



Minimal Realizations of Dirac Neutrino Mass from Generic One-loop and Two-loop Topologies

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Based on: [arxiv: 1904.07407](#) (accepted for publication in EPJC), [arxiv: 1910.xxxx](#)

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❖ Outline

Motivation

Neutrino Mass Models

Minimal Dirac Neutrino Mass Models: One-loop Topology

Minimal Dirac Neutrino Mass Models: Two-loop Topology

Phenomenology of Specific Models

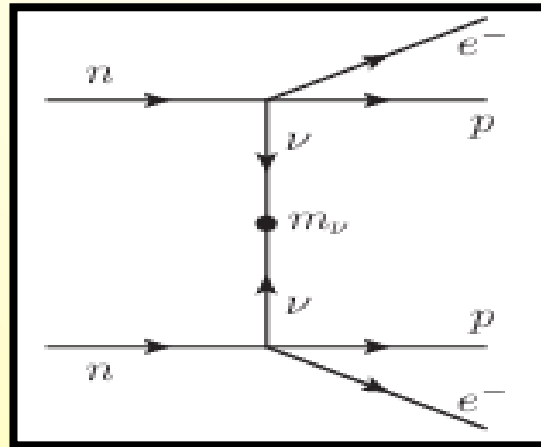
Summary

❖ Neutrino Mass and Nature of Neutrinos

- Neutrino Oscillation data suggests that neutrinos have tiny masses
→ Physics Beyond SM.

Origin of neutrino mass is still unknown

- Neutrinos can be either Dirac type or Majorana type in nature.
Can be resolved by neutrinoless double beta decay.



No Conclusive Evidence

❖ Neutrino Mass Models

- Most of the popular models assumes that the neutrinos are Majorana type in nature.
 - Seesaw : Type I, Type II, Type III, etc...
 - Radiative Mechanism : 1-loop (Zee), 2-loop (Zee-Babu)
- Building Dirac neutrino mass models require RH neutrinos, ν_R singlets under the SM

$$\mathcal{L}_4 = -y_{ij}^\nu \bar{L}_i \tilde{H} \nu_{Rj} + h.c.,$$



$$y_\nu \leq 10^{-11}$$

- Using additional symmetries one can forbid the tree-level mass term as well as Majorana neutrino mass terms at all order
 - Discrete Symmetries: D. Borah et al.
 - In Left-Right Symmetry Model: Babu-He, Branco-Senjanovic
 - SM with $U(1)$ symmetry (global/local): $U(1)_{B-L}$ (Ma), $U(1)_R$ (S.Jana, VPK, S.Saad (arXiv:1904.07407))

❖ Dirac Neutrino Mass: $\mathbf{SM} \times U(1)_{B-L}$

- $\mathbf{SM} \times U(1)_{B-L}$ is anomaly free with three RH neutrinos.
- Anomaly free B-L charge assignment of RH neutrinos:

$$\nu_{Ri} = \{-1, -1, -1\} \text{ or } \{5, -4, -4\}$$

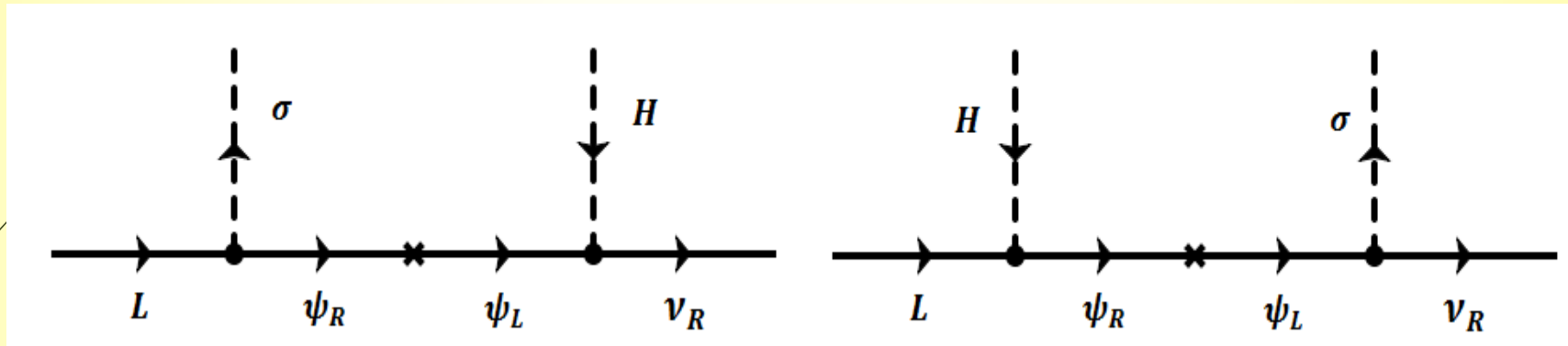
- The second possibility will naturally forbid *tree-level neutrino mass term* as well as *Majorana mass term* at all order.
- Neutrino mass generated by $\mathbf{d} = 5$ effective operator of the form

$$\mathcal{L}_5 = -\frac{h_{ij}}{\Lambda} \bar{L}_i \tilde{H} \nu_{Rj} \sigma + h.c.,$$

σ : SM singlet scalar charged +3 under $U(1)_{B-L}$

❖ Dirac Neutrino Mass: $SM \times U(1)_{B-L}$

- Dirac Neutrino Mass via Seesaw mechanism



With $\psi_{L/R} = (1, 1, 0, -1)$ or $(1, 2, -1/2, -4)$

- Interested in NP at TeV Scale

Radiative Models

❖ Dirac Neutrino Mass: $SM \times U(1)_{B-L}$

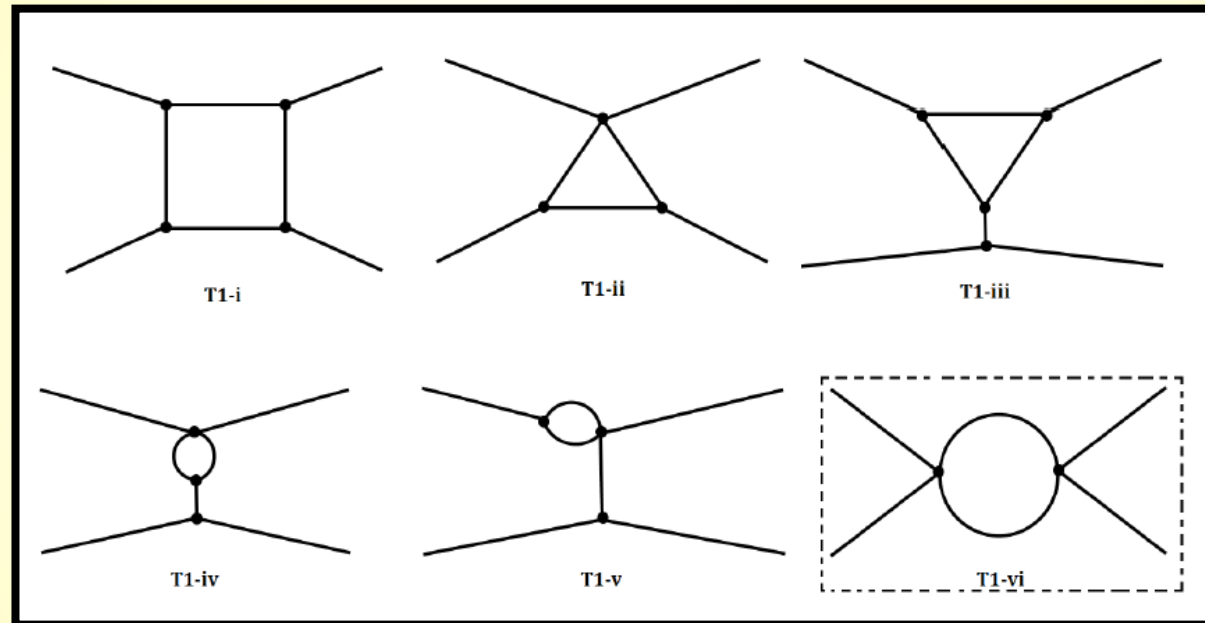
- Systematically search for the minimal Dirac neutrino mass models arise from this $d = 5$ effective operator.
- Strategy: Construct the generic one-loop and two-loop topologies and then build the associated minimal models .
- Minimality refers to

- ✓ Models with minimum number of BSM states are preferable.
- ✓ $SU(2)_L$ singlet BSM states are preferred. If BSM particles are required not to be iso-singlet, then we minimize the number of states that are charged under $SU(2)_L$.
- ✓ If possible, introduction of any BSM fermion is prohibited. If the presence of BSM fermionic state is required, we assume it to be vector like under $SM \times U(1)_{B-L}$.
- ✓ BSM states with lowest dimensional representation under $SU(2)_L$ are preferred.

❖ One-loop topologies: Viable topologies

□ All possible one-loop topologies with four external legs

$$\mathcal{L}_5 = -\frac{h_{ij}}{\Lambda} \bar{L}_i \tilde{H} \nu_{Rj} \sigma + h.c.,$$

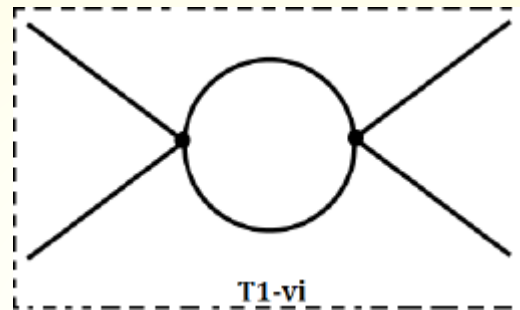


F.Bonnet, M.Hirsch, T. Ota, W. Winter **JHEP 1207 (2012)153**

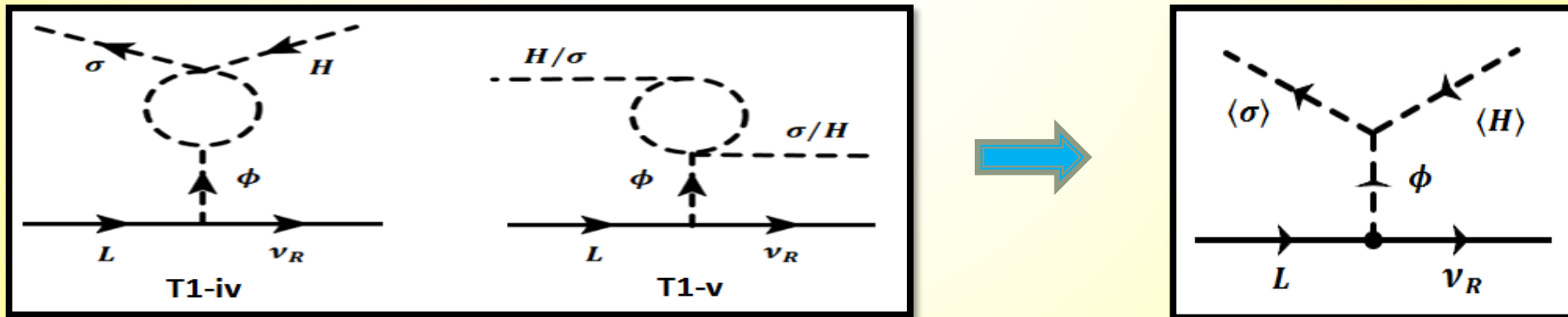
□ Not all these topologies can be lead to successful one-loop neutrino mass in our framework

❖ One-loop topologies: Viable topologies

- **T1 – vi** corresponds to non-renormalizable topologies



- **T1 – iv / v** can't give viable one loop models in our framework.



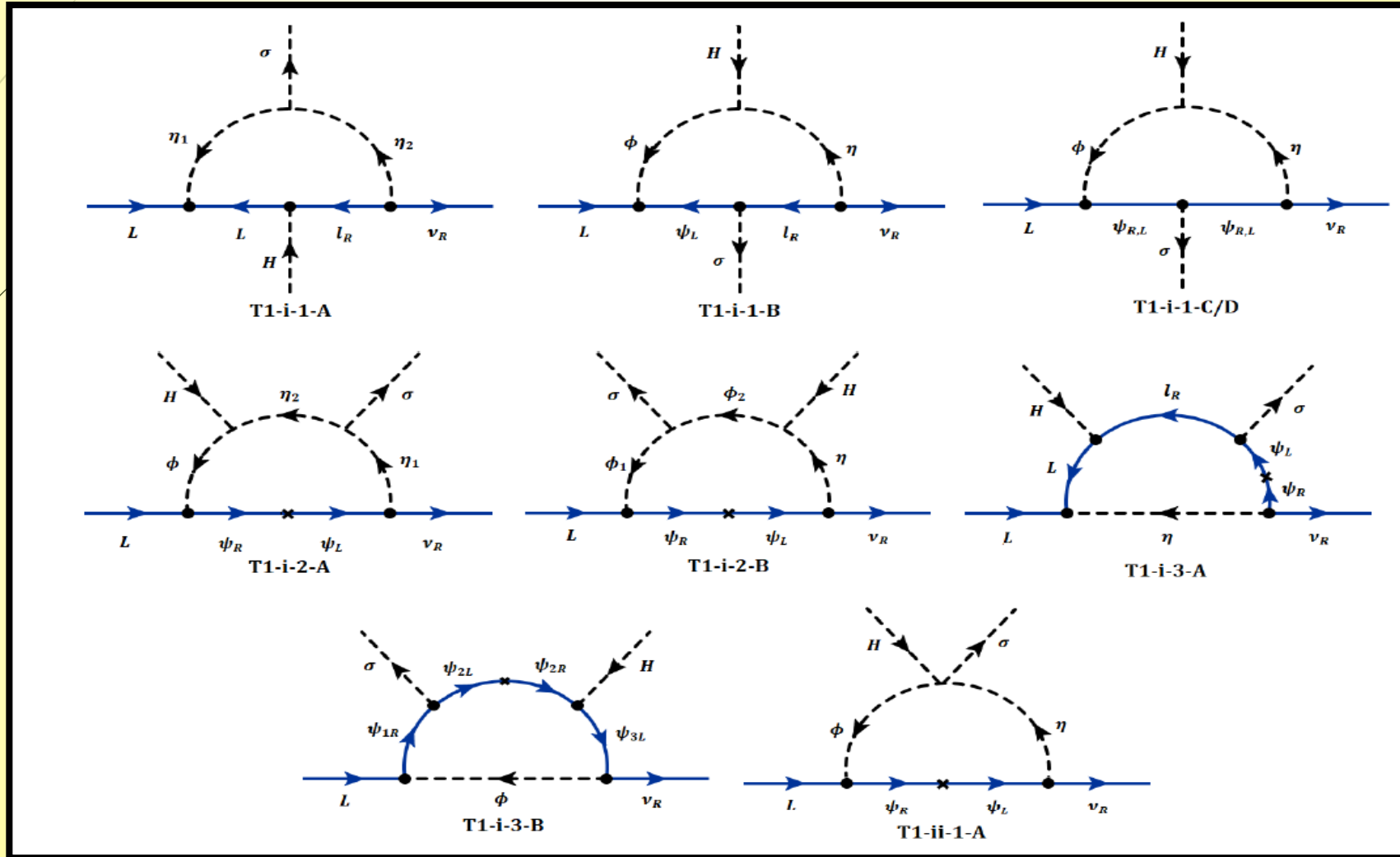
- **T1 – iii** can't give viable one loop models in our framework.

❖ Minimal One-loop models

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- Constructing minimal models arising from $T1 - i, ii$

S.Jana, VPK, S.Saad (arXiv:1910.xxxx)

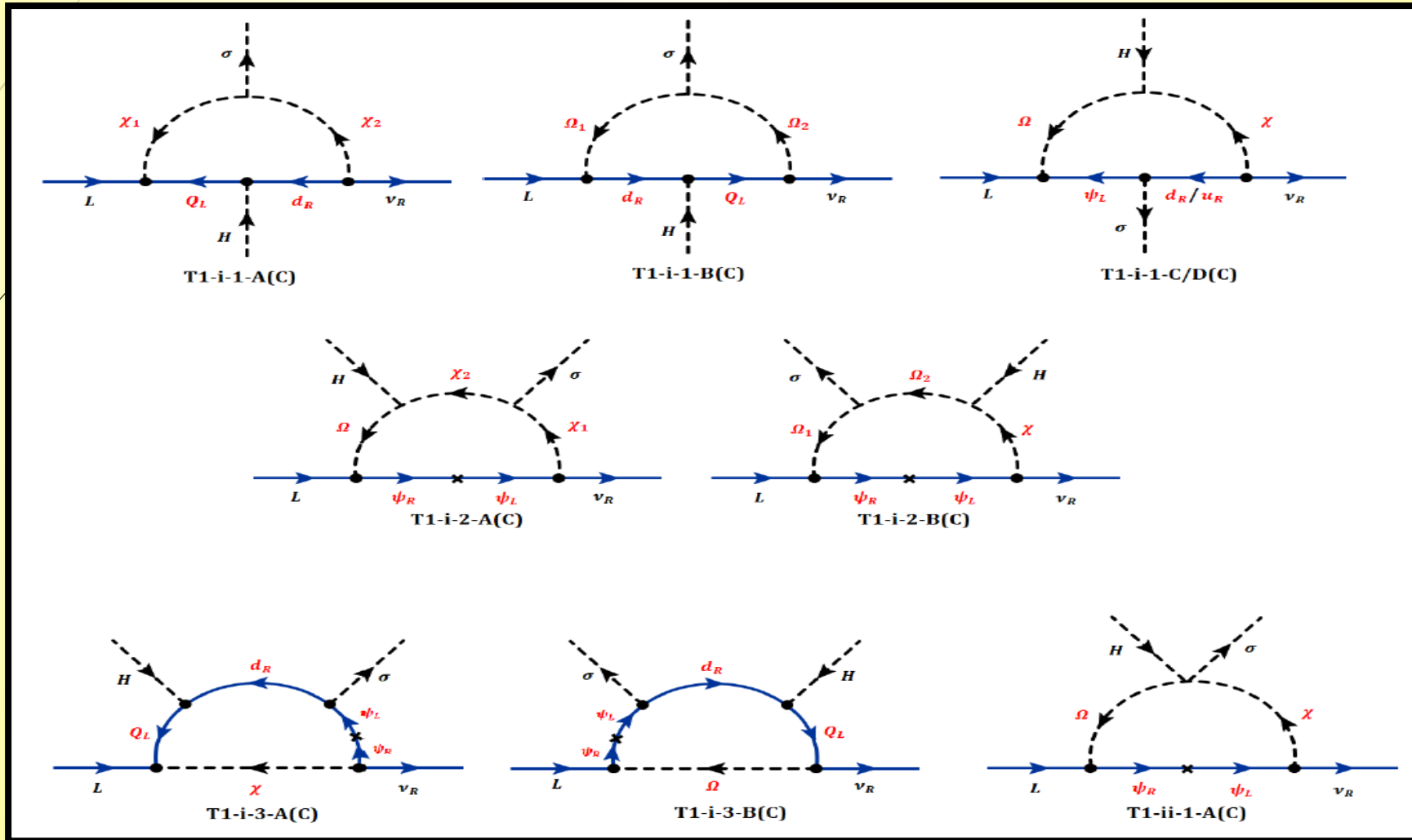


❖ Minimal One-loop models – Colored version

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- Corresponding minimal models for the colored version

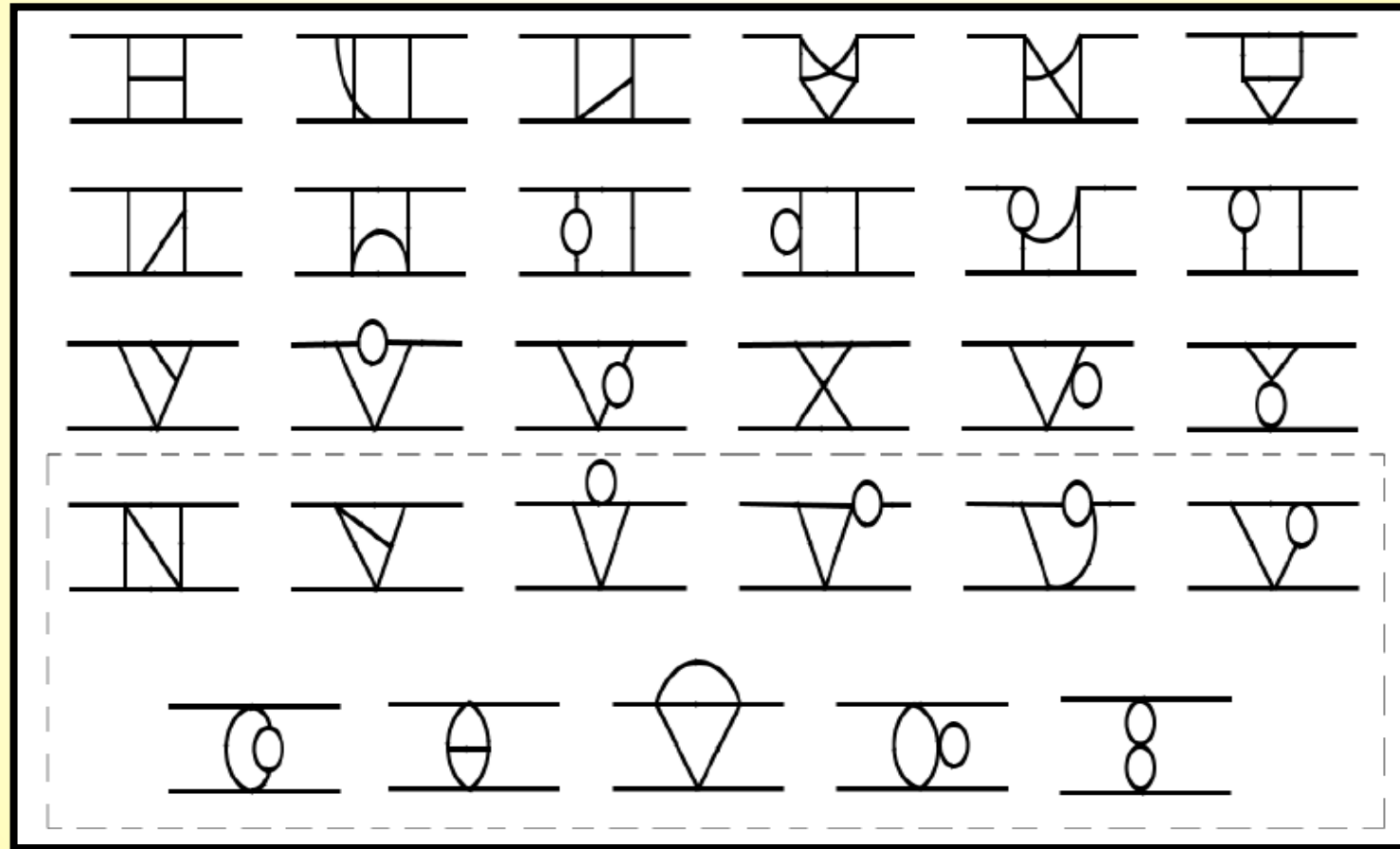
S.Jana, VPK, S.Saad (arXiv:1910.xxxx)



❖ Two-loop topologies

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- All possible two-loop topologies with four external legs



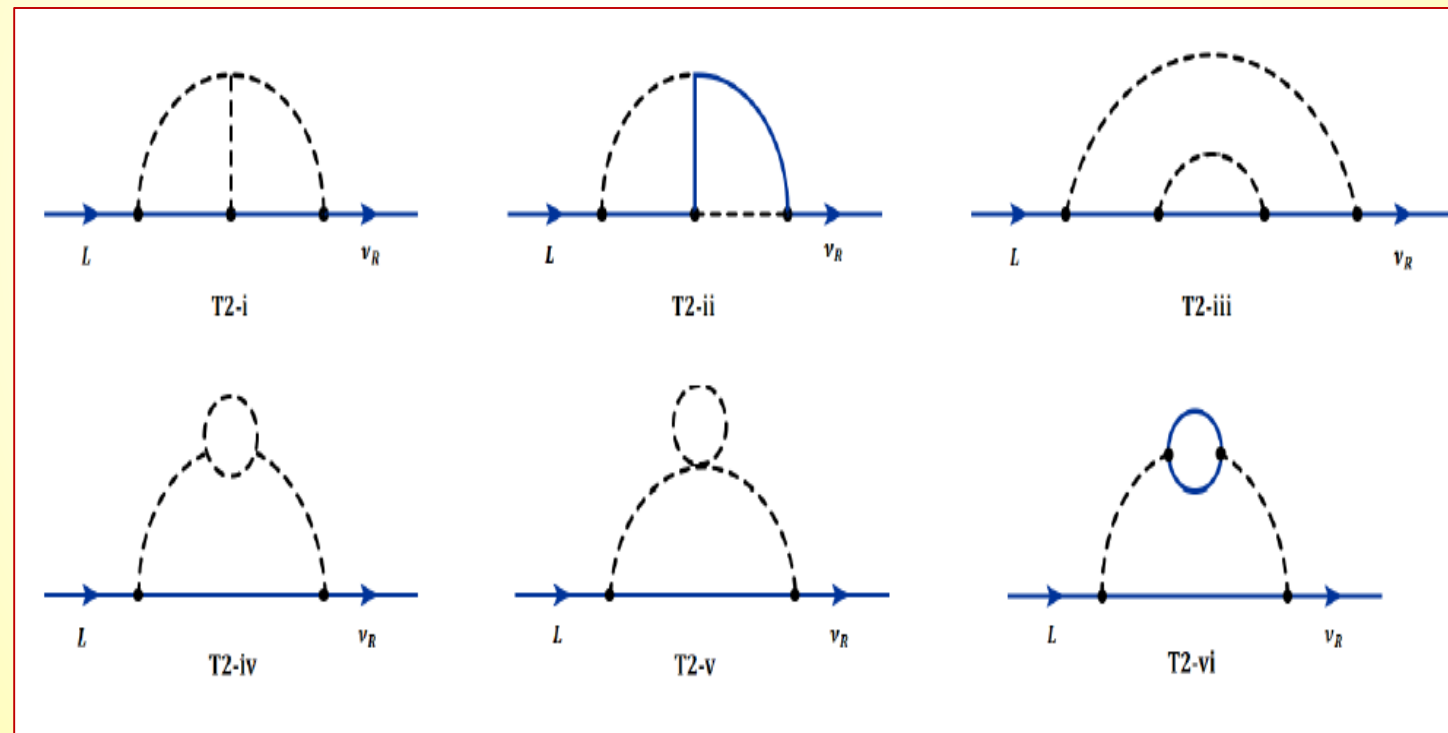
- Not all the topologies can be lead to successful two-loop neutrino mass in our framework. For example, last 11 topologies corresponds to non-renormalizable topologies

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❖ Two-loop Skeleton Diagrams

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- ❑ Remaining 18 topologies can give vast number of diagrams.
- ❑ By suppressing the external scalar legs, all these diagrams can be reduced into six basic diagrams → **Skeleton diagrams**



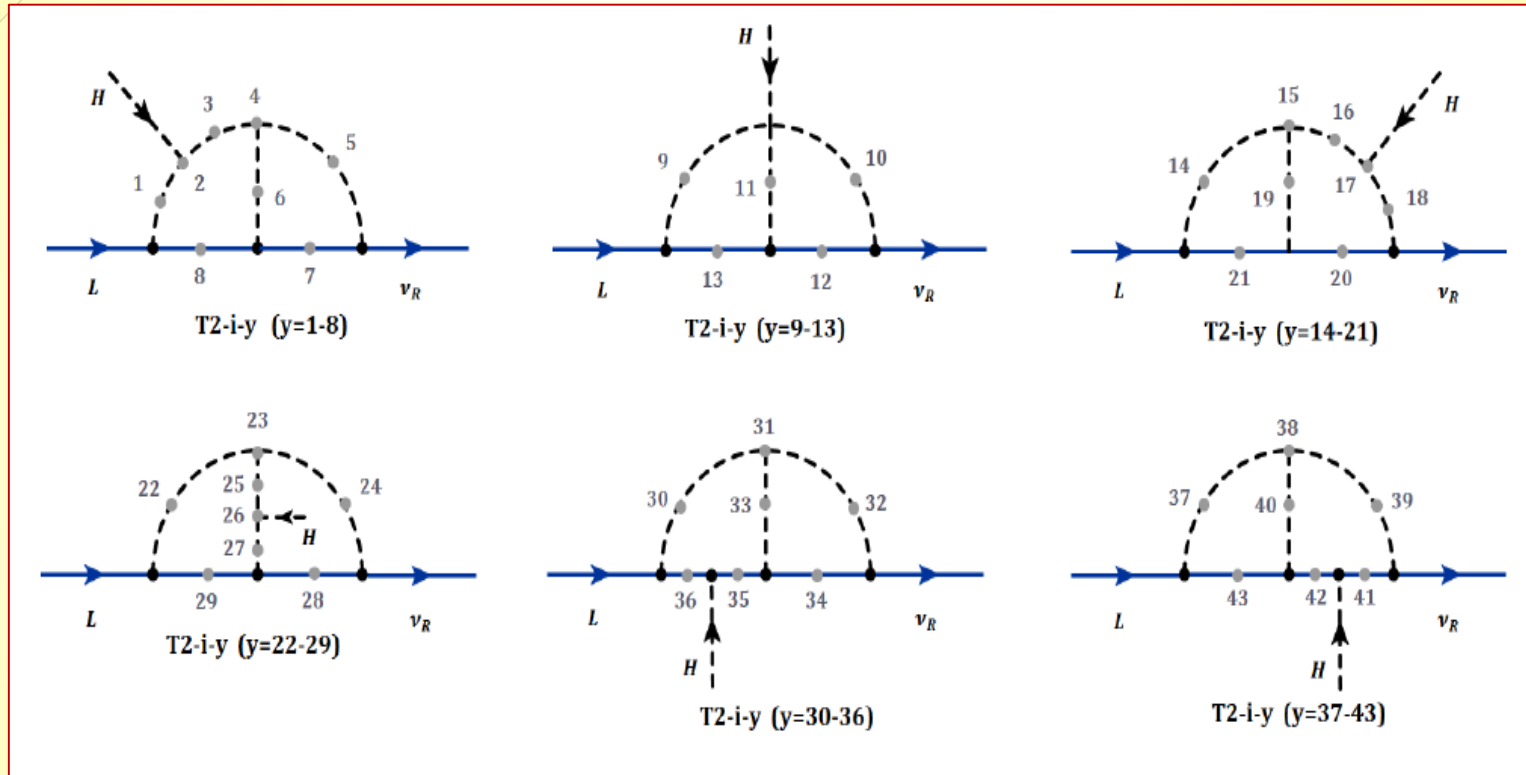
- ❑ Constructing minimal models arising from each of the skeleton diagrams.

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❖ Search for Minimal Two-loop Models

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- All possible diagrams emerging from $T2 - i$



- Discarding 22 viable diagrams, $T2 - i - y$, $y = \{7 - 8, 12 - 13, 20 - 21, 28 - 29, 34 - 36, 41 - 43\}$

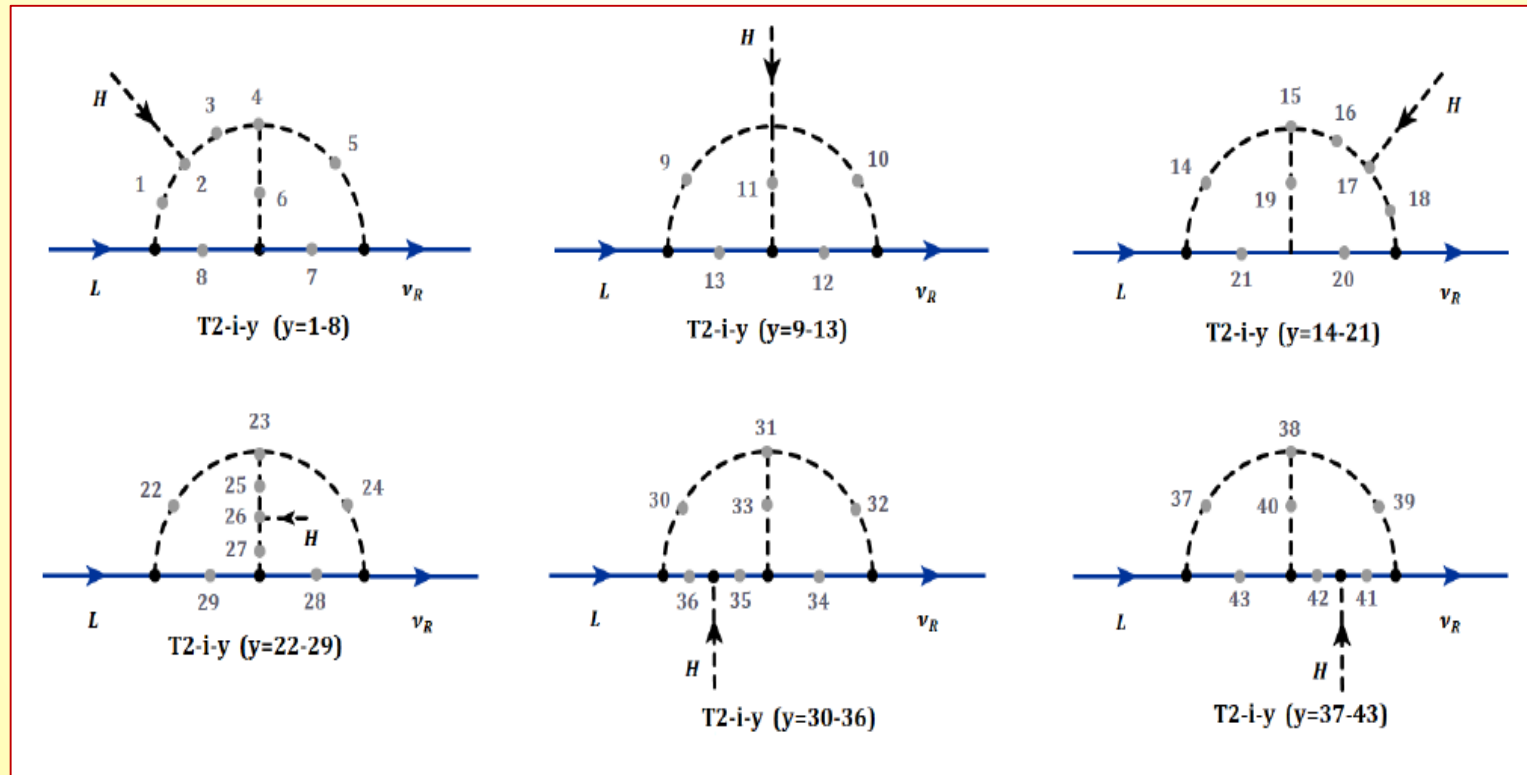
Required introduction of BSM fermions

❖ Search for Minimal Two-loop Models

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❑ Fixing internal fermion lines to be \mathbf{l}_R ,

excluding all diagrams other than $T2 - i - y$, $y = \{2 - 6, 10 - 11\}$, required more than one isodoublet



❑ Among these 7 diagrams selecting diagrams with minimal number of BSM scalars.

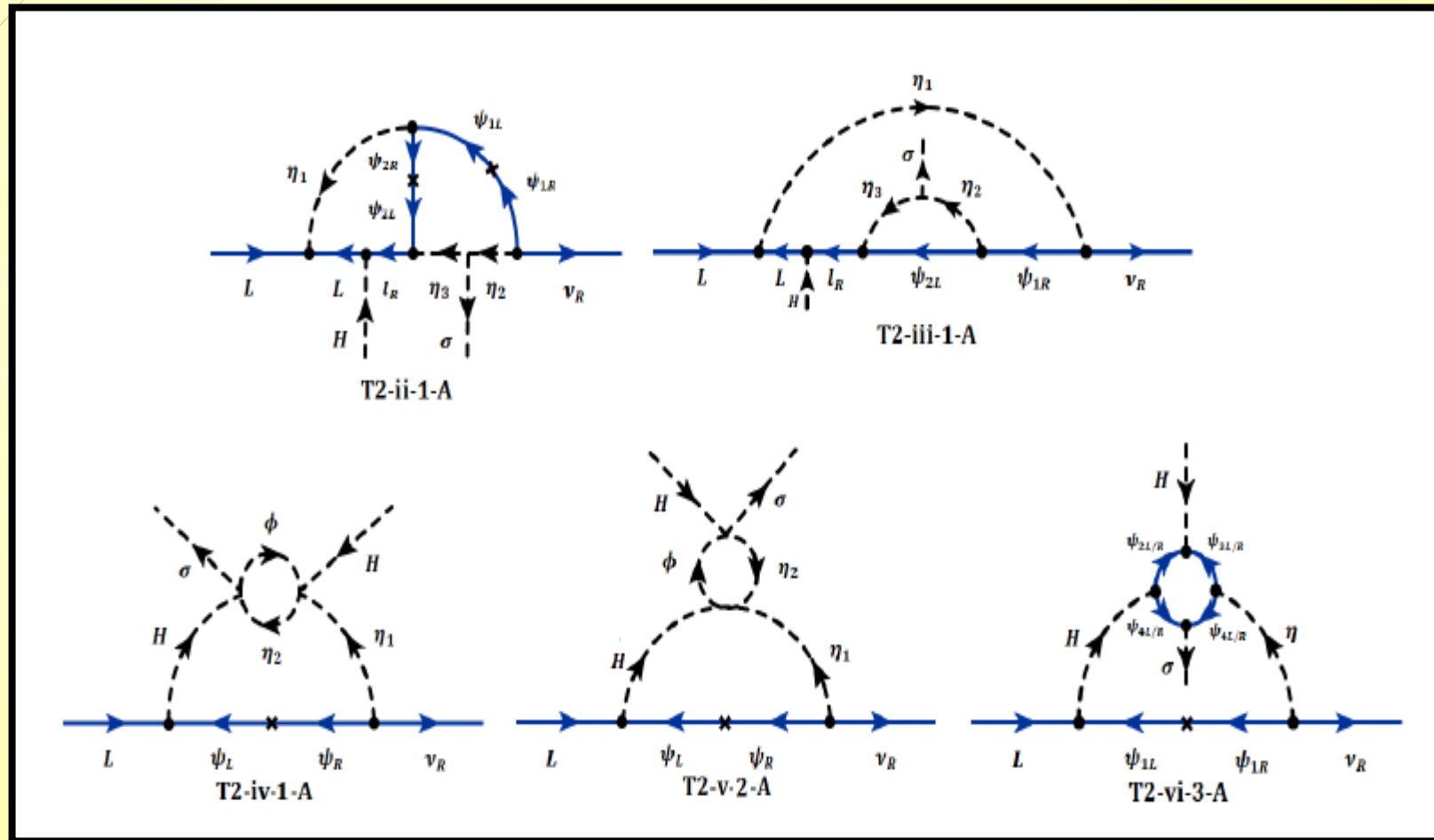
❖ Minimal Two-loop Models

Skeleton	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T2-i	T2-i-2-A	$\eta_1(1, 1, 1, -3)$ $\eta_2(1, 1, 1, 5)$ $\eta_3(1, 1, -2, -2)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_3^* l_R$ $+ y_3 \bar{l}_R^c \eta_2 \nu_R + \mu \eta_1 \eta_2 \eta_3$ $+ \lambda \phi \epsilon H \sigma^* \eta_1^*$	(Saad)
	T2-i-4-A	$\eta_1(1, 1, 1, 0)$ $\eta_2(1, 1, 1, 5)$ $\eta_3(1, 1, -2, -2)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_3^* l_R$ $+ y_3 \bar{l}_R^c \eta_2 \nu_R + \mu \phi \epsilon H \eta_1^*$ $+ \lambda \eta_1 \eta_2 \eta_3 \sigma^*$	✓
	T2-i-11-A	$\eta_1(1, 1, 1, 5)$ $\eta_2(1, 1, -2, -2)$ $\eta_3(1, 1, -2, -5)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_2^* l_R$ $+ y_3 \bar{l}_R^c \eta_1 \nu_R + \mu \eta_2 \eta_3^* \sigma^*$ $+ \lambda \phi \epsilon H \eta_1 \eta_3$	✓

❖ Minimal Two-loop Models

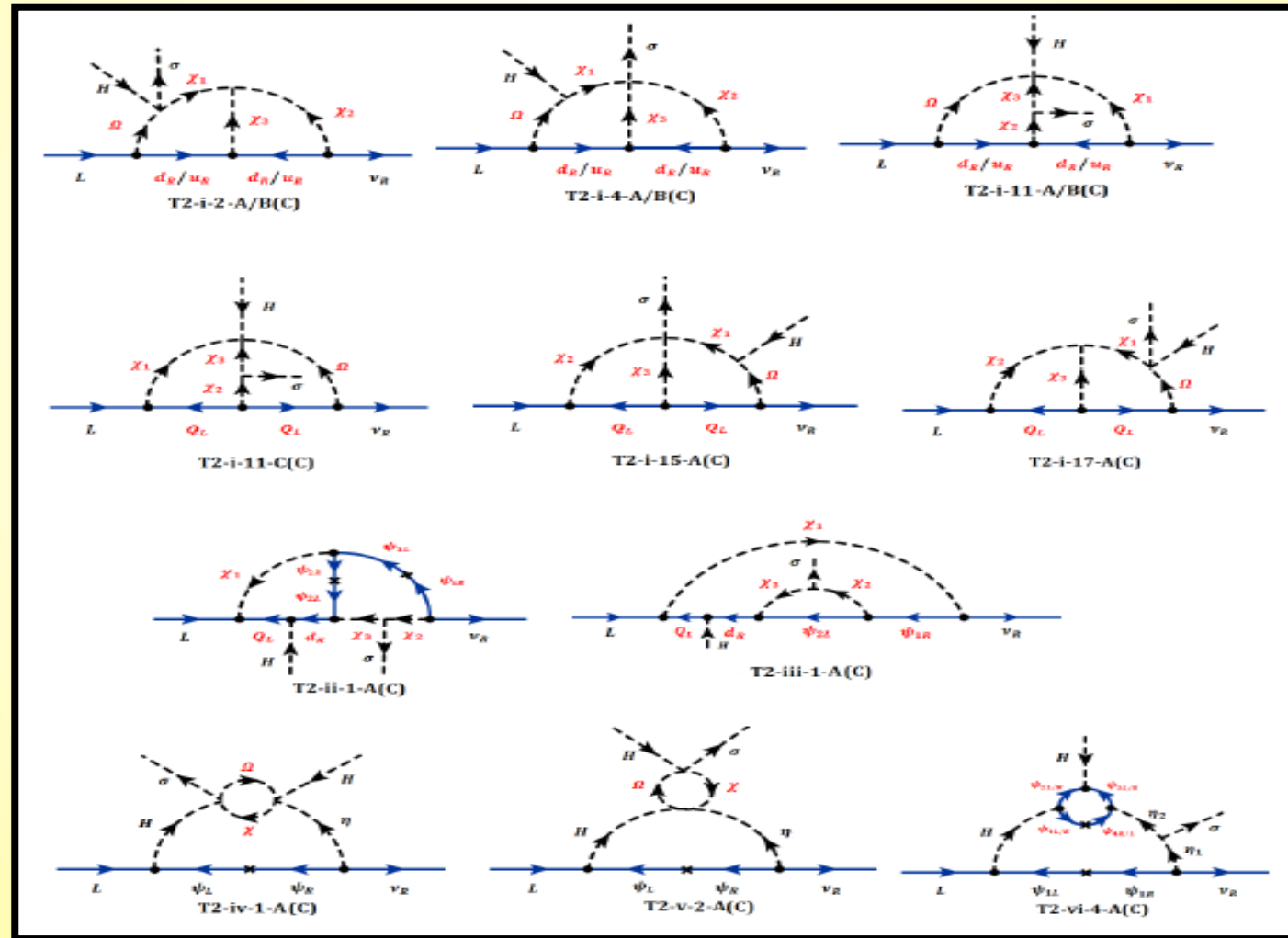
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□ Repeating similar strategy for $T2 - ii$ to vi



❖ Minimal Two-loop models – Colored version

- Corresponding minimal models for the colored version



❖ Possible DM Candidates

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- Spontaneously broken $U(1)_{B-L}$ symmetry, may leave a residual unbroken symmetry that can potentially stabilize the DM particle.

One-Loop Models

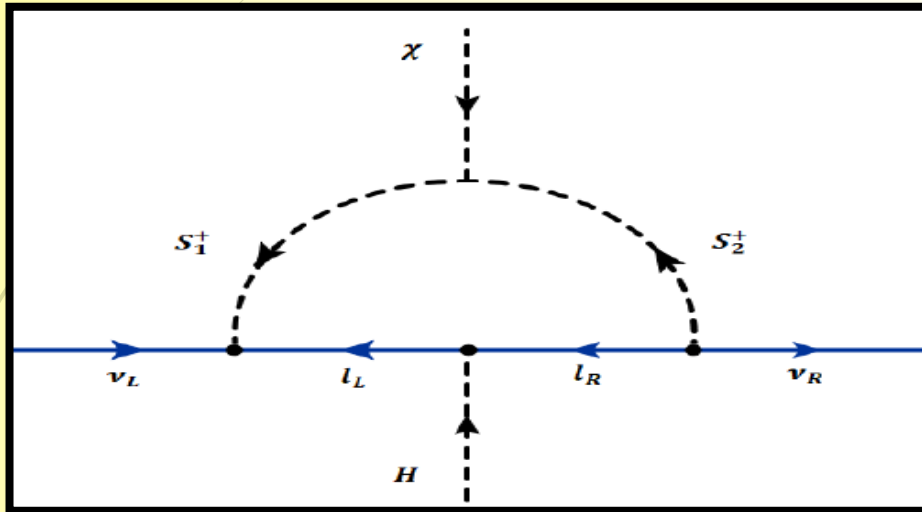
Models	Residual lepton symmetry	Residual dark symmetry	Choice of Y	Possible DM candidate
T1-i-1-A	Z_3	\times	—	\times
T1-i-1-B	Z_3	\times	—	\times
T1-i-1-C/D	Z_6	\checkmark	—	$\psi_{L,R}, \eta, \phi$
T1-i-2-A	Z_6	\checkmark	0	$\psi_{L,R}, \phi, \eta_1, \eta_2$
			-1	ϕ
T1-i-2-B	Z_6	\checkmark	0	$\psi_{L,R}, \phi_1, \phi_2, \eta_1, \eta_2$
			-1	ϕ_1, ϕ_2
T1-i-3-A	Z_3	\times	—	\times
T1-i-3-B	Z_6	\checkmark	0	$\psi_{1L,R}, \psi_{2L,R}, \psi_{3L,R}, \phi$
			-1	$\psi_{3L,R}, \phi$
T1-ii-1-A	Z_6	\checkmark	0	$\psi_{L,R}, \eta, \phi$
			-1	ϕ

Two-Loop Models

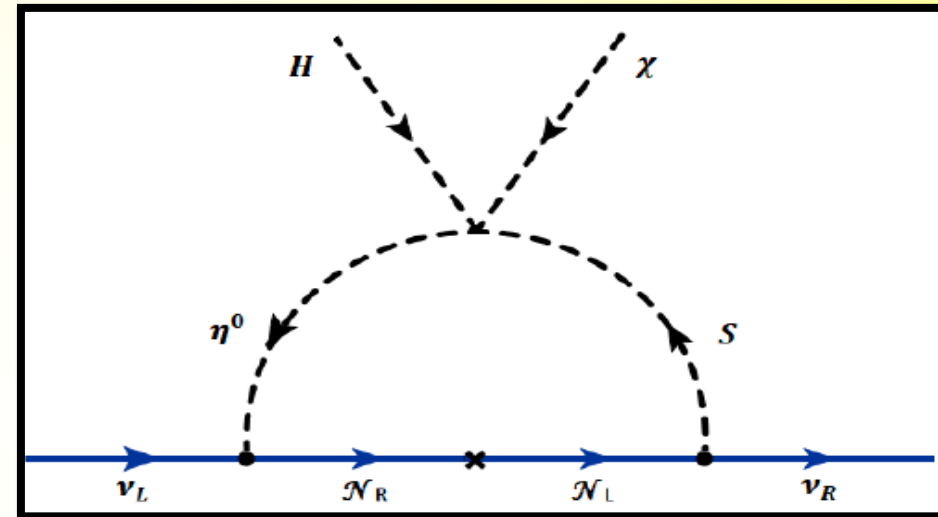
Models	Residual lepton symmetry	Residual dark symmetry	Choice of Y	Possible DM candidate
T2-i-2-A	Z_3	\times	—	\times
T2-i-4-A	Z_3	\times	—	\times
T2-i-11-A	Z_3	\times	—	\times
T2-ii-1-A	Z_6	\checkmark	0	$\psi_{2L,R}$
			-1	η_2, η_3
T2-iii-1-A	Z_6	\checkmark	0	$\psi_{2L,R}$
			-1	η_2, η_3
T2-iv-1-A	Z_6	\checkmark	1/2	ϕ, η
			-1/2	ϕ
T2-v-2-A	Z_6	\checkmark	1/2	ϕ
			-1/2	ϕ, η_2
T2-vi-3-A	Z_6	\checkmark	—	$\psi_{2L,R}, \psi_{4L,R}$

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❖ Phenomenology of Specific Models

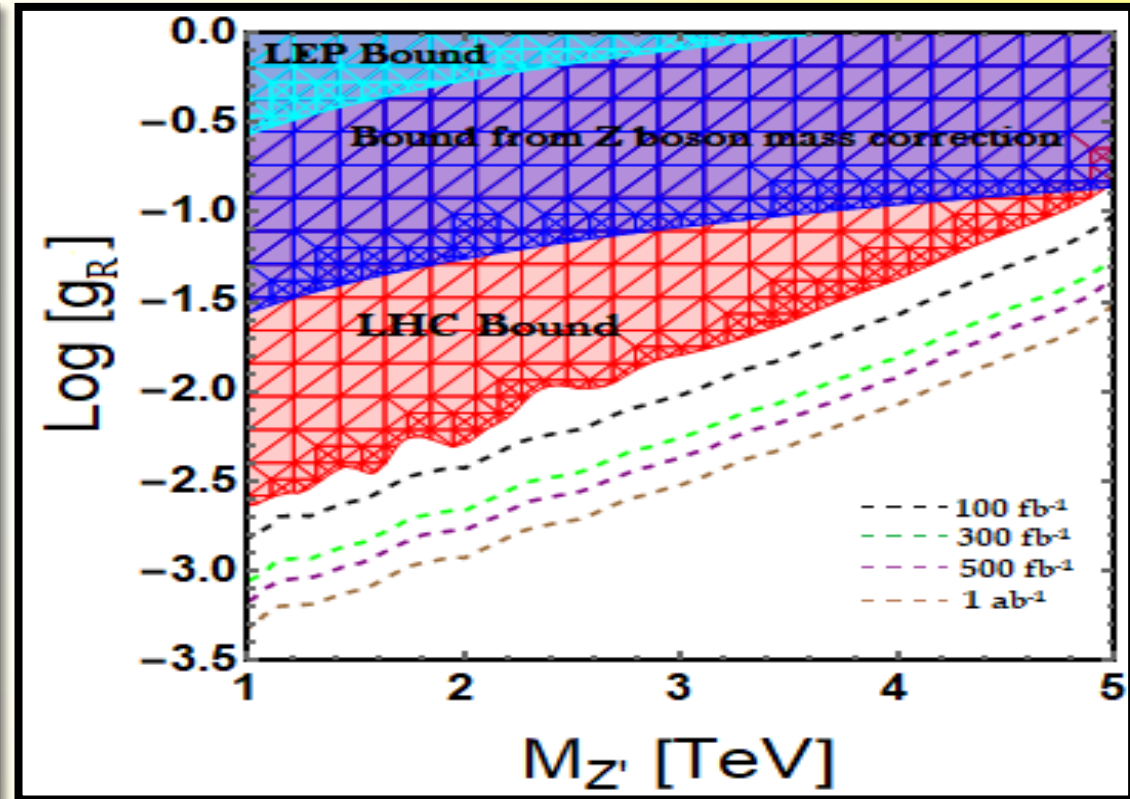
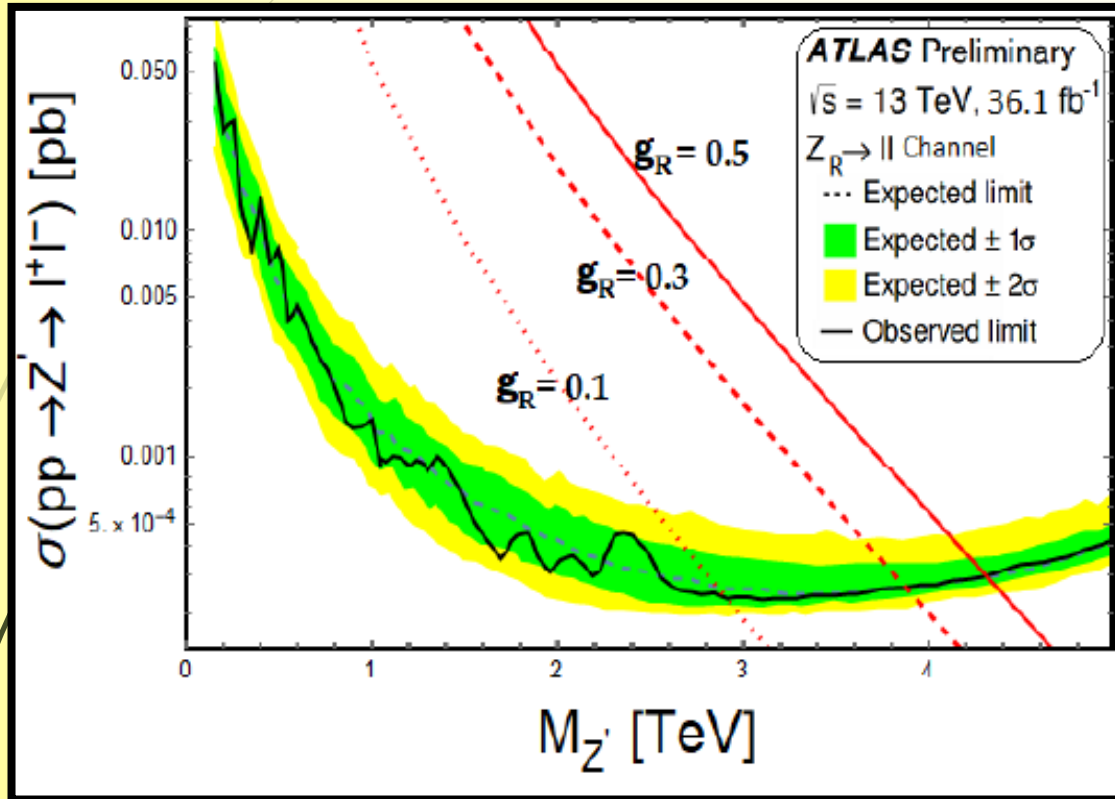


Multiplets	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$
Leptons	$L_{Li}(1, 2, -\frac{1}{2}, 0)$ $\ell_{Ri}(1, 1, -1, -1)$ $\nu_{Ri}(1, 1, 0, \{-5, 4, 4\})$
Scalars	$H(1, 2, \frac{1}{2}, 1)$ $\chi(1, 1, 0, 3)$ $S_1^+(1, 1, 1, 0)$ $S_2^+(1, 1, 1, -3)$



Multiplets	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$
Leptons	$L_{Li}(1, 2, -\frac{1}{2}, 0)$ $\ell_{Ri}(1, 1, -1, -1)$ $\nu_{Ri}(1, 1, 0, \{-5, 4, 4\})$
Scalars	$H(1, 2, \frac{1}{2}, 1)$ $\chi(1, 1, 0, 3)$ $S(1, 1, 0, -\frac{7}{2})$ $\eta(1, 2, \frac{1}{2}, \frac{1}{2})$
Vector-like fermion	$\mathcal{N}_{L,R}(1, 1, 0, \frac{1}{2})$

❖ Constraints from LEP and LHC

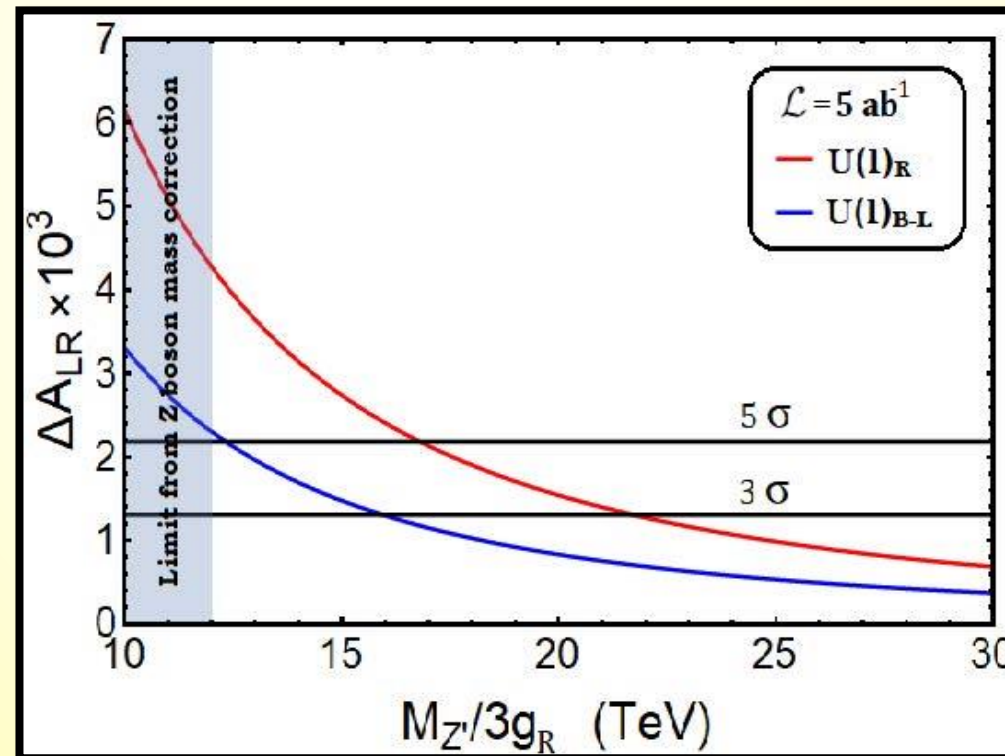


❖ Heavy Gauge Boson Z' at ILC : Left-Right Asymmetry

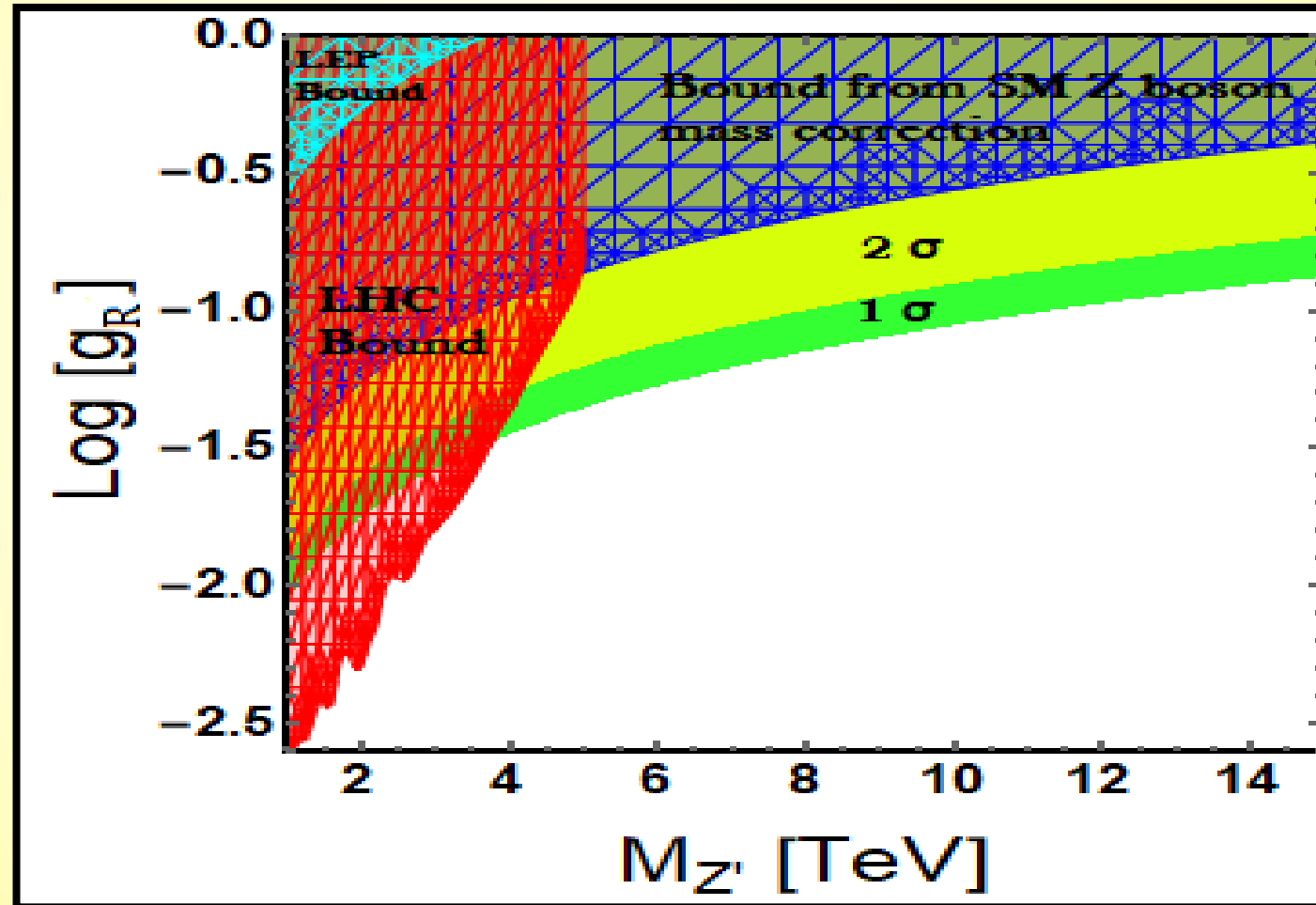
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$$\mathcal{L}_{eff} = \frac{1}{1 + \delta_{ef}} \frac{g_R^2}{M_{Z'}^2} (\bar{e} \gamma^\mu \mathcal{P}_R e) (\bar{f} \gamma_\mu \mathcal{P}_R f)$$

- Analysis with the polarized initial states at ILC can be used to understand the chirality structure of the effective interaction

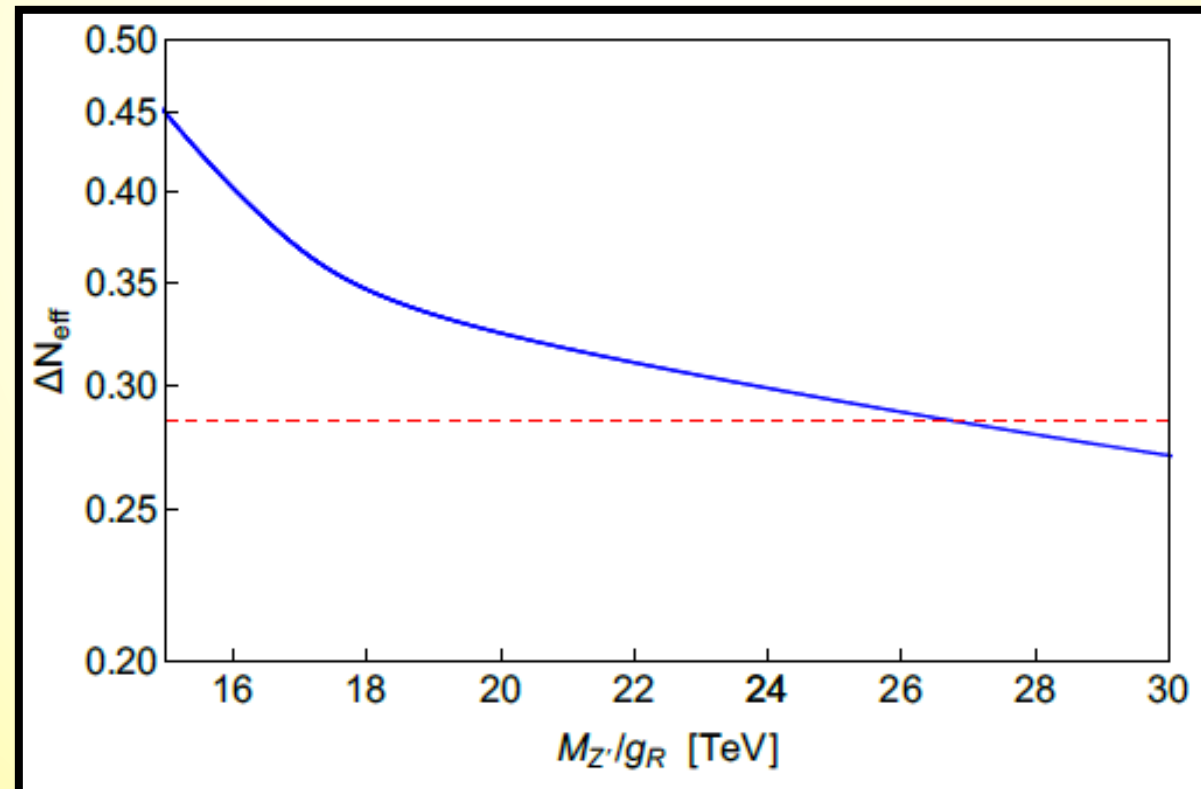


❖ Phenomenology of Z'



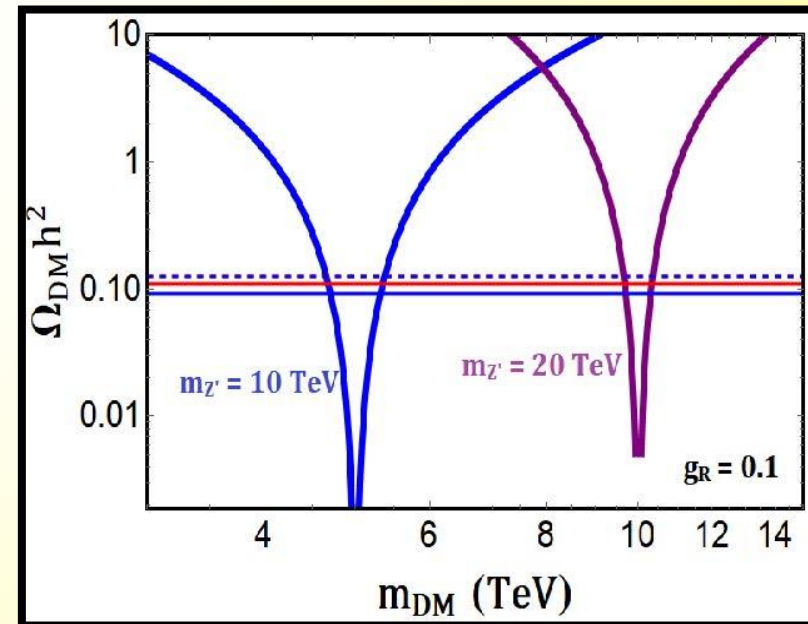
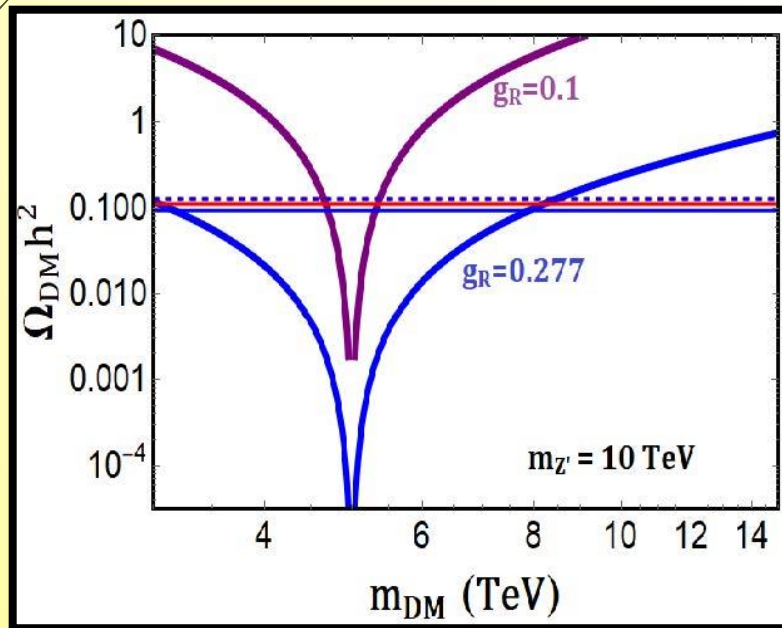
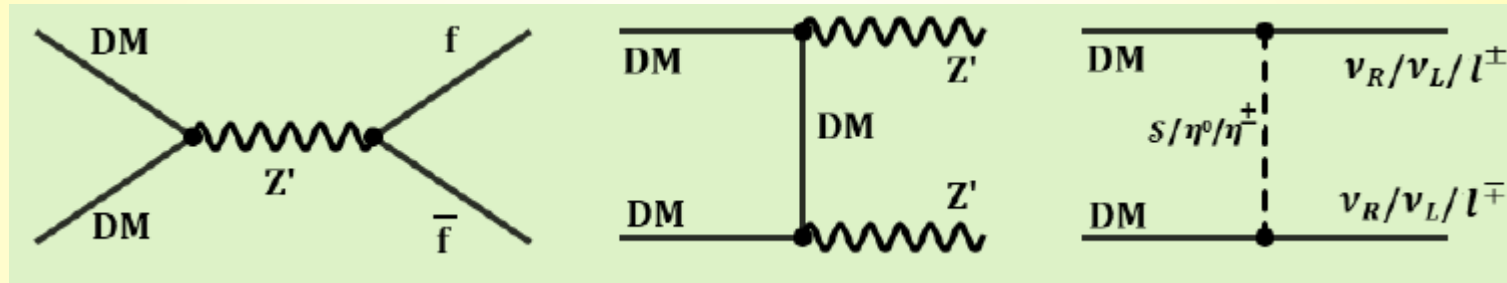
❖ Constraints from Cosmology

- ❑ The Right-handed neutrino ν_R can increase the effective number of relativistic degree of freedom N_{eff} .
- ❑ To be compatible with the current cosmological constraints on N_{eff} , the interaction of ν_R with the primordial plasma must be highly suppressed.



❖ Dark Matter Phenomenology

- Spontaneously broken $U(1)_R$ symmetry, leave a Z_2 residual symmetry that can potentially stabilize the DM particle (considering for the case of vector-like fermion).



❖ Summary

- ❑ We constructed minimal Dirac neutrino mass models arising from generic one-loop and two-loop topologies for both colored and non-colored versions.
- ❑ Out of the 40 models that we proposed 37 of them are new.
- ❑ Out of the 17 non-colored models that we proposed 11 of them can naturally incorporate DM candidate.
- ❑ Every single model that we are presented in this work require no representation higher than the fundamental representation under $SU(2)_L$ and $SU(3)_C$.
- ❑ Our methodology can be implemented to construct new models by utilizing various different symmetries (discrete, global, gauge)
- ❑ Each of the models in this work can have very distinct phenomenology and must be studied case by case.

Thank
you!

❖ One-loop Models

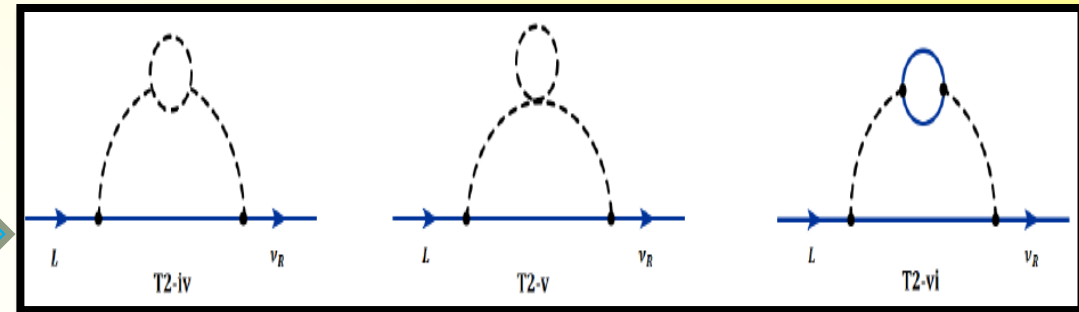
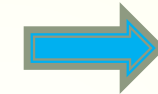
Diagram	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T1-i-1	T1-i-1-A	$\eta_1(1, 1, 1, 2)$ $\eta_2(1, 1, 1, 5)$	—	$y_1 \bar{L}^c \epsilon \eta_1 L + y_e \bar{L} H l_R$ $+ y_2 \bar{l}_R^c \eta_2 \nu_R + \mu \eta_2 \eta_1^* \sigma^*$	(Saad, Zapata)
	T1-i-1-B	$\eta(1, 1, 1, 5)$ $\phi(1, 2, \frac{3}{2}, 5)$	$\psi_{L,R}(1, 1, -1, -4)$	$y_1 \bar{L}^c \epsilon \phi \psi_L + y_2 \bar{\psi}_L \sigma^* l_R$ $+ y_3 \bar{l}_R^c \eta \nu_R + \mu \phi^\dagger H \eta$	✓
	T1-i-1-C	$\eta(1, 1, 0, \frac{5}{2})$ $\phi(1, 2, \frac{1}{2}, \frac{5}{2})$	$\psi_{L,R}(1, 1, 0, \frac{3}{2})$	$y_1 \bar{L} \epsilon \phi^* \psi_R + y_2 \bar{\psi}_R^c \sigma^* \psi_R$ $+ y_3 \bar{\psi}_R^c \eta \nu_R + \mu \phi^\dagger H \eta$	✓
	T1-i-1-D	$\eta(1, 1, 0, \frac{5}{2})$ $\phi(1, 2, \frac{1}{2}, \frac{5}{2})$	$\psi_{L,R}(1, 1, 0, -\frac{3}{2})$	$y_1 \bar{L}^c \epsilon \phi \psi_L + y_2 \bar{\psi}_L^c \sigma \psi_L$ $+ y_3 \bar{\psi}_L \eta \nu_R + \mu \phi^\dagger H \eta$	✓
T1-i-2	T1-i-2-A	$\eta_1(1, 1, Y, 4 + \alpha)$ $\eta_2(1, 1, Y, 1 + \alpha)$ $\phi(1, 2, Y + \frac{1}{2}, 1 + \alpha)$	$\psi_{L,R}(1, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \phi^* \psi_R$ $+ y_2 \bar{\psi}_L \eta_1 \nu_R + \mu_1 \eta_1 \eta_2^* \sigma^*$ $+ \mu_2 \phi^\dagger H \eta_2$	✓
	T1-i-2-B	$\eta(1, 1, Y, 4 + \alpha)$ $\phi_1(1, 2, Y + \frac{1}{2}, 1 + \alpha)$ $\phi_2(1, 2, Y + \frac{1}{2}, 4 + \alpha)$	$\psi_{L,R}(1, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \phi_1^* \psi_R$ $+ y_2 \bar{\psi}_L \eta \nu_R + \mu_1 H \eta \phi_2^*$ $+ \mu_2 \phi_1^\dagger \phi_2 \sigma^*$	✓
T1-i-3	T1-i-3-A	$\eta(1, 1, 1, 2)$	$\psi_{L,R}(1, 1, -1, 2)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L}^c \epsilon \eta L$ $+ y_2 \bar{\psi}_R^c \eta \nu_R + y_3 \bar{\psi}_L \sigma l_R$ $+ y_e \bar{L} H l_R$	✓
	T1-i-3-B	$\phi(1, 2, Y + \frac{1}{2}, 1 + \alpha)$	$\psi_{1L,R}(1, 1, Y, \alpha)$ $\psi_{2L,R}(1, 1, Y, \alpha - 3)$ $\psi_{3L,R}(1, 2, Y + \frac{1}{2}, \alpha - 3)$	$M_{\psi_1} \bar{\psi}_{1L} \psi_{1R} + M_{\psi_2} \bar{\psi}_{2L} \psi_{2R}$ $+ y_1 \bar{L} \epsilon \phi^* \psi_{1R} + y_2 \bar{\psi}_{2L} \sigma^* \psi_{1R}$ $+ y_3 \bar{\psi}_{3L} H \psi_{2R} + y_4 \bar{\psi}_{3L} \phi \nu_R$	✓
T1-ii-1	T1-ii-1-A	$\eta(1, 1, Y, 4 + \alpha)$ $\phi(1, 2, Y + \frac{1}{2}, 1 + \alpha)$	$\psi_{L,R}(1, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \phi^* \psi_R$ $+ y_2 \bar{\psi}_L \eta \nu_R + \lambda \phi^\dagger H \eta \sigma^*$	(Zapata, Srivastava)

❖ One-loop Models: Colored version

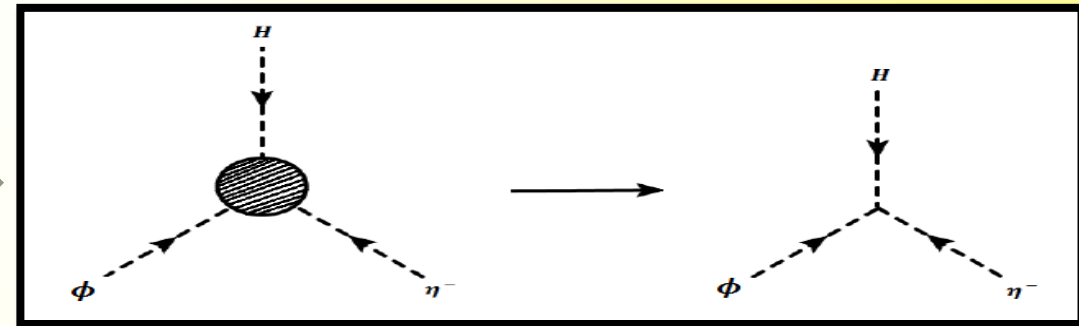
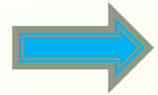
Diagram	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T1-i-1	T1-i-1-A(C)	$\chi_1(\bar{3}, 1, \frac{1}{3}, \frac{2}{3})$ $\chi_2(\bar{3}, 1, \frac{1}{3}, \frac{11}{3})$	—	$y_1 \bar{L}^c \epsilon \chi_1 Q_L + y_d \bar{Q}_L H d_R$ $+ y_2 \bar{d}_R^c \chi_2 \nu_R + \mu \chi_2 \chi_1^* \sigma^*$	✓
	T1-i-1-B(C)	$\Omega_1(3, 2, \frac{1}{6}, \frac{4}{3})$ $\Omega_2(3, 2, \frac{1}{6}, \frac{13}{3})$	—	$y_1 \bar{d}_R \Omega_1 \epsilon L + y_d \bar{Q}_L H d_R$ $+ y_2 \bar{Q}_L \Omega_2 \nu_R + \mu \Omega_1^\dagger \Omega_2 \sigma^*$	✓
	T1-i-1-C(C)	$\chi(\bar{3}, 1, \frac{1}{3}, \frac{11}{3})$ $\Omega(\bar{3}, 2, \frac{5}{6}, \frac{11}{3})$	$\psi_{L,R}(3, 1, -\frac{1}{3}, -\frac{8}{3})$	$y_1 \bar{L}^c \epsilon \Omega \psi_L + y_2 \bar{\psi}_L \sigma^* d_R$ $+ y_3 \bar{d}_R^c \chi \nu_R + \mu \Omega^\dagger H \chi$	✓
	T1-i-1-D(C)	$\chi(\bar{3}, 1, -\frac{2}{3}, \frac{11}{3})$ $\Omega(\bar{3}, 2, -\frac{1}{6}, \frac{11}{3})$	$\psi_{L,R}(3, 1, \frac{2}{3}, -\frac{8}{3})$	$y_1 \bar{L}^c \epsilon \Omega \psi_L + y_2 \bar{\psi}_L \sigma^* u_R$ $+ y_3 \bar{u}_R^c \chi \nu_R + \mu \Omega^\dagger H \chi$	✓
T1-i-2	T1-i-2-A(C)	$\chi_1(3, 1, Y, 4 + \alpha)$ $\chi_2(3, 1, Y, 1 + \alpha)$ $\Omega(3, 2, Y + \frac{1}{2}, 1 + \alpha)$	$\psi_{L,R}(3, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \Omega^* \psi_R$ $+ y_2 \bar{\psi}_L \chi_1 \nu_R + \mu_1 \Omega^\dagger H \chi_2$ $+ \mu_2 \chi_1 \chi_2^* \sigma^*$	✓
	T1-i-2-B(C)	$\chi(3, 1, Y, 4 + \alpha)$ $\Omega_1(3, 2, Y + \frac{1}{2}, 1 + \alpha)$ $\Omega_2(3, 2, Y + \frac{1}{2}, 4 + \alpha)$	$\psi_{L,R}(3, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \Omega_1^* \psi_R$ $+ y_2 \bar{\psi}_L \chi \nu_R + \mu_1 \Omega_2^\dagger H \chi$ $+ \mu_2 \Omega_1^\dagger \Omega_2 \sigma^*$	✓
T1-i-3	T1-i-3-A(C)	$\chi(\bar{3}, 1, \frac{1}{3}, \frac{2}{3})$	$\psi_{L,R}(3, 1, -\frac{1}{3}, \frac{10}{3})$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L}^c \epsilon \chi Q_L$ $+ y_d \bar{Q}_L H d_R + y_2 \bar{\psi}_R^c \chi \nu_R$ $+ y_3 \bar{\psi}_L \sigma d_R$	✓
	T1-i-3-B(C)	$\Omega(3, 2, \frac{1}{6}, \frac{13}{3})$	$\psi_{L,R}(3, 1, -\frac{1}{3}, \frac{10}{3})$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \Omega^* \psi_R$ $+ y_d \bar{Q}_L H d_R + y_2 \bar{Q}_L \Omega \nu_R$ $+ y_3 \bar{\psi}_L \sigma d_R$	✓
T1-ii-1	T1-ii-1-A(C)	$\chi(3, 1, Y, 4 + \alpha)$ $\Omega(3, 2, Y + \frac{1}{2}, 1 + \alpha)$	$\psi_{L,R}(3, 1, Y, \alpha)$	$M_\psi \bar{\psi}_L \psi_R + y_1 \bar{L} \epsilon \Omega^* \psi_R$ $+ y_2 \bar{\psi}_L \chi \nu_R + \lambda \Omega^\dagger H \sigma^* \chi$	✓

❖ Search for Minimal Two-loop Models

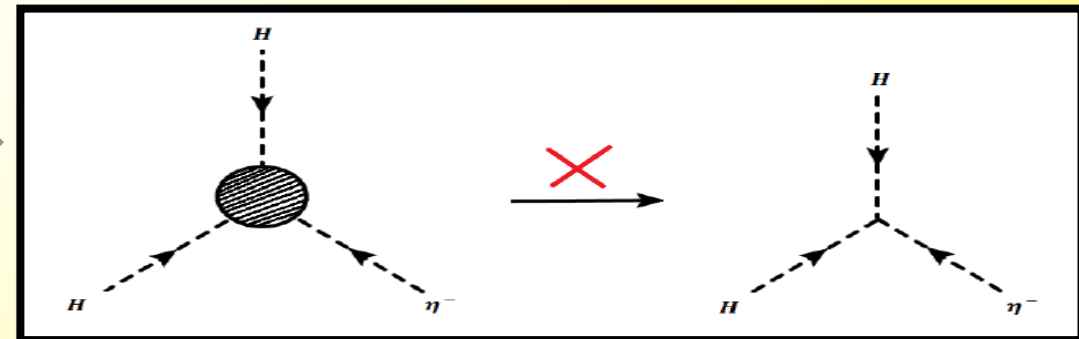
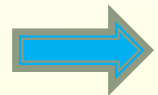
❑ Consider the case of $T2 - iv, v, vi$ skeleton diagrams



❑ In each case the blob can reduce to tree vertex for a general isodoublet \rightarrow no two-loop diagrams



❑ Can be resolved if the isodoublet is Higgs doublet, because of the antisymmetric property of $L \supset H \epsilon H \eta^-$



❖ Two-loop Models

Skeleton	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T2-i	T2-i-2-A	$\eta_1(1, 1, 1, -3)$ $\eta_2(1, 1, 1, 5)$ $\eta_3(1, 1, -2, -2)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_3^* l_R$ $+ y_3 \bar{l}_R^c \eta_2 \nu_R + \mu \eta_1 \eta_2 \eta_3$ $+ \lambda \phi \epsilon H \sigma^* \eta_1^*$	[52]
	T2-i-4-A	$\eta_1(1, 1, 1, 0)$ $\eta_2(1, 1, 1, 5)$ $\eta_3(1, 1, -2, -2)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_3^* l_R$ $+ y_3 \bar{l}_R^c \eta_2 \nu_R + \mu \phi \epsilon H \eta_1^*$ $+ \lambda \eta_1 \eta_2 \eta_3 \sigma^*$	✓
	T2-i-11-A	$\eta_1(1, 1, 1, 5)$ $\eta_2(1, 1, -2, -2)$ $\eta_3(1, 1, -2, -5)$ $\phi(1, 2, \frac{1}{2}, 0)$	—	$y_1 \bar{L} \phi l_R + y_2 \bar{l}_R^c \eta_2^* l_R$ $+ y_3 \bar{l}_R^c \eta_1 \nu_R + \mu \eta_2 \eta_3^* \sigma^*$ $+ \lambda \phi \epsilon H \eta_1 \eta_3$	✓
T2-ii	T2-ii-1-A	$\eta_1(1, 1, 1, 2)$ $\eta_2(1, 1, -(Y+1), 2-\alpha)$ $\eta_3(1, 1, -(Y+1), -(1+\alpha))$	$\psi_{1L,R}(1, 1, Y+1, 2+\alpha)$ $\psi_{2L,R}(1, 1, Y, \alpha)$	$M_{\psi_1} \bar{\psi}_{1L} \psi_{1R} + M_{\psi_2} \bar{\psi}_{2L} \psi_{2R}$ $+ y_1 \bar{L}^c \epsilon \eta_1 L + y_2 \bar{\psi}_{2L} \eta_3^* l_R$ $+ y_3 \bar{\psi}_{1R}^c \eta_2 \nu_R + y_4 \bar{\psi}_{1L} \eta_1 \psi_{2R}$ $+ y_e \bar{L} H l_R + \mu \eta_2 \eta_3^* \sigma^*$	✓
T2-iii	T2-iii-1-A	$\eta_1(1, 1, -1, -2)$ $\eta_2(1, 1, -(Y+1), 2-\alpha)$ $\eta_3(1, 1, -(Y+1), -(1+\alpha))$	$\psi_{1L,R}(1, 1, -1, 2)$ $\psi_{2L,R}(1, 1, Y, \alpha)$	$y_1 \bar{L}^c \eta_1^* L + y_2 \bar{\psi}_{2L} \eta_3^* l_R$ $+ y_3 \bar{\psi}_{2L} \eta_2^* \psi_{1R} + y_4 \bar{\psi}_{1R}^c \eta_1^* \nu_R$ $+ y_e \bar{L} H l_R + \mu \eta_2 \eta_3^* \sigma^*$	✓
T2-iv	T2-iv-1-A	$\eta_1(1, 1, -1, 3)$ $\eta_2(1, 1, Y - \frac{1}{2}, \alpha + 3)$ $\phi(1, 2, Y, \alpha)$	$\psi_{L,R}(1, 1, 1, 1)$	$M_{\psi} \bar{\psi}_L \psi_R + y_1 \bar{L}^c H^* \psi_L$ $+ y_2 \bar{\psi}_R^c \eta_1 \nu_R + \lambda_1 \phi^\dagger H \sigma^* \eta_2$ $+ \lambda_2 \phi \epsilon H \eta_1 \eta_2^*$	✓
T2-v	T2-v-2-A	$\eta_1(1, 1, -1, 3)$ $\eta_2(1, 1, Y + \frac{1}{2}, \alpha - 3)$ $\phi(1, 2, Y, \alpha)$	$\psi_{L,R}(1, 1, 1, 1)$	$M_{\psi} \bar{\psi}_L \psi_R + y_1 \bar{L}^c H^* \psi_L$ $+ y_2 \bar{\psi}_R^c \eta_1 \nu_R + \lambda_1 \phi^\dagger H \eta_1 \eta_2$ $+ \lambda_2 \phi \epsilon H \sigma^* \eta_2^*$	✓
T2-vi	T2-vi-3-A	$\eta(1, 1, -1, 3)$	$\psi_{1L,R}(1, 1, 1, 1)$ $\psi_{2L,R}(1, 2, \frac{1}{2}, -\frac{3}{2})$ $\psi_{3L,R}(1, 1, -1, \frac{3}{2})$ $\psi_{4L,R}(1, 1, 0, \frac{3}{2})$	$M_{\psi_1} \bar{\psi}_{1L} \psi_{1R} + y_1 \bar{L}^c H^* \psi_{1L}$ $+ y_2 \bar{\psi}_{1R}^c \eta \nu_R$ $+ y_3 \bar{\psi}_{4L/R}^c \psi_{4L/R} \sigma^*$ $+ y_4 \bar{\psi}_{2L/R}^c \psi_{4L/R} H^*$ $+ y_5 \bar{\psi}_{2L/R}^c \psi_{3L/R} \epsilon H$ $+ y_6 \bar{\psi}_{3L/R}^c \psi_{4L/R} \eta^*$	✓

❖ Two-loop Models: Colored version

Skeleton	Models	New Fields		Relevant terms in Lagrangian	New ?
		Scalars	Fermions		
T2-i	T2-i-2-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega d_R + y_2 \overline{d}_R^c \chi_3^* d_R$ $+ y_3 \overline{d}_R^c \chi_2 \nu_R + \mu \chi_1 \chi_2 \chi_3$ $+ \lambda \Omega e H \sigma^* \chi_1^\dagger$	✓
	T2-i-2-B(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega u_R + y_2 \overline{u}_R^c \chi_3^* u_R$ $+ y_3 \overline{u}_R^c \chi_2 \nu_R + \mu \chi_1 \chi_2 \chi_3$ $+ \lambda \Omega e H \sigma^* \chi_1^\dagger$	✓
	T2-i-4-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega d_R + y_2 \overline{d}_R^c \chi_3^* d_R$ $+ y_3 \overline{d}_R^c \chi_2 \nu_R + \mu \Omega e H \chi_1^\dagger$ $+ \lambda \chi_1 \chi_2 \chi_3 \sigma^*$	✓
	T2-i-4-B(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega u_R + y_2 \overline{u}_R^c \chi_3^* u_R$ $+ y_3 \overline{u}_R^c \chi_2 \nu_R + \mu \Omega e H \chi_1^\dagger$ $+ \lambda \chi_1 \chi_2 \chi_3 \sigma^*$	✓
	T2-i-11-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega d_R + y_2 \overline{d}_R^c \chi_3^* d_R$ $+ y_3 \overline{d}_R^c \chi_1 \nu_R + \mu \chi_2 \chi_3 \sigma^*$ $+ \lambda \Omega e H \chi_1 \chi_3$	✓
	T2-i-11-B(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, -\frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L} \Omega u_R + y_2 \overline{u}_R^c \chi_3^* u_R$ $+ y_3 \overline{u}_R^c \chi_1 \nu_R + \mu \chi_2 \chi_3 \sigma^*$ $+ \lambda \Omega e H \chi_1 \chi_3$	✓
	T2-i-11-C(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, \frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L}^c \chi_1^\dagger Q_L + y_2 \overline{Q}_L^c Q_L \chi_2$ $+ y_3 \overline{Q}_L^c \Omega \nu_R + \mu \chi_2 \chi_3 \sigma^*$ $+ \lambda \Omega e H \chi_1 \chi_3$	✓
	T2-i-15-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, \frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L}^c \chi_3^\dagger Q_L + y_2 \overline{Q}_L^c Q_L \chi_3$ $+ y_3 \overline{Q}_L^c \Omega \nu_R + \mu \chi_1^\dagger \Omega e H$ $+ \lambda \chi_1 \chi_2 \chi_3 \sigma^*$	✓
T2-i-17-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_3(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\Omega(3, 2, \frac{1}{6}, -\frac{10}{3})$	—	$y_1 \overline{L}^c \chi_3^\dagger Q_L + y_2 \overline{Q}_L^c Q_L \chi_3$ $+ y_3 \overline{Q}_L^c \Omega \nu_R + \mu \chi_1 \chi_2 \chi_3$ $+ \lambda \sigma^* \chi_1^\dagger \Omega e H$	✓	
T2-ii	T2-ii-1-A(C)	$\chi_1(3, 1, \frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, -Y - \frac{1}{3}, -\frac{10}{3} - \alpha)$ $\chi_3(3, 1, -Y - \frac{1}{3}, -\frac{10}{3} - \alpha)$	$\psi_{1L,R}(3, 1, Y + \frac{1}{3}, \alpha + \frac{2}{3})$ $\psi_{2L,R}(3, 1, Y, \alpha)$	$M_{\phi_1} \overline{\psi}_{1L} \psi_{1R} + M_{\phi_2} \overline{\psi}_{2L} \psi_{2R}$ $+ y_1 \overline{L}^c \chi_1 Q_L + y_2 \overline{\psi}_{2L} \chi_3^* d_R$ $+ y_3 \overline{\psi}_{1R}^c \chi_2 \nu_R + y_4 \overline{\psi}_{1L} \chi_1 \psi_{2R}$ $+ y_5 \overline{Q}_L^c H d_R + \mu \chi_2 \chi_3 \sigma^*$	✓
T2-iii	T2-iii-1-A(C)	$\chi_1(3, 1, -\frac{1}{3}, -\frac{10}{3})$ $\chi_2(3, 1, -(Y + \frac{1}{3}), \frac{10}{3} - \alpha)$ $\chi_3(3, 1, -(Y + \frac{1}{3}), \frac{10}{3} - \alpha)$	$\psi_{1L,R}(3, 1, -\frac{1}{3}, \frac{10}{3})$ $\psi_{2L,R}(3, 1, Y, \alpha)$	$y_1 \overline{Q}_L^c \chi_1^\dagger L + y_2 \overline{\psi}_{2L} \chi_3^* d_R$ $+ y_3 \overline{\psi}_{2L} \chi_2^* \psi_{1R} + y_4 \overline{\psi}_{1R}^c \chi_1^\dagger \nu_R$ $+ y_5 \overline{Q}_L^c H d_R + \mu \chi_2 \chi_3 \sigma^*$	✓
T2-iv	T2-iv-1-A(C)	$\eta(1, 1, -1, 3)$ $\chi(3, 1, Y - \frac{1}{3}, \alpha + 3)$ $\Omega(3, 2, Y, \alpha)$	$\psi_{L,R}(1, 1, 1, 1)$	$M_{\phi} \overline{\psi}_L \psi_R + y_1 \overline{L}^c H^* \psi_L$ $+ y_2 \overline{\psi}_R^c \nu_R + \lambda_1 \Omega^3 H \sigma^* \chi$ $+ \lambda_2 \Omega e H \eta \chi^*$	✓
T2-v	T2-v-2-A(C)	$\eta(1, 1, -1, 3)$ $\chi(3, 1, Y + \frac{1}{3}, \alpha - 3)$ $\Omega(3, 2, Y, \alpha)$	$\psi_{L,R}(1, 1, 1, 1)$	$M_{\phi} \overline{\psi}_L \psi_R + y_1 \overline{L}^c H^* \psi_L$ $+ y_2 \overline{\psi}_R^c \nu_R + \lambda_1 \Omega^3 H \eta \chi$ $+ \lambda_2 \Omega e H \sigma^* \chi^*$	✓
T2-vi	T2-vi-4-A(C)	$\eta_1(1, 1, -1, 3)$ $\eta_2(1, 1, -1, 0)$	$\psi_{1L,R}(1, 1, 1, 1)$ $\psi_{2L,R}(3, 2, Y, \alpha)$ $\psi_{3L,R}(3, 1, -Y - \frac{1}{3}, -\alpha)$ $\psi_{4L,R}(3, 1, -Y + \frac{1}{3}, -\alpha)$	$M_{\phi_1} \overline{\psi}_{1L} \psi_{1R} + M_{\phi_2} \overline{\psi}_{4L} \psi_{4R}$ $+ y_1 \overline{L}^c H^* \psi_{1L} + y_2 \overline{\psi}_{1R}^c \eta_1 \nu_R$ $+ y_3 \overline{\psi}_{2L}^c / R \psi_{4L} / R H^*$ $+ y_4 \overline{\psi}_{2L}^c / R \psi_{3L} / R e H$ $+ y_5 \overline{\psi}_{4R} / L \psi_{3L} / R \sigma_2^* + \mu \eta_1 \eta_2^* \sigma^*$	✓