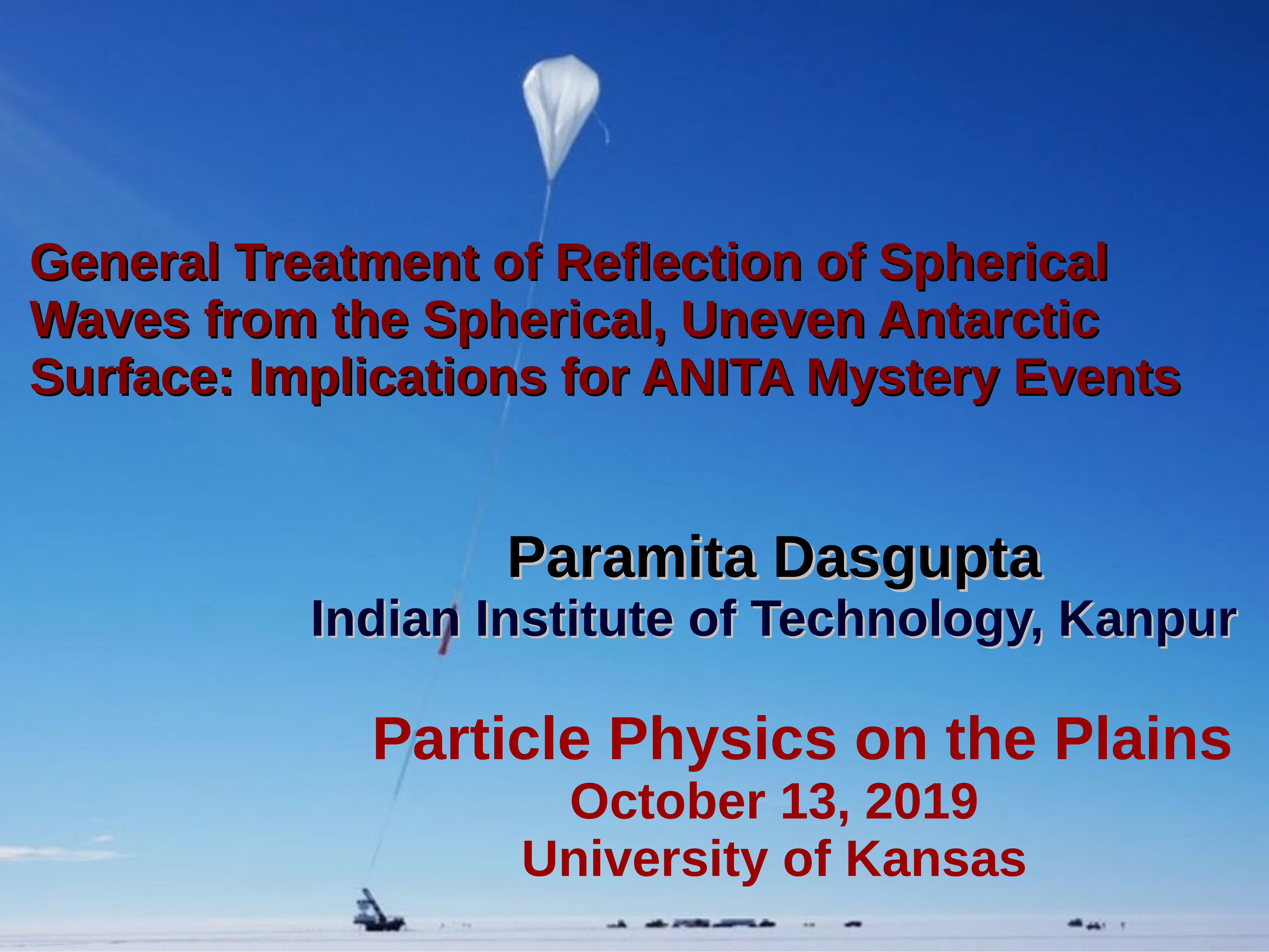


General Treatment of Reflection of Spherical Waves from the Spherical, Uneven Antarctic Surface: Implications for ANITA Mystery Events

Paramita Dasgupta
Indian Institute of Technology, Kanpur

Particle Physics on the Plains
October 13, 2019
University of Kansas

Plan of Talk

- NASA sponsored Balloon-Borne ANITA Experiment at Antarctica
- High Altitude Calibration (HiCal) Experiment
- Working Principle of ANITA and HiCal
- Our work with ANITA-HiCal research group at KU (with Prof. David Besson and Dr. Steven Prohira) ⟶ Rigorous Formalism to Study the Reflection of Spherical EM Waves from a Spherical Surface
- Comparison between our Simulated Result and ANITA-HiCal Data
- Possible explanation for Mystery Events Detected by ANITA
- Summary

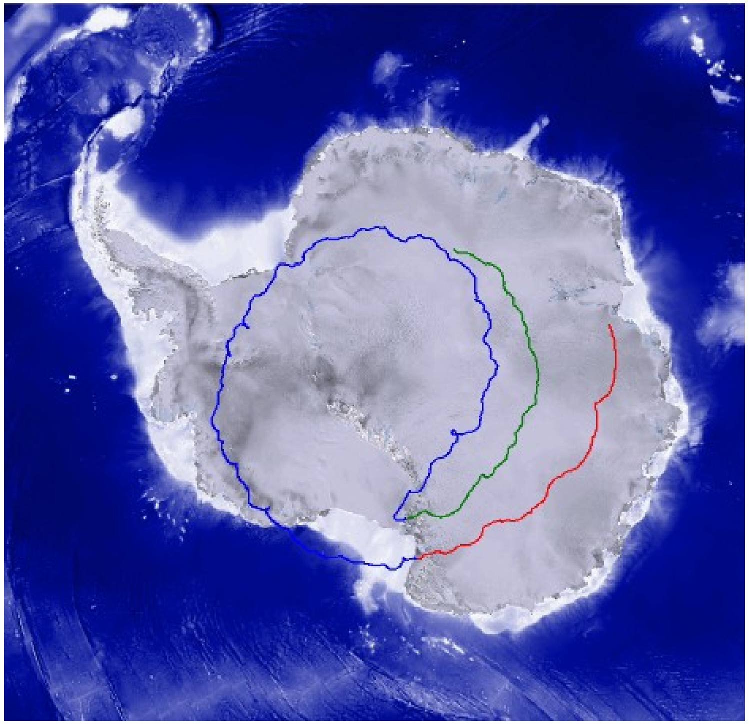
Why Antarctica ?

- Low flux and small cross section requires vast detection area

- Volume ~ 1 Million Km^3 Coverage Radius ~ 700 Km, , Lots of ice !! 1-4 km depth Few People, less noise Ballon Fly at ~ 37 Km above Surface

- Long radio attenuation length (~1 km) in ice
- Negative charge excess of particle shower in ice

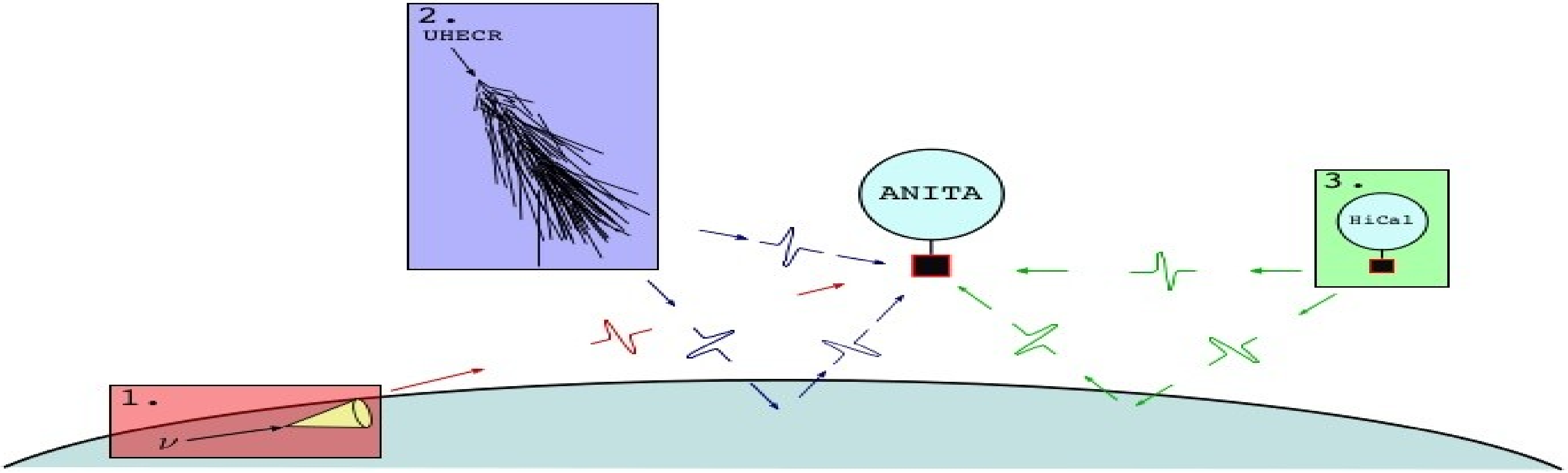
→ Coherent Cherenkov Radiation



Journal of Astronomical Instrumentation VOL. 06, NO. 02 , P. Gorham, D.Z. Besson et. al.

ANITA and HiCal Working Principle

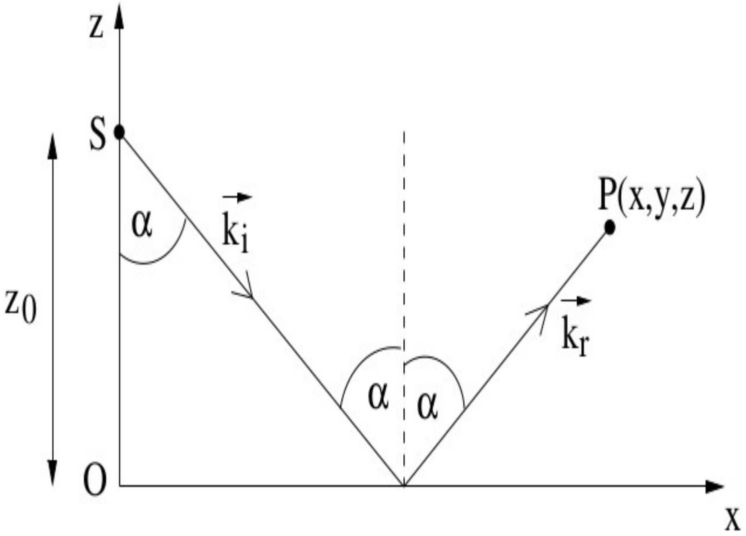
- **Antarctic Impulsive Transient Antenna is a balloon-borne RF Receiver Array.**
- **High Altitude Calibration (HiCal) is a Balloon-Borne RF Transmitter, in concert with the ANITA RF receiver array.**
- **ANITA-HiCal Measures Antarctic Surface Reflectivity in RF regime.**
- **Main Goal of ANITA is to detect UHE neutrinos via Askaryan effect in ice**
- **ANITA also detects Highest Energy Particles via the Radio signals produced by the UHECR interaction with Earth’s atmosphere.**
- **Down Coming Charged Particles produce Mainly H-Pol radiation.**



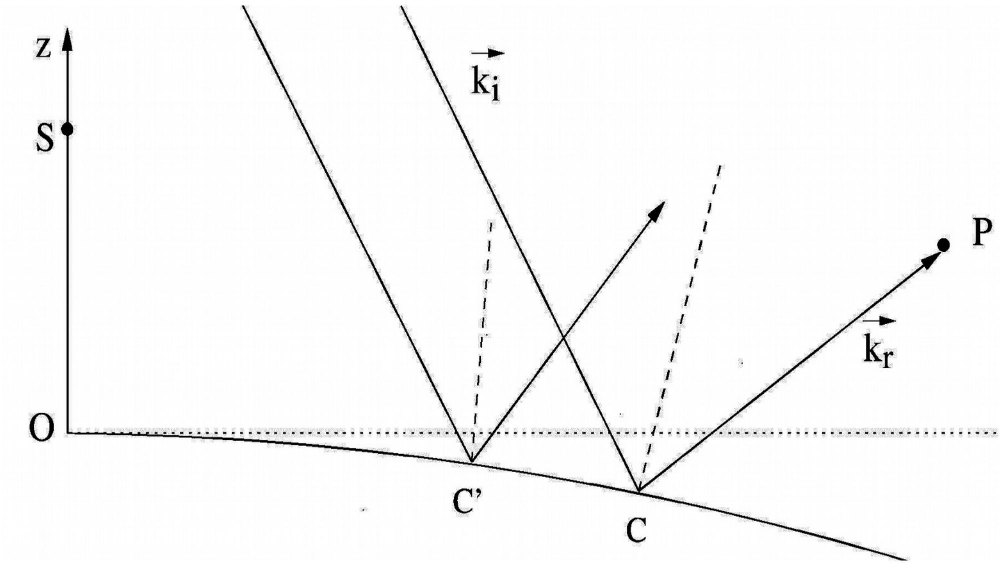
P. W. Gorham et al. arXiv:1710.11175

A Rigorous Framework for ANITA-HiCal

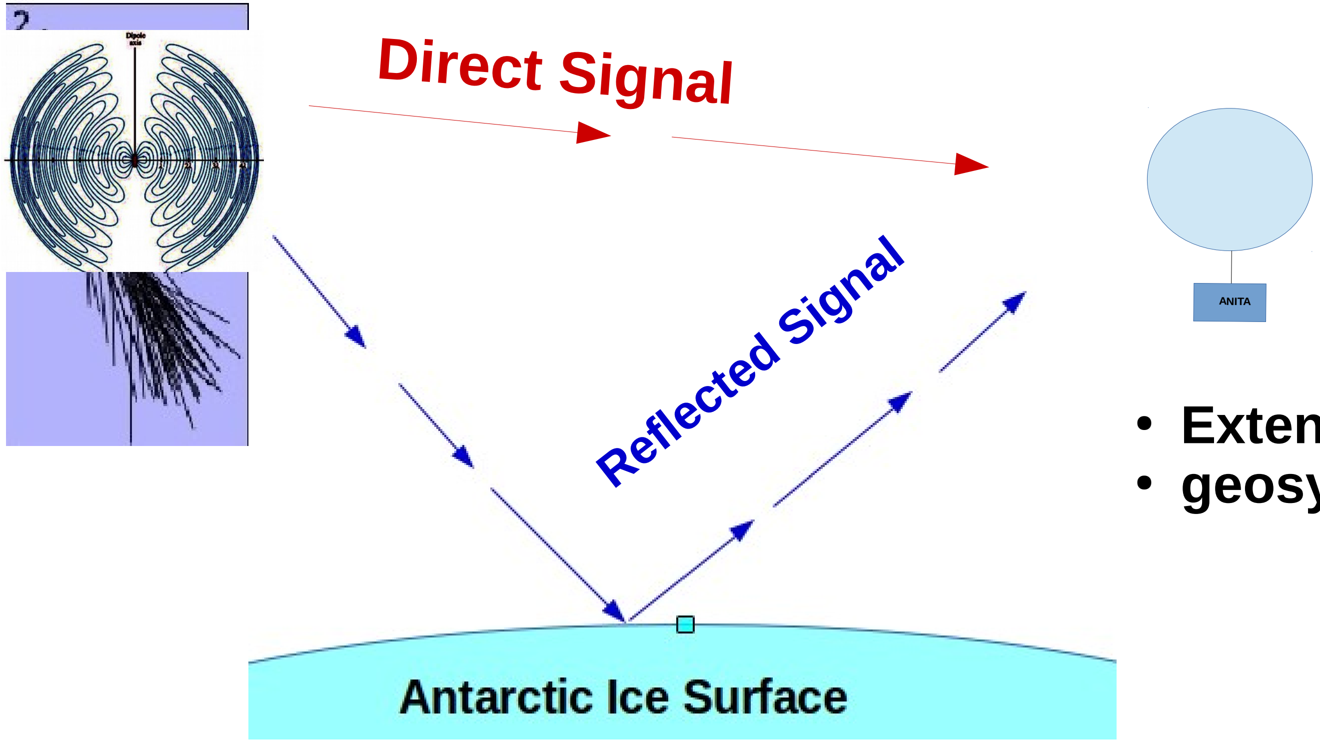
Reflection off a Flat Surface



Reflection off a Spherical Surface

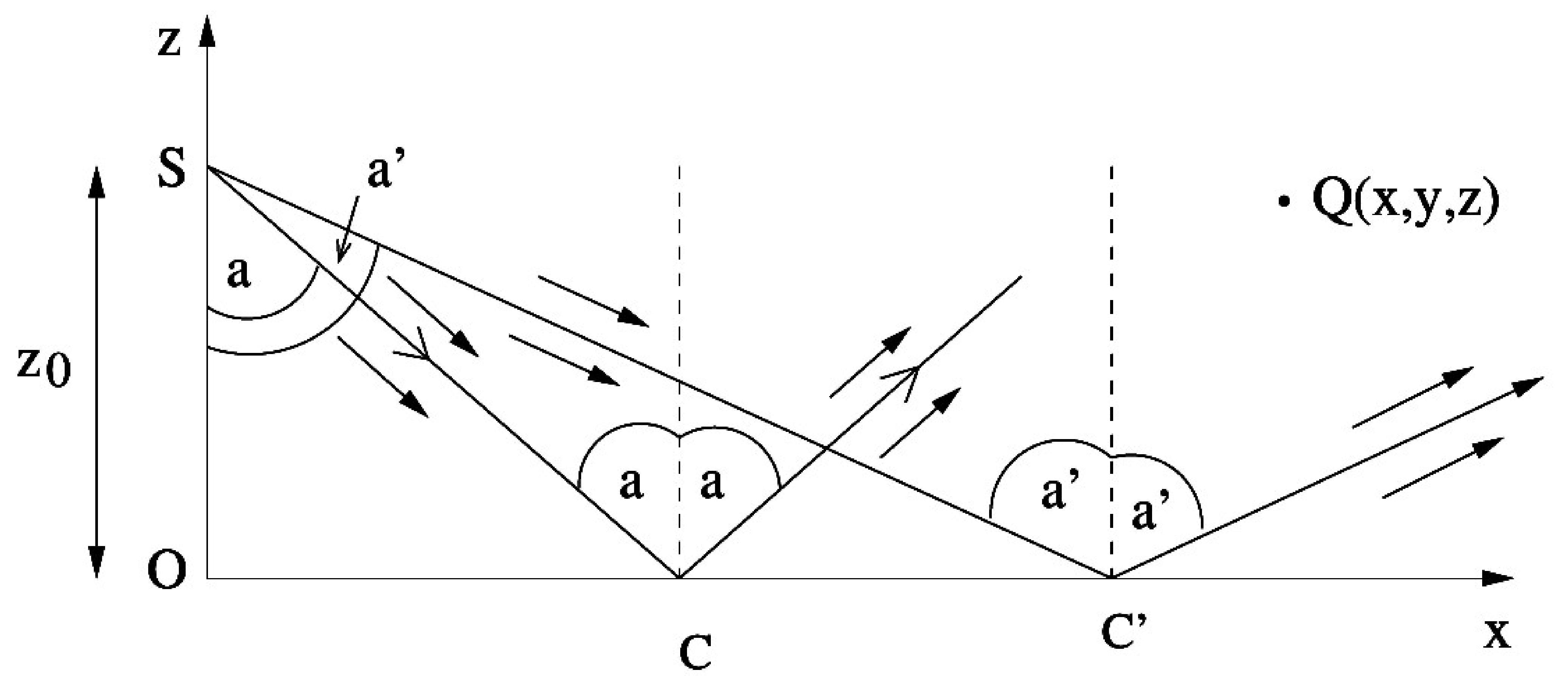


- A dipole radiator on the z axis

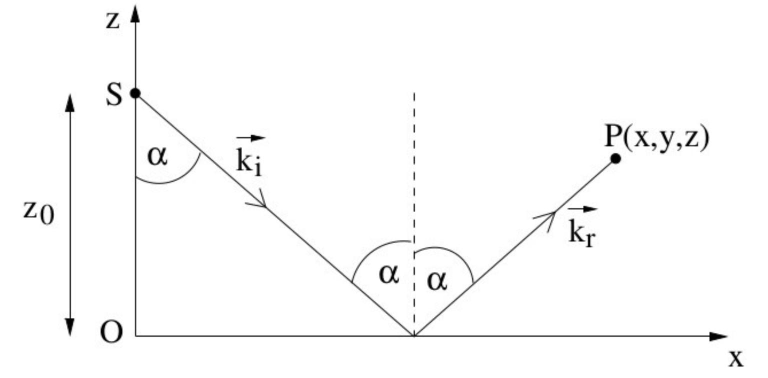


- Extensive Air Shower (EAS)
- geosynchroton radiation

Reflection of plane waves off a flat surface



Weyl Formalism : Decomposition of Spherical Waves into Plane Waves



Start with a Hertz dipole at (0,0,z_0)

Hertz Potential (In Far Field r>> λ)

$$\Pi_y(x,y,z) = \frac{e^{ikR}}{4\pi\epsilon R} + F_1(x,y,z)$$

Primary field **Due to reflection**

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{ik[x\sin\alpha\cos\beta+y\sin\alpha\sin\beta+(z_0-z)\cos\alpha]}\sin\alpha d\alpha d\beta$$

$$\vec{k}_I = k(\sin\alpha\cos\beta\hat{x}+\sin\alpha\sin\beta\hat{y}-\cos\alpha\hat{z})$$

$$\begin{aligned}\vec{E} &= \vec{\nabla}(\vec{\nabla}\cdot\vec{\Pi})+k^2\vec{\Pi}\\ \vec{H} &= \frac{k^2}{i\omega\mu}(\vec{\nabla}\times\vec{\Pi})\end{aligned}$$

$$\begin{aligned}\vec{E}_{inc} &= \frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}\left[-\sin^2\alpha\cos\beta\sin\beta\hat{x}+(1-\sin^2\alpha\sin^2\beta)\hat{y}+(\sin\alpha\sin\beta\cos\alpha)\hat{z}\right]\\ \vec{H}_{inc} &= \frac{ik^2\omega}{8\pi^2}\tilde{\Pi}\left[\cos\alpha\hat{x}+(\cos\beta\sin\alpha)\hat{z}\right] .\end{aligned}$$

* Weyl Formalism (H. Weyl, Ann. Physik, 60,481,1919)

Reflection and Transmission at a Flat Surface

- **Reflected Field components** E^s_{ref} , E^p_{ref} , H^s_{ref} , H^p_{ref}
- Transmitted Field Components E^s_{tran} , E^p_{tran} , H^s_{tran} , H^p_{tran}
- Impose boundary conditions at z=0

$$E_{(ref),y} = \frac{ik^3}{8\epsilon\pi^2}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty}\tilde{\Pi}_{ref}(f_r^s\cos^2\beta - f_r^p\cos^2\alpha\sin^2\beta)\sin\alpha d\alpha d\beta$$

This is Electric fieid perp. to Plane of incidence (H- Pol)

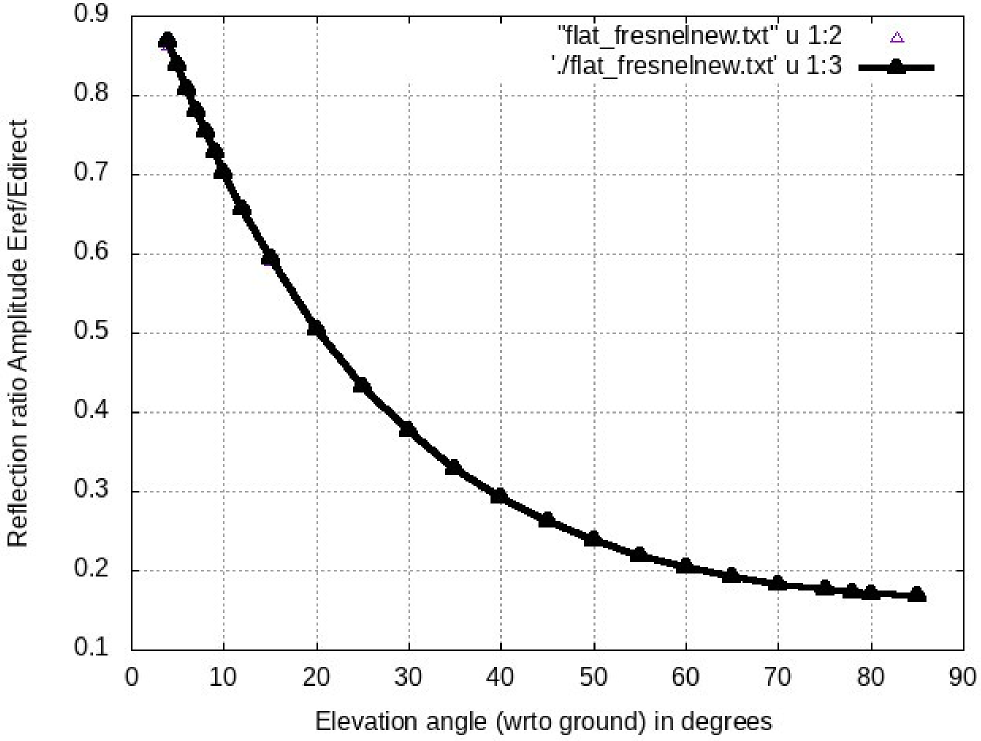
Numerical Solution

Compared with Fresnel refl. coefficients

H-Pol (reflected/direct) amplitude Ratio: **Flat Surface** calculation using our theoretical framework

```
#angle(°)      #amp(flat)       #fresnel
4              0.862801         0.867387
5              0.835699         0.837217
6              0.806814         0.808184
7              0.779242         0.780260
8              0.753162         0.753415
9              0.726390         0.727618
10             0.702397         0.702839
12             0.654847         0.656209
15             0.590581         0.593123
20             0.502614         0.504245
25             0.432435         0.432603
30             0.374581         0.375000
35             0.327991         0.328724
40             0.290877         0.291543
45             0.261001         0.261666
50             0.237114         0.237675
55             0.216842         0.218464
60             0.202191         0.203177
65             0.190783         0.191156
70             0.180672         0.181906
75             0.174257         0.175055
78             0.171902         0.171982
80             0.168961         0.170338
85             0.167334         0.167576
```

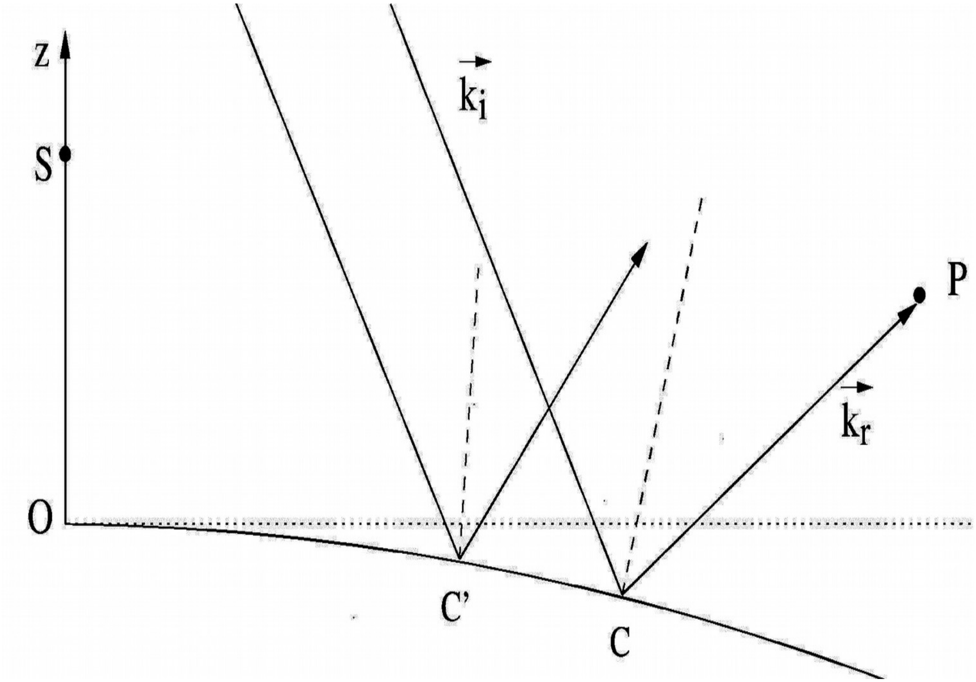
$$E_{(ref),y} = \frac{ik^3}{8\epsilon\pi^2}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty}\tilde{\Pi}_{ref}(f_r^s\cos^2\beta - f_r^p\cos^2\alpha\sin^2\beta)\sin\alpha d\alpha d\beta$$

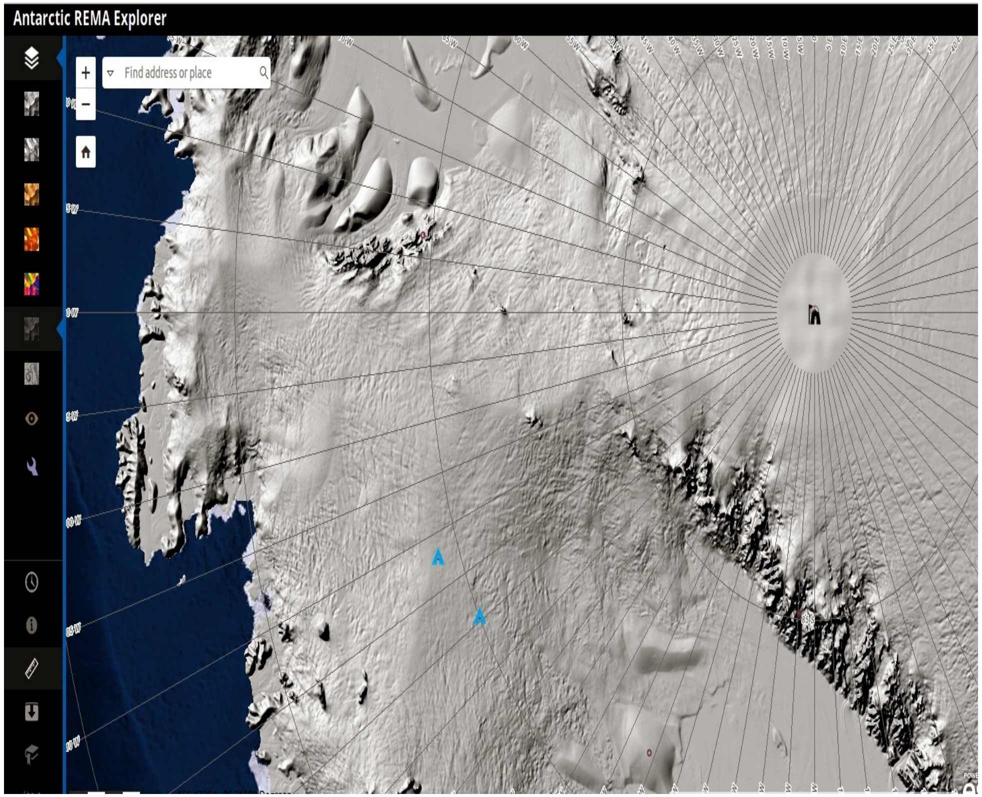


S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018)

This formalism works for a Flat Reflecting Surface !!

Reflection and Transmission of Radio Signals from Spherical, Rough Surface ??

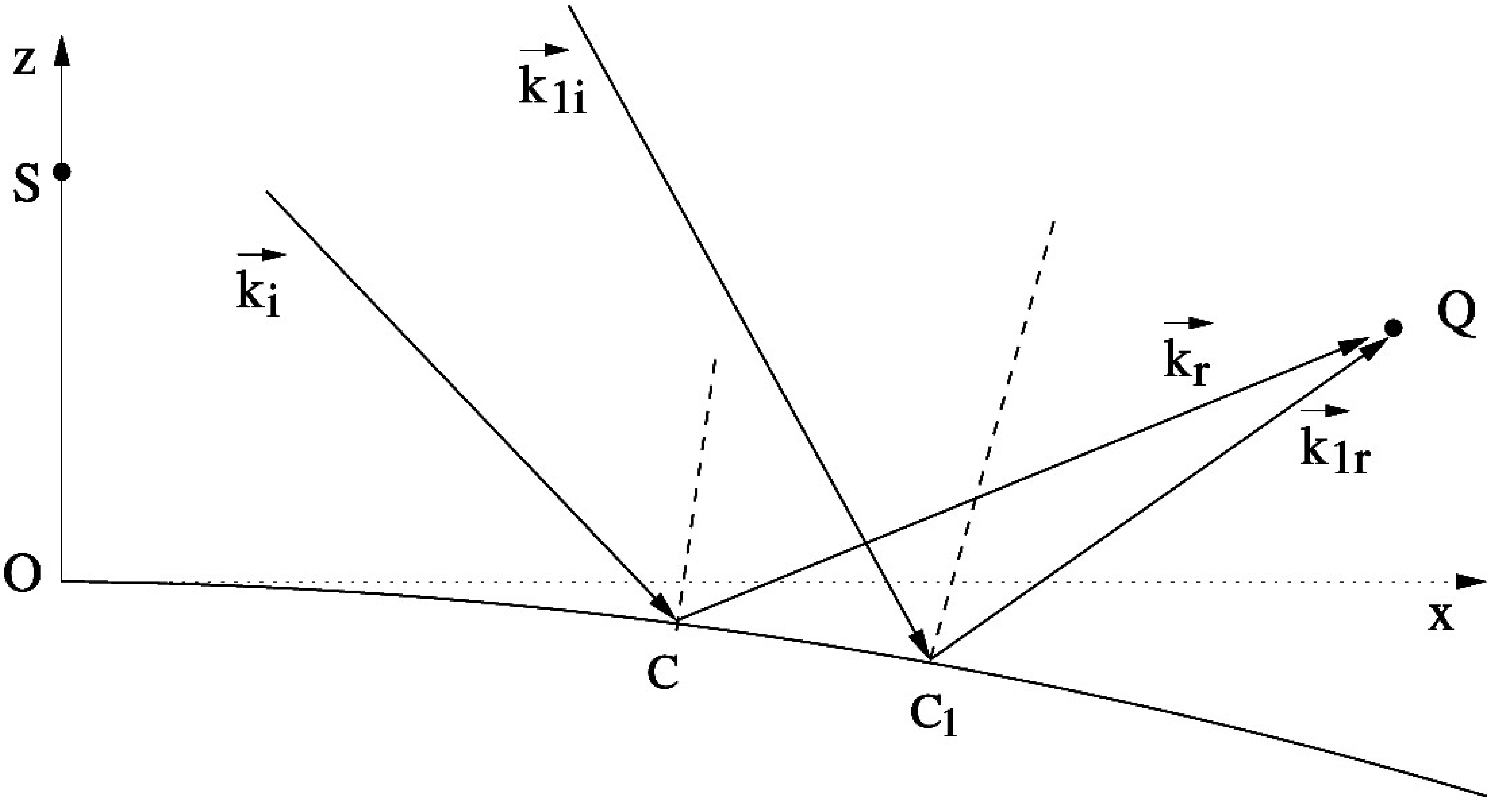




We developed the first complete theoretical framework for a Spherical Uneven Reflecting Surface without making any uncontrolled approximation

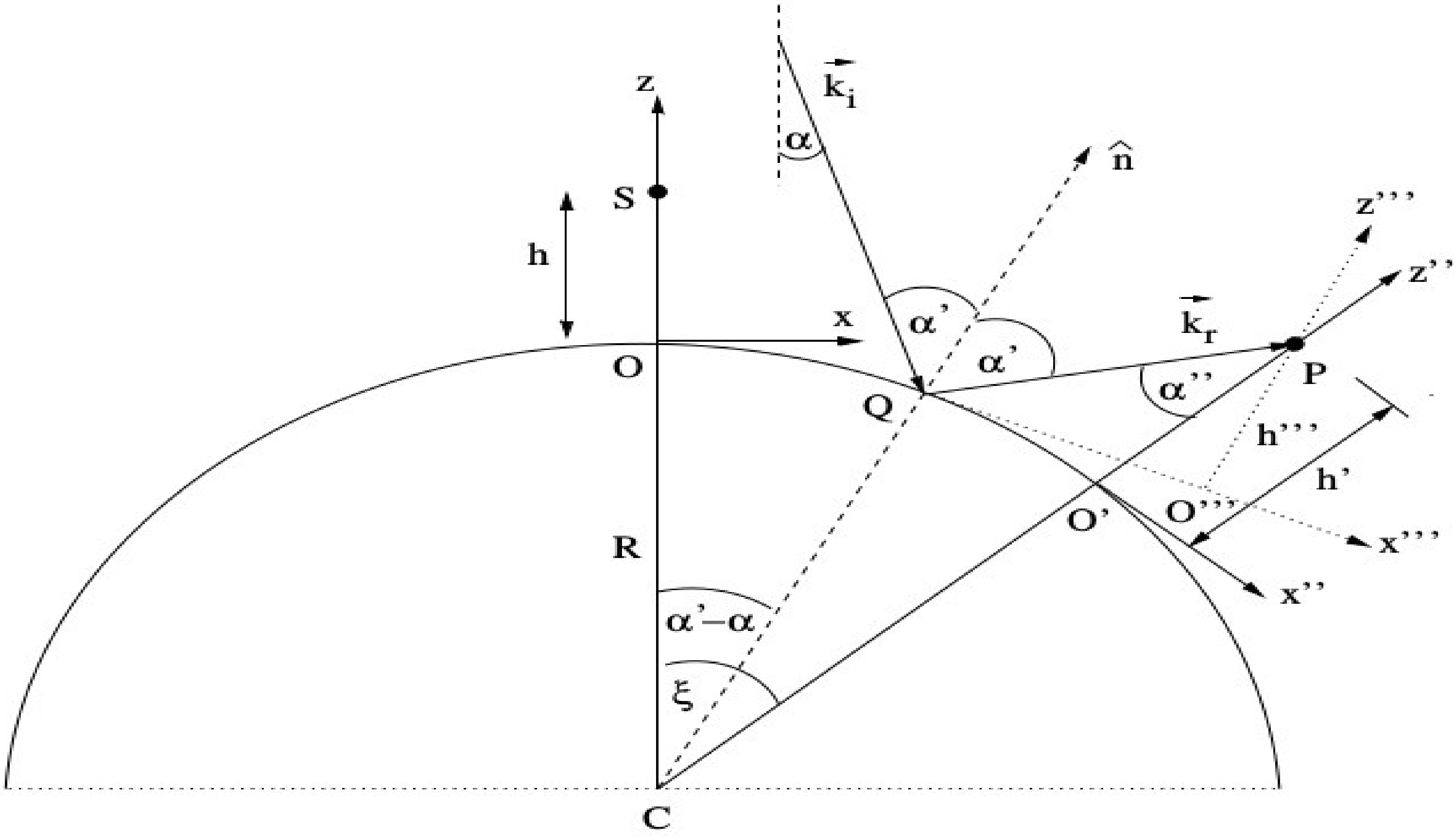
S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018)

Reflection off a Spherical Surface : "Locally Plane Waves"



Paramita Dasgupta and Pankaj Jain arXiv:1811.00900, 2018

Refinement of Framework : “Local Plane Wave Approximation Theory”



$$Rot_1 = \begin{pmatrix} \cos\xi\cos\tilde{\beta} & \sin\tilde{\beta} & -\sin\xi\cos\tilde{\beta} \\ -\cos\xi\sin\tilde{\beta} & \cos\tilde{\beta} & \sin\xi\sin\tilde{\beta} \\ \sin\xi & 0 & \cos\xi \end{pmatrix}$$

$$R_{y'}(\psi) = \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{pmatrix}$$

P. Dasgupta and P Jain arXiv:1811.00900, 2018

Derive s and p component of Reflected E Fields

$$\vec{E}_r'''^{(s)} = f_r'^{(s)} \frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}_{S,r}[\cos\tilde{\beta}\hat{y}'''],$$

$$\vec{E}_r'''^{(p)} = f_r'^{(p)} \frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}_{S,r}\,[-(\sin\tilde{\beta}\cos^2\tilde{\alpha}\cos\psi + \sin\alpha\cos\tilde{\alpha}\sin\beta\sin\psi)\hat{x}'''$$
$$+\,(\sin\alpha\cos\tilde{\alpha}\sin\beta\cos\psi - \cos^2\tilde{\alpha}\sin\tilde{\beta}\sin\psi)\hat{z}''']$$

$$\vec{E}_r''' = \vec{E}_r'''^{(s)} + \vec{E}_r'''^{(p)}$$

$$\vec{E}_r = Rot^{-1}\cdot\vec{E}_r'''$$

Imposing Boundary bonditions at z=0

$$f_r'^{(s)} = \frac{k\cos(\tilde{\alpha}-\psi) - k_1\cos(\tilde{\alpha}_t-\psi)}{k\cos(\tilde{\alpha}-\psi) + k_1\cos(\tilde{\alpha}_t-\psi)},$$

$$f_t'^{(s)} = \left(\frac{k}{k_1}\right)^2 \frac{2k_1\cos(\tilde{\alpha}-\psi)}{k\cos(\tilde{\alpha}-\psi) + k_1\cos(\tilde{\alpha}_t-\psi)}$$

Local Plane Wave Approximation: Electric and Magnetic Field (HPol &VPol)

Peter Gorham's roughness model

$$F(k,\rho,\theta) = \exp\left[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z\right]$$

$$\sigma_h(L) = \sigma_h(L_0)\left(\frac{L}{L_0}\right)^H$$

Reflected fields for a Spherical + Rough Reflecting Surface

$$E_{r,y} = \frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}_{S,r}\left[f_r'^{(s)}\cos^2\tilde{\beta} - f_r'^{(p)}\cos\tilde{\alpha}\cos(\tilde{\alpha}-2\psi)\sin^2\tilde{\beta}\right]$$

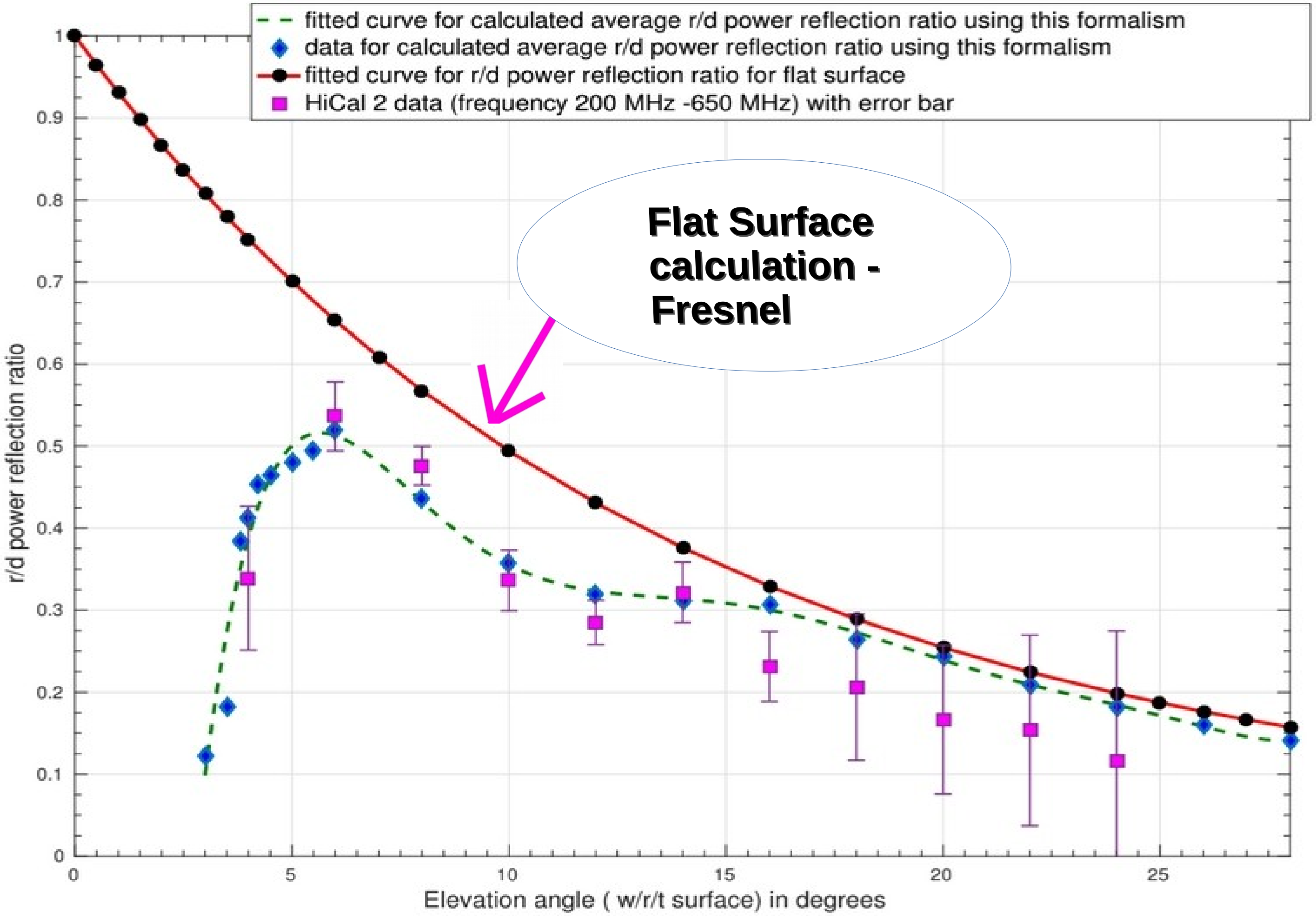
$$E_{(r,total),y} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} F_{rough}E_{r,y}\sin\alpha d\alpha d\beta.$$

Integrating over dΩ gives Total E_{ref} (H-Pol)

P. Dasgupta and P Jain arXiv:1811.00900, November 2018

H-Pol (ref/direct) Power Ratio Compared with HiCal Data for a Spherical Surface: Using Local Plane Wave Calculation

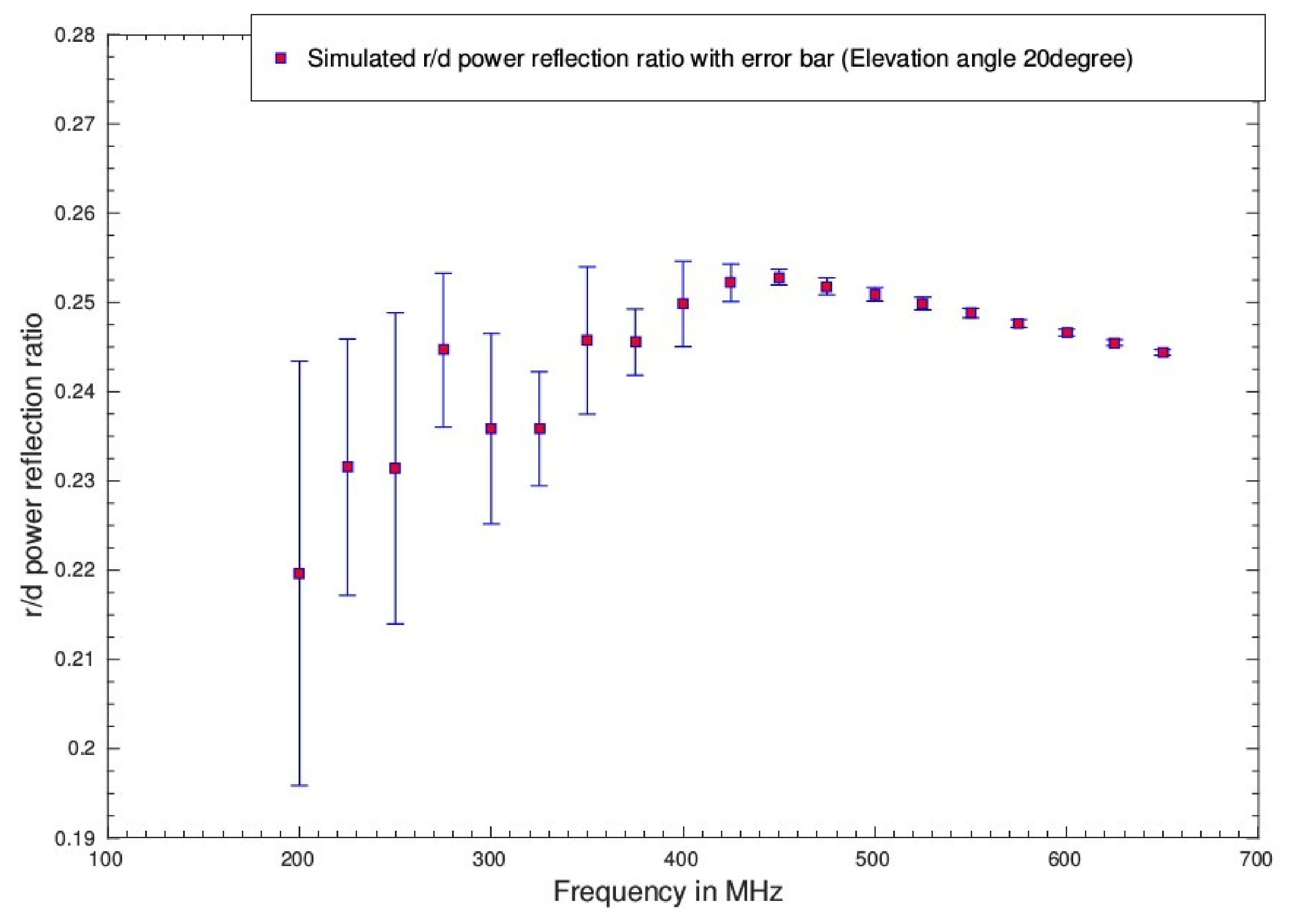
$$E_{r,y} = \frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}_{S,r}\left[f_r'^{(s)}\cos^2\tilde{\beta} - f_r'^{(p)}\cos\tilde{\alpha}\cos(\tilde{\alpha}-2\psi)\sin^2\tilde{\beta}\right] \qquad E_{(r,total),y} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} F_{rough}E_{r,y}\sin\alpha d\alpha d\beta.$$



Flat Surface calculation - Fresnel

Paramita Dasgupta and Pankaj Jain arXiv:1811.00900

Frequency dependence of r/d Power Ratio



Paramita Dasgupta and Pankaj Jain arXiv:1811.00900

Generalizing Framework for Electromagnetic Pulses (Applicable to ANITA events)

We need to consider Real Event: Electromagnetic pulses

$$\tilde{F}(n) = \sum_{p=0}^{N-1} f(p) e^{i\frac{2\pi n}{N}p}$$

HiCal-2 Time Domain Pulse

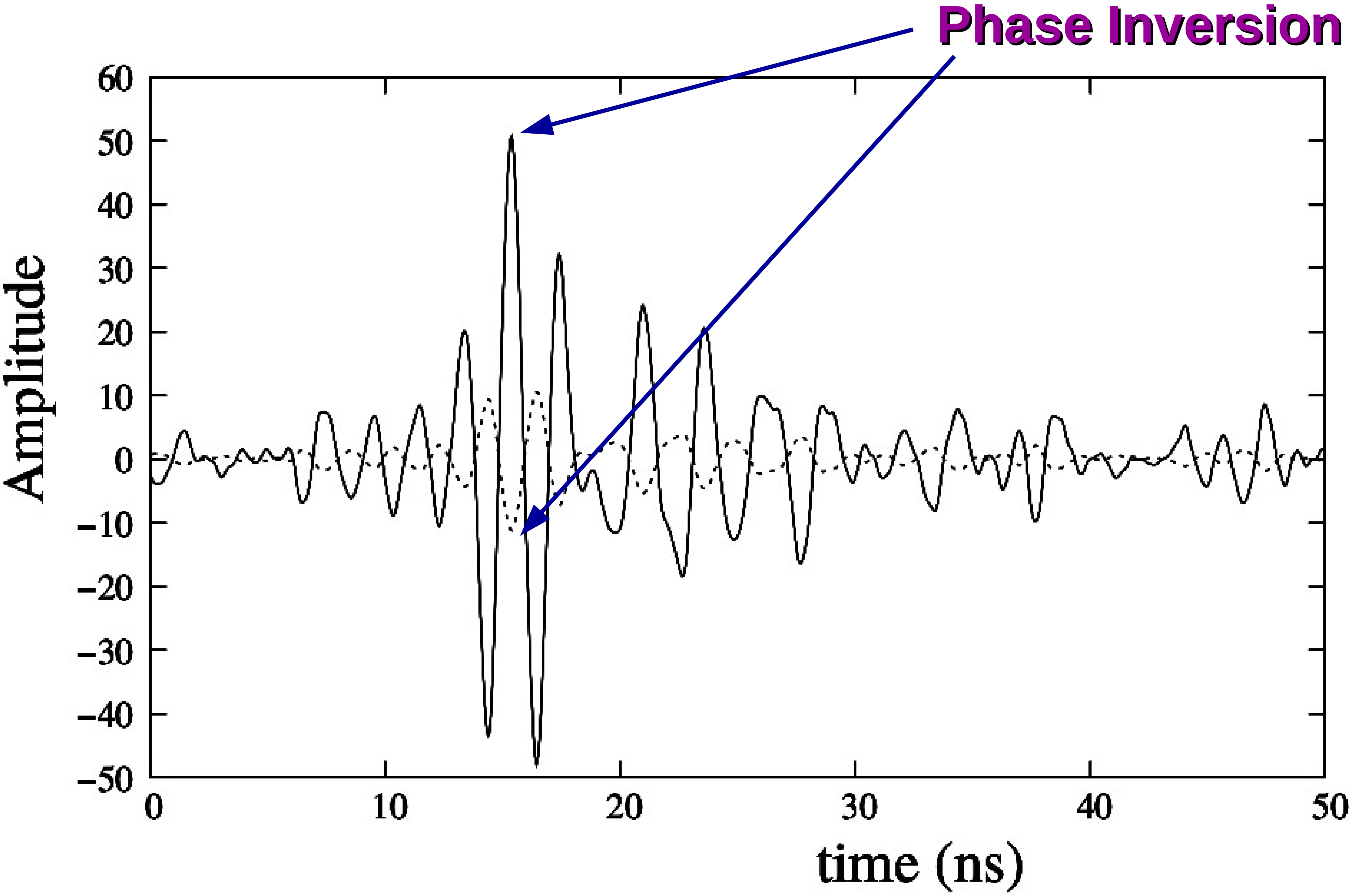
$$E'_{ref,y} = \int_0^{\frac{\pi}{2}-i\infty} \int_0^{2\pi} \int_\omega \frac{ik}{2\pi} \tilde{\Pi}_{S,r} F_{rough}(\omega,\alpha,\beta,\theta_z)\eta(\alpha,\beta,\omega)(\tilde{F}(\omega)e^{-i\omega(t+t_0)})d\omega d\Omega\,.$$

Roughness factor

Superposition of dipole radiation of different frequencies

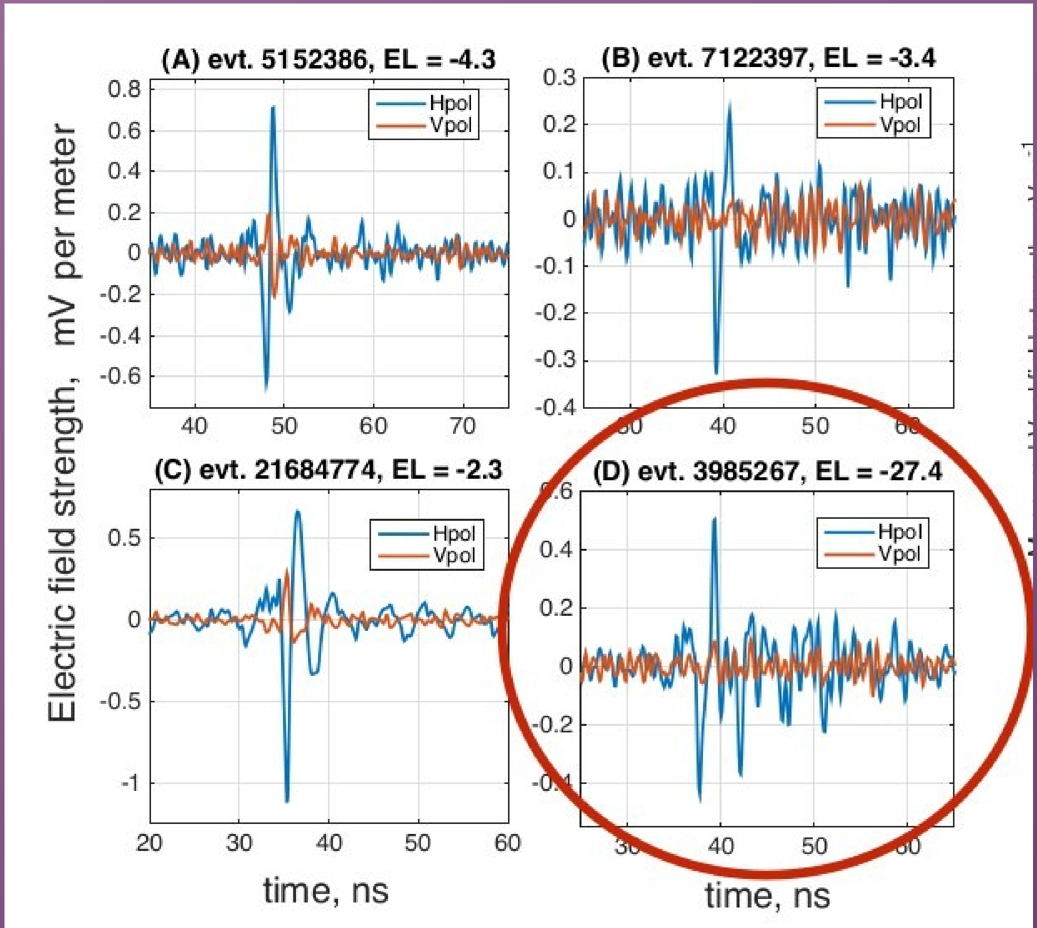
Incorporating our formalism to compute Direct and Reflected pulse profile (using HiCal 2 pulses)

Simulated Direct and Reflected Hical Pulse using this framework


Phase Inversion

ANITA Mystery Events

Unusual steeply pointed up-going air showers with E ~ EeV scale



Phys. Rev. Lett. 117, 071101 (2016)

Surface Roughness Models → ANITA Mystery Events?

We investigated actual Antarctic topograpy: Local Sastrugi

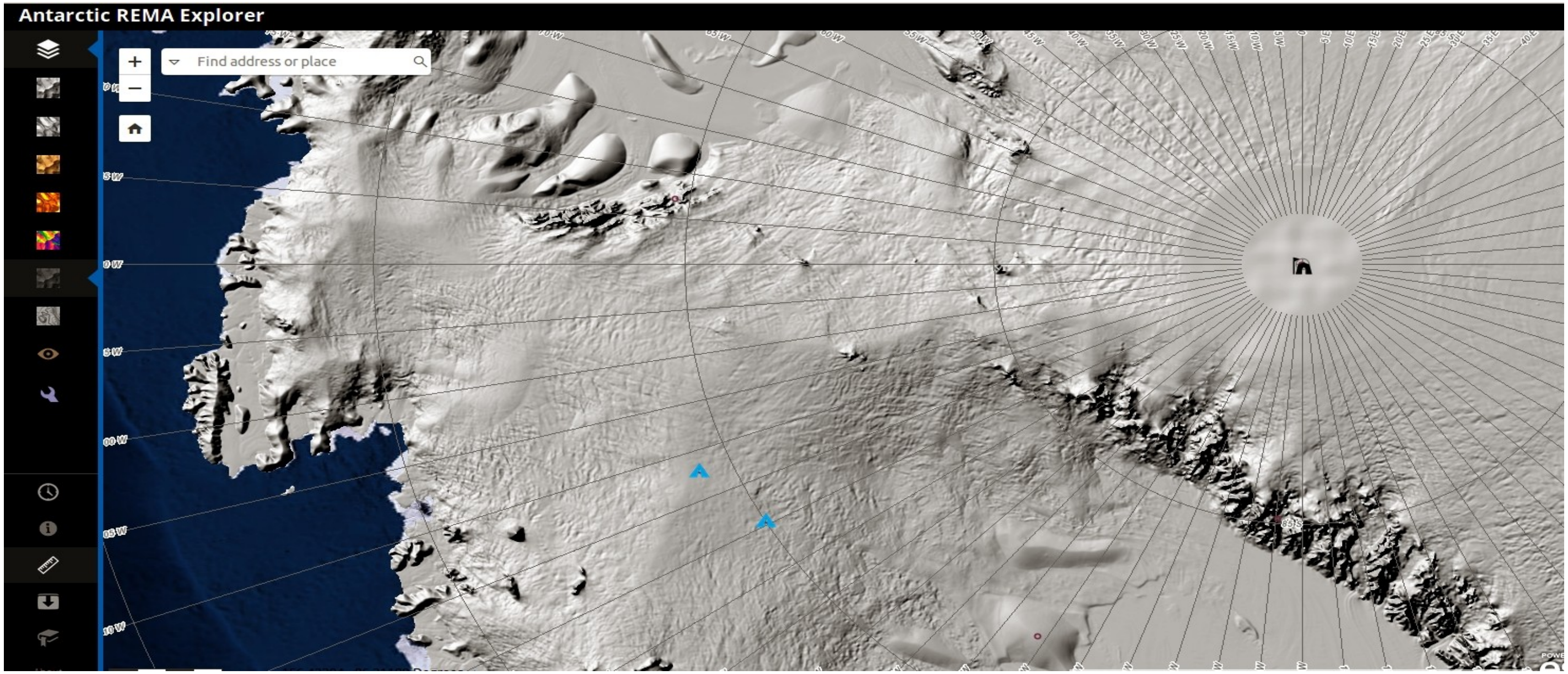
Simulated new Roughness model + Our theoretical Framework

Reality Check: HiCal data

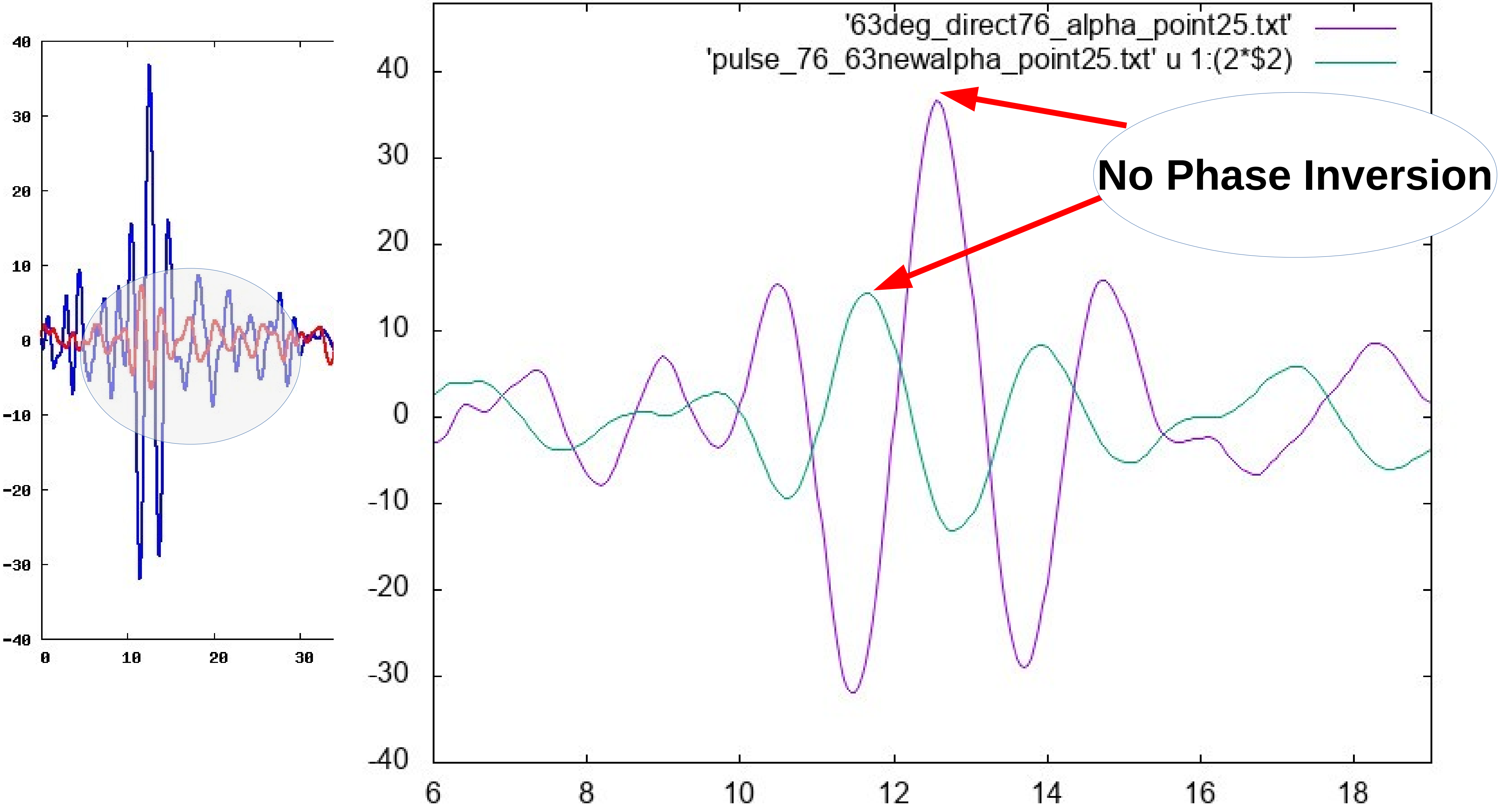
Non Inversion of phase in Reflected signal ??

New Roughness Models

- Antarctic topographic data one side is quite smooth but the other side has lots of variation in altitude. Even in the smooth side we have gently rolling hills and valleys.
- In order to account for this we allowed our incident angle to vary over a range of about 3 Km by about 1 degree.
- We also simply increase the curvature. This will model a hill like structure in the region of mystery events



Possible explanation for ANITA mystery events ?



No Phase Inversion

Acknowledgement

- **Professor Pankaj Jain, Dept of Physics, IIT Kanpur (Thesis Supervisor)**
- **Professor David Besson, Department of Physics and Astronomy, University of Kansas**
- **Dr. Steven Prohira, Ohio State University**

Thank You

BACK UP

Include Non Uniform Roughness Parameter

Previously we incorporated Roughness Factor that assumed a circular region around the specular point

$$F(k,\rho,\theta) = \exp[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z] \qquad \sigma_h(L) = \sigma_h(L_0)\left(\frac{L}{L_0}\right)^H$$

where, sigma(L_0)= 0.041, L_0= 150 m, H= Hurst Parameter=0.65,

← **Circular**

$$X^2 + Y^2 = L^2$$

Now, I use an elliptical region around the specular pt $X^2 + (aY)^2 = L^2$ with a<1

We compute reflected pulses using this assymetric roughness parameter “a”

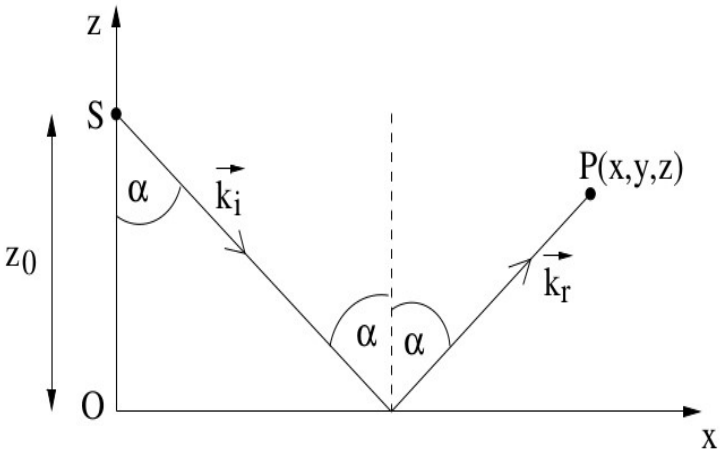
choosing a=0.1, 0.25, 0.5 and a=1 (which is the symmetric case, as given by Peter Gorham's model)

0.041< =sigma(L_0)<=0.071 and L_0 changed accordingly between 150m to 80 m

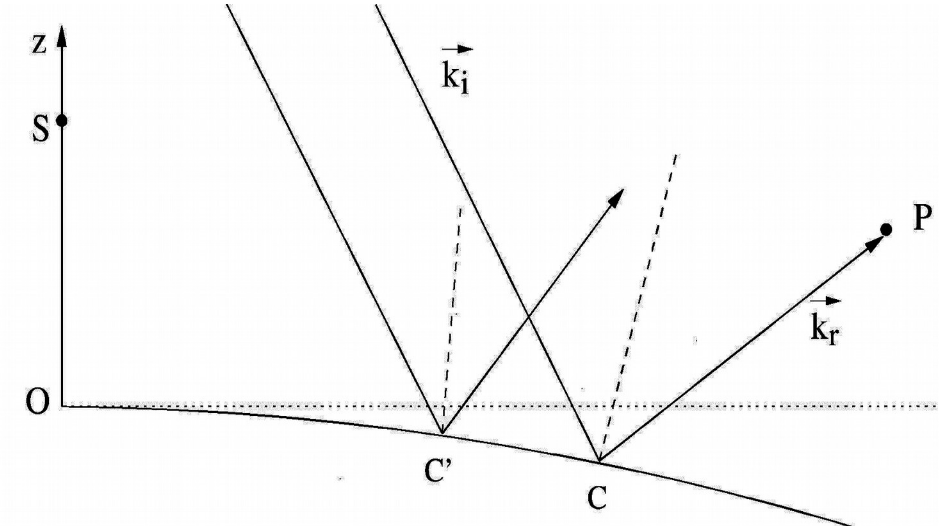
RESULTS with different assymetric factors “a”, and slightly changed sigma(L_0), and L_0 values are obtained.

A Rigorous Formalism to study the Radio Signals Reflecting off a Spherical Surface

Flat Earth Surface



Spherical Earth Surface

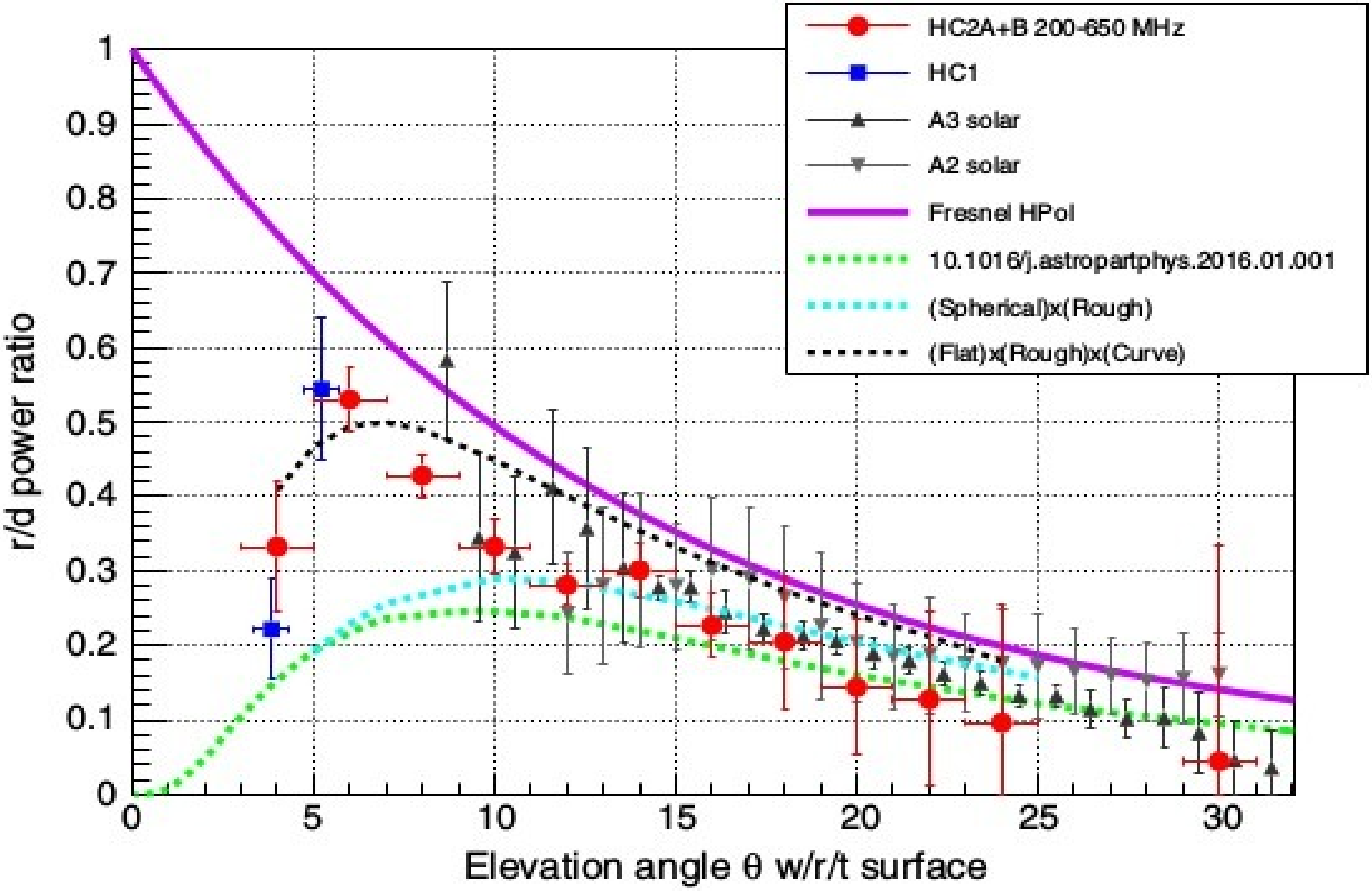


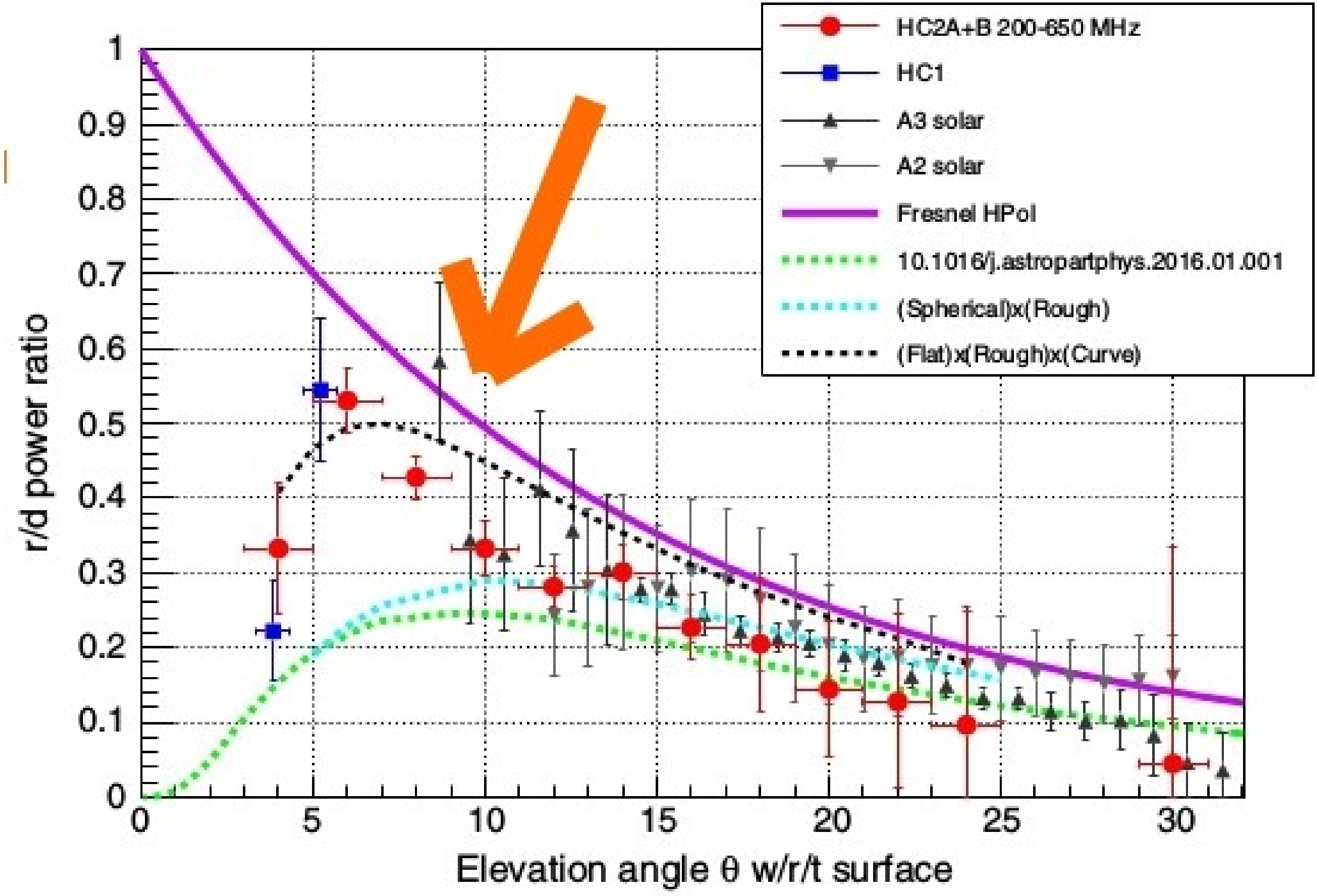
We extend our Formalism for a Spherical Reflecting Surface

We do not make any uncontrolled approximation.

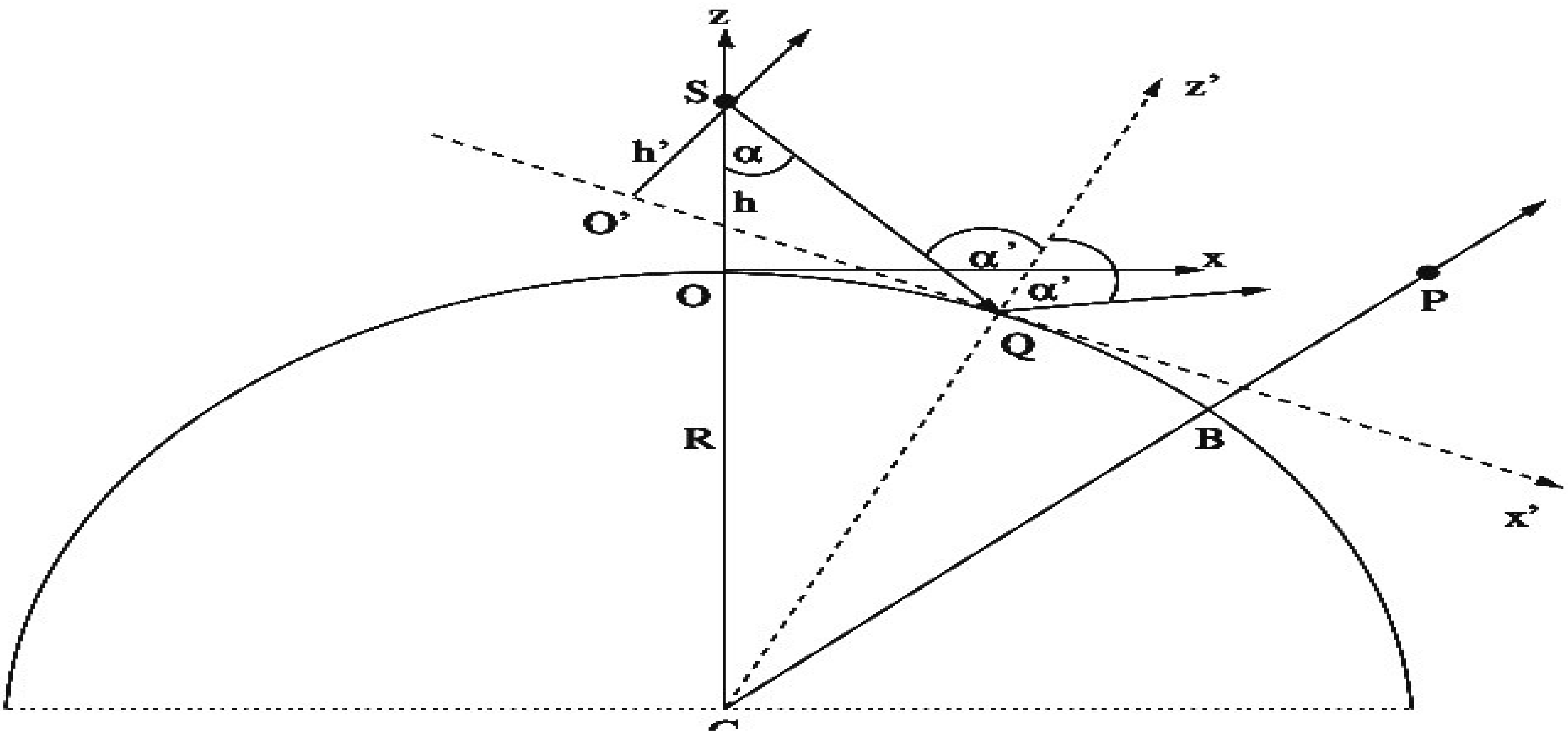
Our framework is a General & Rigorous treatment to handle the Reflection and Transmission of EM waves and pulses from a spherical+uneven surface

S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004


HC2A+B 200-650 MHz
HC1
A3 solar
A2 solar
Fresnel HPol
10.1016/j.astropartphys.2016.01.001
(Spherical)x(Rough)
(Flat)x(Rough)x(Curve)
r/d power ratio
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
Elevation angle θ w/r/t surface


HC2A+B 200-650 MHz
HC1
A3 solar
A2 solar
Fresnel HPol
10.1016/j.astropartphys.2016.01.001
(Spherical)x(Rough)
(Flat)x(Rough)x(Curve)
r/d power ratio
Elevation angle θ w/r/t surface
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30

Reflection of EM waves at Spherical Earth Surface



$$Rot = \begin{pmatrix} \cos(\alpha' - \alpha)\cos\beta & \cos(\alpha' - \alpha)\sin\beta & -\sin(\alpha' - \alpha) \\ -\sin\beta & \cos\beta & 0 \\ \sin(\alpha' - \alpha)\cos\beta & \sin(\alpha' - \alpha)\sin\beta & \cos(\alpha' - \alpha) \end{pmatrix}$$

R_{Earth}~6371 Km
Detector Height z_0~ 37Km

S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004

Antarctic Surface Roughnes Model

Peter Gorham's roughness model

$$F(k,\rho,\theta) = \exp[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z]$$

$$\sigma_h(L) = \sigma_h(L_0)\left(\frac{L}{L_0}\right)^H$$

Reflected fields for a Spherical + Rough Reflecting Surface

$$E_{\mathrm{ref},y} = \frac{1}{2}\frac{ik^3}{8\epsilon\pi^2}\tilde{\Pi}_{S,r}F(k,\rho,\theta)[f_r^{\prime s}(1+\cos 2\beta) - f_r^{\prime p}\cos\alpha\cos(2\alpha' - \alpha)(1-\cos 2\beta)]$$

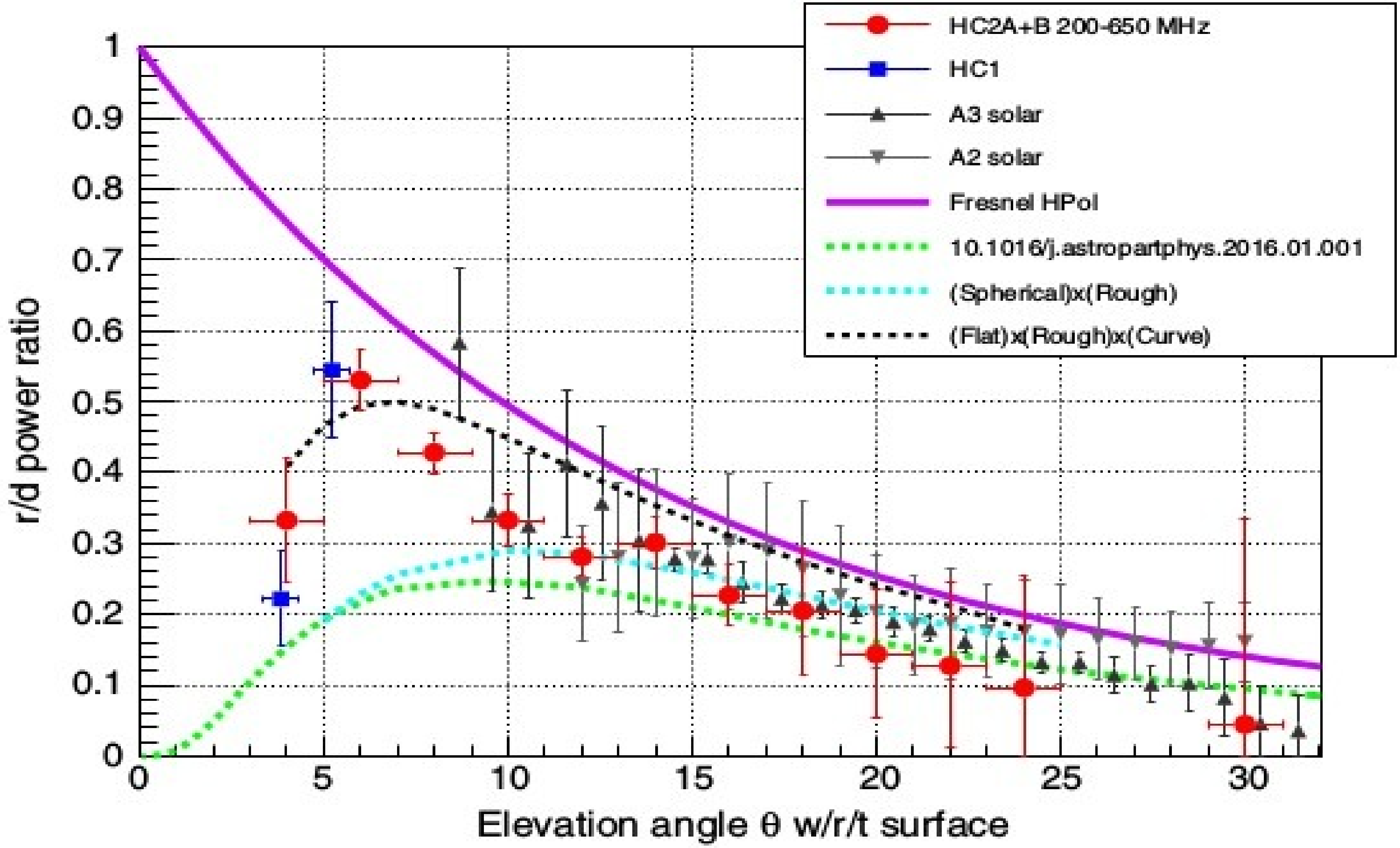
Numerically Solve

Compare with HiCal data

Integrating over dΩ gives Total E_{ref} (H-Pol)

S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018)

H-Pol (ref/direct) Power Ratio Compared with HiCal Data: Using **Spherical Reflecting Surface** Calculation



S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018)

Verifying our Framework with expt. data

- **We Computed Reflected H-Pol & V- Pol Fields and compared with HiCal-2 H-Pol Data**

- **This Formalism works very well for elevation angle >10^0**

- **For elevation angle < 10^0 are off from HiCal-2 data**

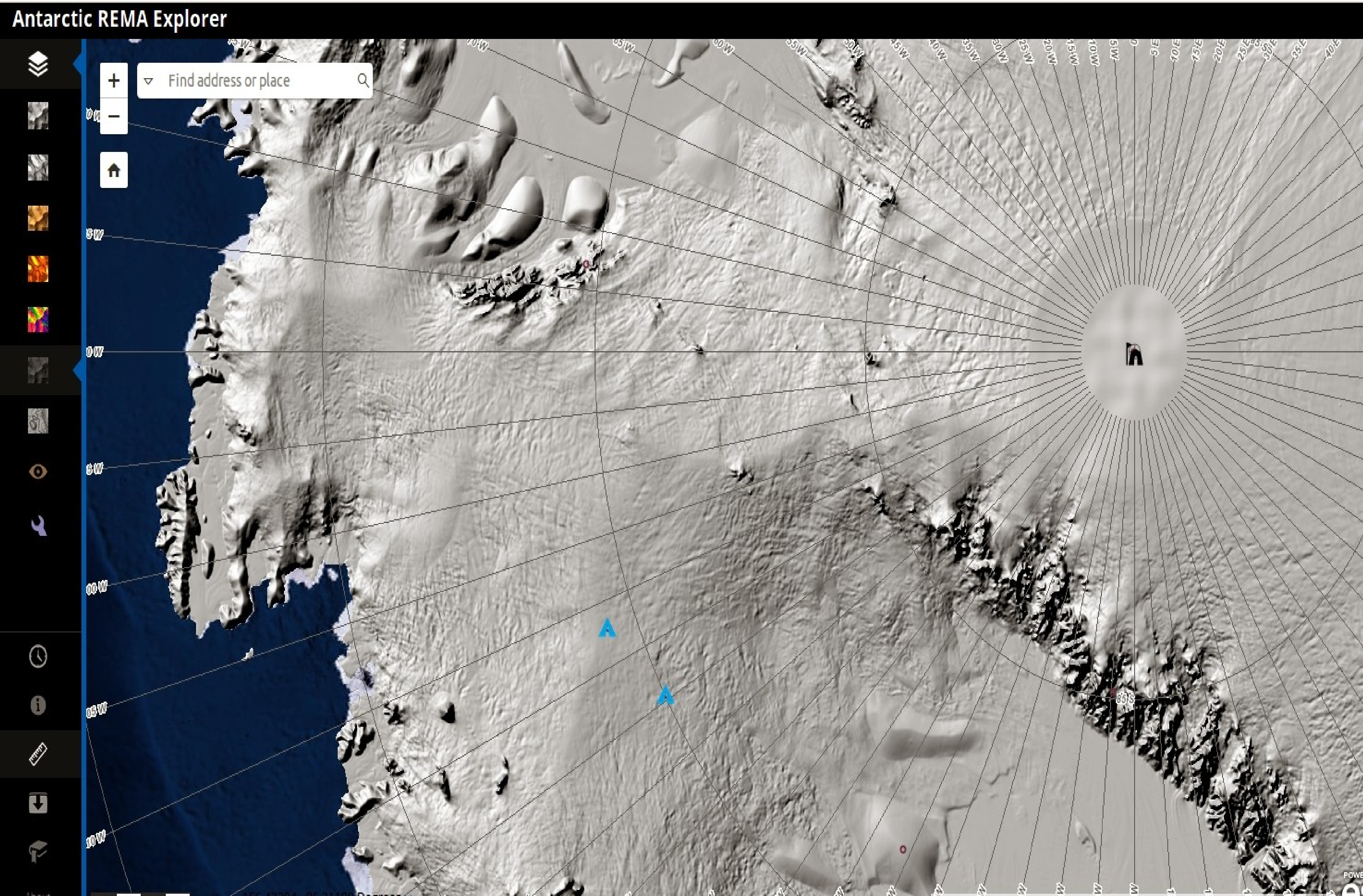
- **Results are given in August 2018 in Physical Review D. Volume 98, 042004**

“Refinement of the Framework is necessary to apply this model for all elevation angles applicable to ANITA payload”

Refinement of Framework

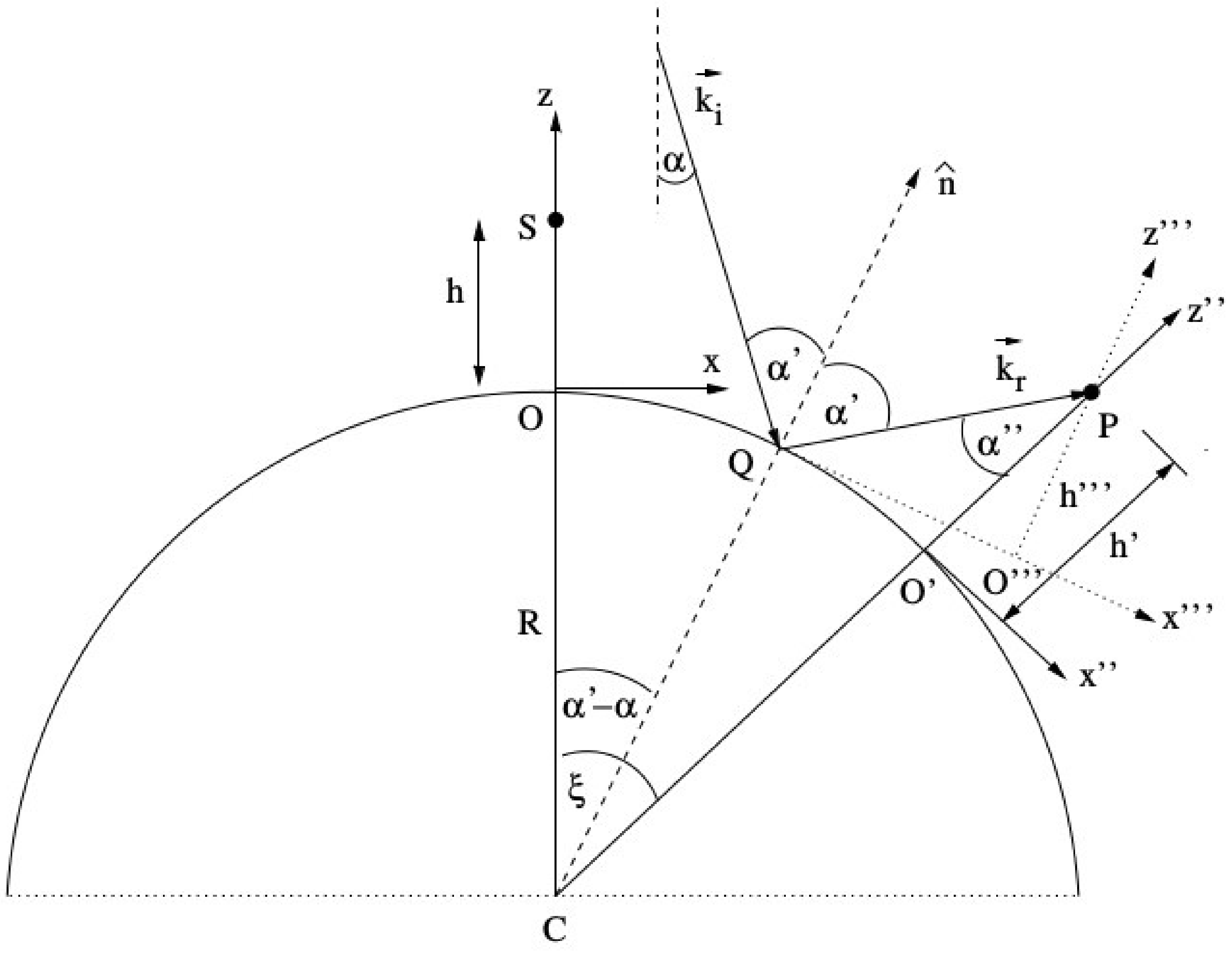
- Next, we developed a modified formalism that we named as **“Local Plane Wave Approximation”** which is a modified version of our theory developed in **Physical Review D, volume 98, 042004**

Antarctic Surface Topography



Antarctic Surface

Refinement of Framework : “Local Plane Wave Approximation Theory”



P. Dasgupta and P Jain arXiv:1811.00900, 2018

s and p component of Transmitted fields

$$\vec{E}_t^{\prime\prime\prime(s)} = f_t^{\prime(s)} \frac{ik_1^3}{8\epsilon_1\pi^2} \tilde{\Pi}_{S,t} \left[\cos\tilde{\beta}_t \hat{y}^{\prime\prime\prime}\right]$$

$$\begin{aligned}\vec{E}_t^{\prime\prime\prime(p)} &= \vec{E}_t^{\prime\prime\prime} - \vec{E}_t^{\prime\prime\prime(s)} \\ &= f_t^{\prime(p)} \frac{ik_1^3}{8\epsilon_1\pi^2} \tilde{\Pi}_{S,t} \left[(\sin\tilde{\beta}_t \cos^2\tilde{\alpha}_t \cos\psi + \sin\alpha_t \cos\tilde{\alpha}_t \sin\beta_t \sin\psi)\hat{x}^{\prime\prime\prime}\right. \\ &\left.+ (\sin\alpha_t \cos\tilde{\alpha}_t \sin\beta_t \cos\psi - \cos^2\tilde{\alpha}_t \sin\tilde{\beta}_t \sin\psi)\hat{z}^{\prime\prime\prime}\right]\end{aligned}$$

$$\vec{H}_t^{\prime\prime\prime(p)} = f_t^{\prime(p)} \frac{ik_1^2\omega}{8\pi^2} \tilde{\Pi}_{S,t} \left[-\cos\tilde{\alpha}_t \sin\tilde{\beta}_t \hat{y}^{\prime\prime\prime}\right]$$

$$\begin{aligned}\vec{H}_t^{\prime\prime\prime(s)} = \vec{H}_t^{\prime\prime\prime} - \vec{H}_t^{\prime\prime\prime(p)} &= f_t^{\prime(s)} \frac{ik_1^2\omega}{8\pi^2} \tilde{\Pi}_{S,t} \left[\cos\tilde{\beta}_t \cos(\tilde{\alpha}_t - \psi)\hat{x}^{\prime\prime\prime}\right. \\ &\left.+ \cos\tilde{\beta}_t \sin(\tilde{\alpha}_t - \psi)\hat{z}^{\prime\prime\prime}\right]\end{aligned}$$

$$\vec{H}_r^{\prime\prime\prime(p)} = f_r^{\prime(p)}\frac{ik^2\omega}{8\pi^2}\tilde{\Pi}_{S,r}[-\cos\tilde{\alpha}\sin\tilde{\beta}\hat{y}^{\prime\prime\prime}]$$

$$\vec{H}_r^{\prime\prime\prime(s)} = f_r^{\prime(s)}\frac{ik^2\omega}{8\pi^2}\tilde{\Pi}_{S,r}[-\cos\tilde{\beta}\cos(\tilde{\alpha}-\psi)\hat{x}^{\prime\prime\prime} + \cos\tilde{\beta}\sin(\tilde{\alpha}-\psi)\hat{z}^{\prime\prime\prime}]$$

- **Incident, Reflected and Transmissted wave vectors in new frame**

- **We derive s and p components of Electric and Magnetic Fields**
  - **Impose Boundary Conditions at z_s'''= 0**

$$f_r'^{(s)} = \frac{k\cos(\tilde{\alpha}-\psi) - k_1\cos(\tilde{\alpha}_t-\psi)}{k\cos(\tilde{\alpha}-\psi) + k_1\cos(\tilde{\alpha}_t-\psi)},$$

$$f_t'^{(s)} = \left(\frac{k}{k_1}\right)^2 \frac{2k_1\cos(\tilde{\alpha}-\psi)}{k\cos(\tilde{\alpha}-\psi) + k_1\cos(\tilde{\alpha}_t-\psi)}$$

$$\vec{k}_i''' = k[\sin(\tilde{\alpha}-\psi)\hat{x}''' - \cos(\tilde{\alpha}-\psi)\hat{z}''']$$

$$\vec{k}_r''' = k[\sin(\alpha''+\psi)\hat{x}''' + \cos(\alpha''+\psi)\hat{z}''']$$

$$\vec{k}_t''' = k_1[\sin(\tilde{\alpha}_t-\psi)\hat{x}''' - \cos(\tilde{\alpha}_t-\psi)\hat{z}''']$$

Transmitted Field: Flat Reflecting Surface

$$E_{(trans),y} = \frac{ik_1^3}{8\epsilon_1\pi^2}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty}\tilde{\Pi}_t(f_t^s\cos^2\beta_t + f_t^p\cos^2\alpha_t\sin^2\beta_t)\sin\alpha d\alpha d\beta$$

H-Pol Component of E_{trans}