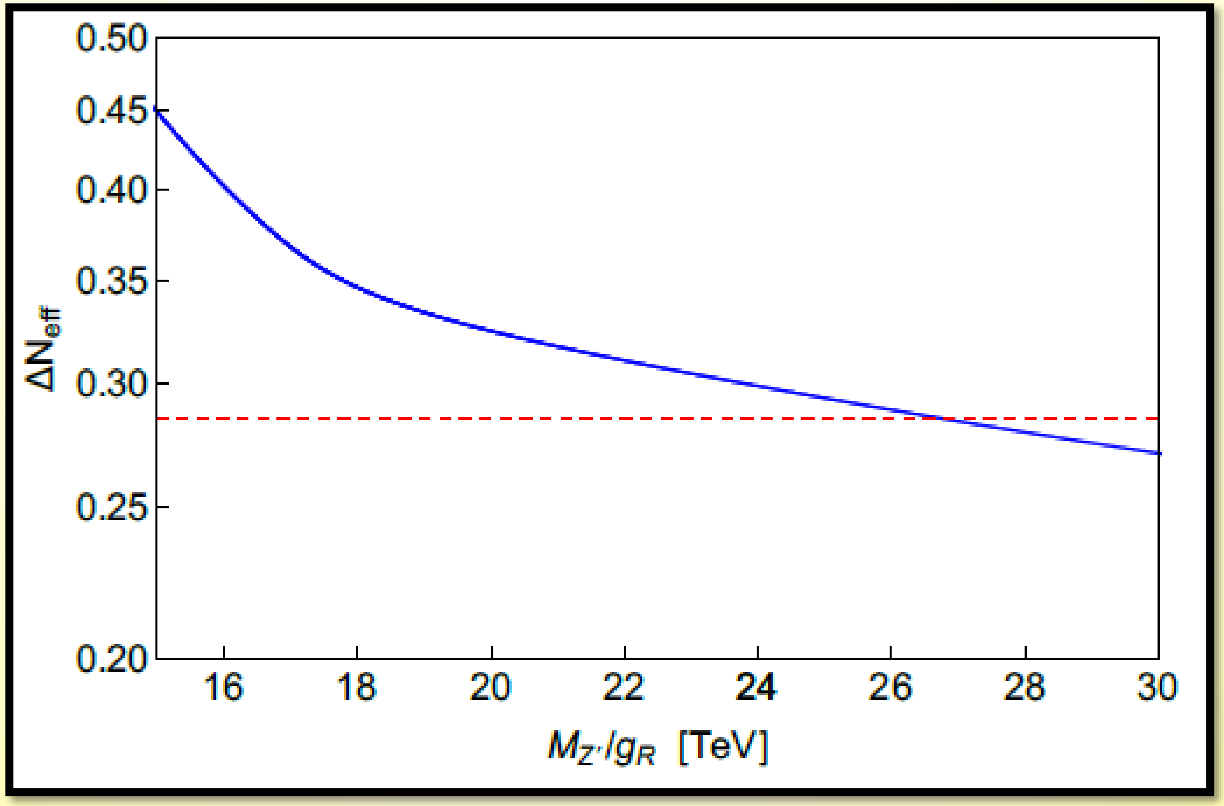


S.Jana,**VPK**,S.Saad (arXiv:1904.07407)

❖ Constraints from Cosmology

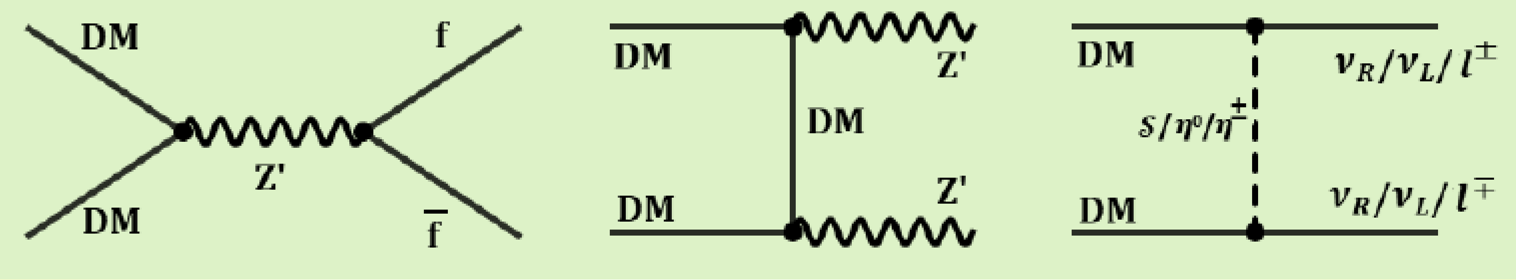
- The Right-handed neutrino $\boldsymbol{\nu_R}$ can increase the effective number of relativistic degree of freedom $\boldsymbol{N_{eff}}$.

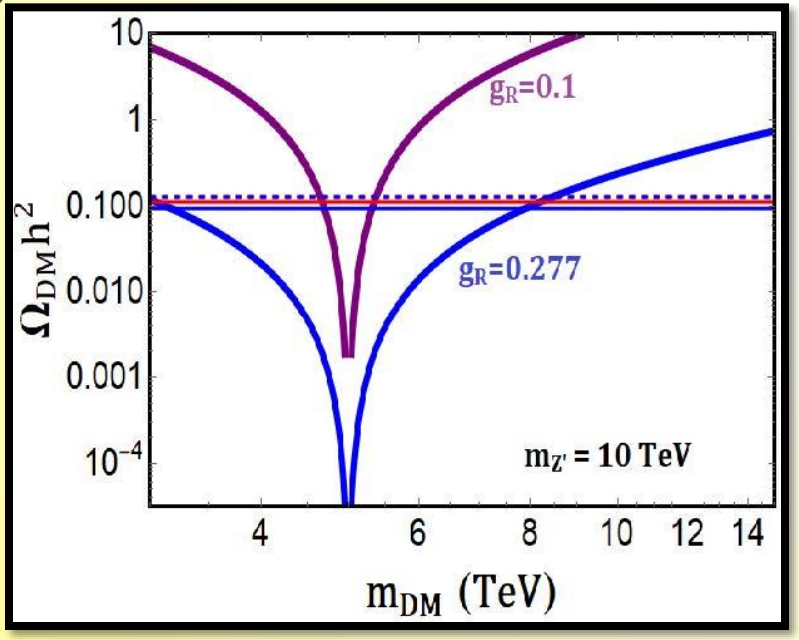
- To be compatible with the current cosmological constraints on $\boldsymbol{N_{eff}}$, the interaction of $\boldsymbol{\nu_R}$ with the primordial plasma must be highly suppressed.

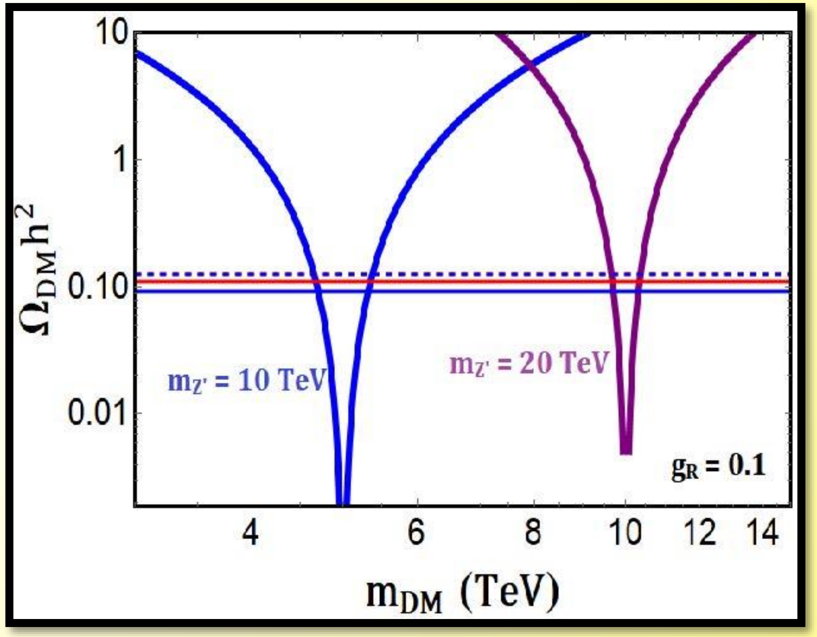


❖ Dark Matter Phenomenology

- Spontaneously broken $U(1)_R$ symmetry, leave a Z_2 residual symmetry that can potentially stabilize the DM particle (considering for the case of vector-like fermion).







❖ Summary

- We constructed minimal Dirac neutrino mass models arising from generic one-loop and two-loop topologies for both colored and non-colored versions.
- Out of the 40 models that we proposed 37 of them are new.
- Out of the 17 non-colored models that we proposed 11 of them can naturally incorporate DM candidate.
- Every single model that we are presented in this work require no representation higher than the fundamental representation under $\boldsymbol{SU(2)_L}$and $\boldsymbol{SU(3)_C}$.
- Our methodology can be implemented to construct new models by utilizing various different symmetries (discrete, global, gauge)
- Each of the models in this work can have very distinct phenomenology and must be studied case by case.

Thank you!

❖ One-loop Models

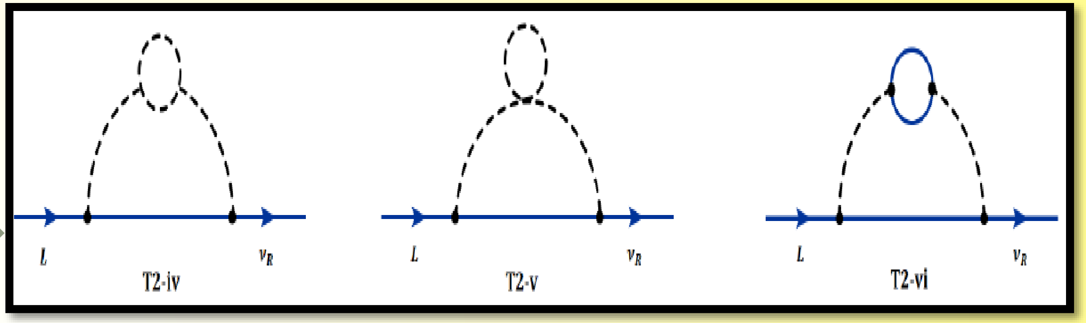
Diagram	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T1-i-1	**T1-i-1-A**	$\eta_1(1,1,1,2)$ $\eta_2(1,1,1,5)$	—	$y_1\overline{L^c}\epsilon\eta_1 L + y_e\overline{L}Hl_R$ $+y_2\overline{l^c_R}\eta_2\nu_R + \mu\eta_2\eta_1^*\sigma^*$	(Saad, Zapata)
	T1-i-1-B	$\eta(1,1,1,5)$ $\phi(1,2,\frac{3}{2},5)$	$\psi_{L,R}(1,1,-1,-4)$	$y_1\overline{L^c}\epsilon\phi\psi_L + y_2\overline{\psi_L}\sigma^* l_R$ $+y_3\overline{l^c_R}\eta\nu_R + \mu\phi^\dagger H\eta$	✔
	T1-i-1-C	$\eta(1,1,0,\frac{5}{2})$ $\phi(1,2,\frac{1}{2},\frac{5}{2})$	$\psi_{L,R}(1,1,0,\frac{3}{2})$	$y_1\overline{L}\epsilon\phi^*\psi_R + y_2\overline{\psi^c_R}\sigma^*\psi_R$ $+y_3\overline{\psi^c_R}\eta\nu_R + \mu\phi^\dagger H\eta$	✔
	T1-i-1-D	$\eta(1,1,0,\frac{5}{2})$ $\phi(1,2,\frac{1}{2},\frac{5}{2})$	$\psi_{L,R}(1,1,0,-\frac{3}{2})$	$y_1\overline{L^c}\epsilon\phi\psi_L + y_2\overline{\psi^c_L}\sigma\psi_L$ $+y_3\overline{\psi_L}\eta\nu_R + \mu\phi^\dagger H\eta$	✔
T1-i-2	**T1-i-2-A**	$\eta_1(1,1,Y,4+\alpha)$ $\eta_2(1,1,Y,1+\alpha)$ $\phi(1,2,Y+\frac{1}{2},1+\alpha)$	$\psi_{L,R}(1,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\phi^*\psi_R$ $+y_2\overline{\psi_L}\eta_1\nu_R + \mu_1\eta_1\eta_2^*\sigma^*$ $+\mu_2\phi^\dagger H\eta_2$	✔
	T1-i-2-B	$\eta(1,1,Y,4+\alpha)$ $\phi_1(1,2,Y+\frac{1}{2},1+\alpha)$ $\phi_2(1,2,Y+\frac{1}{2},4+\alpha)$	$\psi_{L,R}(1,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\phi_1^*\psi_R$ $+y_2\overline{\psi_L}\eta\nu_R + \mu_1 H\eta\phi_2^*$ $+\mu_2\phi_1^\dagger\phi_2\sigma^*$	✔
T1-i-3	**T1-i-3-A**	$\eta(1,1,1,2)$	$\psi_{L,R}(1,1,-1,2)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}\epsilon\eta L$ $+y_2\overline{\psi^c_R}\eta\nu_R + y_3\overline{\psi_L}\sigma l_R$ $+y_e\overline{L}Hl_R$	✔
	T1-i-3-B	$\phi(1,2,Y+\frac{1}{2},1+\alpha)$	$\psi_{1L,R}(1,1,Y,\alpha)$ $\psi_{2L,R}(1,1,Y,\alpha-3)$ $\psi_{3L,R}(1,2,Y+\frac{1}{2},\alpha-3)$	$M_{\psi_1}\overline{\psi_{1L}}\psi_{1R} + M_{\psi_2}\overline{\psi_{2L}}\psi_{2R}$ $+y_1\overline{L}\epsilon\phi^*\psi_{1R} + y_2\overline{\psi_{2L}}\sigma^*\psi_{1R}$ $+y_3\overline{\psi_{3L}}H\psi_{2R} + y_4\overline{\psi_{3L}}\phi\nu_R$	✔
T1-ii-1	**T1-ii-1-A**	$\eta(1,1,Y,4+\alpha)$ $\phi(1,2,Y+\frac{1}{2},1+\alpha)$	$\psi_{L,R}(1,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\phi^*\psi_R$ $+y_2\overline{\psi_L}\eta\nu_R + \lambda\phi^\dagger H\eta\sigma^*$	(Zapata,Srivastava)

❖ One-loop Models: Colored version

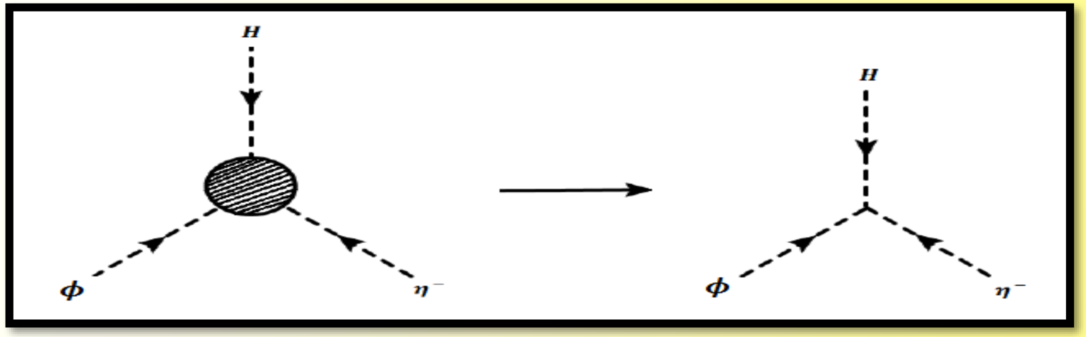
Diagram	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T1-i-1	T1-i-1-A(C)	$\chi_1(\bar{3},1,\frac{1}{3},\frac{2}{3})$ $\chi_2(\bar{3},1,\frac{1}{3},\frac{11}{3})$	—	$y_1\overline{L^c}\epsilon\chi_1 Q_L + y_d\overline{Q_L}Hd_R$ $+y_2\overline{d^c_R}\chi_2\nu_R + \mu\chi_2\chi_1^*\sigma^*$	✔
	T1-i-1-B(C)	$\Omega_1(3,2,\frac{1}{6},\frac{4}{3})$ $\Omega_2(3,2,\frac{1}{6},\frac{13}{3})$	—	$y_1\overline{d_R}\Omega_1\epsilon L + y_d\overline{Q_L}Hd_R$ $+y_2\overline{Q_L}\Omega_2\nu_R + \mu\Omega_1^\dagger\Omega_2\sigma^*$	✔
	T1-i-1-C(C)	$\chi(\bar{3},1,\frac{1}{3},\frac{11}{3})$ $\Omega(\bar{3},2,\frac{5}{6},\frac{11}{3})$	$\psi_{L,R}(3,1,-\frac{1}{3},-\frac{8}{3})$	$y_1\overline{L^c}\epsilon\Omega\psi_L + y_2\overline{\psi_L}\sigma^* d_R$ $+y_3\overline{d^c_R}\chi\nu_R + \mu\Omega^\dagger H\chi$	✔
	T1-i-1-D(C)	$\chi(\bar{3},1,-\frac{2}{3},\frac{11}{3})$ $\Omega(\bar{3},2,-\frac{1}{6},\frac{11}{3})$	$\psi_{L,R}(3,1,\frac{2}{3},-\frac{8}{3})$	$y_1\overline{L^c}\epsilon\Omega\psi_L + y_2\overline{\psi_L}\sigma^* u_R$ $+y_3\overline{u^c_R}\chi\nu_R + \mu\Omega^\dagger H\chi$	✔
T1-i-2	T1-i-2-A(C)	$\chi_1(3,1,Y,4+\alpha)$ $\chi_2(3,1,Y,1+\alpha)$ $\Omega(3,2,Y+\frac{1}{2},1+\alpha)$	$\psi_{L,R}(3,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\Omega^*\psi_R$ $+y_2\overline{\psi_L}\chi_1\nu_R + \mu_1\Omega^\dagger H\chi_2$ $+\mu_2\chi_1\chi_2^*\sigma^*$	✔
	T1-i-2-B(C)	$\chi(3,1,Y,4+\alpha)$ $\Omega_1(3,2,Y+\frac{1}{2},1+\alpha)$ $\Omega_2(3,2,Y+\frac{1}{2},4+\alpha)$	$\psi_{L,R}(3,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\Omega_1^*\psi_R$ $+y_2\overline{\psi_L}\chi\nu_R + \mu_1\Omega_2^\dagger H\chi$ $+\mu_2\Omega_1^\dagger\Omega_2\sigma^*$	✔
T1-i-3	T1-i-3-A(C)	$\chi(\bar{3},1,\frac{1}{3},\frac{2}{3})$	$\psi_{L,R}(3,1,-\frac{1}{3},\frac{10}{3})$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}\epsilon\chi Q_L$ $+y_d\overline{Q_L}Hd_R + y_2\overline{\psi^c_R}\chi\nu_R$ $+y_3\overline{\psi_L}\sigma d_R$	✔
	T1-i-3-B(C)	$\Omega(3,2,\frac{1}{6},\frac{13}{3})$	$\psi_{L,R}(3,1,-\frac{1}{3},\frac{10}{3})$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\Omega^*\psi_R$ $+y_d\overline{Q_L}Hd_R + y_2\overline{Q_L}\Omega\nu_R$ $+y_3\overline{\psi_L}\sigma d_R$	✔
T1-ii-1	T1-ii-1-A(C)	$\chi(3,1,Y,4+\alpha)$ $\Omega(3,2,Y+\frac{1}{2},1+\alpha)$	$\psi_{L,R}(3,1,Y,\alpha)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L}\epsilon\Omega^*\psi_R$ $+y_2\overline{\psi_L}\chi\nu_R + \lambda\Omega^\dagger H\sigma^*\chi$	✔

❖ Search for Minimal Two-loop Models

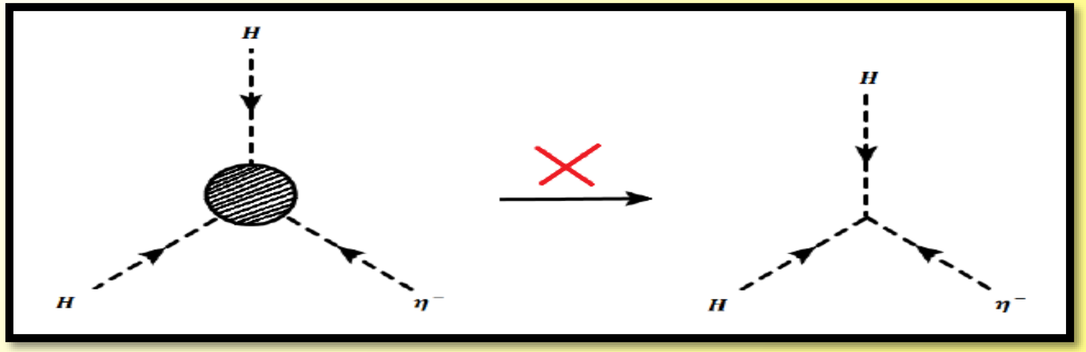
- ❑ Consider the case of $T2 - iv, v, vi$ skeleton diagrams



- ❑ In each case the blob can reduce to tree vertex for a general isodoublet → no two-loop diagrams



- ❑ Can be resolved if the isodoublet is Higgs doublet, because of the antisymmetric property of $L \supset H\varepsilon H\eta^-$



R.Cepedello, R.M. Fonseca,M.Hirsch **JHEP 1906 (2019) 034**

❖ Two-loop Models

Skeleton	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T2-i	**T2-i-2-A**	$\eta_1(1,1,1,-3)$ $\eta_2(1,1,1,5)$ $\eta_3(1,1,-2,-2)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_3^* l_R$ $+y_3\overline{l^c_R}\eta_2\nu_R + \mu\eta_1\eta_2\eta_3$ $+\lambda\phi\epsilon H\sigma^*\eta_1^*$	[52]
	T2-i-4-A	$\eta_1(1,1,1,0)$ $\eta_2(1,1,1,5)$ $\eta_3(1,1,-2,-2)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_3^* l_R$ $+y_3\overline{l^c_R}\eta_2\nu_R + \mu\phi\epsilon H\eta_1^*$ $+\lambda\eta_1\eta_2\eta_3\sigma^*$	✔
	T2-i-11-A	$\eta_1(1,1,1,5)$ $\eta_2(1,1,-2,-2)$ $\eta_3(1,1,-2,-5)$ $\phi(1,2,\frac{1}{2},0)$	—	$y_1\overline{L}\phi l_R + y_2\overline{l^c_R}\eta_2^* l_R$ $+y_3\overline{l^c_R}\eta_1\nu_R + \mu\eta_2\eta_3^*\sigma^*$ $+\lambda\phi\epsilon H\eta_1\eta_3$	✔
T2-ii	**T2-ii-1-A**	$\eta_1(1,1,1,2)$ $\eta_2(1,1,-(Y+1),2-\alpha)$ $\eta_3(1,1,-(Y+1),-(1+\alpha))$	$\psi_{1L,R}(1,1,Y+1,2+\alpha)$ $\psi_{2L,R}(1,1,Y,\alpha)$	$M_{\psi_1}\overline{\psi_{1L}}\psi_{1R} + M_{\psi_2}\overline{\psi_{2L}}\psi_{2R}$ $+y_1\overline{L^c}\epsilon\eta_1 L + y_2\overline{\psi_{2L}}\eta_3^* l_R$ $+y_3\overline{\psi^c_{1R}}\eta_2\nu_R + y_4\overline{\psi_{1L}}\eta_1\psi_{2R}$ $+y_e\overline{L}Hl_R + \mu\eta_2\eta_3^*\sigma^*$	✔
T2-iii	**T2-iii-1-A**	$\eta_1(1,1,-1,-2)$ $\eta_2(1,1,-(Y+1),2-\alpha)$ $\eta_3(1,1,-(Y+1),-(1+\alpha))$	$\psi_{1L,R}(1,1,-1,2)$ $\psi_{2L,R}(1,1,Y,\alpha)$	$y_1\overline{L^c}\eta_1^* L + y_2\overline{\psi_{2L}}\eta_3^* l_R$ $+y_3\overline{\psi_{2L}}\eta_2^*\psi_{1R} + y_4\overline{\psi^c_{1R}}\eta_1^*\nu_R$ $+y_e\overline{L}Hl_R + \mu\eta_2\eta_3^*\sigma^*$	✔
T2-iv	**T2-iv-1-A**	$\eta_1(1,1,-1,3)$ $\eta_2(1,1,Y-\frac{1}{2},\alpha+3)$ $\phi(1,2,Y,\alpha)$	$\psi_{L,R}(1,1,1,1)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}H^*\psi_L$ $+y_2\overline{\psi^c_R}\eta_1\nu_R + \lambda_1\phi^\dagger H\sigma^*\eta_2$ $+\lambda_2\phi\epsilon H\eta_1\eta_2^*$	✔
T2-v	**T2-v-2-A**	$\eta_1(1,1,-1,3)$ $\eta_2(1,1,Y+\frac{1}{2},\alpha-3)$ $\phi(1,2,Y,\alpha)$	$\psi_{L,R}(1,1,1,1)$	$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}H^*\psi_L$ $+y_2\overline{\psi^c_R}\eta_1\nu_R + \lambda_1\phi^\dagger H\eta_1\eta_2$ $+\lambda_2\phi\epsilon H\sigma^*\eta_2^*$	✔
T2-vi	**T2-vi-3-A**	$\eta(1,1,-1,3)$	$\psi_{1L,R}(1,1,1,1)$ $\psi_{2L,R}(1,2,\frac{1}{2},-\frac{3}{2})$ $\psi_{3L,R}(1,1,-1,\frac{3}{2})$ $\psi_{4L,R}(1,1,0,\frac{3}{2})$	$M_{\psi_1}\overline{\psi_{1L}}\psi_{1R} + y_1\overline{L^c}H^*\psi_{1L}$ $+y_2\overline{\psi^c_{1R}}\eta\nu_R$ $+y_3\overline{\psi^c_{4L/R}}\psi_{4L/R}\sigma^*$ $+y_4\overline{\psi^c_{2L/R}}\psi_{4L/R}H^*$ $+y_5\overline{\psi^c_{2L/R}}\psi_{3L/R}\epsilon H$ $+y_6\overline{\psi^c_{3L/R}}\psi_{4L/R}\eta^*$	✔

❖ Two-loop Models: Colored version

<table>
<tr><th rowspan="2">Skeleton</th><th rowspan="2">Models</th><th colspan="2">New Fields</th><th rowspan="2">Relevant terms in Lagrangian</th><th rowspan="2">New ?</th></tr>
<tr><th>Scalars</th><th>Fermions</th></tr>
<tr><td rowspan="9">T2-i</td><td>T2-i-2-A(C)</td><td>$\chi_1(3,1,\frac{1}{3},-\frac{13}{3})$
$\chi_2(3,1,\frac{1}{3},\frac{11}{3})$
$\chi_3(3,1,-\frac{2}{3},\frac{2}{3})$
$\Omega(3,2,-\frac{1}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega d_R + y_2\overline{d^c_R}\chi_3^* d_R$
$+y_3\overline{d^c_R}\chi_2\nu_R + \mu\chi_1\chi_2\chi_3$
$+\lambda\Omega\epsilon H\sigma^*\chi_1^*$</td><td>✓</td></tr>
<tr><td>T2-i-2-B(C)</td><td>$\chi_1(3,1,-\frac{2}{3},-\frac{13}{3})$
$\chi_2(3,1,-\frac{2}{3},\frac{11}{3})$
$\chi_3(3,1,\frac{4}{3},\frac{2}{3})$
$\Omega(3,2,-\frac{7}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega u_R + y_2\overline{u^c_R}\chi_3^* u_R$
$+y_3\overline{u^c_R}\chi_2\nu_R + \mu\chi_1\chi_2\chi_3$
$+\lambda\Omega\epsilon H\sigma^*\chi_1^*$</td><td>✓</td></tr>
<tr><td>T2-i-4-A(C)</td><td>$\chi_1(3,1,\frac{1}{3},-\frac{4}{3})$
$\chi_2(3,1,\frac{1}{3},\frac{11}{3})$
$\chi_3(3,1,-\frac{2}{3},\frac{2}{3})$
$\Omega(3,2,-\frac{1}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega d_R + y_2\overline{d^c_R}\chi_3^* d_R$
$+y_3\overline{d^c_R}\chi_2\nu_R + \mu\Omega\epsilon H\chi_1^*$
$+\lambda\chi_1\chi_2\chi_3\sigma^*$</td><td>✓</td></tr>
<tr><td>T2-i-4-B(C)</td><td>$\chi_1(3,1,-\frac{2}{3},-\frac{4}{3})$
$\chi_2(3,1,-\frac{2}{3},\frac{11}{3})$
$\chi_3(3,1,\frac{4}{3},\frac{2}{3})$
$\Omega(3,2,-\frac{7}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega u_R + y_2\overline{u^c_R}\chi_3^* u_R$
$+y_3\overline{u^c_R}\chi_2\nu_R + \mu\Omega\epsilon H\chi_1^*$
$+\lambda\chi_1\chi_2\chi_3\sigma^*$</td><td>✓</td></tr>
<tr><td>T2-i-11-A(C)</td><td>$\chi_1(3,1,\frac{1}{3},\frac{11}{3})$
$\chi_2(3,1,-\frac{2}{3},\frac{2}{3})$
$\chi_3(3,1,-\frac{2}{3},-\frac{7}{3})$
$\Omega(3,2,-\frac{1}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega d_R + y_2\overline{d^c_R}\chi_2^* d_R$
$+y_3\overline{d^c_R}\chi_1\nu_R + \mu\chi_2\chi_3^*\sigma^*$
$+\lambda\Omega\epsilon H\chi_1\chi_3$</td><td>✓</td></tr>
<tr><td>T2-i-11-B(C)</td><td>$\chi_1(3,1,-\frac{2}{3},\frac{11}{3})$
$\chi_2(3,1,\frac{4}{3},\frac{2}{3})$
$\chi_3(3,1,\frac{4}{3},-\frac{7}{3})$
$\Omega(3,2,-\frac{7}{6},-\frac{4}{3})$</td><td>—</td><td>$y_1\overline{L}\Omega u_R + y_2\overline{u^c_R}\chi_2^* u_R$
$+y_3\overline{u^c_R}\chi_1\nu_R + \mu\chi_2\chi_3^*\sigma^*$
$+\lambda\Omega\epsilon H\chi_1\chi_3$</td><td>✓</td></tr>
<tr><td>T2-i-11-C(C)</td><td>$\chi_1(3,1,-\frac{1}{3},-\frac{2}{3})$
$\chi_2(3,1,-\frac{1}{3},-\frac{2}{3})$
$\chi_3(3,1,-\frac{1}{3},-\frac{11}{3})$
$\Omega(3,2,\frac{1}{6},\frac{13}{3})$</td><td>—</td><td>$y_1\overline{L^c}\epsilon\chi_1^\dagger Q_L + y_2\overline{Q^c_L}\epsilon Q_L\chi_2$
$+y_3\overline{Q_L}\Omega\nu_R + \mu\chi_2\chi_3^*\sigma^*$
$+\lambda\Omega\epsilon H\chi_1\chi_3$</td><td>✓</td></tr>
<tr><td>T2-i-15-A(C)</td><td>$\chi_1(3,1,\frac{2}{3},\frac{13}{3})$
$\chi_2(3,1,-\frac{1}{3},-\frac{2}{3})$
$\chi_3(3,1,-\frac{1}{3},-\frac{2}{3})$
$\Omega(3,2,\frac{1}{6},\frac{13}{3})$</td><td>—</td><td>$y_1\overline{L^c}\epsilon\chi_2^* Q_L + y_2\overline{Q^c_L}\epsilon Q_L\chi_3$
$+y_3\overline{Q_L}\Omega\nu_R + \mu\chi_1^\dagger\Omega\epsilon H$
$+\lambda\chi_1\chi_2\chi_3\sigma^*$</td><td>✓</td></tr>
<tr><td>T2-i-17-A(C)</td><td>$\chi_1(3,1,\frac{2}{3},\frac{4}{3})$
$\chi_2(3,1,-\frac{1}{3},-\frac{2}{3})$
$\chi_3(3,1,-\frac{1}{3},-\frac{2}{3})$
$\Omega(3,2,\frac{1}{6},\frac{13}{3})$</td><td>—</td><td>$y_1\overline{L^c}\epsilon\chi_2^* Q_L + y_2\overline{Q^c_L}\epsilon Q_L\chi_3$
$+y_3\overline{Q_L}\Omega\nu_R + \mu\chi_1\chi_2\chi_3$
$+\lambda\sigma^*\chi_1^\dagger\Omega\epsilon H$</td><td>✓</td></tr>
<tr><td>T2-ii</td><td>T2-ii-1-A(C)</td><td>$\chi_1(3,1,\frac{1}{3},\frac{2}{3})$
$\chi_2(3,1,-Y-\frac{1}{3},\frac{10}{3}-\alpha)$
$\chi_3(3,1,-Y-\frac{1}{3},\frac{1}{3}-\alpha)$</td><td>$\psi_{1L,R}(3,1,Y+\frac{1}{3},\alpha+\frac{2}{3})$
$\psi_{2L,R}(3,1,Y,\alpha)$</td><td>$M_{\psi_1}\overline{\psi_{1L}}\psi_{1R} + M_{\psi_2}\overline{\psi_{2L}}\psi_{2R}$
$+y_1\overline{L^c}\epsilon\chi_1 Q_L + y_2\overline{\psi_{2L}}\chi_3^* d_R$
$+y_3\overline{\psi^c_{1R}}\chi_2\nu_R + y_4\overline{\psi_{1L}}\chi_1\psi_{2R}$
$+y_d\overline{Q_L}Hd_R + \mu\chi_2\chi_3^*\sigma^*$</td><td>✓</td></tr>
<tr><td>T2-iii</td><td>T2-iii-1-A(C)</td><td>$\chi_1(3,1,-\frac{1}{3},-\frac{2}{3})$
$\chi_2(\bar{3},1,-(Y+\frac{1}{3}),\frac{10}{3}-\alpha)$
$\chi_3(\bar{3},1,-(Y+\frac{1}{3}),\frac{1}{3}-\alpha)$</td><td>$\psi_{1L,R}(3,1,-\frac{1}{3},\frac{10}{3})$
$\psi_{2L,R}(3,1,Y,\alpha)$</td><td>$y_1\overline{Q^c_L}\chi_1^\dagger L + y_2\overline{\psi_{2L}}\chi_3^* d_R$
$+y_3\overline{\psi_{2L}}\chi_2^*\psi_{1R} + y_4\overline{\psi^c_{1R}}\chi_1^*\nu_R$
$+y_d\overline{Q_L}Hd_R + \mu\chi_2\chi_3^*\sigma^*$</td><td>✓</td></tr>
<tr><td>T2-iv</td><td>T2-iv-1-A(C)</td><td>$\eta(1,1,-1,3)$
$\chi(3,1,Y-\frac{1}{2},\alpha+3)$
$\Omega(3,2,Y,\alpha)$</td><td>$\psi_{L,R}(1,1,1,1)$</td><td>$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}H^*\psi_L$
$+y_2\overline{\psi^c_R}\eta\nu_R + \lambda_1\Omega^\dagger H\sigma^*\chi$
$+\lambda_2\Omega\epsilon H\eta\chi^*$</td><td>✓</td></tr>
<tr><td>T2-v</td><td>T2-v-2-A(C)</td><td>$\eta_1(1,1,-1,3)$
$\chi(3,1,Y+\frac{1}{2},\alpha-3)$
$\Omega(3,2,Y,\alpha)$</td><td>$\psi_{L,R}(1,1,1,1)$</td><td>$M_\psi\overline{\psi_L}\psi_R + y_1\overline{L^c}H^*\psi_L$
$+y_2\overline{\psi^c_R}\eta\nu_R + \lambda_1\Omega^\dagger H\eta\chi$
$+\lambda_2\Omega\epsilon H\sigma^*\chi^*$</td><td>✓</td></tr>
<tr><td>T2-vi</td><td>T2-vi-4-A(C)</td><td>$\eta_1(1,1,-1,3)$
$\eta_2(1,1,-1,0)$</td><td>$\psi_{1L,R}(1,1,1,1)$
$\psi_{2L,R}(3,2,Y,\alpha)$
$\psi_{3L,R}(\bar{3},1,-Y-\frac{1}{2},-\alpha)$
$\psi_{4L,R}(\bar{3},1,-Y+\frac{1}{2},-\alpha)$</td><td>$M_{\psi_1}\overline{\psi_{1L}}\psi_{1R} + M_{\psi_4}\overline{\psi_{4L}}\psi_{4R}$
$+y_1\overline{L^c}H^*\psi_{1L} + y_2\overline{\psi^c_{1R}}\eta_1\nu_R$
$+y_3\overline{\psi^c_{2L/R}}\psi_{4L/R}H^*$
$+y_4\overline{\psi^c_{2L/R}}\psi_{3L/R}\epsilon H$
$+y_5\overline{\psi_{4R/L}}\psi_{3L/R}\eta_2^* + \mu\eta_1\eta_2^*\sigma^*$</td><td>✓</td></tr>
</table>