

Probing top Higgs Yukawa coupling at the LHC via single top +h production

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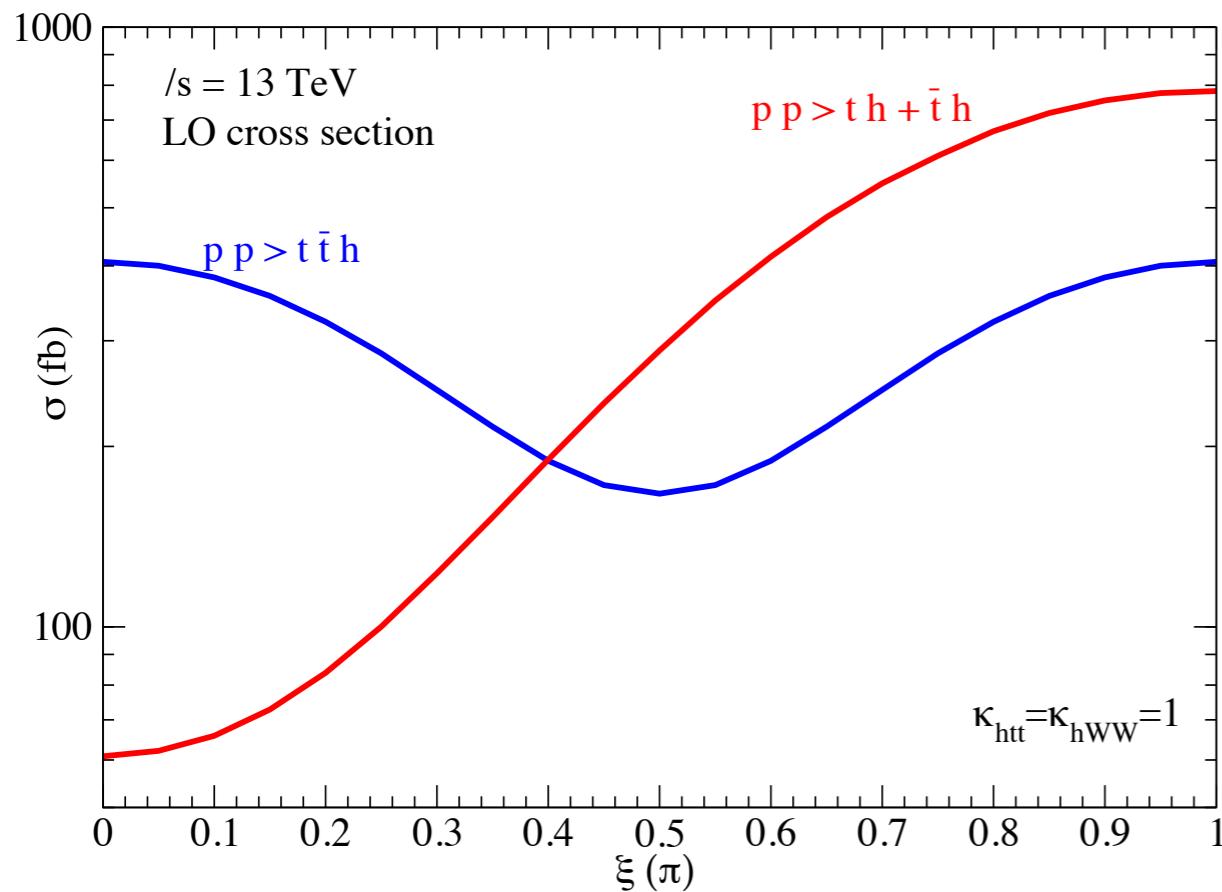
In collaboration with Vernon Barger (U. Wisconsin) and Kaoru Hagiwara (KEK),
PRD99(2019)031701 [arXiv:1807.00281] and work in progress.

Outline

- Top Higgs Yukawa couplings with CP violation
- Helicity amplitudes: $t+h$: $u_b > d_t h$ and $\bar{d}_b > \bar{u}_t h$
 $\bar{t}+h$: $d_{\bar{b}} > u_{\bar{t}} h$ and $\bar{u}_{\bar{b}} > \bar{d}_{\bar{t}} h$
- Single top + Higgs event distributions
- Azimuthal asymmetry A_ϕ of $t/\bar{t}+h$ +jet distributions
- Top quark polarisation P_2
- Summary

Top Yukawa coupling

$$\begin{aligned}
 \mathcal{L} &= -g_{htt} h \bar{t} (\cos \xi_{htt} + i \sin \xi_{htt} \gamma_5) t \\
 &= -g_{htt} h (t_R^\dagger, t_L^\dagger) \begin{pmatrix} e^{-i\xi_{htt}} & 0 \\ 0 & e^{i\xi_{htt}} \end{pmatrix} \begin{pmatrix} t_L \\ t_R \end{pmatrix} \\
 &= -g_{htt} h (e^{-i\xi_{htt}} t_R^\dagger t_L + e^{i\xi_{htt}} t_L^\dagger t_R) \\
 g_{htt} &= \frac{m_t}{v} \kappa_{htt}, \quad \kappa_{htt} > 0, \quad -\pi < \xi_{htt} \leq \pi
 \end{aligned}$$



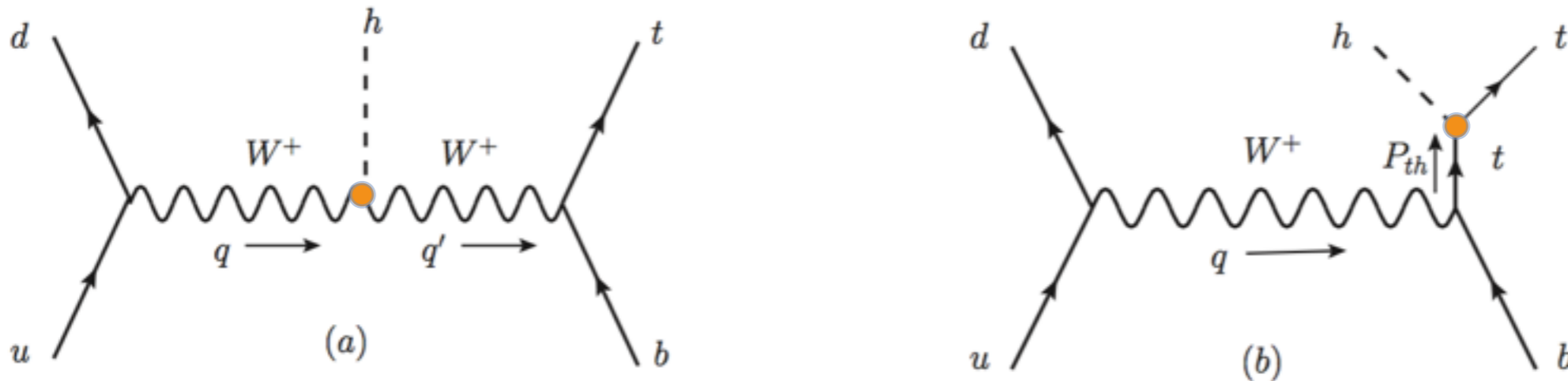
$pp \rightarrow th + \bar{t}h + \text{anything}$

$$\sigma_{tot}(|\xi_{htt}| = \pi) \sim \mathbf{13} \sigma_{tot}^{SM}(\xi_{htt} = 0)$$

change the sign of Yukawa coupling

In the SM, strong destructive interference between the htt and hWW amplitudes

ub > dth amplitudes



$$M_\sigma \sim \underbrace{u_L(p_d)^\dagger \sigma_- u_L(p_u)}_{\text{common to both diagrams}} \underbrace{\frac{-g_{\mu\nu} + q_\mu q_\nu / m_W^2}{q^2 - m_W^2}}_{\text{common to both diagrams}}$$

$$\left(\underbrace{u_R^\dagger(p_t, \sigma), u_L^\dagger(p_t, \sigma)}_{\bar{u}(p_t, \sigma)} \right) \left\{ \underbrace{g_{hWW}}_{\text{common to both diagrams}} \frac{-g_\rho^\nu + q'^\nu q'_\rho / m_W^2}{q'^2 - m_W^2} \right.$$

$$\left. + \frac{g_{htt} \delta_\rho^\nu}{P_{th}^2 - m_t^2} \begin{pmatrix} e^{-i\xi} & 0 \\ 0 & e^{i\xi} \end{pmatrix} \begin{pmatrix} m & P_{th} \cdot \sigma_+ \\ P_{th} \cdot \sigma_- & m \end{pmatrix} \right\} \begin{pmatrix} 0 & \sigma_+^\rho \\ \sigma_-^\rho & 0 \end{pmatrix} \begin{pmatrix} u_L(p_b) \\ 0 \end{pmatrix}$$

$$\cos \xi + i \sin \xi \gamma_5$$

$$P_{th} \cdot \gamma + m$$

$$\gamma^\rho$$

$$\frac{1 - \gamma_5}{2} u(p_b)$$

$$g_{hWW} = \frac{2m_W^2}{v} \kappa_{hWW} \quad (\kappa_{hWW} = 1)$$

The common W propagator

The W propagator $D_W(q)$ can be expressed as a summation over three polarisation states because of current conservation

$$-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2} \rightarrow -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}$$

$$= \sum_{\lambda=\pm 1,0} (-1)^{\lambda+1} \epsilon_\mu^*(q, \lambda) \epsilon_\nu(q, \lambda)$$

$$q_\mu u_L^\dagger(p_d) \sigma_-^\mu u_L(p_u) = 0$$

for $m_u = m_d = 0$

The factor is needed for space like vector boson. e.g. in the Breit frame:

$$q^\mu = (0, 0, 0, Q) \quad (q^2 = -Q^2 < 0)$$

$$-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} = \text{diag}(-1, 1, 1, 1) + \text{diag}(0, 0, 0, -1)$$

$$= \text{diag}(-1, 1, 1, 0)$$

$$\epsilon^\mu(q, \lambda = \pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

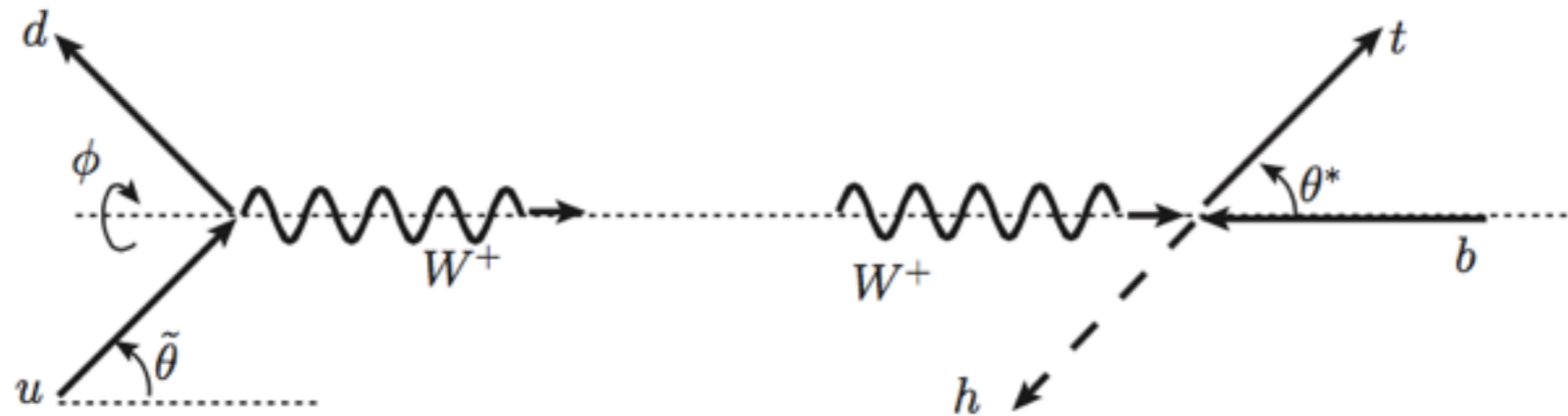
$$\epsilon^\mu(q, \lambda = 0) = (1, 0, 0, 0)$$

$$\sum_{\lambda=\pm 1,0} (-1)^{\lambda+1} \epsilon_\mu^*(q, \lambda) \epsilon_\nu(q, \lambda) = \text{diag}(-1, 1, 1, 0)$$

Amplitudes ($u \rightarrow d+W^+$ emission)

$$M_\sigma = \sum_{\lambda=\pm 1,0} \underbrace{j(u \rightarrow dW_\lambda^+)}_{\text{Breit frame}} \underbrace{\hat{M}(W_\lambda^+ b \rightarrow t_\sigma h)}_{W^+ b=th \text{ rest frame}}$$

W^+ momentum
 \vec{q} direction is along
the positive z -axis



Breit frame

$$p_u^\mu = \tilde{\omega}(1, \sin \tilde{\theta} \cos \phi, -\sin \tilde{\theta} \sin \phi, \cos \tilde{\theta}),$$

$$p_d^\mu = \tilde{\omega}(1, \sin \tilde{\theta} \cos \phi, -\sin \tilde{\theta} \sin \phi, -\cos \tilde{\theta}),$$

$$q^\mu = p_u^\mu - p_d^\mu = (0, 0, 0, 2\tilde{\omega} \cos \tilde{\theta}) = (0, 0, 0, Q)$$

$$\cos \tilde{\theta} = \frac{1}{2\hat{s}/(W^2 + Q^2) - 1}$$

$$\tilde{\omega} = Q/2 (2\hat{s}/(W^2 + Q^2) - 1)$$

$$\tilde{\omega} \cos \tilde{\theta} = Q/2$$

$$j_\lambda = (-1)^{(\lambda+1)} u_L^\dagger(p_d) \sigma_-^\mu u_L(p_u) \epsilon_\mu^*(q, \lambda) = \begin{cases} \pm \sqrt{2} \tilde{\omega} (1 \pm \cos \tilde{\theta}) e^{\pm i\phi}, & \text{if } \lambda = \pm 1 \\ -2\tilde{\omega} \sin \tilde{\theta}, & \text{if } \lambda = 0 \end{cases}$$

Amplitudes (full process $u b \rightarrow d t h$)

$$M_\sigma = \sum_{\lambda=\pm 1,0} j(u \rightarrow dW_\lambda^+) \hat{M}(W_\lambda^+ b \rightarrow t_\sigma h)$$

$$\begin{aligned}
 M_+ &= \frac{1 - \tilde{c}}{2} e^{i\phi} \sin \frac{\theta^*}{2} A \frac{1 + \cos \theta^*}{2} \\
 &+ \frac{1 + \tilde{c}}{2} e^{-i\phi} \sin \frac{\theta^*}{2} \left[A \left(\frac{1 + \cos \theta^*}{2} + \epsilon_1 \right) - B (e^{-i\xi} + \delta\delta' e^{i\xi}) \right] \\
 &+ \frac{\tilde{s}}{2} \cos \frac{\theta^*}{2} \frac{W}{Q} \left[A \left(\frac{q^* E_h^* + q^{0*} p^* \cos \theta^*}{W p^*} + \epsilon_1 \right) - B (e^{-i\xi} + \delta\delta' e^{i\xi}) \right] \\
 M_- &= -\frac{1 - \tilde{c}}{2} e^{i\phi} \cos \frac{\theta^*}{2} A \delta \frac{1 - \cos \theta^*}{2} \\
 &- \frac{1 + \tilde{c}}{2} e^{-i\phi} \cos \frac{\theta^*}{2} \left[A \left(\delta \frac{1 - \cos \theta^*}{2} - \epsilon_2 \right) + B (\delta e^{-i\xi} + \delta' e^{i\xi}) \right] \\
 &- \frac{\tilde{s}}{2} \sin \frac{\theta^*}{2} \frac{W}{Q} \left[A \left(\delta \frac{q^* E_h^* + q^{0*} p^* \cos \theta^*}{W p^*} + \epsilon_2 \right) - B (\delta e^{-i\xi} + \delta' e^{i\xi}) \right]
 \end{aligned}$$

← $\lambda=+1$
 $J_z=3/2$

← $\lambda=-1$
 $J_z=-1/2$

← $\lambda=0$
 $J_z=1/2$

← $\lambda=+1$
 $J_z=3/2$

← $\lambda=-1$
 $J_z=-1/2$

← $\lambda=0$
 $J_z=1/2$

$$A = 2g^2 \underline{D_W}(q) \tilde{\omega} \sqrt{2q^*(E^* + p^*)} \frac{mp^*}{m_W^2} \underline{g_{hWW}} \underline{D_W}(q'), > 0$$

$$B = -2g^2 \underline{D_W}(q) \tilde{\omega} \sqrt{2q^*(E^* + p^*)} W \underline{g_{htt}} \underline{D_t}(P_{th}), > 0$$

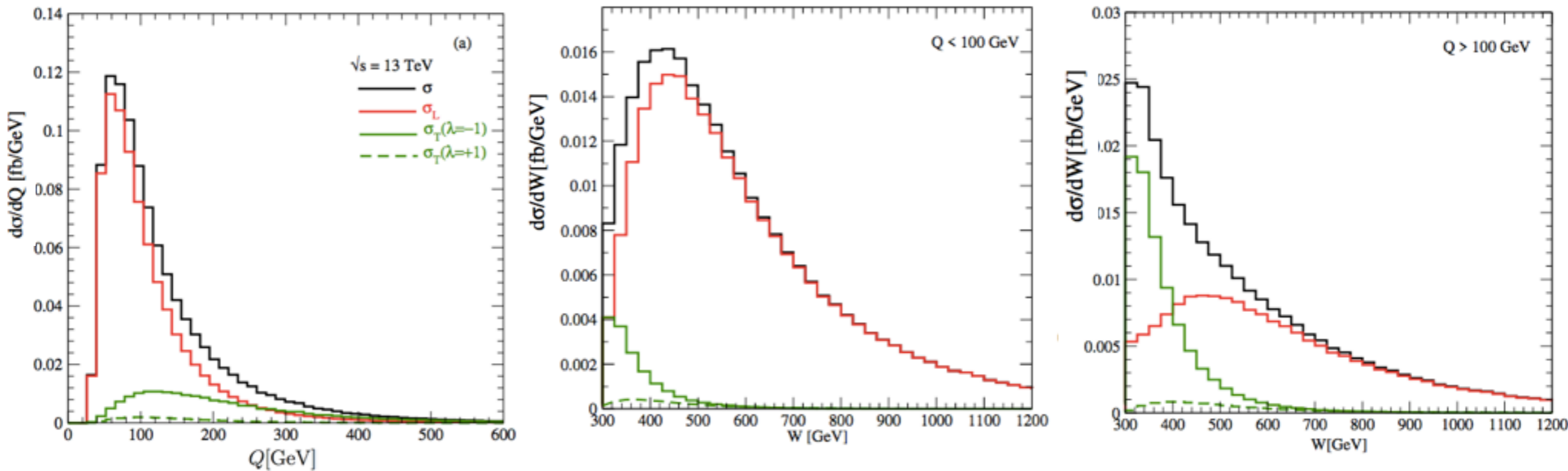
$$\begin{aligned}
 \delta &= m_t / (E^* + p^*) \\
 \delta' &= m_t / W
 \end{aligned}$$

→ $\delta \sim \delta'$
at high energy
high W ($W=m_{th}$)

Q and W distribution

$Q = \sqrt{-q^2}$ invariant momentum transfer of the virtual W^+

$W = \sqrt{P_{th}^2} = m(th)$ the invariant mass of the th system



W_L is dominant in low Q ($Q < 100$ GeV) and large W ($W > 400$ GeV)

W_T is significant in large Q ($Q > 100$ GeV) and small W ($W < 400$ GeV)

Azimuthal angle distribution

$$\frac{d\sigma}{dW d\phi} \sim |M_+|^2 + |M_-|^2$$

For instance, at high W

$$M_+ \sim \frac{1 + \tilde{c}}{2} e^{-i\phi} \sin \frac{\theta^*}{2} \left[A \frac{1 + \cos \theta^*}{2} - \underline{B e^{-i\xi}} \right] \quad \lambda = -1$$

$$+ \frac{\tilde{s}}{2} \cos \frac{\theta^*}{2} \left(\frac{W}{Q} \right) \left[A \frac{1 + \cos \theta^*}{2} - \underline{B e^{-i\xi}} \right] \quad \lambda = 0$$

$|M_+|^2$ contains terms proportional to $\sin \phi \sin \xi$

in the **interference** between $\lambda = -1$ and $\lambda = 0$ terms. $\longrightarrow A_{\phi}$

Asymmetry is large at small W & large Q (W_T is comparable to W_L)
small at large W & small Q (W_L dominates over W_T)

$|M_-|^2$ contains terms proportional to $2 \cos \xi \longrightarrow$ not sensitive to CPV

Azimuthal angle distribution

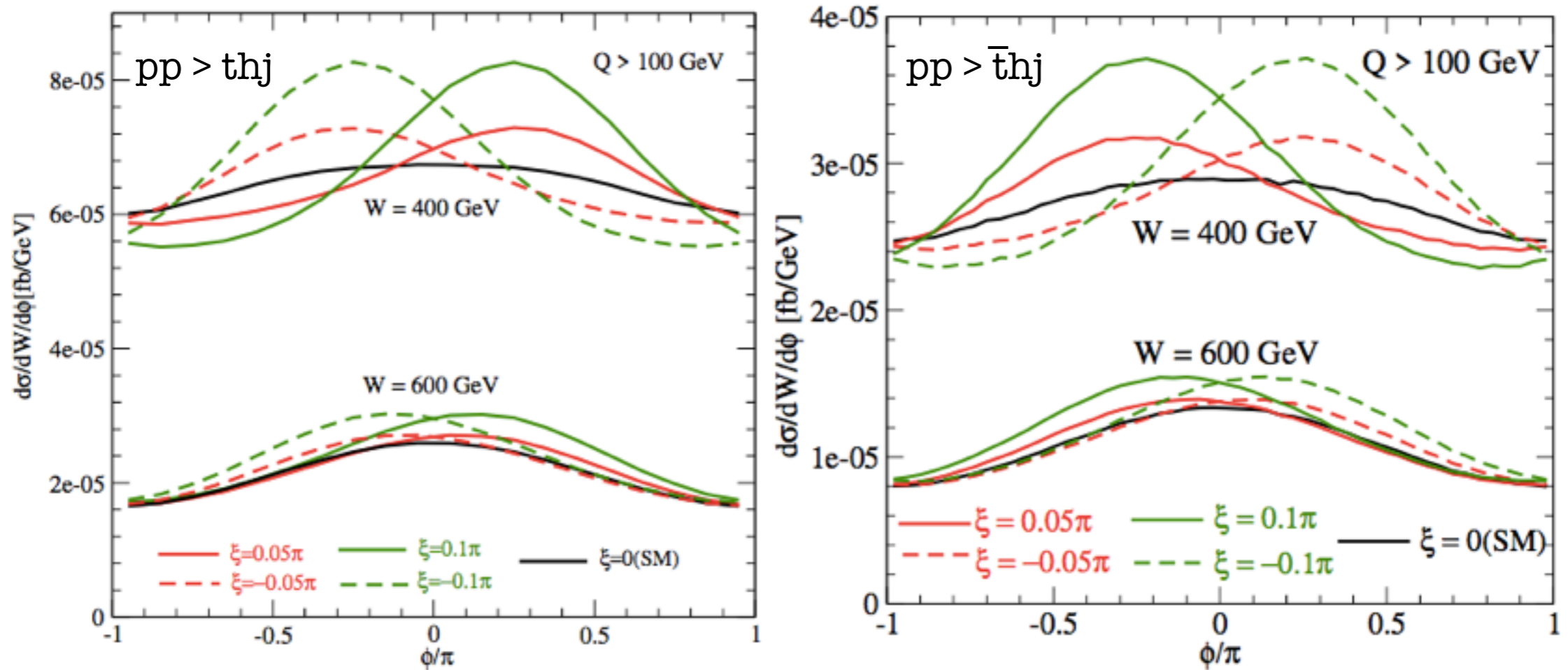


FIG. 8: Left panel: t . Right panel: \bar{t} . $d\sigma/dW/d\phi$ v.s. ϕ at $W = 400$ and 600 GeV for $Q > 100$ GeV. Black, red and green curves are for the SM ($\xi = 0$), $\xi = \pm 0.1\pi$, and $\pm 0.2\pi$. The solid curves are for $\xi \geq 0$, while the dashed curves are for $\xi < 0$.

asymmetry
$$A_\phi(W) = \frac{\int_0^\pi d\sigma/dW/d\phi - \int_{-\pi}^0 d\sigma/dW/d\phi}{\int_0^\pi d\sigma/dW/d\phi + \int_{-\pi}^0 d\sigma/dW/d\phi} \quad \begin{array}{l} > 0 \text{ (} th \text{) and } < 0 \text{ (} \bar{t}h \text{)} \text{ for } \xi > 0 \\ < 0 \text{ (} th \text{) and } > 0 \text{ (} \bar{t}h \text{)} \text{ for } \xi < 0 \end{array}$$

Asymmetry is large at small W & large Q (W_T is comparable to W_L)
 small at large W & small Q (W_L dominates over W_T)

Azimuthal asymmetry A_ϕ

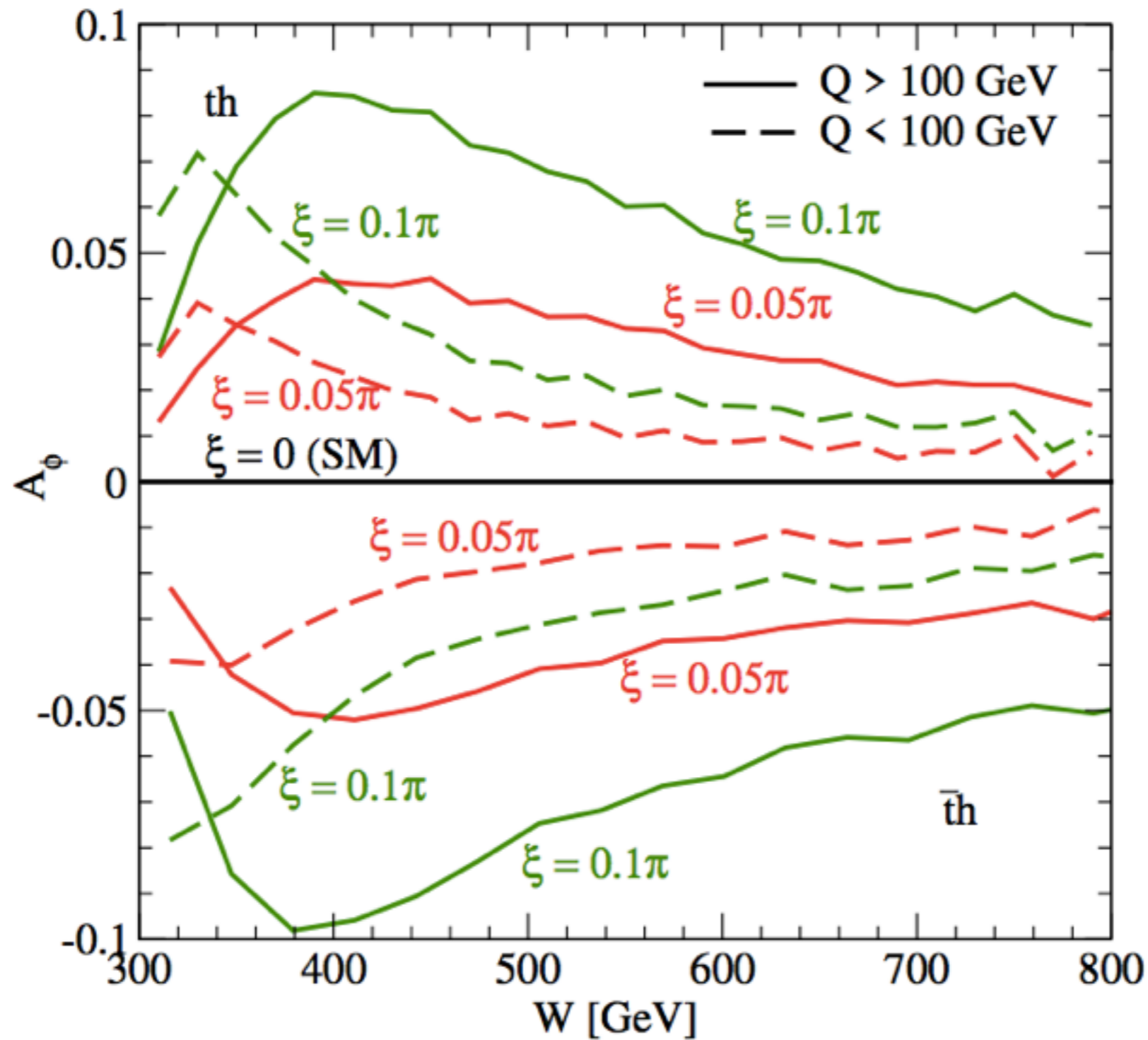


FIG. 11: Asymmetry $A_\phi(W)$ for $pp \rightarrow thj$ and $pp \rightarrow \bar{t}hj$

Top Polarization (pure state)

Because only the left-handed u,d,b quarks contribute to the process $u b \rightarrow d t h$, the helicity amplitudes M_+ and M_- are determined uniquely as complex numbers for the top quark of definite momentum in the $t+h$ rest frame. The produced top quark polarisation state is then expressed as the superposition

$$|t \rangle = \frac{M_+ |J_z = +\frac{1}{2} \rangle + M_- |J_z = -\frac{1}{2} \rangle}{\sqrt{|M_+|^2 + |M_-|^2}}$$

The polarisation direction is determined by M_+ and M_-

$$\begin{aligned} |(\theta, \phi) \rangle &= R_z(\phi) R_y(\theta) |J_z = +\frac{1}{2} \rangle \\ &= \begin{pmatrix} e^{-i\frac{\phi}{2}} & 0 \\ 0 & e^{i\frac{\phi}{2}} \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= e^{-i\frac{\phi}{2}} \cos \frac{\theta}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{i\frac{\phi}{2}} \sin \frac{\theta}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{M_+}{\sqrt{|M_+|^2 + |M_-|^2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{M_-}{\sqrt{|M_+|^2 + |M_-|^2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

The top spin orientation (Θ, φ) in the top rest frame is determined by the top momentum in the Wb to th scattering plane where the helicity amplitudes are obtained. The $\sin\varphi$ component (polarization perpendicular to the scattering plane) appears only when M_+ and M_- have relative complex phase (\rightarrow CPV)

Top Polarization (pure state)

The top quark polarisation can be expressed by a 3-vector $\mathbf{P} = (P_1, P_2, P_3)$ by using the density matrix formalism.

For the pure state, the density matrix reads

$$\begin{aligned} \rho &= \frac{1}{|M_+|^2 + |M_-|^2} \begin{pmatrix} |M_+|^2 & M_+ M_-^* \\ M_- M_+^* & |M_-|^2 \end{pmatrix} = \begin{pmatrix} \cos^2 \frac{\theta}{2} & \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & \sin^2 \frac{\theta}{2} \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 + \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & 1 - \cos \theta \end{pmatrix} \\ &= \frac{1}{2} \left\{ \underbrace{1 + \sin \theta \cos \phi}_{P_1} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\sigma^1} + \underbrace{\sin \theta \sin \phi}_{P_2} \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\sigma^2} + \underbrace{\cos \theta}_{P_3} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\sigma^3} \right\} \end{aligned}$$

$$\vec{P} = (P_1, P_2, P_3) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

Top Polarization (mixed state)

For general mixed state, we introduce differential cross section matrix

$$d\sigma_{\lambda\lambda'} = \int dx_1 \int dx_2 D_{u/p}(x_1) D_{b/p}(x_2) \frac{1}{2\hat{s}} \overline{\sum} M_{\lambda} M_{\lambda'}^* d\Phi_{dth}$$

where the phase space integration can be restricted. For an arbitrary kinematical distributions, $d\sigma = d\sigma_{++} + d\sigma_{--}$, the polarisation density matrix is defined as

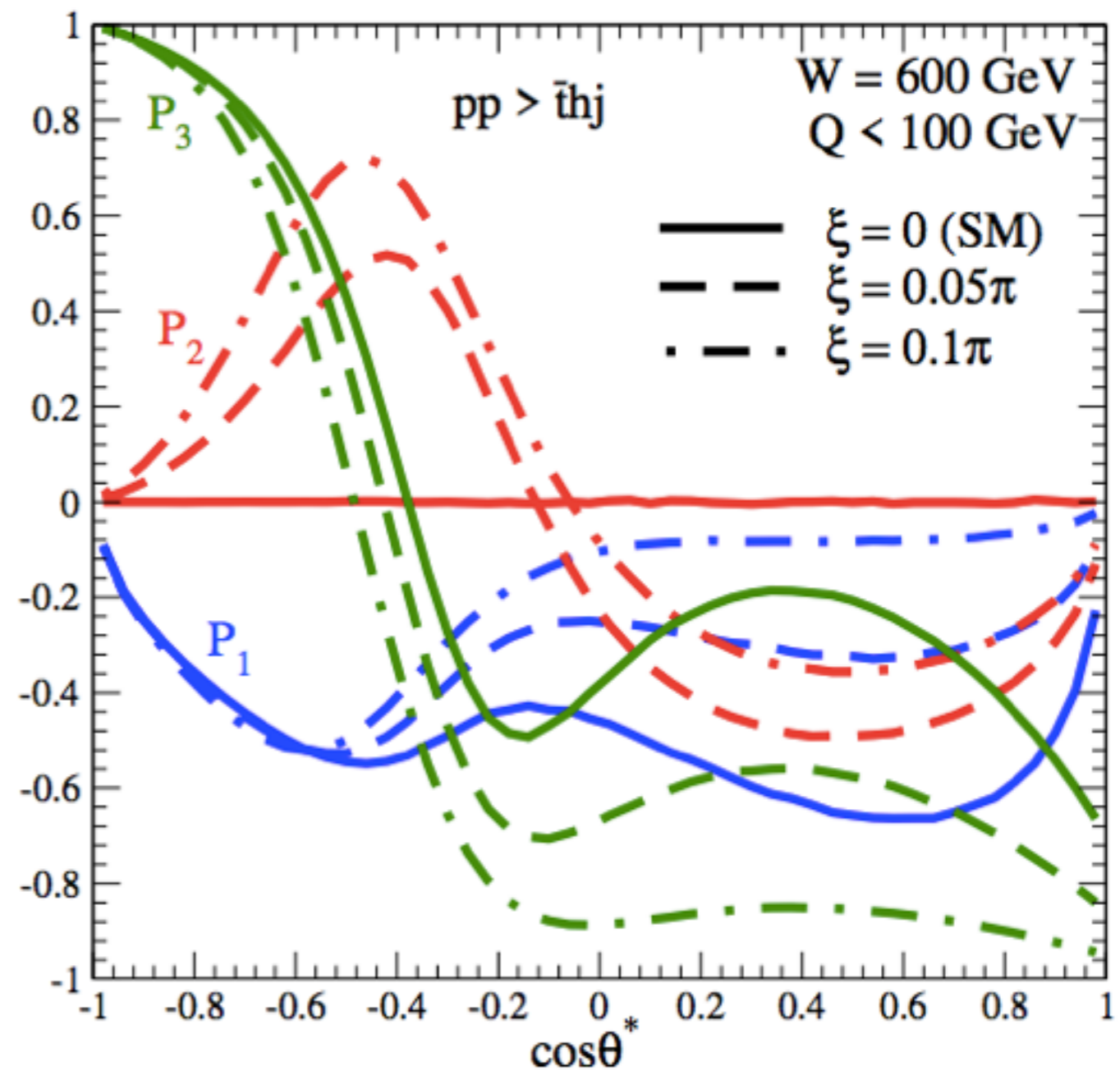
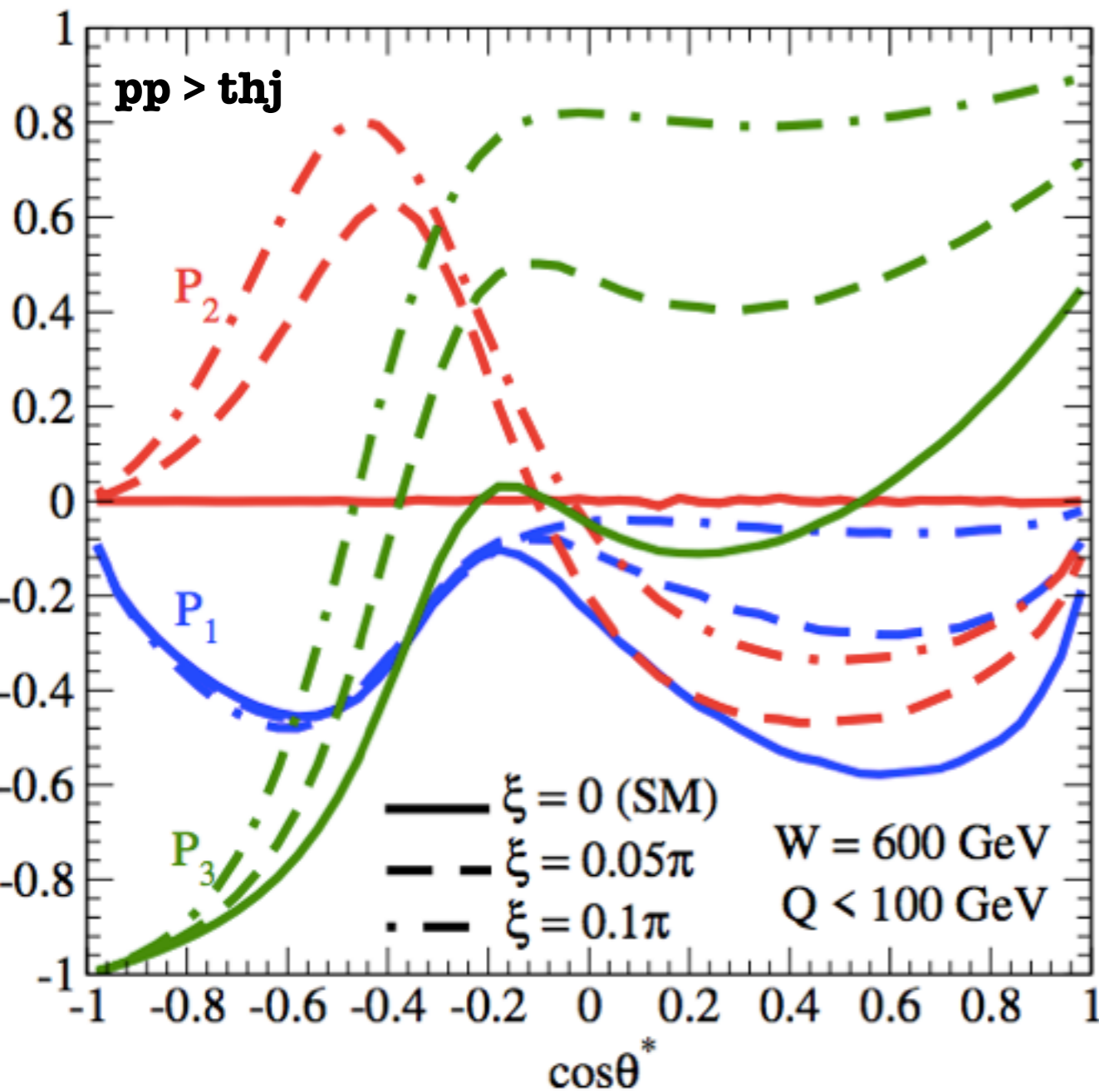
$$\rho_{\lambda\lambda'} = \frac{d\sigma_{\lambda\lambda'}}{d\sigma_{++} + d\sigma_{--}} = \frac{1}{2} \left[\delta_{\lambda\lambda'} + \sum_{k=1}^3 P_k \sigma_{\lambda\lambda'}^k \right]$$

The 3-vector $\mathbf{P} = (P_1, P_2, P_3)$ gives the general polarisation of the top quark. The magnitude $P = |\mathbf{P}|$ gives the degree of polarisation ($P=1$ for 100% polarization, $P=0$ for no polarisation). The orientation gives the direction of the top quark spin in the top rest frame.

$$P_2 = -2\text{Im}(M_+ M_-^*) / (|M_+|^2 + |M_-|^2)$$

We find \mathbf{P} lies in the $W+b \rightarrow th$ scattering plane in the SM ($\xi_i=0$). Polarisation orthogonal to the production plane P_2 appears for nonzero ξ_i . The sign of P_2 determines the sign of ξ_i .

Top Polarization and anti-Top Polarisation $\mathbf{P} = (P_1, P_2, P_3)$



We find large positive P_2 when $\cos\theta^* < 0$, both for t and \bar{t} . We therefore examine P_2 for events with $\cos\theta^* < 0$ in the next slides.

Polarization P_2 of t and $tbar$

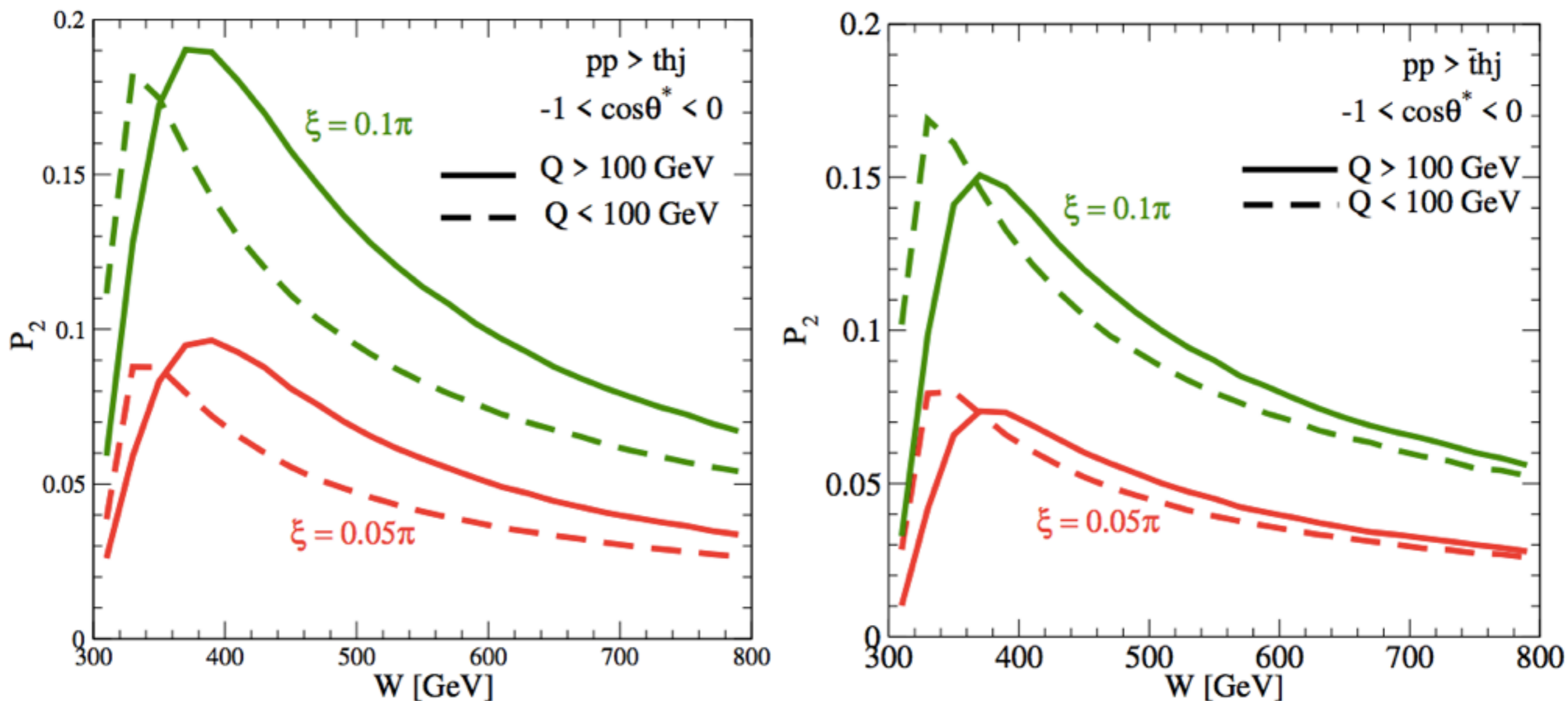


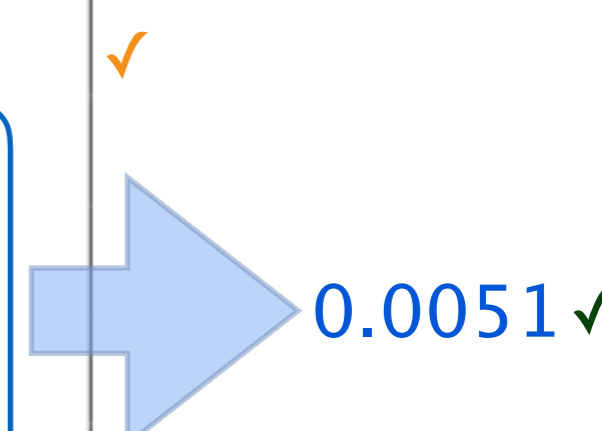
FIG. 15: P_2 v.s. W for $pp \rightarrow thj$ and $pp \rightarrow \bar{t}hj$ in the region $-1 < \cos\theta^* < 0$

Expected number of events @ HL-LHC

	\sqrt{s} 14 TeV	Number of events @ $3ab^{-1}$	Decay channel	Branching Ratio	Number of events	
$\sigma(th)+\sigma(\bar{t}h)$	90 fb	270,000	$(bl\nu)(b\bar{b})$	0.13	34,000	✓✓
			$(bl\nu)(\gamma\gamma, \ell\ell jj, \mu\mu, 4\ell)$	0.0011	300	✓✓
$\sigma(t\bar{t}h)$	613 fb	1,840,000	$(bl\nu)(bjj)(b\bar{b})$	0.17	310,000	✓✓✓
			$(bl\nu)^2(b\bar{b})$	0.028	52,000	✓✓✓
			$(bl\nu)(bjj)(\gamma\gamma, \ell\ell jj, \mu\mu, 4\ell)$	0.0015	2,800	✓✓✓
			$(bl\nu)^2(\gamma\gamma, \ell\ell jj, \mu\mu, 4\ell)$	0.00025	460	✓✓✓

- $t > bl\nu$ mode for CP sensitivity (t vs. \bar{t})
- h decay should not have neutrinos to determine $t(\bar{t})$ frame.

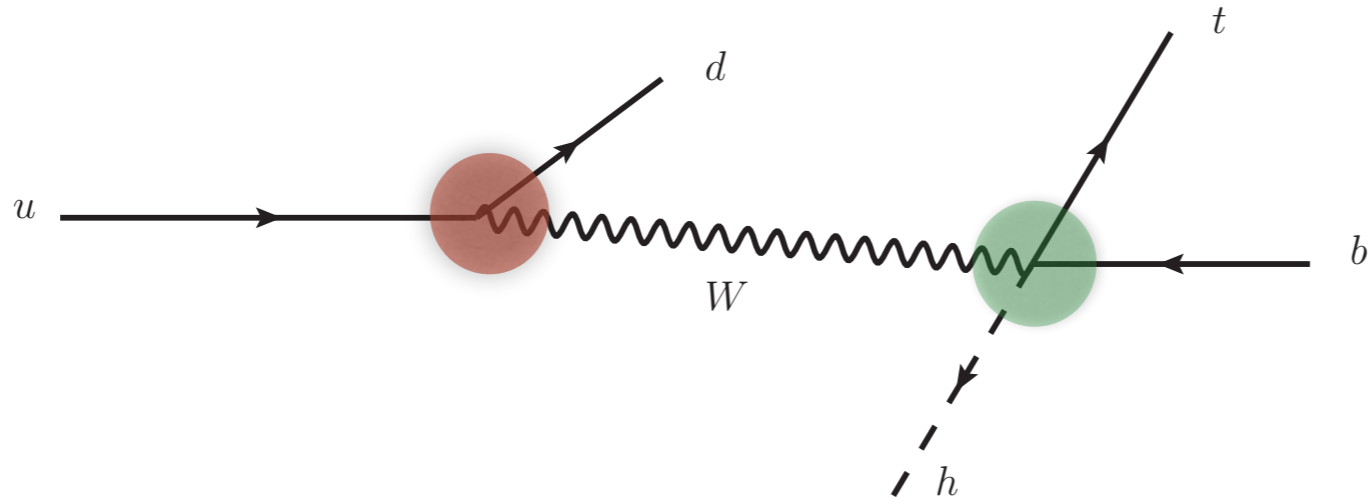
	Decay channel	Branching ratio		Decay channel	Branching Ratio
$t \rightarrow$	bjj	0.67	$h \rightarrow$	$b\bar{b}$	0.58
	$bl\nu(\ell = e, \mu)$ ✓	0.22		$\ell\bar{\ell}jj$	0.0025
	$b\tau\nu$ ✓	0.11		$\gamma\gamma$	0.0023
		$\mu\bar{\mu}$		0.00022	
				4ℓ	0.00012



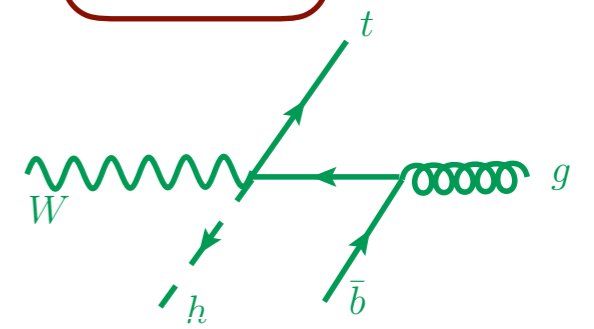
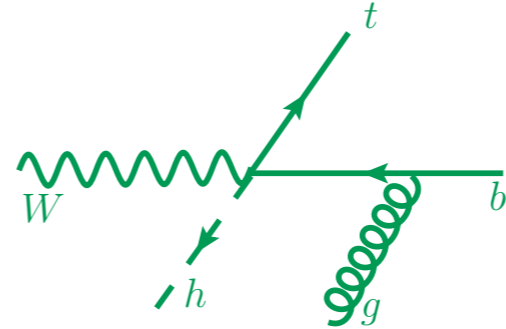
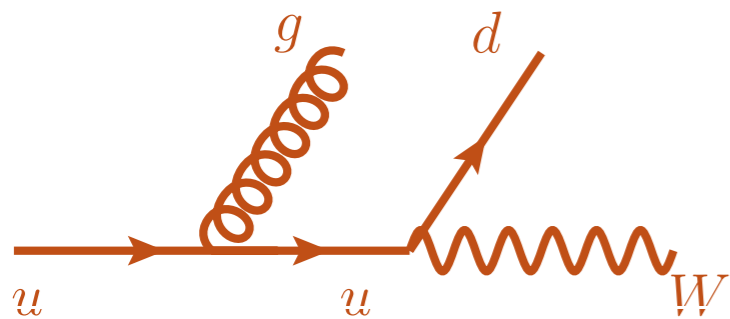
0.0051 ✓

- For a few percent asymmetry measurement, $h > b\bar{b}$ is necessary

Radiative corrections

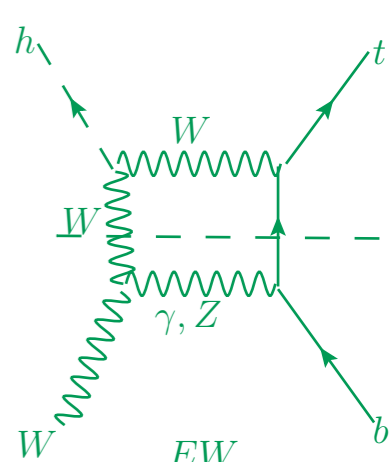
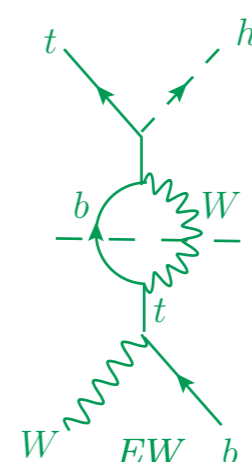
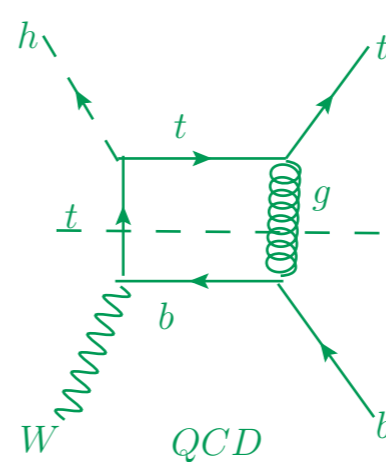
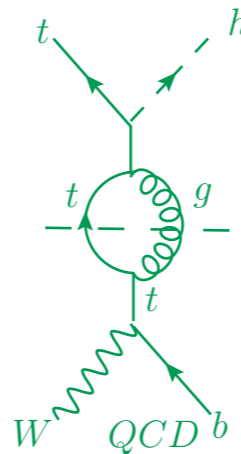


Color singlet (W) exchange factorizes QCD corrections into $q > q'W$ emission and $Wb > th$ production parts @NLO.



- $b > bg$ makes $W(m_{th})$ softer.
- $g > bb$ correction depends on b PDF.
- CP conserving T-odd asymmetry @1-loop.

- NLO corrections are the same as DIS and VBF process.
- g -jet miss-tag washes out A_ϕ and $P_{1,3}^A$ but P_2 is not affected.



Summary

- Single top+Higgs production is an ideal probe of the top Yukawa coupling because the htt and hWW amplitudes interfere strongly.

- Azimuthal asymmetry between the $u \rightarrow dW^+$ emission and the $W^+b \rightarrow th$ production planes probes the sign of CP violating phase.

$$A_\phi \sim \int_0^\pi (|M_+|^2 + |M_-|^2) d\phi - \int_{-\pi}^0 (|M_+|^2 + |M_-|^2) d\phi \propto \sin \xi_{htt}$$

- Polarization can be measured by using the density matrix.

$$\rho_{\lambda\lambda'} = \frac{1}{\int (|M_+|^2 + |M_-|^2) d\Phi} \int \begin{pmatrix} |M_+|^2 & M_+M_-^* \\ M_-M_+^* & |M_-|^2 \end{pmatrix} d\Phi = \frac{1}{2} \left[\delta_{\lambda\lambda'} + \sum_{k=1}^3 P_k \sigma_{\lambda\lambda'}^k \right]$$

- Polarization perpendicular to the scattering plane measures the relative phase between the two helicity amplitudes

$$P_2 = \frac{-2\text{Im}(M_+M_-^*)}{|M_+|^2 + |M_-|^2} \propto \sin \xi_{htt}$$

- We find significant asymmetry reaching $A_\phi \sim +8\%$ (th), -10% (\bar{th}), whereas $P_2 \sim +18\%$ (th), $+15\%$ (\bar{th}) for $\xi=0.1\pi$. All the asymmetries change sign if ξ is negative.